Course Descriptions
Fall 2019

Department of Mathematics
University of Illinois

Math 500

Professor Walton

This will be a fast-paced course that covers graduate-level introductory material in various concepts of Abstract Algebra. We will spend roughly 30% of the course on Group Theory, 20% on Ring Theory, 25% on Modules and Vector Spaces, and 25% of the course on Fields and Galois Theory.

C O U R S E   D E S C R I P T I O N

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FALL 2019
MATH 502

INTRODUCTION TO COMMUTATIVE ALGEBRA
Professor S. P. Dutta
9:30 – 10:50 Tu-Th

This course is intended mainly for students who are going to specialize in
Commutative Algebra, Algebraic Geometry, Algebraic K-theory and Algebraic
Number Theory.
In this course we will mainly focus on Noetherian rings and modules. The topics
will include: Primary decomposition, Artin-Rees Lemma, Flatness, Completion,
Hilbert-Samuel Polynomial, Dimension Theory, Integral extensions, Going-up and
Going-down theorems, Noether's Normalization (its geometric interpretation),
Regular rings and the notion of depth. We would also like to study Cohen-
Macaulayness if time permits.

Prerequisite: Math500, 501
Recommended text: Commutative ring theory by H. Matsumura

Math 510. Riemann Surfaces & Algebraic Curves
Professor Pascaleff

The course focuses on Riemann Surfaces from both the algebraic and function-theoretic points of view.

Topics include:

  Holomorphic and meromorphic differential forms on Riemann surfaces
  Integration of differential forms on Riemann surfaces
  Divisors on Riemann surfaces and linear equivalence
  The genus of a compact Riemann surface
  Canonical divisors on a Riemann surface
  The Riemann-Roch theorem and applications
  Projective algebraic curves
  The canonical mapping

Textbook:

  Miranda, Algebraic curves and Riemann surfaces

FALL 2019
MATH 512
ALGEBRAIC GEOMETRY I

Time: MWF 11:00–11:50
Room: TBA
Instructor: Sheldon Katz
Text: Algebraic Geometry, R. Hartshorne, Graduate Texts in Mathematics
52, Springer NY 1977
Prerequisites: Abstract Algebra II (Math 501), or Introduction to Algebraic
Geometry (Math 511), or permission of the instructor

This course introduces Algebraic Geometry from both the algebraic and geometric viewpoints. Affine and projective varieties will be developed from both the classical viewpoint as well as using the language of schemes and sheaves.

The course will focus on the first two chapters of the text: Varieties, and Schemes, primarily the second chapter on Schemes. The text will be frequently supplemented with additional materials designed to enhance geometric intuition. Some of the topics will presented differently than in the text.

Comments on prerequisites: students can profitably take this course after either Abstract Algebra II (Math 501) or Introduction to Algebraic Geometry (Math 511). It is not necessary to have taken both courses. We will be freely using the language of commutative algebra for the most part, including notions such as prime ideals, localization, and tensor product. This is not the only route to algebraic geometry but is a choice being made largely by the choice of textbook. Many of the ideas from Math 511 are developed in a more general and abstract setting in Math 512, so 511 will be helpful. But since the ideas will be developed independently, 511 is not necessary. Familiarity with Riemann Surfaces and Algebraic Curves (Math 510) is not necessary but will be helpful, as a smooth 1-dimensional projective variety over the complex numbers is equivalent to a compact Riemann surface.

Math 518

Professor Lerman

Prerequisites

Point set topology and linear algebra.
If you have any questions or concerns, please contact me by e-mail.

Course outline

1. Manifolds: Definitions and examples including projective spaces and Lie groups; smooth functions and mappings; submanifolds; Inverse Function Theorem and its applications including transversality; (co)tangent vectors and bundles; vector bundles; operations on vector bundles; manifolds with boundary; orientations.

2. Calculus on Manifolds: Vector fields, flows, and Lie derivative/bracket; differential forms and the exterior algebra of forms; orientations again; exterior derivative, contraction, and Lie derivative of forms; integration and Stokes Theorem, DeRham cohomology.

Recommended Texts

- *Introduction to Smooth Manifolds* by John M. Lee, Springer, ISBN: 978-1-4419-9981-8 (Print) 978-1-4419-9982-5 (Online) [free online access from a campus computer or through a VPN if off-campus]

Grades

The course grade will be based on weekly homework (35%), a midterm (25%) and a final (40%).

MATH 520 Symplectic Geometry

Professor Fernandes

Course Description: This course will introduce the foundational tools, ideas, examples and theorems of Symplectic Geometry. The goal of the course is to prepare students to conduct re-search in this or one of the many related fields of mathematics and physics.

Prerequisites: Math 518 or consent of the instructor.

Suggested Texts:

- *Lectures on Symplectic Geometry*, Ana Cannas da Silva Lecture Notes in Mathematics, 1764; **ISBN:** 978-3540421955.

- *Introduction to Symplectic Topology*, Third Edition, Dusa McDuff and Dietmar Salamon Oxford Graduate Texts in Mathematics, 27 (2017); **ISBN:** 978-0198794905.

Principle Topics:

1. Linear Symplectic Geometry

    - symplectic vector spaces, the symplectic linear group, Lagrangian subspaces, Maslov index, complex structures

2. Symplectic Manifolds

    - symplectic forms, examples and constructions, Darboux's Theorem, Lagrangian subman- ifolds, Weinstein's tubular neighborhood theorem, blowing up and down, symplectic cuts and sums

3. Symplectomorphisms

    - fixed point theorems, Hamiltonian flows, Poisson brackets, integrable systems, the group of symplectomorphisms

4. Contact Manifolds

    - Contact structures, contact forms, Gray's theorem, Reeb flows

5. Almost Complex Structures

    - almost complex structures, integrability, complex manifolds, Kähler forms, compact Kähler Manifolds

6. Hamiltonian Group Actions and Reduction
    - group actions, moment maps, Marsden-Weinstein-Meyer reduction, Toric   Manifolds

7. Advanced Topics (to be determined by instructor and student interests)

    - Example 1. Delzant's classification of symplectic toric manifolds.

    - Example 2. Generalized complex geometry.

    - Example 3. *J*-holomorphic curves and Gromov's nonsqueezing theorem.

Work required of students: Students are expected to complete 10 problem sets. Based on instructor preference, students will also be required to either take a final examination or to write   a 5-10 page expository paper surveying a more advanced topic.

Grading Scheme: The problem set scores will make up 70% of the total grade. The remaining 30% will be based on the Final Exam / Survey paper.

MATH 526 Algebraic Topology II
Fall 2019

Instructor: Vesna Stojanoska

Course description: This is the second semester of the algebraic topology sequence, and for the most part will concentrate on studying singular cohomology, its structure and applications. The first part of the course will concentrate on the cup product in cohomology, Poincaré duality, and various applications. Then we will study vector bundles, characteristic classes, and cohomology operations, and if time permits, we'll cover some basics related to complex K-theory.

Prerequisites: MATH 525 or consent of instructor.

Textbooks: The main textbooks will be

- *Algebraic Topology*, by Hatcher. (Free pdf version is available at `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`)

- *Geometry and Topology*, by Bredon. (Free pdf version is available through the library.)

- *Characteristic Classes*, by Milnor and Stasheff. (A free pdf version is available through the library.)

Other helpful references include:

- *Algebraic Topology*, by Switzer,

- *A Concise Course in Algebraic Topology*, by May.

Assignments: There will be homework every 2-3 weeks, and a final project.

MATH 531: ANALYTIC NUMBER THEORY, I
THE DISTRIBUTION OF PRIME NUMBERS

Kevin Ford (304 Altgeld, 265-6255, `http://www.math.uiuc.edu/~ford/`)

Prime number theory has witnessed many exciting new developments in the past few years:

- The primes contain arbitrarily long arithmetic progressions (Green and Tao, 2005)

- Bounded gaps between primes exist infinitely often (Yitang Zhang, 2013)

- Every odd number greater than 5 is the sum of three primes (Harald Helfgott, 2014)

All of these rely on **analytic methods**, that is, methods stemming from some kind of analysis (broadly speaking, this included real analysis, complex analysis, and harmonic analysis).

Main goals: Become familiar with fundamental principles of real and complex analytic methods for studying the distribution of arithmetic functions (functions which capture interesting number theoretic information, e.g. Euler's function) and prime numbers. Throughout the semester, we'll discuss some of the newest theorems (e.g. the three mentioned above, recent progress on large gaps between primes, etc) and the role of famous conjectures in number theory such the Generalized Riemann Hypothesis, the Twin Prime Conjecture, the Elliott-Halberstam Conjecture.

Syllabus:

1. Arithmetic functions: theory of multiplicative and additive functions, Dirichlet convolution, Möbius inversion, average order of magnitude estimates. Big-O and little-o notation.

2. Elementary theory of the distribution of primes. Statements equivalent to the prime number theorem, estimates of Chebyshev and Mertens.

3. Study of arithmetic functions via the analytic theory of Dirichlet series, Euler products and Perron's inversion formula.

4. Analytic methods for the distribution of primes. Theory of the Riemann Zeta function; connection between zeros of the zeta function and primes; analytic proof of the Prime Number Theorem. Why do people believe the Riemann Hypothesis? Why is it important?

5. Dirichlet Characters and Dirichlet's theorem on primes in arithmetic progressions. Counting integers that are the sum of two squares.

6. As time permits, a brief "sneak preview" of other further topics in analytic number theory, such as exponential sums, L-functions, sieve methods, modular forms.

Text: Highly recommended for purchase:

G. Tenenbaum, *Introduction to analytic and probabilistic number theory, 3rd ed.*, 2015

GENERAL TOPOLOGY

Igor Mineyev. Math 535, Fall 2019. 2pm, MWF.
`https://faculty.math.illinois.edu/~mineyev/class/19f/535/`

    This course will present foundations of general topology, which is a subject used in many areas of mathematics. We will roughly follow Part 1 of the textbook and discuss additional topics from algebraic topology and functional analysis. Any other information will appear on the class website.

    Also consider taking the other, related, topics course that I will be teaching in the second half of the Fall semester: Math 595 *Open problems in group theory and L^2-cohomology*. That course will only run if sufficiently many students sign up.
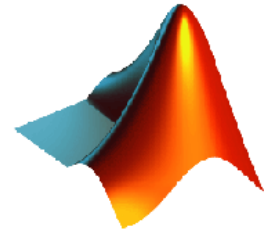
Textbook:

- James Munkres, *Topology*, second edition.

- For additional material, feel free to look up the *Algebraic topology* book by Hatcher freely available online.

Math 540 Real Analysis - Fall 2019

Prof. Richard Laugesen <Laugesen@illinois.edu>

Real analysis is the study of functions, especially
their integrability and differentiability properties.

Classical real analysis, as taught at the undergraduate level in terms
of Riemann integration and continuously differentiable functions, is
inadequate for the modern needs of differential equations, functional
analysis, probability theory, and so on.

This course develops modern integration theory (in Euclidean spaces
and abstract measure spaces), and modern differentiation theory for
functions of bounded variation. We explore L^p theory, which
provides a one-parameter family of norms for measuring the "size" of
functions, and Hilbert space theory, which leads to orthonormal
expansions in function spaces.

Prerequisites: Math 447 is the official prerequisite. Unofficially,
students need a certain amount of mathematical maturity. If you have
not studied metric spaces, then you should take Math 535 before
attempting Math 540.

Assessment: Two midterms and a final exam. Weekly homework.

Textbook
Richard Bass, *Real Analysis for Graduate Students*, Version 3.1.
Free online http://bass.math.uconn.edu/real.html

Supplementary reading (not required to purchase)
Gerald Folland, *Real Analysis: Modern Techniques and Their
Applications*, 2nd edition.

Math 542 Complex Variables I, Fall 2019

Contact info:

Instructor: M. Burak Erdoğan

E-mail: berdogan@illinois.edu

Office: 347 Illini Hall

Webpage: https://faculty.math.illinois.edu/~berdogan/

Textbook: An Introduction to Complex Function Theory, B. Palka.

Prerequisites: MATH 446 and MATH 447, or MATH 448.

Outline: This is an introductory course in complex analysis. Topics will include:

1. Complex number system: Basic definitions, topology of the complex plane, Riemann sphere, stereographic projection.

2. Differentiability: Basic properties, Cauchy-Riemann equations, analytic functions.

3. Elementary functions: Fundamental algebraic, analytic, and geometric properties. Basic conformal mappings.

4. Contour integration: The Cauchy integral theorem, consequences.

5. Sequences and series: Uniform convergence, power series.

6. The local theory: Zeros, Liouville's theorem, Maximum modulus theorem, Schwarz's Lemma.

7. Laurent series: Classification of isolated singular points, Riemann's theorem, the Casorati-Weierstrass theorem.

8. Residue theory: The residue theorem, evaluation of improper integrals, argument principle, Rouche's theorem, the local mapping theorem.

9. The global theory: Winding number, general Cauchy theorem and integral formula, simply connected domains.

10. Uniform convergence on compact sets: Ascoli-Arzela theorem, normal families, theorems of Montel and Hurwitz, the Riemann mapping theorem.

11. Infinite products. Weierstrass factorization theorem. Gamma function, Riemann-Zeta function.

12. Runge's theorem. Applications.

13. Harmonic functions: Basic properties, Laplace's equation, analytic completion, the Dirichlet problem.

MATH 550

Dynamical Systems I

Professor Rapti

An introduction to the study of dynamical systems. Considers continuous and discrete dynamical systems: linear and nonlinear differential equations, flows and maps on Euclidean space and other manifolds. Among other things, we will study the existence and uniqueness of solutions, dependence on initial conditions and parameters, linearization, stable and center manifold theorems.
Discrete dynamics includes Bernoulli shifts, elementary Anosov diffeomorphisms and surfaces of sections of flows.
Bifurcation phenomena in both continuous and discrete dynamics will be studied.

Prerequisite: MATH 489 or consent of instructor.

Textbook information is below.

Title: Ordinary Differential Equations with Applications
Author: Carmen Charles Chicone
Series: Texts in Applied Mathematics
Publisher: Springer

Fall 2019. MATH 554. <u>Linear Analysis and Partial Differential Equations.</u>

The course covers and develops techniques from functional analysis along with their implementation in the theory of partial differential equations. Time permitting we will cover the following topics.

Coverage: Short introduction to Banach and Hilbert space theory. Banach Fixed Point Theorem and applications to differential and integral equations.

The theory of L^p spaces. Completeness, Duality, Reflexivity, Convolutions and Mollification, along with the basic inequalities that we use in PDEs.

Introduction to the spectral theory of bounded linear operators. The spectrum of compact operators, Fredholm Alternative and applications to Layer Potentials. Introduction to the class of unbounded operators. Criteria for self-adjointness.

Distributions and Fourier Transform on $\mathbb{R}^n$. L^2-based Sobolev spaces, Fundamental Solutions, Green's Functions. Heat, Schrödinger or Linear Wave equation on $\mathbb{R}^n$ by inverting the Fourier transform.

Weak derivatives and Sobolev spaces on bounded and unbounded domains. Embedding Theorems, Rellich-Kondrachov Theorem. Elliptic regularity. Lax-Milgram and applications to linear evolution equations (non constant coefficients). Galerkin approximation. Examples on Elliptic, Parabolic and Hyperbolic equations.

Introduction to nonlinear partial differential equations.

Prerequisites: Math 447, Math 489, or consent of instructor. Math 540 would be useful, but is not required.

The course will be based on my lecture notes. The notes will be detailed and self contained. Recommended texts:
1. L. E. Evans, Partial Differential Equations.
2. M. B. Erdogan and N. Tzirakis: Dispersive Partial Differential Equations: Wellposedness and Applications.

For students which are not familiar with modern analysis techniques it may be helpful to consult from time to time the following books:
1. G. B. Folland, Real Analysis.
2. P. D. Lax, Functional Analysis.

The grade will be based on regular homework, participation and attendance.

Math 562 (Probability II)

Instructor: Renming Song
Office: 338 Illini Hall
Phone number: 217 244 6604

Text: Jean-Francois Le Gall : Brownian Motion, Martingales and Stochastic Calculus, 2016, Springer

Course Topics: This is the second half of the basic graduate course in
probability theory. This course will concentrate on stochastic calculus
and its applications. In particular, we will cover, among other things,
the following topics: Brownian motion, stochastic integrals, Ito's
formula, martingale representation theorem, Girsanov's theorem, stochastic
differential equations, connections to partial differential equations. If time allows,
I will also present some applications to mathematical finance.

Math 561 is a prerequisite for this course. However, if you have not taken
Math 561, but are willing to invest some extra time to pick up the
necessary materials from 561, you may register for this course.

Grading Policy: Your grade will depend on homework assignment and a possible final exam.
6.

FALL 2019

MATH 564

APPLIED STOCHASTIC PROCESSES

Professor DeVille

Course Topics: This course is designed for both math and non-math graduate students. Measure theory is NOT a prerequisite for this course. However, you do need a basic knowledge of probability theory (math 461 or its equivalent).

The goal of this course is to reach fairly rigorous understanding of Markov chains and Markov processes. We are going to cover most of the materials from Norris' book, augmenting the text when necessary. Below is a rough list of some of the topics we will cover:

Strong Markov properties, recurrence and transience, invariant distributions, convergence and ergodicity, time reversal, Q-matrices, holding time, forward and backward equations, martingales and potential theory, queuing networks, Markov decision processes, Markov Chain and Monte Carlo techniques. Depending on the interest of the audience, we may cover some additional materials.

Text: J.R.Norris, Markov Chains, Cambridge University Press

Mathematics 570: MATHEMATICAL LOGIC

Fall 2019

Instructor: Lou van den Dries

MWF, from 12 to 12:50, Altgeld Hall 443

This course gives an introduction to *First Order Logic* (Predicate Logic). No previous study of logic is assumed. Included in the course are:

- The completeness and compactness theorems for first order logic. The first says that provability from a set of axioms is equivalent to validity in all models of the axioms. The second of these is basic to model theory.

- Elements of model theory: theorem of Skolem-Löwenheim, complete theories, back-and-forth, quantifier elimination, Presburger arithmetic.

- Elements of computability theory and Gödel's incompleteness theorem. The latter says that no system of effectively given axioms that includes some basic arithmetic can be complete. Also a discussion of decidable and undecidable theories with examples.

There will be a midsemester exam, a final exam, and regular homework. The part that will be graded will usually be assigned on Wednesday, and is always due on Monday **at the beginning of class** (except for Labour Day). These count to your course grade as follows:

HW: 20/100, midterm: 30/100, final: 50/100.

Prerequisites: For undergraduates, Math 414 or consent of the instructor. Some knowledge of (naive) set theory is desirable.

Lecture notes can be downloaded as follows: go to the webpage of the math department, click on "directory", click on "faculty", click on my name (Lou van den Dries, under D), click on "website", and download "Logic Notes". Alternatively, send me an email and I can send you the pdf file of these notes. My email: vddries@illinois.edu

Office Hours: Monday, 10-11.

Course webpage: you can access it by going to: learn.illinois.edu
and then it will be listed on the left as: MATH 570 X FA18. This will contain all kinds of extra information, such as homework assignments, solutions, and little essays and extra material that are not in the Lecture Notes but that might be instructive to some of you.

DESCRIPTIVE SET THEORY

Fall 2019   Math 574-dst   Anush Tserunyan

TR 11am–12:20pm   29 English Bldg

Classical, as well as contemporary, descriptive set theory (DST) is part of analysis, rather than set theory[1]. It combines techniques from analysis, topology, combinatorics, recursion theory, set theory, and other areas of mathematics to study *definable* subsets of (and functions on) Polish spaces (e.g. $\mathbb{R}^n$, $\mathbb{N}^{\mathbb{N}}$, $L^p(\mathbb{R})$). Examples of such sets include Borel, analytic (projections of Borel[2]), co-analytic (complement of analytic), etc. A typical example (perhaps the first) of a theorem in DST is Cantor's Perfect Set Theorem, which states that any uncountable Polish space (e.g. $\mathbb{R} \setminus \mathbb{Q}$) contains a Cantor set.
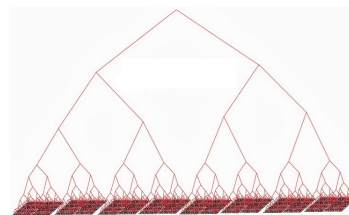
Cantor space

At its earlier stage, a central interest in DST was investigating the regularity properties of definable sets such as the perfect set property, measurability, and the Baire measurability. At the heart of this lies the theory of *infinite games*, which we will study.

For the past 30 years, a major focus of descriptive set theory has been the study of *equivalence relations* on Polish spaces that are definable when viewed as sets of pairs. Such equivalence relations arise naturally all over mathematics since many mathematical objects (such as Riemann surfaces, Banach spaces, measure-preserving transformations, etc.) can be parameterized as points in Polish spaces. Classifying these points (e.g. Banach spaces) up to some equivalence relation (e.g. isomorphism) means understanding the (Borel) complexity of this equivalence relation. DST provides a rigorous framework for this, as well as tools.

The theory of definable equivalence relations is intertwined with *actions of Polish groups* (e.g. all countable groups, Lie groups, many automorphism groups) and the combinatorics of definable (typically, analytic or Borel) *graphs on Polish spaces*, thus lying in the nexus of ergodic theory, topological dynamics, measured group theory, and combinatorics.

Prerequisites: Basic pointset topology and real analysis

Course material: A. Tserunyan, *Introduction to Descriptive Set Theory*, lecture notes [link]

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[1]The name is a rather historical artifact.

[2]Lebesgue famously delayed the development of DST by a decade publishing a false proof that projections of Borel sets are Borel — they typically aren't!

Math 580

Professor Balogh

Syllabus: This is a rigorous, graduate level introduction to combinatorics. It does not assume
prior study, but requires mathematical maturity; it moves at a fast pace. The first half of
the course is on enumeration. The second half covers graph theory. There are some topics that are treated
more in depth in advanced graduate courses
(Math 581, 582, 583, 584, 585): Ramsey theory, partially ordered sets, the probabilistic method
and combinatorial designs (as time permits).

Textbook: The FALL 2019 edition of the text COMBINATORIAL MATHEMATICS
(by Douglas West) will available at TIS Bookstore.

REQUIREMENTS: A raw score of 80% or higher guarantees an A while a score of 60% or
higher guarantees a B- (grade drops by 5%). Additionally, for an A, in the final exam minimum 50%, for
B+ (passing com) 40% required. (Near) weekly assignments. Each assignment
will have 6 problems of your choice of 5/6 are graded.
There are 9 homework assignments, each worth 4.44%, two tests, each 15%, and a final exam for
30%.
The gradings: 80%− : A, 75%− : A−, 70%− : B+, 65%− : B, etc.
Note that the writings of the solutions must have a high quality and typed, if the argument
is messy or not typed then even if the solution is correct it could be returned without grading
with 0 points.

Late homework policy: In case the homework is not submitted on time, it could be submitted
for the next class, with losing 10% of the score. If there is offical or medical reason
then try to notify me in advance via e-mail.
RESOURCES: Electronic mail is a medium for announcements and questions.
PREREQUISITES: There are no official prerequisites, but students need the mathematical
maturity and background for graduate-level mathematics.

MATH 595: CATEGORICAL LOGIC (FALL 2019)

Instructor: Ruiyuan (Ronnie) Chen

Time and location: TBD

Course description: Broadly speaking, a *logic* consists of syntactical expressions (e.g., "$\forall x \neq 0 \, \exists y \, (xy = 1)$"), rules for manipulating them, and rules for giving them mathematical meaning. Categorical logic provides a framework for addressing the question: how well does the syntax of a logic match its semantics? That is, is every semantic "thing" (be it truth or data) syntactically expressible, and irredundantly so? In the best cases, the syntax of a logic is encoded in an algebraic structure[1] which is precisely equivalent to the space of all possible semantics via a duality theorem. We will study several instances of this phenomenon, including

- Stone duality for propositional logic/Boolean algebras/Stone spaces;
- Gabriel–Ulmer duality for Cartesian logic/finitely complete categories/locally finitely presentable categories;
- Makkai duality for first-order logic/pretoposes/ultracategories;
- (time permitting) Joyal–Tierney representation theory, $\mathcal{L}_{\kappa\omega}$, and Grothendieck toposes, and/or other topics of interest to participants.

Each such duality theorem manifests simultaneously as a very strong completeness/definability[2] theorem for the logic, a representation theorem for the syntactic algebra, and an axiomatization of the "function algebra" on the corresponding space of possible semantics.

Prerequisites:

- being comfortable with general abstract structures (monoids, posets, etc.) and related notions (homomorphisms, presentations, etc.)
- basic familiarity with point-set topology
- some background in category theory would be helpful but will *not* be assumed; we will cover everything needed during the course
- ditto for model theory

Coursework: There will be occasional (optional, but highly recommended) exercises. Grading will be based on attendance.

References: I will be posting course notes online. Other good references:
- P. Johnstone, *Sketches of an elephant*, Part D.
- B. Jacobs, *Categorical logic and type theory*.
- J. Lurie, lecture notes.
- A. Pillay and friends, lecture notes.

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[1] versions include the "Lindenbaum–Tarski algebra" and "syntactic category"

[2] sometimes called "strong conceptual completeness"

Math 595

Professor Dey

Title: Concentration inequalities and Stein's Method for Distributional Approximation

Description: Concentration inequalities bound the probability that a function of several random variables differs from its mean by more than a certain amount. The search for such inequalites has been a popular topic of research in the last deacades because of their importance in numeruos applications in discrete mathematics, statistical mechanics, information theory, high-dimensional geometry, random matrices and others.

This course will cover several different approaches to answering the question of finding useful concentration bounds. In particular, we will study martingale method, entropy method, transportation method, isoperimetric method, Stein's method and will cover examples from current research, including dimension reduction, random matrices, Boolean analysis, spin glasses and statistical estimation. We will also look at distributional approximation techniques using Stein's method.

Students are expected to have working knowledge of basic probability theory.

MATH 595 FALL 2019
THE MATHEMATICS OF THE ADS/CFT CORRESPONDENCE

One of the most exciting topics in modern physics with applications as diverse as Quantum Gravity and Condensed Matter Theory is the so called AdS/CFT correspondence. Loosely put, AdS/CFT, is a duality that establishes a correspondence between string theory in an asymptotically AdS background and a conformal field theory on the conformal boundary of the bulk. The basic connection between the (dimensionless) parameters g_s, l_s (resp. string coupling and string lenght) and curvature radius R of the bulk theory are related to basic (dimensionless) parameters of the field theory via $g^2 = 4\pi g_s$ and $g^2 N = \frac{R^4}{l_s^4}$. Here N is the number of colors and for large N, the t' Hooft coupling $\lambda = g^2 N$ controls the perturbation theory. In the regime in which the curvature radius R is much larger then the string length (and therefore, due to $g^2 N = \frac{R^4}{l_s^4}$, in the regime where the number of colors N is large) and when g_s is very small, it is well known that the string theory reduces to classical supergravity. Therefore one of the first striking applications of the theory is that one can perform (semi)classical calculations in $d+1$-dimensional can be used to deduce properties of strongly correlated Conformal Field theories. One striking example of this is the celebrated Ryu-Takayanagi formula which computes the entanglement entropy of a region A (and its complement) in the boundary CFT via the renormalized are of a bulk minimal submanifold sharing the boundary of the region A.

The topic of the course is to explain all this in mathematically sound fashion (when possible) and also explain many mathematical and physical consequences and current research topics.

- Introduction to String Theory and CFT: type II string theory and SUSY
- Analysis of PDE's (Einstein equations, unique continuation results for systems of PDE's and their application to the diffeomorphic structure of conformally compact Einstein manifolds, fractional Laplacians and non-local PDE's)
- General introduction to CFT's
- Minimal surfaces in asymptotically hyperbolic manifolds and their normalized volumes and their application to Entanglement entropy
- Energy/Thermodynamic formulation of (quantum) gravity: Einstein equations out of boundary data and entanglement entropy (via Ryu-Takayanagi formulae)
- Quantum chaos

OPEN PROBLEMS IN
GROUP THEORY AND L^2-COHOMOLOGY

Igor Mineyev. Math 595, second half of Fall 2019 (October 19 - December 19)

MWF 3pm, 140 Henry Administration Bldg

`https://faculty.math.illinois.edu/~mineyev/class/19f/595/`

The course will be a gentle introduction to L^2-cohomology and L^2-invariants for groups. Those are the "Hilbert space versions" of the usual cohomology and the usual Betti numbers. The subject is quite modern and is related to geometry, topology of manifolds and other spaces, group theory, group algebras, analysis, measure theory. Some familiarity with algebra, homological algebra, algebraic topology, functional analysis might be helpful, but there are no formal required prerequisites. No textbook will be required for the course. One reference might be the book and articles by Lück on L^2 invariants, and the article by Cheeger and Gromov on L^2-cohomology. The purpose is to present a reasonably short overview of the subject.

Important and interesting open problems in these areas will be discussed. One is the Atiyah problem, which can be thought of as an analytic analog of the Kaplansky zero-divisor conjecture coming from ring theory. The strengthened Hanna Neumann conjecture, a question about free groups (now solved), can also be restated in this language. This allows asking more general questions: submultiplicativity of ℓ^2-Betti numbers. Still another related open problem is the Singer conjecture, about manifolds.

This course will only run if sufficiently many students sign up.

MATH 595: p-ADIC INTEGRATION AND COHOMOLOGY, FALL 2019

Course Meets: First half of the semester, day/time TBA

Instructor: Thomas Nevins (`nevins@illinois.edu`)

Prerequisites: Math 500; Math 510 or Math 511 or some basic knowledge of algebraic varieties.

Course Web Page: `http://www.math.uiuc.edu/∼nevins/courses/aut20/m595p-adic.html`

p-adic integration was developed as an answer to the question of how to integrate differential forms on manifolds over the field $\mathbb{Q}_p$. This course will provide an economical introduction to the theory with selected cohomological applications.

There are multiple versions of "p-adic integrals." This course will focus on the one developed in works by Tate, Serre, etc. and used in the 1990s by Batyrev to prove that birational, smooth projective Calabi-Yau varieties have the same Betti numbers. The course will not focus on maximal generality, and will not assume any familiarity with p-adic numbers or real analysis beyond Math 540. Instead, its goal is to provide students with basic tools that can then be used to understand various applications in algebraic geometry, number theory, and representation theory.

A principal goal of the course will be to develop enough about p-adic integrals to prove Batyrev's theorem on K-invariance of Betti numbers. An added benefit is that the course should prepare interested students, postdocs, and faculty to participate in a working seminar to understand more recent work of Groechenig-Wyss-Ziegler on mirror symmetry for Hitchin systems and Ngô's "Geometric stabilization theorem."

The course will rely on freely available sources as its supporting texts.

MATH 595: INTRODUCTION TO ALGEBRAIC DIFFERENTIAL OPERATORS AND $\mathcal{D}$-MODULES, FALL 2020

Course Meets: Second half of the semester, day/time TBA

Instructor: Thomas Nevins (`nevins@illinois.edu`)

Prerequisites: Math 500.

Course Web Page: `http://www.math.uiuc.edu/∼nevins/courses/aut20/m595-Dmod.html`

Rings of algebraic differential operators form basic examples in noncommutative algebra. Their modules, known as $\mathcal{D}$-modules, lie at the interfaces of algebraic geometry, representation theory, real and complex analysis, topology, and noncommutative ring theory.

This course will provide an introduction to rings of algebraic differential operators and $\mathcal{D}$-modules. It will focus, more than is common in most standard texts, on understanding concrete examples and calculations. Goals of the course will depend on the interests of students registered, but might include a treatment of holonomic $\mathcal{D}$-modules and the Riemann-Hilbert correspondence.

The main text for the course will be "$\mathcal{D}$-Modules, Perverse Sheaves, and Representation Theory" by Hotta, Takeuchi, and Tanisaki, available electronically at:
  `https://vufind.carli.illinois.edu/vf-uiu/Record/uiu_5635867`
   There are many other good references freely available, including lecture notes by Bernstein (classic!), Kashiwara (classic!), Ginzburg, and Gaitsgory, and the book "Primer on Algebraic D-Modules" by Coutinho. But the course will aim to be largely self-contained.

No background will be assumed beyond Math 500 and an eagerness to learn.

Quiver representations
Eddie Nijholt

Quiver representations play an increasingly important role in many areas of mathematics. Indeed, this predominantly algebraic topic has seen applications ranging from mathematical physics to data analysis. Conceptually, a quiver may be seen as simply a directed graph. A representation of such a graph then means assigning a vector space to every node, and a linear map between the source and target spaces of any arrow. We will introduce quiver representations and study some important concepts, such as maps between representations, the path algebra and indecomposable representations. Our main goal will be to prove Gabriel's theorem: a result that classifies all quivers with only finitely many indecomposable representations. It turns out these are exactly all graphs that, when one forgets about the direction of the arrows, equal the ADE Dynkin diagrams. As prerequisites we only assume knowledge of linear algebra and some familiarity with the most basic algebraic concepts.

FALL 2019, MATH 595, EXPONENTIAL SUMS

INSTRUCTOR: ALEXANDRU ZAHARESCU

Exponential sums play an important role in many questions in number theory, and also in some problems arising from other fields. The first part of the course will cover classical material. For this part we will follow selected chapters from Montgomery's book. In the second part of the course we will study some recent papers on the distribution of zeros of the Riemann zeta function, points on curves over finite fields, billiards, lattice points and Farey fractions, where exponential sums play a central role.

Prerequisite: MATH 531.

Recommended Textbook:

H. L. Montgomery, *Ten lectures on the interface between analytic number theory and harmonic analysis*, CBMS Regional Conference Series in Mathematics, 84. Providence, RI, 1994.

There will be no exams. Students registered for this course will be expected to give one or two lectures on some topics related to the content of the course. In addition some homework problems will be assigned.

Office hours by appointment.

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