Física I – Sears, Zemansky, Young & Freedman.

PHYSICS ACT.

Capítulo 1

1.1: $1\,\text{mi}\times(5280\,\text{ft/mi})\times(12\,\text{in./ft})\times(2.54\,\text{cm/in.})\times(1\,\text{km}/10^5\,\text{cm})=1.61\,\text{km}$
Although rounded to three figures, this conversion is exact because the given conversion from inches to centimeters defines the inch.

1.2: $$0.473\,\text{L}\times\left(\frac{1000\,\text{cm}^3}{1\,\text{L}}\right)\times\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)^3=28.9\,\text{in}^3.$$

1.3: The time required for light to travel any distance in a vacuum is the distance divided by the speed of light;
$$\frac{10^3\,\text{m}}{3.00\times10^8\,\text{m/s}}=3.33\times10^{-6}\text{s}=3.33\times10^3\,\text{ns}.$$

1.4: $$11.3\ \frac{\text{g}}{\text{cm}^3}\times\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\times\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3=1.13\times10^4\ \frac{\text{kg}}{\text{m}^3}.$$

1.5: $(327\,\text{in}^3)\times(2.54\,\text{cm/in})^3\times(1\,\text{L}/1000\,\text{cm}^3)=5.36\,\text{L}.$

1.6: $$1\ \text{m}^3\times\left(\frac{1000\ \text{L}}{1\ \text{m}^3}\right)\times\left(\frac{1\ \text{gal}}{3.788\ \text{L}}\right)\times\left(\frac{128\ \text{oz.}}{1\ \text{gal}}\right)\times\left(\frac{1\ \text{bottle}}{16\ \text{oz.}}\right).$$
$= 2111.9\,\text{bottles}\approx 2112\ \text{bottles}$

The daily consumption must then be
$$2.11\times10^3\ \frac{\text{bottles}}{\text{yr}}\times\left(\frac{1\ \text{yr}}{365.24\ \text{da}}\right)=5.78\ \frac{\text{bottles}}{\text{da}}.$$

1.7: $(1450\,\text{mi/hr})\times(1.61\,\text{km/mi})=2330\,\text{km/hr}.$
$2330\,\text{km/hr}\times(10^3\,\text{m/km})\times(1\,\text{hr}/3600\,\text{s})=648\,\text{m/s}.$

1.8: $$180{,}000\frac{\text{furlongs}}{\text{fortnight}}\times\left(\frac{1\ \text{mile}}{8\ \text{furlongs}}\right)\times\left(\frac{1\ \text{fortnight}}{14\ \text{day}}\right)\times\left(\frac{1\ \text{day}}{24\ \text{h}}\right)=67\ \frac{\text{mi}}{\text{h}}.$$

1.9: $$15.0\frac{\text{km}}{\text{L}}\times\left(\frac{1\,\text{mi}}{1.609\,\text{km}}\right)\times\left(\frac{3.788\,\text{L}}{1\,\text{gal}}\right)=35.3\frac{\text{mi}}{\text{gal}}.$$

1.10: a) $\left(60\dfrac{\text{mi}}{\text{hr}}\right)\left(\dfrac{1\,\text{h}}{3600\,\text{s}}\right)\left(\dfrac{5280\,\text{ft}}{1\,\text{mi}}\right)=88\dfrac{\text{ft}}{\text{s}}$

b) $\left(32\dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48\,\text{cm}}{1\,\text{ft}}\right)\left(\dfrac{1\,\text{m}}{100\,\text{cm}}\right)=9.8\dfrac{\text{m}}{\text{s}^2}$

c) $\left(1.0\dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100\,\text{cm}}{1\,\text{m}}\right)^3\left(\dfrac{1\,\text{kg}}{1000\,\text{g}}\right)=10^3\,\dfrac{\text{kg}}{\text{m}^3}$

1.11: The density is mass per unit volume, so the volume is mass divided by density.
$V=\left(60\times10^3\,\text{g}\right)/\left(19.5\,\text{g/cm}^3\right)=3077\,\text{cm}^3$

Use the formula for the volume of a sphere, $V=\dfrac{4}{3}\pi r^3$,

to calculate $r: r=\left(3V/4\pi\right)^{1/3}=9.0\,\text{cm}$

1.12: $(3.16\times10^7\,\text{s}-\pi\times10^7\,\text{s})/(3.16\times10^7\,\text{s})\times100=0.58\%$

1.13: a) $\dfrac{10\,\text{m}}{890\times10^3\,\text{m}}=1.1\times10^{-3}\%.$

b) Since the distance was given as 890 km, the total distance should be 890,000 meters.

To report the total distance as 890,010 meters, the distance should be given as 890.01 km.

1.14: a) $\left(12\,\text{mm}\right)\times\left(5.98\,\text{mm}\right)=72\,\text{mm}^2$ (two significant figures).

b) $\frac{5.98\,\text{mm}}{12\,\text{mm}}$ = 0.50 (also two significant figures).

c) 36 mm (to the nearest millimeter).

d) 6 mm.

e) 2.0.

1.15: a) If a meter stick can measure to the nearest millimeter, the error will be about 0.13%. b) If the chemical balance can measure to the nearest milligram, the error will be about $8.3\times10^{-3}\%$. c) If a handheld stopwatch (as opposed to electric timing devices) can measure to the nearest tenth of a second, the error will be about $2.8\times10^{-2}\%$.

1.16: The area is $9.69 \pm 0.07\ \text{cm}^2$, where the extreme values in the piece's length and width are used to find the uncertainty in the area. The fractional uncertainty in the area is $\frac{0.07\,\text{cm}^2}{9.69\,\text{cm}^2} = 0.72\%$, and the fractional uncertainties in the length and width are $\frac{0.01\,\text{cm}}{5.10\,\text{cm}} = 0.20\%$ and $\frac{0.01\,\text{cm}}{1.9\,\text{cm}} = 0.53\%$.

1.17: a) The average volume is

$$\pi\frac{(8.50\ \text{cm})^2}{4}(0.050\ \text{cm}) = 2.8\ \text{cm}^3$$

(two significant figures) and the uncertainty in the volume, found from the extreme values of the diameter and thickness, is about $0.3\ \text{cm}^3$, and so the volume of a cookie is $2.8\pm0.3\ \text{cm}^3$. (This method does not use the usual form for progation of errors, which is not addressed in the text. The fractional uncertainty in the thickness is so much greater than the fractional uncertainty in the diameter that the fractional uncertainty in the volume is 10%, reflected in the above answer.)

b) $\frac{8.50}{.05} = 170\pm20$.

1.18: (Number of cars × miles/car·day)/mi/gal = gallons/day
(2×10^8 cars × 10000 mi/yr/car × 1 yr/365 days)/(20 mi/gal) = 2.75×10^8 gal/day

1.19: Ten thousand; if it were to contain ten million, each sheet would be on the order of a millionth of an inch thick.

1.20: If it takes about four kernels to fill 1 cm^3, a 2-L bottle will hold about 8000 kernels.

1.21: Assuming the two-volume edition, there are approximately a thousand pages, and each page has between 500 and a thousand words (counting captions and the smaller print, such as the end-of-chapter exercise and problems), so an estimate for the number of words is about 10^6.

1.22: Assuming about 10 breaths per minutes, 24×60 minutes per day, 365 days per year, and a lifespan of fourscore (80) years, the total volume of air breathed in a lifetime is about $2\times 10^5\ \text{m}^3$. This is the volume of a room $100\,\text{m}\times 100\,\text{m}\times 20\,\text{m}$, which is kind of tight for a major-league baseball game, but it's the same order of magnitude as the volume of the Astrodome.

1.23: This will vary from person to person, but should be of the order of 1×10^5.

1.24: With a pulse rate of a bit more than one beat per second, a heart will beat 10^5 times per day. With 365 days in a year and the above lifespan of 80 years, the number of beats in a lifetime is about 3×10^9. With $\frac{1}{20}$ L (50 cm^3) per beat, and about $\frac{1}{4}$ gallon per liter, this comes to about 4×10^7 gallons.

1.25: The shape of the pile is not given, but gold coins stacked in a pile might well be in the shape of a pyramid, say with a height of $2\ \text{m}$ and a base $3\ \text{m}\times 3\ \text{m}$. The volume of such a pile is $6\ \text{m}^3$, and the calculations of Example 1-4 indicate that the value of this volume is $\$6\times 10^8$.

1.26: The surface area of the earth is about $4\pi R^2 = 5\times 10^{14}\ \text{m}^2$, where R is the radius of the earth, about $6\times 10^6\ \text{m}$, so the surface area of all the oceans is about $4\times 10^{14}\ \text{m}^2$. An average depth of about 10 km gives a volume of $4\times 10^{18}\ \text{m}^3 = 4\times 10^{24}\text{cm}^3$. Characterizing the size of a "drop" is a personal matter, but $25\ \text{drops/cm}^3$ is reasonable, giving a total of 10^{26} drops of water in the oceans.

1.27: This will of course depend on the size of the school and who is considered a "student". A school of thousand students, each of whom averages ten pizzas a year (perhaps an underestimate) will total 10^4 pizzas, as will a school of 250 students averaging 40 pizzas a year each.

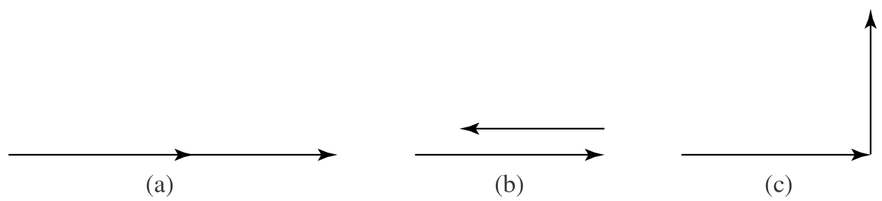
1.28: The moon is about $4 \times 10^8 \text{ m} = 4 \times 10^{11} \text{ mm}$ away. Depending on age, dollar bills can be stacked with about 2-3 per millimeter, so the number of bills in a stack to the moon would be about 10^{12}. The value of these bills would be $1 trillion (1 terabuck).

1.29: $(\text{Area of USA})/(\text{Area/bill}) = \text{number of bills.}$

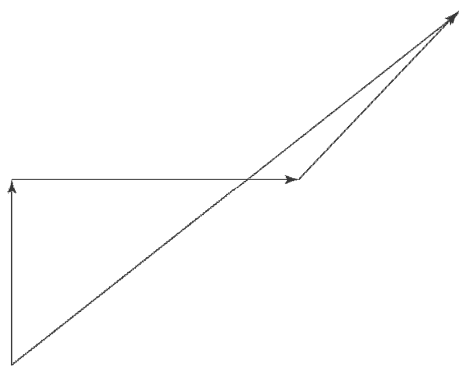
$$\left(9{,}372{,}571 \text{ km}^2 \times 10^6 \text{ m}^2/\text{km}^2\right)/\left(15.6 \text{ cm} \times 6.7 \text{ cm} \times 1 \text{ m}^2/10^4 \text{ cm}^2\right) = 9 \times 10^{14} \text{ bills}$$

$$9 \times 10^{14} \text{ bills}/2.5\ \ 10^8 \text{ inhabitants} = \$3.6 \text{ million/inhabitant.}$$

1.30:

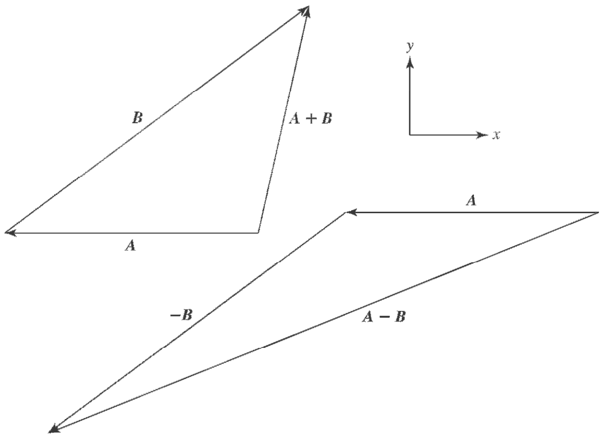


1.31:



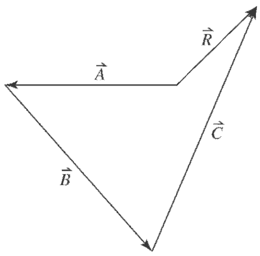
$7.8 \text{ km}, 38^\circ$ north of east

1.32:



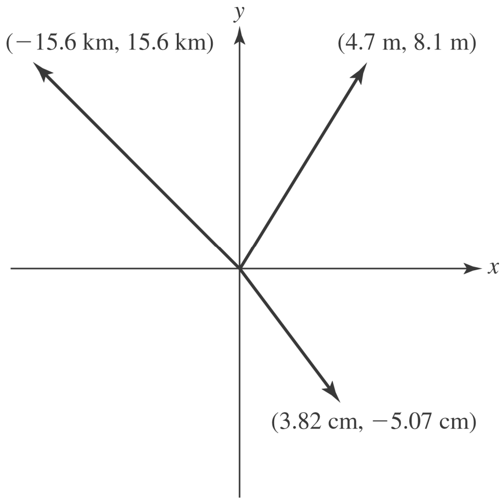
a) 11.1 m @ 77.6°
b) 28.5 m @ 202°
c) 11.1 m @ 258°
d) 28.5 m @ 22°

1.33:



144 m, 41° south of west.

1.34:



1.35: $\vec{\boldsymbol{A}}; A_x = (12.0\ \text{m})\sin 37.0^\circ = 7.2\ \text{m},\ A_y = (12.0\ \text{m})\cos 37.0^\circ = 9.6\ \text{m}.$

$\vec{\boldsymbol{B}}; B_x = (15.0\ \text{m})\cos 40.0^\circ = 11.5\ \text{m},\ B_y = -(15.0\ \text{m})\sin 40.0^\circ = -9.6\ \text{m}.$

$\vec{\boldsymbol{C}}; C_x = -(6.0\ \text{m})\cos 60.0^\circ = -3.0\ \text{m},\ C_y = -(6.0\ \text{m})\sin 60.0^\circ = -5.2\ \text{m}.$

1.36: (a)
$$\tan\theta = \frac{A_y}{A_X} = \frac{-1.00\ \text{m}}{2.00\ \text{m}} = -0.500$$

$$\theta = \tan^{-1}(-0.500) = 360^\circ - 26.6^\circ = 333^\circ$$

(b)
$$\tan\theta = \frac{A_y}{A_x} = \frac{1.00\ \text{m}}{2.00\ \text{m}} = 0.500$$

$$\theta = \tan^{-1}(0.500) = 26.6^\circ$$

(*c*)
$$\tan\theta = \frac{A_y}{A_x} = \frac{1.00\ \text{m}}{-2.00\ \text{m}} = -0.500$$

$$\theta = \tan^{-1}(-0.500) = 180^\circ - 26.6^\circ = 153^\circ$$

(d)
$$\tan\theta = \frac{A_y}{A_x} = \frac{-1.00\ \text{m}}{-2.00\ \text{m}} = 0.500$$

$$\theta = \tan^{-1}(0.500) = 180^\circ + 26.6^\circ = 207^\circ$$

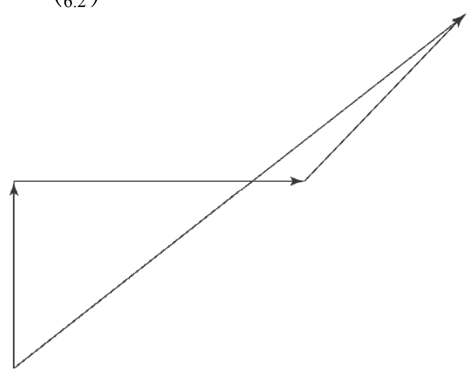
1.37: Take the $+x$-direction to be forward and the $+y$-direction to be upward. Then the second force has components $F_{2x} = F_2 \cos 32.4^\circ = 433\,\text{N}$ and $F_{2y} = F_2 \sin 32.4^\circ = 275\,\text{N}$. The first force has components $F_{1x} = 725\,\text{N}$ and $F_{1y} = 0$.

$F_x = F_{1x} + F_{2x} = 1158\,\text{N}$ and $F_y = F_{1y} + F_{2y} = 275\,\text{N}$

The resultant force is 1190 N in the direction 13.4° above the forward direction.

1.38: (The figure is given with the solution to Exercise 1.31).

The net northward displacement is (2.6 km) + (3.1 km) sin 45^o = 4.8 km, and the net eastward displacement is (4.0 km) + (3.1 km) cos 45^o = 6.2 km. The magnitude of the resultant displacement is $\sqrt{(4.8\text{ km})^2 + (6.2\text{ km})^2}$ = 7.8 km, and the direction is arctan $\left(\frac{4.8}{6.2}\right)$ = 38^o north of east.



1.39: Using components as a check for any graphical method, the components of $\vec{B}$ are $B_x = 14.4\ \text{m}$ and $B_y = 10.8\ \text{m}$, $\vec{A}$ has one component, $A_x = -12\ \text{m}$.

a) The x- and y-components of the sum are 2.4 m and 10.8 m, for a magnitude of $\sqrt{(2.4\ \text{m})^2 + (10.8\ \text{m})^2} = 11.1\ \text{m}$, , and an angle of $\left(\frac{10.8}{2.4}\right) = 77.6^\circ$.

b) The magnitude and direction of **A** + **B** are the same as **B** + **A**.

c) The *x*- and *y*-components of the vector difference are – 26.4 m and $-10.8\ \text{m}$, for a magnitude of $28.5\ \text{m}$ and a direction arctan $\left(\frac{-10.8}{-26.4}\right) = 202^\circ$. Note that 180° must be added to $\arctan\left(\frac{-10.8}{-26.4}\right) = \arctan\left(\frac{10.8}{26.4}\right) = 22^\circ$ in order to give an angle in the third quadrant.

d) $\vec{B} - \vec{A} = 14.4\ \text{m}\hat{\boldsymbol{i}} + 10.8\ \text{m}\hat{\boldsymbol{j}} + 12.0\ \text{m}\hat{\boldsymbol{i}} = 26.4\ \text{m}\hat{\boldsymbol{i}} + 10.8\ \text{m}\hat{\boldsymbol{j}}$.

$\text{Magnitude} = \sqrt{(26.4\ \text{m})^2 + (10.8\ \text{m})^2} = 28.5\ \text{m}$ at and angle of $\arctan\left(\frac{10.8}{26.4}\right) = 22.2^\circ$.

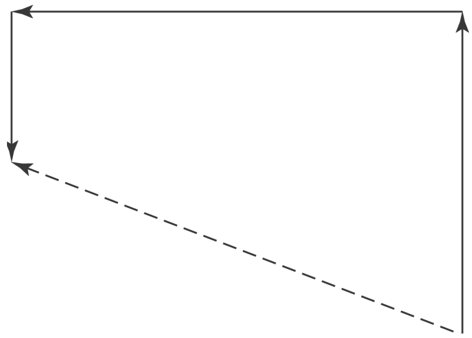
1.40: Using Equations (1.8) and (1.9), the magnitude and direction of each of the given vectors is:

a) $\sqrt{(-8.6\ \text{cm})^2 + (5.20\ \text{cm})^2}$ = 10.0 cm, arctan $\left(\frac{5.20}{-8.60}\right)$ = 148.8° (which is 180° – 31.2°).

b) $\sqrt{(-9.7\ \text{m})^2 + (-2.45\ \text{m})^2}$ = 10.0 m, arctan $\left(\frac{-2.45}{-9.7}\right)$ = 14° + 180° = 194°.

c) $\sqrt{(7.75\ \text{km})^2 + (-2.70\ \text{km})^2}$ = 8.21 km, arctan $\left(\frac{-2.7}{7.75}\right)$ = 340.8° (which is 360° – 19.2°).

1.41:



The total northward displacement is $3.25\,\text{km} - 1.50\,\text{km} = 1.75\,\text{km}$, , and the total westward displacement is $4.75\,\text{km}$. The magnitude of the net displacement is $\sqrt{(1.75\,\text{km})^2 + (4.75\,\text{km})^2} = 5.06\,\text{km}$. The south and west displacements are the same, so The direction of the net displacement is 69.80° West of North.

1.42: a) The x- and y-components of the sum are 1.30 cm + 4.10 cm = 5.40 cm, 2.25 cm + (–3.75 cm) = –1.50 cm.

b) Using Equations (1-8) and (1-9),

$$\sqrt{(5.40\,\text{cm})^2\,(-1.50\text{ cm})^2} = 5.60\text{ cm},\ \arctan\left(\tfrac{-1.50}{+5.40}\right) = 344.5^\circ\text{ ccw.}$$

c) Similarly, 4.10 cm – (1.30 cm) = 2.80 cm, –3.75 cm – (2.25 cm) = –6.00 cm.

d) $\sqrt{(2.80\,\text{cm})^2 + (-6.0\,\text{cm})^2} = 6.62\text{ cm},\ \arctan\left(\tfrac{-6.00}{2.80}\right) = 295^\circ$ (which is 360° – 65

1.43: a) The magnitude of $\vec{A}+\vec{B}$ is

$$\sqrt{\begin{pmatrix} \left((2.80\text{ cm})\cos 60.0^\circ + (1.90\text{ cm})\cos 60.0^\circ\right)^2 \\ +\left((2.80\text{ cm})\sin 60.0^\circ - (1.90\text{ cm})\sin 60.0^\circ\right)^2 \end{pmatrix}} = 2.48\text{ cm}$$

and the angle is

$$\arctan\left(\frac{(2.80\text{ cm})\sin 60.0^\circ - (1.90\text{ cm})\sin 60.0^\circ}{(2.80\text{ cm})\cos 60.0^\circ + (1.90\text{ cm})\cos 60.0^\circ}\right) = 18^\circ$$

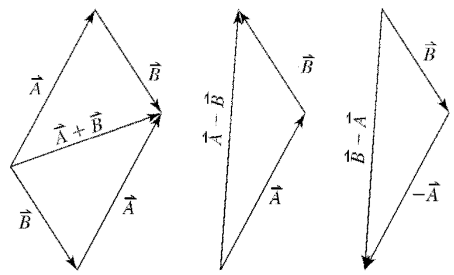
b) The magnitude of $\vec{A}-\vec{B}$ is

$$\sqrt{\begin{pmatrix} \left((2.80\text{ cm})\cos 60.0^\circ - (1.90\text{ cm})\cos 60.0^\circ\right)^2 \\ +\left((2.80\text{ cm})\sin 60.0^\circ + (1.90\text{ cm})\sin 60.0^\circ\right)^2 \end{pmatrix}} = 4.10\text{ cm}$$

and the angle is

$$\arctan\left(\frac{(2.80\text{ cm})\sin 60.0^\circ + (1.90\text{ cm})\sin 60.0^\circ}{(2.80\text{ cm})\cos 60.0^\circ - (1.90\text{ cm})\cos 60.0^\circ}\right) = 84^\circ$$

c) $\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$; the magnitude is 4.10 cm and the angle is $84^\circ + 180^\circ = 264^\circ$.



1.44: $\vec{A} = (-12.0\text{ m})\,\hat{\imath}$. More precisely,

$$\vec{A} = (12.0\text{ m})(\cos 180^\circ)\hat{\imath} + (12.0\text{ m})(\sin 180^\circ)\hat{\jmath}.$$

$$\vec{B} = (18.0\text{ m})(\cos 37^\circ)\hat{\imath} + (18.0\text{ m})(\sin 37^\circ)\hat{\jmath} = (14.4\text{ m})\hat{\imath} + (10.8\text{ m})\hat{\jmath}$$

1.45:

$$\vec{A} = (12.0\text{ m})\sin 37.0^\circ\,\hat{\imath} + (12.0\text{ m})\cos 37.0^\circ\,\hat{\jmath} = (7.2\text{ m})\hat{\imath} + (9.6\text{ m})\hat{\jmath}$$
$$\vec{B} = (15.0\text{ m})\cos 40.0^\circ\,\hat{\imath} - (15.0\text{ m})\sin 40.0^\circ\,\hat{\jmath} = (11.5\text{ m})\hat{\imath} - (9.6\text{ m})\hat{\jmath}$$
$$\vec{C} = -(6.0\text{ m})\cos 60.0^\circ\,\hat{\imath} - (6.0\text{ m})\sin 60.0^\circ\,\hat{\jmath} = -(3.0\text{ m})\hat{\imath} - (5.2\text{ m})\hat{\jmath}$$

1.46: a) $\vec{A} = (3.60\text{ m})\cos 70.0^\circ \hat{i} + (3.60\text{ m})\sin 70.0^\circ \hat{j} = (1.23\text{ m})\hat{i} + (3.38\text{ m})\hat{j}$

$\vec{B} = -(2.40\text{ m})\cos 30.0^\circ \hat{i} - (2.40\text{ m})\sin 30.0^\circ \hat{j} = (-2.08\text{ m})\hat{i} + (-1.20\text{ m})\hat{j}$

b)

$$\begin{aligned}\vec{C} &= (3.00)\,\vec{A} - (4.00)\vec{B} \\ &= (3.00)(1.23\text{ m})\hat{i} + (3.00)(3.38\text{ m})\hat{j} - (4.00)(-2.08\text{ m})\hat{i} - (4.00)(-1.20\text{ m})\hat{j} \\ &= (12.01\text{ m})\hat{i} + (14.94)\hat{j}\end{aligned}$$

(Note that in adding components, the fourth figure becomes significant.)

c) From Equations (1.8) and (1.9),

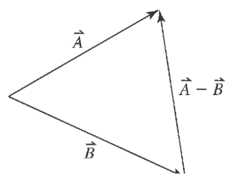
$$C = \sqrt{(12.01\text{ m})^2 + (14.94\text{ m})^2} = 19.17\text{ m}, \arctan\left(\frac{14.94\text{ m}}{12.01\text{ m}}\right) = 51.2^\circ$$

1.47: a) $A = \sqrt{(4.00)^2 + (3.00)^2} = 5.00,\ B = \sqrt{(5.00)^2 + (2.00)^2} = 5.39$

b) $\vec{A} - \vec{B} = (4.00 - 3.00)\hat{i} + (5.00 - (-2.00))\hat{j} = (-1.00)\hat{i} + (5.00)\hat{j}$

c) $\sqrt{(1.00)^2 + (5.00)^2} = 5.10, \quad \arctan\left(\frac{5.00}{-1.00}\right) = 101.3^\circ$

d)



1.48: a) $\left|\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right| = \sqrt{1^2+1^2+1^2} = \sqrt{3} \neq 1$ so it is not a unit vector

b) $$\left|\vec{\boldsymbol{A}}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

If any component is greater than + 1 or less than –1, $\left|\vec{\boldsymbol{A}}\right| \geq 1$, so it cannot be a unit vector. $\vec{\boldsymbol{A}}$ can have negative components since the minus sign goes away when the component is squared.

c)

$$\left|\vec{\boldsymbol{A}}\right| = 1$$

$$\sqrt{a^2(3.0)^2 + a^2(4.0)^2} = 1$$

$$\sqrt{a^2}\sqrt{25} = 1$$

$$a = \pm\frac{1}{5.0} = \pm 0.20$$

1.49: a) Let $\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}}$, $\quad \vec{\boldsymbol{B}} = B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}}$.

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = (A_x + B_x)\hat{\boldsymbol{i}} + (A_y + B_y)\hat{\boldsymbol{j}}$$
$$\vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} = (B_x + A_x)\hat{\boldsymbol{i}} + (B_y + A_y)\hat{\boldsymbol{j}}$$

Scalar addition is commutative, so $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = A_xB_x + A_yB_y$$
$$\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = B_xA_x + B_yA_y$$

Scalar multiplication is commutative, so $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$

b) $$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = (A_yB_z - A_zB_y)\hat{\boldsymbol{i}} + (A_zB_x - A_xB_z)\hat{\boldsymbol{j}} + (A_xB_y - A_yB_x)\hat{\boldsymbol{k}}$$
$$\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}} = (B_yA_z - B_zA_y)\hat{\boldsymbol{i}} + (B_zA_x - B_xA_z)\hat{\boldsymbol{j}} + (B_xA_y - B_yA_x)\hat{\boldsymbol{k}}$$

Comparison of each component in each vector product shows that one is the negative of the other.

1.50: Method 1: (Product of magnitudes $\times \cos\theta$)

$$\text{AB}\cos\theta = (12\text{ m} \times 15\text{ m})\cos 93^\circ = -9.4\text{ m}^2$$

$$\text{BC}\cos\theta = (15\text{ m} \times 6\text{ m})\cos 80^\circ = 15.6\text{ m}^2$$

$$\text{AC}\cos\theta = (12\text{ m} \times 6\text{ m})\cos 187^\circ = -71.5\text{ m}^2$$

Method 2: (Sum of products of components)

$$\mathbf{A}\cdot\mathbf{B} = (7.22)(11.49) + (9.58)(-9.64) = -9.4\text{ m}^2$$

$$\mathbf{B}\cdot\mathbf{C} = (11.49)(-3.0) + (-9.64)(-5.20) = 15.6\text{ m}^2$$

$$\mathbf{A}\cdot\mathbf{C} = (7.22)(-3.0) + (9.58)(-5.20) = -71.5\text{ m}^2$$

1.51: a) From Eq.(1.21),

$$\vec{A}\cdot\vec{B} = (4.00)(5.00) + (3.00)(-2.00) = 14.00.$$

b) $\mathbf{A}\cdot\mathbf{B} = \text{AB}\cos\theta$, so $\theta = \arccos[(14.00)/(5.00\times 5.39)] = \arccos(.5195) = 58.7^\circ$.

1.52: For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as

$$\phi = \arccos\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\frac{A_x B_x + A_y B_y}{AB}\right).$$

In the intermediate calculations given here, the significant figures in the dot products and in the magnitudes of the vectors are suppressed.

a) $\vec{A}\cdot\vec{B} = -22,\ A = \sqrt{40},\ B = \sqrt{13},$ **and so**

$$\phi = \arccos\left(\frac{-22}{\sqrt{40}\sqrt{13}}\right) = 165^\circ.$$

b) $\vec{A}\cdot\vec{B} = 60,\ A = \sqrt{34},\ B = \sqrt{136},\quad \phi = \arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right) = 28^\circ.$

c) $\vec{A}\cdot\vec{B} = 0,\ \phi = 90.$

1.53: Use of the right-hand rule to find cross products gives (a) out of the page and b) into the page.

1.54: a) From Eq. (1.22), the magnitude of the cross product is

$$(12.0\,\mathrm{m})(18.0\,\mathrm{m})\sin\left(180^\circ - 37^\circ\right) = 130\,\mathrm{m}^2$$

The right-hand rule gives the direction as being into the page, or the – z-direction. Using Eq. (1.27), the only non-vanishing component of the cross product is

$$C_z = A_x B_y = (-12\,\mathrm{m})\left((18.0\,\mathrm{m})\sin 37^\circ\right) = -130\,\mathrm{m}^2$$

b) The same method used in part (a) can be used, but the relation given in Eq. (1.23) gives the result directly: same magnitude (130 m^2), but the opposite direction (+z-direction).

1.55: In Eq. (1.27), the only non-vanishing component of the cross product is

$$C_z = A_x B_y - A_y B_x = (4.00)(-2.00) - (3.00)(5.00) = -23.00,$$

so $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = -(23.00)\hat{\boldsymbol{k}}$, and the magnitude of the vector product is 23.00.

1.56: a) From the right-hand rule, the direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is, from Eq. (1.22),

$$AB\ \sin\phi = (2.80\ \text{cm})(1.90\ \text{cm})\sin 120^\circ = 4.61\ \text{cm}^2.$$

Or, using Eq. (1.27) and noting that the only non-vanishing component is

$$\begin{aligned} C_z &= A_x B_y - A_y B_x \\ &= (2.80\ \text{cm})\cos 60.0^\circ\,(-1.90\ \text{cm})\sin\ 60^\circ \\ &\quad -(2.80\ \text{cm})\sin 60.0^\circ\,(1.90\ \text{cm})\cos\ 60.0^\circ \\ &= -4.61\ \text{cm}^2 \end{aligned}$$

gives the same result.

b) Rather than repeat the calculations, Eq. (1-23) may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

1.57: a) The area of one acre is $\frac{1}{8}\ \text{mi}\times\frac{1}{80}\ \text{mi} = \frac{1}{640}\ \text{mi}^2$, so there are 640 acres to a square mile.

b) $$(1\ \text{acre})\times\left(\frac{1\ \text{mi}^2}{640\ \text{acre}}\right)\times\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)^2 = 43{,}560\ \text{ft}^2$$

(all of the above conversions are exact).

c) $$(1\ \text{acre-foot}) = (43{,}560\ \text{ft}^3)\times\left(\frac{7.477\ \text{gal}}{1\ \text{ft}^3}\right) = 3.26\times 10^5\ \text{gal},$$

which is rounded to three significant figures.

1.58: a) $(\$4{,}950{,}000/102\ \text{acres})\times(1\,\text{acre}/43560\,\text{ft}^2)\times(10.77\,\text{ft}^2/\text{m}^2) = \$12/\text{m}^2$.

b) $(\$12/\text{m}^2)\times(2.54\,\text{cm}/\text{in})^2\times(1\,\text{m}/100\,\text{cm})^2 = \$.008/\text{in}^2$.

c) $\$.008/\text{in}^2\times(1\,\text{in}\times 7/8\,\text{in}) = \$.007$ for postage stamp sized parcel.

1.59: a) To three significant figures, the time for one cycle is

$$\frac{1}{1.420\times10^{9}\ \text{Hz}}=7.04\times10^{-10}\ \text{s}.$$

b) $$\left(1.420\times10^{9}\ \frac{\text{cycles}}{\text{s}}\right)\times\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=5.11\times10^{12}\ \frac{\text{cycles}}{\text{h}}$$

c) Using the conversion from years to seconds given in Appendix F,

$$\left(1.42\times10^{9}\ \text{Hz}\right)\times\left(\frac{3.156\times10^{7}\ \text{s}}{1\ \text{y}}\right)\times\left(4.600\times10^{9}\ \text{y}\right)=2.06\times10^{26}.$$

d) $4.600\times10^{9}\ \text{y}=\left(4.60\times10^{4}\right)\left(1.00\times10^{5}\ \text{y}\right)$, so the clock would be off by 4.60×10^{4} s.

1.60: Assume a 70-kg person, and the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: 18.015 u × 1.661 × 10^{-27} kg/u = 2.992 × 10^{-26} kg/molecule. (70 kg/2.992 × 10^{-26} kg/molecule) = 2.34 × 10^{27} molecules. (Assuming carbon to be the most common atom gives 3 × 10^{27} molecules.

1.61: a) Estimate the volume as that of a sphere of diameter 10 cm:

$$V=\frac{4}{3}\pi r^{3}=5.2\times10^{-4}\,\text{m}^{3}$$

Mass is density times volume, and the density of water is $1000\ \text{kg/m}^3$, so

$$m=(0.98)\left(1000\ \text{kg/m}^3\right)\left(5.2\times10^{-4}\,\text{m}^3\right)=0.5\ \text{kg}$$

b) Approximate as a sphere of radius $r=0.25\mu\text{m}$ (probably an over estimate)

$$V=\frac{4}{3}\pi r^{3}=6.5\times10^{-20}\,\text{m}^{3}$$

$$m=(0.98)\left(1000\ \text{kg/m}^3\right)\left(6.5\times10^{-20}\,\text{m}^3\right)=6\times10^{-17}\,\text{kg}=6\times10^{-14}\,\text{g}$$

c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm:

$$V=\pi r^{2}l=2.8\times10^{-7}\,\text{m}^{3}$$

$$m=(0.98)\left(1000\ \text{kg/m}^3\right)\left(2.8\times10^{-7}\,\text{m}^3\right)=3\times10^{-4}\,\text{kg}=0.3\ \text{g}$$

1.62: a) $$\rho = \frac{M}{V}, \text{so } V = \frac{M}{\rho}$$

$$x^3 = \frac{0.200\ \text{kg}}{7.86 \times 10^3\ \text{kg/m}^3} = 2.54 \times 10^{-5}\ \text{m}^3$$

$$x = 2.94 \times 10^{-2}\ \text{m} = 2.94\ \text{cm}$$

b) $$\frac{4}{3}\pi R^3 = 2.54 \times 10^{-5}\ \text{m}^3$$

$$R = 1.82 \times 10^{-2}\ \text{m} = 1.82\ \text{cm}$$

1.63: Assume each person sees the dentist twice a year for checkups, for 2 hours. Assume 2 more hours for restorative work. Assuming most dentists work less than 2000 hours per year, this gives $2000\ \text{hours}/4\ \text{hours per patient} = 500\ \text{patients per dentist}$. Assuming only half of the people who should go to a dentist do, there should be about 1 dentist per 1000 inhabitants. Note: A dental assistant in an office with more than one treatment room could increase the number of patients seen in a single dental office.

1.64: a) $$(6.0 \times 10^{24}\ \text{kg}) \times \left(\frac{6.0 \times 10^{23}\ \frac{\text{atoms}}{\text{mole}}}{14 \times 10^{-3}\ \frac{\text{kg}}{\text{mole}}} \right) = 2.6 \times 10^{50}\ \text{atoms.}$$

b) The number of neutrons is the mass of the neutron star divided by the mass of a neutron:

$$\frac{(2)(2.0 \times 10^{30}\ \text{kg})}{(1.7 \times 10^{-27}\ \text{kg/neutron})} = 2.4 \times 10^{57}\ \text{neutrons.}$$

c) The average mass of a particle is essentially $\frac{2}{3}$ the mass of either the proton or the neutron, 1.7×10^{-27} kg. The total number of particles is the total mass divided by this average, and the total mass is the volume times the average density. Denoting the density by ρ (the notation introduced in Chapter 14).

$$\frac{M}{m_{\text{ave}}} = \frac{\frac{4}{3}\pi R^3 \rho}{\frac{2}{3} m_{\text{p}}} = \frac{(2\pi)(1.5 \times 10^{11}\ \text{m})^3\ (10^{18}\ \text{kg/m}^3)}{(1.7 \times 10^{-27}\ \text{kg})} = 1.2 \times 10^{79}.$$

Note the conversion from g/cm^3 to kg/m^3.

1.65: Let $\vec{D}$ be the fourth force.

$$\vec{A}+\vec{B}+\vec{C}+\vec{D}=0, \text{so } \vec{D}=-\left(\vec{A}+\vec{B}+\vec{C}\right)$$

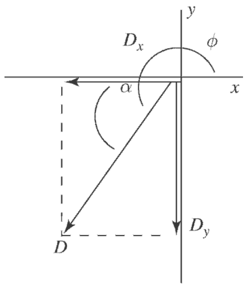
$$A_x=+A\cos 30.0^\circ=+86.6\,\text{N}, \quad A_y=+A\cos 30.0^\circ=+50.00\,\text{N}$$

$$B_x=-B\sin 30.0^\circ=-40.00\,\text{N}, \; B_y=+B\cos 30.0^\circ=+69.28\,\text{N}$$

$$C_x=+C\cos 53.0^\circ=-24.07\,\text{N}, \; C_y=-C\sin 53.0^\circ=-31.90\,\text{N}$$

Then $D_x=-22.53$ N, $\qquad D_y=-87.34\,\text{N}$

$$D=\sqrt{D_x^2+D_y^2}=90.2\ \text{N};$$

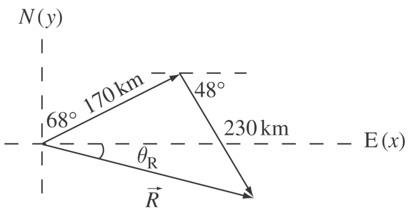


$$\tan\alpha=\left|D_y/D_x\right|=87.34/22.53$$

$$\alpha=75.54^\circ$$

$$\varphi=180^\circ+\alpha=256^\circ, \text{counterclockwise from} +x\text{-axis}$$

1.66:



$$R_x=A_x+B_x=(170\,\text{km})\sin 68^\circ+(230\,\text{km})\cos 48^\circ=311.5\,\text{km}$$

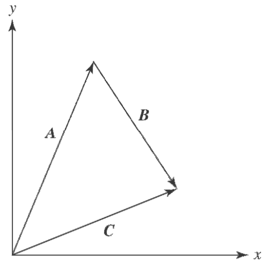
$$R_y=A_y+B_y=(170\,\text{km})\cos 68^\circ-(230\,\text{km})\sin 48^\circ=-107.2\,\text{km}$$

$$R=\sqrt{R_x^2+R_y^2}=\sqrt{(311.5\,\text{km})^2+(-107.2\,\text{km})^2}=330\,\text{km}$$

$$\tan\theta_R=\left|\frac{R_y}{R_x}\right|=\frac{107.2\,\text{km}}{311.5\,\text{km}}=0.344$$

$\theta_R=19^\circ$ south of east

1.67: a)



b) Algebraically, $\vec{A} = \vec{C} - \vec{B}$, and so the components of $\vec{A}$ are

$$A_x = C_x - B_x = (6.40\text{ cm})\cos 22.0^\circ - (6.40\text{ cm})\cos 63.0^\circ = 3.03\text{ cm}$$

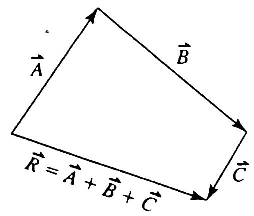
$$A_y = C_y - B_y = (6.40\text{ cm})\sin 22.0^\circ + (6.40\text{ cm})\sin 63.0^\circ = 8.10\text{ cm}.$$

c) $A = \sqrt{(3.03\text{ cm})^2 + (8.10\text{ cm})^2} = 8.65\text{ cm}, \quad \arctan\left(\dfrac{8.10\text{ cm}}{3.03\text{ cm}}\right) = 69.5^\circ$

1.68: a) $R_x = A_x + B_x + C_x$

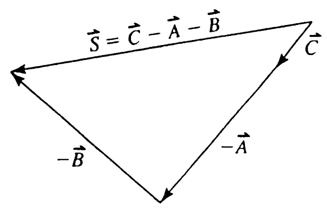
$$= (12.0\text{ m})\cos(90^\circ - 37^\circ) + (15.00\text{ m})\cos(-40^\circ) + (6.0\text{ m})\cos(180^\circ + 60^\circ)$$
$$= 15.7\text{ m, and}$$
$$R_y = A_y + B_y + C_y$$
$$= (12.0\text{ m})\sin(90^\circ - 37^\circ) + (15.00\text{ m})\sin(-40^\circ) + (6.0\text{ m})\sin(180^\circ + 60^\circ)$$
$$= -5.3\text{ m.}$$

The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2} = 16.6\text{ m}$, and the direction from the positive x-axis is arctan $\left(\frac{-5.3}{15.7}\right) = -18.6^\circ$. Keeping extra significant figures in the intermediate calculations gives an angle of $-18.49°$, which when considered as a positive counterclockwise angle from the positive x-axis and rounded to the nearest degree is 342°.

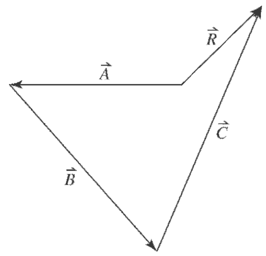


b) $S_x = -3.00\text{ m} - 7.22\text{ m} - 11.49\text{ m} = -21.71\text{ m};$

$$S_y = -5.20\text{ m} - (-9.64\text{ m}) - 9.58\text{ m} = -5.14\text{ m};$$
$$\theta = \arctan\left[\frac{(-5.14)}{(-21.71)}\right] = 13.3^\circ$$
$$S = \sqrt{(-21.71\text{ m})^2 + (-5.14\text{ m})^2} = 22.3\text{ m}$$



1.69:



Take the east direction to be the x - direction and the north direction to be the y - direction. The x- and y-components of the resultant displacement of the first three displacements are then

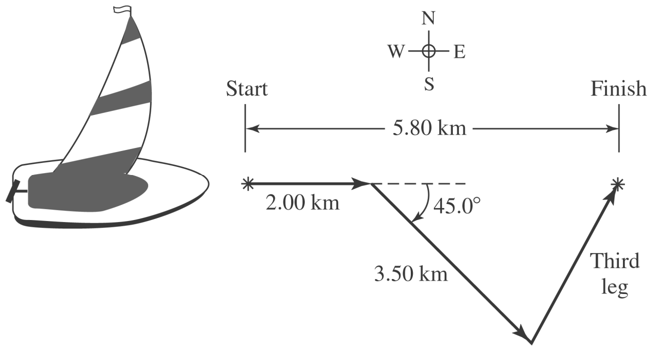
$$(-180\text{ m})+(210\text{ m})\sin 45^\circ+(280\text{ m})\sin 30^\circ=108\text{ m},$$
$$-(210\text{ m})\cos 45^\circ+(280\text{ m})\cos 30^\circ=+94.0\text{ m},$$

keeping an extra significant figure. The magnitude and direction of this net displacement are

$$\sqrt{(108\text{ m})^2+(94.0\text{ m})^2}=144\text{ m},\quad \arctan\left(\frac{94\text{ m}}{108\text{ m}}\right)=40.9^\circ.$$

The fourth displacement must then be 144 m in a direction 40.9° south of west.

1.70:



The third leg must have taken the sailor east a distance

$$(5.80\text{ km}) - (3.50\text{ km})\cos 45^\circ - (2.00\text{ km}) = 1.33\text{ km}$$

and a distance north

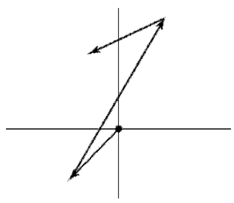
$$(3.5\text{ km})\sin 45^\circ = (2.47\text{ km})$$

The magnitude of the displacement is

$$\sqrt{(1.33\text{ km})^2 + (2.47\text{ km})^2} = 2.81\text{ km}$$

and the direction is arctan $\left(\frac{2.47}{1.33}\right) = 62°$ north of east, which is $90° - 62° = 28°$ east of north. A more precise answer will require retaining extra significant figures in the intermediate calculations.

1.71: a)



b) The net east displacement is $-(2.80\text{ km})\sin 45^\circ + (7.40\text{ km})\cos 30^\circ - (3.30\text{ km})\cos 22^\circ = 1.37\text{ km}$, and the net north displacement is $-(2.80\text{ km})\cos 45^\circ + (7.40\text{ km})\sin 30^\circ - (3.30\text{ km})\sin 22.0^\circ = 0.48\text{ km}$, and so the distance traveled is $\sqrt{(1.37\text{ km})^2 + (0.48\text{ km})^2} = 1.45\text{ km}$.

1.72: The eastward displacement of Manhattan from Lincoln is

$$(147\text{ km})\sin 85^\circ + (106\text{ km})\sin 167^\circ + (166\text{ km})\sin 235^\circ = 34.3\text{ km}$$

and the northward displacement is

$$(147\text{ km})\cos 85^\circ + (106\text{ km})\cos 167^\circ + (166\text{ km})\cos 235^\circ = -185.7\text{ km}$$

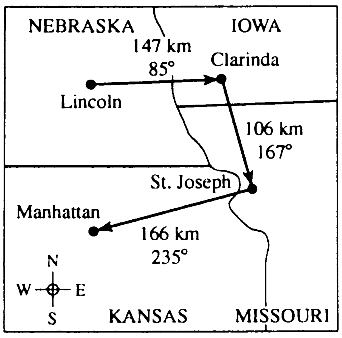
(A negative northward displacement is a southward displacement, as indicated in Fig. (1.33). Extra figures have been kept in the intermediate calculations.)

a) $$\sqrt{(34.3\text{ km})^2 + (185.7\text{ km})^2} = 189\text{ km}$$

b) The direction from Lincoln to Manhattan, relative to the north, is

$$\arctan\left(\frac{34.3\text{ km}}{-185.7\text{ km}}\right) = 169.5^\circ$$

and so the direction to fly in order to return to Lincoln is $169.5^\circ + 180^\circ + 349.5^\circ$.

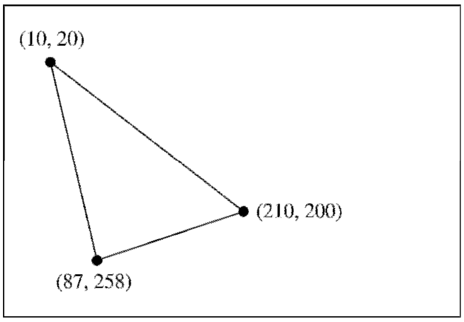


1.73: a) Angle of first line is $\theta = \tan^{-1}\left(\frac{200-20}{210-10}\right) = 42^\circ$. Angle of second line is $42^\circ + 30^\circ = 72^\circ$. Therefore

$$X = 10 + 250\cos 72^\circ = 87$$
$$Y = 20 + 250\sin 72^\circ = 258$$

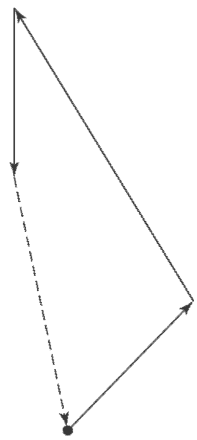
for a final point of (87,258).

b) The computer screen now looks something like this:



The length of the bottom line is $\sqrt{(210-87)^2 + (200-258)^2} = 136$ and its direction is $\tan^{-1}\left(\frac{258-200}{210-87}\right) = 25^\circ$ below straight left.

1.74: a)



b) To use the method of components, let the east direction be the x-direction and the north direction be the y-direction. Then, the explorer's net x-displacement is, in units of his step size,

$$(40)\cos 45^\circ - (80)\cos 60^\circ = -11.7$$

and the y-displacement is

$$(40)\sin 45^\circ + (80)\sin 60^\circ - 50 = 47.6.$$

The magnitude and direction of the displacement are

$$\sqrt{(-11.7)^2 + (47.6)^2} = 49, \quad \arctan\left(\frac{47.6}{-11.7}\right) = 104^\circ.$$

(More precision in the angle is not warranted, as the given measurements are to the nearest degree.) To return to the hut, the explorer must take 49 steps in a direction $104^\circ - 90^\circ = 14^\circ$ east of south.

1.75: Let $+x$ be east and $+y$ be north. Let $\vec{A}$ be the displacement 285 km at 40.0° north of west and let $\vec{B}$ be the unknown displacement.

$\vec{A} + \vec{B} = \vec{R}$, where $\vec{R} = 115\,\text{km}$, east

$\vec{B} = \vec{R} - \vec{A}$

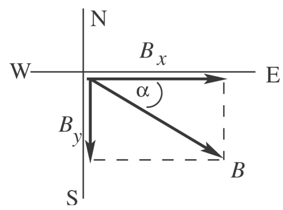
$B_x = R_x - A_x,\ B_y = R_y - A_y$

$A_x = -A\cos 40.0^\circ = -218.3\,\text{km},\ A_y = +A\sin 40.0^\circ = +183.2\,\text{km}$

$R_x = 115\,\text{km},\ R_y = 0$

Then $B_x = 333.3\,\text{km},\ B_y = -183.2\,\text{km}.$

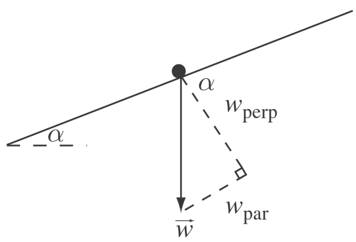
$B = \sqrt{B_x^2 + B_y^2} = 380\,\text{km}.$



$\tan\alpha = \left| B_y / B_x \right| = (183.2\,\text{km})/(333.3\,\text{km})$

$\alpha = 28.8^\circ$, south of east

1.76:



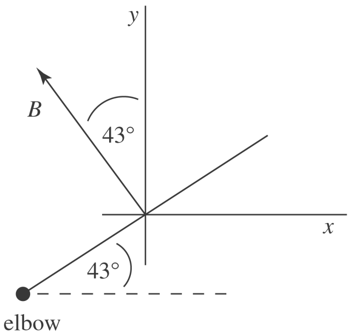
(a) $\omega_{\text{par}} = \omega \sin\alpha$

(b) $\omega_{\text{perp}} = \omega \cos\alpha$

(c) $\omega_{\text{par}} = \omega \sin\alpha$

$$\omega = \frac{\omega_{\text{par}}}{\sin\alpha} = \frac{550\ \text{N}}{\sin 35.0^0} = 960\ \text{N}$$

1.77:



$\vec{B}$ is the force the biceps exerts.

$\vec{E}$ is the force the elbow exerts.

$\vec{E} + \vec{B} = \vec{R}$, where $R = 132.5$ N and is upward.

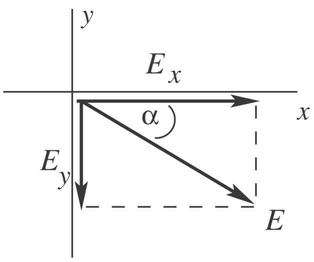
$E_x = R_x - B_x, \quad E_y = R_y - B_y$

$B_x = -B \sin 43^\circ = -158.2$ N, $B_y = +B \cos 43^\circ = +169.7$N

$R_x = 0, R_y = +132.5$N

Then $E_x = +158.2$N, $E_y = -37.2$N

$E = \sqrt{E_x^2 + E_y^2} = 160$N;



$\tan\alpha = |E_y / E_x| = 37.2/158.2$

$\alpha = 13^\circ$, below horizontal

1.78: (a) Take the beginning of the journey as the origin, with north being the y-direction, east the x-direction, and the z-axis vertical. The first displacement is then $-30\hat{\boldsymbol{k}}$, the second is $-15\hat{\boldsymbol{j}}$, the third is $200\hat{\boldsymbol{i}}$ $(0.2\text{ km} = 200\text{ m})$, and the fourth is $100\hat{\boldsymbol{j}}$. Adding the four:

$$-30\hat{\boldsymbol{k}} - 15\hat{\boldsymbol{j}} + 200\hat{\boldsymbol{i}} + 100\hat{\boldsymbol{j}} = 200\hat{\boldsymbol{i}} + 85\hat{\boldsymbol{j}} - 30\hat{\boldsymbol{k}}$$

(b) The total distance traveled is the sum of the distances of the individual segments: 30 + 15 + 200 + 100 = 345 m. The magnitude of the total displacement is:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{200^2 + 85^2 + (-30)^2} = 219\text{ m}$$

1.79: Let the displacement from your camp to the store be $\vec{\boldsymbol{A}}$.

$A = 240\text{ m}, 32^\circ$ south of east

$\vec{\boldsymbol{B}}$ is 32° south of west and $\vec{\boldsymbol{C}}$ is 62° south of west

Let $+x$ be east and $+y$ be north

$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = \boldsymbol{0}$

$A_x + B_x + C_x = 0$, so $A\cos 32^\circ - B\cos 48^\circ - C\cos 62^\circ = 0$

$A_y + B_y + C_y = 0$, so $-A\sin 32^\circ + B\sin 48^\circ - C\sin 62^\circ = 0$

A is known so we have two equations in the two unknowns B and C. Solving gives B = 255 m and C = 70 m.

1.80: Take your tent's position as the origin. The displacement vector for Joe's tent is $(21\cos 23^\circ)\hat{\boldsymbol{i}} - (21\sin 23^\circ)\hat{\boldsymbol{j}} = 19.33\hat{\boldsymbol{i}} - 8.205\hat{\boldsymbol{j}}$. The displacement vector for Karl's tent is $(32\cos 37^\circ)\hat{\boldsymbol{i}} + (32\sin 37^\circ)\hat{\boldsymbol{j}} = 25.56\hat{\boldsymbol{i}} + 19.26\hat{\boldsymbol{j}}$. The difference between the two displacements is:

$$(19.33 - 25.56)\hat{\boldsymbol{i}} + (-8.205 - 19.25)\hat{\boldsymbol{j}} = -6.23\hat{\boldsymbol{i}} - 27.46\hat{\boldsymbol{j}}.$$

The magnitude of this vector is the distance between the two tents:

$$D = \sqrt{(-6.23)^2 + (-27.46)^2} = 28.2\text{ m}$$

1.81: a) With $A_z = B_z = 0$, Eq. (1.22) becomes

$$\begin{aligned} A_x B_x + A_y B_y &= (A\cos\theta_A)(\mathrm{B}\cos\theta_B) + (A\sin\theta_A)(B\sin\theta_B) \\ &= AB(\cos\theta_A\cos\theta_B + \sin\theta_A\sin\theta_B) \\ &= AB\cos(\theta_A - \theta_B) \\ &= AB\cos\phi \end{aligned}$$

where the expression for the cosine of the difference between two angles has been used (see Appendix B).

b) With $A_z = B_z = 0$, $\vec{C} = C_z\hat{k}$ and $C = |C_z|$. From Eq. (1.27),

$$\begin{aligned} |C| &= |A_x B_x - A_y B_x| \\ &= |(A\cos\theta_A)(B\cos\theta_B) - (A\sin\theta_A)(B\cos\theta_A)| \\ &= AB|\cos\theta_A\sin\theta_B - \sin\theta_A\cos\theta_B| \\ &= AB|\sin(\theta_B - \theta_A)| \\ &= AB\sin\phi \end{aligned}$$

2: a) The angle between the vectors is $210^\circ - 70^\circ = 140^\circ$, and so Eq. (1.18) gives $\vec{A}\cdot\vec{B} = (3.60\ \mathrm{m})(2.40\ \mathrm{m})\cos 140^\circ = -6.62\ \mathrm{m}^2$ Or, Eq. (1.21) gives

$$\begin{aligned} \vec{A}\cdot\vec{B} &= A_x B_x + A_y B_y \\ &= (3.60\ \mathrm{m})\cos 70^\circ (2.4\ \mathrm{m})\cos 210^\circ + (3.6\ \mathrm{m})\sin 70^\circ (2.4\ \mathrm{m})\sin 210^\circ \\ &= -6.62\ \mathrm{m}^2 \end{aligned}$$

b) From Eq. (1.22), the magnitude of the cross product is

$$(3.60\ \mathrm{m})(2.40\ \mathrm{m})\sin 140^\circ = 5.55\ \mathrm{m}^2,$$

and the direction, from the right-hand rule, is out of the page (the $+z$-direction). From Eq. (1-30), with the z-components of $\vec{A}$ and $\vec{B}$ vanishing, the z-component of the cross product is

$$\begin{aligned} A_x B_y - A_y B_x &= (3.60\ \mathrm{m})\cos 70^\circ (2.40\ \mathrm{m})\sin 210^\circ \\ &\quad - (3.60\ \mathrm{m})\sin 70^\circ (2.40\ \mathrm{m})\cos 210^\circ \\ &= 5.55\ \mathrm{m}^2 \end{aligned}$$

1.83: a) Parallelogram area $= 2 \times$ area of triangle ABC

Triangle area $= 1/2\left(\text{base}\right)\left(\text{height}\right) = 1/2\left(\text{B}\right)\left(\text{A} \sin\theta\right)$

Parellogram area $= BA\sin\theta$

b) 90°

1.84: With the $+x$-axis to the right, $+y$-axis toward the top of the page, and $+z$-axis out of the page, $\left(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right)_x = 87.8\ \text{cm}^2$, $\left(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right)_y = 68.9\ \text{cm}^2$, $\left(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}\right)_z = 0$.

1.85: a)

$$A = \sqrt{(2.00)^2 + (3.00)^2 + (4.00)^2} = 5.39.$$
$$B = \sqrt{(3.00)^2 + (1.00)^2 + (3.00)^2} = 4.36.$$

b)

$$\begin{aligned}\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} &= (A_x - B_x)\hat{\boldsymbol{i}} + (A_y - B_y)\hat{\boldsymbol{j}} + (A_z - B_z)\hat{\boldsymbol{k}} \\ &= (-5.00)\hat{\boldsymbol{i}} + (2.00)\hat{\boldsymbol{j}} + (7.00)\hat{\boldsymbol{k}}\end{aligned}$$

c) $\sqrt{(5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83,$

and this will be the magnitude of $\vec{\boldsymbol{B}} - \vec{\boldsymbol{A}}$ as well.

1.86: The direction vectors each have magnitude $\sqrt{3}$, and their dot product is (1) (1) + (1) (–1) + (1) (–1) = –1, so from Eq. (1-18) the angle between the bonds is arccos $= \left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) =$ arccos $\left(-\frac{1}{3}\right) = 109^\circ$.

1.87: The best way to show these results is to use the result of part (a) of Problem 1-65, a restatement of the law of cosines. We know that

$$C^2 = A^2 + B^2 + 2AB\cos\phi,$$

where ϕ is the angle between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

a) If $C^2 = A^2 + B^2$, $\cos\phi = 0$, and the angle between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is 90° (the vectors are perpendicular).

b) If $C^2 < A^2 + B^2$, $\cos\phi < 0$, and the angle between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is greater than 90°.

c) If $C^2 > A^2 + B^2$, $\cos\phi > 0$, and the angle between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is less than 90°.

1.88: a) This is a statement of the law of cosines, and there are many ways to derive it. The most straightforward way, using vector algebra, is to assume the linearity of the dot product (a point used, but not explicitly mentioned in the text) to show that the square of the magnitude of the sum $\vec{A} + \vec{B}$ is

$$\begin{aligned}
\left(\vec{A} + \vec{B}\right)\cdot\left(\vec{A} + \vec{B}\right) &= \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B} \\
&= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + 2\,\vec{A}\cdot\vec{B} \\
&= A^2 + B^2 + 2\vec{A}\cdot\vec{B} \\
&= A^2 + B^2 + 2AB\cos\phi
\end{aligned}$$

Using components, if the vectors make angles θ_A and θ_B with the x-axis, the components of the vector sum are $A\cos\theta_A + B\cos\theta_B$ and $A\sin\theta_A + B\sin\theta_B$, and the square of the magnitude is

$$\begin{aligned}
&(A\cos\theta_A + B\cos\theta_B)^2 + (A\sin\theta_A + B\sin\theta_B)^2 \\
&\quad = A^2\left(\cos^2\theta_A + \sin^2\theta_A\right) + B^2\left(\cos^2\theta_B + \sin^2\theta_B\right) \\
&\qquad\qquad + 2AB\left(\cos\theta_A\cos\theta_B + \sin\theta_A\sin\theta_B\right) \\
&\quad = A^2 + B^2 + 2AB\cos(\theta_A - \theta_B) \\
&\quad = A^2 + B^2 + 2AB\cos\phi
\end{aligned}$$

where $\phi = \theta_A - \theta_B$ is the angle between the vectors.

b) A geometric consideration shows that the vectors $\vec{A}$, $\vec{B}$ and the sum $\vec{A}$ + l must be the sides of an equilateral triangle. The angle *between* $\vec{A}$, and $\vec{B}$ i 120°, since one vector must shift to add head-to-tail. Using the result of pa (a), with $A = B$, the condition is that $A^2 = A^2 + A^2 + 2A^2\cos\phi$, which solv for $1 = 2 + 2\cos\phi$, $\cos\phi = -\frac{1}{2}$, and $\phi = 120°$.

c) Either method of derivation will have the angle ϕ replaced by $180° - \phi$, so the cosine will change sign, and the result is $\sqrt{A^2 + B^2 - 2AB\cos\phi}$.

d) Similar to what is done in part (b), when the vector *difference* has the same magnitude, the angle between the vectors is 60°. Algebraically, ϕ is obtain from $1 = 2 - 2\cos\phi$, so $\cos\phi = \frac{1}{2}$ and $\phi = 60°$.

1.89: Take the length of a side of the cube to be L, and denote the vectors from a to b, a to c and a to d as $\vec{\boldsymbol{B}}, \vec{\boldsymbol{C}}$, and $\vec{\boldsymbol{D}}$. In terms of unit vectors,

$$\vec{\boldsymbol{B}} = L\hat{\boldsymbol{k}}, \qquad \vec{\boldsymbol{C}} = L\left(\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right), \qquad \vec{\boldsymbol{D}} = L\left(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right)$$

Using Eq. (1.18),

$$\arccos\left(\frac{\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{D}}}{BD}\right) = \arccos\left(\frac{L^2}{(L)\left(L\sqrt{3}\right)}\right) = 54.7^\circ,$$

$$\arccos\left(\frac{\vec{\boldsymbol{C}} \cdot \vec{\boldsymbol{D}}}{CD}\right) = \arccos\left(\frac{2L^2}{\left(L\sqrt{2}\right)\left(L\sqrt{3}\right)}\right) = 35.3^\circ.$$

1.90: From Eq. (1.27), the cross product is

$$(-13.00)\ \hat{\boldsymbol{i}} + (6.00)\ \hat{\boldsymbol{j}} + (-11.00)\ \hat{\boldsymbol{k}} = 13\left[-(1.00)\ \hat{\boldsymbol{i}} + \left(\frac{6.00}{13.00}\right)\hat{\boldsymbol{j}} - \frac{11.00}{13.00}\hat{\boldsymbol{k}}\right].$$

The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction (which is necessarily perpendicular to both $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$) is

$$\left[\frac{-(1.00)\hat{\boldsymbol{i}} + (6.00/13.00)\hat{\boldsymbol{j}} - (11.00/13)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{\boldsymbol{i}} - (6.00/13.00)\hat{\boldsymbol{j}} + (11.00/13)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

1.91: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C}=0$. $A_xC_x+A_yC_y=0$, , which gives $5.0C_x-6.5C_y=0$.

$\vec{B}\cdot\vec{C}=15.0$, so $-3.5C_x+7.0C_y=15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x=8.0$ and $C_y=6.1$.

1.92:

$$\left|\vec{A}\times\vec{B}\right|=AB\sin\theta$$

$$\sin\theta=\frac{\left|\vec{A}\times\vec{B}\right|}{AB}=\frac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)}=0.5984$$

$$\theta=\sin^{-1}(0.5984)=36.8^\circ$$

1.93: a) Using Equations (1.21) and (1.27), and recognizing that the vectors $\vec{A},\vec{B},$and $\vec{C}$ do not have the same meanings as they do in those equations,

$$\begin{aligned}\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}&=\left((A_yB_z-A_zB_y)\hat{\boldsymbol{i}}+(A_zB_x-A_xB_z)\hat{\boldsymbol{j}}+(A_xB_y-A_yB_x)\hat{\boldsymbol{k}}\right)\cdot\vec{C}\\&=A_yB_zC_x-A_zB_yC_x+A_zB_xC_y-A_xB_zC_y+A_xB_yC_z-A_yB_xC_z.\end{aligned}$$

A similar calculation shows that

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=A_xB_yC_z-A_xB_zC_y+A_yB_zC_x-A_yB_xC_z+A_zB_xC_y-A_zB_yC_x$$

and a comparison of the expressions shows that they are the same.

b) Although the above expression could be used, the form given allows for ready compuation of $\vec{A}\times\vec{B}$ the magnitude is $AB\sin\phi=(20.00)\sin 37.0^\circ$ and the direction is, from the right-hand rule, in the +*z*-direction, and so

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}=+(20.00)\sin 37.0^\circ(6.00)=+72.2.$$

1.94: a) The maximum and minimum areas are

$$(L + l)\,(W + w) = LW + lW + Lw, \quad (L - l)\,(W - w) = LW - lW - Lw,$$

where the common terms wl have been omitted. The area and its uncertainty are then $WL \pm (lW + Lw)$, so the uncertainty in the area is $a = lW + Lw$.

b) The fractional uncertainty in the area is

$$\frac{a}{A} = \frac{lW + Wl}{WL} = \frac{l}{L} + \frac{w}{W},$$

the sum of the fractional uncertainties in the length and width.

c) The similar calculation to find the uncertainty v in the volume will involve neglecting the terms lwH, lWh and Lwh as well as lwh; the uncertainty in the volume is $v = lWH + LwH + LWh$, and the fractional uncertainty in the volume is

$$\frac{v}{V} = \frac{lWH + LwH + LWh}{LW\ H} = \frac{l}{L} + \frac{w}{W} + \frac{h}{H},$$

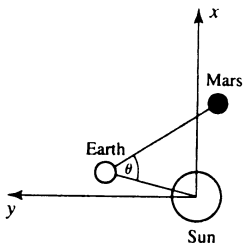
the sum of the fractional uncertainties in the length, width and height.

1.95: The receiver's position is

$$(+1.0 + 9.0 - 6.0 + 12.0)\hat{\boldsymbol{i}} + (-5.0 + 11.0 + 4.0 + 18.0)\hat{\boldsymbol{j}} = (16.0)\hat{\boldsymbol{i}} + (28.0)\hat{\boldsymbol{j}}.$$

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0)\hat{\boldsymbol{i}} + (35.0)\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0)^2 + (35.0)^2} = 38.5$, given as being in yards. The angle is $\arctan\left(\frac{16.0}{35.0}\right) = 24.6^\circ$ to the right of downfield.

1.96: a)



b) i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857.$

ii) In AU, $\sqrt{(1.3087)^2 + (-.4423)^2 + (-.0414)^2} = 1.3820$

iii) In AU,

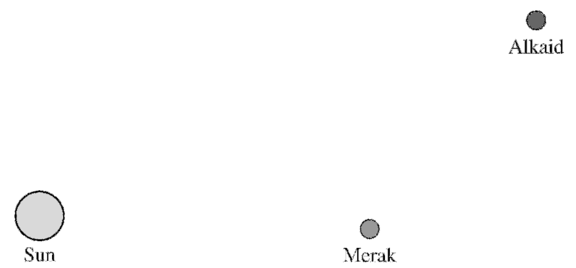
$$\sqrt{(0.3182 - (1.3087))^2 + (0.9329 - (-.4423))^2 + (0.0414)^2} = 1.695.$$

c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Equations (1-18) and (1.21),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329) \cdot}{(0.9857)(1.695)}\right.$$

d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

1.97: a)



The law of cosines (see Problem 1.88) gives the distance as

$$\sqrt{(138\ \text{ly})^2 + (77\ \text{ly})^2 + 2(138\ \text{ly})(77\ \text{ly})\cos 154.4^\circ} = 76.2\ \text{ly},$$

where the supplement of 25.6° has been used for the angle between the direction vectors.

b) Although the law of cosines could be used again, it's far more convenient to use the law of sines (Appendix B), and the angle is given by

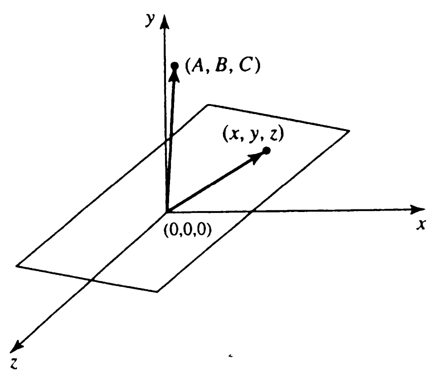
$$\arcsin\left(\frac{\sin 25.6^\circ}{76.2\ \text{ly}} 138\ \text{ly}\right) = 51.5^\circ,\ 180^\circ - 51.5^\circ = 129^\circ,$$

where the appropriate angle in the second quadrant is used.

1.98: Define $\vec{S} = A\hat{\boldsymbol{i}} + B\hat{\boldsymbol{j}} + C\hat{\boldsymbol{k}}$

$$\begin{aligned}\vec{r} \cdot \vec{S} &= (x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}}) \cdot (A\hat{\boldsymbol{i}} + B\hat{\boldsymbol{j}} + C\hat{\boldsymbol{k}}) \\ &= Ax + By + Cz\end{aligned}$$

If the points satisfy $Ax + By + Cz = 0$, then $\vec{r} \cdot \vec{S} = 0$ and all points $\vec{r}$ are perpendicular to $\vec{S}$.



Capítulo 2

2.1: a) During the later 4.75-s interval, the rocket moves a distance $1.00\times10^{3}\text{ m}-63\text{ m}$, and so the magnitude of the average velocity is

$$\frac{1.00\times10^{3}\text{ m}-63\text{ m}}{4.75\text{ s}}=197\text{ m/s}.$$

b) $\frac{1.00\times10^{3}\text{ m}}{5.90\text{ s}}=169\text{ m/s}$

2.2: a) The magnitude of the average velocity on the return flight is

$$\frac{(5150\times10^{3}\text{ m})}{(13.5\text{ da})(86,400\text{ s/da})}=4.42\text{ m/s}.$$

The direction has been defined to be the $-x$-direction $(-\hat{\boldsymbol{i}})$.

b) Because the bird ends up at the starting point, the average velocity for the round trip is **0**.

2.3: Although the distance could be found, the intermediate calculation can be avoided by considering that the time will be inversely proportional to the speed, and the extra time will be

$$(140\text{ min})\left(\frac{105\text{ km/hr}}{70\text{ km/hr}}-1\right)=70\text{ min}.$$

2.4: The eastward run takes $(200\text{ m}/5.0\text{ m/s})=40.0\text{ s}$ and the westward run takes $(280\text{ m}/4.0\text{ m/s})=70.0\text{ s}$. a) $(200\text{ m}+280\text{ m})/(40.0\text{ s}+70.0\text{ s})=4.4\text{ m/s}$ to two significant figures. b) The net displacement is 80 m west, so the average velocity is $(80\text{ m}/110.0\text{ s})=0.73\text{ m/s}$ in the $-x$-direction $(-\hat{\boldsymbol{i}})$.

2.5: In time t the fast runner has traveled 200 m farther than the slow runner:
$(5.50\text{ m/s})t+200\text{ m}=(6.20\text{ m/s})t$, so $t=286\text{ s}$.
Fast runner has run $(6.20\text{ m/s})t=1770\text{ m}$.
Slow runner has run $(5.50\text{ m/s})t=1570\text{ m}$.

2.6: The s-waves travel slower, so they arrive 33 s after the p-waves.

$$t_{\mathrm{s}} = t_{\mathrm{p}} + 33\,\mathrm{s}$$

$$d = vt \rightarrow t = \frac{d}{v}$$

$$\frac{d}{v_{\mathrm{s}}} = \frac{d}{v_{\mathrm{p}}} + 33\,\mathrm{s}$$

$$\frac{d}{3.5\,\frac{\mathrm{km}}{\mathrm{s}}} = \frac{d}{6.5\,\frac{\mathrm{km}}{\mathrm{s}}} + 33\,\mathrm{s}$$

$$d = 250\,\mathrm{km}$$

2.7: a) The van will travel 480 m for the first 60 s and 1200 m for the next 60 s, for a total distance of 1680 m in 120 s and an average speed of $14.0\ \mathrm{m/s}$. b) The first stage of the journey takes $\frac{240\ \mathrm{m}}{8.0\ \mathrm{m/s}} = 30\ \mathrm{s}$ and the second stage of the journey takes $(240\ \mathrm{m}/20\ \ \mathrm{m/s}) = 12\,\mathrm{s}$, so the time for the 480-m trip is 42 s, for an average speed of $11.4\ \mathrm{m/s}$. c) The first case (part (a)); the average speed will be the numerical average only if the time intervals are the same.

2.8: From the expression for $x(t)$, $x(0) = 0$, $x(2.00\ \mathrm{s}) = 5.60$ m and $x(4.00\ \mathrm{s}) = 20.8$ m. a) $\frac{5.60\ \mathrm{m}-0}{2.00\ \mathrm{s}} = 2.80\ \mathrm{m/s}$ b) $\frac{20.8\ \mathrm{m}-0}{4.00\ \mathrm{s}} = 5.2\ \mathrm{m/s}$ c) $\frac{20.8\ \mathrm{m}-5.60\ \mathrm{m}}{2.00\ \mathrm{s}} = 7.6\ \mathrm{m/s}$

2.9: a) At $t_1 = 0,\ \ x_1 = 0$, so Eq (2.2) gives

$$v_{\mathrm{av}} = \frac{x_2}{t_2} = \frac{(2.4\ \mathrm{m/s^2})(10.0\,\mathrm{s})^2 - (0.120\ \mathrm{m/s^3})(10.0\,\mathrm{s})^3}{(10.0\,\mathrm{s})} = 12.0\ \mathrm{m/s}.$$

b) From Eq. (2.3), the instantaneous velocity as a function of time is

$$v_x = 2bt - 3ct^2 = (4.80\ \mathrm{m/s^2})t - (0.360\ \mathrm{m/s^3})t^2,$$

so i) $v_x(0) = 0$,

ii) $v_x(5.0\,\mathrm{s}) = (4.80\ \mathrm{m/s^2})(5.0\,\mathrm{s}) - (0.360\ \mathrm{m/s^3})(5.0\,\mathrm{s})^2 = 15.0\ \mathrm{m/s}$,

and iii) $v_x(10.0\,\mathrm{s}) = (4.80\ \mathrm{m/s^2})(10.0\,\mathrm{s}) - (0.360\ \mathrm{m/s^3})(10.0\,\mathrm{s})^2 = 12.0\,\mathrm{m/s}$.

c) The car is at rest when $v_x = 0$. Therefore $(4.80\ \mathrm{m/s^2})t - (0.360\ \mathrm{m/s^3})t^2 = 0$. The only time after $t = 0$ when the car is at rest is $t = \frac{4.80\ \mathrm{m/s^2}}{0.360\ \mathrm{m/s^3}} = 13.3\,\mathrm{s}$

2.10: a) IV: The curve is horizontal; this corresponds to the time when she stops. b) I: This is the time when the curve is most nearly straight and tilted upward (indicating postive velocity). c) V: Here the curve is plainly straight, tilted downward (negative velocity). d) II: The curve has a postive slope that is increasing. e) III: The curve is still tilted upward (positive slope and positive velocity), but becoming less so.

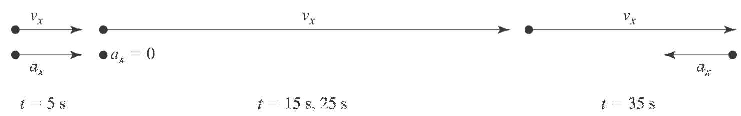
2.11:

Time (s)	0	2	4	6	8	10	12	14	16
Acceleration (m/s^2)	0	1	2	2	3	1.5	1.5	0	

a) The acceleration is not constant, but is approximately constant between the times $t = 4\,\text{s}$ and $t = 8\,\text{s}$.

2.12: The cruising speed of the car is 60 km/hr = 16.7 m/s. a) $\frac{16.7\,\text{m/s}}{10\,\text{s}} = 1.7\,\text{m/s}^2$ (to two significant figures). b) $\frac{0-16.7\,\text{m/s}}{10\,\text{s}} = -1.7\,\text{m/s}^2$ c) No change in speed, so the acceleration is zero. d) The final speed is the same as the initial speed, so the average acceleration is zero.

2.13: a) The plot of the velocity seems to be the most curved upward near t = 5 s. b) The only negative acceleration (downward-sloping part of the plot) is between t = 30 s and t = 40 s. c) At t = 20 s, the plot is level, and in Exercise 2.12 the car is said to be cruising at constant speed, and so the acceleration is zero. d) The plot is very nearly a straight line, and the acceleration is that found in part (b) of Exercise 2.12, $-1.7\,\text{m/s}^2$.
e)



2.14: (a) The displacement vector is:

$$\vec{r}(t) = -(5.0 \text{ m/s})t\hat{\boldsymbol{i}} + (10.0 \text{ m/s})t\hat{\boldsymbol{j}} + \left((7.0 \text{ m/s})t - (3.0 \text{ m/s}^2)t^2\right)\hat{\boldsymbol{k}}$$

The velocity vector is the time derivative of the displacement vector:

$$\frac{d\vec{r}(t)}{dt} = (-5.0 \text{ m/s})\hat{\boldsymbol{i}} + (10.0 \text{ m/s})\hat{\boldsymbol{j}} + (7.0 \text{ m/s} - 2(3.0 \text{ m/s}^2)t)\,\hat{\boldsymbol{k}}$$

and the acceleration vector is the time derivative of the velocity vector:

$$\frac{d^2\vec{r}(t)}{dt^2} = -6.0 \text{ m/s}^2\,\hat{\boldsymbol{k}}$$

At $t = 5.0$ s:

$$\begin{aligned}
\vec{r}(t) &= -(5.0 \text{ m/s})(5.0\,\text{s})\hat{\boldsymbol{i}} + (10.0 \text{ m/s})(5.0\,\text{s})\hat{\boldsymbol{j}} + \left((7.0\,\text{m/s})(5.0\,\text{s}) - (-3.0 \text{ m/s}^2)(25.0\,\text{s}^2)\right)\hat{\boldsymbol{k}} \\
&= (-25.0\,\text{m})\hat{\boldsymbol{i}} + (50.0\,\text{m})\hat{\boldsymbol{j}} - (40.0\,\text{m})\hat{\boldsymbol{k}} \\
\frac{d\,\vec{r}(t)}{dt} &= (-5.0 \text{ m/s})\hat{\boldsymbol{i}} + (10.0 \text{ m/s})\hat{\boldsymbol{j}} + ((7.0 \text{ m/s} - (6.0 \text{ m/s}^2)(5.0\,\text{s}))\,\hat{\boldsymbol{k}} \\
&= (-5.0 \text{ m/s})\hat{\boldsymbol{i}} + (10.0 \text{ m/s})\hat{\boldsymbol{j}} - (23.0 \text{ m/s})\,\hat{\boldsymbol{k}} \\
\frac{d^2\vec{r}(t)}{dt^2} &= -6.0 \text{ m/s}^2\,\hat{\boldsymbol{k}}
\end{aligned}$$

(b) The velocity in both the x- and the y-directions is constant and nonzero; thus the overall velocity can never be zero.

(c) The object's acceleration is constant, since t does not appear in the acceleration vector.

2.15: $v_x = \dfrac{dx}{dt} = 2.00\ \text{cm/s} - (0.125\ \text{cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\ \text{cm/s}^2$

a) At $t = 0, x = 50.0\,\text{cm}, v_x = 2.00\,\text{cm/s}, a_x = -0.125\,\text{cm/s}^2$.

b) Set $v_x = 0$ and solve for $t: t = 16.0\,\text{s}$.

c) Set $x = 50.0$ cm and solve for t. This gives $t = 0$ and $t = 32.0\text{s}$. The turtle returns to the starting point after 32.0 s.

d) Turtle is 10.0 cm from starting point when x = 60.0 cm or x = 40.0 cm.
Set $x = 60.0$ cm and solve for $t : t = 6.20$ s and $t = 25.8$ s.

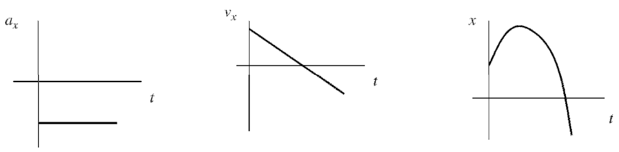
At $t = 6.20\,\text{s}, v_x = +1.23\ \text{cm/s}$.

At $t = 25.8\,\text{s}, v_x = -1.23\ \text{cm/s}$.

Set $x = 40.0\,\text{cm}$ and solve for $t : t = 36.4\,\text{s}$ (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4\,\text{s}$, $v_x = -2.55\ \text{cm/s}$.

e)



2.16: Use of Eq. (2.5), with Δt = 10 s in all cases,

a) $((5.0\ \text{m/s}) - (15.0\ \text{m/s}))/(10\ \text{s}) = -1.0\ \text{m/s}^2$

b) $((-15.0\ \text{m/s}) - (-5.0\ \text{m/s}))/(10\ \text{s}) = -1.0\ \text{m/s}^2$

c) $((-15.0\ \text{m/s}) - (-15.0\ \text{m/s}))/(10\ \text{s}) = -3.0\ \text{m/s}^2$.

In all cases, the negative acceleration indicates an acceleration to the left.

2.17: a) Assuming the car comes to rest from 65 mph (29 m/s) in 4 seconds,
$a_x = (29\ \text{m/s} - 0)/(4\ \text{s}) = 7.25\ \text{m/s}^2$.

b) Since the car is coming to a stop, the acceleration is in the direction opposite to the velocity. If the velocity is in the positive direction, the acceleration is negative; if the velocity is in the negative direction, the acceleration is positive.

2.18: a) The velocity at $t = 0$ is

$$(3.00\ \text{m/s}) + (0.100\ \text{m/s}^3)\ (0) = 3.00\ \text{m/s},$$

and the velocity at $t = 5.00$ s is

$$(3.00\ \text{m/s}) + (0.100\ \text{m/s}^3)\ (5.00\ \text{s})^2 = 5.50\ \text{m/s},$$

so Eq. (2.4) gives the average acceleration as

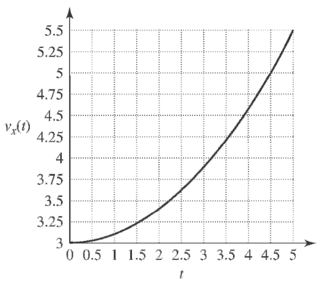
$$\frac{(5.50\ \text{m/s}) - (3.00\ \text{m/s})}{(5.00\,\text{s})} = .50\ \text{m/s}^2.$$

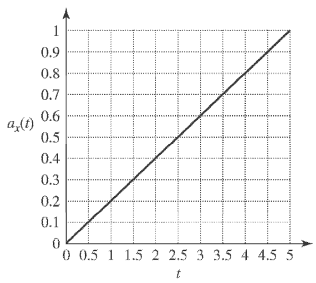
b) The instantaneous acceleration is obtained by using Eq. (2.5),

$$a_x = \frac{dv}{dt} = 2\beta t = (0.2\ \text{m/s}^3)t.$$

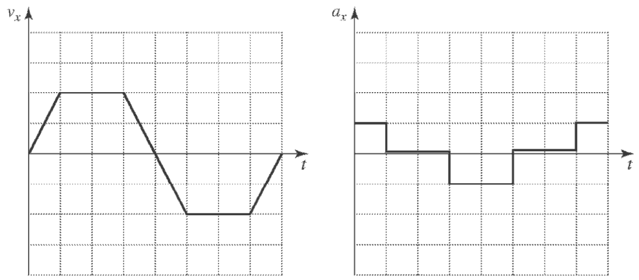
Then, i) at $t = 0$, $a_x = (0.2\ \text{m/s}^3)\ (0) = 0$, and

ii) at $t = 5.00$ s, $a_x = (0.2\ \text{m/s}^3)\ (5.00\ \text{s}) = 1.0\ \text{m/s}^2$.

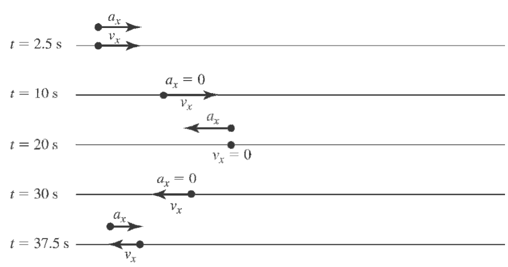




2.19: a)


v_x
t
a_x
t

b)


$t = 2.5$ s
a_x
v_x
$t = 10$ s
$a_x = 0$
v_x
$t = 20$ s
a_x
$v_x = 0$
$t = 30$ s
$a_x = 0$
v_x
$t = 37.5$ s
a_x
v_x

2.20: a) The bumper's velocity and acceleration are given as functions of time by

$$v_x = \frac{dx}{dt} = (9.60 \text{ m/s}^2)t - (0.600 \text{ m/s}^6)t^5$$

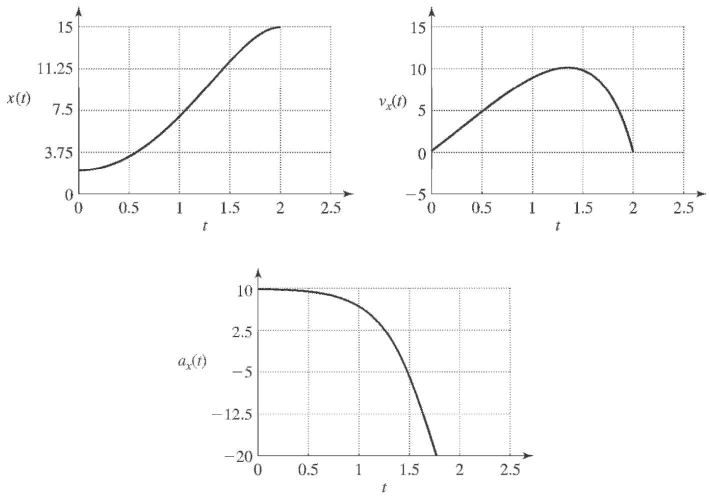
$$a_x = \frac{dv}{dt} = (9.60 \text{ m/s}^2) - (3.000 \text{ m/s}^6)t^4.$$

There are two times at which $v = 0$ (three if negative times are considered), given by $t =$ 0 and $t^4 = 16 \text{ s}^4$. At $t = 0$, $x = 2.17$ m and $a_x = 9.60 \text{ m/s}^2$. When $t^4 = 16 \text{ s}^4$,

$$x = (2.17 \text{ m}) + (4.80 \text{ m/s}^2)\sqrt{(16 \text{ s}^4)} - (0.100) \text{ m/s}^6)(16 \text{ s}^4)^{3/2} = 14.97 \text{ m},$$

$$a_x = (9.60 \text{ m/s}^2) - (3.000 \text{ m/s}^6)(16 \text{ s}^4) = -38.4 \text{ m/s}^2.$$

b)



2.21: a) Equating Equations (2.9) and (2.10) and solving for v_0,

$$v_{0x} = \frac{2(x - x_0)}{t} - v_x = \frac{2(70 \text{ m})}{7.00 \text{ s}} - 15.0 \text{ m/s} = 5.00 \text{ m/s}.$$

b) The above result for v_{0x} may be used to find

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{15.0 \text{ m/s} - 5.00 \text{ m/s}}{7.00 \text{ s}} = 1.43 \text{ m/s}^2,$$

or the intermediate calculation can be avoided by combining Eqs. (2.8) and (2.12) to eliminate v_{0x} and solving for a_x,

$$a_x = 2\left(\frac{v_x}{t} - \frac{x - x_0}{t^2}\right) = 2\left(\frac{15.0 \text{ m/s}}{7.00 \text{ s}} - \frac{70.0 \text{ m}}{(7.00 \text{ s})^2}\right) = 1.43 \text{ m/s}^2.$$

2.22: a) The acceleration is found from Eq. (2.13), which $v_{0x} = 0$;

$$a_x = \frac{v_x^2}{2(x - x_0)} = \frac{\left((173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)\right)^2}{2\left((307\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\right)} = 32.0\ \text{m/s}^2,$$

where the conversions are from Appendix E.

b) The time can be found from the above acceleration,

$$t = \frac{v_x}{a_x} = \frac{(173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)}{32.0\ \text{m/s}^2} = 2.42\ \text{s}.$$

The intermediate calculation may be avoided by using Eq. (2.14), again with $v_{0x} = 0$,

$$t = \frac{2(x - x_0)}{v_x} = \frac{2\left((307\ \text{ft}\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\right)}{(173\ \text{mi/hr})\left(\frac{0.4470\ \text{m/s}}{1\ \text{mi/hr}}\right)} = 2.42\ \text{s}.$$

2.23: From Eq. (2.13), with $v_x = 0$, $a_x = \frac{v_0^2}{2(x - x_0)} < a_{\text{max}}$. Taking $x_0 = 0$,

$$x > \frac{v_{0x}^2}{2a_{\text{max}}} = \frac{((105\ \text{km/hr})(1\ \text{m/s})(3.6\ \text{km/hr}))^2}{2(250\ \text{m/s}^2)} = 1.70\ \text{m}.$$

2.24: In Eq. (2.14), with $x - x_0$ being the length of the runway, and $v_{0x} = 0$ (the plane starts from rest), $v_x = 2\frac{x - x_0}{t} = 2\frac{280\ \text{m}}{8\ \text{s}} = 70.0\ \text{m/s}$.

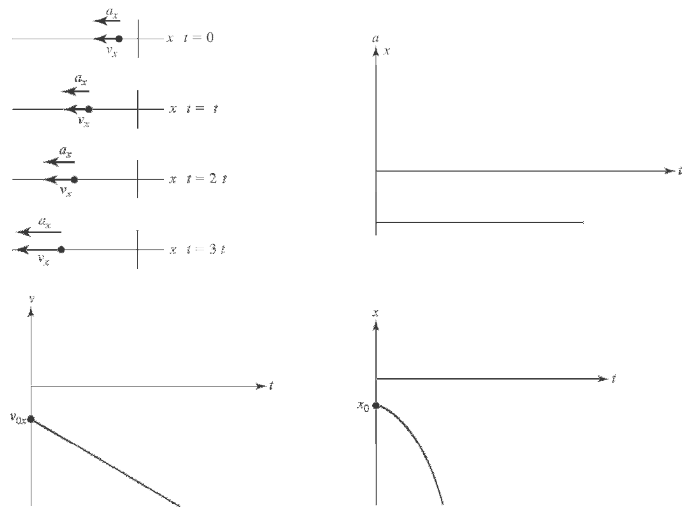
2.25: a) From Eq. (2.13), with $v_{0x} = 0$,

$$a_x = \frac{v_x^2}{2(x - x_0)} = \frac{(20\ \text{m/s})^2}{2(120\ \text{m})} = 1.67\ \text{m/s}^2.$$

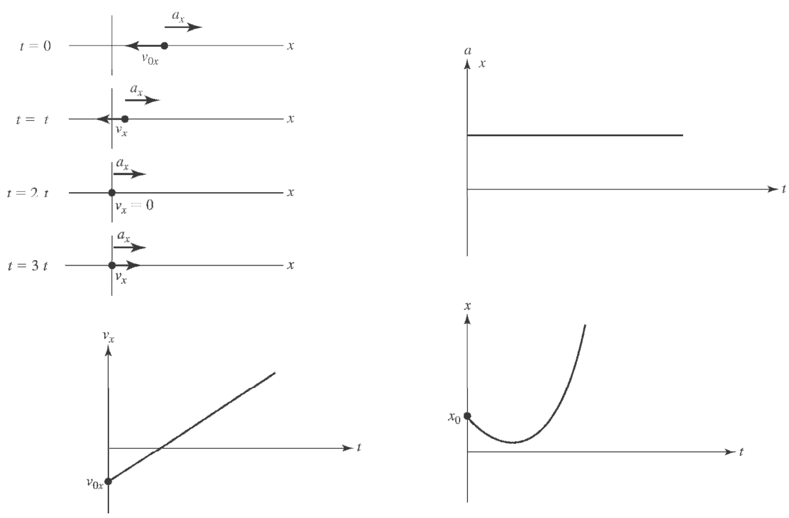
b) Using Eq. (2.14), $t = 2(x - x_0)/v = 2(120\ \text{m})/(20\ \text{m/s}) = 12\ \text{s}$.

c) $(12\ \text{s})(20\ \text{m/s}) = 240\ \text{m}$.

2.26: a) $x_0 < 0$, $v_{0x} < 0$, $a_x < 0$



b) $x_0 > 0$, $v_{0x} < 0$, $a_x > 0$



c) $x_0 > 0$, $v_{0x} > 0$, $a_x < 0$

2.27: a) speeding up:

$x - x_0 = 1320\,\text{ft}, v_{0x} = 0,\ t = 19.9\,\text{s}, a_x = ?$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = 6.67\ \text{ft/s}^2$

slowing down:

$x - x_0 = 146\,\text{ft}, v_{0x} = 88.0\ \text{ft/s},\ v_x = 0,\ a_x = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = -26.5\ \text{ft/s}^2$.

b) $x - x_0 = 1320\,\text{ft}, v_{0x} = 0, a_x = 6.67\ \text{ft/s}^2,\ v_x = ?$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 133\ \text{ft/s} = 90.5\,\text{mph}$.
a_x must not be constant.

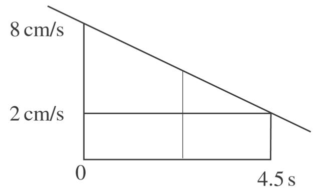
c) $v_{0x} = 88.0\ \text{ft/s},\ a_x = -26.5\ \text{ft/s}^2,\ v_x = 0,\ t = ?$
$v_x = v_{0x} + a_x t$ gives $t = 3.32\,\text{s}$.

2.28: a) Interpolating from the graph:

$$\text{At } 4.0\,\text{s},\ v = +2.7\ \text{cm/s (to the right)}$$
$$\text{At } 7.0\,\text{s},\ v = -1.3\ \text{cm/s (to the left)}$$

b) $a = \text{slope of } v\text{-}t \text{ graph} = -\frac{8.0\ \text{cm/s}}{6.0\ \text{s}} = -1.3\ \text{cm/s}^2$ which is constant

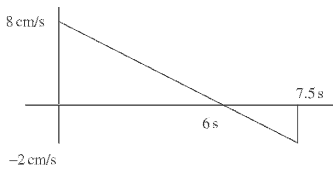
c) $\Delta x =$ area under v-t graph



First 4.5 s:

$$\Delta x = A_{\text{Rectangle}} + A_{\text{Triangle}}$$
$$= (4.5\ \text{s})\left(2\frac{\text{cm}}{\text{s}}\right) + \frac{1}{2}(4.5\ \text{s})\left(6\frac{\text{cm}}{\text{s}}\right) = 22.5\ \text{cm}$$

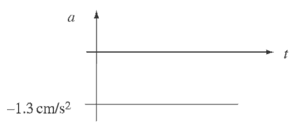
From 0 to 7.5 s:

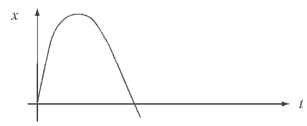


The *distance* is the sum of the magnitudes of the areas.

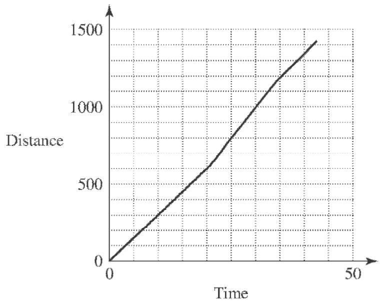
$$d = \frac{1}{2}(6\ \text{s})\left(8\frac{\text{cm}}{\text{s}}\right) + \frac{1}{2}(1.5\ \text{s})\left(2\frac{\text{cm}}{\text{s}}\right) = 25.5\ \text{cm}$$

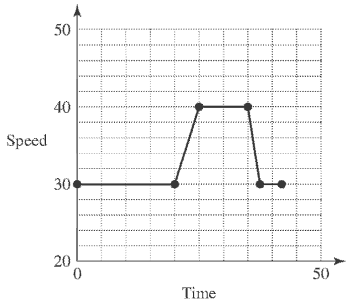
d)



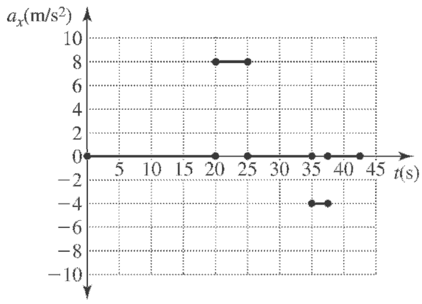


2.29: a)

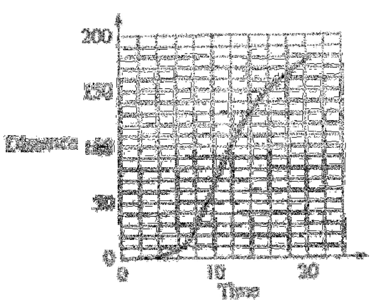

1500
1000
500
0
Distance
0
50
Time

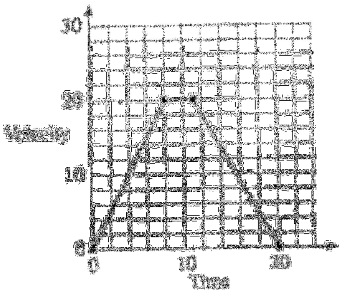

50
40
30
20
Speed
0
50
Time

b)


a_x(m/s^2)
10
8
6
4
2
0
−2
−4
−6
−8
−10
5 10 15 20 25 30 35 40 45
t(s)

2.30: a)

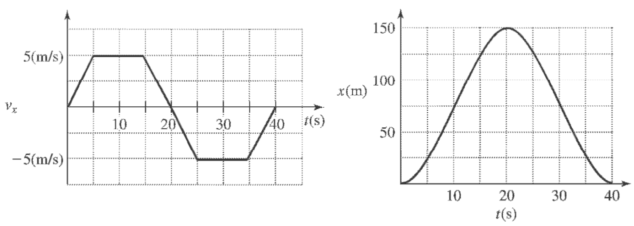

200
0
0
10
20


30
10
0
0

2.31: a) At t = 3 s the graph is horizontal and the acceleration is 0. From t = 5 s to t = 9 s, the acceleration is constant (from the graph) and equal to $\frac{45\,\text{m/s}-20\,\text{m/s}}{4\,\text{s}} = 6.3\,\text{m/s}^2$. From t = 9 s to t = 13 s the acceleration is constant and equal to $\frac{0-45\,\text{m/s}}{4\,\text{s}} = -11.2\,\text{m/s}^2$.

b) In the first five seconds, the area under the graph is the area of the rectangle, (20 m)(5 s) = 100 m. Between t = 5 s and t = 9 s, the area under the trapezoid is (1/2)(45 m/s + 20 m/s)(4 s) = 130 m (compare to Eq. (2.14)), and so the total distance in the first 9 s is 230 m. Between t = 9 s and t = 13 s, the area under the triangle is $(1/2)(45\,\text{m/s})(4\,\text{s}) = 90\,\text{m}$, and so the total distance in the first 13 s is 320 m.

2.32:



2.33: a) The maximum speed will be that after the first 10 min (or 600 s), at which time the speed will be

$$(20.0\,\text{m/s}^2)(900\,\text{s}) = 1.8\times10^4\,\text{m/s} = 18\,\text{km/s}.$$

b) During the first 15 minutes (and also during the last 15 minutes), the ship will travel $(1/2)(18\,\text{km/s})(900\,\text{s}) = 8100\,\text{km}$, so the distance traveled at non-constant speed is 16,200 km and the fraction of the distance traveled at constant speed is

$$1-\frac{16{,}200\,\text{km}}{384{,}000\,\text{km}} = 0.958,$$

keeping an extra significant figure.

c) The time spent at constant speed is $\frac{384{,}000\,\text{km}-16{,}200\,\text{km}}{18\,\text{km/s}} = 2.04\times10^4\,\text{s}$ and the time spent during both the period of acceleration and deceleration is 900 s, so the total time required for the trip is $2.22\times10^4\,\text{s}$, about 6.2 hr.

2.34: After the initial acceleration, the train has traveled

$$\frac{1}{2}(1.60\ \mathrm{m/s^2})(14.0\,\mathrm{s})^2 = 156.8\,\mathrm{m}$$

(from Eq. (2.12), with $x_0 = 0$, $v_{0x} = 0$), and has attained a speed of

$$(1.60\ \mathrm{m/s^2})(14.0\,\mathrm{s}) = 22.4\ \mathrm{m/s}.$$

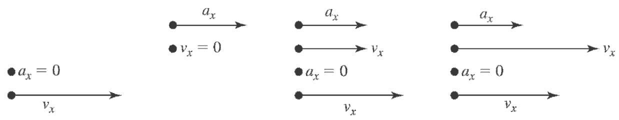
During the 70-second period when the train moves with constant speed, the train travels $(22.4\,\mathrm{m/s})(70\,\mathrm{s}) = 1568\,\mathrm{m}$. The distance traveled during deceleration is given by Eq. (2.13), with $v_x = 0, v_{0x} = 22.4\,\mathrm{m/s}$ and $a_x = -3.50\,\mathrm{m/s^2}$, so the train moves a distance $x - x_0 = \frac{-(22.4\ \mathrm{m/s})^2}{2(-3.50\ \mathrm{m/s^2})} = 71.68\,\mathrm{m}$. The total distance covered in then 156.8 m + 1568 m + 71.7 m = 1.8 km.

In terms of the initial acceleration a_1, the initial acceleration time t_1, the time t_2 during which the train moves at constant speed and the magnitude a_2 of the final acceleration, the total distance x_T is given by

$$x_\mathrm{T} = \frac{1}{2}a_1 t_1^2 + (a_1 t_1)t_2 + \frac{1}{2}\frac{(a_1 t_1)^2}{|a_2|} = \left(\frac{a_1 t_1}{2}\right)\left(t_1 + 2t_2 + \frac{a_1 t_1}{|a_2|}\right),$$

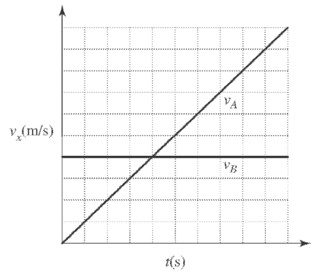
which yields the same result.

2.35: a)



b) From the graph (Fig. (2.35)), the curves for A and B intersect at $t = 1$ s and $t = 3$ s.

c)



d) From Fig. (2.35), the graphs have the same slope at $t = 2$ s . e) Car A passes car B when they have the same position and the slope of curve A is greater than that of curve B in Fig. (2.30); this is at $t = 3$ s. f) Car B passes car A when they have the same position and the slope of curve B is greater than that of curve A; this is at $t = 1$ s.

2.36: a) The truck's position as a function of time is given by $x_T = v_T t$, with v_T being the truck's constant speed, and the car's position is given by $x_C = (1/2)\, a_C t^2$. Equating the two expressions and dividing by a factor of t (this reflects the fact that the car and the truck are at the same place at $t = 0$) and solving for t yields

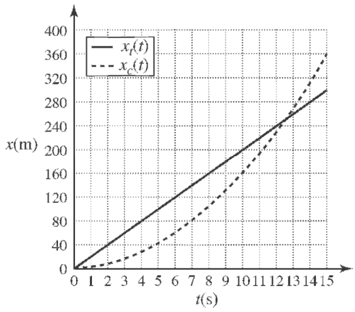
$$t = \frac{2v_T}{a_C} = \frac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s}$$

and at this time

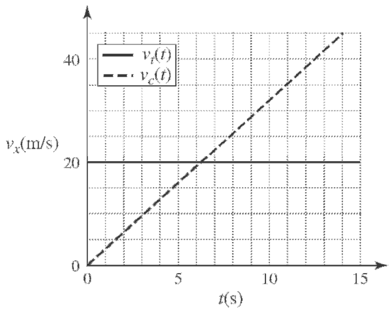
$$x_T = x_C = 250 \text{ m}.$$

b) $a_C t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40.0 \text{ m/s}$ (See Exercise 2.37 for a discussion of why the car's speed at this time is twice the truck's speed.)

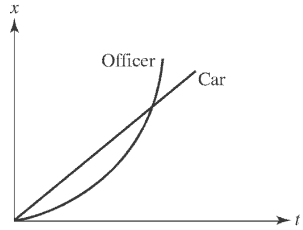
c)



d)



2.37: a)



The car and the motorcycle have gone the same distance during the same time, so their average speeds are the same. The car's average speed is its constant speed v_C, and for constant acceleration from rest, the motorcycle's speed is always twice its average, or $2v_C$. b) From the above, the motorcyle's speed will be v_C after half the time needed to catch the car. For motion from rest with constant acceleration, the distance traveled is proportional to the square of the time, so for half the time one-fourth of the total distance has been covered, or $d/4$.

2.38: a) An initial height of 200 m gives a speed of 60 $\mathrm{m/s}$ when rounded to one significant figure. This is approximately 200 km/hr or approximately 150 $\mathrm{mi/hr}$. (Different values of the approximate height will give different answers; the above may be interpreted as slightly better than order of magnitude answers.) b) Personal experience will vary, but speeds on the order of one or two meters per second are reasonable. c) Air resistance may certainly not be neglected.

2.39: a) From Eq. (2.13), with $v_y = 0$ and $a_y = -g$,

$$v_{0y} = \sqrt{2g(y - y_0)} = \sqrt{2(9.80\ \mathrm{m/s^2})(0.440\ \mathrm{m})} = 2.94\ \mathrm{m/s},$$

which is probably too precise for the speed of a flea; rounding down, the speed is about $2.9\ \mathrm{m/s}$.

b) The time the flea is rising is the above speed divided by g, and the total time is twice this; symbolically,

$$t = 2\frac{\sqrt{2g(y - y_0)}}{g} = 2\sqrt{\frac{2(y - y_0)}{g}} = 2\sqrt{\frac{2(0.440\ \mathrm{m})}{(9.80\ \mathrm{m/s^2})}} = 0.599\ \mathrm{s},$$

or about 0.60 s.

2.40: Using Eq. (2.13), with downward velocities and accelerations being positive, v_y^2 = $(0.8\ \mathrm{m/s})^2 + 2(1.6\ \mathrm{m/s^2})(5.0\ \mathrm{m}) = 16.64\ \mathrm{m^2/s^2}$ (keeping extra significant figures), so v_y = $4.1\ \mathrm{m/s}$.

2.41: a) If the meter stick is in free fall, the distance d is related to the reaction time t by $d = (1/2)gt^2$, so $t = \sqrt{2d/g}$. If d is measured in centimeters, the reaction time is

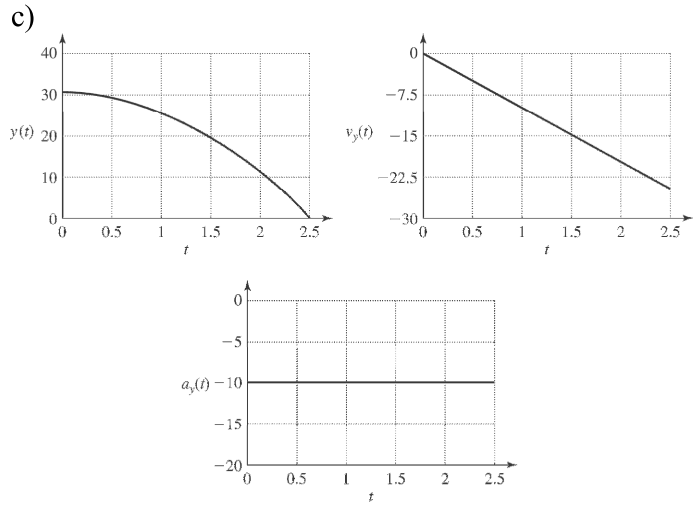
$$t = \sqrt{\frac{2}{g}}\sqrt{d} = \sqrt{\frac{2}{980\ \text{cm/s}^2}}\sqrt{d} = (4.52\times10^{-2}\ \text{s})\sqrt{d/(1\,\text{cm})}.$$

b) Using the above result, $(4.52\times10^{-2}\,\text{s})\sqrt{17.6} = 0.190\,\text{s}$.

2.42: a) $(1/2)gt^2 = (1/2)(9.80\ \text{m/s}^2)(2.5\,\text{s})^2 = 30.6\,\text{m}$.

b) $gt = (9.80\,\text{m/s}^2)(2.5\,\text{s}) = 24.5.\text{m/s}$.

c)



2.43: a) Using the method of Example 2.8, the time the ring is in the air is

$$t = \frac{v_{0y} + \sqrt{v_{0y}^2 - 2g(y - y_0)}}{g}$$

$$= \frac{(5.00\ \mathrm{m/s}) + \sqrt{(5.00\ \mathrm{m/s})^2 - 2(9.80\ \mathrm{m/s^2})(-12.0\ \mathrm{m})}}{(9.80\ \mathrm{m/s^2})}$$

$$= 2.156\ \mathrm{s},$$

keeping an extra significant figure. The average velocity is then $\frac{12.0\ \mathrm{m}}{2.156\ \mathrm{s}} = 5.57\ \mathrm{m/s}$, down. As an alternative to using the quadratic formula, the speed of the ring when it hits the ground may be obtained from $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, and the average velocity found from $\frac{v_y + v_{0y}}{2}$; this is algebraically identical to the result obtained by the quadratic formula.

b) While the ring is in free fall, the average acceleration is the constant acceleration due to gravity, $9.80\,\mathrm{m/s^2}$ down.

c)
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

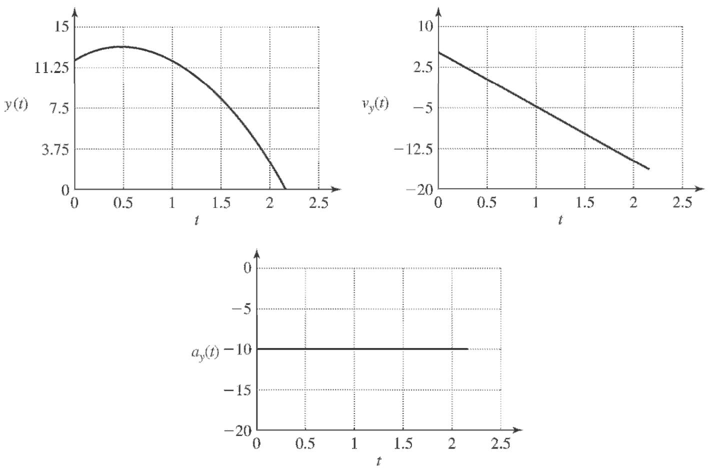
$$0 = 12.0\mathrm{m} + (5.00\mathrm{m/s})t - \frac{1}{2}(9.8\mathrm{m/s^2})t^2$$

Solve this quadratic as in part a) to obtain t = 2.156 s.

d) $v_y^2 = v_{0y}^2 - 2g(y - y_0) = (5.00\ \mathrm{m/s})^2 - 2(9.8\ \mathrm{m/s^2})(-12.0\ \mathrm{m})$

$|v_y| = 16.1\ \mathrm{m/s}$

e)



2.44: a) Using $a_y = -g$, $v_{0y} = 5.00\ \text{m/s}$ and $y_0 = 40.0$ m in Eqs. (2.8) and (2.12) gives i) at $t = 0.250$ s,

$$y = (40.0\ \text{m}) + (5.00\ \text{m/s})(0.250\ \text{s}) - (1/2)(9.80\ \text{m/s}^2)(0.250\ \text{s})^2 = 40.9\ \text{m},$$
$$v_y = (5.00\ \text{m/s}) - (9.80\ \text{m/s}^2)(0.250\ \text{s}) = 2.55\ \text{m/s}$$

and ii) at $t = 1.00$ s,

$$y = (40.0\ \text{m}) + (5.00\ \text{m/s})(1.00\ \text{s}) - (1/2)(9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 40.1\ \text{m},$$
$$v_y = (5.00\ \text{m/s}) - (9.80\ \text{m/s}^2)(1.00\ \text{s}) = -4.80\ \text{m/s}.$$

b) Using the result derived in Example 2.8, the time is

$$t = \frac{(5.00\ \text{m/s}) + \sqrt{(5.00\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(0 - 40.0\ \text{m})}}{(9.80\ \text{m/s}^2)} = 3.41\ \text{s}.$$

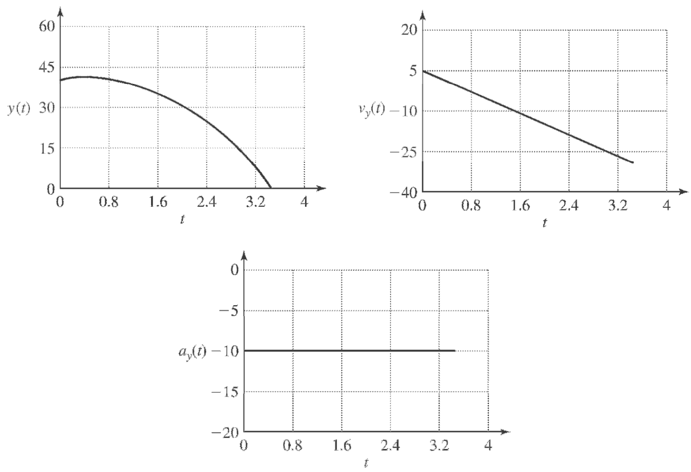
c) Either using the above time in Eq. (2.8) or avoiding the intermediate calculation by using Eq. (2.13),

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) = (5.00\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(-40.0\ \text{m}) = 809\ \text{m}^2/\text{s}^2,$$
$$v_y = 28.4\ \text{m/s}.$$

d) Using $v_y = 0$ in Eq. (2.13) gives

$$y = \frac{v_0^2}{2g} + y_0 = \frac{(5.00\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} + 40.0\ \text{m} = 41.2\ \text{m}.$$

e)



2.45: a) $v_y = v_{0y} - gt = (-6.00\ \text{m/s}) - (9.80\ \text{m/s}^2)(2.00\ \text{s}) = -25.6\ \text{m/s}$, so the speed is $25.6\ \text{m/s}$.

b) $y = v_{0y}t - \frac{1}{2}gt^2 = (-6.00\ \text{m/s})(2.00\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2 = -31.6\ \text{m}$, with the minus sign indicating that the balloon has indeed fallen.

c)
$v_y^2 = v_{0y}^2 - 2g(y_0 - y) = (6.00\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(-10.0\ \text{m}) = 232\ \text{m}^2/\text{s}^2$, so $v_y = 15.2$ r

2.46: a) The vertical distance from the initial position is given by

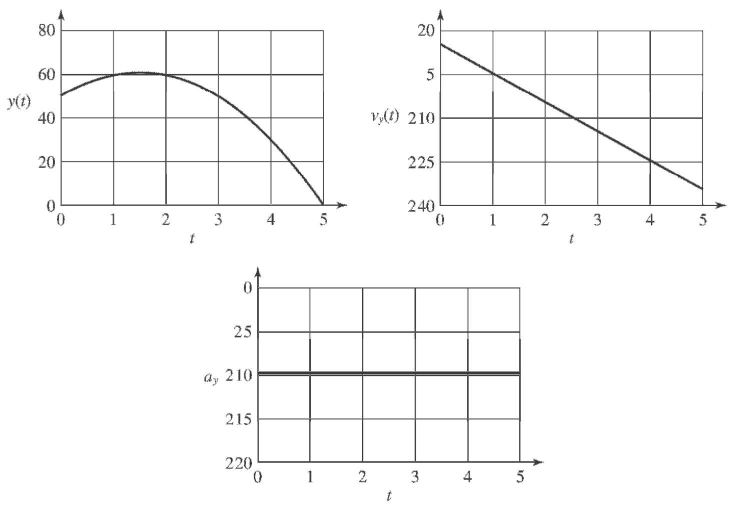
$$y = v_{0y}t - \frac{1}{2}gt^2;$$

solving for v_{0y},

$$v_{0y} = \frac{y}{t} + \frac{1}{2}gt = \frac{(-50.0\ \text{m})}{(5.00\ \text{s})} + \frac{1}{2}(9.80\ \text{m/s}^2)(5.00\ \text{s}) = 14.5\ \text{m/s}.$$

b) The above result could be used in $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, with $v = 0$, to solve for $y - y_0 = 10.7$ m (this requires retention of two extra significant figures in the calculation for v_{0y}). c) 0 d) 9.8 m/s^2, down.

e) Assume the top of the building is 50 m above the ground for purposes of graphing:



2.47: a) $(224\ \text{m/s})/(0.9\ \text{s}) = 249\ \text{m/s}^2$. b) $\frac{249\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 25.4$. c) The most direct way to find the distance is $v_{\text{ave}}t = ((224\ \text{m/s})/2)(0.9\ \text{s}) = 101\ \text{m}$.

d) $(283\ \text{m/s})/(1.40\ \text{s}) = 202\ \text{m/s}^2$ but $40g = 392\ \text{m/s}^2$, so the figures are not consistent.

2.48: a) From Eq. (2.8), solving for t gives $(40.0\ \text{m/s} - 20.0\ \text{m/s})/9.80\ \text{m/s}^2 = 2.04\ \text{s}$.

b) Again from Eq. (2.8),

$$\frac{40.0\ \text{m/s} - (-20.0\ \text{m/s})}{9.80\ \text{m/s}^2} = 6.12\ \text{s}.$$

c) The displacement will be zero when the ball has returned to its original vertical position, with velocity opposite to the original velocity. From Eq. (2.8),

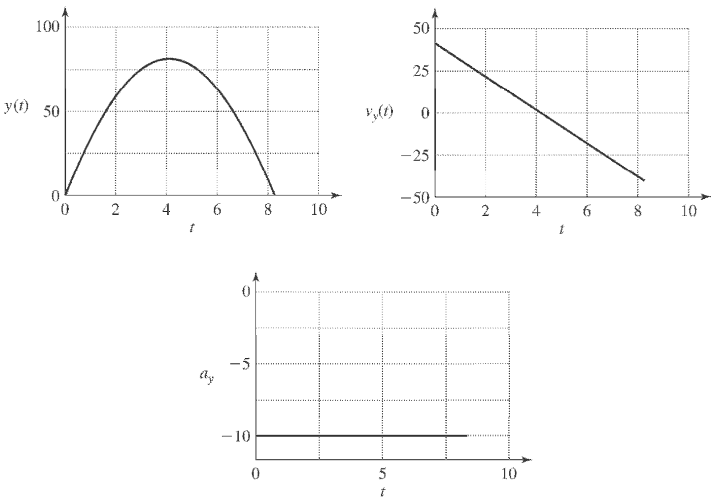
$$\frac{40\ \text{m/s} - (-40\ \text{m/s})}{9.80\ \text{m/s}^2} = 8.16\ \text{s}.$$

(This ignores the $t = 0$ solution.)

d) Again from Eq. (2.8), $(40\ \text{m/s})/(9.80\ \text{m/s}^2) = 4.08\ \text{s}$. This is, of course, half the time found in part (c).

e) $9.80\ \text{m/s}^2$, down, in all cases.

f)



2.49: a) For a given initial upward speed, the height would be inversely proportional to the magnitude of g, and with g one-tenth as large, the height would be ten times higher, or 7.5 m. b) Similarly, if the ball is thrown with the same upward speed, it would go ten times as high, or 180 m. c) The maximum height is determined by the speed when hitting the ground; if this speed is to be the same, the maximum height would be ten times as large, or 20 m.

2.50: a) From Eq. (2.15), the velocity v_2 at time t

$$v_2 = v_1 + \int_{t_1}^{t} \alpha t \, dt$$

$$= v_1 + \frac{\alpha}{2}(t^2 - t_1^2)$$

$$= v_1 - \frac{\alpha}{2}t_1^2 + \frac{\alpha}{2}t^2$$

$$= (5.0 \text{ m/s}) - (0.6 \text{ m/s}^3)(1.0 \text{ s})^2 + (0.6 \text{ m/s}^3)\, t^2$$

$$= (4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)\, t^2.$$

At $t_2 = 2.0$ s, the velocity is $v_2 = (4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.80$ m/s, or 6.8 m/s to two significant figures.

b) From Eq. (2.16), the position x_2 as a function of time is

$$x_2 = x_1 + \int_{t_1}^{t} v_x \, dt$$

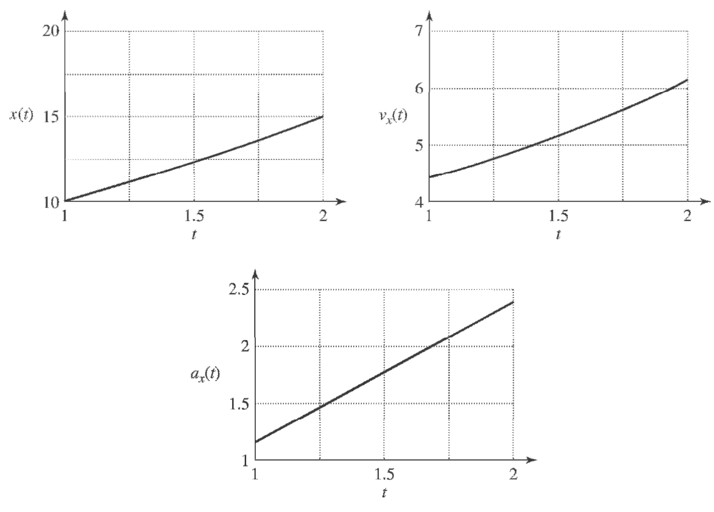
$$= (6.0 \text{ m}) + \int_{t_1}^{t} ((4.40 \text{ m/s}) + (0.6 \text{ m/s}^3)t^2)dt$$

$$= (6.0 \text{ m}) + (4.40 \text{ m/s})(t - t_1) + \frac{(0.6 \text{ m/s}^3)}{3}(t^3 - t_1^3).$$

At $t = 2.0$ s, and with $t_1 = 1.0$ s,

$$x = (6.0 \text{ m}) + (4.40 \text{ m/s})((2.0 \text{ s}) - (1.0 \text{ s})) + (0.20 \text{ m/s}^3)((2.0 \text{ s})^3 - (1.0 \text{ s})^3)$$
$$= 11.8 \text{ m}.$$

c)



2.51: a) From Eqs. (2.17) and (2.18), with v_0=0 and x_0=0,

$$v_x = \int_0^t (At - Bt^2)\,dt = \frac{A}{2}t^2 - \frac{B}{3}t^2 = (0.75\ \mathrm{m/s^3})t^3 - (0.040\ \mathrm{m/s^4})t^3$$

$$x = \int_0^t \left(\frac{A}{2}t^2 - \frac{B}{3}t^3\right)dt = \frac{A}{6}t^3 - \frac{B}{12}t^4 = (0.25\ \mathrm{m/s^3})t^3 - (0.010\ \mathrm{m/s^4})t^4.$$

b) For the velocity to be a maximum, the acceleration must be zero; this occurs at t=0 and $t = \frac{A}{B} = 12.5\ \mathrm{s}$. At t=0 the velocity is a minimum, and at t=12.5 s the velocity is

$$v_x = (0.75\ \mathrm{m/s^3})(12.5\ \mathrm{s})^2 - (0.040\ \mathrm{m/s^4})(12.5\ \mathrm{s})^3 = 39.1\ \mathrm{m/s}.$$

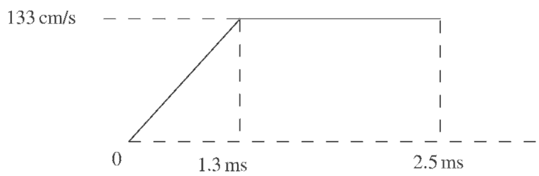
2.52: a) Slope $= a = 0$ for $t \geq 1.3\,\text{ms}$

b) $h_{\max} = \text{Area under } v - t \text{ graph}$

$$\approx A_{\text{Triangle}} + A_{\text{Rectangle}}$$

$$\approx \frac{1}{2}(1.3\,\text{ms})\left(133\frac{\text{cm}}{\text{s}}\right) + (2.5\,\text{ms} - 1.3\,\text{ms})(133\ \text{cm/s})$$

$$\approx 0.25\,\text{cm}$$



c) a = slope of v– t graph

$$a(0.5\,\text{ms}) \approx a(1.0\,\text{ms}) \approx \frac{133\ \text{cm/s}}{1.3\,\text{ms}} = 1.0\times10^{5}\ \text{cm/s}^2$$

$a(1.5\,\text{ms}) = 0$ because the slope is zero.

d) h = area under v– t graph

$$h(0.5\,\text{ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5\,\text{ms})\left(33\frac{\text{cm}}{\text{s}}\right) = 8.3\times10^{-3}\ \text{cm}$$

$$h(1.0\,\text{ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0\,\text{ms})(100\ \text{cm/s}) = 5.0\times10^{-2}\ \text{cm}$$

$$h(1.5\,\text{ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3\,\text{ms})\left(133\frac{\text{cm}}{\text{s}}\right) + (0.2\,\text{ms})(1.33)$$

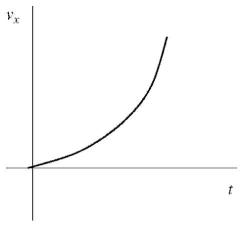
$$= 0.11\,\text{cm}$$

2.53: a) The change in speed is the area under the a_x versus t curve between vertical lines at $t = 2.5\,\text{s}$ and $t = 7.5\,\text{s}$. This area is

$\frac{1}{2}(4.00\ \text{cm/s}^2 + 8.00\,\text{cm/s}^2)(7.5\ \text{s} - 2.5\ \text{s}) = 30.0\ \text{cm/s}$

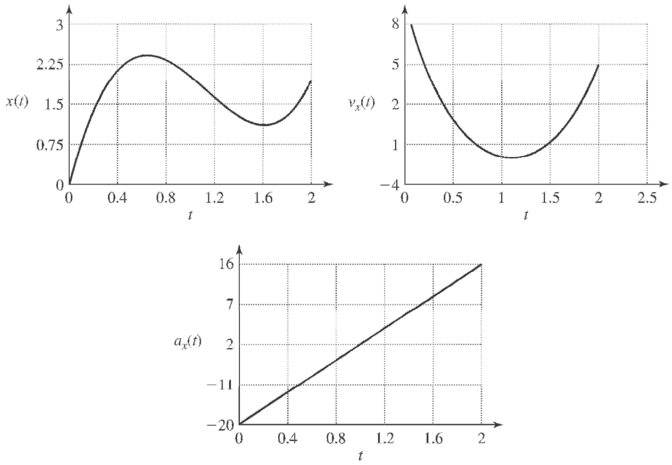
This acceleration is positive so the change in velocity is positive.

b) Slope of v_x versus t is positive and increasing with t.



2.54: a) To average $4\ \text{mi/hr}$, the total time for the twenty-mile ride must be five hours, so the second ten miles must be covered in 3.75 hours, for an average of 2.7 mi/hr. b) To average $12\ \text{mi/hr}$, the second ten miles must be covered in 25 minutes and the average speed must be $24\ \text{mi/hr}$. c) After the first hour, only ten of the twenty miles have been covered, and $16\ \text{mi/hr}$ is not possible as the average speed.

2.55: a)



The velocity and acceleration of the particle as functions of time are

$$v_x(t) = (9.00\ \text{m/s}^3)t^2 - (20.00\ \text{m/s}^2)t + (9.00\ \text{m/s})$$
$$a_x(t) = (18.0\ \text{m/s}^3)t - (20.00\ \text{m/s}^2).$$

b) The particle is at rest when the velocity is zero; setting $v = 0$ in the above expression and using the quadratic formula to solve for the time t,

$$t = \frac{(20.0\ \text{m/s}^3) \pm \sqrt{(20.0\ \text{m/s}^3)^2 - 4(9.0\ \text{m/s}^3)(9.0\ \text{m/s})}}{2(9.0\ \text{m/s}^3)}$$

and the times are 0.63 s and 1.60 s. c) The acceleration is negative at the earlier time and positive at the later time. d) The velocity is instantaneously not changing when the acceleration is zero; solving the above expression for $a_x(t) = 0$ gives

$$\frac{20.00\ \text{m/s}^2}{18.00\ \text{m/s}^3} = 1.11\,\text{s}.$$

Note that this time is the numerical average of the times found in part (c). e) The greatest distance is the position of the particle when the velocity is zero and the acceleration is negative; this occurs at 0.63 s, and at that time the particle is at

$$(3.00\ \text{m/s}^3)(0.63\,\text{s})^3 - (10.0\ \text{m/s}^2)(0.63\,\text{s})^2 + (9.00\ \text{m/s})(0.63\,\text{s}) = 2.45\,\text{m}.$$

(In this case, retaining extra significant figures in evaluating the roots of the quadratic equation does not change the answer in the third place.) f) The acceleration is negative at $t = 0$ and is increasing, so the particle is speeding up at the greatest rate at $t = 2.00$ s and slowing down at the greatest rate at $t = 0$. This is a situation where the extreme values of a function (in the case the acceleration) occur not at times when $\frac{da}{dt} = 0$ but at the endpoints of the given range.

2.56: a) $\frac{25.0\,\text{m}}{20.0\,\text{s}} = 1.25\,\text{m/s}$.

b) $\frac{25\,\text{m}}{15\,\text{s}} = 1.67\,\text{m/s}$.

c) Her net displacement is zero, so the average velocity has zero magnitude.

d) $\frac{50.0\,\text{m}}{35.0\,\text{s}} = 1.43\ \text{m/s}$. Note that the answer to part (d) is the *harmonic* mean, not the arithmetic mean, of the answers to parts (a) and (b). (See Exercise 2.5).

2.57: Denote the times, speeds and lengths of the two parts of the trip as t_1 and t_2, v_1 and v_2, and l_1 and l_2.

a) The average speed for the whole trip is

$$\frac{l_1 + l_2}{t_1 + t_2} = \frac{l_1 + l_2}{(l_1/v_1) + (l_2/v_2)} = \frac{(76\ \text{km}) + (34\ \text{km})}{\left(\frac{76\ \text{km}}{88\ \text{km/h}}\right) + \left(\frac{34\ \text{km}}{72\ \text{km/h}}\right)} = 82\ \text{km/h},$$

or 82.3 km/h, keeping an extra significant figure.

b) Assuming nearly straight-line motion (a common feature of Nebraska highways), the total distance traveled is $l_1 – l_2$ and

$$|v_{\text{ave}}| = \frac{l_1 - l_2}{t_1 + t_2} = \frac{(76\ \text{km}) - (34\ \text{km})}{\left(\frac{76\ \text{km}}{88\ \text{km/h}}\right) + \left(\frac{34\ \text{km}}{72\ \text{km/h}}\right)} = 31\ \text{km/h}.$$

($31.4\ \text{km/hr}$ to three significant figures.)

2.58: a) The space per vehicle is the speed divided by the frequency with which the cars pass a given point;

$$\frac{96\ \text{km/h}}{2400\ \text{vehicles/h}} = 40\ \text{m/vehicle}.$$

An average vehicle is given to be 4.5 m long, so the average spacing is 40.0 m – 4.6 m = 35.4 m.

b) An average spacing of 9.2 m gives a space per vehicle of 13.8 m, and the traffic flow rate is

$$\frac{96000\ \text{m/h}}{13.8\ \text{m/vehicle}} = 6960\ \text{vehicle/h}.$$

2.59: (a) Denote the time for the acceleration (4.0 s) as t_1 and the time spent running at constant speed (5.1 s) as t_2. The constant speed is then at_1, where a is the unknown acceleration. The total l is then given in terms of a, t_1 and t_2 by

$$l = \frac{1}{2}at_1^2 + at_1t_2,$$

and solving for a gives

$$a = \frac{l}{(1/2)t_1^2 + t_1t_2} = \frac{(100\text{ m})}{(1/2)(4.0\text{ s})^2 + (4.0\text{ s})(5.1\text{ s})} = 3.5\text{ m/s}^2.$$

(b) During the 5.1 s interval, the runner is not accelerating, so $a = 0$.

(c) $\Delta v/\Delta t = [(3.5\text{ m/s}^2)(4\text{ s})]/(9.1\text{ s}) = 1.54\text{ m/s}^2$.

(d) The runner was moving at constant velocity for the last 5.1 s.

2.60: a) Simple subtraction and division gives average speeds during the 2-second intervals as 5.6, 7.2 and 8.8 m/s.

b) The average speed increased by 1.6 m/s during each 2-second interval, so the acceleration is 0.8 m/s^2.

c) From Eq. (2.13), with $v_0 = 0$, $v = \sqrt{2(0.8\text{ m/s}^2)(14.4\text{ m})} = 4.8\text{ m/s}$. Or, recognizing that for constant acceleration the average speed of 5.6 m/s is the speed one second after passing the 14.4-m mark, $5.6\text{ m/s} - (0.8\text{ m/s}^2)(1.0\text{ s}) = 4.8\text{ m/s}$.

d) With both the acceleration and the speed at the 14.4-m known, either Eq. (2.8) or Eq. (2.12) gives the time as 6.0 s.

e) From Eq. (2.12), $x - x_0 = (4.8\text{ m/s})(1.0\text{ s}) + \frac{1}{2}(0.8\text{ m/s}^2)(1.0\text{ s})^2 = 5.2\text{ m}$. This is also the average velocity $(1/2)(5.6\text{ m/s} + 4.8\text{ m/s})$ times the time interval of 1.0 s.

2.61: If the driver steps on the gas, the car will travel

$$(20\text{ m/s})(3.0\text{ s}) + (1/2)(2.3\text{ m/s}^2)(3.0\text{ s})^2 = 70.4\text{ m}.$$

If the brake is applied, the car will travel

$$(20\text{ m/s})(3.0\text{ s}) + (1/2)(-3.8\text{ m/s}^2)(3.0\text{ s})^2 = 42.9\text{ m},$$

so the driver should apply the brake.

2.62: a) $$d = ct = (3.0\times10^{8}\ \frac{\text{m}}{\text{s}})(1\,\text{y})\left(\frac{365\frac{1}{4}\,\text{d}}{1\,\text{y}}\right)\left(\frac{24\,\text{h}}{1\,\text{d}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)$$
$$= 9.5\times10^{15}\ \text{m}$$

b) $$d = ct = (3.0\times10^{8}\ \frac{\text{m}}{\text{s}})(10^{-9}\ \text{s}) = 0.30\ \text{m}$$

c) $$t = \frac{d}{c} = \frac{1.5\times10^{11}\ \text{m}}{3.0\times10^{8}\ \text{m/s}} = 500\ \text{s} = 8.33\ \text{min}$$

d) $$t = \frac{d}{c} = \frac{2(3.84\times10^{8}\ \text{m})}{3.0\times10^{8}\ \text{m/s}} = 2.6\ \text{s}$$

e) $$t = \frac{d}{c} = \frac{3\times10^{9}\ \text{mi}}{186{,}000\ \text{mi/s}} = 16{,}100\ \text{s} = 4.5\ \text{h}$$

2.63: a) $v = 2\pi R_{\text{E}}/t = 464\,\text{m/s}$

b) $v = 2\pi r/t = 2.99\times10^{4}\ \text{m/s}$ (r is the radius of the earth's orbit)

c) Let c be the speed of light, then in one second light travels a distance $c(1.00\,\text{s})$. The number of times around the earth to which this corresponds is $c(1.00\,\text{s})/2\pi R_{\text{E}} = 7.48$

2.64: Taking the start of the race as the origin, runner A's speed at the end of 30 m can be found from:

$$v_{\mathrm{A}}^2 = v_{0\mathrm{A}}^2 + 2a_{\mathrm{A}}(x - x_0) = 0 + 2(1.6\ \mathrm{m/s^2})(30\ \mathrm{m}) = 96\ \mathrm{m^2/s^2}$$

$$v_{\mathrm{A}} = \sqrt{96\ \mathrm{m^2/s^2}} = 9.80\ \mathrm{m/s}$$

A's time to cover the first 30 m is thus:

$$t = \frac{v_{\mathrm{A}} - v_{0\mathrm{A}}}{a_{\mathrm{A}}} = \frac{9.80\ \mathrm{m/s}}{1.6\ \mathrm{m/s^2}} = 6.13\ \mathrm{s}$$

and A's total time for the race is:

$$6.13\ \mathrm{s} + \frac{(350 - 30)\ \mathrm{m}}{9.80\ \mathrm{m/s}} = 38.8\ \mathrm{s}$$

B's speed at the end of 30 m is found from:

$$v_{\mathrm{B}}^2 = v_{0\mathrm{B}}^2 + 2a_{\mathrm{B}}(x - x_0) = 0 + 2(2.0\ \mathrm{m/s^2})(30\ \mathrm{m}) = 120\ \mathrm{m^2/s^2}$$

$$v_{\mathrm{B}} = \sqrt{120\ \mathrm{m^2/s^2}} = 10.95\ \mathrm{m/s}$$

B's time for the first 30 m is thus

$$t = \frac{v_{\mathrm{B}} - v_{0\mathrm{B}}}{a_{\mathrm{B}}} = \frac{10.95\ \mathrm{m/s}}{2.0\ \mathrm{m/s^2}} = 5.48\ \mathrm{s}$$

and B's total time for the race is:

$$5.48\ \mathrm{s} + \frac{(350 - 30)\ \mathrm{m}}{10.95\ \mathrm{m/s}} = 34.7\ \mathrm{s}$$

B can thus nap for $38.8 - 34.7 = 4.1\ \mathrm{s}$ and still finish at the same time as A.

2.65: For the first 5.0 s of the motion, $v_{0x} = 0,\ t = 5.0\ \mathrm{s}$.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\ \mathrm{s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\ \mathrm{s}),\ t = 5.0\ \mathrm{s},\ x - x_0 = 150\ \mathrm{m}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $150\ \mathrm{m} = (25\ \mathrm{s^2})a_x + (12.5\ \mathrm{s^2})a_x$ and $a_x = 4.0\ \mathrm{m/s^2}$

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(4.0\ \mathrm{m/s^2})(5.0\ \mathrm{s})^2 = 50.0\ \mathrm{m}$.

2.66: a) The simplest way to do this is to go to a frame in which the freight train (which moves with constant velocity) is stationary. Then, the passenger train has an initial relative velocity of $v_{\text{rel},0} = 10\ \text{m/s}$. This relative speed would be decreased to zero after the relative separation had decreased to $\frac{v_{\text{rel},0}^2}{2a_{\text{rel}}} = +500\ \text{m}$. Since this is larger in magnitude than the original relative separation of 200 m, there will be a collision. b) The time at which the relative separation goes to zero (*i.e.*, the collision time) is found by solving a quadratic (see Problems 2.35 & 2.36 or Example 2.8). The time is given by

$$\begin{aligned} t &= \frac{1}{a}\left(v_{\text{rel},0} - \sqrt{v_{\text{rel},0}^2 + 2ax_{\text{rel},0}}\right) \\ &= (10\ \text{s}^2/\text{m})(10\ \text{m/s} - \sqrt{100\ \text{m}^2/\text{s}^2 - 40\ \text{m}^2/\text{s}^2}) \\ &= (100\ \text{s})\left((1 - \sqrt{0.6}\right). \end{aligned}$$

Substitution of this time into Eq. (2.12), with $x_0 = 0$, yields 538 m as the distance the passenger train moves before the collision.

2.67: The total distance you cover is $1.20\ \text{m} + 0.90\ \text{m} = 2.10\ \text{m}$ and the time available is $\frac{1.20\ \text{m}}{1.50\ \text{m/s}} = 0.80\ \text{s}$. Solving Eq. (2.12) for a_x,

$$a_x = 2\frac{(x - x_0) - v_{0x}t}{t^2} = 2\frac{(2.10\ \text{m}) - (0.80\ \text{m/s})(0.80\ \text{s})}{(0.80\ \text{s})^2} = 4.56\ \text{m/s}^2.$$

2.68: One convenient way to do the problem is to do part (b) first; the time spent accelerating from rest to the maximum speed is $\frac{20\ \mathrm{m/s}}{2.5\ \mathrm{m/s^2}} = 80$ s.

At this time, the officer is

$$x_1 = \frac{v_1^2}{2a} = \frac{(20\ \mathrm{m/s})^2}{2(2.5\ \mathrm{m/s^2})} = 80.0\ \mathrm{m}.$$

This could also be found from $(1/2)a_1t_1^2$, where t_1 is the time found for the acceleration. At this time the car has moved $(15\ \mathrm{m/s})(8.0\ \mathrm{s}) = 120$ m, so the officer is 40 m behind the car.

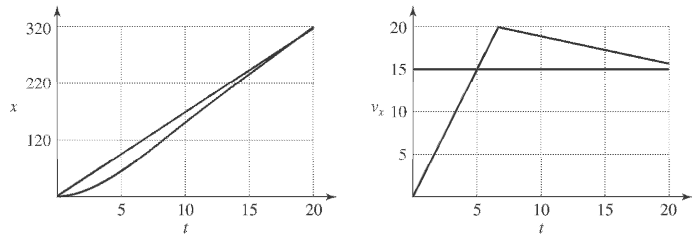
a) The remaining distance to be covered is 300 m $- x_1$ and the average speed is $(1/2)(v_1 + v_2) = 17.5\ \mathrm{m/s}$, so the time needed to slow down is

$$\frac{360\ \mathrm{m} - 80\ \mathrm{m}}{17.5\ \mathrm{m/s}} = 16.0\ \mathrm{s},$$

and the total time is 24.0 s.

c) The officer slows from $20\ \mathrm{m/s}$ to $15\ \mathrm{m/s}$ in 16.0 s (the time found in part (a)), so the acceleration is $-0.31\ \mathrm{m/s^2}$.

d), e)



2.69: a) $x_T = (1/2)a_T t^2$, and with $x_T = 40.0\,\text{m}$, solving for the time gives $t = \sqrt{\frac{2(40.0\text{ m})}{(2.10\text{ m/s})}} = 6.17\,\text{s}$

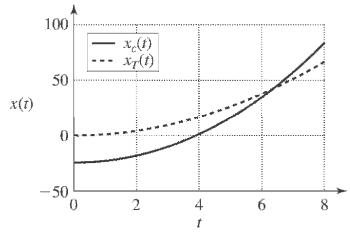
b) The car has moved a distance

$$\frac{1}{2}a_C t^2 = \frac{a_C}{a_T}x_1 = \frac{3.40\text{ m/s}^2}{2.10\text{ m/s}^2}40.0\,\text{m} = 64.8\,\text{m},$$

and so the truck was initially 24.8 m in front of the car.

c) The speeds are $a_T t = 13\text{ m/s}$ and $a_C t = 21\text{ m/s}$.

d)



2.70: The position of the cars as functions of time (taking $x_1 = 0$ at $t = 0$) are

$$x_1 = \frac{1}{2}at^2, \qquad x_2 = D - v_0 t.$$

The cars collide when $x_1 = x_2$; setting the expressions equal yields a quadratic in t,

$$\frac{1}{2}at^2 + v_0 t - D = 0,$$

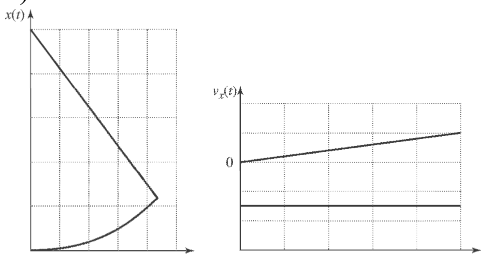
the solutions to which are

$$t = \frac{1}{a}\left(\sqrt{v_0^2 + 2aD} - v_0\right), \qquad t = \frac{1}{a}\left(-\sqrt{v_0^2 + 2aD} - v_0\right).$$

The second of these times is negative and does not represent the physical situation.

b) $$v_1 = at = \left(\sqrt{v_0^2 + 2aD} - v_0\right)$$

c)



2.71: a) Travelling at $20\ \text{m/s}$, Juan is $x_1 = 37\ \text{m} - (20\ \text{m/s})(0.80\ \text{s}) = 21\ \text{m}$ from the spreader when the brakes are applied, and the magnitude of the acceleration will be $a = \frac{v_1^2}{2x_1}$. Travelling at $25\ \text{m/s}$, Juan is $x_2 = 37\ \text{m} - (25\ \text{m/s})(0.80\ \text{s}) = 17\ \text{m}$ from the spreader, and the speed of the car (and Juan) at the collision is obtained from

$$v_x^2 = v_{0x}^2 - 2a_x x_2 = v_{0x}^2 - 2\left(\frac{v_1^2}{2x_1}\right)x_2 = v_{0x}^2 - v_1^2\left(\frac{x_2}{x_1}\right) = (25\ \text{m/s})^2 - (20\ \text{m/s})^2\left(\frac{17\ \text{m}}{21\ \text{m}}\right)$$
$$= 301\ \text{m}^2/\text{s}^2$$

and so $v_x = 17.4\ \text{m/s}$.

b) The time is the reaction time plus the magnitude of the change in speed $(v_0 - v)$ divided by the magnitude of the acceleration, or

$$t_{\text{flash}} = t_{\text{reaction}} + 2\frac{v_0 - v}{v_0^2}x_1 = (0.80\ \text{s}) + 2\frac{25\ \text{m/s} - 17.4\ \text{m/s}}{(20\ \text{m/s})^2}(21\ \text{m}) = 1.60\ \text{s}.$$

2.72: a) There are many ways to find the result using extensive algebra, but the most straightforward way is to note that between the time the truck first passes the police car and the time the police car catches up to the truck, both the truck and the car have travelled the same distance in the same time, and hence have the same average velocity over that time. Since the truck had initial speed $\frac{3}{2}v_{\text{p}}$ and the average speed is v_{p}, the truck's final speed must be $\frac{1}{2}v_{\text{p}}$.

2.73: a) The most direct way to find the time is to consider that the truck and the car are initially moving at the same speed, and the time of the acceleration must be that which gives a difference between the truck's position and the car's position as $24\ \text{m} + 21\ \text{m} + 26\ \text{m} + 4.5\ \text{m} = 75.5\ \text{m}$, or $t = \sqrt{2(75.5\ \text{m})/(0.600\ \text{m/s}^2)} = 15.9\ \text{s}$.

b) $v_{0x}t + (1/2)a_x t^2 = (20.0\ \text{m/s})(15.9\ \text{s}) + (1/2)(0.600\ \text{m/s}^2)(15.9\ \text{s})^2 = 394\ \text{m}$.

c) $v_{0x} + a_x t = (20.0\ \text{m/s}) + (0.600\ \text{m/s}^2)(15.9\ \text{s}) = 29.5\ \text{m/s}$.

2.74: a) From Eq. (2.17), $x(t) = \alpha t - \frac{\beta}{3}t^3 = (4.00\ \text{m/s})t - (0.667\ \text{m/s}^3)t^3$. From Eq. (2.5), the acceleration is $a(t) = -2\beta t = (-4.00\ \text{m/s}^3)t$.

b) The velocity is zero at $t = \pm\sqrt{\frac{\alpha}{\beta}}$ ($a = 0$ at $t = 0$, but this is an inflection point, not an extreme). The extreme values of x are then

$$x = \pm\left(\alpha\sqrt{\frac{\alpha}{\beta}} - \frac{\beta}{3}\sqrt{\frac{\alpha^3}{\beta^3}}\right) = \pm\frac{2}{3}\sqrt{\frac{\alpha^3}{\beta}}.$$

The positive value is then

$$x = \frac{2}{3}\left(\frac{(4.00\ \text{m/s})^3}{2.00\ \text{m/s}^3}\right)^{\frac{1}{2}} = \frac{2}{3}\sqrt{32\ \text{m}^2} = 3.77\ \text{m}.$$

2.75: a) The particle's velocity and position as functions of time are

$$\begin{aligned} v_x(t) &= v_{0x} + \int_0^t ((-2.00\,\text{m/s}^2) + (3.00\,\text{m/s}^3)t)\,dt \\ &= v_{0x} - (2.00\,\text{m/s}^2)t + \left(\frac{3.00\,\text{m/s}^3}{2}\right)t^2, \\ x(t) &= \int_0^t v_x(t)dt = v_{0x}t - (1.00\ \text{m/s}^2)t^2 + (0.50\ \text{m/s}^3)t^3 \\ &= t(v_{0x} - (1.00\,\text{m/s}^2)t + (0.50\ \text{m/s}^3)t^2), \end{aligned}$$

where x_0 has been set to 0. Then, $x(0) = 0$, and to have $x(4\ \text{s}) = 0$,

$$v_{0x} - (1.00\,\text{m/s}^2)(4.00\,\text{s}) + (0.50\,\text{m/s}^3)(4.00\text{s})^2 = 0,$$

which is solved for $v_{0x} = -4.0\,\text{m/s}$. b) $v_x(4\,\text{s}) = 12.0\,\text{m/s}$.

2.76: The time needed for the egg to fall is

$$t = \sqrt{\frac{2\Delta h}{9}} = \sqrt{\frac{2(46.0\ \text{m} - 1.80\ \text{m})}{(9.80\ \text{m/s}^2)}} = 3.00\ \text{s},$$

and so the professor should be a distance $v_y t = (1.20\ \text{m/s})(3.00\ \text{s}) = 3.60\ \text{m}$.

2.77: Let t_1 be the fall for the watermelon, and t_2 be the travel time for the sound to return. The total time is $T = t_1 + t_2 = 2.5\,\text{s}$. Let y be the height of the building, then, $y = \frac{1}{2}gt_1^2$ and $y = v_s t_2$. There are three equations and three unknowns. Eliminate t_2, solve for t_1, and use the result to find y. A quadratic results: $\frac{1}{2}gt_1^2 + v_s t_1 - v_s T = 0$. If $at^2 + bt + c = 0$, then $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Here, $t = t_1, a = 1/2\,g = 4.9\ \text{m/s}^2, b = v_s = 340\ \text{m/s}$, and $c = -v_s T = -(340\ \text{m/s})(2.5\,\text{s}) = -850\,\text{m}$

Then upon substituting these values into the quadratic formula,

$$t_1 = \frac{-(340\ \text{m/s}) \pm \sqrt{(340\,\text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-850\,\text{m})}}{2(4.9\ \text{m/s}^2)}$$

$t_1 = \frac{-(340\,\text{m/s}) \pm (363.7\,\text{m/s})}{2(4.9\,\text{m/s}^2)} = 2.42\,\text{s}$. The other solution, –71.8 s has no real physical meaning. Then, $y = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(2.42\,\text{s})^2 = 28.6\,\text{m}$. Check: $(28.6\,\text{m})/(340\ \text{m/s}) = .08\,\text{s}$, the time for the sound to return.

2.78: The elevators to the observation deck of the Sears Tower in Chicago move from the ground floor to the 103rd floor observation deck in about 70 s. Estimating a single floor to be about 3.5 m (11.5 ft), the average speed of the elevator is $\frac{(103)(3.5\,\text{m})}{70\,\text{s}} = 5.15\ \text{m/s}$. Estimating that the elevator must come to rest in the space of one floor, the acceleration is about $\frac{0^2 - (5.15\,\text{m/s})^2}{2(3.5\,\text{m})} = -3.80\,\text{m/s}^2$.

2.79: a) $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(21.3\,\text{m})} = 20.4\ \text{m/s}$; the announcer is mistaken.

b) The required speed would be

$$v_0 = \sqrt{v^2 + 2g(y - y_0)} = \sqrt{(25\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(-21.3\text{m})} = 14.4\ \text{m/s},$$

which is not possible for a leaping diver.

2.80: If the speed of the flowerpot at the top of the window is v_0, height h of the window is

$$h = v_{\text{ave}}t = v_0 t + (1/2)gt^2, \text{ or } v_0 = \frac{h}{t} - (1/2)gt.$$

The distance l from the roof to the top of the window is then

$$l = \frac{v_0^2}{2g} = \frac{((1.90\text{ m})/(0.420\text{ s}) - (1/2)(9.80\text{ m/s}^2)(0.420\text{ s}))^2}{2(9.80\text{ m/s}^2)} = 0.310\text{ m}.$$

An alternative but more complicated algebraic method is to note that t is the difference between the times taken to fall the heights $l + h$ and h, so that

$$t = \sqrt{\frac{2(l+h)}{g}} - \sqrt{\frac{2l}{g}}, \quad \sqrt{gt^2/2} + \sqrt{l} = \sqrt{l + h}.$$

Squaring the second expression allows cancelation of the l terms,

$$(1/2)gt^2 + 2\sqrt{gt^2 l/2} = h,$$

which is solved for

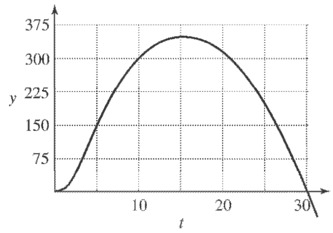
$$l = \frac{1}{2g}\left(\frac{h}{t} - (1/2)gt\right)^2,$$

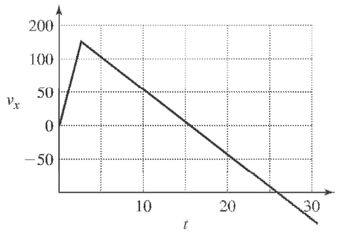
which is the same as the previous expression.

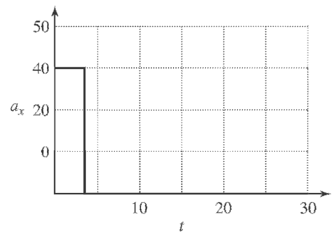
2.81: a) The football will go an additional $\frac{v^2}{2g} = \frac{(5.00\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.27\text{ m}$ above the window, so the greatest height is 13.27 m or 13.3 m to the given precision.

b) The time needed to reach this height is $\sqrt{2(13.3\text{m})/(9.80\text{ m/s}^2)} = 1.65\text{ s}.$

2.82: a)







2.83: a) From Eq. (2.14), with v_0=0,

$$v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(45.0\ \text{m/s}^2)(0.640\,\text{m})} = 7.59\,\text{m/s}.$$

b) The height above the release point is also found from Eq. (2.14), with $v_{0y} = 7.59\,\text{m/s}, v_y = 0$ and $a_y = -g$,

$$h = \frac{v_{0y}^2}{2g} = \frac{(7.59\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 2.94\,\text{m}$$

(Note that this is also (64.0 cm)$\left(\frac{45\,\text{m/s}^2}{g}\right)$.The height above the ground is then 5.14 m.

c) See Problems 2.46 & 2.48 or Example 2.8: The shot moves a total distance 2.20 m –1.83 m = 0.37 m, and the time is

$$\frac{(7.59\ \text{m/s}) + \sqrt{(7.59\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(0.37\,\text{m})}}{(9.80\ \text{m/s}^2)} = 1.60\,\text{s}.$$

2.84: a) In 3.0 seconds the teacher falls a distance

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(9.0\ \text{s}^2) = 44.1\ \text{m}$$

To reach her ears after 3.0 s, the sound must therefore have traveled a total distance of $h + (h - 44.1)\ \text{m} = 2h - 44.1\ \text{m}$, where h is the height of the cliff. Given 340 m/s for the speed of sound: $2h - 44.1\ \text{m} = (340\ \text{m/s})(3.0\ \text{s}) = 1020\ \text{m}$, which gives $h = 532\ \text{m}$ or $530\ \text{m}$ to the given precision.

b) We can use $v_y^2 = v_{0y}^2 + 2g(y - y_0)$ to find the teacher's final velocity. This gives $v_y^2 = 2(9.8\ \text{m/s}^2)(532\ \text{m}) = 10427\ \text{m}^2/\text{s}^2$ and $v_y = 102\ \text{m/s}$.

2.85: a) Let $+y$ be upward.

At ceiling, $v_y = 0, y - y_0 = 3.0\ \text{m}, a_y = -9.80\ \text{m/s}^2$. Solve for v_{0y}.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = 7.7\ \text{m/s}$.

b) $v_y = v_{0y} + a_y t$ with the information from part (a) gives $t = 0.78\ \text{s}$.

c) Let the first ball travel downward a distance d in time t. It starts from its maximum height, so $v_{0y} = 0$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.9\ \text{m/s}^2)t^2$

The second ball has $v_{0y} = \frac{1}{3}(7.7\ \text{m/s}) = 5.1\ \text{m/s}$. In time t it must travel upward $3.0\ \text{m} - d$ to be at the same place as the first ball.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $3.0\ \text{m} - d = (5.1\ \text{m/s})t - (4.9\ \text{m/s}^2)t^2$.

We have two equations in two unknowns, d and t. Solving gives $t = 0.59\ \text{s}$ and $d = 1.7\ \text{m}$.

d) $3.0\ \text{m} - d = 1.3\ \text{m}$

2.86: a) The helicopter accelerates from rest for 10.0 s at a constant $5.0\ \text{m/s}^2$. It thus reaches an upward velocity of

$$v_y = v_{0y} + a_y t = (5.0\ \text{m/s}^2)(10.0\ \text{s}) = 50.0\ \text{m/s}$$

and a height of $y = \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0\ \text{m/s}^2)(10.0\ \text{s})^2 = 250\ \text{m}$ at the moment the engine is shut off. To find the helicopter's maximum height use

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Taking $y_0 = 250\ \text{m}$, where the engine shut off, and since $v_y^2 = 0$ at the maximum height:

$$y_{\text{max}} - y_0 = \frac{-v_{0y}^2}{2g}$$

$$y_{\text{max}} = 250\ \text{m} - \frac{(50.0\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 378\ \text{m}$$

or 380 m to the given precision.

b) The time for the helicopter to crash from the height of 250 m where Powers stepped out and the engine shut off can be found from:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 250\ \text{m} + (50.0\ \text{m/s})t + \tfrac{1}{2}(-9.8\ \text{m/s}^2)t^2 = 0$$

where we now take the ground as $y = 0$. The quadratic formula gives solutions of $t = 3.67\ \text{s}$ and 13.88 s, of which the first is physically impossible in this situation. Powers' position 7.0 seconds after the engine shutoff is given by:

$$y = 250\ \text{m} + (50.0\ \text{m/s})(7.0\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(49.0\ \text{s}^2) = 359.9\ \text{m}$$

at which time his velocity is

$$v_y = v_{0y} + gt = 50.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(7.0\ \text{s}) = -18.6\ \text{m/s}$$

Powers thus has $13.88 - 7.0 = 6.88\ \text{s}$ more time to fall before the helicopter crashes, at his constant downward acceleration of $2.0\ \text{m/s}^2$. His position at crash time is thus:

$$\begin{aligned} y &= y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \\ &= 359.9\ \text{m} + (-18.6\ \text{m/s})(6.88\ \text{s}) + \frac{1}{2}(-2.0\ \text{m/s}^2)(6.88\ \text{s})^2 \\ &= 184.6\ \text{m} \end{aligned}$$

or 180 m to the given precision.

2.87: Take $+y$ to be downward.
Last 1.0 s of fall:
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h/4 = v_{0y}(1.0\ \text{s}) + (4.9\ \text{m/s}^2)(1.0\ \text{s})^2$
v_{0y} is his speed at the start of this time interval.
Motion from roof to $y - y_0 = 3h/4$:
$v_{0y} = 0, v_y = ?$
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80\ \text{m/s}^2)(3h/4)} = 3.834\sqrt{h}\ \text{m/s}$
This is v_y for the last 1.0 s of fall. Using this in the equation for the first 1.0 s gives
$h/4 = 3.834\sqrt{h} + 4.9$
Let $h = u^2$ and solve for $u : u = 16.5$. Then $h = u^2 = 270\ \text{m}$.

2.88: a)

$$t_{\text{fall}} + t_{\text{sound return}} = 10.0\ \text{s}$$

$$t_{\text{f}} + t_{\text{s}} = 10.0\ \text{s} \qquad (1)$$

$$d_{\text{Rock}} = d_{\text{Sound}}$$

$$\frac{1}{2}gt_{\text{f}}^2 = v_{\text{s}}t_{\text{s}}$$

$$\frac{1}{2}(9.8\ \text{m/s}^2)t_{\text{f}}^2 = (330\ \text{m/s})t_{\text{s}} \qquad (2)$$

Combine (1) and (2): $t_{\text{f}} = 8.84\ \text{s}, t_{\text{s}} = 1.16\ \text{s}$

$$h = v_{\text{s}}t_{\text{s}} = (330\frac{\text{m}}{\text{s}})(1.16\ \text{s}) = 383\ \text{m}$$

b) You would think that the rock fell for 10 s, not 8.84 s, so you would have thought it fell farther. Therefore your answer would be an *overestimate* of the cliff's height.

2.89: a) Let $+y$ be upward.
$y - y_0 = -15.0\ \text{m}$, $t = 3.25\ \text{s}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 11.31\ \text{m/s}$
Use this v_{0y} in $v_y = v_{0y} + a_y t$ to solve for $v_y : v_y = -20.5\ \text{m/s}$

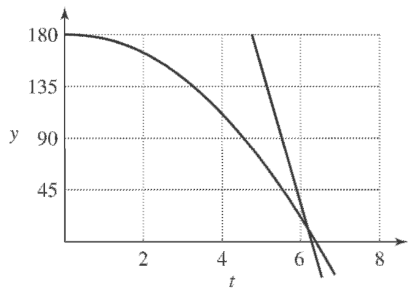
b) Find the maximum height of the can, above the point where it falls from the scaffolding:
$v_y = 0$, $v_{0y} = +11.31\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = ?$
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 6.53\ \text{m}$
The can will pass the location of the other painter. Yes, he gets a chance.

2.90: a) Suppose that Superman falls for a time t, and that the student has been falling for a time t_0 before Superman's leap (in this case, $t_0 = 5$ s). Then, the height h of the building is related to t and t_0 in two different ways:

$$\begin{aligned} -h &= v_{0y}t - \frac{1}{2}gt^2 \\ &= -\frac{1}{2}g(t + t_0)^2, \end{aligned}$$

where v_{0y} is Superman's initial velocity. Solving the second t gives $t = \sqrt{\frac{2h}{g}} - t_0$. Solving the first for v_{0y} gives $v_{0y} = -\frac{h}{t} + \frac{g}{2}t$, and substitution of numerical values gives $t = 1.06$ s and $v_{0y} = -165\ \text{m/s}$, with the minus sign indicating a downward initial velocity.

b)



c) If the skyscraper is so short that the student is already on the ground, then $h = \frac{1}{2}gt_0^2 = 123$ m.

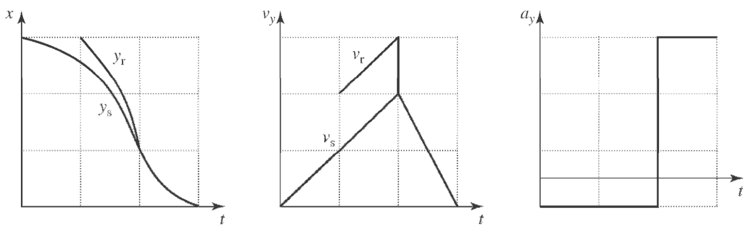
2.91: a) The final speed of the first part of the fall (free fall) is the same as the initial speed of the second part of the fall (with the Rocketeer supplying the upward acceleration), and assuming the student is a rest both at the top of the tower and at the ground, the distances fallen during the first and second parts of the fall are $\frac{v_1^2}{2g}$ and $\frac{v_1^2}{10g}$, where v_1 is the student's speed when the Rocketeer catches him. The distance fallen in free fall is then five times the distance from the ground when caught, and so the distance above the ground when caught is one-sixth of the height of the tower, or 92.2 m. b) The student falls a distance $5H/6$ in time $t = \sqrt{5H/3g}$, and the Rocketeer falls the same distance in time $t–t_0$, where t_0=5.00 s (assigning three significant figures to t_0 is more or less arbitrary). Then,

$$\frac{5H}{6} = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2, \text{or}$$

$$v_0 = \frac{5H/6}{(t - t_0)} - \frac{1}{2}g(t - t_0).$$

At this point, there is no great advantage in expressing t in terms of H and g algebraically; $t - t_0 = \sqrt{5(553\text{ m})/29.40\text{ m/s}^2} - 5.00\text{ s} = 4.698\text{ s}$, from which $v_0 = 75.1\text{m/s}$.

c)



2.92: a) The time is the initial separation divided by the initial relative speed, H/v_0. More precisely, if the positions of the balls are described by

$$y_1 = v_0t - (1/2)gt^2, \quad y_2 = H - (1/2)gt^2,$$

setting $y_1 = y_2$ gives $H = v_0t$. b) The first ball will be at the highest point of its motion if at the collision time t found in part (a) its velocity has been reduced from v_0 to 0, or $gt = gH/v_0 = v_0$, or $H = v_0^2/g$.

2.93: The velocities are $v_A = \alpha + 2\beta t$ and $v_B = 2\gamma t - 3\delta t^2$ a) Since v_B is zero at $t = 0$, car A takes the early lead. b) The cars are both at the origin at $t = 0$. The non-trivial solution is found by setting $x_A = x_B$, cancelling the common factor of t, and solving the quadratic for

$$t = \frac{1}{2\delta}\left[(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\alpha\delta}\right]$$

Substitution of numerical values gives 2.27 s, 5.73 s. The use of the term "starting point" can be taken to mean that negative times are to be neglected. c) Setting $v_A = v_B$ leads to a different quadratic, the positive solution to which is

$$t = -\frac{1}{6\delta}\left[(2\beta - 2\gamma) - \sqrt{(2\beta - 2\gamma)^2 - 12\alpha\delta}\right]$$

Substitution of numerical results gives 1.00 s and 4.33 s.

d) Taking the second derivative of x_A and x_B and setting them equal, yields, $2\beta = 2\gamma - 6\delta t$. Solving, $t = 2.67\,\mathrm{s}$.

2.94: a) The speed of any object falling a distance $H - h$ in free fall is $\sqrt{2g(H - h)}$. b) The acceleration needed to bring an object from speed v to rest over a distance h is $\frac{v^2}{2h} = \frac{2g(H - h)}{2h} = g\left(\frac{H}{h} - 1\right)$.

2.95: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then

$$x_1 = v_0 t, \qquad x_2 = x_0 + (1/2)at^2.$$

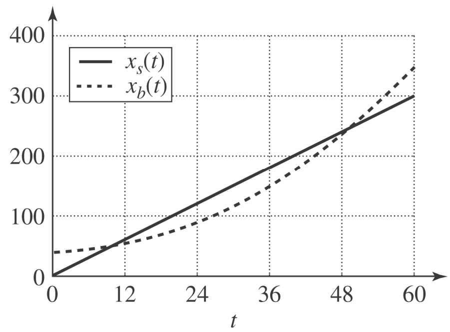
a) Setting $x_1 = x_2$ and solving for the times t gives

$$\begin{aligned} t &= \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right) \\ &= \frac{1}{(0.170\ \mathrm{m/s^2})}\left((5.0\ \mathrm{m/s}) \pm \sqrt{(5.0\ \mathrm{m/s})^2 - 2(0.170\ \mathrm{m/s^2})(40.0\ \mathrm{m})}\right) \\ &= 9.55\ \mathrm{s},\ 49.3\ \mathrm{s}. \end{aligned}$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \mathrm{m/s})(9.55\ \mathrm{s}) = 47.8\ \mathrm{m}$.

b) The speed of the bus is $(0.170\ \mathrm{m/s^2})(9.55\ \mathrm{s}) = 1.62\ \mathrm{m/s}$.

c) The results can be verified by noting that the x lines for the student and the bus intersect at two points:



d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \mathrm{m/s^2})(49.3\ \mathrm{s}) = 8.38\ \mathrm{m/s}$.

2.96: The time spent above $y_{\max}/2$ is $\frac{1}{\sqrt{2}}$ the total time spent in the air, as the time is proportional to the square root of the change in height. Therefore the ratio is

$$\frac{1/\sqrt{2}}{1-1/\sqrt{2}}=\frac{1}{\sqrt{2}-1}=2.4.$$

2.97: For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is,

let $y_1 = h + v_0 t - \frac{1}{2} g t^2, y_2 = h - \frac{1}{2} g (t - t_0)^2$. In this case, $t_0 = 1.00\ \text{s}$. Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2} g t^2$ yields $v_0 t = g t_0 t - \frac{1}{2} g t_0^2$, which can be solved for *t*;

$$t = \frac{\frac{1}{2} g t_0^2}{g t_0 - v_0} = \frac{t_0}{2} \frac{1}{1 - \frac{v_0}{g t_0}}.$$

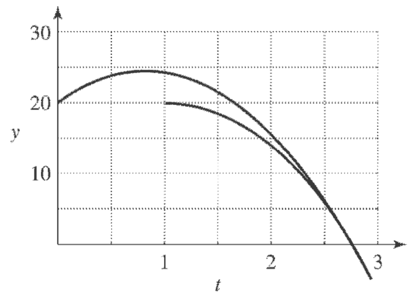
Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for *h* as a function of v_0 yields, after some algebra,

$$h = \frac{1}{2} g t_0^2 \frac{\left(\frac{1}{2} g t_0 - v_0\right)^2}{(g t_0 - v_0)^2}.$$

a) Using the given value $t_0 = 1.00\ \text{s}$ and $g = 9.80\ \text{m/s}^2$,

$$h = 20.0\ \text{m} = (4.9\ \text{m}) \left(\frac{4.9\ \text{m/s} - v_0}{9.8\ \text{m/s} - v_0} \right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields $8.2\ \text{m/s}$.



b) The above expression gives for i), 0.411 m and for ii) 1.15 km. c) As v_0 approaches $9.8\ \text{m/s}$, the height *h* becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\ \text{m/s}$, the first ball can never catch the second ball. d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 > 4.9\ \text{m/s}$, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

2.98: a) Let the height be h and denote the 1.30-s interval as Δt; the simultaneous equations $h = \frac{1}{2}gt^2$, $\frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$ can be solved for t. Eliminating h and taking the square root, $\frac{t}{t-\Delta t} = \sqrt{\frac{3}{2}}$, and $t = \frac{\Delta t}{1-\sqrt{2/3}}$, and substitution into $h = \frac{1}{2}gt^2$ gives $h = 246$ m. This method avoids use of the quadratic formula; the quadratic formula is a generalization of the method of "completing the square", and in the above form, $\frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$, the square is already completed.

b) The above method assumed that $t > 0$ when the square root was taken. The negative root (with $\Delta t = 0$) gives an answer of 2.51 m, clearly not a "cliff". This would correspond to an object that was initially near the bottom of this "cliff" being thrown upward and taking 1.30 s to rise to the top and fall to the bottom. Although physically possible, the conditions of the problem preclude this answer.

Capítulo 3

3.1: a)

$$v_{x,\text{ave}} = \frac{(5.3\,\text{m}) - (1.1\,\text{m})}{(3.0\,\text{s})} = 1.4\ \text{m/s},$$
$$v_{y,\text{ave}} = \frac{(-0.5\,\text{m}) - (3.4\,\text{m})}{(3.0\,\text{s})} = -1.3\ \text{m/s}.$$

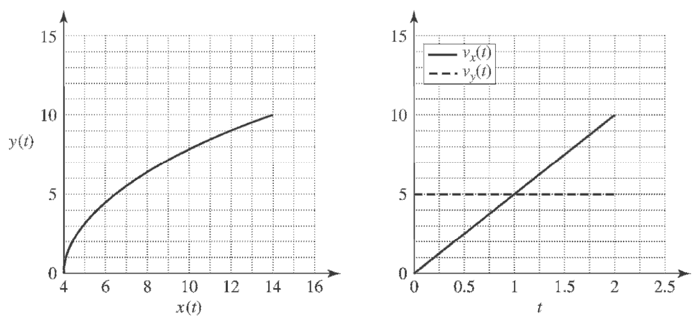
b) $v_{\text{ave}} = \sqrt{(1.4\ \text{m/s})^2 + (-1.3\ \text{m/s})^2} = 1.91\ \text{m/s}$, or $1.9\ \text{m/s}$ to two significant figures, $\theta = \arctan\left(\frac{-1.3}{1.4}\right) = -43°$.

3.2: a)

$$x = (v_{x,\text{ave}})\Delta t = (-3.8\ \text{m/s})(12.0\,\text{s}) = -45.6\,\text{m} \quad \text{and}$$
$$y = (v_{y,\text{ave}})\Delta t = (4.9\ \text{m/s})(12.0\,\text{s}) = 58.8\,\text{m}.$$

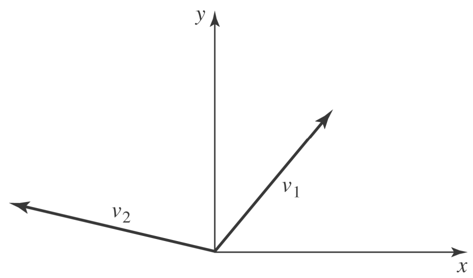
b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\,\text{m})^2 + (58.8\,\text{m})^2} = 74.4\,\text{m}.$

3.3: The position is given by $\vec{r} = [4.0\ \text{cm} + (2.5\ \text{cm/s}^2)t^2]\hat{\boldsymbol{i}} + (5.0\ \text{cm/s})t\hat{\boldsymbol{j}}$.
(a) $\mathbf{r}(0) = [4.0\,\text{cm}]\hat{\boldsymbol{i}}$, and
$\mathbf{r}(2\text{s}) = [4.0\,\text{cm} + (2.5\ \text{cm/s}^2)(2\,\text{s})^2]\hat{\boldsymbol{i}} + (5.0\ \text{cm/s})(2\,\text{s})\hat{\boldsymbol{j}} = (14.0\,\text{cm})\hat{\boldsymbol{i}} + (10.0\,\text{cm})\hat{\boldsymbol{j}}$. Then using the definition of average velocity, $\vec{\boldsymbol{v}}_{\text{ave}} = \frac{(14\,\text{cm} - 4\,\text{cm})\hat{\boldsymbol{i}} + (10\,\text{cm} - 0)\hat{\boldsymbol{j}}}{2\,\text{s}} = (5\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}.$
$v_{\text{ave}} = 7.1\ \text{cm/s}$ at an angle of $45°$.
b) $\vec{\boldsymbol{v}} = \frac{d\vec{r}}{dt} = (2)(2.5\ \text{cm/s})t\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}} = (5\ \text{cm/s})t\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$. Substituting for $t = 0, 1\,\text{s}$, and 2 s, gives:
$\vec{\boldsymbol{v}}(0) = (5\ \text{cm/s})\hat{\boldsymbol{j}}, \vec{\boldsymbol{v}}(1\,\text{s}) = (5\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$, and $\vec{\boldsymbol{v}}(2\,\text{s}) = (10\ \text{cm/s})\hat{\boldsymbol{i}} + (5\ \text{cm/s})\hat{\boldsymbol{j}}$.
The magnitude and direction of $\vec{\boldsymbol{v}}$ at each time therefore are: $t = 0 : 5.0\ \text{cm/s}$ at $90°$; $t = 1.05 : 7.1\ \text{cm/s}$ at $45°$; $t = 2.05 : 11\,\text{cm/s}$ at $27°$.
c)



3.4: $\vec{v} = 2bt\hat{\boldsymbol{i}} + 3ct^2\hat{\boldsymbol{j}}$. This vector will make a $45°$-angle with both axes when the x- and y-components are equal; in terms of the parameters, this time is $2b/3c$.

3.5: a)



b)
$$a_{x,\text{ave}} = \frac{(-170 \text{ m/s}) - (90 \text{ m/s})}{(30.0 \text{ s})} = -8.7 \text{ m/s}^2,$$
$$a_{y,\text{ave}} = \frac{(40 \text{ m/s}) - (110 \text{ m/s})}{(30.0 \text{ s})} = -2.3 \text{ m/s}^2.$$

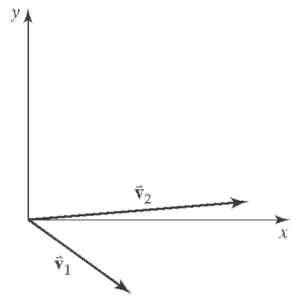
c) $\sqrt{(-8.7 \text{ m/s}^2)^2 + (-2.3 \text{ m/s}^2)^2} = 9.0 \text{ m/s}^2$, arctan $\left(\frac{-2.3}{-8.7}\right) = 14.8° + 180° = 195°$.

3.6: a) $a_x = (0.45 \text{ m/s}^2)\cos 31.0° = 0.39 \text{ m/s}^2$, $a_y = (0.45 \text{ m/s}^2)\sin 31.0° = 0.23 \text{ m/s}^2$, so $v_x = 2.6 \text{ m/s} + (0.39 \text{ m/s}^2)(10.0 \text{ s}) = 6.5 \text{ m/s}$ and

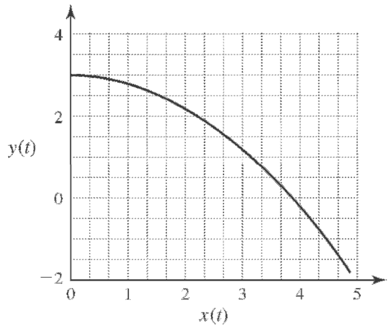
$v_y = -1.8 \text{ m/s} + (0.23 \text{ m/s}^2)(10.0 \text{ s}) = 0.52 \text{ m/s}$.

b) $v = \sqrt{(6.5 \text{ m/s})^2 + (0.52 \text{ m/s})^2} = 6.48 \text{ m/s}$, at an angle of arctan$\left(\frac{0.52}{6.5}\right) = 4.6°$ above the horizontal.

c)



3.7: a)



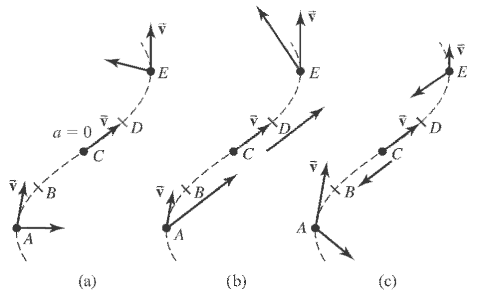
b)

$$\vec{v} = \alpha\hat{i} - 2\beta t\hat{j} = (2.4\,\text{m/s})\hat{i} - [(2.4\,\text{m/s}^2)t]\hat{j}$$

$$\vec{a} = -2\beta\hat{j} = (-2.4\,\text{m/s}^2)\,\hat{j}.$$

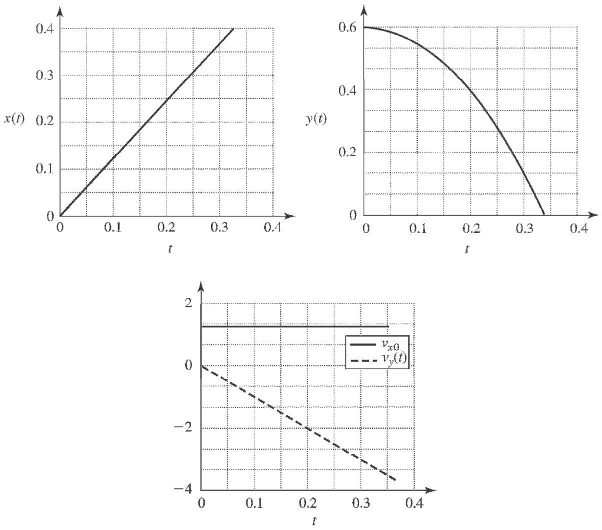
c) At $t = 2.0\,\text{s}$, the velocity is $\vec{v} = (2.4\text{ m/s})\hat{i} - (4.8\text{ m/s})\hat{j}$; the magnitude is $\sqrt{(2.4\text{ m/s})^2 + (-4.8\text{ m/s})^2} = 5.4\text{ m/s}$, and the direction is $\arctan\left(\frac{-4.8}{2.4}\right) = -63°$. The acceleration is constant, with magnitude 2.4 m/s^2 in the $-y$-direction. d) The velocity vector has a component parallel to the acceleration, so the bird is speeding up. The bird is turning toward the $-y$ -direction, which would be to the bird's right (taking the $+z$ -direction to be vertical).

3.8:



3.9: a) Solving Eq. (3.18) with $y = 0$, $v_{0y} = 0$ and $t = 0.350\,\text{s}$ gives $y_0 = 0.600\,\text{m}$.

b) $v_x t = 0.385\,\text{m}$ c) $v_x = v_{0x} = 1.10\ \text{m/s}$, $v_y = -gt = -3.43\ \text{m/s}$, $v = 3.60\ \text{m/s}, 72.2°$ below the horizontal.

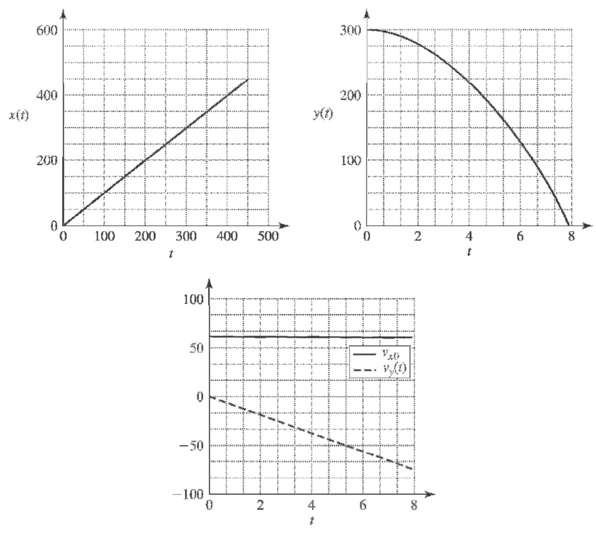


3.10: a) The time t is given by $t = \sqrt{\frac{2h}{g}} = 7.82\,\text{s}$.

b) The bomb's constant horizontal velocity will be that of the plane, so the bomb travels a horizontal distance $x = v_x t = (60\ \text{m/s})(7.82\,\text{s}) = 470\,\text{m}$.

c) The bomb's horizontal component of velocity is 60 m/s, and its vertical component is $-gt = -76.7\ \text{m/s}$.

d)



e) Because the airplane and the bomb always have the same x-component of velocity *and* position, the plane will be 300 m above the bomb at impact.

3.11: Take $+y$ to be upward.
Use Chirpy's motion to find the height of the cliff.
$v_{0y} = 0,\ a_y = -9.80\ \text{m/s}^2,\ y - y_0 = -h,\ t = 3.50\,\text{s}$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = 60.0\,\text{m}$
Milada: Use vertical motion to find time in the air.
$v_{0y} = v_0 \sin 32.0°,\ y - y_0 = -60.0\,\text{m},\ a_y = -9.80\ \text{m/s}^2,\ t = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 3.55\,\text{s}$
Then $v_{0x} = v_0 \cos 32.0°,\ a_x = 0,\ t = 3.55\,\text{s}$ gives $x - x_0 = 2.86\,\text{m}$.

3.12: Time to fall 9.00 m from rest:

$$y = \frac{1}{2}gt^2$$
$$9.00\,\text{m} = \frac{1}{2}(9.8\,\text{m/s}^2)t^2$$
$$t = 1.36\,\text{s}$$

Speed to travel 1.75 m horizontally:

$$x = v_0 t$$
$$1.75\,\text{m} = v_0(1.36\,\text{s})$$
$$v_0 = 1.3\,\text{m/s}$$

3.13: Take $+y$ to be upward.
Use the vertical motion to find the time in the air:
$v_{0y} = 0,\ a_y = -9.80\ \text{m/s}^2,\ y - y_0 = -(21.3\,\text{m} - 1.8\,\text{m}) = -19.5\,\text{m},\ t = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 1.995\,\text{s}$
Then $x - x_0 = 61.0\,\text{m},\ a_x = 0,\ t = 1.995\,\text{s}, v_{0x} = ?$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_{0x} = 30.6\ \text{m/s}$.
b) $v_x = 30.6\ \text{m/s}$ since $a_x = 0$
$v_y = v_{0y} + a_y t = -19.6\ \text{m/s}$
$v = \sqrt{v_x^2 + v_y^2} = 36.3\ \text{m/s}$

3.14: To make this prediction, the student needs the ball's horizontal velocity at the moment it leaves the tabletop and the time it will take for the ball to reach the floor (or rather, the rim of the cup). The latter can be determined simply by measuring the height of the tabletop above the rim of the cup and using $y = \frac{1}{2}gt^2$ to calculate the falling time. The horizontal velocity can be determined (although with significant uncertainty) by timing the ball's roll for a measured distance before it leaves the table, assuming that its speed doesn't change much on the hard tabletop. The horizontal distance traveled while the ball is in flight will simply be horizontal velocity $\times$ falling time. The cup should be placed at this distance (or a slightly shorter distance, to allow for the slowing of the ball on the tabletop and to make sure it clears the rim of the cup) from a point vertically below the edge of the table.

3.15: a) Solving Eq. (3.17) for $v_y = 0$, with $v_{0y} = (15.0 \text{ m/s})\sin 45.0°$,

$$T = \frac{(15.0 \text{ m/s})\sin 45°}{9.80 \text{ m/s}^2} = 1.08 \text{ s}.$$

b) Using Equations (3.20) and (3.21) gives at t_1, $(x, y) = (6.18 \text{ m}, 4.52 \text{ m})$: t_2, $(11.5 \text{ m}, 5.74 \text{ m})$: t_3, $(16.8 \text{ m}, 4.52 \text{ m})$.

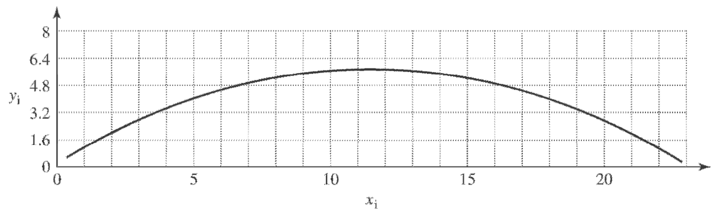
c) Using Equations (3.22) and (3.23) gives at t_1, $(v_x, v_y) = (10.6 \text{ m/s}, 4.9 \text{ m/s})$: t_2, $(10.6 \text{ m/s}, 0)$ t_3: $(10.6 \text{ m/s}, -4.9 \text{ m/s})$, for velocities, respectively, of 11.7 m/s @ 24.8°, 10.6 m/s @ 0° and 11.7 m/s @ –24.8°. Note that v_x is the same for all times, and that the y-component of velocity at t_3 is negative that at t_1.

d) The parallel and perpendicular components of the acceleration are obtained from

$$\vec{a}_{\parallel} = \frac{(\vec{a} \cdot \vec{v})\vec{v}}{v^2}, |\vec{a}_{\parallel}| = \frac{|\vec{a} \cdot \vec{v}|}{v}, |\vec{a}_{\perp}| = \sqrt{|\vec{a}| - |\vec{a}_{\parallel}|}.$$

For projectile motion, $\vec{a} = -g\hat{j}$, so $\vec{a} \cdot \vec{v} = -gv_y$, and the components of acceleration parallel and perpendicular to the velocity are t_1: -4.1 m/s^2, 8.9 m/s^2. t_2: 0, 9.8 m/s^2. t_3: 4.1 m/s^2, 8.9 m/s^2.

e)



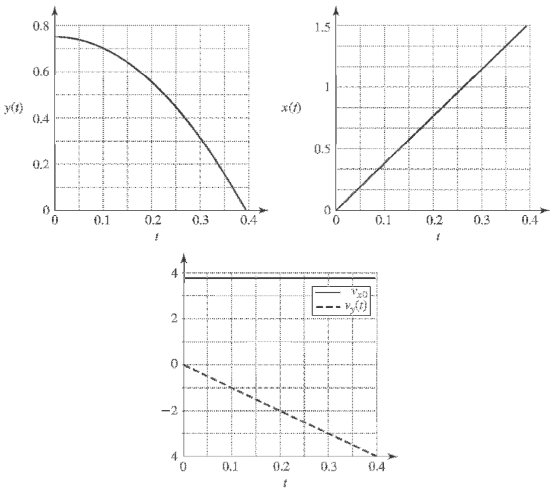
f) At t_1, the projectile is moving upward but slowing down; at t_2 the motion is instantaneously horizontal, but the vertical component of velocity is decreasing; at t_3, the projectile is falling down and its speed is increasing. The horizontal component of velocity is constant.

3.16: a) Solving Eq. (3.18) with $y = 0,\ y_0 = 0.75\,\text{m}$ gives $t = 0.391\,\text{s}$.

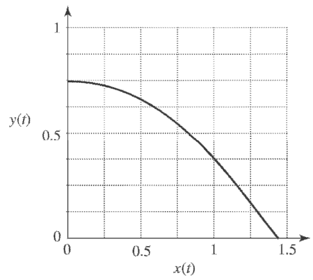
b) Assuming a horizontal tabletop, $v_{0y} = 0$, and from Eq. (3.16), $v_{0x} = (x - x_0)/t = 3.58\,\text{m/s}$.

c) On striking the floor, $v_y = -gt = -\sqrt{2gy_0} = -3.83\,\text{m/s}$, and so the ball has a velocity of magnitude $5.24\,\text{m/s}$, directed $46.9°$ below the horizontal.

d)



Although not asked for in the problem, this *y* vs. *x* graph shows the trajectory of the tennis ball as viewed from the side.



3.17: The range of a projectile is given in Example 3.11, $R = v_0^2 \sin 2\alpha_0/g$.

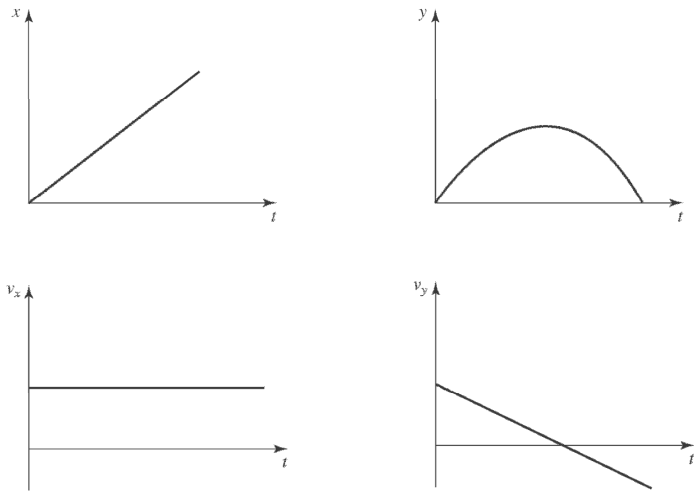
a) $(120\ \text{m/s})^2 \sin 110°/(9.80\ \text{m/s}^2) = 1.38\ \text{km}$. b) $(120\ \text{m/s})^2 \sin 110°/(1.6\ \text{m/s}^2) = 8.4\ \text{km}$.

3.18: a) The time t is $\frac{v_{y0}}{g} = \frac{16.0\,\text{m/s}}{9.80\,\text{m/s}^2} = 1.63\ \text{s}$.

b) $\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \frac{v_{y0}^2}{2g} = 13.1\ \text{m}$.

c) Regardless of how the algebra is done, the time will be twice that found in part (a), or 3.27 s d) v_x is constant at $20.0\ \text{m/s}$, so $(20.0\ \text{m/s})(3.27\ \text{s}) = 65.3\ \text{m}$.

e)



3.19: a) $v_{0y} = (30.0\ \text{m/s})\sin 36.9° = 18.0\ \text{m/s}$; solving Eq. (3.18) for t with $y_0 = 0$ and $y = 10.0\ \text{m}$ gives

$$t = \frac{(18.0\ \text{m/s}) \pm \sqrt{(18.0\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(10.0\ \text{m})}}{9.80\ \text{m/s}^2} = 0.68\ \text{s}, 2.99\ \text{s}$$

b) The x-component of velocity will be $(30.0\ \text{m/s})\cos 36.9° = 24.0\ \text{m/s}$ at all times. The y-component, obtained from Eq. (3.17), is $11.3\ \text{m/s}$ at the earlier time and $-11.3\ \text{m/s}$ at the later.

c) The magnitude is the same, $30.0\ \text{m/s}$, but the direction is now $36.9°$ below the horizontal.

3.20: a) If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically.

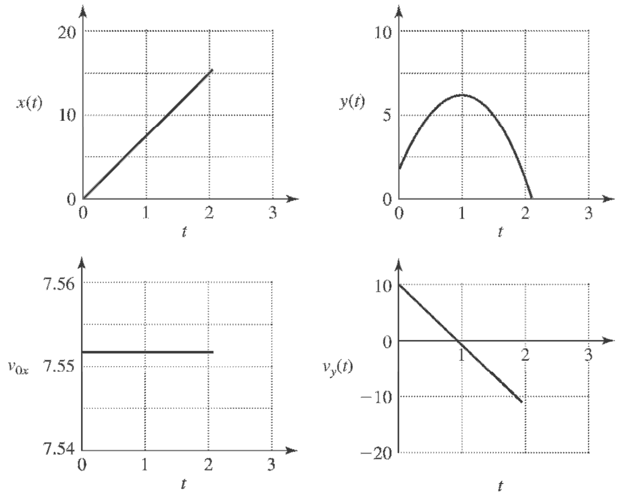
b) The x-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0° = 7.55\ \text{m/s}$. The y-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0° = 9.32\ \text{m/s}$ at release and $v_{0y} - gt = (10.57\ \text{m/s}) - (9.80\ \text{m/s}^2)(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.

c) $v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.

d) The initial and final heights are not the same.

e) With $y = 0$ and v_{0y} as found above, solving Eq. (3.18) for $y_0 = 1.81\text{m}$.

f)



3.21: a) The time the quarter is in the air is the horizontal distance divided by the horizontal component of velocity. Using this time in Eq. (3.18),

$$\begin{aligned} y - y_0 &= v_{0y}\frac{x}{v_{0x}} - \frac{gx^2}{2v_{0x}^2} \\ &= \tan\alpha_0 x - \frac{gx^2}{v_0^2 2\cos^2\alpha_0} \\ &= \tan 60°(2.1\text{m}) - \frac{(9.80\ \text{m/s}^2)(2.1\ \text{m})^2}{2(6.4\ \text{m/s})^2\cos^2 60°} = 1.53\ \text{m rounded.} \end{aligned}$$

b) Using the same expression for the time in terms of the horizontal distance in Eq. (3.17),

$$v_y = v_0\sin\alpha_0 - \frac{gx}{v_0\cos\alpha_0} = (6.4\ \text{m/s})\sin 60° - \frac{(9.80\ \text{m/s}^2)(2.1\ \text{m})}{(6.4\ \text{m/s})\cos 60°} = -0.89\ \text{m/s}.$$

3.22: Substituting for t in terms of d in the expression for y_{dart} gives

$$y_{\text{dart}} = d\left(\tan\alpha_0 - \frac{gd}{2v_0^2\cos^2\alpha_0}\right).$$

Using the given values for d and α_0 to express this as a function of v_0,

$$y = (3.00\text{ m})\left(0.90 - \frac{26.62\text{ m}^2/\text{s}^2}{v_0^2}\right).$$

Then, a) $y = 2.14\text{m}$, b) $y = 1.45\text{m}$, c) $y = -2.29\text{m}$. In the last case, the dart was fired with so slow a speed that it hit the ground before traveling the 3-meter horizontal distance.

3.23: a) With $v_y = 0$ in Eq. (3.17), solving for t and substituting into Eq. (3.18) gives

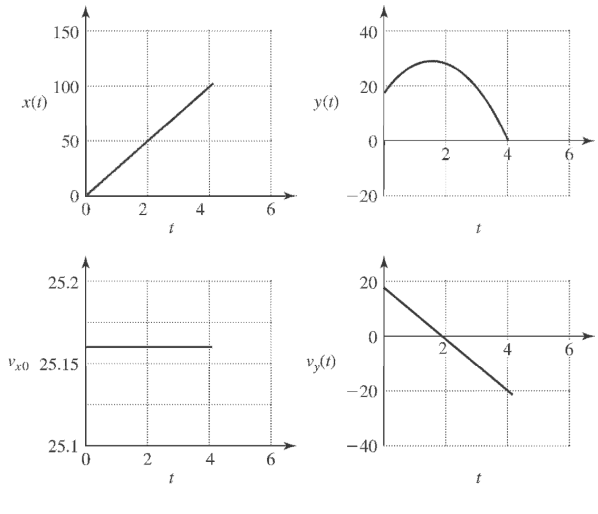
$$(y - y_0) = \frac{v_{0y}^2}{2g} = \frac{v_0^2\sin^2\alpha_0}{2g} = \frac{(30.0\text{ m/s})^2\sin^2 33.0°}{2(9.80\text{ m/s}^2)} = 13.6\text{ m}$$

b) Rather than solving a quadratic, the above height may be used to find the time the rock takes to fall from its greatest height to the ground, and hence the vertical component of velocity, $v_y = \sqrt{2yg} = \sqrt{2(28.6\text{ m})(9.80\text{ m/s}^2)} = 23.7\text{ m/s}$, and so the speed of the rock is $\sqrt{(23.7\text{ m/s})^2 + ((30.0\text{ m/s})(\cos 33.0°))^2} = 34.6\text{ m/s}$.

c) The time the rock is in the air is given by the change in the vertical component of velocity divided by the acceleration $-g$; the distance is the constant horizontal component of velocity multiplied by this time, or

$$x = (30.0\text{ m/s})\cos 33.0°\frac{(-23.7\text{ m/s} - ((30.0\text{ m/s})\sin 33.0°))}{(-9.80\text{ m/s}^2)} = 103\text{ m}.$$

d)



3.24: a)

$$v_0 \cos \alpha t = 45.0\,\text{m}$$

$$\cos \alpha = \frac{45.0\,\text{m}}{(25.0\,\text{m/s})(3.00\,\text{s})} = 0.600$$

$$\alpha = 53.1°$$

b)

$$v_x = (25.0\,\text{m/s})\cos 53.1° = 15.0\,\text{m/s}$$
$$v_y = 0$$
$$v = 15.0\,\text{m/s}$$
$$a = 9.80\,\text{m/s}^2\ \text{downward}$$

c) Find y when $t = 3.00\,\text{s}$

$$y = v_0 \sin \alpha t - \frac{1}{2} g t^2$$
$$= (25.0\,\text{m/s})(\sin 53.1°)(3.00\text{s}) - \frac{1}{2}(9.80\,\text{m/s}^2)(3.00\,\text{s})^2$$
$$= 15.9\,\text{m}$$
$$v_x = 15.0\,\text{m/s} = \text{constant}$$
$$v_y = v_0 \sin \alpha - gt = (25.0\,\text{m/s})(\sin 53.1°) - (9.80\,\text{m/s}^2)(3.00\,\text{s}) = -9.41$$
$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(15.0\,\text{m/s})^2 + (-9.41\,\text{m/s}^2} = 17.7\,\text{m/s}$$

3.25: Take $+y$ to be downward.

a) Use the vertical motion of the rock to find the initial height.

$t = 6.00\,\text{s},\ v_{0y} = +20.0\,\text{s},\ a_y = +9.80\,\text{m/s}^2,\ y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 296\,\text{m}$

b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\,\text{s})(6.00\,\text{s}) = 120\,\text{m}$. So, its height above ground when the rock hits is $296\,\text{m} - 120\,\text{m} = 176\,\text{m}$.

c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\,\text{m})^2 + (90\,\text{m})^2} = 198\,\text{m}$ from the basket when it hits the ground.

d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket.

Just before the rock hits the ground, its vertical component of velocity is

$v_y = v_{0y} + a_y t = 20.0\,\text{s} + (9.80\,\text{m/s}^2)(6.00\,\text{s}) = 78.8\,\text{m/s}$, downward, relative to the ground.

The basket is moving downward at 20.0 m/s, so relative to the basket the rock has downward component of velocity 58.8 m/s.

e) horizontal: 15.0 m/s; vertical: 78.8 m/s

3.26: a) horizontal motion: $x - x_0 = v_{0x}t$ so $t = \frac{60.0\,\text{m}}{(v_0 \cos 43°)t}$

vertical motion (take $+y$ to be upward):

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $25.0\,\text{m} = (v_0 \sin 43.0°)t + \frac{1}{2}(-9.80\,\text{m/s}^2)t^2$

Solving these two simultaneous equations for v_0 and t gives $v_0 = 3.26\,\text{m/s}$ and $t = 2.51\,\text{s}$.

b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6\,\text{m/s}) \sin 43.0° - (9.80\,\text{m/s}^2)(2.51\,\text{s}) = -2.4\,\text{m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

3.27: Take $+y$ to be upward.

Use the vertical motion to find the time it takes the suitcase to reach the ground:

$v_{0y} = v_0 \sin 23°,\ a_y = -9.80\,\text{m/s}^2,\ y - y_0 = -114\,\text{m}, t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60\,\text{s}$

The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795\,\text{m}$

3.28: For any item in the washer, the centripetal acceleration will be proportional to the square of the frequency, and hence inversely proportional to the square of the rotational period; tripling the centripetal acceleration involves decreasing the period by a factor of $\sqrt{3}$, so that the new period T' is given in terms of the previous period T by $T' = T/\sqrt{3}$.

3.29: Using the given values in Eq. (3.30),

$$a_{\text{rad}} = \frac{4\pi^2(6.38\times10^6\ \text{m})}{((24\ \text{h})(3600\ \text{s/h}))^2} = 0.034\ \ \text{m/s}^2 = 3.4\times10^{-3}\,g.$$

(Using the time for the siderial day instead of the solar day will give an answer that differs in the third place.) b) Solving Eq. (3.30) for the period T with $a_{\text{rad}} = g$,

$$T = \sqrt{\frac{4\pi^2(6.38\times10^6\text{m})}{9.80\ \text{m/s}^2}} = 5070\ \text{s} \sim 1.4\ \text{h}.$$

3.30: $550\ \text{rev/min} = 9.17\ \text{rev/s}$, corresponding to a period of 0.109 s. a) From Eq. (3.29), $v = \frac{2\pi R}{T} = 196\ \text{m/s}$. b) From either Eq. (3.30) or Eq. (3.31), $a_{\text{rad}} = 1.13\times10^4\ \text{m/s}^2 = 1.15\times10^3\ \text{g}$.

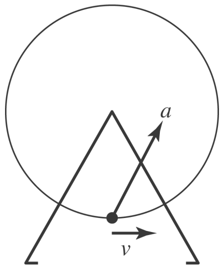
3.31: Solving Eq. (3.30) for T in terms of R and a_{rad},
a) $\sqrt{4\pi^2(7.0\ \text{m})/(3.0)(9.80\ \text{m/s}^2)} = 3.07\ \text{s}$. b) 1.68 s.

3.32: a) Using Eq. (3.31), $\frac{2\pi R}{T} = 2.97\times10^4\ \text{m/s}$. b) Either Eq. (3.30) or Eq. (3.31) gives $a_{\text{rad}} = 5.91\times10^{-3}\ \text{m/s}^2$. c) $v = 4.78\times10^4\ \text{m/s}$, and $a = 3.97\times10^{-2}\ \text{m/s}^2$.

3.33: a) From Eq. (3.31), $a = (7.00\ \text{m/s})^2/(15.0\ \text{m}) = 3.50\ \text{m/s}^2$. The acceleration at the bottom of the circle is toward the center, up.
b) $a = 3.50\ \text{m/s}^2$, the same as part (a), but is directed *down*, and still towards the center.
c) From Eq. (3.29), $T = 2\pi R/v = 2\pi(15.0\ \text{m})/(7.00\ \text{m/s}) = 12.6\ \text{s}$.

3.34: a) $a_{\text{rad}} = (3\text{ m/s})^2/(14\text{ m}) = 0.643\text{ m/s}^2$, and $a_{\tan} = 0.5\text{ m/s}^2$. So,
$a = ((0.643\text{ m/s}^2)^2 + (0.5\text{ m/s}^2)^2)^{1/2} = 0.814\text{ m/s}^2, 37.9°$ to the right of vertical.

b)



3.35: b) No. Only in a circle would a_{rad} point to the center (See planetary motion in Chapter 12).

c) Where the car is farthest from the center of the ellipse.

3.36: Repeated use of Eq. (3.33) gives a) $5.0 = \text{m/s}$ to the right, b) 16.0 m/s to the left, and c) $13.0 = \text{m/s}$ to the left.

3.37: a) The speed relative to the ground is $1.5\text{ m/s} + 1.0\text{ m/s} = 2.5\text{ m/s}$, and the time is $35.0\text{ m}/2.5\text{ m/s} = 14.0\text{ s}$. b) The speed relative to the ground is 0.5 m/s, and the time is 70 s.

3.38: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min). The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream, so the total time the rower takes is

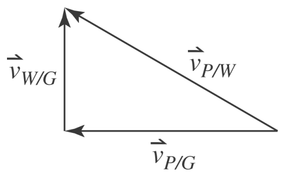
$$\frac{1.5\text{ km}}{6.8\text{ km/h}} + \frac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ hr} = 88\text{ min}.$$

3.39: The velocity components are

$$-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2}\,\text{east} \text{ and } (0.40\text{ m/s})/\sqrt{2}\,\text{south},$$

for a velocity relative to the earth of 0.36 m/s, $52.5°$ south of west.

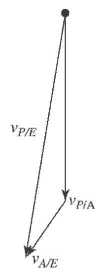
3.40: a) The plane's northward component of velocity relative to the air must be 80.0 km/h, so the heading must be $\arcsin \frac{80.8}{320} = 14°$ north of west. b) Using the angle found in part (a), $(320 \text{ km/h}) \cos 14° = 310 \text{ km/h}$. Equivalently,
$\sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310 \text{ km/h}$.



3.41: a) $\sqrt{(2.0 \text{ m/s})^2 + (4.2 \text{ m/s})^2} = 4.7 \text{ m/s}, \arctan \frac{2.0}{4.2} = 25.5°$, south of east.
b) $800 \text{ m}/4.2 \text{ m/s} = 190 \text{ s}$.
c) $2.0 \text{ m/s} \times 190 \text{ s} = 381 \text{ m}$.

3.42: a) The speed relative to the water is still 4.2 m/s; the necessary heading of the boat is $\arcsin \frac{2.0}{4.2} = 28°$ north of east. b) $\sqrt{(4.2 \text{ m/s})^2 - (2.0 \text{ m/s})^2} = 3.7 \text{ m/s}$, east. d) $800 \text{ m}/3.7 \text{ m/s} = 217 \text{ s}$, rounded to three significant figures.

3.43: a)



b) $x: -(10 \text{ m/s}) \cos 45° = -7.1 \text{ m/s}. \; y := -(35 \text{ m/s}) - (10 \text{ m/s}) \sin 45° = -42.1 \text{ m/s}$.
c) $\sqrt{(-7.1 \text{ m/s})^2 + (-42.1 \text{ m/s})^2} = 42.7 \text{ m/s}, \arctan \frac{-42.1}{-7.1} = 80°$, south of west.

3.44: a) Using generalizations of Equations 2.17 and 2.18,
$v_x = v_{0x} + \frac{\alpha}{3} t^3, v_y = v_{0y} + \beta t - \frac{\gamma}{2} t^2$, and $x = v_{0x} t + \frac{\alpha}{12} t^4, y = v_{0y} t + \frac{\beta}{2} t^2 - \frac{\gamma}{6} t^3$. b) Setting $v_y = 0$ yields a quadratic in $t, 0 = v_{0y} + \beta t - \frac{\gamma}{2} t^2$, which has as the positive solution

$$t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2 v_0 \gamma}\right] = 13.59 \text{ s},$$

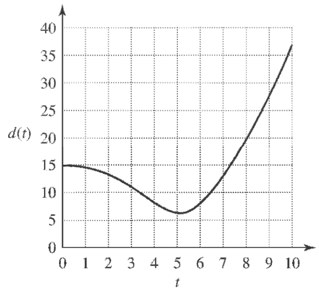
keeping an extra place in the intermediate calculation. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

3.45: a) The $a_x = 0$ and $a_y = -2\beta$, so the velocity and the acceleration will be perpendicular only when $v_y = 0$, which occurs at $t = 0$.

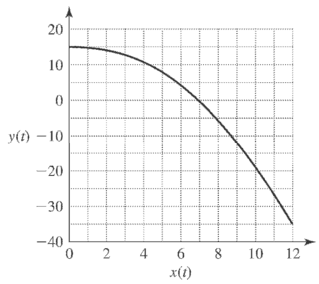
b) The speed is $v = (\alpha^2 + 4\beta^2 t^2)^{1/2}$, $dv/dt = 0$ at $t = 0$. (See part d below.)

c) r and v are perpendicular when their dot product is 0:
$(\alpha t)(\alpha) + (15.0\text{ m} - \beta t^2) \times (-2\beta t) = \alpha^2 t - (30.0\text{ m})\beta t + 2\beta^2 t^3 = 0$. Solve this for t:
$t = \pm\sqrt{\frac{(30.0\text{ m})(0.500\text{ m/s}^2) - (1.2\text{ m/s})^2}{2(0.500\text{ m/s}^2)^2}} = +5.208\text{ s}$, and 0 s, at which times the student is at (6.25 m, 1.44 m) and (0 m, 15.0 m), respectively.

d) At $t = 5.208\text{ s}$, the student is 6.41 m from the origin, at an angle of 13° from the x-axis. A plot of $d(t) = (x(t)^2 + y(t)^2)^{1/2}$ shows the minimum distance of 6.41 m at 5.208 s:



e) In the x - y plane the student's path is:



3.46: a) Integrating, $\vec{r} = (\alpha t - \frac{\beta}{3}t^3)\hat{i} + (\frac{\gamma}{2}t^2)\hat{j}$. Differentiating, $\vec{a} = (-2\beta)\hat{i} + \gamma\hat{j}$.

b) The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the y-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\text{ m/s})(4.0\text{ m/s}^2)}{2(1.6\text{ m/s}^3)} = 9.0\text{ m}$$

.

3.47: a) The acceleration is

$$a=\frac{v^2}{2x}=\frac{((88\ \text{km/h})(1\ \text{m/s})/(3.6\ \text{km/h}))^2}{2(300\ \text{m})}=0.996\ \text{m/s}^2\approx 1\ \text{m/s}^2$$

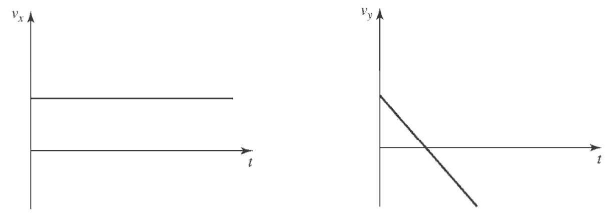
b) $\arctan\left(\frac{15\ \text{m}}{460\ \text{m}-300\ \text{m}}\right)=5.4°$. c) The vertical component of the velocity is $(88\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\frac{15\ \text{m}}{160\ \text{m}}=2.3\ \text{m/s}$. d) The average speed for the first 300 m is 44 km/h, so the elapsed time is

$$\frac{300\ \text{m}}{(44\ \text{km/h})(1\ \text{m/s})(3.6\ \text{km/h})}+\frac{160\ \text{m}}{(88\ \text{km/h})(1\ \text{m/s})\cos 5.4°/(3.6\ \text{km/h})}=31.1\ \text{s},$$

or 31 s to two places.

3.48:

a)



The equations of motions are:

$$y = h + (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

$$x = (v_0 \cos\alpha)t$$

$$v_y = v_0 \sin\alpha - gt$$

$$v_x = v_0 \cos\alpha$$

Note that the angle of 36.9° results in $\sin 36.9° = 3/5$ and $\cos 36.9° = 4/5$.

b) At the top of the trajectory, $v_y = 0$. Solve this for *t* and use in the equation for *y* to find the maximum height: $t = \frac{v_0 \sin\alpha}{g}$. Then, $y = h + (v_0 \sin\alpha)\left(\frac{v_0 \sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\alpha}{g}\right)^2$, which reduces to $y = h + \frac{v_0^2 \sin^2\alpha}{2g}$. Using $v_0 = \sqrt{25gh/8}$, and $\sin\alpha = 3/5$, this becomes $y = h + \frac{(25gh/8)(3/5)^2}{2g} = h + \frac{9}{16}h$, or $y = \frac{25}{16}h$. Note: This answer assumes that $y_0 = h$. Taking $y_0 = 0$ will give a result of $y = \frac{9}{16}h$ (above the roof).

c) The total time of flight can be found from the *y* equation by setting $y = 0$, assuming $y_0 = h$, solving the quadratic for *t* and inserting the total flight time in the *x* equation to find the range. The quadratic is $\frac{1}{2}gt^2 - \frac{3}{5}v_0 - h = 0$. Using the quadratic formula gives $t = \frac{(3/5)v_0 \pm \sqrt{(-(3/5)v_0)^2 - 4(\frac{1}{2}g)(-h)}}{2(\frac{1}{2}g)}$. Substituting $v_0 = \sqrt{25gh/8}$ gives $t = \frac{(3/5)\sqrt{25gh/8} \pm \sqrt{\frac{9}{25} \bullet \frac{25gh}{8} + \frac{16gh}{8}}}{g}$. Collecting terms gives *t*: $t = \frac{1}{2}\left(\sqrt{\frac{9h}{2g}} \pm \sqrt{\frac{25h}{2g}}\right) = \frac{1}{2}\left(3\sqrt{\frac{h}{2g}} \pm 5\sqrt{\frac{h}{2g}}\right)$. Only the positive root is meaningful and so $t = 4\sqrt{\frac{h}{2g}}$. Then, using $x = (v_0 \cos\alpha)t$, $x = \sqrt{\frac{25gh}{8}}\left(\frac{4}{5}\right)\left(4\sqrt{\frac{h}{2g}}\right) = 4h$.

3.49: The range for a projectile that lands at the same height from which it was launched is $R = \frac{v_0^2 \sin 2\alpha}{g}$. Assuming $\alpha = 45°$, and $R = 50$ m, $v_0 = \sqrt{gR} = 22$ m/s.

3.50: The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(8.00\text{ m})}{5.00\text{ s}} = \frac{50.27\text{ m}}{5.00\text{ s}} = 10.05\text{ m/s}$$

Thus its velocity consists of the components $v_x = 10.05\text{ m/s}$ and $v_y = 3.00\text{ m/s}$. The speed relative to the ground is then

$$v = v_x^2 + v_y^2 = 10.052 + 3.002 = 10.49\text{ m/s or } 10.5\text{ m/s}$$

(b) The bird's speed is constant, so its acceleration is strictly centripetal–entirely in the horizontal direction, toward the center of its spiral path–and has magnitude

$$a_c = \frac{v_x^2}{r} = \frac{(10.05\text{ m/s})^2}{8.00\text{ m}} = 12.63\text{ m/s}^2 \quad \text{or} \quad 12.6\text{ m/s}^2$$

(c) Using the vertical and horizontal velocity components:

$$\theta = \tan^{-1}\frac{3.00\text{ m/s}}{10.05\text{ m/s}} = 16.6°$$

3.51: Take $+y$ to be downward.

Use the vertical motion to find the time in the air:

$v_{0y} = 0, a_y = 9.80\text{ m/s}^2, y - y_0 = 25\text{ m}, t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259\text{ s}$

During this time the dart must travel 90 m horizontally, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{90\text{ m}}{2.25\text{ s}} = 40\text{ m/s}$$

3.52: a) Setting $y = -h$ in Eq. (3.27) (h being the stuntwoman's initial height above the ground) and rearranging gives

$$x^2 - \frac{2v_0^2 \sin\alpha_0 \cos\alpha_0}{g}x - \frac{2v_{0x}^2}{g}h = 0,$$

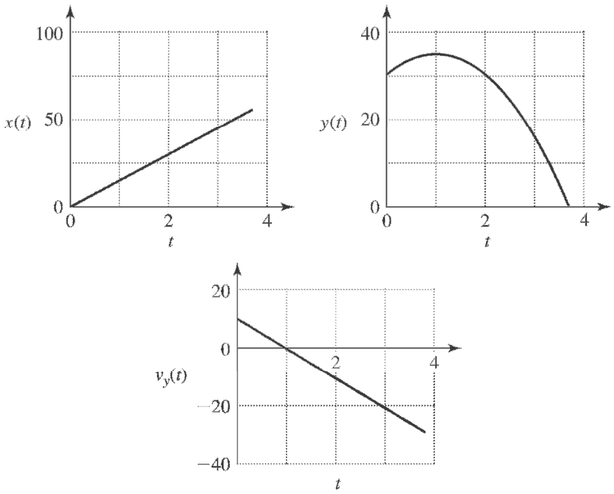
The easier thing to do here is to recognize that this can be put in the form

$$x^2 - \frac{2v_{0x}v_{0y}}{g}x - \frac{2v_{0x}^2}{g}h = 0,$$

the solution to which is

$$x = \frac{v_{0x}}{g}\left[v_{0y} + \sqrt{v_{0y}^2 + 2gh}\right] = 55.5 \text{ m (south)}.$$

b) The graph of $v_x(t)$ is a horizontal line.



3.53: The distance is the horizontal speed times the time of free fall,

$$v_x\sqrt{\frac{2y}{g}} = (64.0 \text{ m/s})\sqrt{\frac{2(90 \text{ m})}{(9.80 \text{ m/s}^2)}} = 274 \text{ m}.$$

3.54: In terms of the range R and the time t that the balloon is in the air, the car's original distance is $d = R + v_{\text{car}}t$. The time t can be expressed in terms of the range and the horizontal component of velocity, $t = \frac{R}{v_0 \cos\alpha_0}$, so $d = R\left(1 + \frac{v_{\text{car}}}{v_0 \cos\alpha_0}\right)$. Using $R = v_0^2 \sin 2\alpha_0 / g$ and the given values yields $d = 29.5 \text{ m}$.

3.55: a) With $\alpha_0 = 45°$, Eq. (3.27) is solved for $v_0^2 = \frac{gx^2}{x-y}$. In this case, $y = -0.9\ \text{m}$ is the change in height. Substitution of numerical values gives $v_0 = 42.8\ \text{m/s}$. b) Using the above algebraic expression for v_0 in Eq. (3.27) gives

$$y = x - \left(\frac{x}{188\ \text{m}}\right)^2 (188.9\ \text{m})$$

Using $x = 116\ \text{m}$ gives $y = 44.1\ \text{m}$ above the initial height, or 45.0 m above the ground, which is 42.0 m above the fence.

3.56: The equations of motions are $y = (v_0 \sin\alpha)t - 1/2gt^2$ and $x = (v_0 \cos\alpha)t$, assuming the match starts out at $x = 0$ and $y = 0$. When the match goes in the wastebasket for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the match goes in the wastebasket for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$..

To reach the *minimum* distance: $6D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for *t* gives $t = \frac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\frac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for *t* gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\frac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

3.57: The range for a projectile that lands at the same height from which it was launched is $R = \frac{v_0^2 \sin 2\alpha}{g}$, and the maximum height that it reaches is $H = \frac{v_0^2 \sin^2\alpha}{2g}$. We must find *R* when $H = D$ and $v_0 = \sqrt{6gD}$. Solving the height equation for $\sin\alpha$, $D = \frac{6gD \sin^2\alpha}{2g}$, or $\sin\alpha = (1/3)^{1/2}$. Then, $R = \frac{6gD\sin(70.72°)}{g}$, or $R = 5.6569D = 4\sqrt{2}D$.

3.58: Equation 3.27 relates the vertical and horizontal components of position for a given set of initial values.

a) Solving for v_0 gives

$$v_0^2 = \frac{gx^2 / 2\cos^2 \alpha_0}{x \tan \alpha_0 - y}.$$

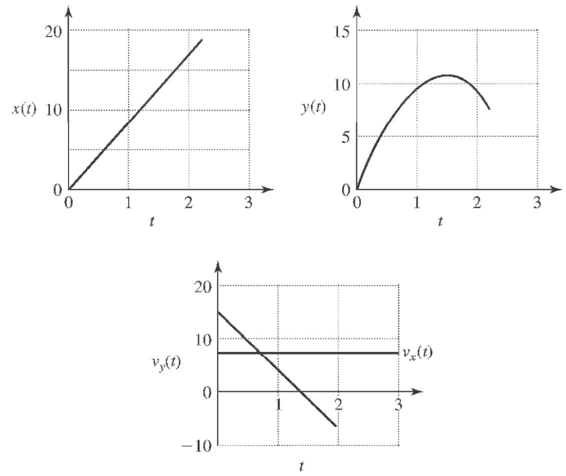
Insertion of numerical values gives $v_0 = 16.6 \text{ m/s}$.

b) Eliminating *t* between Equations 3.20 and 3.23 gives v_y as a function of x,

$$v_y = v_0 \sin \alpha_0 - \frac{gx}{v_0 \cos \alpha_0}.$$

Using the given values yields $v_x = v_0 \cos \alpha_0 = 8.28 \text{ m/s}, v_y = -6.98 \text{ m/s}$, so $v = \sqrt{(8.28 \text{ m/s})^2 + (-6.98 \text{ m/s})^2} = 10.8 \text{ m/s}$, at an angle of $\arctan\left(\frac{-6.98}{8.24}\right) = -40.1°$, with the negative sign indicating a direction *below* the horizontal.

c) The graph of $v_x(t)$ is a horizontal line.



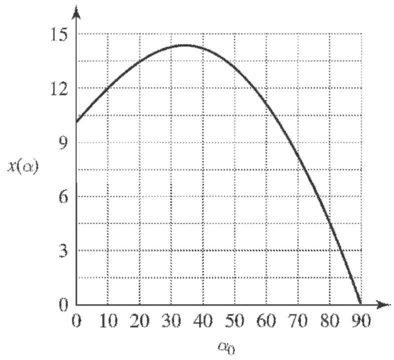
3.59: a) In Eq. (3.27), the change in height is $y=-h$. This gives a quadratic equation in x, the solution to which is

$$x=\frac{v_0^2\cos\alpha_0}{g}\left[\tan^2\alpha_0+\frac{2gh}{v_0^2\cos\alpha_0}\right]$$

$$=\frac{v_0\cos\alpha_0}{g}\left[v_0\sin\alpha_0+\sqrt{v_0^2\sin^2\alpha_0+2gh}\right]$$

If $h=0$, the square root reduces to $v_0\sin\alpha_0$, and $x=R$. b) The expression for x becomes $x=(10.2\text{ m})\cos\alpha_0+[\sin^2\alpha_0+\sqrt{\sin^2\alpha_0+0.98}]$

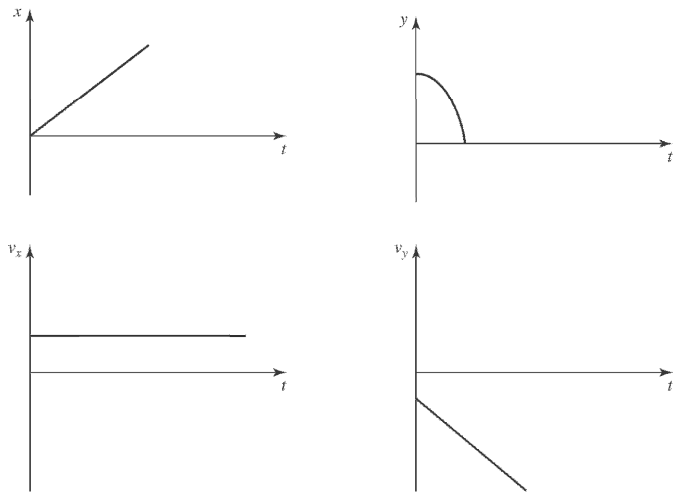
The angle $\alpha_0=90°$ corresponds to the projectile being launched straight up, and there is no horizontal motion. If $\alpha_0=0$, the projectile moves horizontally until it has fallen the distance h.



c) The maximum range occurs for an angle less than $45°$, and in this case the angle is about $36°$.

3.60: a) This may be done by a direct application of the result of Problem 3.59; with $\alpha_0 = -40°$, substitution into the expression for *x* gives 6.93 m.

b)



c) Using $(14.0 \text{ m} - 1.9 \text{ m})$ instead of *h* in the above calculation gives $x = 6.3 \text{ m}$, so the man will not be hit.

3.61: a) The expression for the range, as derived in Example 3.10, involves the sine of twice the launch angle, and

$$\sin(2(90° - \alpha_0)) = \sin(180° - 2\alpha_0) = \sin 180° \cos 2\alpha_0 - \cos 180° \sin 2\alpha_0 = \sin 2\alpha_0,$$

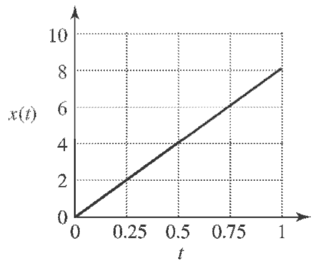
and so the range is the same. As an alternative, using $\sin(90° - \alpha_0) = \cos\alpha$ and $\cos(90° - \alpha_0) = \sin\alpha_0$ in the expression for the range that involves the product of the sine and cosine of α_0 gives the same result.

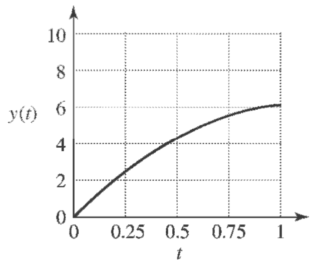
b) The range equation is $R = \frac{v_0^2 \sin 2\alpha}{g}$. In this case, $v_0 = 2.2 \text{ m/s}$ and $R = 0.25 \text{ m}$. Hence $\sin 2\alpha = (9.8 \text{ m/s}^2)(0.25 \text{ m})/(2.2 \text{ m/s}^2)$, or $\sin 2\alpha = 0.5062$; and $\alpha = 15.2°$ or $74.8°$.

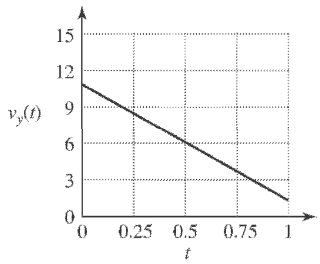
3.62: a) Using the same algebra as in Problem 3.58(a), $v_0 = 13.8 \text{ m/s}$.

b) Again, the algebra is the same as that used in Problem 3.58; $v = 8.4 \text{ m/s}$, at an angle of $9.1°$, this time above the horizontal.

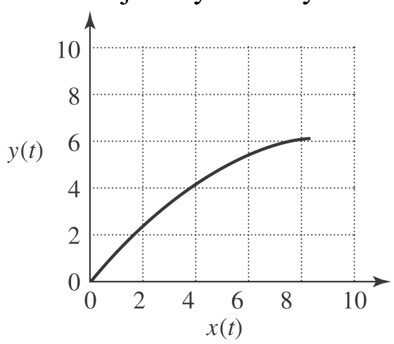
c) The graph of $v_x(t)$ is a horizontal line.







A graph of $y(t)$ vs. $x(t)$ shows the trajectory of Mary Belle as viewed from the side:



d) In this situation it's convenient to use Eq. (3.27), which becomes $y = (1.327)x - (0.071115 \text{ m}^{-1})x^2$. Use of the quadratic formula gives $x = 23.8 \text{ m}$.

3.63: a) The algebra is the same as that for Problem 3.58,

$$v_0^2 = \frac{gx^2}{2\cos^2\alpha_0(x\tan\alpha_0 - y)}.$$

In this case, the value for y is -15.0 m, the change in height. Substitution of numerical values gives 17.8 m/s. b) 28.4 m from the near bank (i.e., in the water!).

3.64: Combining equations 3.25, 3.22 and 3.23 gives

$$\begin{aligned}
v^2 &= v_0^2\cos^2\alpha_0 + (v_0\sin\alpha_0 - gt)^2 \\
&= v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0\sin\alpha_0 gt + (gt)^2 \\
&= v_0^2 - 2g(v_0\sin\alpha_0 t - \frac{1}{2}gt^2) \\
&= v_0^2 - 2gy,
\end{aligned}$$

where Eq. (3.21) has been used to eliminate t in favor of y. This result, which will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any y, positive, negative or zero, as long as $v^2 > 0$. For the case of a rock thrown from the roof of a building of height h, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh}$, which is independent of α_0.

3.65: Take $+y$ to be upward. The vertical motion of the rocket is unaffected by its horizontal velocity.

a) $v_y = 0$ (at maximum height), $v_{0y} = +40.0\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 81.6\ \text{m}$

b) Both the cart and the rocket have the same constant horizontal velocity, so both travel the same horizontal distance while the rocket is in the air and the rocket lands in the cart.

c) Use the vertical motion of the rocket to find the time it is in the air.

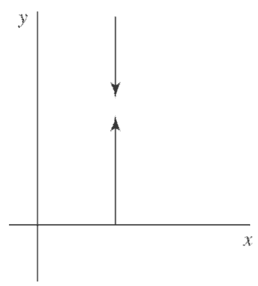
$v_{0y} = 40\ \text{m/s}, a_y = -9.80\ \text{m/s}^2, v_y = -40\ \text{m/s}, t = ?$

$v_y = v_{0y} + a_y t$ gives $t = 8.164\ \text{s}$

Then $x - x_0 = v_{0x}t = (30.0\ \text{m/s})(8.164\ \text{s}) = 245\ \text{m}.$

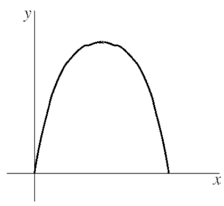
d) Relative to the ground the rocket has initial velocity components $v_{0x} = 30.0\ \text{m/s}$ and $v_{0y} = 40.0\ \text{m/s}$, so it is traveling at $53.1°$ above the horizontal.

e) (i)



Relative to the cart, the rocket travels straight up and then straight down

(ii)



Relative to the ground the rocket travels in a parabola.

3.66: (a)

$$
\begin{aligned}
v_x(\text{runner}) &= v_x(\text{ball}) \\
6.00\ \text{m/s} &= (20.0\ \text{m/s})\ \cos\theta \\
\cos\theta &= 0.300 \\
\theta &= 72.5°
\end{aligned}
$$

Time the ball is in the air:

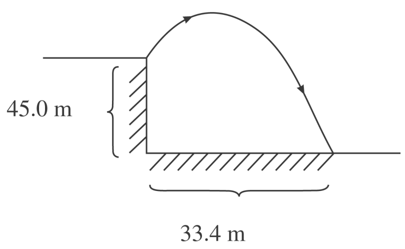
$$
\begin{aligned}
y &= v_0 \sin\theta t - \tfrac{1}{2} g t^2 \\
-45.0\ \text{m} &= (20.0\ \text{m/s})(\sin 72.5°)t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2
\end{aligned}
$$

Solve for t: $t = 5.549\ \text{s}$.

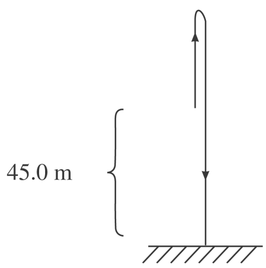
$$
\begin{aligned}
x &= v_0 \cos\theta t = (20.0\ \text{m/s})(\cos 72.5°)(5.549\ \text{s}) \\
&= 33.4\ \text{m}
\end{aligned}
$$

(b)

(i)



(ii)



3.67: Take $+y$ to be downward.

a) Use the vertical motion of the boulder to find the time it takes it to fall 20 m to the level of the surface of the water.

$v_{0y} = 0, a_y = 9.80\ \text{m/s}^2, y - y_0 = 20\ \text{m}, t = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = 2.02\ \text{s}$

The rock must travel 100 m horizontally during this time, so

$$v_{0x} = \frac{x - x_0}{t} = \frac{100\ \text{m}}{2.20\ \text{s}} = 49\ \text{m/s}.$$

b) The rock travels downward 45 m in going from the cliff to the plain. Use this vertical motion to find the time:

$v_{0y} = 0, a_y = 9.80\ \text{m/s}^2, y - y_0 = 45\ \text{m}, t = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = 3.03\ \text{s}$

During this time the rock travels horizontally

$x - x_0 = v_{0x}t = (49\ \text{m/s})(3.03\ \text{s}) = 150\ \text{m}$

The rock lands 50 m past the foot of the dam.

3.68: (a) When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 43.9 m from rest. Get the time to fall:

$$y = \frac{1}{2}gt^2$$

$$43.9\ \text{m} = \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$t = 2.99\ \text{s}$$

So she has been jogging for $9.00\ \text{s} + 2.99\ \text{s} = 12.0\ \text{s}$. During this time she has gone $x = vt = (3.05\ \text{m/s})(12.0\ \text{s}) = 36.6\ \text{m}$. Bruce must throw the bagels so they travel 36.6 m horizontally in 2.99 s

$$x = vt$$

$$36.6\ \text{m} = v(2.99\ \text{s})$$

$$v = 12.2\ \text{m/s}$$

(b) 36.6 m from the building.

3.69: Take $+y$ to be upward.

a) The vertical motion of the shell is unaffected by the horizontal motion of the tank. Use the vertical motion of the shell to find the time the shell is in the air:
$v_{0y} = v_0 \sin\alpha = 43.4\ \text{m/s},\ a_y = -9.80\ \text{m/s}^2,\ y - y_0 = 0$ (returns to initial height), $t = ?$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 8.86\ \text{s}$

Relative to tank #1 the shell has a constant horizontal velocity $v_0 \cos\alpha = 246.2\ \text{m/s}$. Relative to the ground the horizontal velocity component is $246.2\ \text{m/s} + 15.0\ \text{m/s} = 261.2\ \text{m/s}$. Relative to tank #2 the shell has horizontal velocity component $261.2\ \text{m/s} - 35.0\ \text{m/s} = 226.2\ \text{m/s}$. The distance between the tanks when the shell was fired is the $(226.2\ \text{m/s})(8.86\ \text{s}) = 2000\ \text{m}$ that the shell travels relative to tank #2 during the 8.86 s that the shell is in the air.

b) The tanks are initially 2000 m apart. In 8.86 s tank #1 travels 133 m and tank #2 travels 310 m, in the same direction. Therefore, their separation increases by $310\ \text{m} - 183\ \text{m} = 177\ \text{m}$. So, the separation becomes 2180 m (rounding to 3 significant figures).

3.70: The firecracker's falling time can be found from the usual

$$t = \sqrt{\frac{2h}{g}}$$

The firecracker's horizontal position at any time t (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2 = 0$ when cracker hits the ground, from which we can find that $t = 2v/a$. Combining this with the expression for the falling time:

$$\frac{2v}{a} = \sqrt{\frac{2h}{g}}$$

so

$$h = \frac{2v^2 g}{a^2}$$

3.71: a) The height above the player's hand will be $\frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2 \alpha_0}{2g} = 0.40\ \text{m}$, so the maximum height above the floor is 2.23 m. b) Use of the result of Problem 3.59 gives 3.84 m. c) The algebra is the same as that for Problems 3.58 and 3.62. The distance y is $3.05\ \text{m} - 1.83\ \text{m} = 1.22\ \text{m}$, and

$$v_0 = \sqrt{\frac{(9.80\ \text{m/s}^2)(4.21\ \text{m})^2}{2\cos^2 35°((4.21\ \text{m})\tan 35° - 1.22\ \text{m})}} = 8.65\ \text{m/s}.$$

d) As in part (a), but with the larger speed,

$$1.83\ \text{m} + (8.65\ \text{m/s})^2 \sin^2 35°/2(9.80\ \text{m/s}^2) = 3.09\ \text{m}.$$

The distance from the basket is the distance from the foul line to the basket, minus half the range, or

$$4.21\ \text{m} - (8.655\ \text{m/s})^2 \sin 70°/2(9.80\ \text{m/s}^2) = 0.62\ \text{m}.$$

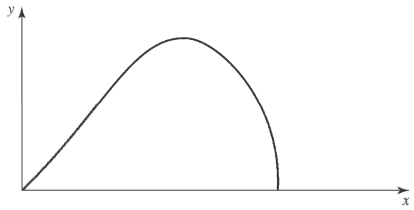
Note that an extra figure in the intermediate calculation was kept to avoid roundoff error.

3.72: The initial y-component of the velocity is $v_{0y} = \sqrt{2gy}$, and the time the pebble is in flight is $t = \sqrt{2y/g}$. The initial x-component is $v_{0x} = x/t = \sqrt{x^2 g/2y}$. The magnitude of the initial velocity is then

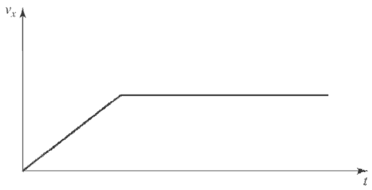
$$v_0 = \sqrt{2gy + \frac{x^2 g}{2y}} = \sqrt{2gy}\sqrt{1 + \left(\frac{x}{2y}\right)^2},$$

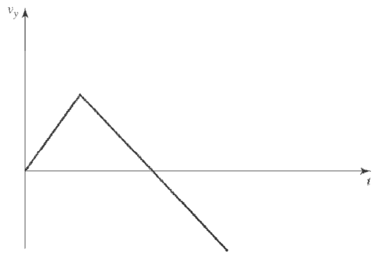
and the angle is $\arctan\left(\frac{v_{0y}}{v_{0x}}\right) = \arctan\,(2y/x)$.

3.73: a) The acceleration is given as g at an angle of $53.1°$ to the horizontal. This is a 3-4-5 triangle, and thus, $a_x = (3/5)g$ and $a_y = (4/5)g$ during the "boost" phase of the flight. Hence this portion of the flight is a straight line at an angle of $53.1°$ to the horizontal. After time *T*, the rocket is in free flight, the acceleration is $a_x = 0$ and $a_y = g$, and the familiar equations of projectile motion apply. During this coasting phase of the flight, the trajectory is the familiar parabola.



b) During the boost phase, the velocities are: $v_x = (3/5)gt$ and $v_y = (4/5)gt$, both straight lines. After $t = T$, the velocities are $v_x = (3/5)gT$, a horizontal line, and $v_y = (4/5)gT - g(t - T)$, a negatively sloping line which crosses the axis at the time of the maximum height.





c) To find the maximum height of the rocket, set $v_y = 0$, and solve for *t*, where $t = 0$ when the engines are cut off, use this time in the familiar equation for *y*. Thus, using $t = (4/5)T$ and

$y_{\max} = y_0 + v_{0y}t - \frac{1}{2}gt^2$, $y_{\max} = \frac{2}{5}gT^2 + \frac{4}{5}gT(\frac{4}{5}T) - \frac{1}{2}g(\frac{4}{5}T)^2$, $y_{\max} = \frac{2}{5}gT^2 + \frac{16}{25}gT^2 - \frac{8}{25}g$

Combining terms, $y_{\max} = \frac{18}{25}gT^2$.

d) To find the total horizontal distance, break the problem into three parts: The boost phase, the rise to maximum, and the fall back to earth. The fall time back to earth can be found from the answer to part (c), $(18/25)gT^2 = (1/2)gt^2$, or $t = (6/5)T$. Then, multiplying these times and the velocity, $x = \frac{3}{10}gT^2 + (\frac{3}{5}gT)(\frac{4}{5}T) + (\frac{3}{5}gT)(\frac{6}{5}T)$, or $x = \frac{3}{10}gT^2 + \frac{12}{25}gT^2 + \frac{18}{25}gT^2$. Combining terms gives $x = \frac{3}{2}gT^2$.

3.74: In the frame of the hero, the range of the object must be the initial separation plus the amount the enemy has pulled away in that time. Symbolically, $R = x_0 + v_{\text{E/H}}t = x_0 + v_{\text{E/H}}\frac{R}{v_{0x}}$, where $v_{\text{E/H}}$ is the velocity of the enemy relative to the hero, t is the time of flight, $v_{0\text{x}}$ is the (constant) x-component of the grenade's velocity, as measured by the hero, and R is the range of the grenade, also as measured by the hero. Using Eq. (3-29) for R, with $\sin 2\alpha_0 = 1$ and $v_{0x} = v_0/\sqrt{2}$,

$$\frac{v_0^2}{g} = x_0 + v_{\text{E/H}}\frac{v_0}{g}\sqrt{2}, \quad \text{or} \quad v_0^2 - (\sqrt{2}v_{\text{E/H}})v_0 - gx_0 = 0.$$

This quadratic is solved for

$$v_0 = \frac{1}{2}(\sqrt{2}v_{\text{E/H}} + \sqrt{2v_{\text{E/H}}^2 + 4gx_0}) = 61.1\,\text{km/h},$$

where the units for g and x_0 have been properly converted. Relative to the earth, the x-component of velocity is $90.0\,\text{km/h} + (61.1\,\text{km/h})\cos 45° = 133.2\,\text{km/h}$, the y-component, the same in both frames, is $(61.1\,\text{km/h})\sin 45° = 43.2\,\text{km/h}$, and the magnitude of the velocity is then 140 km/h.

3.75: a) $x^2 + y^2 = (R\cos\omega t)^2 + (R\sin\omega t)^2 = R^2(\cos^2\omega t + \sin^2\omega t) = R^2$, so the radius is R.

b)
$$v_x = -\omega R\sin\omega t,\ v_y = \omega R\cos\omega t,$$

and so the dot product

$$\begin{aligned}\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{v}} &= xv_x + yv_y\\ &= (R\cos\omega t)(-\omega R\sin\omega t) + (R\sin\omega t)(\omega R\cos\omega t)\\ &= \omega R(-\cos\omega t\sin\omega t + \sin\omega t\cos\omega t)\\ &= 0.\end{aligned}$$

c) $a_x = -\omega^2 R\cos\omega t = -\omega^2 x, \quad a_y = \omega^2 R\sin\omega t = -\omega^2 y,$

and so $\vec{\boldsymbol{a}} = -\omega^2\vec{\boldsymbol{r}}$ and $a = \omega^2 R.$

d) $v^2 = v_x^2 + v_y^2 = (-\omega R\sin\omega t)^2 + (\omega R\cos\omega t)^2 = \omega^2 R^2(\sin^2\omega t + \cos^2\omega t) = \omega^2 R^2$, and so $v = \omega R$.

e)
$$a = \omega^2 R = \frac{(\omega R)^2}{R} = \frac{v^2}{R}.$$

3.76: a)

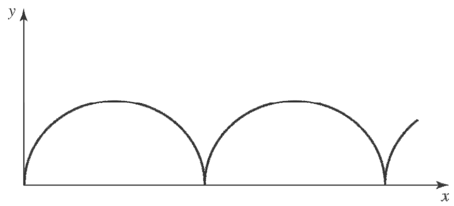
$$\frac{dv}{dt}=\frac{d}{dt}\sqrt{v_x^2+v_y^2}$$
$$=\frac{(1/2)\frac{d}{dt}(v_x^2+v_y^2)}{\sqrt{v_x^2+v_y^2}}$$
$$=\frac{v_xa_x+v_ya_y}{\sqrt{v_x^2+v_y^2}}.$$

b) Using the numbers from Example 3.1 and 3.2,

$$\frac{dv}{dt}=\frac{(-1.0\text{ m/s})(-0.50\text{ m/s}^2)+(1.3\text{ m/s})(0.30\text{ m/s}^2)}{\sqrt{(-1.0\text{ m/s})^2+(1.3\text{ m/s})^2}}=0.54\text{ m/s}.$$

The acceleration is due to changing both the magnitude and direction of the velocity. If the direction of the velocity is changing, the magnitude of the acceleration is larger than the rate of change of speed. c) $\vec{v}\cdot\vec{a}=v_xa_x+v_ya_y,\ v=\sqrt{v_x^2+v_y^2}$, and so the above form for $\frac{dv}{dt}$ is seen to be $\vec{v}\cdot\vec{a}/v.$

3.77: a) The path is a cycloid.



b) To find the velocity components, take the derivative of x and y with respect to time: $v_x=R\omega(1-\cos\omega t)$, and $v_y=R\omega\sin\omega t.$ To find the acceleration components, take the derivative of v_x and v_y with respect to time: $a_x=R\omega^2\sin\omega t,$ and $a_y=R\omega^2\cos\omega t.$

c) The particle is at rest $(v_y=v_x=0)$ every period, namely at $t=0, 2\pi/\omega, 4\pi/\omega, \ldots.$ At that time, $x=0, 2\pi R, 4\pi R, \ldots;$ and $y=0.$ The acceleration is $a=R\omega^2$ in the $+y$ - direction.

d) No, since $a=\left[\left(R\omega^2\sin\omega t\right)^2+\left(R\omega^2\cos\omega t\right)^2\right]^{1/2}=R\omega^2.$

3.78: A direct way to find the angle is to consider the velocity relative to the air and the velocity relative to the ground as forming two sides of an isosceles triangle. The wind direction relative to north is half of the included angle, or $\arcsin(10/50)=11.53°,$ east of north.

3.79: Finding the infinite series consisting of the times between meeting with the brothers is possible, and even entertaining, but hardly necessary. The relative speed of the brothers is 70 km/h, and as they are initially 42 km apart, they will reach each other in six-tenths of an hour, during which time the pigeon flies 30 km.

3.80: a) The drops are given as falling vertically, so their horizontal component of velocity with respect to the earth is zero. With respect to the train, their horizontal component of velocity is 12.0 m/s, west (as the train is moving eastward). b) The vertical component, in either frame, is $(12.0\ \text{m/s})/(\tan 30°) = 20.8\ \text{m/s},$ and this is the magnitude of the velocity in the frame of the earth. The magnitude of the velocity in the frame of the train is $\sqrt{(12.0\ \text{m/s})^2 + (20.8\ \text{m/s})^2} = 24\ \text{m/s}.$ This is, of course, the same as $(12.0\ \text{m/s})/\sin 30°.$

3.81: a) With no wind, the plane would be 110 km west of the starting point; the wind has blown the plane 10 km west and 20 km south in half an hour, so the wind velocity is $\sqrt{(20\ \text{km/h})^2 + (40\ \text{km/h})^2} = 44.7\ \text{km/h}$ at a direction of $\arctan(40/20) = 63°$ south of west. b) $\arcsin(40/220) = 10.5°$ north of west.

3.82: a) $2D/v$ b) $2Dv/(v^2 - w^2)$ c) $2D/\sqrt{v^2 - w^2}$ d) 1.50 h, 1.60 h, 1.55 h.

3.83: a) The position of the bolt is $3.00\ \text{m} + (2.50\ \text{m/s})t - 1/2(9.80\ \text{m/s}^2)t^2,$ and the position of the floor is (2.50 m/s)t. Equating the two, $3.00\ \text{m} = (4.90\ \text{m/s}^2)t^2.$ Therefore $t = 0.782\ \text{s}.$ b) The velocity of the bolt is $2.50\ \text{m/s} - (9.80\ \text{m/s}^2)(0.782\ \text{s}) = -5.17\ \text{m/s}$ relative to Earth, therefore, relative to an observer in the elevator $v = -5.17\ \text{m/s} - 2.50\ \text{m/s} = -7.67\ \text{m/s}.$ c) As calculated in part (b), the speed relative to Earth is 5.17 m/s. d) Relative to Earth, the distance the bolt travelled is $(2.50\ \text{m/s})t - 1/2(9.80\ \text{m/s}^2)t^2 = (2.50\ \text{m/s})(0.782\ \text{s}) - (4.90\ \text{m/s}^2)(0.782\ \text{s})^2 = -1.04\ \text{m}$

3.84: Air speed of plane $= \frac{5310\ \text{km}}{6.60\ \text{h}} = 804.5\ \text{km/h}$

With wind from A to B:

$$t_{\text{AB}} + t_{\text{BA}} = 6.70\ \text{h}$$

Same distance both ways:

$$(804.5\ \text{km/h} + v_{\text{w}})t_{\text{AB}} = \frac{5310\ \text{km}}{2} = 2655\ \text{km}$$

$$(804.5\ \text{km/h} + v_{\text{w}})t_{\text{BA}} = 2655\ \text{km}$$

Solve (1), (2), and (3) to obtain wind speed v_{w} :

$$v_{\text{w}} = 98.1\ \text{km/h}$$

3.85: The three relative velocities are:

$\vec{v}_{J/G,}$ Juan relative to the ground. This velocity is due north and has magnitude

$v_{J/G} = 8.00\,\text{m/s}.$

$\vec{v}_{B/G,}$ the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude

$v_{B/G} = 12.0\,\text{m/s}.$

$\vec{v}_{B/J,}$ the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{B/G} = \vec{v}_{B/J} + \vec{v}_{J/G,}$ so $\vec{v}_{B/J} = \vec{v}_{B/G} - \vec{v}_{J/G}$.

Take $+y$ to be north and $+x$ to be east.

$v_{B/Jx} = +v_{B/G}\sin 37.0° = 7.222\,\text{m/s}$

$v_{B/Jy} = +v_{B/G}\cos 37.0° - v_{J/G} = 1.584\,\text{m/s}$

These two components give $v_{B/J} = 7.39\,\text{m/s}$ at $12.4°$ north of east.

3.86: a) $v_{0y} = \sqrt{2gh} = \sqrt{2(9.80\,\text{m/s}^2)(4.90\,\text{m})} = 9.80\,\text{m/s}.$ b) $v_{0y}/g = 1.00\,\text{s}.$ c) The speed relative to the man is $\sqrt{(10.8\,\text{m/s})^2 - (9.80\,\text{m/s})^2} = 4.54\,\text{m/s},$ and the speed relative to the hoop is 13.6 m/s (rounding to three figures), and so the man must be 13.6 m in front of the hoop at release. d) Relative to the flat car, the ball is projected at an angle $\theta = \tan^{-1}\left(\frac{9.80\text{ m/s}}{4.54\text{ m/s}}\right) = 65°$. Relative to the ground the angle is $\theta = \tan^{-1}\left(\frac{9.80\text{ m/s}}{4.54\text{ m/s}+9.10\text{ m/s}}\right) = 35.7°$

3.87: a) $(150\,\text{m/s})^2\sin 2°/9.80\,\text{m/s}^2 = 80\,\text{m}.$

b) $1000 \times \frac{\pi(10\times10^{-2}\text{ m})^2}{\pi(80\text{ m})^2} = 1.6\times10^{-3}.$

c) The slower rise will tend to reduce the time in the air and hence reduce the radius. The slower horizontal velocity will also reduce the radius. The lower speed would tend to increase the time of descent, hence increasing the radius. As the bullets fall, the friction effect is smaller than when they were rising, and the overall effect is to decrease the radius.

3.88: Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to *t*, and solve for the value of *t* when this derivative is zero. If the discriminant is zero or negative, the distance *D* will never decrease. Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.

3.89: a) The trajectory of the projectile is given by Eq. (3.27), with $\alpha_0 = \theta + \varphi$, and the equation describing the incline is $y = x \tan\theta$. Setting these equal and factoring out the $x = 0$ root (where the projectile is on the incline) gives a value for x_0; the range measured along the incline is

$$x/\cos\theta = \left[\frac{2v_0^2}{g}\right][\tan(\theta+\varphi) - \tan\theta]\left[\frac{\cos^2(\theta+\varphi)}{\cos\theta}\right].$$

b) Of the many ways to approach this problem, a convenient way is to use the same sort of "trick", involving double angles, as was used to derive the expression for the range along a horizontal incline. Specifically, write the above in terms of $\alpha = \theta + \varphi$, as

$$R = \left[\frac{2v_0^2}{g\cos^2\theta}\right][\sin\alpha\cos\alpha\cos\theta - \cos^2\alpha\sin\theta].$$

The dependence on α and hence φ is in the second term. Using the identities $\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$ and $\cos^2\alpha = (1/2)(1+\cos 2\alpha)$, this term becomes

$$(1/2)[\cos\theta\sin 2\alpha - \sin\theta\cos 2\alpha - \sin\theta] = (1/2)[\sin(2\alpha-\theta) - \sin\theta].$$

This will be a maximum when $\sin(2\alpha - \theta)$ is a maximum, at $2\alpha - \theta = 2\varphi + \theta = 90°$, or $\varphi = 45° - \theta/2$. Note that this reduces to the expected forms when $\theta = 0$ (a flat incline, $\varphi = 45°$ and when $\theta = -90°$ (a vertical cliff), when a horizontal launch gives the greatest distance).

3.90: As in the previous problem, the horizontal distance x in terms of the angles is

$$\tan\theta = \tan(\theta+\phi) - \left(\frac{gx}{2v_0^2}\right)\frac{1}{\cos^2(\theta+\phi)}.$$

Denote the dimensionless quantity $gx/2v_0^2$ by β; in this case

$$\beta = \frac{(9.80\ \text{m/s}^2)(60.0\ \text{m})\cos 30.0°}{2(32.0\ \text{m/s})^2} = 0.2486.$$

The above relation can then be written, on multiplying both sides by the product $\cos\theta\cos(\theta+\phi)$,

$$\sin\theta\cos(\theta+\phi) = \sin(\theta+\phi)\cos\theta - \frac{\beta\cos\theta}{\cos(\theta+\phi)},$$

and so

$$\sin(\theta+\phi)\cos\theta - \cos(\theta+\phi)\sin\theta = \frac{\beta\cos\theta}{\cos(\theta+\phi)}.$$

The term on the left is $\sin((\theta+\phi)-\theta) = \sin\phi,$ so the result of this combination is

$$\sin\phi\cos(\theta+\phi) = \beta\cos\theta.$$

Although this can be done numerically (by iteration, trial-and-error, or other methods), the expansion $\sin a\cos b = \frac{1}{2}(\sin(a+b)+\sin(a-b))$ allows the angle ϕ to be isolated; specifically, then

$$\frac{1}{2}(\sin(2\phi+\theta)+\sin(-\theta)) = \beta\cos\theta,$$

with the net result that

$$\sin(2\phi+\theta) = 2\beta\cos\theta + \sin\theta.$$

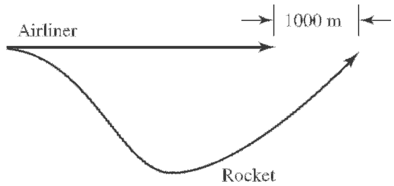
a) For $\theta = 30°,$ and β as found above, $\phi = 19.3°$ and the angle above the horizontal is $\theta+\phi = 49.3°.$ For level ground, using $\beta = 0.2871,$ gives $\phi = 17.5°.$ b) For $\theta = -30°,$ the same β as with $\theta = 30°$ may be used $(\cos 30° = \cos(-30°)),$ giving $\phi = 13.0°$ and $\phi+\theta = -17.0°.$

3.91: In a time $\Delta t,$ the velocity vector has moved through an angle (in radians) $\Delta\phi = \frac{v\Delta t}{R}$ (see Figure 3.23). By considering the isosceles triangle formed by the two velocity vectors, the magnitude $|\Delta\vec{\boldsymbol{v}}|$ is seen to be $2v\sin(\phi/2),$ so that

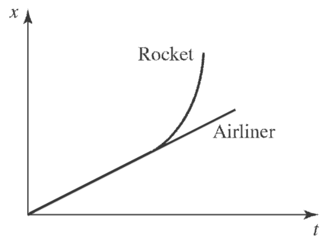
$$|\vec{\boldsymbol{a}}_{\text{ave}}| = 2\frac{v}{\Delta t}\sin\left(\frac{v\Delta t}{2R}\right) = \frac{10\,\text{m/s}}{\Delta \text{t}}\sin(1.0/\text{s}\cdot\Delta\text{t})$$

Using the given values gives magnitudes of $9.59\,\text{m/s}^2, 9.98\,\text{m/s}^2$ and $10.0\,\text{m/s}^2$. The instantaneous acceleration magnitude, $v^2/R = (5.00\,\text{m/s})^2/(2.50\,\text{m}) = 10.0\,\text{m/s}^2$ is indeed approached in the limit at $\Delta t \to 0.$ The changes in direction of the velocity vectors are given by $\Delta\theta = \frac{v\Delta t}{R}$ and are, respectively, 1.0 rad, 0.2 rad, and 0.1 rad. Therefore, the angle of the average acceleration vector with the original velocity vector is $\frac{\pi+\Delta\theta}{2} = \pi/2 + 1/2\,\text{rad}(118.6°), \pi/2 + 0.1\,\text{rad}(95.7°),$ and $\pi/2 + 0.05\,\text{rad}(92.9°).$

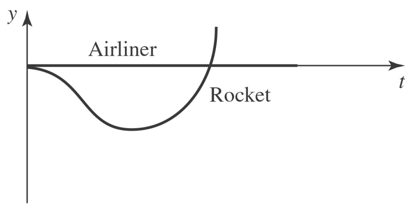
3.92:



The x-position of the plane is $(236\ \text{m/s})t$ and the x-position of the rocket is $(236\ \text{m/s})t + 1/2(3.00)(9.80\ \text{m/s}^2)\cos 30°(t-T)^2$. The graphs of these two have the form,



If we take $y = 0$ to be the altitude of the airliner, then $y(t) = -1/2gT^2 - gT(t-T) + 1/2(3.00)(9.80\ \text{m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. This graph looks like



By setting $y = 0$ for the rocket, we can solve for t in terms of $T, 0 = -(4.90\ \text{m/s}^2)T^2 - (9.80\ \text{m/s}^2)T(t-T) + (7.35\ \text{m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x = t - T,$ we find

$$x = t - T = \frac{(9.80\ \text{m/s}^2)T + \sqrt{(9.80\ \text{m/s}^2 T)^2 + (4)(7.35\ \text{m/s}^2)(4.9)T^2}}{2(7.35\ \text{m/s}^2)}$$ or $t = 2.72\,T$. Now, using the condition that $x_{\text{rocket}} - x_{\text{plane}} = 1000\ \text{m},$ we find $(236\ \text{m/s})t + (12.7\ \text{m/s}^2) \times (t-T)^2 - (236\ \text{m/s})t = 1000\ \text{m},$ or $(1.72T)^2 = 78.6\ \text{s}^2$. Therefore $T = 5.15\ \text{s}$.

3.93: a) Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{c/w} - v_{w/G} = 2$ where $v_{c/w}$ is the speed of the curve relative to water and $v_{w/G}$ is the speed of the water relative to the ground. We know that heading downstream for a time $t, (v_{c/w} + v_{w/G})t = 5$. We also know that for the bottle $v_{w/G}(t+1) = 3$. Solving these three equations for $v_{w/G} = x, v_{c/w} = 2 + x,$ therefore $(2 + x + x)t = 5$ or $(2 + 2x)t = 5$. Also $t = 3/x - 1,$ so $(2 + 2x)(\frac{3}{x} - 1) = 5$ or $2x^2 + x - 6 = 0$. The positive solution is $x = v_{w/G} = 1.5\ \text{km/h}$.

b) $v_{c/w} = 2\ \text{km/h} + v_{w/G} = 3.5\ \text{km/h}$.

Capítulo 4

4.1: a) For the magnitude of the sum to be the sum of the magnitudes, the forces must be parallel, and the angle between them is zero. b) The forces form the sides of a right isosceles triangle, and the angle between them is $90°$. Alternatively, the law of cosines may be used as

$$F^2 + F^2 = \left(\sqrt{2}F\right)^2 - 2F^2 \cos\theta,$$

from which $\cos\theta = 0$, and the forces are perpendicular. c) For the sum to have 0 magnitude, the forces must be antiparallel, and the angle between them is $180°$.

4.2: In the new coordinates, the 120-N force acts at an angle of $53°$ from the $-x$-axis, or $233°$ from the $+x$-axis, and the 50-N force acts at an angle of $323°$ from the $+x$-axis.

a) The components of the net force are

$$R_x = (120\ \text{N})\cos 233° + (50\ \text{N})\cos 323° = -32\ \text{N}$$

$$R_y = (250\ \text{N}) + (120\ \text{N})\sin 233° + (50\ \text{N})\sin 323° = 124\ \text{N}.$$

b) $R = \sqrt{R_x^2 + R_y^2} = 128\ \text{N}$, $\arctan\left(\frac{124}{-32}\right) = 104°$. The results have the same magnitude, and the angle has been changed by the amount $(37°)$ that the coordinates have been rotated.

4.3: The horizontal component of the force is $(10\ \text{N})\cos 45° = 7.1\ \text{N}$ to the right and the vertical component is $(10\ \text{N})\sin 45° = 7.1\ \text{N}$ down.

4.4: a) $F_x = F\cos\theta$, where θ is the angle that the rope makes with the ramp ($\theta = 30°$ in this problem), so $F = \left|\vec{\boldsymbol{F}}\right| = \frac{F_x}{\cos\theta} = \frac{60.0\ \text{N}}{\cos 30°} = 69.3\ \text{N}$.

b) $F_y = F\sin\theta = F_x \tan\theta = 34.6\ \text{N}$.

4.5: Of the many ways to do this problem, two are presented here.

Geometric: From the law of cosines, the magnitude of the resultant is

$$R=\sqrt{(270\text{ N})^2+(300\text{ N})^2+2(270\text{ N})(300\text{ N})\cos 60^\circ}=494\text{ N}.$$

The angle between the resultant and dog A's rope (the angle opposite the side corresponding to the 250-N force in a vector diagram) is then

$$\arcsin\left(\frac{\sin 120^\circ(300\text{ N})}{(494\text{ N})}\right)=31.7^\circ.$$

Components: Taking the $+x$-direction to be along dog A's rope, the components of the resultant are

$$R_x=(270\text{ N})+(300\text{ N})\cos 60^\circ=420\text{ N}$$
$$R_y=(300\text{ N})\sin 60^\circ=259.8\text{ N},$$

so $R=\sqrt{(420\text{ N})^2+(259.8\text{ N})^2}=494\text{ N}, \theta=\arctan\left(\frac{259.8}{420}\right)=31.7^\circ.$

4.6: a) $F_{1x}+F_{2x}=(9.00\text{ N})\cos 120^\circ+(6.00\text{ N})\cos(-126.9^\circ)=-8.10\text{ N}$

$F_{1y}+F_{2y}=(9.00\text{ N})\sin 120^\circ+(6.00\text{ N})\sin(-126.9^\circ)=+3.00\text{ N}.$

b) $R=\sqrt{R_x^2+R_y^2}=\sqrt{(8.10\text{ N})^2+(3.00\text{ N})^2}=8.64\text{ N}.$

4.7: $a=F/m=(132\text{ N})/(60\text{ kg})=2.2\text{ m/s}^2$ (to two places).

4.8: $F=ma=(135\text{ kg})(1.40\text{ m/s}^2)=189\text{ N}.$

4.9: $m=F/a=(48.0\text{ N})/(3.00\text{ m/s}^2)=16.00\text{ kg}.$

4.10: a) The acceleration is $a=\frac{2x}{t^2}=\frac{2(11.0\text{m})}{(5.00\text{ s})^2}=0.88\text{ m/s}^2$. The mass is then $m=\frac{F}{a}=\frac{80.0\text{ N}}{0.88\text{ m/s}^2}=90.9\text{ kg}.$

b) The speed at the end of the first 5.00 seconds is $at=4.4\text{ m/s}$, and the block on the frictionless surface will continue to move at this speed, so it will move another $vt=22.0\text{m}$ in the next 5.00 s.

4.11: a) During the first 2.00 s, the acceleration of the puck is $F/m = 1.563\ \text{m/s}^2$ (keeping an extra figure). At $t = 2.00\ \text{s}$, the speed is $at = 3.13\ \text{m/s}$ and the position is $at^2/2 = vt/2 = 3.13\ \text{m}$. b) The acceleration during this period is also $1.563\ \text{m/s}^2$, and the speed at 7.00 s is $3.13\ \text{m/s} + (1.563\ \text{m/s}^2)(2.00\ \text{s}) = 6.26\ \text{m/s}$. The position at $t = 5.00\text{s}$ is $x = 3.13\ \text{m} + (3.13\ \text{m/s})(5.00\ \text{s} - 2.00\ \text{s}) = 125\ \text{m}$, and at $t = 7.00\ \text{s}$ is

$$12.5\ \text{m} + (3.13\ \text{m/s})(2.00\ \text{s}) + (1/2)(1.563\ \text{m/s}^2)(2.00\ \text{s})^2 = 21.89\ \text{m},$$

or 21.9 m to three places.

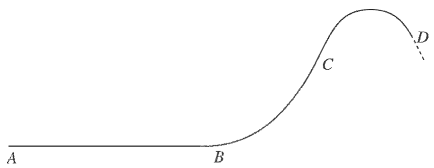
4.12: a) $a_x = F/m = 140\ \text{N}/32.5\ \text{kg} = 4.31\ \text{m/s}^2.$

b) With $v_{0x} = 0, x = \frac{1}{2}at^2 = 215\ \text{m}$.

c) With $v_{0x} = 0, v_x = a_x t = 2x/t = 43.0\ \text{m/s}$.

4.13: a) $\sum \vec{\boldsymbol{F}} = \boldsymbol{0}$

b), c), d)



4.14: a) With $v_{0x} = 0$,

$$a_x = \frac{v_x^2}{2x} = \frac{(3.00 \times 10^6\ \text{m/s})^2}{2(1.80 \times 10^{-2}\ \text{m})} = 2.50 \times 10^{14}\ \text{m/s}^2.$$

b) $t = \frac{v_x}{a_x} = \frac{3.00 \times 10^6\ \text{m/s}}{2.50 \times 10^{14}\ \text{m/s}^2} = 1.20 \times 10^{-8}\ \text{s}$. Note that this time is also the distance divided by the *average* speed.

c) $F = ma = (9.11 \times 10^{-31}\ \text{kg})(2.50 \times 10^{14}\ \text{m/s}^2) = 2.28 \times 10^{-16}\ \text{N}.$

4.15: $F = ma = w(a/g) = (2400\ \text{N})(12\ \text{m/s}^2)(9.80\ \text{m/s}^2) = 2.94 \times 10^3\ \text{N}.$

4.16:
$$a = \frac{F}{m} = \frac{F}{w/g} = \frac{F}{w}g = \left(\frac{160}{71.2}\right)(9.80\ \text{m/s}^2) = 22.0\ \text{m/s}^2.$$

4.17: a) $m = w/g = (44.0\ \text{N})/(9.80\ \text{m/s}^2) = 4.49\ \text{kg}$ b) The mass is the same, 4.49 kg, and the weight is $(4.49\ \text{kg})(1.81\ \text{m/s}^2) = 8.13\ \text{N}.$

4.18: a) From Eq. (4.9), $m = w/g = (3.20\ \text{N})/(9.80\ \text{m/s}^2) = 0.327\ \text{kg}$.
b) $w = mg = (14.0\ \text{kg})(9.80\ \text{m/s}^2) = 137\ \text{N}$.

4.19: $F = ma = (55\ \text{kg})(15\ \text{m/s}^2) = 825\ \text{N}$. The net forward force on the sprinter is exerted by the blocks. (The sprinter exerts a backward force on the blocks.)

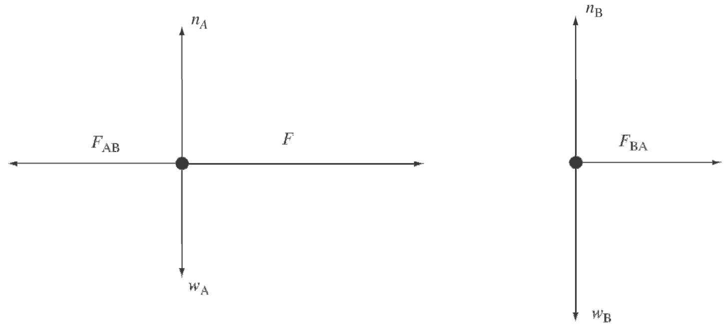
4.20: a) the earth (gravity) b) 4 N, the book c) no d) 4 N, the earth, the book, up e) 4 N, the hand, the book, down f) second g) third h) no i) no j) yes k) yes l) one (gravity) m) no

4.21: a) When air resistance is not neglected, the net force on the bottle is the weight of the bottle plus the force of air resistance. b) The bottle exerts an upward force on the earth, and a downward force on the air.

4.22: The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N. $\frac{\sum F}{m} = \frac{620\ \text{N} - 650\ \text{N}}{650\ \text{N}/9.80\ \text{m/s}^2} = -0.452\ \text{m/s}^2$. The passenger's acceleration is $0.452\ \text{m/s}^2$, downward.

4.23: $a_\text{E} = \dfrac{F}{m_\text{E}} = \dfrac{mg}{m_\text{E}} = \dfrac{(45\ \text{kg})(9.80\ \text{m/s}^2)}{(6.0\times 10^{24}\ \text{kg})} = 7.4\times 10^{-23}\ \text{m/s}^2$.

4.24: (a) Each crate can be considered a single particle:

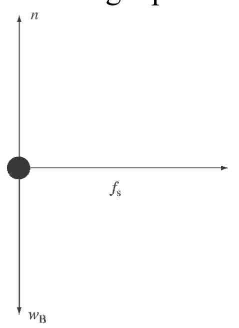


F_{AB} (the force on m_A due to m_B) and F_{BA} (the force on m_B due to m_A) form an action-reaction pair.

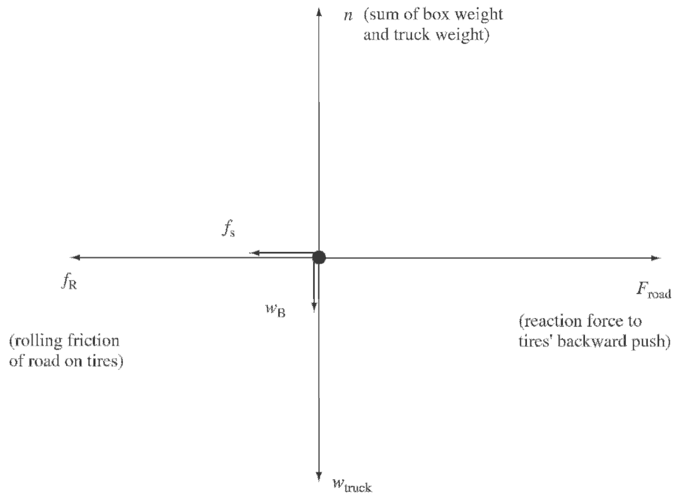
(b) Since there is no horizontal force opposing F, any value of F, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

4.25: The ball must accelerate eastward with the same acceleration as the train. There must be an eastward component of the tension to provide this acceleration, so the ball hangs at an angle relative to the vertical. The net force on the ball is not zero.

4.26: The box can be considered a single particle.

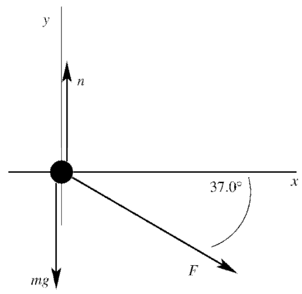


For the truck:



The box's friction force on the truck bed and the truck bed's friction force on the box form an action-reaction pair. There would also be some small air-resistance force action to the left, presumably negligible at this speed.

4.27: a)

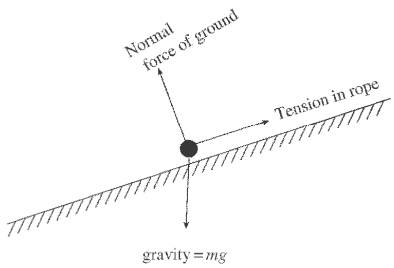


b) For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives

$n - mg - F\sin 37° = 0$

$n = 142\,\text{N}$

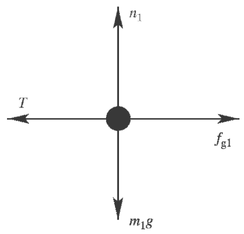
4.28: a)



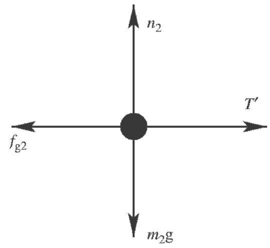
b)

$$\begin{aligned} T &= mg\sin\theta \\ &= (65.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 26.0° = 279\,\text{N} \end{aligned}$$

4.29: tricycle and Frank



T is the force exerted by the rope and f_g is the force the ground exerts on the tricycle.
spot and the wagon



T' is the force exerted by the rope. T and T' form a third-law action-reaction pair, $\vec{\boldsymbol{T}} = -\vec{\boldsymbol{T}}'$.

4.30: a) The stopping time is $\frac{x}{v_{ave}} = \frac{x}{(v_0/2)} = \frac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43\times10^{-4}$ s.

b) $F = ma = (1.80\times10^{-3}\text{ kg})\frac{(350\text{ m/s})}{(7.43\times10^{-4}\text{ s})} = 848\text{ N}$. (Using $a = v_0^2/2x$ gives the same result.)

4.31: Take the $+x$-direction to be along $\vec{\boldsymbol{F}}_1$ and the $+y$-direction to be along $\vec{\boldsymbol{R}}$. Then $F_{2x} = -1300\text{ N}$ and $F_{2y} = 1300\text{ N}$, so $F_2 = 1838\text{ N}$, at an angle of $135°$ from $\vec{\boldsymbol{F}}_1$.

4.32: Get g on X:

$$y = \frac{1}{2}gt^2$$

$$10.0\text{m} = \frac{1}{2}g(2.2\text{ s})^2$$

$$g = 4.13\text{ m/s}^2$$

$$w_\text{X} = mg_\text{X} = (0.100\text{ kg})(4.03\text{ m/s}^2) = 0.41\text{ N}$$

4.33: a) The resultant must have no y-component, and so the child must push with a force with y-component $(140\ \text{N})\sin 30° - (100\ \text{N})\sin 60° = -16.6\ \text{N}$. For the child to exert the smallest possible force, that force will have no x-component, so the smallest possible force has magnitude 16.6 N and is at an angle of $270°$, or $90°$ clockwise from the $+x$-direction.

b) $m = \frac{\sum F}{a} = \frac{100\ \text{N}\cos 60° + 140\ \text{N}\cos 30°}{2.0\ \text{m/s}^2} = 85.6\ \text{kg}$. $w = mg = (85.6\ \text{kg})(9.80\ \text{m/s}^2) = 840\ \text{N}$..

4.34: The ship would go a distance

$$\frac{v_0^2}{2a} = \frac{v_0^2}{2(F/m)} = \frac{mv_0^2}{2F} = \frac{(3.6\times10^7\ \text{kg})(1.5\ \text{m/s})^2}{2(8.0\times10^4\ \text{N})} = 506.25\ \text{m},$$

so the ship would hit the reef. The speed when the tanker hits the reef is also found from

$$v = \sqrt{v_0^2 - (2Fx/m)} = \sqrt{(1.5\ \text{m/s})^2 - \frac{2(8.0\times10^4\ \text{N})(500\ \text{m})}{(3.6\times10^7\ \text{kg})}} = 0.17\ \text{m/s},$$

so the oil should be safe.

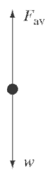
4.35: a) Motion after he leaves the floor: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

$v_y = 0$ at the maximum height, $y - y_0 = 1.2\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, so

$v_{0y} = 4.85\ \text{m/s}$.

b) $a_{\text{av}} = \Delta v/\Delta t = (4.85\ \text{m/s})/(0.300\ \text{s}) = 16.2\ \text{m/s}^2$.

c)



$$F_{\text{av}} - w = ma_{\text{av}}$$

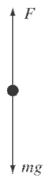
$$F_{\text{av}} = w + ma_{\text{av}} = 890\ \text{N} + (890\ \text{N}/9.80\ \text{m/s}^2)(16.2\ \text{m/s}^2)$$

$$F_{\text{av}} = 2.36\times10^3\ \text{N}$$

4.36:

$$F = ma = m\frac{v_0^2}{2x} = (850\ \text{kg})\frac{(12.5\ \text{m/s})^2}{2(1.8\times10^{-2}\ \text{m})} = 3.7\times10^6\ \text{N}.$$

4.37: a)



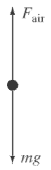
$$F_{\text{net}} = F - mg \text{ (upward)}$$

b) When the upward force has its maximum magnitude F_{max} (the breaking strength), the net upward force will be $F_{\text{max}} - mg$ and the upward acceleration will be

$$a = \frac{F_{\text{max}} - mg}{m} = \frac{F_{\text{max}}}{m} - g = \frac{75.0\ \text{N}}{4.80\ \text{kg}} - 9.80\ \text{m/s}^2 = 5.83\ \text{m/s}^2.$$

4.38: a) $w = mg = 539\ \text{N}$

b)



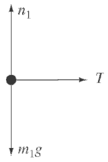
Downward velocity is decreasing so $\vec{a}$ is upward and the net force should be upward. $F_{\text{air}} > mg$, so the net force is upward.

c) Taking the upward direction as positive, the acceleration is

$$a = \frac{F}{m} = \frac{F_{\text{air}} - mg}{m} = \frac{F_{\text{air}}}{m} - 9.80\ \text{m/s}^2 = \frac{620\ \text{N}}{55.0\ \text{kg}} - 9.80\ \text{m/s}^2 = 1.47\ \text{m/s}^2.$$

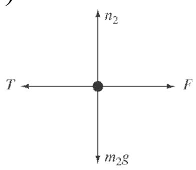
4.39: a) Both crates moves together, so $a = 2.50\,\text{m/s}^2$

b)



$T = m_1 a = (4.00\,\text{kg})(2.50\,\text{m/s}^2) = 10.0\,\text{N}$

c)



$F > T$ and the net force is to the right, in the direction of $\vec{a}$.

d) $F - T = m_2 a$

$F = T + m_2 a = 10.0\,\text{N} + (6.00\,\text{kg})(2.50\,\text{m/s}^2) = 25.0\,\text{N}$

4.40: a) The force the astronaut exerts on the rope and the force that the rope exerts on the astronaut are an action-reaction pair, so the rope exerts a force of 80.0 N on the astronaut. b) The cable is under tension. c) $a = \frac{F}{m} = \frac{80.0\text{ N}}{105.0\text{ kg}} = 0.762\,\text{m/s}^2$. d) There is no net force on the massless rope, so the force that the shuttle exerts on the rope must be 80.0 N (this is *not* an action-reaction pair). Thus, the force that the rope exerts on the shuttle must be 80.0 N. e) $a = \frac{F}{m} = \frac{80.0\text{ N}}{9.05\times10^4\text{ kg}} = 8.84\times10^{-4}\,\text{m/s}^2$.

4.41: a) $x\,(0.025\,\text{s}) = (9.0\times10^3\,\text{m/s}^2)(0.025\,\text{s})^2 - (8.0\times10^4\,\text{m/s}^3)(0.025\,\text{s})^3 = 4.4\,\text{m}$.

b) Differentiating, the velocity as a function of time is

$$v(t) = (1.80\times10^4\,\text{m/s}^2)t - (2.40\times10^5\,\text{m/s}^3)t^2,\text{ so}$$

$$\begin{aligned} v(0.025\,\text{s}) &= (1.80\times10^4\,\text{m/s}^2)(0.025\,\text{s}) - (2.40\times10^5\,\text{m/s}^3)(0.025\,\text{s})^2 \\ &= 3.0\times10^2\,\text{m/s}. \end{aligned}$$

c) The acceleration as a function of time is

$$a(t) = 1.80\times10^4\,\text{m/s}^2 - (4.80\times10^5\,\text{m/s}^3)t,$$

so (i) at $t = 0$, $a = 1.8\times10^4\,\text{m/s}^2$, and (ii) $a(0.025\text{s}) = 6.0\times10^3\,\text{m/s}^2$, and the forces are (i) $ma = 2.7\times10^4$ N and (ii) $ma = 9.0\times10^3$ N.

4.42: a) The velocity of the spacecraft is downward. When it is slowing down, the acceleration is upward. When it is speeding up, the acceleration is downward.

b)

speeding up: $w > F$ and the net force is downward
slowing down: $w < F$ and the net force is upward

c) Denote the y-component of the acceleration when the thrust is F_1 by a_1 and the y-component of the acceleration when the thrust is F_2 by a_2. The forces and accelerations are then related by

$$F_1 - w = ma_1, \quad F_2 - w = ma_2.$$

Dividing the first of these by the second to eliminate the mass gives

$$\frac{F_1 - w}{F_2 - w} = \frac{a_1}{a_2},$$

and solving for the weight w gives

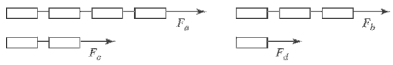
$$w = \frac{a_1 F_2 - a_2 F_1}{a_1 - a_2}.$$

In this form, it does not matter which thrust and acceleration are denoted by 1 and which by 2, and the acceleration due to gravity at the surface of Mercury need not be found. Substituting the given numbers, with $+y$ upward, gives

$$w = \frac{(1.20\,\mathrm{m/s^2})(10.0\times 10^3\ \mathrm{N}) - (-0.80\,\mathrm{m/s^2})(25.0\times 10^3\ \mathrm{N})}{1.20\,\mathrm{m/s^2} - (-0.80\,\mathrm{m/s^2})} = 16.0\times 10^3\ \mathrm{N}.$$

In the above, note that the upward direction is taken to be positive, so that a_2 is negative. Also note that although a_2 is known to two places, the sums in both numerator and denominator are known to three places.

4.43:



a) The engine is pulling four cars, and so the force that the engine exerts on the first car is $4m|\vec{a}|$. b), c), d): Similarly, the forces the cars exert on the car behind are $3m|\vec{a}|$, $2m|\vec{a}|$ and $-m|\vec{a}|$. e) The direction of the acceleration, and hence the direction of the forces, would change but the magnitudes would not; the answers are the same.

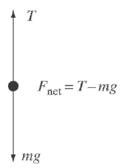
4.44: a) If the gymnast climbs at a constant rate, there is no net force on the gymnast, so the tension must equal the weight; $T = mg$.

b) No motion is no acceleration, so the tension is again the gymnast's weight.

c) $T - w = T - mg = ma = m|\vec{a}|$ (the acceleration is upward, the same direction as the tension), so $T = m(g + |\vec{a}|)$.

d) $T - w = T - mg = ma = -m|\vec{a}|$ (the acceleration is downward, the same opposite as the tension), so $T = m(g - |\vec{a}|)$.

4.45: a)



The maximum acceleration would occur when the tension in the cables is a maximum,

$$a = \frac{F_{\text{net}}}{m} = \frac{T - mg}{m} = \frac{T}{m} - g = \frac{28{,}000\ \text{N}}{2200\ \text{kg}} - 9.80\ \text{m/s}^2 = 2.93\ \text{m/s}^2.$$

b)

$$\frac{28{,}000\ \text{N}}{2200\ \text{kg}} - 1.62\ \text{m/s}^2 = 11.1\ \text{m/s}^2.$$

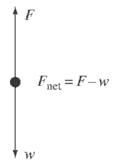
4.46: a) His speed as he touches the ground is

$$v=\sqrt{2gh}=\sqrt{2(9.80\,\text{m/s}^2)(3.10\,\text{m})}=7.80\,\text{m/s}.$$

b) The acceleration while the knees are bending is

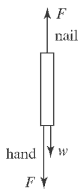
$$a=\frac{v^2}{2y}=\frac{(7.80\,\text{m/s})^2}{2(0.60\,\text{m})}=50.6\,\text{m/s}^2.$$

c)



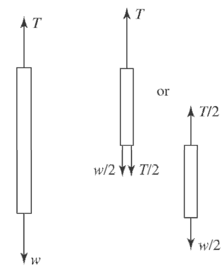
The net force that the feet exert on the ground is the force that the ground exerts on the feet (an action-reaction pair). This force is related to the weight and acceleration by $F-w=F-mg=ma$, so $F=m(a+g)=(75.0\,\text{kg})(50.6\,\text{m/s}^2+9.80\,\text{m/s}^2)=4532\,\text{N}$. As a fraction of his weight, this force is $\frac{F}{mg}=\left(\frac{a}{g}+1\right)=6.16$ (keeping an extra figure in the intermediate calculation of a). Note that this result is the same algebraically as $\left(\frac{3.10\text{ m}}{0.60\text{ m}}+1\right)$.

4.47: a)



b) The acceleration of the hammer head will be the same as the nail, $a=v_0^2/2x=(3.2\,\text{m/s})^2/2(0.45\,\text{cm})=1.138\times10^3\,\text{m/s}^2$. The mass of the hammer head is its weight divided by g, $4.9\,\text{N}/9.80\,\text{m/s}^2=0.50\,\text{kg}$, and so the net force on the hammer head is $(0.50\,\text{kg})(1.138\times10^3\,\text{m/s}^2)=570\,\text{N}$. This is the sum of the forces on the hammer head; the upward force that the nail exerts, the downward weight and the downward 15-N force. The force that the nail exerts is then 590 N, and this must be the magnitude of the force that the hammer head exerts on the nail. c) The distance the nail moves is .12 m, so the acceleration will be $4267\,\text{m/s}^2$, and the net force on the hammer head will be 2133 N. The magnitude of the force that the nail exerts on the hammer head, and hence the magnitude of the force that the hammer head exerts on the nail, is 2153 N, or about 2200 N.

4.48:



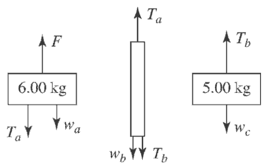
a) The net force on a point of the cable at the top is zero; the tension in the cable must be equal to the weight *w*.

b) The net force on the cable must be zero; the difference between the tensions at the top and bottom must be equal to the weight *w*, and with the result of part (a), there is no tension at the bottom.

c) The net force on the bottom half of the cable must be zero, and so the tension in the cable at the middle must be half the weight, $w/2$. Equivalently, the net force on the upper half of the cable must be zero. From part (a) the tension at the top is *w*, the weight of the top half is $w/2$ and so the tension in the cable at the middle must be $w - w/2 = w/2$.

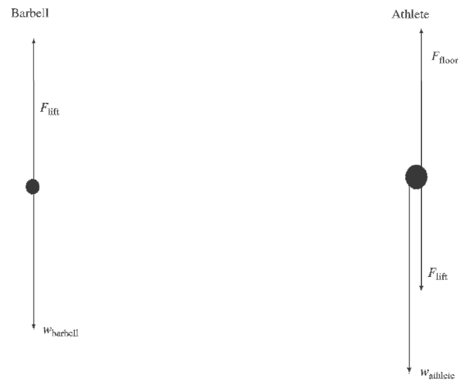
d) A graph of *T* vs. distance will be a negatively sloped line.

4.49: a)



b) The net force on the system is $200\ \text{N} - (15.00\ \text{kg})(9.80\ \text{m/s}^2) = 53.0\ \text{N}$ (keeping three figures), and so the acceleration is $(53.0\ \text{N})/(15.0\ \text{kg}) = 3.53\ \text{m/s}^2$, up. c) The net force on the 6-kg block is $(6.00\ \text{kg})(3.53\ \text{m/s}^2) = 21.2\ \text{N}$, so the tension is found from $F - T - mg = 21.2\ \text{N}$, or $T = (200\ \text{N}) - (6.00\ \text{kg})(9.80\ \text{m/s}^2) - 21.2\ \text{N} = 120\ \text{N}$. Equivalently, the tension at the top of the rope causes the upward acceleration of the rope and the bottom block, so $T - (9.00\ \text{kg})g = (9.00\ \text{kg})a$, which also gives $T = 120\ \text{N}$. d) The same analysis of part (c) is applicable, but using $6.00\ \text{kg} + 2.00\ \text{kg}$ instead of the mass of the top block, or 7.00 kg instead of the mass of the bottom block. Either way gives $T = 93.3\ \text{N}$.

4.50: a)



b) The athlete's weight is $mg = (90.0\,\text{kg})(9.80\,\text{m/s}^2) = 882\,\text{N}$. The acceleration of the barbell is found from $v_{\text{av}} = 0.60\,\text{m}/1.6\,\text{s} = 0.375\,\text{m/s}$. Its final velocity is thus $(2)(0.375\,\text{m/s}) = 0.750\,\text{m/s}$, and its acceleration is

$$a = \frac{v - v_0}{t} = \frac{0.750\,\text{m/s}}{1.65} = 0.469\,\text{m/s}^2$$

The force needed to lift the barbell is given by:

$$F_{\text{net}} = F_{\text{lift}} - w_{\text{barbell}} = ma$$

The barbell's mass is $(490\,\text{N})/(9.80\,\text{m/s}^2) = 50.0\,\text{kg}$, so

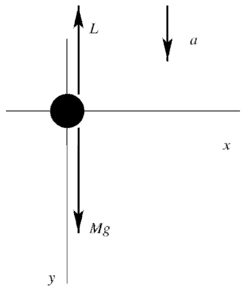
$$\begin{aligned} F_{\text{lift}} = w_{\text{barbell}} + ma &= 490\,\text{N} + (50.0\,\text{kg})(0.469\,\text{m/s}^2) \\ &= 490\,\text{N} + 23\,\text{N} = 513\,\text{N} \end{aligned}$$

The athlete is not accelerating, so:

$$F_{\text{net}} = F_{\text{floor}} - F_{\text{lift}} - w_{\text{athlete}} = 0$$

$$F_{\text{floor}} = F_{\text{lift}} + w_{\text{athlete}} = 513\,\text{N} + 882\,\text{N} = 1395\,\text{N}$$

4.51: a)



L is the lift force

b) $\sum F_y = ma_y$

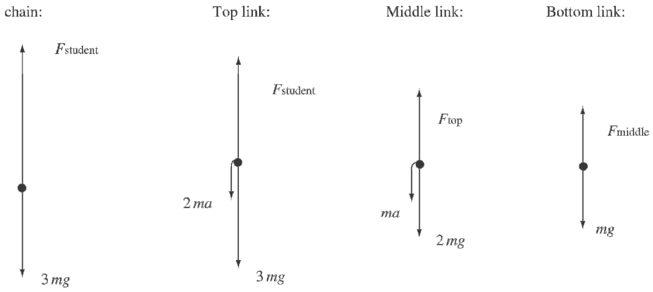
$Mg - L = M(g/3)$

$L = 2Mg/3$

c) $L - mg = m(g/2)$, where m is the mass remaining.

$L = 2Mg/3$, so $m = 4M/9$. Mass $5M/9$ must be dropped overboard.

4.52: a) $m =$ mass of one link



The downward forces of magnitude $2ma$ and ma for the top and middle links are the reaction forces to the upward force needed to accelerate the links below.

b) (i) The weight of each link is $mg = (0.300\,\text{kg})(9.80\,\text{m/s}^2) = 2.94\,\text{N}$. Using the free-body diagram for the whole chain:

$$a = \frac{F_{\text{net}}}{3m} = \frac{12\,\text{N} - 3(2.94\,\text{N})}{0.900\,\text{kg}} = \frac{3.18\,\text{N}}{0.900\,\text{kg}} = 3.53\,\text{m/s}^2 \text{ or } 3.5\,\text{m/s}^2$$

(ii) The second link also accelerates at $3.53\,\text{m/s}^2$, so:

$$F_{\text{net}} = F_{\text{top}} - ma - 2mg = ma$$
$$F_{\text{top}} = 2ma + 2mg = 2(0.300\,\text{kg})(3.53\,\text{m/s}^2) + 2(2.94\,\text{N})$$
$$= 2.12\,\text{N} + 5.88\,\text{N} = 8.0\,\text{N}$$

4.53: Differentiating twice, the acceleration of the helicopter as a function of time is

$$\vec{a} = (0.120\,\text{m/s}^3)t\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}},$$

and at $t = 5.0\,\text{s}$, the acceleration is

$$\vec{a} = (0.60\,\text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}}.$$

The force is then

$$F = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times10^5\,\text{N})}{(9.80\,\text{m/s}^2)}\left[(0.60\,\text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\,\text{m/s}^2)\hat{\boldsymbol{k}}\right]$$
$$= (1.7\times10^4\,\text{N})\hat{\boldsymbol{i}} - (3.4\times10^3\,\text{N})\hat{\boldsymbol{k}}.$$

4.54: The velocity as a function of time is $v(t) = A - 3Bt^2$ and the acceleration as a function of time is $a(t) = -6Bt$, and so the Force as a function of time is $F(t) = ma(t) = -6mBt$.

4.55:

$$\vec{\boldsymbol{v}}(t) = \frac{1}{m}\int_0^t \vec{\boldsymbol{a}}\, dt = \frac{1}{m}\left(k_1 t\hat{\boldsymbol{i}} + \frac{k^2}{4}t^4\hat{\boldsymbol{j}}\right).$$

4.56: a) The equation of motion, $-Cv^2 = m\frac{dv}{dt}$ cannot be integrated with respect to time, as the unknown function $v(t)$ is part of the integrand. The equation must be *separated* before integration; that is,

$$-\frac{C}{m}dt = \frac{dv}{v^2}$$

$$-\frac{Ct}{m} = -\frac{1}{v} + \frac{1}{v_0},$$

where v_0 is the constant of integration that gives $v = v_0$ at $t = 0$. Note that this form shows that if $v_0 = 0$, there is no motion. This expression may be rewritten as

$$v = \frac{dx}{dt} = \left(\frac{1}{v_0} + \frac{Ct}{m}\right)^{-1},$$

which may be integrated to obtain

$$x - x_0 = \frac{m}{C}\ln\left[1 + \frac{Ctv_0}{m}\right].$$

To obtain x as a function of v, the time t must be eliminated in favor of v; from the expression obtained after the first integration, $\frac{Ctv_0}{m} = \frac{v_0}{v} - 1$, so

$$x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$$

b) By the chain rule,

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dv}{dt} = \frac{dv}{dx}v,$$

and using the given expression for the net force,

$$-Cv^2 = \left(v\frac{dv}{dx}\right)m$$

$$-\frac{C}{m}dx = \frac{dv}{v}$$

$$-\frac{C}{m}(x - x_0) = \ln\left(\frac{v}{v_0}\right)$$

$$x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$$

4.57: In this situation, the *x*-component of force depends explicitly on the *y*-component of position. As the *y*-component of force is given as an explicit function of time, v_y and *y* can be found as functions of time. Specifically, $a_y = (k_3/m)t$, so $v_y = (k_3/2m)t^2$ and $y = (k_3/6m)t^3$, where the initial conditions $v_{0y} = 0, y_0 = 0$ have been used. Then, the expressions for a_x, v_x and *x* are obtained as functions of time:

$$a_x = \frac{k_1}{m} + \frac{k_2k_3}{6m^2}t^3$$

$$v_x = \frac{k_1}{m}t + \frac{k_2k_3}{24m^2}t^4$$

$$x = \frac{k_1}{2m}t^2 + \frac{k_2k_3}{120m^2}t^5.$$

In vector form,

$$\vec{\boldsymbol{r}} = \left(\frac{k_1}{2m}t^2 + \frac{k_2k_3}{120m^2}t^5\right)\hat{\boldsymbol{i}} + \left(\frac{k_3}{6m}t^3\right)\hat{\boldsymbol{j}}$$

$$\vec{\boldsymbol{v}} = \left(\frac{k_1}{m}t + \frac{k_2k_3}{24m^2}t^4\right)\hat{\boldsymbol{i}} + \left(\frac{k_3}{2m}t^2\right)\hat{\boldsymbol{j}}.$$

Capítulo 5

5.1: a) The tension in the rope must be equal to each suspended weight, 25.0 N. b) If the mass of the light pulley may be neglected, the net force on the pulley is the vector sum of the tension in the chain and the tensions in the two parts of the rope; for the pulley to be in equilibrium, the tension in the chain is twice the tension in the rope, or 50.0 N.

5.2: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w. Two forces act on each mass: w down and $T(=w)$ up.

5.3: a) The two sides of the rope each exert a force with vertical component $T\sin\theta$, and the sum of these components is the hero's weight. Solving for the tension T,

$$T = \frac{w}{2\sin\theta} = \frac{(90.0\ \text{kg})(9.80\ \text{m/s}^2)}{2\sin 10.0°} = 2.54\times10^3\ \text{N}.$$

b) When the tension is at its maximum value, solving the above equation for the angle θ gives

$$\theta = \arcsin\left(\frac{w}{2T}\right) = \arcsin\left(\frac{(90.0\ \text{kg})(9.80\ \text{m/s}^2)}{2(2.50\times10^4\ \text{N})}\right) = 1.01°.$$

5.4: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical, so if the weight is $w, \frac{w}{2} = \frac{3w}{4}\cos\theta$ and $\theta = \arccos\frac{2}{3} = 48°$.

5.5: With the positive y-direction up and the positive x-direction to the right, the free-body diagram of Fig. 5.4(b) will have the forces labeled n and T resolved into x- and y-components, and setting the net force equal to zero,

$$F_x = T\cos\alpha - n\sin\alpha = 0$$

$$F_y = n\cos\alpha + T\sin\alpha - w = 0.$$

Solving the first for $n = T\cot\alpha$ and substituting into the second gives

$$T\frac{\cos^2\alpha}{\sin\alpha} + T\sin\alpha = T\left(\frac{\cos^2\alpha}{\sin\alpha} + \frac{\sin^2\alpha}{\sin\alpha}\right) = \frac{T}{\sin\alpha} = w$$

and so $n = T\cot\alpha = w\sin\alpha\cot\alpha = w\cos\alpha$, as in Example 5.4.

5.6: $w\sin\alpha = mg\sin\alpha = (1390\ \text{kg})(9.80\ \text{m/s}^2)\sin 17.5° = 4.10\times10^3\ \text{N}.$

5.7: a) $T_B \cos\theta = W$, or $T_B = W/\cos\theta = \frac{(4090\text{ kg})(9.8\text{ m/s}^2)}{\cos 40°} = 5.23\times10^4$ N.
b) $T_A = T_B \sin\theta = (5.23\times10^4\text{ N})\sin 40° = 3.36\times10^4$ N.

5.8: a) $T_C = w, T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so $T_A = \frac{w}{1.366} = 0.732w$. Then, $T_B = T_A \frac{\cos 30°}{\cos 45°} = 0.897w$.
b) Similar to part (a), $T_C = w, -T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$. Again adding the last two, $T_A = \frac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_B \frac{\sin 60°}{\cos 45°} = 3.35w$.

5.9: The resistive force is $w \sin\alpha = (1600\text{ kg})(9.80\text{ m/s}^2)(200\text{ m}/6000\text{ m}) = 523$ N..

5.10: The magnitude of the force must be equal to the component of the weight along the incline, or $W \sin\theta = (180\text{ kg})(9.80\text{ m/s}^2)\sin 11.0° = 337$ N.

5.11: a) $W = 60\text{ N}, T\sin\theta = W$, so $T = (60\text{ N})/\sin 45°$, or $T = 85$ N.
b) $F_1 = F_2 = T\cos\theta, F_1 = F_2 = 85\text{ N}\cos 45° = 60$ N.

5.12: If the rope makes an angle θ with the vertical, then $\sin\theta = \frac{0.110}{1.51} = 0.073$ (the denominator is the sum of the length of the rope and the radius of the ball). The weight is then the tension times the cosine of this angle, or

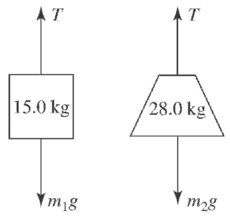
$$T = \frac{w}{\cos\theta} = \frac{mg}{\cos(\arcsin(.073))} = \frac{(0.270\text{ kg})(9.80\text{ m/s}^2)}{0.998} = 2.65\text{ N}.$$

The force of the pole on the ball is the tension times $\sin\theta$, or $(0.073)T = 0.193$ N.

5.13: a) In the absence of friction, the force that the rope between the blocks exerts on block B will be the component of the weight along the direction of the incline, $T = w\sin\alpha$. b) The tension in the upper rope will be the sum of the tension in the lower rope and the component of block A's weight along the incline, $w\sin\alpha + w\sin\alpha = 2w\sin\alpha$. c) In each case, the normal force is $w\cos\alpha$. d) When $\alpha = 0, n = w$, when $\alpha = 90°, n = 0$.

5.14: a) In level flight, the thrust and drag are horizontal, and the lift and weight are vertical. At constant speed, the net force is zero, and so $F = f$ and $w = L$. b) When the plane attains the new constant speed, it is again in equilibrium and so the new values of the thrust and drag, F' and f', are related by $F' = f'$; if $F' = 2F, f' = 2f$. c) In order to increase the magnitude of the drag force by a factor of 2, the speed must increase by a factor of $\sqrt{2}$.

5.15: a)



The tension is related to the masses and accelerations by

$$T - m_1g = m_1a_1$$

$$T - m_2g = m_2a_2.$$

b) For the bricks accelerating upward, let $a_1 = -a_2 = a$ (the counterweight will accelerate down). Then, subtracting the two equations to eliminate the tension gives

$$(m_2 - m_1)g = (m_1 + m_2)a, \text{ or}$$

$$a = g\frac{m_2 - m_1}{m_2 + m_1} = 9.80\ \text{m/s}^2\left(\frac{28.0\ \text{kg} - 15.0\ \text{kg}}{28.0\ \text{kg} + 15.0\text{kg}}\right) = 2.96\ \text{m/s}^2.$$

c) The result of part (b) may be substituted into either of the above expressions to find the tension $T = 191\ \text{N}$. As an alternative, the expressions may be manipulated to eliminate a algebraically by multiplying the first by m_2 and the second by m_1 and adding (with $a_2 = -a_1$) to give

$$T(m_1 + m_2) - 2m_1m_2g = 0, \text{ or}$$

$$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2(15.0\ \text{kg})\,(28.0\ \text{kg})\,(9.80\ \text{m/s}^2)}{(15.0\ \text{kg} + 28.0\ \text{kg})} = 191\ \text{N}.$$

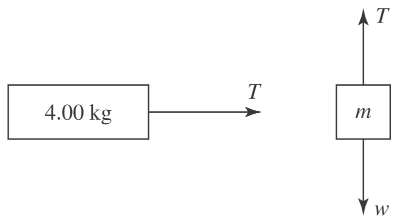
In terms of the weights, the tension is

$$T = w_1\frac{2m_2}{m_1 + m_2} = w_2\frac{2m_1}{m_1 + m_2}.$$

If, as in this case, $m_2 > m_1, 2m_2 > m_1 + m_2$ and $2m_1 < m_1 + m_2$, so the tension is greater than w_1 and less than w_2; this must be the case, since the load of bricks rises and the counterweight drops.

5.16: Use Second Law and kinematics: $a = g\sin\theta, 2ax = v^2$, solve for θ.
$g\sin\theta = v^2/2x$, or
$\theta = \arcsin\left(v^2/2gx\right) = \arcsin[(2.5\ \text{m/s})^2/[(2)(9.8\ \text{m/s}^2)(1.5\ \text{m})]], \theta = 12.3°.$

5.17: a)



b) In the absence of friction, the net force on the 4.00-kg block is the tension, and so the acceleration will be $(10.0\ \text{N})/(4.00\ \text{kg}) = 2.50\ \text{m/s}^2$. c) The net upward force on the suspended block is $T - mg = ma$, or $m = T/(g + a)$. The block is accelerating downward, so $a = -2.50\ \text{m/s}^2$, and so $m = (10.0\ \text{N})/(9.80\ \text{m/s}^2 - 2.50\ \text{m/s}^2) = 1.37\ \text{kg}$.
d) $T = ma + mg$, so $T < mg$, because $a < 0$.

5.18: The maximum net force on the glider combination is

$$12{,}000\ \text{N} - 2 \times 2500\ \text{N} = 7000\ \text{N},$$

so the maximum acceleration is $a_{\text{max}} = \frac{7000\ \text{N}}{1400\ \text{kg}} = 5.0\ \text{m/s}^2$.

a) In terms of the runway length L and takoff speed $v, a = \frac{v^2}{2L} < a_{\text{max}}$, so

$$L > \frac{v^2}{2a_{\text{max}}} = \frac{(40\ \text{m/s})^2}{2(5.0\ \text{m/s}^2)} = 160\ \text{m}.$$

b) If the gliders are accelerating at a_{max}, from
$T - F_{\text{drag}} = ma, T = ma + F_{\text{drag}} = (700\ \text{kg})(5.0\ \text{m/s}^2) + 2500\ \text{N} = 6000\ \text{N}$. Note that this is exactly half of the maximum tension in the towrope between the plane and the first glider.

5.19: Denote the scale reading as F, and take positive directions to be upward. Then,

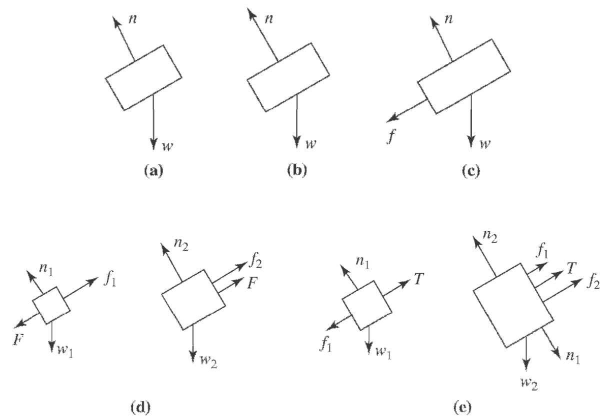
$$F - w = ma = \frac{w}{g}a, \text{ or } a = g\left(\frac{F}{w} - 1\right).$$

a) $a = (9.80\ \text{m/s}^2)((450\ \text{N})/(550\ \text{N}) - 1) = -1.78\ \text{m/s}^2$, down.
b) $a = (9.80\ \text{m/s}^2)((670\ \text{N})/(550\ \text{N}) - 1) = 2.14\ \text{m/s}^2$, up. c) If $F = 0, a = -g$ and the student, scale, and elevator are in free fall. The student should worry.

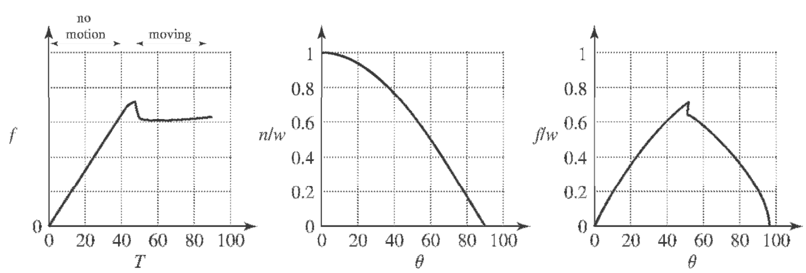
5.20: Similar to Exercise 5.16, the angle is $\arcsin(\frac{2L}{gt^2})$, , but here the time is found in terms of velocity along the table, $t = \frac{x}{v_0}, x$ being the length of the table and v_0 the velocity component along the table. Then,

$$\arcsin\left(\frac{2L}{g(x/v_0)^2}\right) = \arcsin\left(\frac{2Lv_0^2}{gx^2}\right)$$
$$= \arcsin\left(\frac{2(2.50\times10^{-2}\,\mathrm{m})(3.80\,\mathrm{m/s})^2}{(9.80\,\mathrm{m/s^2})(1.75\,\mathrm{m})^2}\right) = 1.38°.$$

5.21:



5.22:



5.23: a) For the net force to be zero, the applied force is

$$F = f_k = \mu_k n = \mu_k mg = (0.20)\,(11.2\,\mathrm{kg})\,(9.80\,\mathrm{m/s^2}) = 22.0\ \mathrm{N}.$$

b) The acceleration is $\mu_k g$, and $2ax = v^2$, so $x = v^2/2\mu_k g$, or $x = 3.13\,\mathrm{m}$.

5.24: a) If there is no applied horizontal force, no friction force is needed to keep the box in equilibrium. b) The maximum static friction force is, from Eq. (5.6), $\mu_s n = \mu_s w = (0.40)\,(40.0\ \text{N}) = 16.0\ \text{N},$ so the box will not move and the friction force balances the applied force of 6.0 N. c) The maximum friction force found in part (b), 16.0 N. d) From Eq. (5.5), $\mu_k n = (0.20)(40.0\,\text{N}) = 8.0\,\text{N}$ e) The applied force is enough to either start the box moving or to keep it moving. The answer to part (d), from Eq. (5.5), is independent of speed (as long as the box is moving), so the friction force is 8.0 N. The acceleration is $(F - f_k)/m = 2.45\ \text{m/s}^2.$

5.25: a) At constant speed, the net force is zero, and the magnitude of the applied force must equal the magnitude of the kinetic friction force,

$$\left|\vec{F}\right| = f_k = \mu_k n = \mu_k mg = (0.12)\,(6.00\ \text{kg})\,(9.80\ \text{m/s}^2) = 7\ \text{N}.$$

b) $\left|\vec{F}\right| - f_k = ma,$ so

$$\left|\vec{F}\right| = ma + f_k = ma = \mu_k mg = m(a + \mu_k g)$$

$$= (6.00\ \text{kg})(0.180\ \text{m/s}^2 + (0.12)9.80\ \text{m/s}^2) = 8\ \text{N}.$$

c) Replacing $g = 9.80\ \text{m/s}^2$ with $1.62\ \text{m/s}^2$ gives 1.2 N and 2.2 N.

5.26: The coefficient of kinetic friction is the ratio $\frac{f_k}{n}$, and the normal force has magnitude $85\ \text{N} + 25\ \text{N} = 110\ \text{N}$. The friction force, from $F_H - f_k = ma = w\frac{a}{g}$ is

$$f_k = F_H - w\frac{a}{g} = 20\ \text{N} - 85\ \text{N}\left(\frac{-0.9\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = 28\ \text{N}$$

(note that the acceleration is negative), and so $\mu_k = \frac{28\ \text{N}}{110\ \text{N}} = 0.25.$

5.27: As in Example 5.17, the friction force is $\mu_k n = \mu_k w \cos\alpha$ and the component of the weight down the skids is $w \sin\alpha$. In this case, the angle α is $\arcsin(2.00/20.0) = 5.7°$. The ratio of the forces is $\frac{\mu_k \cos\alpha}{\sin\alpha} = \frac{\mu_k}{\tan\alpha} = \frac{0.25}{0.10} > 1,$ so the friction force holds the safe back, and another force is needed to move the safe down the skids.

b) The difference between the downward component of gravity and the kinetic friction force is

$$w(\sin\alpha - \mu_k \cos\alpha) = (260\ \text{kg})\,(9.80\ \text{m/s}^2)\,(\sin 5.7° - (0.25)\cos 5.7°) = -381\ \text{N}.$$

5.28: a) The stopping distance is

$$\frac{v^2}{2a} = \frac{v^2}{2\mu_k g} = \frac{(28.7\ \text{m/s})^2}{2(0.80)\,(9.80\ \text{m/s}^2)} = 53\ \text{m}.$$

b) The stopping distance is inversely proportional to the coefficient of friction and proportional to the square of the speed, so to stop in the same distance the initial speed should not exceed

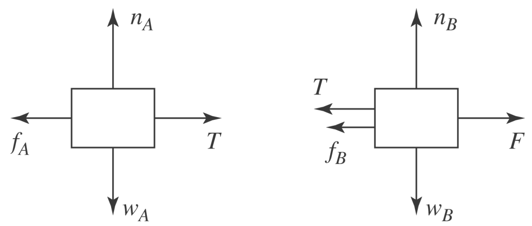
$$v\sqrt{\frac{\mu_{\text{k, wet}}}{\mu_{\text{k,dry}}}} = (28.7\ \text{m/s})\sqrt{\frac{0.25}{0.80}} == 16\ \text{m/s}.$$

5.29: For a given initial speed, the distance traveled is inversely proportional to the coefficient of kinetic friction. From Table 5.1, the ratio of the distances is then $\frac{0.44}{0.04} = 11$.

5.30: (a) If the block descends at constant speed, the tension in the connecting string must be equal to the hanging block's weight, w_B. Therefore, the friction force $\mu_k w_A$ on block A must be equal to w_B, and $w_B = \mu_k w_A$.

(b) With the cat on board, $a = g(w_B - \mu_k 2w_A)/(w_B + 2w_A)$.

5.31:



a) For the blocks to have no acceleration, each is subject to zero net force. Considering the horizontal components,

$$T = f_A, \left|\vec{F}\right| = T + f_B, \text{ or}$$

$$\left|\vec{F}\right| = f_A + f_B.$$

Using $f_A = \mu_k g m_A$ and $f_B = \mu_k g m_B$ gives $\left|\vec{F}\right| = \mu_k g(m_A + m_B)$.

b) $T = f_A = \mu_k g m_A$.

5.32:

$$\mu_r = \frac{a}{g} = \frac{v_0^2 - v^2}{2Lg} = \frac{v_0^2 - \frac{1}{4}v_0^2}{2Lg} = \frac{3}{8}\frac{v_0^2}{Lg},$$

where L is the distance covered before the wheel's speed is reduced to half its original speed. Low pressure, $L = 18.1\ \text{m}; \frac{3}{8}\frac{(3.50\ \text{m/s})^2}{(18.1\ \text{m})(9.80\ \text{m/s}^2)} = 0.0259.$ High pressure,

$L = 92.9\ \text{m}; \frac{3}{8}\frac{(3.50\ \text{m/s})^2}{(92.9\text{m})(9.80\text{m/s}^2)} = 0.00505.$

5.33: Without the dolly: $n = mg$ and $F - \mu_k n = 0$ ($a_x = 0$ since speed is constant).

$$m = \frac{F}{\mu_k g} = \frac{160\ \text{N}}{(0.47)\,(9.80\ \text{m/s}^2)} = 34.74\ \text{kg}$$

With the dolly: the total mass is $34.7\ \text{kg} + 5.3\ \text{kg} = 40.04\ \text{kg}$ and friction now is rolling friction, $f_r = \mu_r mg.$

$$F - \mu_r mg = ma$$

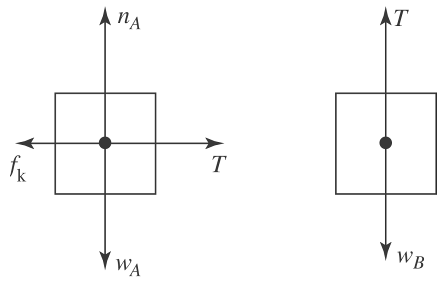
$$a = \frac{F - \mu_r mg}{m} = 3.82\ \text{m/s}^2$$

5.34: Since the speed is constant and we are neglecting air resistance, we can ignore the 2.4 m/s, and F_{net} in the horizontal direction must be zero. Therefore $f_r = \mu_r n =$ $= F_{\text{horiz}} = 200$ N before the weight and pressure changes are made. After the changes, $(0.81)\,(1.42n) = F_{\text{horiz}},$ because the speed is still constant and $F_{\text{net}} = 0$. We can simply divide the two equations**:**

$$\frac{(0.81\mu_r)(1.42n)}{\mu_r n} = \frac{F_{\text{horiz}}}{200\ \text{N.}}$$

$$(0.81)\,(1.42)\,(200\ \text{N}) = F_{\text{horiz}} = 230\ \text{N}$$

5.35: First, determine the acceleration from the freebody diagrams.



There are two equations and two unknowns, a and T:

$$-\mu_k m_A g + T = m_A a$$

$$m_B g - T = m_B a$$

Add and solve for $a: a = g(m_B - \mu_k m_A)/(m_B + m_A), a = 0.79\ \mathrm{m/s^2}$.

(a) $v = (2ax)^{1/2} = 0.22\ \mathrm{m/s}$.

(b) Solving either equation for the tension gives $T = 11.7\ \mathrm{N}$.

5.36: a) The normal force will be $w\cos\theta$ and the component of the gravitational force along the ramp is $w\sin\theta$. The box begins to slip when $w\sin\theta > \mu_s w\cos\theta$, or $\tan\theta > \mu_s = 0.35$, so slipping occurs at $\theta = \arctan(0.35) = 19.3°$, or $19°$ to two figures.

b) When moving, the friction force along the ramp is $\mu_k w\cos\theta$, the component of the gravitational force along the ramp is $w\sin\theta$, so the acceleration is

$$(w\sin\theta) - w\mu_k\cos\theta)/m = g(\sin\theta - \mu_k\cos\theta) = 0.92\ \mathrm{m/s^2}.$$

(c) $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\,\mathrm{m/s^2})(5\ \mathrm{m})]^{1/2} = 3\ \mathrm{m}$.

5.37: a) The magnitude of the normal force is $mg + |\vec{F}|\sin\theta$. The horizontal component of $\vec{F}, |\vec{F}|\cos\theta$ must balance the frictional force, so

$$|\vec{F}|\cos\theta = \mu_k(mg + |\vec{F}|\sin\theta);$$

solving for $|\vec{F}|$ gives

$$|\vec{F}| = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta}$$

b) If the crate remains at rest, the above expression, with μ_s instead of μ_k, gives the force that must be applied in order to start the crate moving. If $\cot\theta < \mu_s$, the needed force is infinite, and so the critical value is $\mu_s = \cot\theta$.

5.38: a) There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k(mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k(mg - F\sin\theta)$, and so at constant speed,

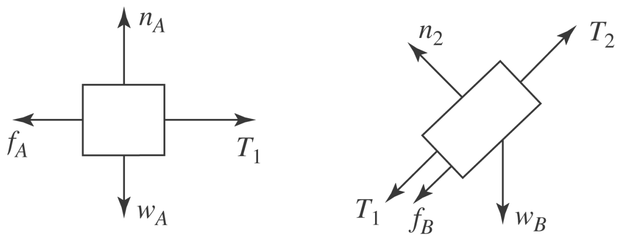
$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

b) Using the given values,

$$F = \frac{(0.35)(90\ \text{kg})(9.80\ \text{m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 293\ \text{N},$$

or 290 N to two figures.

5.39: a)



b) The blocks move with constant speed, so there is no net force on block *A*; the tension in the rope connecting *A* and *B* must be equal to the frictional force on block *A*, $\mu_k = (0.35)\,(25.0\ \text{N}) = 9\ \text{N}$. c) The weight of block *C* will be the tension in the rope connecting *B* and *C*; this is found by considering the forces on block *B*. The components of force along the ramp are the tension in the first rope (9 N, from part (a)), the component of the weight along the ramp, the friction on block *B* and the tension in the second rope. Thus, the weight of block *C* is

$$\begin{aligned} w_C &= 9\ \text{N} + w_B(\sin 36.9° + \mu_k \cos 36.9°) \\ &= 9\ \text{N} + (25.0\ \text{N})(\sin 36.9° + (0.35)\cos 36.9°) = 31.0\ \text{N}, \end{aligned}$$

or 31 N to two figures. The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight *w* of blocks *A* and *B*,

$$w_C = w(\mu_k + (\sin\theta + \mu_k \cos\theta)),$$

giving the same result.

(d) Applying Newton's Second Law to the remaining masses (*B* and *C*) gives**:**

$$a = g(w_c - \mu_k w_B \cos\theta - w_B \sin\theta)/(w_B + w_c) = 1.54\ \text{m/s}^2.$$

5.40: Differentiating Eq. (5.10) with respect to time gives the acceleration

$$a = v_{\mathrm{t}}\left(\frac{k}{m}\right)e^{-(k/m)t} = ge^{-(k/m)t},$$

where Eq. (5.9), $v_{\mathrm{t}} = mg/k$ has been used.

Integrating Eq. (5.10) with respect to time with $y_0 = 0$ gives

$$\begin{aligned} y &= \int_0^t v_{\mathrm{t}}[1 - e^{-(k/m)t}]\,dt \\ &= v_{\mathrm{t}}\left[t + \left(\frac{m}{k}\right)e^{-(k/m)t}\right] - v_{\mathrm{t}}\left(\frac{m}{k}\right) \\ &= v_{\mathrm{t}}\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right]. \end{aligned}$$

5.41: a) Solving for *D* in terms of v_t,

$$D = \frac{mg}{v_t^2} = \frac{(80\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})}{(42\ \mathrm{m/s})^2} = 0.44\ \mathrm{kg/m}.$$

b)

$$v_{\mathrm{t}} = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(45\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(0.25\ \mathrm{kg/m})}} = 42\ \mathrm{m/s}.$$

5.42: At half the terminal speed, the magnitude of the frictional force is one-fourth the weight. a) If the ball is moving up, the frictional force is down, so the magnitude of the net force is (5/4)*w* and the acceleration is (5/4)*g*, down. b) While moving down, the frictional force is up, and the magnitude of the net force is (3/4)*w* and the acceleration is (3/4)*g*, down.

5.43: Setting F_{net} equal to the maximum tension in Eq. (5.17) and solving for the speed *v* gives

$$v = \sqrt{\frac{F_{\mathrm{net}}R}{m}} = \sqrt{\frac{(600\ \mathrm{N})(0.90\ \mathrm{m})}{(0.80\ \mathrm{kg})}} = 26.0\ \mathrm{m/s},$$

or 26 m/s to two figures.

5.44: This is the same situation as Example 5.23. Solving for μ_{s} yields

$$\mu_{\mathrm{s}} = \frac{v^2}{Rg} = \frac{(25.0\ \mathrm{m/s})^2}{(220\ \mathrm{m})(9.80\ \mathrm{m/s^2})} = 0.290.$$

5.45: a) The magnitude of the force F is given to be equal to $3.8w$. "Level flight" means that the net vertical force is zero, so $F\cos\beta = (3.8)w\cos\beta = w$, , and $\beta = \arccos(1/3.8) = 75°$.

(b) The angle does not depend on speed.

5.46: a) The analysis of Example 5.22 may be used to obtain $\tan\beta = (v^2/gR)$, but the subsequent algebra expressing R in terms of L is not valid. Denoting the length of the horizontal arm as r and the length of the cable as $l, R = r + l\sin\beta$. The relation $v = \frac{2\pi R}{T}$ is still valid, so $\tan\beta = \frac{4\pi^2 R}{gT^2} = \frac{4\pi^2(r+l\sin\beta)}{gT^2}$. Solving for the period T,

$$T = \sqrt{\frac{4\pi^2(r + l\sin\beta)}{g\tan\beta}} = \sqrt{\frac{4\pi^2(3.00\text{ m} + (5.00\text{ m})\sin 30°)}{(9.80\text{ m/s}^2)\tan 30°}} = 6.19\text{ s}.$$

Note that in the analysis of Example 5.22, β is the angle that the support (string or cable) makes with the vertical (see Figure 5.30(b)). b) To the extent that the cable can be considered massless, the angle will be independent of the rider's weight. The tension in the cable will depend on the rider's mass.

5.47: This is the same situation as Example 5.22, with the lift force replacing the tension in the string. As in that example, the angle β is related to the speed and the turning radius by $\tan\beta = \frac{v^2}{gR}$. Solving for β,

$$\beta = \arctan\left(\frac{v^2}{gR}\right) = \arctan\left(\frac{(240\text{ km/h} \times ((1\text{m/s})/(3.6\text{ km/h})))^2}{(9.80\text{ m/s}^2)(1200\text{ m})}\right) = 20.7°.$$

5.48: a) This situation is equivalent to that of Example 5.23 and Problem 5.44, so $\mu_s = \frac{v^2}{Rg}$. Expressing v in terms of the period T, $v = \frac{2\pi R}{T}$, so $\mu_S = \frac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so

$$\mu_s = \frac{4\pi^2(0.150\text{ m})}{(1.50\text{ s})^2(9.80\text{ m/s}^2)} = 0.269.$$

b) For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved further out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is (0.150 m) $\left(\frac{40.0}{60.0}\right)^2 = 0.067\text{ m}$.

5.49: a) Setting $a_{\text{rad}} = g$ in Eq. (5.16) and solving for the period T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\text{ m}}{9.80\text{ m/s}^2}} = 40.1\text{ s},$$

so the number of revolutions per minute is $(60\text{ s/min})/(40.1\text{ s}) = 1.5\text{ rev/min}$.

b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\text{ rev/min}) \times \sqrt{3.70/9.8} = 0.92\text{ rev/min.}$.

5.50: a) $2\pi R/T = 2\pi(50.0\text{ m})/(60.0\text{ s}) = 5.24\text{ m/s}$. b) The magnitude of the radial force is $mv^2/R = m4\pi^2 R/T^2 = w(4\pi^2 R/gT^2) = 49\text{ N}$ (to the nearest Newton), so the apparent weight at the top is $882\text{ N} - 49\text{ N} = 833\text{ N}$, and at the bottom is $882\text{ N} + 49\text{ N} = 931\text{ N}$. c) For apparent weightlessness, the radial acceleration at the top is equal to g in magnitude. Using this in Eq. (5.16) and solving for T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{50.0\text{ m}}{9.80\text{ m/s}^2}} = 14\text{ s}.$$

d) At the bottom, the apparent weight is twice the weight, or 1760 N.

5.51: a) If the pilot feels weightless, he is in free fall, and $a = g = v^2/R$, so $v = \sqrt{Rg} = \sqrt{(150\text{ m})(9.80\text{ m/s}^2)} = 38.3\text{ m/s}$, or 138 km/h. b) The apparent weight is the sum of the net inward (upward) force and the pilot's weight, or

$$\begin{aligned} w + ma &= w\left(1 + \frac{a}{g}\right) \\ &= (700\text{ N})\left(1 + \frac{(280\text{ km/h})^2}{(3.6(\text{km/h})/(\text{m/s}))^2(9.80\text{ m/s}^2)(150\text{ m})}\right) \\ &= 3581\text{ N}, \end{aligned}$$

or 3580 N to three places.

5.52: a) Solving Eq. (5.14) for R,

$$R = v^2/a = v^2/4g = (95.0\text{ m/s})^2/(4 \times 9.80\text{ m/s}^2) = 230\text{ m}.$$

b) The apparent weight will be five times the actual weight,

$$5\,mg = 5\,(50.0\text{ kg})\,(9.80\text{ m/s}^2) = 2450\ \text{ N}$$

to three figures.

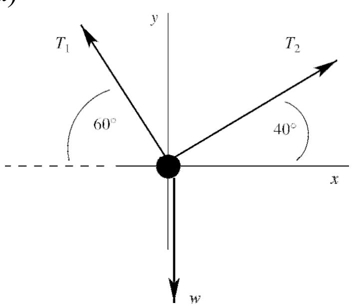
5.53: For no water to spill, the magnitude of the downward (radial) acceleration must be at least that of gravity; from Eq. (5.14), $v > \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)(0.600\ \text{m})} = 2.42\ \text{m/s}$.

5.54: a) The inward (upward, radial) acceleration will be $\frac{v^2}{R} = \frac{(4.2\ \text{m/s})^2}{(3.80\ \text{m})} = 4.64\ \text{m/s}^2$. At the bottom of the circle, the inward direction is upward.

b) The forces on the ball are tension and gravity, so $T - mg = ma$,

$$T = m(a+g) = w\left(\frac{a}{g}+1\right) = (71.2\ \text{N})\left(\frac{4.64\ \text{m/s}^2}{9.80\ \text{m/s}^2}+1\right) = 105\ \text{N}.$$

5.55: a)



T_1 is more vertical so supports more of the weight and is larger.

You can also see this from $\sum F_x = ma_x$:

$T_2 \cos 40° - T_1 \cos 60° = 0$

$$T_1 = \left(\frac{\cos 40°}{\cos 60°}\right)T_2 = 1.532T_2$$

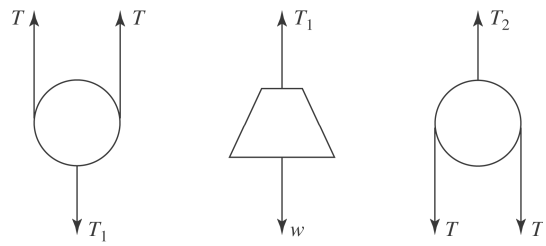
b) T_1 is larger so set $T_1 = 5000\ \text{N}$. Then $T_2 = T_1/1.532 = 3263.5\ \text{N}$,

$\sum F_y = ma_y$

$T_1 \sin 60° + T_2 \sin 40° = w$

$w = 6400\ \text{N}$

5.56:



The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w*, and so the tension in the rope is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

5.57: In the absence of friction, the only forces along the ramp are the component of the weight along the ramp, $w \sin \alpha$, and the component of $\vec{F}$ along the ramp, $|\vec{F}| \cos \alpha = F \cos \alpha$. These forces must sum to zero, so $F = w \tan \alpha$.

Considering horizontal and vertical components, the normal force must have horizontal component equal to $n \sin \alpha$, which must be equal to *F*; the vertical component must balance the weight, $n \cos \alpha = w$. Eliminating *n* gives the same result.

5.58: The hooks exert forces on the ends of the rope. At each hook, the force that the hook exerts and the force due to the tension in the rope are an action-reaction pair.
The vertical forces that the hooks exert must balance the weight of the rope, so each hook exerts an upward vertical force of $w/2$ on the rope. Therefore, the downward force that the rope exerts at each end is $T_{\text{end}} \sin \theta = w/2$, so $T_{\text{end}} = w/(2 \sin \theta) = Mg/(2 \sin \theta)$.
b) Each half of the rope is itself in equilibrium, so the tension in the middle must balance the horizontal force that each hook exerts, which is the same as the horizontal component of the force due to the tension at the end; $T_{\text{end}} \cos \theta = T_{\text{middle}}$, so $T_{\text{middle}} = Mg \cos \theta/(2 \sin \theta) = Mg/(2 \tan \theta)$.
(c) Mathematically speaking, $\theta \neq 0$ because this would cause a division by zero in the equation for T_{end} or T_{middle}. Physically speaking, we would need an infinite tension to keep a non-massless rope perfectly straight.

5.59: Consider a point a distance x from the top of the rope. The forces acting in this point are T up and $\left(M+\frac{m(L-x)}{L}\right)g$ downwards. Newton's Second Law becomes $T-\left(M+\frac{m(L-x)}{L}\right)g=\left(M+\frac{m(L-x)}{L}\right)a$. Since $a=\frac{F-(M+m)g}{M+m}, T=\left(M+\frac{m(L-x)}{L}\right)\left(\frac{F}{M+m}\right)$. At $x=0, T=F$, and at $x=L, T=\frac{MF}{M+m}=M(a+g)$ as expected.

5.60: a) The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g=\left(m_1 g\sin\alpha+\mu_k m_1 g\cos\alpha\right)$ and $m_2=m_1(\sin\alpha+\mu_k\cos\alpha)$.

b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g=(m_1 g\sin\alpha-\mu_k m_1 g\cos\alpha)$, or $m_2=m_1(\sin\alpha-\mu_k\cos\alpha)$.

c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha+\mu_s\cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha-\mu_s\cos\alpha)$.

5.61: For an angle of $45.0°$, the tensions in the horizontal and vertical wires will be the same. a) The tension in the vertical wire will be equal to the weight $w=12.0\text{ N}$; this must be the tension in the horizontal wire, and hence the friction force on block A is also 12.0 N. b) The maximum frictional force is $\mu_s w_A=(0.25)(60.0\text{ N})=15\text{ N}$; this will be the tension in both the horizontal and vertical parts of the wire, so the maximum weight is 15 N.

5.62: a) The most direct way to do part (a) is to consider the blocks as a unit, with total weight 4.80 N. Then the normal force between block B and the lower surface is 4.80 N, and the friction force that must be overcome by the force F is $\mu_k n=(0.30)(4.80\text{ N})=1.440\text{ N}$, or 1.44 N, to three figures. b) The normal force between block B and the lower surface is still 4.80 N, but since block A is moving *relative to block B*, there is a friction force between the blocks, of magnitude $(0.30)(1.20\text{ N})=0.360\text{ N}$, so the total friction force that the force F must overcome is $1.440\text{ N}+0.360\text{ N}=1.80\text{ N}$. (An extra figure was kept in these calculations for clarity.)

5.63: (Denote $|\vec{F}|$ by F.) a) The force normal to the surface is $n = F\cos\theta$; the vertical component of the applied force must be equal to the weight of the brush plus the friction force, so that $F\sin\theta = w + \mu_k F\cos\theta$, and

$$F = \frac{w}{\sin\theta - \mu_k\cos\theta} = \frac{12.00\ \text{N}}{\sin 53.1° - (0.51)\cos 53.1°} = 16.9\ \text{N},$$

keeping an extra figure. b) $F\cos\theta = (16.91\ \text{N})\cos 53.1° = 10.2\ \text{N}$.

5.64: a)

$$\begin{aligned}\sum F &= ma = m(62.5g) = 62.5mg\\ &= (62.5)(210\times10^{-6}\,\text{g})(980\ \text{cm/s}^2)\\ &= 13\ \text{dynes} = 1.3\times10^{-4}\ \text{N}\end{aligned}$$

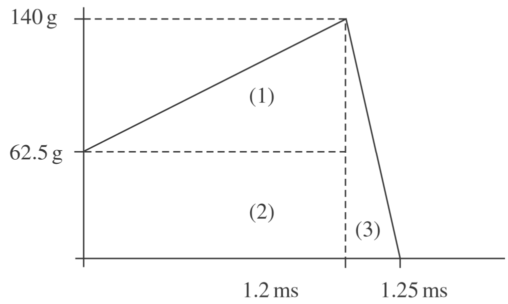
This force is 62.5 times the flea's weight.

b)

$$\begin{aligned}F_{\max} &= ma_{\max} = m(140g) = 140mg\\ &= 29\ \text{dynes} = 2.9\times10^{-4}\ \text{N}\end{aligned}$$

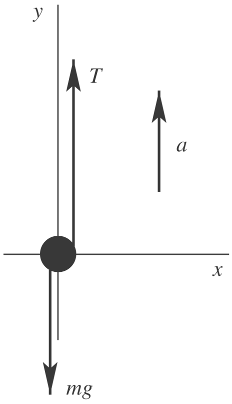
Occurs at approximately 1.2 ms.

c) $\Delta v = v - v_0 = v - 0 = v =$ area under *a-t* graph. Approximate area as shown:



$$\begin{aligned}A &= A(1) + A(2) + A(3)\\ &= \frac{1}{2}(1.2\ \text{ms})(77.5\ \text{g}) + (1.2\ \text{ms})(62.5\ \text{g})\\ &\quad + \frac{1}{2}(0.05\ \text{ms})(140\ \text{g})\\ &= 120\ \text{cm/s} = 1.2\ \text{m/s}\end{aligned}$$

5.65: a) The instrument has mass $m = w/g = 1.531\,\text{kg}$. Forces on the instrument:



$$\Sigma F_y = ma_y$$
$$T - mg = ma$$
$$a = \frac{T - mg}{m} = 13.07\ \text{m/s}^2$$

$v_{0y} = 0, v_y = 330\ \text{m/s}, a_y = 13.07\ \text{m/s}^2, t = ?$

$v_y = v_{0y} + a_y t$ gives $t = 25.3\ \text{s}$

Consider forces on the rocket; rocket has the same a_y. Let F be the thrust of the rocket engines.

$F - mg = ma$

$$F = m(g + a) = (25{,}000\ \text{kg})\,(9.80\ \text{m/s}^2 + 13.07\ \text{m/s}^2) = 5.72 \times 10^5\ \text{N}$$

b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 4170\ \text{m}$.

5.66: The elevator's acceleration is:

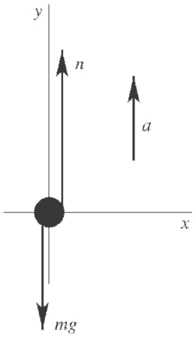
$$a = \frac{dv(t)}{dt} = 3.0\ \text{m/s}^2 + 2(0.20\ \text{m/s}^3)t = 3.0\ \text{m/s}^2 + (0.40\ \text{m/s}^3)t$$

At $t = 4.0\ \text{s}, a = 3.0\ \text{m/s}^2 + (0.40\ \text{m/s}^3)(4.0\ \text{s}) = 4.6\ \text{m/s}^2$. From Newton's Second Law, the net force on you is

$$F_{\text{net}} = F_{\text{scale}} - w = ma$$

$$F_{\text{scale}} = \text{apparent weight} = w + ma = (72\ \text{kg})(9.8\ \text{m/s}^2) + (72\ \text{kg})(4.6\ \text{m/s}^2)$$
$$= 1036.8\ \text{N or } 1040\ \text{N}$$

5.67: Consider the forces on the person:



$$\sum F_y = ma_y$$
$$n - mg = ma$$
$$n = 1.6mg \text{ so } a = 0.60\ g = 5.88\ \text{m/s}^2$$

$y - y_0 = 3.0\ \text{m},\ a_y = 5.88\ \text{m/s}^2,\ v_{0y} = 0, v_y = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.0\ \text{m/s}$

5.68: (a) Choosing upslope as the positive direction:

$$F_{\text{net}} = -mg \sin 37° - f_{\text{k}} = -mg \sin 37° - \mu_{\text{k}} mg \cos 37° = ma$$

and

$$a = -(9.8\ \text{m/s}^2)(0.602 + (0.30)(0.799)) = -8.25\ \text{m/s}^2$$

Since we know the length of the slope, we can use $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $v = 0$ at the top.

$$v_0^2 = -2ax = -2(-8.25\ \text{m/s}^2)(8.0\ \text{m}) = 132\ \text{m}^2/\text{s}^2$$
$$v_0 = \sqrt{132\ \text{m}^2/\text{s}^2} = 11.5\ \text{m/s or } 11\ \text{m/s}$$

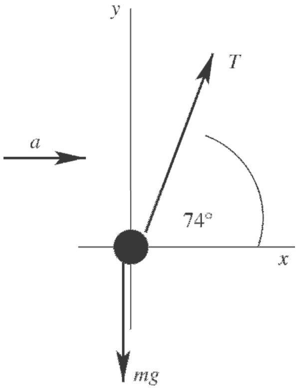
(b) For the trip back down the slope, gravity and the friction force operate in opposite directions**:**

$$F_{\text{net}} = -mg \sin 37° + \mu_{\text{k}} mg \cos 37° = ma$$
$$a = g(-\sin 37° + 0.30 \cos 37°) = (9.8\ \text{m/s}^2)((-0.602) + (0.30)(0.799)) = -3.55\ \text{m/s}^2$$

Now

$$v_0 = 0, x_0 = -8.0\ \text{m}, x = 0, \text{ and}$$
$$v^2 = v_0^2 + 2a(x - x_0) = 0 + 2(-3.55\ \text{m/s}^2)(-8.0\ \text{m})$$
$$= 56.8\ \text{m}^2/\text{s}^2$$
$$v = \sqrt{56.8\ \text{m}^2/\text{s}^2} = 7.54\ \text{m/s or } 7.5\ \text{m/s}$$

5.69: Forces on the hammer**:**



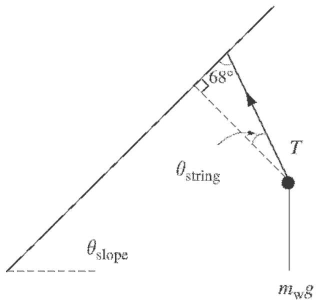
$\sum F_y = ma_y$ gives $T\sin 74° - mg = 0$ so $T\sin 74° = mg$

$\sum F_x = ma_x$ gives $T\cos 74° - ma$

Divide the second equation by the first**:**

$\dfrac{a}{g} = \dfrac{1}{\tan 74°}$ and $a = 2.8\ \mathrm{m/s^2}$

5.70:



It's interesting to look at the string's angle measured from the perpendicular to the top of the crate. This angle is of course $90°$—angle measured from the top of the crate. The free-body diagram for the washer then leads to the following equations, using Newton's Second Law and taking the upslope direction as positive:

$$-m_{\text{w}}g\sin\theta_{\text{slope}} + T\sin\theta_{\text{string}} = m_{\text{w}}a$$

$$-m_{\text{w}}g\cos\theta_{\text{slope}} + T\cos\theta_{\text{string}} = 0$$

$$T\sin\theta_{\text{string}} = m_{\text{w}}(a + g\sin\theta_{\text{slope}})$$

$$T\cos\theta_{\text{string}} = m_{\text{w}}g\cos\theta_{\text{slope}}$$

Dividing the two equations:

$$\tan\theta_{\text{string}} = \frac{a + g\sin\theta_{\text{slope}}}{g\cos 0_{\text{slope}}}$$

For the crate, the component of the weight along the slope is $-m_{\text{c}}g\sin\theta_{\text{slope}}$ and the normal force is $m_{\text{c}}g\cos\theta_{\text{slope}}$. Using Newton's Second Law again:

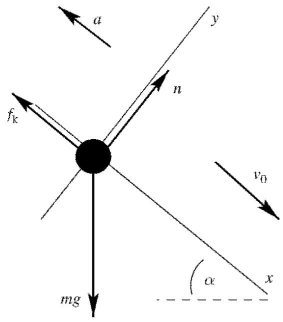
$$-m_{\text{c}}g\sin\theta_{\text{slope}} + \mu_{\text{k}}m_{\text{c}}g\cos\theta_{\text{slope}} = m_{\text{c}}a$$

$$\mu_{\text{k}} = \frac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$$

which leads to the interesting observation that the string will hang at an angle whose tangent is equal to the coefficient of kinetic friction**:**

$$\mu_{\text{k}} = \tan\theta_{\text{string}} = \tan(90° - 68°) = \tan 22° = 0.40$$

5.71: a) Forces on you**:**



$$\sum F_y = ma_y \text{ gives } n = mg\ \cos\alpha$$
$$\sum F_x = ma_x$$
$$mg\sin\alpha - f_k = ma$$
$$a = g(\sin\alpha - \mu_k \cos\alpha) = -3.094\ \text{m/s}^2$$

Find your stopping distance

$v_x = 0, a_x = -3.094\ \text{m/s}^2, v_{0x} = 20\ \text{m/s}, x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = 64.6\ \text{m},$ which is greater than 40 m. You don't stop before you reach the hole, so you fall into it.

b) $a_x = -3.094\ \text{m/s}^2, x - x_0 = 40\ \text{m}, v_x = 0, v_{0x} = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = 16\ \text{m/s}.$

5.72: The key idea in solving this problem is to recognize that if the system is accelerating, the tension that block A exerts on the rope is different from the tension that block B exerts on the rope. (Otherwise the net force on the rope would be zero, and the rope couldn't accelerate.) Also, treat the rope as if it is just another object. Taking the "clockwise" direction to be positive, the Second Law equations for the three different parts of the system are**:**

Block A (The only horizontal forces on A are tension to the right, and friction to the left):

$$-\mu_k m_A g + T_A = m_A a.$$

Block B (The only vertical forces on B are gravity down, and tension up):

$$m_B g - T_B = m_B a.$$

Rope (The forces on the rope along the direction of its motion are the tensions at either end and the weight of the portion of the rope that hangs vertically):

$$m_R\left(\tfrac{d}{L}\right)g + T_B - T_A = m_R a.$$

To solve for a and eliminate the tensions, add the left hand sides and right hand sides of the three equations**:** $-\mu_k m_A g + m_B g + m_R\left(\tfrac{d}{L}\right)g = (m_A + m_B + m_R)a$, or $a = g\frac{m_B + m_R\frac{d}{L} - \mu_k m_A}{(m_A + m_B + m_R)}$.

(a) When $\mu_k = 0, a = g\frac{m_B + m_R\left(\frac{d}{L}\right)}{(m_A + m_B + m_R)}$. As the system moves, d will increase, approaching L as a limit, and thus the acceleration will approach a maximum value of $a = g\frac{m_B + m_R}{(m_A + m_B + m_R)}$.

(b) For the blocks to just begin moving, $a > 0$, so solve $0 = [m_B + m_R\left(\frac{d}{L}\right) - \mu_s m_A]$ for d. Note that we must use static friction to find d for when the block will *begin* to move. Solving for d, $d = \frac{L}{m_R}(\mu_s m_A - m_B)$, or $d = \frac{1.0\text{ m}}{.160\text{ kg}}(.25(2\text{ kg}) - .4\text{ kg}) = .63\text{ m}$..

(c) When $m_R = .04\text{ kg}$, $d = \frac{1.0\text{ m}}{.04\text{ kg}}(.25(2\text{ kg}) - .4\text{ kg}) = 2.50\text{ m}$. This is not a physically possible situation since $d > L$. The blocks won't move, no matter what portion of the rope hangs over the edge.

5.73: For a rope of length L, and weight w, assume that a length rL is on the table, so that a length $(1-r)L$ is hanging. The tension in the rope at the edge of the table is then $(1-r)w$, and the friction force on the part of the rope on the table is $f_s = \mu_s rw$. This must be the same as the tension in the rope at the edge of the table, so $\mu_s rw = (1-r)w$ and $r = 1/(1+\mu_s)$. Note that this result is independent of L and w for a uniform rope. The fraction that hangs over the edge is $1 - r = \mu_s/(1+\mu_s)$; note that if $\mu_s = 0, r = 1$ and $1 - r = 0$.

5.74: a) The normal force will be $mg\cos\alpha + F\sin\alpha$, and the net force along (up) the ramp is

$F\cos\alpha - mg\sin\alpha - \mu_s(mg\cos\alpha + F\sin\alpha) = F(\cos\alpha - \mu_s\sin\alpha) - mg(\sin\alpha + \mu_s\cos\alpha).$

In order to move the box, this net force must be greater than zero. Solving for *F*,

$$F > mg\frac{\sin\alpha + \mu_s\cos\alpha}{\cos\alpha - \mu_s\sin\alpha}.$$

Since *F* is the magnitude of a force, *F* must be positive, and so the denominator of this expression must be positive, or $\cos\alpha > \mu_s\sin\alpha$, and $\mu_s < \cot\alpha$. b) Replacing μ_s with μ_k with in the above expression, and making the inequality an equality,

$$F = mg\frac{\sin\alpha + \mu_k\cos\alpha}{\cos\alpha - \mu_k\sin\alpha}.$$

5.75: a) The product $\mu_s g = 2.94\ \mathrm{m/s^2}$ is greater than the magnitude of the acceleration of the truck, so static friction can supply sufficient force to keep the case stationary relative to the truck; the crate accelerates north at $2.20\ \mathrm{m/s^2}$, due to the friction force of $ma = 66.0\ \mathrm{N}$. b) In this situation, the static friction force is insufficient to maintain the case at rest relative to the truck, and so the friction force is the kinetic friction force, $\mu_k n = \mu_k mg = 59\ \mathrm{N}$.

5.76: To answer the question, v_0 must be found and compared with $20\ \mathrm{m/s}\ (72\ \mathrm{km/hr})$. The kinematics relationship $2ax = -v_0^2$ is useful, but we also need *a*. The acceleration must be large enough to cause the box to begin sliding, and so we must use the force of static friction in Newton's Second Law: $-\mu_s mg \le ma$, or $a = -\mu_s g$. Then, $2(-\mu_s g)x = -v_0^2$, or $v_0 = \sqrt{2\mu_s gx} = \sqrt{2(.30)(9.8\ \mathrm{m/s^2})(47\ \mathrm{m})}$. Hence, $v_0 = 16.6\ \mathrm{m/s} = 60\ \mathrm{km/h}$, which is less than 72 km/h, so do you not go to jail.

5.77: See Exercise 5.40. a) The maximum tension and the weight are related by

$$T_{\max} \cos\beta = \mu_k (w - T_{\max} \sin\beta),$$

and solving for the weight w gives

$$w = T_{\max}\left(\frac{\cos\beta}{\mu_k} + \sin\beta\right).$$

This will be a maximum when the quantity in parentheses is a maximum. Differentiating with respect to β,

$$\frac{d}{d\beta}\left(\frac{\cos\beta}{\mu_k} + \sin\beta\right) = -\frac{\sin\beta}{\mu_k} + \cos\beta = 0,$$

or $\tan\theta = \mu_k$, where θ is the value of β that maximizes the weight. Substituting for μ_k in terms of θ,

$$\begin{aligned} w &= T_{\max}\left(\frac{\cos\theta}{\sin\theta/\cos\theta} + \sin\theta\right) \\ &= T_{\max}\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}\right) \\ &= \frac{T_{\max}}{\sin\theta}. \end{aligned}$$

b) In the absence of friction, any non-zero horizontal component of force will be enough to accelerate the crate, but slowly.

5.78: a) Taking components along the direction of the plane's descent, $f = w\sin\alpha$ and $L = w\cos\alpha$. b) Dividing one of these relations by the other cancels the weight, so $\tan\alpha = f/L$. c) The distance will be the initial altitude divided by the tangent of α. $f = L\tan\alpha$ and $L = w\cos\alpha$, therefore $\sin\alpha = f/w = \frac{1300\ \text{N}}{12{,}900\ \text{N}} g$ and so $\alpha = 5.78°$. This makes the horizontal distance $(2500\ \text{m})/\tan(5.78°) = 24.7\ \text{km}$. d) If the drag is reduced, the angle α is reduced, and the plane goes further.

5.79: If the plane is flying at a constant speed of $36.1\ \text{m/s}$, then $\sum F = 0$, or $T - w\sin\alpha - f = 0$. The rate of climb and the speed give the angle α, $\alpha = \arcsin(5/36.1) = 7.96°$. Then, $T = w\sin\alpha + f$. $T = (12{,}900\ \text{N})\sin 7.96° + 1300\ \text{N} = 3087\ \text{N}$. Note that in level flight $(\alpha = 0)$, the thrust only needs to overcome the drag force to maintain the constant speed of $36.1\ \text{m/s}$.

5.80: If the block were to remain at rest relative to the truck, the friction force would need to cause an acceleration of $2.20\ \mathrm{m/s^2}$; however, the maximum acceleration possible due to static friction is $(0.19)(9.80\ \mathrm{m/s^2}) = 1.86\ \mathrm{m/s^2}$, and so the block will move relative to the truck; the acceleration of the box would be $\mu_\mathrm{k} g = (0.15)(9.80\ \mathrm{m/s^2}) = 1.47\ \mathrm{m/s^2}$. The difference between the distance the truck moves and the distance the box moves (*i.e.*, the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_\mathrm{truck} - a_\mathrm{box}}} = \sqrt{\frac{2(1.80\ \mathrm{m})}{(2.20\ \mathrm{m/s^2} - 1.47\ \mathrm{m/s^2})}} = 2.22\ \mathrm{s}.$$

In this time, the truck moves $\frac{1}{2} a_\mathrm{truck} t^2 = \frac{1}{2}(2.20\ \mathrm{m/s^2})(2.221\ \mathrm{s})^2 = 5.43\ \mathrm{m}$. Note that an extra figure was kept in the intermediate calculation to avoid roundoff error.

5.81: The friction force *on* block A is $\mu_\mathrm{k} w_A = (0.30)(1.40\ \mathrm{N}) = 0.420\ \mathrm{N}$, as in Problem 5-68. This is the magnitude of the friction force that block A exerts on block B, as well as the tension in the string. The force F must then have magnitude

$$\begin{aligned} F &= \mu_\mathrm{k}(w_B + w_A) + \mu_\mathrm{k} w_A + T = \mu_\mathrm{k}(w_B + 3w_A) \\ &= (0.30)(4.20\ \mathrm{N} + 3(1.40\ \mathrm{N})) = 2.52\ \ \mathrm{N}. \end{aligned}$$

Note that the normal force exerted on block B by the table is the sum of the weights of the blocks.

5.82: We take the upward direction as positive. The explorer's vertical acceleration is $-3.7\ \text{m/s}^2$ for the first 20 s. Thus at the end of that time her vertical velocity will be $v_y = at = (-3.7\ \text{m/s}^2)(20\ \text{s}) = -74\ \text{m/s}$. She will have fallen a distance

$$d = v_{\text{av}}t = \left(\frac{-74\ \text{m/s}}{2}\right)(20\ \text{s}) = -740\ \text{m}$$

and will thus be $1200 - 740 = 460\ \text{m}$ above the surface. Her vertical velocity must reach zero as she touches the ground; therefore, taking the ignition point of the PAPS as $y_0 = 0$,

$$v_y^2 = v_0^2 + 2a(y - y_0)$$

$$a = \frac{v_y^2 - v_0^2}{2(y - y_0)} = \frac{0 - (-74\ \text{m/s})^2}{-460} = 5.95\ \text{m/s}^2 \text{ or } 6.0\ \text{m/s}^2$$

which is the vertical acceleration that must be provided by the PAPS. The time it takes to reach the ground is given by

$$t = \frac{v - v_0}{a} = \frac{0 - (-74\ \text{m/s})}{5.95\ \text{m/s}^2} = 12.4\ \text{s}$$

Using Newton's Second Law for the vertical direction

$$F_{\text{PAPSv}} + mg = ma$$

$$\begin{aligned} F_{\text{PAPSv}} &= ma - mg = m(a + g) = (150\text{kg})(5.95 - (-3.7))\ \text{m/s}^2 \\ &= 1447.5\ \text{N or } 1400\ \text{N} \end{aligned}$$

which is the vertical component of the PAPS force. The vehicle must also be brought to a stop horizontally in 12.4 seconds; the acceleration needed to do this is

$$a = \frac{v - v_0}{t} = \frac{0 - 33\ \text{m/s}^2}{12.4\ \text{s}} = 2.66\ \text{m/s}^2$$

and the force needed is $F_{\text{PAPSh}} = ma = (150\ \text{kg})(2.66\ \text{m/s}^2) = 399\ \text{N}$ or 400 N, since there are no other horizontal forces.

5.83: Let the tension in the cord attached to block A be T_A and the tension in the cord attached to block C be T_C. The equations of motion are then

$$T_A - m_A g = m_A a$$
$$T_C - \mu_k m_B g - T_A = m_B a$$
$$m_C g - T_C = m_C a.$$

a) Adding these three equations to eliminate the tensions gives

$$a(m_A + m_B + m_C) = g(m_C - m_A - \mu_k m_B),$$

solving for m_C gives

$$m_C = \frac{m_A(a+g) + m_B(a + \mu_k g)}{g - a},$$

and substitution of numerical values gives $m_C = 12.9\ \text{kg}$.

b) $T_A = m_A(g+a) = 47.2\ \text{N}, T_C = m_C(g-a) = 101\ \text{N}$.

5.84: Considering positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block), the forces along the inclines and the accelerations are related by $T - (100\ \text{kg})g\sin 30° = (100\ \text{kg})a, (50\ \text{kg})g\sin 53° - T = (50\ \text{kg})a,$ where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50\ \text{kg}\sin 53° - 100\ \text{kg}\sin 30°)g = (50\ \text{kg} + 100\ \text{kg})a$, or $a = -0.067\ g$. a) Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067\ g$. b) $0.067(9.80\ \text{m/s}^2) = 0.658\ \text{m/s}^2$.

c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

5.85: Denote the magnitude of the acceleration of the block with mass m_1 as a; the block of mass m_2 will descend with acceleration $a/2$. If the tension in the rope is T, the equations of motion are then

$$T = m_1 a$$
$$m_2 g - 2T = m_2\, a/2.$$

Multiplying the first of these by 2 and adding to eliminate T, and then solving for a gives

$$a = \frac{m_2 g}{2m_1 + m_2/2} = g\frac{2m_2}{4m_1 + m_2}.$$

The acceleration of the block of mass m_2 is half of this, or $g\, m_2/(4m_1 + m_2)$.

5.86: Denote the common magnitude of the maximum acceleration as a. For block A to remain at rest with respect to block B, $a < \mu_s g$. The tension in the cord is then $T = (m_A + m_B)a + \mu_k g(m_A + m_B) = (m_A + m_B)(a + \mu_k g)$. This tension is related to the mass m_C by $T = m_C(g - a)$. Solving for a yields

$$a = g\frac{m_C - \mu_k(m_A + m_B)}{m_A + m_B + m_C} < \mu_s g.$$

Solving the inequality for m_C yields

$$m_C < \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s}.$$

5.87: See Exercise 5.15 (Atwood's machine). The 2.00-kg block will accelerate upward at $g\frac{5.00\,\text{kg} - 2.00\,\text{kg}}{5.00\,\text{kg} + 2.00\,\text{kg}} = 3g/7$, and the 5.00-kg block will accelerate downward at $3g/7$. Let the initial height above the ground be h_0; when the large block hits the ground, the small block will be at a height $2h_0$, and moving upward with a speed given by $v_0^2 = 2ah_0 = 6gh_0/7$. The small block will continue to rise a distance $v_0^2/2g = 3h_0/7$, and so the maximum height reached will be $2h_0 + 3h_0/7 = 17h_0/7 = 1.46\,\text{m}$, which is 0.860 m above its initial height.

5.88: The floor exerts an upward force n on the box, obtained from $n - mg = ma$, or $n = m(a+g)$. The friction force that needs to be balanced is

$$\mu_k n = \mu_k m(a+g)\,(0.32)(28.0\text{ kg})(1.90\text{ m/s}^2 + 9.80\text{ m/s}^2) = 105\text{ N}.$$

5.89: The upward friction force must be $f_s = \mu_s n = m_A g$, and the normal force, which is the only horizontal force on block A, must be $n = m_A a$, and so $a = g/\mu_s$. An observer on the cart would "feel" a backwards force, and would say that a similar force acts on the block, thereby creating the need for a normal force.

5.90: Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$, a) Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a, a = 2.21\text{ m/s}^2$ (note that an extra figure was kept in the intermediate calculation to avoid roundoff error). b) Substitution into either of the above relations gives $T = 2.27$ N. Equivalently, dividing the second relation by 2 and subtracting from the first gives $\frac{3}{2}T = 11.11\text{ N} - \frac{15.44\text{ N}}{2}$, giving the same result. c) The string will be slack. The 4.00-kg block will have $a = 2.78\text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93\text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

5.91: a) Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and $\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452.$

5.92: (a) There is a contact force n between the man (mass M) and the platform (mass m). The equation of motion for the man is $T + n - Mg = Ma$, where T is the tension in the rope, and for the platform, $T - n - mg = ma$. Adding to eliminate n, and rearranging, $T = \frac{1}{2}(M + m)(a + g)$. This result could be found directly by considering the man-platform combination as a unit, with mass $m + M$, being pulled upward with a force $2T$ due to the *two* ropes on the combination. The tension T in the rope is the same as the force that the man applies to the rope. Numerically,

$$T = \frac{1}{2}(70.0\ \text{kg} + 25.0\ \text{kg})(1.80\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 551\ \text{N}.$$

(b) The end of the rope moves downward 2 m when the platform moves up 1 m, so $a_{\text{rope}} = -2a_{\text{platform}}$. Relative to the man, the acceleration of the rope is $3a = 5.40\ \text{m/s}^2$, downward.

5.93: a) The only horizontal force on the two-block combination is the horizontal component of $\vec{F}, F\cos\alpha$. The blocks will accelerate with $a = F\cos\alpha/(m_1 + m_2)$. b) The normal force between the blocks is $m_1g + F\sin\alpha$, for the blocks to move together, the product of this force and μ_s must be greater than the horizontal force that the lower block exerts on the upper block. That horizontal force is one of an action-reaction pair; the reaction to this force accelerates the lower block. Thus, for the blocks to stay together, $m_2a \le \mu_s(m_1g + F\sin\alpha)$. Using the result of part (a),

$$m_2\frac{F\cos\alpha}{\text{m}_1 + m_2} \le \mu_s(m_1g + F\sin\alpha).$$

Solving the inequality for F gives the desired result.

5.94: The banked angle of the track has the same form as that found in Example 5.24, $\tan\beta = \frac{v_0^2}{gR}$, where v_0 is the ideal speed, $20\ \text{m/s}$ in this case. For speeds larger than v_0, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force n and the friction force f; $n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \frac{v^2}{R} = \frac{(1.5v_0)^2}{R} = 2.25\, g\tan\beta$, these two relations become

$$n\sin\beta + \mu_s n\cos\beta = 2.25\, mg\tan\beta,$$
$$n\cos\beta - \mu_s n\sin\beta = mg.$$

Dividing to cancel n gives

$$\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta.$$

Solving for μ_s and simplifying yields

$$\mu_s = \frac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}.$$

Using $\beta = \arctan\left(\frac{(20\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(120\ \text{m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

5.95: a) The same analysis as in Problem 5.90 applies, but with the speed v an unknown. The equations of motion become

$$n\sin\beta + \mu_s n\cos\beta = mv^2/R,$$
$$n\cos\beta - \mu_s n\sin\beta = mg.$$

Dividing to cancel n gives

$$\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = \frac{v^2}{Rg}.$$

Solving for v and substituting numerical values gives $v = 20.9\ \text{m/s}$ (note that the value for the coefficient of static friction must be used).

b) The same analysis applies, but the friction force must be directed up the bank; this has the same algebraic effect as replacing f with $-f$, or replacing μ_s with $-\mu_s$ (although coefficients of friction may certainly never be negative). The result is

$$v^2 = (gR)\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta},$$

and substitution of numerical values gives $v = 8.5\ \text{m/s}$.

5.96: (a) $80\ \text{mi/h}$ is $35.7\ \text{m/s}$ in SI units. The centripetal force needed to keep the car on the road is provided by friction; thus

$$\mu_s mg = \tfrac{mv^2}{r}$$

$$r = \frac{v^2}{\mu_s g} = \frac{(35.7\ \text{m/s})^2}{(0.76)(9.8\ \text{m/s}^2)} = 171\ \text{m} \ \text{ or } 170\ \text{m}$$

(b) If $\mu_s = 0.20$:

$$v^2 = r\mu_s g = (171\ \text{m})\,(0.20)\,(9.8\ \text{m/s}^2) = 335.2\ \text{m}^2/\text{s}^2$$

$$v = 18.3\ \text{m/s or about } 41\ \text{mi/h}$$

(c) If $\mu_s = 0.37$:

$$v^2 = (171\ \text{m})\,(0.37)\,(9.8\ \text{m/s}^2) = 620\ \text{m}^2/\text{s}^2$$

$$v = 24.9\ \text{m/s or about } 56\ \text{mi/h}$$

The speed limit is evidently designed for these conditions.

5.97: a) The static friction force between the tires and the road must provide the centripetal acceleration for motion in the circle.

$$\mu_s mg = m\frac{v^2}{r}$$

m, g, and r are constant so $\dfrac{v_1}{\sqrt{\mu_{s1}}} = \dfrac{v_2}{\sqrt{\mu_{s2}}}$, where 1 refers to dry road and 2 to wet road.

$\mu_{s2} = \frac{1}{2}\mu_{s1}$, so $v_2 = (27\ \text{m/s})/\sqrt{2} = 19\ \text{m/s}$

b) Calculate the time it takes you to reach the curve

$v_{0x} = 27\ \text{m/s},\ v_x = 19\ \text{m/s},\ x - x_0 = 800\ \text{m},\ t = ?$

$x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right) t$ gives $t = 34.7\ \text{s}$

During this time the other car will travel $x - x_0 = v_{0x}t = (36\ \text{m/s})\,(34.7\text{s}) = 1250\ \text{m}$. The other car will be 50 m behind you as you enter the curve, and will be traveling at nearly twice your speed, so it is likely it will skid into you.

5.98: The analysis of this problem is the same as that of Example 5.22; solving for v in terms of β and R, $v = \sqrt{gR\tan\beta} = \sqrt{(9.80\ \text{m/s}^2)\,(50.0)\tan 30.0^\circ} = 16.8\ \text{m/s}$, about $60.6\ \text{km/h}$.

5.99: The point to this problem is that the monkey and the bananas have the same weight, and the tension in the string is the same at the point where the bananas are suspended and where the monkey is pulling; in all cases, the monkey and bananas will have the same net force and hence the same acceleration, direction and magnitude. a) The bananas move up. b) The monkey and bananas always move at the same velocity, so the distance between them stays the same. c) Both the monkey and bananas are in free fall, and as they have the same initial velocity, the distance bewteen them doesn't change. d) The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

5.100: The separated equation of motion has a lower limit of $3v_t$ instead of 0; specifically,

$$\int_{3v_t}^{v} \frac{dv}{v - v_t} = \ln\frac{v_t - v}{-2v_t} = \ln\left(\frac{v}{2v_t} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or}$$

$$v = 2v_t\left[\frac{1}{2} + e^{-(k/m)t}\right].$$

Note that the speed is always greater than v_t.

5.101: a) The rock is released from rest, and so there is initially no resistive force and $a_0 = (18.0\ \text{N})/(3.00\ \text{kg}) = 6.00\ \text{m/s}^2$.

b) $(18.0\ \text{N} - (2.20\ \text{N}\cdot\text{s/m})\,(3.00\ \text{m/s}))/(3.00\ \text{kg}) = 3.80\ \text{m/s}^2$. c) The net force must be 1.80 N, so $kv = 16.2\ \ \text{N}$ and $v = (16.2\ \ \text{N})/(2.20\ \text{N}\cdot\text{s/m}) = 7.36\ \text{m/s}$. d) When the net force is equal to zero, and hence the acceleration is zero, $kv_\text{t} = 18.0\ \ \text{N}$ and $v_\text{t} = (18.0\ \ \text{N})/(2.20\ \text{N}\cdot\text{s/m}) = 8.18\ \text{m/s}$. e) From Eq. (5.12),

$$y = (8.18\ \text{m/s})\left[(2.00\ \text{s}) - \frac{3.00\ \text{kg}}{2.20\ \text{N}\cdot\text{s/m}}(1 - e^{-((2.20\ \text{N}\cdot\text{s/m})/(3.00\ \text{kg}))(2.00\ \text{s})})\right]$$
$$= +7.78\ \text{m}.$$

From Eq. (5.10),

$$v = (8.18\ \text{m/s})[1 - e^{-((2.2\ \text{N}\cdot\text{s/m})/(3.00\ \text{kg}))(2.00\ \text{s})}]$$
$$= 6.29\ \text{m/s}.$$

From Eq. (5.11), but with a_0 instead of g,

$$a = (6.00\ \text{m/s}^2)e^{-((2.20\ \text{N}\cdot\text{s/m})/(3.00\ \text{kg}))(2.00\ \text{s})} = 1.38\ \text{m/s}^2.$$

.

f)

$$1 - \frac{v}{v_\text{t}} = 0.1 = e^{-(k/m)t},\ \text{so}$$

$$t = \frac{m}{k}\ln(10) = 3.14\ \text{s}.$$

5.102: (a) The retarding force of the surface is the only horizontal force acting. Thus

$$a = \frac{F_{\text{net}}}{m} = \frac{F_R}{m} = \frac{-kv^{1/2}}{m} = \frac{dv}{dt}$$

$$\frac{dv}{v^{1/2}} = -\frac{k}{m}dt$$

$$\int_{v_0}^{v} \frac{dv}{v^{1/2}} = \frac{k}{m}\int_0^t dt$$

$$2v^{1/2}\Big|_{v_0}^{v} = -\frac{kt}{m}$$

which gives

$$v = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{4m^2}$$

For the rock's position**:**

$$\frac{dx}{dt} = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{4m^2}$$

$$dx = v_0dt - \frac{v_0^{1/2}ktdt}{m} + \frac{k^2t^2dt}{4m^2}$$

and integrating gives

$$x = v_0t - \frac{v_0^{1/2}kt^2}{2m} + \frac{k^2t^3}{12m^2}$$

(b)

$$v = 0 = v_0 - \frac{v_0^{1/2}kt}{m} + \frac{k^2t^2}{2m^2}$$

This is a quadratic equation in t; from the quadratic formula we can find the single solution**:**

$$t = \frac{2mv_0^{1/2}}{k}$$

(c) Substituting the expression for t into the equation for x**:**

$$x = v_0 \cdot \frac{2mv_0^{1/2}}{k} - \frac{v_0^{1/2}k}{2m} \cdot \frac{4m^2v_0}{k^2} + \frac{k^2}{12m^2} \cdot \frac{8m^3v_0^{3/2}}{k^3}$$

$$= \frac{2mv_0^{3/2}}{3k}$$

5.103: Without buoyancy, $kv_{\mathrm{t}} = mg$, so $k = \dfrac{mg}{v_{\mathrm{t}}} = \dfrac{mg}{0.36\ \mathrm{s}}$.

With buoyancy included there is the additional upward buoyancy force B, so

$$B - kv_{\mathrm{t}} = mg$$

$$B = mg - kv_{\mathrm{t}} = mg\left(1 - \frac{0.24\ \mathrm{m/s}}{0.36\ \mathrm{m/s}}\right) = mg/3$$

5.104: Recognizing the geometry of a 3-4-5 right triangle simplifies the calculation. For instance, the radius of the circle of the mass' motion is 0.75 m.

a) Balancing the vertical force, $T_{\mathrm{U}}\frac{4}{5} - T_{\mathrm{L}}\frac{4}{5} = w$, so

$$T_{\mathrm{L}} = T_{\mathrm{U}} - \frac{5}{4}w = 80.0\ \mathrm{N} - \frac{5}{4}(4.00\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2}) = 31.0\ \mathrm{N}.$$

b) The net inward force is $F = \frac{3}{5}T_{\mathrm{U}} + \frac{3}{5}T_{\mathrm{L}} = 66.6\ \mathrm{N}$. Solving $F = ma_{\mathrm{rad}} = m\frac{4\pi^2 R}{T^2}$ for the period T,

$$T = 2\pi\sqrt{\frac{mR}{F}} = 2\pi\sqrt{\frac{(4.00\ \mathrm{kg})\,(0.75\ \mathrm{m})}{(66.6\ \mathrm{N})}} = 1.334\ \mathrm{s},$$

or 0.02223 min, so the system makes 45.0 rev/min. c) When the lower string becomes slack, the system is the same as the conical pendulum considered in Example 5.22. With $\cos\beta = 0.800$, the period is $T = 2\pi\sqrt{(1.25\ \mathrm{m})\,(0.800)/(9.80\ \mathrm{m/s^2})} = 2.007\ \mathrm{s}$, which is the same as $29.9\ \mathrm{rev/min}$. d) The system will still be the same as a conical pendulum, but the block will drop to a smaller angle.

5.105: a) Newton's 2nd law gives

$$m\frac{dv_y}{dt} = mg - kv_y, \text{ where } \frac{mg}{k} = v_t$$

$$\int_{v_0}^{v_y} \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$$

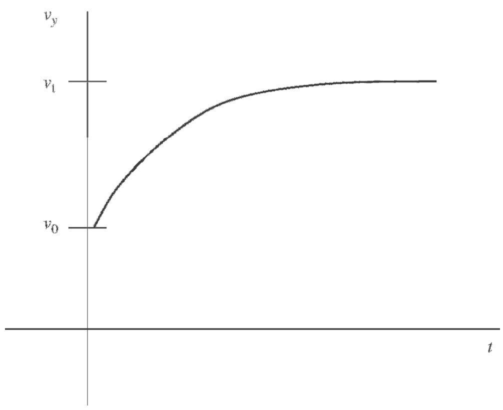
This is the same expression used in the derivation of Eq. (5.10), except the lower limit in the velocity integral is the initial speed v_0 instead of zero.

Evaluating the integrals and rearranging gives

$$v = v_0 e^{-kt/m} + v_t(1 - e^{-kt/m})$$

Note that at $t = 0$ this expression says $v_y = v_0$ and at $t \to \alpha$ it says $v_y \to v_t$.

b) The downward gravity force is larger than the upward fluid resistance force so the acceleration is downward, until the fluid resistance force equals gravity when the terminal speed is reached. The object speeds up until $v_y = v_t$. Take $+y$ to be downward.



c) The upward resistance force is larger than the downward gravity force so the acceleration is upward and the object slows down, until the fluid resistance force equals gravity when the terminal speed is reached. Take $+y$ to be downward.

5.106: (a) To find find the maximum height and time to the top without fluid resistance**:**

$$v_y^2 = v_{0y}^2 + 2a(y - y_0)$$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a} = \frac{0 - (6.0\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 1.84\ \text{m or } 1.8\ \text{m}$$

$$t = \frac{v - v_0}{a} = \frac{0 - 6.0\ \text{m/s}}{-9.8\ \text{m/s}^2} = 0.61\ \text{s}$$

(b) Starting from Newton's Second Law for this situation

$$m\frac{dv}{dt} = mg - kv$$

we rearrange and integrate, taking downward as positive as in the text and noting that the velocity at the top of the rock's "flight" is zero**:**

$$\int_v^0 \frac{dv}{v - v_\text{t}} = -\frac{k}{m}t$$

$$\ln(v - v_\text{t})\Big|_v^0 = \ln\frac{-v_\text{t}}{v - v_\text{t}} = \ln\frac{-2.0\ \text{m/s}}{-6.0\ \text{m/s} - 2.0\ \text{m/s}} = \ln(0.25) = -1.386$$

From Eq. 5.9, $m/k = v_\text{t}/g = (2.0\ \text{m/s}^2)/(9.8\ \text{m/s}^2) = 0.204\ \text{s}$, and $t = -\frac{m}{k}(-1.386) = (0.204\ \text{s})\,(1.386) = 0.283\ \text{s}$ to the top. Equation 5.10 in the text gives us

$$\frac{dx}{dt} = v_\text{t}(1 - e^{-(\text{k/m})t}) = v_\text{t} - v_\text{t}e^{-(\text{k/m})t}$$

$$\int_0^x dx = \int_0^t v_\text{t}dt - \int_0^t v_\text{t}e^{-(\text{k/m})t}dt$$

$$= v_\text{t}t + \frac{v_\text{t}m}{k}(e^{-(\text{k/m})t} - 1)$$

$$= (2.0\ \text{m/s})\,(0.283\ \text{s}) + (2.0\ \text{m/s})\,(0.204\ \text{s})(\text{e}^{-1.387} - 1)$$

$$= 0.26\ \text{m}$$

5.107: a) The forces on the car are the air drag force $f_{\mathrm{D}} = Dv^2$ and the rolling friction force $\mu_{\mathrm{r}} mg$. Take the velocity to be in the $+x$-direction. The forces are opposite in direction to the velocity. $\sum F_x = ma_x$ gives

$$-Dv^2 - \mu_{\mathrm{r}} mg = ma$$

We can write this equation twice, once with $v = 32\ \mathrm{m/s}$ and $a = -0.42\ \mathrm{m/s^2}$ and once with $v = 24\ \mathrm{m/s}$ and $a = -0.30\ \mathrm{m/s^2}$. Solving these two simultaneous equations in the unknowns D and μ_{r} gives $\mu_{\mathrm{r}} = 0.015$ and $D = 0.36\ \mathrm{N \cdot s^2/m^2}$.

b) $n = mg\cos\beta$ and the component of gravity parallel to the incline is $mg\sin\beta$, where $\beta = 2.2°$.

For constant speed, $mg \sin 2.2° - \mu_{\mathrm{r}} mg \cos 2.2° - Dv^2 = 0$.

Solving for v gives $v = 29\ \mathrm{m/s}$.

c) For angle $\beta, mg\sin\beta - \mu_{\mathrm{r}} mg \cos\beta - Dv^2 = 0$

and $v = \sqrt{\dfrac{mg(\sin\beta - \mu_{\mathrm{r}} \cos\beta)}{D}}$

The terminal speed for a falling object is derived from $Dv_{\mathrm{t}}^2 - mg = 0$, so

$v_{\mathrm{t}} = \sqrt{mg/D}.$

$v/v_{\mathrm{t}} = \sqrt{\sin\beta - \mu_{\mathrm{r}} \cos\beta}$

And since $\mu_{\mathrm{r}} = 0.015, v/v_{\mathrm{t}} = \sqrt{\sin\beta - (0.015)\cos\beta}$

5.108: (a) One way of looking at this is that the apparent weight, which is the same as the upward force on the person, is the actual weight of the person minus the centripetal force needed to keep him moving in its circular path**:**

$$w_{\mathrm{app}} = mg - \frac{mv^2}{R} = (70\ \mathrm{kg})\left[(9.8\ \mathrm{m/s^2}) - \frac{(12\ \mathrm{m/s})^2}{40\ \mathrm{m}}\right]$$
$$= 434\ \mathrm{N}$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it**:**

$$mg - \frac{mv^2}{R} = 0$$
$$v^2 = Rg = (40\ \mathrm{m})(9.8\ \mathrm{m/s^2}) = 392\ \mathrm{m^2/s^2}$$
$$v = 19.8\ \mathrm{m/s}\ \text{or}\ 20\ \mathrm{m/s}$$

The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so is its weight, which provides the centripetal force in this situation.

5.109: a) For the same rotation rate, the magnitude of the radial acceleration is proportional to the radius, and for twins of the same mass, the needed force is proportional to the radius; Jackie is twice as far away from the center, and so must hold on with twice as much force as Jena, or 120 N.

b) $\sum F_{\text{Jackie}} = mv^2/r.$

$$v = \sqrt{\frac{(120\ \text{N})\,(3.6\ \text{m})}{30\ \text{kg}}} = 3.8\ \text{m/s}.$$

5.110: The passenger's velocity is $v = 2\pi R/t = 8.80\ \text{m/s}$. The vertical component of the seat's force must balance the passenger's weight and the horizontal component must provide the centripetal force. Therefore**:**

$$F_{\text{seat}} \sin\theta = mg = 833\ \text{N}$$

$$F_{\text{seat}} \cos\theta = \frac{mv^2}{R} = 188\ \text{N}$$

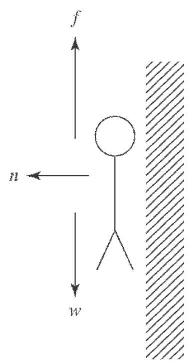
Therefore $\tan\theta = 833\ \text{N}/188\ \text{N} = 4.43; \theta = 77.3°$ above the horizontal. The magnitude of the net force exerted by the seat (note that this is not the net force on the passenger) is

$$F_{\text{seat}} = \sqrt{(mg)^2 + \left(\frac{mv^2}{R}\right)^2} = \sqrt{(833\ \text{N})^2 + (188\ \text{N})^2}$$

$$= 854\ \text{N}$$

(b) The magnitude of the force is the same, but the horizontal component is reversed.

5.111: a)



b) The upward friction force must be equal to the weight, so $\mu_s n = \mu_s m(4\pi^2 R/T^2) \geq mg$ and

$$\mu_s > \frac{gT^2}{4\pi^2 R} = \frac{(9.80\ \text{m/s}^2)\,(1\text{s}/0.60\ \text{rev})^2}{4\pi^2(2.5\ \text{m})} = 0.28.$$

c) No; both the weight and the required normal force are proportional to the rider's mass.

5.112: a) For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed mg, so $m\frac{v^2}{R} > mg$, and $v > \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)(13.0\ \text{m})} = 11.3\ \text{m/s}$.

b) The (upward) acceleration will then be $4g$, so the upward normal force must be $5\,mg = 5(110\ \text{kg})(9.80\ \text{m/s}^2) = 5390\ \text{N}$.

5.113: a) What really happens (according to a nosy observer on the ground) is that you slide closer to the passenger by turning to the right. b) The analysis is the same as that of Example 5.23. In this case, the friction force should be insufficient to provide the inward radial acceleration, and so $\mu_s mg < mv^2/R$, or

$$R < \frac{v^2}{\mu_s g} = \frac{(20\ \text{m/s})^2}{(0.35)(9.80\ \text{m/s}^2)} = 120\ \text{m}$$

to two places. Why the passenger is not wearing a seat belt is another question.

5.114: The tension F in the string must be the same as the weight of the hanging block, and must also provide the resultant force necessary to keep the block on the table in uniform circular motion; $Mg = F = m\frac{v^2}{r}$, so $v = \sqrt{gr\,M/m}$.

5.115: a) The analysis is the same as that for the conical pendulum of Example 5.22, and so

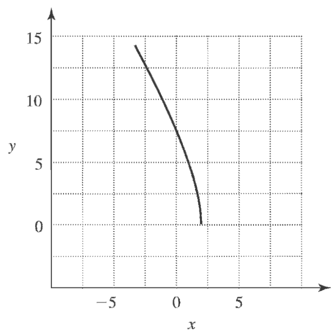
$$\beta = \arccos\left(\frac{gT^2}{4\pi^2 L}\right) = \arccos\left(\frac{(9.80\ \text{m/s}^2)(1/4.00\ \text{s})^2}{4\pi^2(0.100\ \text{m})}\right) = 81.0°.$$

b) For the bead to be at the same elevation as the center of the hoop, $\beta = 90°$ and $\cos\beta = 0$, which would mean $T = 0$, the speed of the bead would be infinite, and this is not possible. c) The expression for $\cos\beta$ gives $\cos\beta = 2.48$, which is not possible. In deriving the expression for $\cos\beta$, a factor of $\sin\beta$ was canceled, precluding the possibility that $\beta = 0$. For this situation, $\beta = 0$ is the only physical possibility.

5.116: a) Differentiating twice, $a_x = -6\beta t$ and $a_y = -2\delta$, so

$$F_x = ma_x = (2.20\text{ kg})\,(-0.72\text{ N/s})t = -(1.58\text{ N/s})t$$
$$F_y = ma_y = (2.20\text{ kg})\,(-2.00\text{ m/s}^2) = -4.40\text{ N}.$$

b)

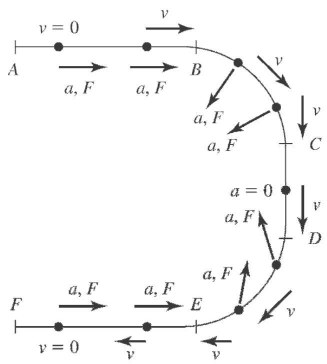


c) At $t = 3.00\text{ s}, F_x = -4.75\text{ N}$ and $F_y = -4.40\text{ N}$, so

$$F = \sqrt{(-4.75\text{ N})^2 + (-4.40\ \text{ N})^2} = 6.48\ \text{ N},$$

at an angle of $\arctan\left(\frac{-4.40}{-4.75}\right) = 223°$.

5.117:



5.118: See Example 5.25.

a) $F_A = m\left(g + \frac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{m/s}^2 + \frac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

b) $F_B = m\left(g - \frac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{m/s}^2 - \frac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N}.$, where the minus sign indicates that the track pushes *down* on the car. The magnitude of this force is 30.4 N.

5.119: The analysis is the same as for Problem 5.95; in the case of the cone, the speed is related to the period by $v = 2\pi R/T = 2\pi h \tan\beta/T$, or $T = 2\pi h \tan\beta/v$. The maximum and minimum speeds are the same as those found in Problem 5.95,

$$v_{\max} = \sqrt{gh \tan\beta \frac{\cos\beta + \mu_s \sin\beta}{\sin\beta - \mu_s \cos\beta}}$$

$$v_{\min} = \sqrt{gh \tan\beta \frac{\cos\beta - \mu_s \sin\beta}{\sin\beta + \mu_s \cos\beta}}.$$

The minimum and maximum values of the period T are then

$$T_{\min} = 2\pi \sqrt{\frac{h \tan\beta}{g} \frac{\sin\beta - \mu_s \cos\beta}{\cos\beta + \mu_s \sin\beta}}$$

$$T_{\max} = 2\pi \sqrt{\frac{h \tan\beta}{g} \frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}}.$$

5.120: a) There are many ways to do these sorts of problems; the method presented is fairly straightforward in terms of application of Newton's laws, but involves a good deal of algebra. For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y. The equations of motion are then

$$MA = -n\sin\alpha$$

$$ma_x = n\sin\alpha$$

$$ma_y = n\cos\alpha - mg.$$

Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y.

An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is

$$\frac{a_y}{a_x - A} = -\tan\alpha.$$

At this point, algebra is unavoidable. Symbolic-manipulation programs may save some solution time. A possible approach is to eliminate a_x by noting that $a_x = -\frac{M}{m}A$ (a result that anticipates conservation of momentum), using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+\mathrm{m})\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

(b) When $M >> m, A \to 0,$ as expected (the large block won't move). Also, $a_x \to \frac{g}{\tan\alpha + (1/\tan\alpha)} = g\frac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$, which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha.$

(c) The trajectory is a spiral.

5.121: If the block is not to move vertically, the acceleration must be horizontal. The common acceleration is $a = g \tan\theta,$ so the applied force must be $(M + m)a = (M + m)g \tan\theta.$

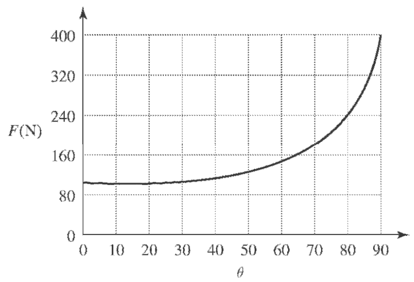
5.122: The normal force that the ramp exerts on the box will be $n = w\cos\alpha - T\sin\theta.$ The rope provides a force of $T\cos\theta$ up the ramp, and the component of the weight down the ramp is $w\sin\alpha.$ Thus, the net force up the ramp is

$$\begin{aligned} F &= T\cos\theta - w\sin\alpha - \mu_k(w\cos\alpha - T\sin\theta) \\ &= T(\cos\theta + \mu_k \sin\theta) - w(\sin\alpha + \mu_k \cos\alpha). \end{aligned}$$

The acceleration will be the greatest when the first term in parantheses is greatest; as in Problems 5.77 and 5.123, this occurs when $\tan\theta = \mu_k.$

5.123: a) See Exercise 5.38; $F = \mu_k w/(\cos\theta + \mu_k \sin\theta).$

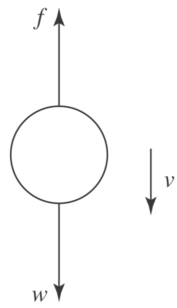
b)



c) The expression for F is a minimum when the denominator is a maximum; the calculus is identical to that of Problem 5.77 (maximizing w for a given F gives the same result as minimizing F for a given w), and so F is minimized at $\tan\theta = \mu_k.$ For $\mu_k = 0.25, \theta = 14.0°,$ keeping an extra figure.

5.124: For convenience, take the positive direction to be down, so that for the baseball released from rest, the acceleration and velocity will be positive, and the speed of the baseball is the same as its positive component of velocity. Then the resisting force, directed against the velocity, is upward and hence negative.

a)



b) Newton's Second Law is then $ma = mg - Dv^2$. Initially, when $v = 0$, the acceleration is g, and the speed increases. As the speed increases, the resistive force increases and hence the acceleration decreases. This continues as the speed approaches the terminal speed. c) At terminal velocity, $a = 0$, so $v_t = \sqrt{\frac{mg}{D}}$, in agreement with Eq. (5.13). d) The equation of motion may be rewritten as $\frac{dv}{dt} = \frac{g}{v_t^2}(v_t^2 - v^2)$. This is a separable equation and may be expressed as

$$\int \frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\int dt, \quad \text{or}$$

$$\frac{1}{v_t}\operatorname{arctanh}\left(\frac{v}{v_t}\right) = \frac{gt}{v_t^2},$$

so $v = v_t \tanh(gt/v_t)$.

Note: If inverse hyperbolic functions are unknown or undesirable, the integral can be done by partial fractions, in that

$$\frac{1}{v_t^2 - v^2} = \frac{1}{2v_t}\left[\frac{1}{v_t - v} + \frac{1}{v_t + v}\right],$$

and the resulting logarithms in the integrals can be solved for $v(t)$ in terms of exponentials.

5.125: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley *B* is a_B, then $a_B = -a_3$, and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3$. There can be no net force on the massless pulley *B*, so $T_C = 2T_A$. The five equations to be solved are then

$$m_1 g - T_A = m_1 a_1$$
$$m_2 g - T_A = m_2 a_2$$
$$m_3 g - T_C = m_3 a_3$$
$$a_1 + a_2 + 2a_3 = 0$$
$$2T_A - T_C = 0.$$

These are five equations in five unknowns, and may be solved by standard means. A symbolic-manipulation program is of great use here.

a) The accelerations a_1 and a_2 may be eliminated by using

$$2a_3 = -(a_1 + a_2) = -(2g - T_A((1/m_1) + (1/m_2))).$$

The tension T_A may be eliminated by using

$$T_A = (1/2)T_C = (1/2)m_3(g - a_3).$$

Combining and solving for a_3 gives

$$a_3 = g\frac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

b) The acceleration of the pulley *B* has the same magnitude as a_3 and is in the opposite direction.

c) $$a_1 = g - \frac{T_A}{m_1} = g - \frac{T_C}{2m_1} = g - \frac{m_3}{2m_1}(g - a_3).$$

Substituting the above expression for a_3 gives

$$a_1 = g\frac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

d) A similar analysis (or, interchanging the labels 1 and 2) gives

$$a_2 = g\frac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

e) & f) Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving

$$T_A = g\frac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3},\quad \mathrm{T_C} = g\frac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

g) If $m_1 = m_2 = m$ and $m_3 = 2m$, all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg$. All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

5.126: In all cases, the tension in the string will be half of F.

a) $F/2 = 62\text{ N}$, which is insufficient to raise either block; $a_1 = a_2 = 0$.

b) $F/2 = 62\text{ N}$. The larger block (of weight 196 N) will not move, so $a_1 = 0$, but the smaller block, of weight 98 N, has a net upward force of 49 N applied to it, and so will accelerate upwards with $a_2 = \frac{49\text{ N}}{10.0\text{ kg}} = 4.9\text{ m/s}^2$.

c) $F/2 = 212\text{ N}$, so the net upward force on block A is 16 N and that on block B is 114 N, so $a_1 = \frac{16\text{ N}}{20.0\text{ kg}} = 0.8\text{ m/s}^2$ and $a_2 = \frac{114\text{ N}}{10.0\text{ kg}} = 11.4\text{ m/s}^2$.

5.127: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$, or $T_A = w/\cos\beta$. At point B, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$, and the ratio $(T_B/T_A) = \cos^2\beta$.

Capítulo 6

6.1: a) $(2.40\ \text{N})\,(1.5\ \text{m}) = 3.60\ \text{J}$ b) $(-0.600\ \text{N})(1.50\ \text{m}) = -0.900\ \text{J}$
c) $3.60\ \text{J} - 0.720\ \text{J} = 2.70\ \text{J}$.

6.2: a) "Pulling slowly" can be taken to mean that the bucket rises at constant speed, so the tension in the rope may be taken to be the bucket's weight. In pulling a given length of rope, from Eq. (6.1),

$$W = Fs = mgs = (6.75\ \text{kg})\,(9.80\ \text{m/s}^2)(4.00\ \text{m}) = 264.6\ \text{J}.$$

b) Gravity is directed opposite to the direction of the bucket's motion, so Eq. (6.2) gives the negative of the result of part (a), or $-265\ \text{J}$. c) The net work done on the bucket is zero.

6.3: $(25.0\ \text{N})(12.0\ \text{m}) = 300\ \text{J}$.

6.4: a) The friction force to be overcome is

$$f = \mu_k n = \mu_k mg = (0.25)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 73.5\ \text{N},$$

or 74 N to two figures.

b) From Eq. (6.1), $Fs = (73.5\ \text{N})(4.5\ \text{m}) = 331\ \text{J}$. The work is positive, since the worker is pushing in the same direction as the crate's motion.

c) Since f and s are oppositely directed, Eq. (6.2) gives

$$-fs = -(73.5\ \text{N})(4.5\ \text{m}) = -331\ \text{J}.$$

d) Both the normal force and gravity act perpendicular to the direction of motion, so neither force does work. e) The net work done is zero.

6.5: a) See Exercise 5.37. The needed force is

$$F = \frac{\mu_k mg}{\cos\phi - \mu_k \sin\phi} = \frac{(0.25)(30\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\ \text{N},$$

keeping extra figures. b) $Fs\cos\phi = (99.2\ \text{N})(4.50\ \text{m})\cos 30° = 386.5\ \text{J}$, again keeping an extra figure. c) The normal force is $mg + F\sin\phi$, and so the work done by friction is $-(4.50\ \text{m})(0.25)((30\ \text{kg})(9.80\ \text{m/s}^2) + (99.2\ \text{N})\sin 30°) = -386.5\ \text{J}$. d) Both the normal force and gravity act perpendicular to the direction of motion, so neither force does work. e) The net work done is zero.

6.6: From Eq. (6.2),

$$Fs\cos\phi = (180\ \text{N})(300\ \text{m})\cos 15.0° = 5.22\times10^4\ \text{J}.$$

6.7: $2Fs\cos\phi = 2(1.80\times10^6\ \text{N})(0.75\times10^3\ \text{m})\cos 14° = 2.62\times10^9\ \text{J}$, or $2.6\times10^9\ \text{J}$ to two places.

6.8: The work you do is:

$$\begin{aligned}\vec{F}\cdot\vec{s} &= ((30\text{N})\hat{\imath} - (40\text{N})\hat{\jmath})\cdot((-9.0\text{m})\hat{\imath} - (3.0\text{m})\hat{\jmath})\\ &= (30\ \text{N})(-9.0\ \text{m}) + (-40\ \text{N})(-3.0\ \text{m})\\ &= -270\ \text{N}\cdot\text{m} + 120\ \text{N}\cdot\text{m} = -150\ \text{J}\end{aligned}$$

6.9: a) (i) Tension force is always perpendicular to the displacement and does no work.
(ii) Work done by gravity is $-mg(y_2 - y_1)$. When $y_1 = y_2$, $W_{mg} = 0$.
b) (i) Tension does no work.
(ii) Let l be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1\ \text{J}$
The displacement is upward and the gravity force is downward, so it does negative work.

6.10: a) From Eq. (6.6),

$$K = \frac{1}{2}(1600\,\text{kg})\left((50.0\,\text{km/h})\left(\frac{1}{3.6}\frac{\text{m/s}}{\text{km/h}}\right)\right)^2 = 1.54\times10^5\ \text{J}.$$

b) Equation (6.5) gives the explicit dependence of kinetic energy on speed; doubling the speed of any object increases the kinetic energy by a factor of four.

6.11: For the T-Rex, $K = \frac{1}{2}(7000\,\text{kg})((4\,\text{km/hr})\frac{1\,\text{m/s}}{3.6\,\text{km/hr}})^2 = 4.32\times10^3\ \text{J}$. The person's velocity would be $v = \sqrt{2(4.32\times10^3\ \text{J})/70\,\text{kg}} = 11.1\,\text{m/s}$, or about 40 km/h.

6.12: (a) Estimate: $v \approx 1\,\text{m/s}$ (walking)

$v \approx 2\,\text{m/s}$ (running)

$m \approx 70\,\text{kg}$

Walking: $KE = \frac{1}{2}mv^2 = \frac{1}{2}(70\,\text{kg})(1\,\text{m/s})^2 = 35\,\text{J}$

Running: $KE = \frac{1}{2}(70\,\text{kg})(2\,\text{m/s})^2 = 140\,\text{J}$

(b) Estimate: $v \approx 60\,\text{mph} = 88\,\text{ft/s} \approx 30\,\text{m/s}$

$m \approx 2000\,\text{kg}$

$KE = \frac{1}{2}(2000\,\text{kg})(30\,\text{m/s})^2 = 9\times10^5\ \text{J}$

(c) $KE = W_{\text{gravity}} = mgh$ Estimate $h \approx 2\,\text{m}$

$KE = (1\,\text{kg})(9.8\,\text{m/s}^2)(2\,\text{m}) \approx 20\,\text{J}$

6.13: Let point 1 be at the bottom of the incline and let point 2 be at the skier.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \dfrac{1}{2}mv_0^2,\ K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem and solving for v_0 gives

$v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$

6.14: (a)

$$\begin{aligned}W &= \Delta KE\\ -mgh &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\\ v_0 &= \sqrt{v_f^2 + 2gh}\\ &= \sqrt{(25.0\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(15.0\,\text{m})}\\ &= 30.3\,\text{m/s}\end{aligned}$$

(b)

$$\begin{aligned}W &= \Delta KE\\ -mgh &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\\ h &= \frac{v_0^2 - v_f^2}{2g} = \frac{(30.3\,\text{m/s})^2 - 0^2}{2(9.80\,\text{ms/s}^2)}\\ &= 46.8\,\text{m}\end{aligned}$$

6.15: a) parallel to incline: force component $= mg\sin\alpha$, down incline; displacement $= h/\sin\alpha$, down incline

$W_{\parallel} = (mg\sin\alpha)(h/\sin\alpha) = mgh$ perpendicular to incline: no displacement in this direction, so $W_{\perp} = 0$.

$W_{mg} = W_{\parallel} + W_{\perp} = mgh$, same as falling height *h*.

b) $W_{\text{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height *h*. The work done by gravity depends only on the vertical displacement of the object.

When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

c) $h = 15.0\,\text{m}$, so $v = \sqrt{2gh} = 17.1\,\text{s}$

6.16: Doubling the speed increases the kinetic energy, and hence the magnitude of the work done by friction, by a factor of four. With the stopping force given as being independent of speed, the distance must also increase by a factor of four.

6.17: Barring a balk, the initial kinetic energy of the ball is zero, and so

$$W = (1/2)mv^2 = (1/2)(0.145\,\text{kg})(32.0\,\text{m/s})^2 = 74.2\,\text{J}.$$

6.18: As the example explains, the boats have the same kinetic energy K at the finish line, so $(1/2)m_A v_A^2 = (1/2)m_B v_B^2$, or, with $m_B = 2m_A, v_A^2 = 2v_B^2$. a) Solving for the ratio of the speeds, $v_A / v_B = \sqrt{2}$. b) The boats are said to start from rest, so the elapsed time is the distance divided by the average speed. The ratio of the average speeds is the same as the ratio of the final speeds, so the ratio of the elapsed times is $t_B / t_A = v_A / v_B = \sqrt{2}$.

6.19: a) From Eq. (6.5), $K_2 = K_1/16$, and from Eq. (6.6), $W = -(15/16)K_1$. b) No; kinetic energies depend on the magnitudes of velocities only.

6.20: From Equations (6.1), (6.5) and (6.6), and solving for F,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(8.00\,\text{kg})((6.00\,\text{m/s})^2 - (4.00\,\text{m/s})^2)}{(2.50\,\text{m})} = 32.0\,\text{N}.$$

6.21:

$$s = \frac{\Delta K}{F} = \frac{\frac{1}{2}(0.420\,\text{kg})((6.00\,\text{m/s})^2 - (2.00\,\text{m/s})^2)}{(40.0\,\text{N})} = 16.8\,\text{cm}$$

6.22: a) If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0\,\text{N})(1.20\,\text{m})}{(4.30\,\text{kg})}} = 4.48\,\text{m/s}.$$

b) The net work is $Fs - f_k s = (F - \mu_k mg)s$, so

$$\begin{aligned} v &= \sqrt{\frac{2(F - \mu_k mg)s}{m}} \\ &= \sqrt{\frac{2(36.0\,\text{N} - (0.30)(4.30\,\text{kg})(9.80\,\text{m/s}^2))(1.20\,\text{m})}{(4.30\,\text{kg})}} \\ &= 3.61\,\text{m/s}. \end{aligned}$$

(Note that even though the coefficient of friction is known to only two places, the difference of the forces is still known to three places.)

6.23: a) On the way up, gravity is opposed to the direction of motion, and so $W = -mgs = -(0.145\,\text{kg})(9.80\,\text{m/s}^2)(20.0\,\text{m}) = -28.4\,\text{J}$.

b) $$v_2 = \sqrt{v_1^2 + 2\frac{W}{m}} = \sqrt{(25.0\,\text{m/s})^2 + \frac{2(-28.4\,\text{J})}{(0.145\,\text{kg})}} = 15.26\,\text{m/s}.$$

c) No; in the absence of air resistance, the ball will have the same speed on the way down as on the way up. On the way down, gravity will have done both negative and positive work on the ball, but the net work will be the same.

6.24: a) Gravity acts in the same direction as the watermelon's motion, so Eq. (6.1) gives

$$W = Fs = mgs = (4.80\,\text{kg})(9.80\,\text{m/s}^2)(25.0\,\text{m}) = 1176\,\text{J}.$$

b) Since the melon is released from rest, $K_1 = 0$, and Eq. (6.6) gives

$$K = K_2 = W = 1176\,\text{J}.$$

6.25: a) Combining Equations (6.5) and (6.6) and solving for v_2 algebraically,

$$v_2 = \sqrt{v_1^2 + 2\frac{W_{\text{tot}}}{m}} = \sqrt{(4.00\,\text{m/s})^2 + \frac{2(10.0\,\text{N})(3.0\,\text{m})}{(7.00\,\text{kg})}} = 4.96\,\text{m/s}.$$

Keeping extra figures in the intermediate calculations, the acceleration is $a = (10.0\,\text{kg}\cdot\text{m/s}^2)/(7.00\,\text{kg}) = 1.429\,\text{m/s}^2$. From Eq. (2.13), with appropriate change in notation,

$$v_2^2 = v_1^2 + 2as = (4.00\,\text{m/s})^2 + 2(1.429\,\text{m/s}^2)(3.0\,\text{m}),$$

giving the same result.

6.26: The normal force does no work. The work-energy theorem, along with Eq. (6.5), gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta},$$

where $h = L\sin\theta$ is the vertical distance the block has dropped, and θ is the angle the plane makes with the horizontal. Using the given numbers,

$$v = \sqrt{2(9.80\,\text{m/s}^2)(0.75\,\text{m})\sin 36.9^\circ} = 2.97\,\text{m/s}.$$

6.27: a) The friction force is $\mu_k mg$, which is directed against the car's motion, so the net work done is $-\mu_k mgs$. The change in kinetic energy is $\Delta K = -K_1 = -(1/2)mv_0^2$, and so $s = v_0^2 / 2\mu_k g$. b) From the result of part (a), the stopping distance is proportional to the square of the initial speed, and so for an initial speed of 60 km/h, $s = (91.2\,\text{m})(60.0/80.0)^2 = 51.3\,\text{m}$. (This method avoids the intermediate calculation of μ_k, which in this case is about 0.279.)

6.28: The intermediate calculation of the spring constant may be avoided by using Eq. (6.9) to see that the work is proportional to the square of the extension; the work needed to compress the spring 4.00 cm is $(12.0\,\text{J})\left(\frac{4.00\,\text{cm}}{3.00\,\text{cm}}\right)^2 = 21.3\,\text{J}$.

6.29: a) The magnitude of the force is proportional to the magnitude of the extension or compression;

$$(160\,\text{N})(0.015\,\text{m}/0.050\,\text{m}) = 48\,\text{N}, \qquad (160\,\text{N})(0.020\,\text{m}/0.050\,\text{m}) = 64\,\text{N}.$$

b) There are many equivalent ways to do the necessary algebra. One way is to note that to stretch the spring the original 0.050 m requires $\frac{1}{2}\left(\frac{169\ \text{N}}{0.050\ \text{m}}\right) = (0.050\,\text{m})^2 = 4\,\text{J}$, so that stretching 0.015 m requires $(4\,\text{J})(0.015/0.050)^2 = 0.360\,\text{J}$ and compressing 0.020 m requires $(4\,\text{J})(0.020/0.050)^2 = 0.64\,\text{J}$. Another is to find the spring constant $k = (160\,\text{N}) \div (0.050\,\text{m}) = 3.20\times 10^3\,\text{N/m}$, from which $(1/2)(3.20\times 10^3\,\text{N/m})(0.015\,\text{m})^2 = 0.360\,\text{J}$ and $(1/2)(3.20\times 10^3\,\text{N/m})(0.020\,\text{m})^2 = 0.64\,\text{J}$.

6.30: The work can be found by finding the area under the graph, being careful of the sign of the force. The area under each triangle is 1/2 $\text{base}\times\text{height}$.

a) $1/2\,(8\,\text{m})(10\,\text{N}) = 40\,\text{J}$.

b) $1/2\,(4\,\text{m})(10\,\text{N}) = +20\,\text{J}$.

c) $1/2\,(12\,\text{m})(10\,\text{N}) = 60\,\text{J}$.

6.31: Use the Work-Energy Theorem and the results of Problem 6.30.

a) $v = \sqrt{\dfrac{(2)(40\,\text{J})}{10\,\text{kg}}} = 2.83\,\text{m/s}$

b) At $x = 12\,\text{m}$, the 40 Joules of kinetic energy will have been increased by 20 J, so

$$v = \sqrt{\frac{(2)(60\,\text{J})}{10\,\text{kg}}} = 3.46\,\text{m/s}.$$

6.32: The work you do with your changing force is

$$\int_0^{6.9} F(x)\,dx = \int_0^{6.9} (-20.0\,\text{N})\,dx - \int_0^{6.9} 3.0\frac{\text{N}}{\text{m}}\,x dx$$

$$= (-20.0\,\text{N})x\,|_0^{6.9} - (3.0\frac{\text{N}}{\text{m}})(x^2/2)\,|_0^{6.9}$$

$$= -138\,\text{N}\cdot\text{m} - 71.4\,\text{N}\cdot\text{m} = -209.4\,\text{J} \quad \text{or} \quad -209\,\text{J}$$

The work is negative because the cow continues to advance as you vainly attempt to push her backward.

6.33: $W_{\text{tot}} = K_2 - K_1$

$$K_1 = \frac{1}{2}mv_0^2, \quad K_2 = 0$$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx^2$, where x is the amount the spring is compressed.

$$-\frac{1}{2}kx^2 = -\frac{1}{2}mv_0^2 \quad \text{and} \quad x = v_0\sqrt{m/k} = 8.5\,\text{cm}$$

6.34: a) The average force is $(80.0\,\text{J})/(0.200\,\text{m}) = 400\,\text{N}$, and the force needed to hold the platform in place is twice this, or 800 N. b) From Eq. (6.9), doubling the distance quadruples the work so an extra 240 J of work must be done. The maximum force is quadrupled, 1600 N.

Both parts may of course be done by solving for the spring constant $k = 2(80.0\text{ J}) \div (0.200\text{ m})^2 = 4.00\times10^3\text{ N/m}$, giving the same results.

6.35: a) The static friction force would need to be equal in magnitude to the spring force, $\mu_s mg = kd$ or $\mu_s = \frac{(20.0\ \mathrm{N/m})(0.086\ \mathrm{m})}{(0.100\ \mathrm{kg})(9.80\ \mathrm{m/s^2})} = 1.76$, which is quite large. (Keeping extra figures in the intermediate calculation for *d* gives a different answer.) b) In Example 6.6, the relation

$$\mu_k mgd + \frac{1}{2}kd^2 = \frac{1}{2}mv_1^2$$

was obtained, and *d* was found in terms of the known initial speed v_1. In this case, the condition on *d* is that the static friction force at maximum extension just balances the spring force, or $kd = \mu_s mg$. Solving for v_1^2 and substituting,

$$\begin{aligned} v_1^2 &= \frac{k}{m}d^2 + 2gd\mu_k d \\ &= \frac{k}{m}\left(\frac{\mu_s mg}{k}\right)^2 + 2\mu_k g\left(\frac{\mu_s mg}{k}\right) \\ &= \frac{mg^2}{k}(\mu_s^2 + 2\mu_s\mu_k) \\ &= \left(\frac{(0.10\,\mathrm{kg})(9.80\,\mathrm{m/s^2})^2}{(20.0\,\mathrm{N/m})}\right)((0.60)^2 + 2(0.60)(0.47)), \end{aligned}$$

from which $v_1 = 0.67\,\mathrm{m/s}$.

6.36: a) The spring is pushing on the block in its direction of motion, so the work is positive, and equal to the work done in compressing the spring. From either Eq. (6.9) or Eq. (6.10), $W = \frac{1}{2}kx^2 = \frac{1}{2}(200\,\mathrm{N/m})(0.025\,\mathrm{m})^2 = 0.06\,\mathrm{J}$.

b) The work-energy theorem gives

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.06\,\mathrm{J})}{(4.0\,\mathrm{kg})}} = 0.18\,\mathrm{m/s}.$$

6.37: The work done in any interval is the area under the curve, easily calculated when the areas are unions of triangles and rectangles. a) The area under the trapezoid is $4.0\,\mathrm{N\cdot m} = 4.0\,\mathrm{J}$. b) No force is applied in this interval, so the work done is zero. c) The area of the triangle is $1.0\,\mathrm{N\cdot m} = 1.0\,\mathrm{J}$, and since the curve is *below* the axis $(F_x < 0)$, the work is negative, or $-1.0\,\mathrm{J}$. d) The net work is the sum of the results of parts (a), (b) and (c), 3.0 J. (e) $+1.0\,\mathrm{J} - 2.0\,\mathrm{J} = -1.0\,\mathrm{J}$.

6.38: a) $K = 4.0\,\text{J}$, so $v = \sqrt{2K/m} = \sqrt{2(4.0\,\text{J})/(2.0\,\text{kg})} = 2.00\,\text{m/s}$. b) No work is done between $x = 3.0\,\text{m}$ and $x = 4.0\,\text{m}$, so the speed is the same, 2.00 m/s. c) $K = 3.0\,\text{J}$, so $v = \sqrt{2K/m} = \sqrt{2(3.0\,\text{J})/(2.0\,\text{kg})} = 1.73\,\text{m/s}$.

6.39: a) The spring does positive work on the sled and rider; $(1/2)kx^2 = (1/2)mv^2$, or $v = x\sqrt{k/m} = (0.375\,\text{m})\sqrt{(4000\,\text{N/m})/(70\,\text{kg})} = 2.83\,\text{m/s}$. b) The net work done by the spring is $(1/2)k(x_1^2 - x_2^2)$, so the final speed is

$$v = \sqrt{\frac{k}{m}(x_1^2 - x_2^2)} = \sqrt{\frac{(4000\,\text{N/m}}{(70\,\text{kg})}((0.375\,\text{m})^2 - (0.200\,\text{m})^2)} = 2.40\,\text{m/s}.$$

6.40: a) From Eq. (6.14), with $dl = R d\phi$,

$$W = \int_{P_1}^{P_2} F\cos\phi\, dl = 2wR\int_0^{\theta_0} \cos\phi\, d\phi = 2wR\sin\theta_0.$$

In an equivalent geometric treatment, when $\vec{F}$ is horizontal, $\vec{F}\cdot d\vec{l} = F dx$, and the total work is $F = 2w$ times the horizontal distance, in this case (see Fig. 6.20(a)) $R\sin\theta_0$, giving the same result. b) The ratio of the forces is $\frac{2w}{w\tan\theta_0} = 2\cot\theta_0$.

c) $$\frac{2wR\sin\theta_0}{wR(1-\cos\theta_0)} = 2\frac{\sin\theta_0}{(1-\cos\theta_0)} = 2\cot\frac{\theta_0}{2}.$$

6.41: a) The initial and final (at the maximum distance) kinetic energy is zero, so the positive work done by the spring, $(1/2)kx^2$, must be the opposite of the negative work done by gravity, $-mgL\sin\theta$, or

$$x = \sqrt{\frac{2mgL\sin\theta}{k}} = \sqrt{\frac{2(0.0900\,\text{kg})(9.80\,\text{m/s}^2)(1.80\,\text{m})\sin 40.0°}{(640\,\text{N/m})}} = 5.7\,\text{cm}.$$

At this point the glider is no longer in contact with the spring. b) The intermediate calculation of the initial compression can be avoided by considering that between the point 0.80 m from the launch to the maximum distance, gravity does a negative amount of work given by $-(0.0900\,\text{kg})(9.80\,\text{m/s}^2)(1.80\,\text{m} - 0.80\,\text{m})\sin 40.0° = -0.567\,\text{J}$, and so the kinetic energy of the glider at this point is 0.567 J. At this point the glider is no longer in contact with the spring.

6.42: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or $d = \sqrt{2mgh/k} = \sqrt{2(1.80\,\text{kg})(9.80\,\text{m/s}^2)(3.6\,\text{m})/(450\,\text{N/m})} = 0.53\,\text{m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length.

6.43: Energy = (power)(time) = $(100\,\text{W})(3600\,\text{s}) = 3.6\times10^5\,\text{J}$

$$K = \frac{1}{2}mv^2 \quad \text{so} \quad v = \sqrt{2K/m} = 100\,\text{s} \quad \text{for} \quad m = 70\,\text{kg}.$$

6.44: Set time to stop:

$$\Sigma F = ma : \mu_k mg = ma$$
$$a = \mu_k g = (0.200)(9.80\,\text{m/s}^2) = 1.96\,\text{m/s}^2$$
$$v = v_0 + at$$
$$0 = 8.00\,\text{m/s} - (1.96\,\text{m/s}^2)t$$
$$t = 4.08\,\text{s}$$
$$P = \frac{KE}{t} = \frac{\frac{1}{2}mv^2}{t}$$
$$= \frac{\frac{1}{2}(20.0\,\text{kg})(8.00\,\text{m/s}^2)}{4.08\,\text{s}} = 157\,\text{W}$$

6.45: The total power is $(165\,\text{N})(9.00\,\text{m/s}) = 1.485\times10^3\,\text{W}$, so the power per rider is 742.5 W, or about 1.0 hp (which is a very large output, and cannot be sustained for long periods).

6.46: a) $$\frac{(1.0\times10^{19}\,\text{J/yr})}{(3.16\times10^7\,\text{s/yr})} = 3.2\times10^{11}\,\text{W}.$$

b) $$\frac{3.2\times10^{11}\,\text{W}}{2.6\times10^8\,\text{folks}} = 1.2\,\text{kW/person}.$$

c) $$\frac{3.2\times10^{11}\,\text{W}}{(0.40)1.0\times10^3\,\text{W/m}^2} = 8.0\times10^8\,\text{m}^2 = 800\,\text{km}^2.$$

6.47: The power is $P = F \cdot v$. F is the weight, mg, so
$P = (700\ \text{kg})\,(9.8\ \text{m/s}^2)\,(2.5\ \text{m/s}) = 17.15\ \text{kW}.$ So, $17.15\ \text{kW}/75\ \text{kW}. = 0.23,$ or about 23% of the engine power is used in climbing.

6.48: a) The number per minute would be the average power divided by the work (mgh) required to lift one box,

$$\frac{(0.50\ \text{hp})\,(746\ \text{W/hp})}{(30\ \text{kg})\,(9.80\ \text{m/s}^2)\,(0.90\ \text{m})} = 1.41\,/\text{s},$$

or $84.6\,/\text{min}$. b) Similarly,

$$\frac{(100\ \text{W})}{(30\ \text{kg})\,(9.80\ \text{m/s}^2)\,(0.90\ \text{m})} = 0.378\,/\text{s},$$

or $22.7\,/\text{min}$.

6.49: The total mass that can be raised is

$$\frac{(40.0\ \text{hp})\,(746\ \text{W/hp})(16.0\ \text{s})}{(9.80\ \text{m/s}^2)\,(20.0\ \text{m})} = 2436\ \text{kg},$$

so the maximum number of passengers is $\frac{1836\ \text{kg}}{65.0\ \text{kg}} = 28.$

6.50: From any of Equations (6.15), (6.16), (6.18) or (6.19),

$$P = \frac{Wh}{t} = \frac{(3800\ \text{N})\,(2.80\ \text{m})}{(4.00\ \text{s})} = 2.66\times 10^3\ \text{W} = 3.57\ \text{hp}.$$

6.51: $F = \dfrac{(0.70)\,P_{\text{ave}}}{v} = \dfrac{(0.70)\,(280{,}000\ \text{hp})(746\ \text{W/hp})}{(65\ \text{km/h})\,((1\ \text{km/h})/(3.6\ \text{m/s}))} = 8.1\times 10^6\ \text{N}.$

6.52: Here, Eq. (6.19) is the most direct. Gravity is doing negative work, so the rope must do positive work to lift the skiers. The force $\vec{F}$ is gravity, and $F = Nmg$, where N is the number of skiers on the rope. The power is then

$$\begin{aligned} P &= (Nmg)\,(v)\cos\phi \\ &= (50)\,(70\ \text{kg})\,(9.80\ \text{m/s}^2)\,(12.0\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\cos(90.0^\circ - 15.0^\circ) \\ &= 2.96\times 10^4\ \text{W}. \end{aligned}$$

Note that Eq. (1.18) uses ϕ as the angle between the force and velocity vectors; in this case, the force is vertical, but the angle 15.0° is measured from the horizontal, so $\phi = 90.0^\circ - 15.0^\circ$ is used.

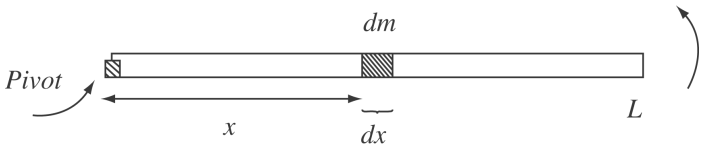
6.53: a) In terms of the acceleration a and the time t since the force was applied, the speed is $v = at$ and the force is ma, so the power is $P = Fv = (ma)(at) = ma^2t$. b) The power at a given time is proportional to the square of the acceleration, tripling the acceleration would mean increasing the power by a factor of nine. c) If the magnitude of the net force is the same, the acceleration will be the same, and the needed power is proportional to the time. At $t = 15.0\ \text{s}$, the needed power is three times that at 5.0 s, or 108 W.

6.54:

$$
\begin{aligned}
\frac{dK}{dt} &= \frac{d}{dt}\left(\frac{1}{2}mv^2\right) \\
&= mv\frac{dv}{dt} \\
&= mva = mav \\
&= Fv = P.
\end{aligned}
$$

6.55: Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t = 100\,Fs/t$
$t = 1.00\ \text{s}$, $F = 2mg$ and $s = 0.010\ \text{m}$. $P_{\text{av}} = 0.20\ \text{W}$.

6.56:



Let M = total mass and T = time for one revolution

$$KE = \int \frac{1}{2}(dm)v^2$$
$$dm = \frac{M}{L}dx$$
$$v = \frac{2\pi x}{T}$$
$$KE = \int_0^L \frac{1}{2}\left(\frac{M}{L}dx\right)\left(\frac{2\pi x}{T}\right)^2$$
$$= \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\int_0^L x^2 dx$$
$$= \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\left(\frac{L^3}{3}\right) = \frac{2}{3}\pi^2 ML^2/T^2$$

5 revolutions in 3 seconds $\rightarrow T = 3/5\text{ s}$

$$KE = \frac{2}{3}\pi^2(12.0\text{ kg})(2.00\text{ m})^2/(3/5\text{ s})^2 = 877\text{ J}.$$

6.57: a) $(140\text{ N})(3.80\text{ m}) = 532\text{ J}$ b) $(20.0\text{ kg})(9.80\text{ m/s}^2)(3.80\text{ m})(-\sin 25°) = -315\text{ J}$
c) The normal force does no work.
d)

$$W_f = -f_k s = -\mu_k ns = -\mu_k mgs\cos\theta$$
$$= -(0.30)(20.0\text{ kg})(9.80\text{ m/s}^2)(3.80\text{ m})\cos 25° = -203\text{ J}$$

e) $532\text{ J} - 315\text{ J} - 203\text{ J} = 15\text{ J}$ (14.7 J to three figures).

f) The result of part (e) is the kinetic energy at the top of the ramp, so the speed is $v = \sqrt{2K/m} = \sqrt{2(14.7\text{ J})/(20.0\text{ kg})} = 1.21\text{ m/s}$.

6.58: The work per unit mass is $(W/m) = gh.$

a) The man does work, $(9.8\ \text{N/kg})\,(0.4\ \text{m}) = 3.92\ \text{J/kg}.$

b) $(3.92\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 5.6\%.$

c) The child does work, $(9.8\ \text{N/kg})\,(0.2\ \text{m}) = 1.96\ \text{J/kg}.$

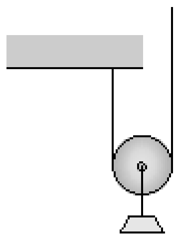
$(1.96\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 2.8\%.$

d) If both the man and the child can do work at the rate of $70\ \text{J/kg},$ and if the child only needs to use $1.96\ \text{J/kg}$ instead of $3.92\ \text{J/kg},$ the child should be able to do more pull ups.

6.59: a) Moving a distance L along the ramp, $s_{\text{in}} = L,\ s_{\text{out}} = L\sin\alpha,$ so $IMA = \frac{1}{\sin\alpha}.$

b) If $AMA = IMA, (F_{\text{out}}/F_{\text{in}}) = (s_{\text{in}}/s_{\text{out}})$ and so $(F_{\text{out}})\,(s_{\text{out}}) = (F_{\text{in}})\,(s_{\text{in}})$, or $W_{\text{out}} = W_{\text{in}}.$

c)



d)

$$E = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{(F_{\text{out}})(s_{\text{out}})}{(F_{\text{in}})(s_{\text{in}})} = \frac{F_{\text{out}}/F_{\text{in}}}{s_{\text{in}}/s_{\text{out}}} = \frac{AMA}{IMA}.$$

6.60: a) $m = \dfrac{w}{g} = \dfrac{-W_{\text{g}}/s}{g} = \dfrac{(7.35\times 10^3\ \text{J})}{(9.80\ \text{m/s}^2)\,(18.0\ \text{m})} = 41.7\ \text{kg}.$

b) $n = \dfrac{W_{\text{n}}}{s} = \dfrac{8.25\times 10^3\ \text{J}}{18.0\ \text{m}} = 458\ \text{N}.$

c) The weight is $mg = \frac{Wg}{s} = 408\ \text{N},$ so the acceleration is the net force divided by the mass, $\dfrac{458\ \text{N} - 408\ \text{N}}{41.7\ \text{kg}} = 1.2\ \text{m/s}^2.$

6.61: a)

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2\pi R}{T}\right)^2 = \frac{1}{2}(86{,}400\ \text{kg})\left(\frac{2\pi(6.66\times 10^6\ \text{m})}{(90.1\ \text{min})\,(60\ \text{s/min})}\right)^2 = 2.59\times 10^{12}\ \text{J}.$$

b) $(1/2)\,mv^2 = (1/2)\,(86{,}400\ \text{kg})\,((1.00\ \text{m})/(3.00\ \text{s}))^2 = 4.80\times 10^3\ \text{J}.$

6.62: a)

$$W_f = -f_k s = -\mu_k mg \cos\theta s$$
$$= -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 12.0°(1.50\text{ m}) = -22.3\text{ J}$$

(keeping an extra figure) b) $(5.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.0°(1.50\text{ m}) = 15.3$ J.
c) The normal force does no work. d) $15.3\text{ J} - 22.3\text{ J} = -7.0$ J.
e) $K_2 = K_1 + W = (1/2)(5.00\text{ kg})(2.2\text{ m/s})^2 - 7.0\text{ J} = 5.1\text{ J}$, and so $v_2 = \sqrt{2(5.1\text{ J})/(5.00\text{ kg})} = 1.4\text{ m/s}$.

6.63: See Problem 6.62: The work done is negative, and is proportional to the distance s that the package slides along the ramp, $W = mg(\sin\theta - \mu_k\cos\theta)s$. Setting this equal to the (negative) change in kinetic energy and solving for s gives

$$s = -\frac{(1/2)mv_1^2}{mg(\sin\theta - \mu_k\cos\theta)} = \frac{v_1^2}{2g(\sin\theta - \mu_k\cos\theta)}$$
$$= \frac{(2.2\text{ m/s})^2}{2(9.80\text{ m/s}^2)(\sin 12° - (0.31)\cos 12°)} = 2.6\text{ m}.$$

As a check of the result of Problem 6.62, $(2.2\text{ m/s})\sqrt{1-(1.5\text{ m})/(2.6\text{ m})} = 1.4\text{ m/s}$.

6.64: a) From Eq. (6.7),

$$W = \int_{x_1}^{x_2} F_x dx = -k\int_{x_1}^{x_2}\frac{dx}{x^2} = -k\left[-\frac{1}{x}\right]_{x_1}^{x_2} = k\left(\frac{1}{x_2} - \frac{1}{x_1}\right).$$

The force is given to be attractive, so $F_x < 0$, and k must be positive. If $x_2 > x_1$, $\frac{1}{x_2} < \frac{1}{x_1}$, and $W < 0$. b) Taking "slowly" to be constant speed, the net force on the object is zero, so the force applied by the hand is opposite F_x, and the work done is negative of that found in part (a), or $k\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$, which is positive if $x_2 > x_1$. c) The answers have the same magnitude but opposite signs; this is to be expected, in that the net work done is zero.

6.65: $F = mg(R_E/r)^2$

$$W = -\int_1^2 F ds = -\int_\infty^{R_E}\left(\frac{mgR_E^2}{r^2}\right)dr = -mgR_E^2(-(1/r)|_\infty^{R_E}) = mgR_E$$

$W_{tot} = K_2 - K_1, K_1 = 0$

This gives $K_2 = mgR_E = 1.25\times 10^{12}$ J

$K_2 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2K_2/m} = 11{,}000\text{ m/s}$

6.66: Let x be the distance past P.

$$\mu_k = 0.100 + Ax$$

when $x = 12.5\,\text{m}$, $\mu_k = 0.600$

$$A = 0.500/12.5\,\text{m} = 0.0400/\text{m}$$

(a)

$$W = \Delta KE : W_f = KE_f - KE_i$$

$$-\int \mu_k mgdx = 0 - \frac{1}{2}mv_i^2$$

$$g\int_0^{x_f}(0.100 + Ax)dx = \frac{1}{2}v_i^2$$

$$g\left[(0.100)x_f + A\frac{x_f^2}{2}\right] = \frac{1}{2}v_i^2$$

$$(9.80\,\text{m/s}^2)\left[(0.100)x_f + (0.0400/\text{m})\frac{x_f^2}{2}\right] = \frac{1}{2}(4.50\,\text{m/s})^2$$

Solve for $x_f : x_f = 5.11\text{m}$

(b) $\mu_k = 0.100 + (0.0400/\text{m})(5.11\,\text{m}) = 0.304$

(c) $W_f = KE_f - KE_i$

$$-\mu_k mgx = 0 - \frac{1}{2}mv_1^2$$

$$x = v_i^2/2\mu_k g = \frac{(4.50\,\text{m/s})^2}{2(0.100)(9.80\,\text{m/s}^2)} = 10.3\,\text{m}$$

6.67: a) $\alpha x_a^3 = (4.00\ \text{N/m}^3)(1.00\,\text{m})^3 = 4.00\,\text{N}$.

b) $\alpha x_b^3 = (4.00\ \text{N/m}^3)(2.00\,\text{m})^3 = 32.0\,\text{N}$. c) Equation 6.7 gives the work needed to move an object against the force; the work done by the force is the negative of this,

$$-\int_{x_1}^{x_2}\alpha x^3 dx = -\frac{\alpha}{4}(x_2^4 - x_1^4).$$

With $x_1 = x_a = 1.00\,\text{m}$ and $x_2 = x_b = 2.00\,\text{m}$, $W = -15.0\,\text{J}$, this work is negative.

6.68: From Eq. (6.7), with $x_1 = 0$,

$$W = \int_0^{x_2} F dx = \int_0^{x_2} (kx - bx^2 + cx^3) dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$$
$$= (50.0\ \mathrm{N/m})x_2^2 - (233\ \mathrm{N/m^2})x_2^3 + (3000\ \mathrm{N/m^3})x_2^4$$

a) When $x_2 = 0.050\ \mathrm{m}$, $W = 0.115\ \mathrm{J}$, or 0.12 J to two figures. b) When $x_2 = -0.050$ m, $W = 0.173\ \mathrm{J}$, or 0.17 J to two figures. c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

6.69: a) $T = ma_{\mathrm{rad}} = m\frac{v^2}{R} = (0.120\mathrm{kg})\frac{(0.70\ \mathrm{m/s})^2}{(0.40\ \mathrm{m})} = 0.147\ \mathrm{N}$, or 0.15 N to two figures. b) At the later radius and speed, the tension is $(0.120\mathrm{kg})\frac{(2.80\ \mathrm{m/s})^2}{(0.10\ \mathrm{m})} = 9.41\ \mathrm{N}$, or 9.4 N to two figures. c) The surface is frictionless and horizontal, so the net work is the work done by the cord. For a massless and frictionless cord, this is the same as the work done by the person, and is equal to the change in the block's kinetic energy, $K_2 - K_1 = (1/2)m(v_2^2 - v_1^2) = (1/2)(0.120\ \mathrm{kg})((2.80\ \mathrm{m/s})^2 - (0.70\ \mathrm{m/s})^2) = 0.441\ \mathrm{J}$. Note that in this case, the tension cannot be perpendicular to the block's velocity at all times; the cord is in the radial direction, and for the radius to change, the block must have some non-zero component of velocity in the radial direction.

6.70: a) This is similar to Problem 6.64, but here $\alpha > 0$ (the force is repulsive), and $x_2 < x_1$, so the work done is again negative;

$$W = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = (2.12\times 10^{-26}\ \mathrm{N\cdot m^2}((0.200\ \mathrm{m^{-1}}) - (1.25\times 10^9\ \mathrm{m^{-1}}))$$
$$= -2.65\times 10^{-17}\ \mathrm{J}.$$

Note that x_1 is so large compared to x^2 that the term $\frac{1}{x_1}$ is negligible. Then, using Eq. (6.13)) and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times 10^5\ \mathrm{m/s})^2 + \frac{2(-2.65\times 10^{-17}\ \mathrm{J})}{(1.67\times 10^{-27}\ \mathrm{kg})}} = 2.41\times 10^5\ \mathrm{m/s}.$$

b) With $K_2 = 0, W = -K_1$. Using $W = -\frac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times 10^{-26}\ \mathrm{N\cdot m^2})}{(1.67\times 10^{-27}\mathrm{kg})(3.00\times 10^5\ \mathrm{m/s})^2} = 2.82\times 10^{-10}\ \mathrm{m}.$$

c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is $3.00\times 10^5\ \mathrm{m/s}$.

6.71: The velocity and acceleration as functions of time are

$$v(t) = \frac{dx}{dt} = 2\alpha t + 3\beta t^2, \quad a(t) = 2\alpha + 6\beta t$$

a) $v(t = 4.00\,\text{s}) = 2(0.20\,\text{m/s}^2)(4.00\text{s}) + 3(0.02\,\text{m/s}^3)(4.00\text{s})^2 = 2.56\,\text{m/s}.$

b) $ma = (6.00\,\text{kg})(2(0.20\,\text{m/s}^2) + 6(0.02\,\text{m/s}^3)(4.00\,\text{s}) = 5.28\,\text{N}.$

c) $W = K_2 - K_1 = K_2 = (1/2)(6.00\,\text{kg})(256\,\text{m/s})^2 = 19.7\,\text{J}.$

6.72: In Eq. (6.14), $dl = dx$ and $\phi = 31.0°$ is constant, and so

$$W = \int_{P_1}^{P_2} F\cos\phi dl = \int_{x_1}^{x_2} F\cos\phi dx$$

$$= (5.00\,\text{N/m}^2)\cos 31.0° \int_{1.00\text{ m}}^{1.50\text{ m}} x^2 dx = 3.39\,\text{J}.$$

The final speed of the object is then

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(4.00\,\text{m/s})^2 + \frac{2(3.39\,\text{J})}{(0.250\,\text{kg})}} = 6.57\,\text{m/s}.$$

6.73: a) $K_2 - K_1 = (1/2)m(v_2^2 - v_1^2)$

$$= (1/2)(80.0\,\text{kg})((1.50\,\text{m/s})^2 - (5.00\,\text{m/s})^2) = -910\,\text{J}.$$

b) The work done by gravity is $-mgh = -(80.0\,\text{kg})(9.80\,\text{m/s}^2)(5.20\,\text{m}) = -4.08\times10^3\,\text{J}$, so the work done by the rider is $-910\,\text{J} - (-4.08\times10^3\,\text{J}) = 3.17\times10^3\,\text{J}$.

6.74: a) $W = \int_{x_0}^{\infty} \frac{b}{x^n} dx = \left.\frac{b}{(-n-1))x^{n-1}}\right|_{x_0}^{\infty} = \frac{b}{(n-1)x_0^{n-1}}.$

Note that for this part, for $n > 1$, $x^{1-n} \to 0$ as $x \to \infty$. b) When $0 < n < 1$, the improper integral must be used,

$$W = \lim_{x_2\to\infty}\left[\frac{b}{(n-1)}(x_2^{n-1} - x_0^{n-1})\right],$$

and because the exponent on the x_2^{n-1} is positive, the limit does not exist, and the integral diverges. This is interpreted as the force F doing an infinite amount of work, even though $F \to 0$ as $x_2 \to \infty$.

6.75: Setting the (negative) work done by the spring to the needed (negative) change in kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$, and solving for the spring constant,

$$k = \frac{mv_0^2}{x^2} = \frac{(1200\,\text{kg})(0.65\,\text{m/s})^2}{(0.070\,\text{m})^2} = 1.03\times10^5\,\text{N/m}.$$

6.76: a) Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\ \mathrm{N/m}}{0.0300\ \mathrm{kg}}}(0.060\ \mathrm{m}) = 6.93\ \mathrm{m/s}.$$

b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\mathrm{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where *f* is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2\boldsymbol{f}}{m}x_0}$$

$$= \sqrt{\frac{400\ \mathrm{N/m}}{0.0300\ \mathrm{kg}}(0.06\ \mathrm{m})^2 - \frac{2(6.00\ \mathrm{N})}{(0.0300\ \mathrm{kg})}(0.06\ \mathrm{m})} = 4.90\ \mathrm{m/s}.$$

c) The greatest speed occurs when the acceleration (and the net force) are zero, or $kx = f$, $x = \frac{f}{k} = \frac{6.00\ \mathrm{N}}{400\ \mathrm{N/m}} = 0.0150\ \mathrm{m}$. To find the speed, the net work is $W_{\mathrm{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$v_{\mathrm{max}} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}$$

$$= \sqrt{\frac{400\mathrm{N/m}}{(0.0300\mathrm{kg})}((0.060\mathrm{m})^2 - (0.0150\mathrm{m})^2) - \frac{2(6.00\mathrm{N})}{(0.0300\mathrm{kg})}(0.060\mathrm{m} - 0.0150\ \mathrm{m})}$$

$$= 5.20\ \mathrm{m/s},$$

which is larger than the result of part (b) but smaller than the result of part (a).

6.77: Denote the initial compression of the spring by *x* and the distance from the initial position by *L*. Then, the work done by the spring is $\frac{1}{2}kx^2$ and the work done by friction is $-\mu_k mg(x + L)$; this form takes into account the fact that while the spring is compressed, the frictional force is still present (see Problem 6.76). The initial and final kinetic energies are both zero, so the net work done is zero, and $\frac{1}{2}kx^2 = \mu_k mg(x + L)$. Solving for *L*,

$$L = \frac{(1/2)kx^2}{\mu_k mg} - x = \frac{(1/2)(250\ \mathrm{N/m})(0.250\mathrm{m})^2}{(0.30)(2.50\ \mathrm{kg})(9.80\mathrm{m/s^2})} - (0.250\mathrm{m}) = 0.813\ \mathrm{m},$$

or 0.81 m to two figures. Thus the book moves $.81\ \mathrm{m} + .25\ \mathrm{m} = 1.06\ \mathrm{m}$, or about 1.1 m.

6.78: The work done by gravity is $W_g = -mgL\sin\theta$ (negative since the cat is moving up), and the work done by the applied force is *FL*, where *F* is the magnitude of the applied force. The total work is

$$W_{\mathrm{tot}} = (100\ \mathrm{N})(2.00\ \mathrm{m}) - (7.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{m})\sin 30° = 131.4\ \mathrm{J}.$$

The cat's initial kinetic energy is $\frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\ \mathrm{kg})(2.40\ \mathrm{m/s})^2 = 20.2\ \mathrm{J}$, and

$$v_2 = \sqrt{\frac{2(K_1 + W)}{m}} = \sqrt{\frac{2(20.2\ \mathrm{J} + 131.4\ \mathrm{J})}{(7.00\ \mathrm{kg})}} = 6.58\ \mathrm{m/s}.$$

6.79: In terms of the bumper compression x and the initial speed v_0, the necessary relations are

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2, \quad kx < 5\,mg.$$

Combining to eliminate k and then x, the two inequalties are

$$x > \frac{v^2}{5g} \quad \text{and} \quad k < 25\frac{mg^2}{v^2}.$$

a) Using the given numbers,

$$x > \frac{(20.0\,\text{m/s})^2}{5(9.80\,\text{m/s}^2)} = 8.16\,\text{m},$$

$$k < 25\frac{(1700\,\text{kg})(9.80\,\text{m/s}^2)^2}{(20.0\,\text{m/s})^2} = 1.02\times10^4\ \text{N/m}.$$

b) A distance of 8 m is not commonly available as space in which to stop a car.

6.80: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $60.0°$ with the chair's motion, so the total work done is $W_{\text{tot}} = ((600\,\text{N})\cos 30.0° - (85.0\,\text{kg})(9.80\,\text{m/s}^2)\cos 60.0°)(2.50\,\text{m}) = 257.8\,\text{J}$, and so the speed at the top of the ramp is

$$v_2 = \sqrt{v_1^2 + \frac{2W_{\text{tot}}}{m}} = \sqrt{(2.00\,\text{m/s})^2 + \frac{2(257.8\,\text{J})}{(85.0\,\text{kg})}} = 3.17\,\text{m/s}.$$

Note that extra figures were kept in the intermediate calculation to avoid roundoff error.

6.81: a) At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy. Therefore, the work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv^2$, or

$$X = \sqrt{\frac{m}{k}}v = \sqrt{\frac{5.00\,\text{kg}}{500\,\text{N/m}}}(6.00\,\text{m/s}) = 0.600\,\text{m}.$$

b) Solving for v in terms of a known X,

$$v = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\,\text{N/m}}{5.00\,\text{kg}}}(0.150\,\text{m}) = 1.50\,\text{m/s}.$$

6.82: The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table). The work done by gravity is (6.00 kg) *gh* and the work done by friction is $-\mu_k(8.00\ \text{kg})\, gh$, so

$$W_{\text{tot}} = (6.00\ \text{kg} - (0.25)(8.00\ \text{kg})\,(9.80\ \text{m/s}^2)\,(1.50\ \text{m}) = 58.8\ \text{J}.$$

This work increases the kinetic energy of both blocks;

$$W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2,$$

so

$$v = \sqrt{\frac{2(58.8\ \text{J})}{(14.00\ \text{kg})}} = 2.90\ \text{m/s}.$$

6.83: See Problem 6.82. Gravity does positive work, while friction does negative work. Setting the net (negative) work equal to the (negative) change in kinetic energy,

$$(m_1 - \mu_k m_2)gh = -\frac{1}{2}(m_1 + m_2)v^2,$$

and solving for μ_k gives

$$\begin{aligned}\mu_k &= \frac{m_1 + (1/2)\,(m_1 + m_2)v^2/gh}{m_2} \\ &= \frac{(6.00\ \text{kg}) + (1/2)\,(14.00\ \text{kg})\,(0.900\ \text{m/s})^2/((9.80\ \text{m/s}^2)\,(2.00\ \text{m}))}{(8.00\ \text{kg})} \\ &= 0.79.\end{aligned}$$

6.84: The arrow will acquire the energy that was used in drawing the bow (*i.e.*, the work done by the archer), which will be the area under the curve that represents the force as a function of distance. One possible way of estimating this work is to approximate the *F vs. x* curve as a parabola which goes to zero at $x = 0$ and $x = x_0$, and has a maximum of F_0 at $x = \frac{x_0}{2}$, so that $F(x) = \frac{4F_0}{x_0^2}x(x_0 - x)$. This may seem like a crude approximation to the figure, but it has the ultimate advantage of being easy to integrate;

$$\int_0^{x_0} F dx = \frac{4F_0}{x_0^2}\int_0^{x_0}(x_0 x - x^2)\,dx = \frac{4F_0}{x_0^2}\left(x_0\frac{x_0^2}{2} - \frac{x_0^3}{3}\right) = \frac{2}{3}F_0 x_0.$$

With $F_0 = 200\ \text{N}$ and $x_0 = 0.75\ \text{m}, W = 100\ \text{J}$. The speed of the arrow is then $\sqrt{\frac{2W}{m}} = \sqrt{\frac{2(100\ \text{J})}{(0.025\ \text{kg})}} = 89\ \text{m/s}$. Other ways of finding the area under the curve in Fig. (6.28) should give similar results.

6.85: $f_k = 0.25\,mg$ so $W_f = W_{tot} = -(0.25\,mg)s,$ where s is the length of the rough patch.

$W_{tot} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.45v_0^2) = 0.2025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.2025 - 1)\frac{1}{2}mv_0^2$

The mass divides out and solving gives $s = 1.5\ \text{m}.$

6.86: Your friend's average acceleration is

$$a = \frac{v - v_0}{t} = \frac{6.00\ \text{m/s}}{3.00\ \text{s}} = 2.00\ \text{m/s}^2$$

Since there are no other horizontal forces acting, the force you exert on her is given by

$$F_{net} = ma = (65.0\ \text{kg})(2.00\ \text{m/s}^2) = 130\ \text{N}$$

Her average velocity during your pull is 3.00 m/s, and the distance she travels is thus 9.00 m. The work you do is $Fx = (130\ \text{N})(9.00\ \text{m}) = 1170\ \text{J}$, and the average power is therefore $1170\ \text{J}/3.00\ \text{s} = 390\ \text{W}.$ The work can also be calculated as the change in the kinetic energy.

6.87: a) $(800\ \text{kg})(9.80\ \text{m/s}^2)(14.0\ \text{m}) = 1.098\times10^5\ \text{J, or } 1.10\times10^5\ \text{J to three figures.}$

b) $(1/2)(800\ \text{kg})(18.0\ \text{m/s}^2) = 1.30\times10^5\ \text{J}.$

c) $\dfrac{1.10\times10^5\ \text{J} + 1.30\times10^5\ \text{J}}{60\ \text{s}} = 3.99\ \text{kW}.$

6.88:

$$\begin{aligned} P &= Fv = mav \\ &= m(2\alpha + 6\beta t)(2\alpha t + 3\beta t^2) \\ &= m(4\alpha^2 t + 18\alpha\beta t^2 + 18\beta^2 t^3) \\ &= (0.96\ \text{N/s})t + (0.43\ \text{N/s}^2)t^2 + (0.043\ \text{N/s}^3)t^3. \end{aligned}$$

At $t = 400\ \text{s},$ the power output is 13.5 W.

6.89: Let t equal the number of seconds she walks every day. Then, $(280\ \text{J/s})t + (100\ \text{J/s})(86400\ \text{s} - t) = 1.1\times10^7\ \text{J}.$ Solving for t, $t = 13{,}111\ \text{s} = 3.6\ \text{hours}.$

6.90: a) The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

b) The steady output of the athlete is $500\ \text{W}/70\ \text{kg} = 7\ \text{W/kg}$, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend $1400\ \text{W}/70\ \text{kg} = 20\ \text{W/kg}$ for short periods of time, no human-powered aircraft could stay aloft for very long. Movies of early attempts at human-powered flight bear out this observation.

6.91: From the chain rule, $P = \frac{d}{dt}W = \frac{d}{dt}(mgh) = \frac{dm}{dt}gh$, for ideal efficiency. Expressing the mass rate in terms of the volume rate and solving gives

$$\frac{(2000\times 10^{6}\ \text{W})}{(0.92)(9.80\ \text{m/s}^2)(170\ \text{m})(1000\ \text{kg/m}^3)} = 1.30\times 10^{3}\ \frac{\text{m}^3}{\text{s}}.$$

6.92: a) The power P is related to the speed by $Pt = K = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2Pt}{m}}$.

b) $$a = \frac{dv}{dt} = \frac{d}{dt}\sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}\frac{d}{dt}\sqrt{t} = \sqrt{\frac{2P}{m}}\frac{1}{2\sqrt{t}} = \sqrt{\frac{P}{2mt}}..$$

a) $$x - x_0 = \int v\,dt = \sqrt{\frac{2P}{m}}\int t^{\frac{1}{2}}\,dt = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{\frac{3}{2}} = \sqrt{\frac{8P}{9m}}\frac{2}{3}t^{\frac{3}{2}}.$$

6.93: a) $(7500\times 10^{-3}\ \text{kg}^3)(1.05\times 10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times 10^{5}\ \text{J}.$

b) $(1.26\times 10^{5}\ \text{J})/(86{,}400\ \text{s}) = 1.46\ \text{W}.$

6.94: a) The number of cars is the total power available divided by the power needed per car,

$$\frac{13.4\times 10^6\ \text{W}}{(2.8\times 10^3\ \text{N})(27\ \text{m/s})}=177,$$

rounding down to the nearest integer.

b) To accelerate a total mass M at an acceleration a and speed v, the extra power needed is Mav. To climb a hill of angle α, the extra power needed is $Mg \sin \alpha v$. These will be nearly the same if $a \sim g \sin \alpha$; if $g \sin \alpha \sim g \tan \alpha \sim 0.10\ \text{m/s}^2$, the power is about the same as that needed to accelerate at $0.10\ \text{m/s}^2$.

c) $(1.10\times 10^6\ \text{kg})(9.80\ \text{m/s}^2)(0.010)(27\ \text{m/s}) = 2.9\ \text{MW}$. d) The power per car needed is that used in part (a), plus that found in part (c) with M being the mass of a single car. The total number of cars is then

$$\frac{13.4\times 10^6\ \text{W}-2.9\times 10^6\ \text{W}}{(2.8\times 10^3\ \text{N}+(8.2\times 10^4\ \text{kg})(9.80\ \text{m/s}^2)(0.010))(27\ \text{m/s})}=36,$$

rounding to the nearest integer.

6.95: a) $P_0 = Fv = (53\times 10^3\ \text{N})(45\ \text{m/s}) = 2.4\ \text{MW}$.

b) $P_1 = mav = (9.1\times 10^5\ \text{kg})(1.5\ \text{m/s}^2)(45\ \text{m/s}) = 61\ \text{MW}$.

c) Approximating $\sin \alpha$, by $\tan \alpha$, and using the component of gravity down the incline as $mg \sin \alpha$,

$P_2 = (mg \sin \alpha)v = (9.1\times 10^5\ \text{kg})(9.80\ \text{m/s}^2)(0.015)(45\ \text{m/s}) = 6.0\ \text{MW}$.

6.96: a) Along this path, y is constant, and the displacement is parallel to the force, so $W = \alpha y\int x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\frac{(2.00\ \text{m})^2}{2} = 15.0\ \text{J}$.

b) Since the force has no y-component, no work is done moving in the y-direction.

c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W=\int F_x dx = 1.5\alpha\int x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(200\ \text{m})^3}{3}=10.0\ \text{J}.$$

6.97: a)

$$\begin{aligned} P &= Fv = (F_{\text{roll}} + F_{\text{air}})v \\ &= ((0.0045)(62.0\text{ kg})(9.80\text{ m/s}^2) \\ &\quad + (1/2)(1.00)(0.463\text{ m}^2)(1.2\text{ kg/m}^3)(12.0\text{ m/s})^2)(12.0\text{ m/s}) \\ &= 513\text{ W}. \end{aligned}$$

6.98: a) $$F = \frac{P}{v} = \frac{28.0 \times 10^3\text{ W}}{(60.0\text{ km/h})((1\text{ m/s})/(3.6\text{ km/h}))} = 1.68 \times 10^3\text{ N}.$$

b) The speed is lowered by a factor of one-half, and the resisting force is lowered by a factor of $(0.65 + 0.35/4)$, and so the power at the lower speed is

$$(28.0\text{ kW})(0.50)(0.65 + 0.35/4) = 10.3\text{ kW} = 13.8\text{ hp}.$$

c) Similarly, at the higher speed,

$$(28.0\text{ kW})(2.0)(0.65 + 0.35 \times 4) = 114.8\text{ kW} = 154\text{ hp}.$$

6.99: a) $$\frac{(8.00 \text{ hp})(746 \text{ W/hp})}{(60.0 \text{ km/h})((1 \text{ m/s})/(3.6 \text{ km/h}))} = 358 \text{ N}.$$

b) The extra power needed is

$$mgv_{\parallel} = (1800 \text{ kg})(9.80 \text{ m/s}^2)\frac{60.6 \text{ km/h}}{3.6 \frac{\text{km/h}}{\text{m/s}}}\sin(\arctan(1/10)) = 29.3 \text{ kW} = 39.2 \text{ hp},$$

so the total power is 47.2 hp. (Note: If the sine of the angle is approximated by the tangent, the third place will be different.) c) Similarly,

$$mgv_{\parallel} = (1800 \text{ kg})(9.80 \text{ m/s}^2)\frac{60.0 \text{ km/h}}{3.6 \frac{\text{km/h}}{\text{m/s}}}\sin(\arctan(0.010)) = 2.94 \text{ kW} = 3.94 \text{ hp},$$

This is the rate at which work is done on the car by gravity. The engine must do work on the car at a rate of 4.06 hp. d) In this case, approximating the sine of the slope by the tangent is appropriate, and the grade is

$$\frac{(8.00 \text{ hp})(746 \text{ W/hp})}{(1800 \text{ kg})(9.80 \text{ m/s}^2)(60.0 \text{ km/h})((1 \text{ m/s})/(3.6 \text{ km/h}))} = 0.0203,$$

very close to a 2% grade.

6.100: Use the Work–Energy Theorem, $W = \Delta KE,$ and integrate to find the work.

$$\Delta KE = 0 - \frac{1}{2}mv_0^2 \text{ and } W = \int_0^x (-\text{mg} \sin\alpha - \mu\text{mg} \cos\alpha)dx.$$

Then,

$$W = -\text{mg}\int_0^x (\sin\alpha + \text{A}x \cos\alpha)dx, W = -\text{mg}\left[\sin\alpha x + \frac{\text{A}x^2}{2}\cos\alpha\right].$$

Set $W = \Delta KE.$

$$-\frac{1}{2}mv_0^2 = -\text{mg}\left[\sin\alpha x + \frac{\text{A}x^2}{2}\cos\alpha\right].$$

To eliminate x, note that the box comes to a rest when the force of static friction balances the component of the weight directed down the plane. So, $\text{mg} \sin\alpha = \text{A}x \,\text{mg} \cos\alpha;$ solve this for x and substitute into the previous equation.

$$x = \frac{\sin\alpha}{\text{A}\cos\alpha}.$$

Then,

$$\frac{1}{2}v_0^2 = +g\left[\sin\alpha \frac{\sin\alpha}{\text{A}\cos\alpha} + \frac{\text{A}\left(\frac{\sin\alpha}{\text{A}\cos\alpha}\right)^2}{2}\cos\alpha\right],$$

and upon canceling factors and collecting terms, $v_0^2 = \dfrac{3g\sin^2\alpha}{\text{A}\cos\alpha}.$ Or the box will remain stationary whenever $v_0^2 \geq \dfrac{3g\sin^2\alpha}{\text{A}\cos\alpha}.$

6.101: a) Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v\frac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). The mass of a piece of length dl is $dm = \frac{M}{L}dl$, and so

$$dK = \frac{1}{2}dmu^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl,$$

and

$$K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}.$$

b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)}x = \sqrt{(3200\ \text{N/m})/(0.053\ \text{kg})}(2.50\times10^{-2}\ \text{m}) = 6.1\ \text{m/s}$. .
c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for v,

$$v = \sqrt{\frac{k}{m + M/3}}x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50\times10^{-2}\text{m}) = 3.9\ \text{m/s}.$$

d) Algebraically,

$$\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\ \text{J} \ \text{ and}$$

$$\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\ \text{J}.$$

6.102: In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. In terms of the range *R* and the (presumed) constant speed *v*,

$$W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right).$$

In terms of the time of flight $T, R = vt,$ so

$$W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right).$$

a) Rather than solve for *R* as a function of *v*, differentiate the first of these relations with respect to *v*, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0.$ For the maximum range, $\frac{dR}{dv} = 0,$ so $\frac{dF}{dv} = 0.$ Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0,$ which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{0.30\ \mathrm{N\cdot s^2/m^2}}\right)^{1/4} = 32.9\ \mathrm{m/s} = 118\ \mathrm{km/h}.$$

b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0;$ performing the differentiation, $3\alpha v^2 - \beta/v^2 = 0,$ which is solved for

$$v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \mathrm{N\cdot m^2/s^2}}{3(0.30\ \mathrm{N\cdot s^2/m^2}}\right)^{1/4} = 25\ \mathrm{m/s} = 90\ \mathrm{km/h}.$$

6.103: a) The walk will take one-fifth of an hour, 12 min. From the graph, the oxygen consumption rate appears to be about $12\ \mathrm{cm^3/kg\cdot min},$ and so the total energy is

$$(12\ \mathrm{cm^3/kg\cdot min})\,(70\ \mathrm{kg})\,(12\ \mathrm{min})\,(20\ \mathrm{J/cm^3}) = 2.0\times10^5\ \mathrm{J}.$$

b) The run will take 6 min. Using an estimation of the rate from the graph of about $33\ \mathrm{cm^3/kg\cdot min}$ gives an energy consumption of about 2.8×10^5 J. c) The run takes 4 min, and with an estimated rate of about $50\ \mathrm{cm^3/kg\cdot min},$ the energy used is about 2.8×10^5 J. d) Walking is the most efficient way to go. In general, the point where the slope of the line from the origin to the point on the graph is the smallest is the most efficient speed; about 5 km/h.

6.104: From $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}, F_x = ma_x, F_y = ma_y$ and $F_z = ma_z$. The generalization of Eq. (6.11) is then

$$a_x = v_x \frac{dv_x}{dx}, a_y = v_y \frac{dv_y}{dy},\ a_z = v_z \frac{dv_z}{dz}.$$

The total work is then

$$\begin{aligned}
W_{\text{tot}} &= \int_{(x_1, y_1, z_1)}^{(x_2, y_2, z_2)} F_x dx + F_y dy + F_z dz \\
&= m\left(\int_{x_1}^{x_2} v_x \frac{dv_x}{dx} dx + \int_{y_1}^{y_2} v_y \frac{dv_y}{dy} dy + \int_{z_1}^{z_2} v_z \frac{dv_z}{dz} dz \right) \\
&= m\left(\int_{v_{x1}}^{v_{x2}} v_x dv_x + \int_{v_{y1}}^{v_{y2}} v_y dv_y + \int_{v_{z2}}^{v_{z2}} v_z dv_z \right) \\
&= \frac{1}{2} m(v_{x2}^2 - v_{x1}^2 + v_{y2}^2 - v_{y1}^2 + v_{z2}^2 - v_{z1}^2 \\
&= \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2.
\end{aligned}$$

Capítulo 7

7.1: From Eq. (7.2),

$$mgy = (800\ \text{kg})\,(9.80\ \text{m/s}^2)\,(440\ \text{m}) = 3.45 \times 10^6\ \text{J} = 3.45\ \ \text{MJ}.$$

7.2: a) For constant speed, the net force is zero, so the required force is the sack's weight, $(5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49\ \text{N}$. b) The lifting force acts in the same direction as the sack's motion, so the work is equal to the weight times the distance, $(49.00\ \text{N})\,(15.0\,\text{m}) = 735\ \text{J};$ this work becomes potential energy. Note that the result is independent of the speed, and that an extra figure was kept in part (b) to avoid roundoff error.

7.3: In Eq. (7.7), taking $K_1 = 0$ (as in Example 6.4) and $U_2 = 0,\ K_2 = U_1 + W_{\text{other}}.$ Friction does negative work $-fy,$ so $K_2 = mgy - fy;$ solving for the speed $v_2,$

$$v_2 = \sqrt{\frac{2(mg - f)y}{m}} = \sqrt{\frac{2((200\ \text{kg})\,(9.80\ \text{m/s}^2) - 60\ \text{N})\,(3.00\ \text{m})}{(200\ \text{kg})}} = 7.55\ \text{m/s}.$$

7.4: a) The rope makes an angle of $\arcsin\left(\frac{3.0\ \text{m}}{6.0\ \text{m}}\right) = 30°$ with the vertical. The needed horizontal force is then $w \tan\theta = (120\ \text{kg})\,(9.80\ \text{m/s}^2)\tan 30° = 679\ \text{N},$ or $6.8 \times 10^2\ \text{N}$ to two figures. b) In moving the bag, the rope does no work, so the worker does an amount of work equal to the change in potential energy, $(120\ \text{kg})\,(9.80\ \text{m/s}^2)\,(6.0\ \text{m})\,(1 - \cos 30°) = 0.95 \times 10^3\ \ \text{J}.$ Note that this is not the product of the result of part (a) and the horizontal displacement; the force needed to keep the bag in equilibrium varies as the angle is changed.

7.5: a) In the absence of air resistance, Eq. (7.5) is applicable. With $y_1 - y_2 = 22.0\ \text{m},$ solving for v_2 gives

$$v_2 = \sqrt{v_1^2 + 2g(y_2 - y_1)} = \sqrt{(12.0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(22.0\ \text{m})} = 24.0\ \text{m/s}.$$

b) The result of part (a), and any application of Eq. (7.5), depends only on the magnitude of the velocities, not the directions, so the speed is again $24.0\ \text{m/s}.$ c) The ball thrown upward would be in the air for a longer time and would be slowed more by air resistance.

7.6: a) (Denote the top of the ramp as point 2.) In Eq. (7.7), $K_2 = 0, W_{\text{other}} = -(35\ \text{N}) \times (2.5\ \text{m}) = -87.5\ \text{J}$, and taking $U_1 = 0$ and $U_2 = mgy_2 = (12\ \text{kg})\,(9.80\ \text{m/s}^2)\,(2.5\ \text{m} \sin 30°) = 147\ \text{J}, v_1 = \sqrt{\frac{2(147\ \text{J}+87.5\ \text{J})}{12\ \text{kg}}} = 6.25\ \text{m/s}$, or $6.3\ \text{m/s}$ to two figures. Or, the work done by friction and the change in potential energy are both proportional to the distance the crate moves up the ramp, and so the initial speed is proportional to the square root of the distance up the ramp; $(5.0\ \text{m/s})\sqrt{\frac{2.5\ \text{m}}{1.6\ \text{m}}} = 6.25\ \text{m/s}$.

b) In part a), we calculated W_{other} and U_2. Using Eq. (7.7),
$K_2 = \frac{1}{2}(12\ \text{kg})\,(11.0\ \text{m/s})^2 - 87.5\ \text{J} - 147\ \text{J} = 491.5\ \text{J}$

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(491.5\ \text{J})}{(12\ \text{kg})}} = 9.05\ \text{m/s}.$$

7.7: As in Example 7.7, $K_2 = 0$, $U_2 = 94\ \text{J}$, and $U_3 = 0$. The work done by friction is

$$-(35\ \text{N})\,(1.6\ \text{m}) = -56\ \text{J},\ \text{and so}\ K_3 = 38\ \text{J},\ \text{and}\ v_3 = \sqrt{\frac{2(38\ \text{J})}{12\ \text{Kg}}} = 2.5\ \text{m/s}.$$

7.8: The speed is v and the kinetic energy is $4K$. The work done by friction is proportional to the normal force, and hence the mass, and so each term in Eq. (7.7) is proportional to the total mass of the crate, and the speed at the bottom is the same for any mass. The kinetic energy is proportional to the mass, and for the same speed but four times the mass, the kinetic energy is quadrupled.

7.9: In Eq. (7.7), $K_1 = 0, W_{\text{other}}$ is given as $-0.22\ \text{J}$, and taking $U_2 = 0, K_2 = mgR - 0.22\ \text{J}$, so

$$v_2 = \sqrt{2\left((9.80\ \text{m/s}^2)\,(0.50\ \text{m}) - \frac{0.22\ \text{J}}{0.20\ \text{kg}}\right)} = 2.8\ \text{m/s}.$$

7.10: (a) The flea leaves the ground with an upward velocity of 1.3 m/s and then is in free-fall with acceleration $9.8\ \mathrm{m/s^2}$ downward. The maximum height it reaches is therefore $(v_y^2 - v_{0y}^2)/2(-g) = 9.0\ \mathrm{cm}$. The distance it travels in the first 1.25 ms can be ignored.

(b)

$$
\begin{aligned}
W = KE &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(210\times10^{-6}\ \mathrm{g})\,(130\ \mathrm{cm/s})^2 \\
&= 1.8\ \mathrm{ergs} = 1.8\times10^{-7}\ \mathrm{J}
\end{aligned}
$$

7.11: Take $y = 0$ at point A. Let point 1 be A and point 2 be B.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = 0,\ U_2 = mg(2R) = 28{,}224\ \mathrm{J},\ W_{\text{other}} = W_f$

$K_1 = \dfrac{1}{2}mv_1^2 = 37{,}500\ \mathrm{J},\ K_2 = \tfrac{1}{2}mv_2^2 = 3840\ \mathrm{J}$

The work - energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400\ \mathrm{J}$.

7.12: Tarzan is lower than his original height by a distance $l(\cos 30 - \cos 45)$, so his speed is

$$v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9\ \mathrm{m/s},$$

a bit quick for conversation.

7.13: a) The force is applied parallel to the ramp, and hence parallel to the oven's motion, and so $W = Fs = (110\ \text{N})\,(8.0\ \text{m}) = 880\ \ \text{J}$. b) Because the applied force $\vec{F}$ is parallel to the ramp, the normal force is just that needed to balance the component of the weight perpendicular to the ramp, $n = w\cos\alpha$, and so the friction force is $f_\text{k} = \mu_\text{k} mg\cos\alpha$ and the work done by friction is $W_\text{f} = -\mu_\text{k} mg\cos\alpha\, s = -(0.25)\,(10.0\ \text{kg})\,(9.80\ \text{m/s}^2)\cos 37°(8.0\ \text{m}) = -157\ \ \text{J}$, keeping an extra figure. c) $mgs\sin\alpha = (10.0\ \text{kg})(9.80\ \text{m/s}^2)(8.0\ \text{m})\sin 37° = 472\ \text{J}$, again keeping an extra figure. d) $880\ \text{J} - 472\ \text{J} - 157\ \text{J} = 251\ \text{J}$. e) In the direction up the ramp, the net force is

$$\begin{aligned} F - mg\sin\alpha - \mu_\text{k} mg\cos\alpha \\ &= 110\ \text{N} - (10.0\ \text{kg})(9.80\ \text{m/s}^2)(\sin 37° + (0.25)\cos 37°) \\ &= 31.46\ \text{N}, \end{aligned}$$

so the acceleration is $(31.46\ \text{N})/10.0\ \text{kg}) = 3.15\ \text{m/s}^2$. The speed after moving up the ramp is $v = \sqrt{2as} = \sqrt{2(3.15\ \text{m/s}^2)\,(8.0\ \text{m})} = 7.09\ \text{m/s}$, and the kinetic energy is $(1/2)mv^2 = 252\ \text{J}$. (In the above, numerical results of specific parts may differ in the third place if extra figures are not kept in the intermediate calculations.)

7.14: a) At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, or $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\ \text{m/s}^2)\,(0.80\ \text{m})\,(1-\cos 45°)} = 2.1\ \text{m/s}$. b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\ \text{kg})\,(9.80\ \text{m/s}^2)\cos 45° = 0.83\ \text{N}$. c) At the bottom of the circle, the tension is the sum of the weight and the radial acceleration,

$$mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.86\ \text{N},$$

or 1.9 N to two figures. Note that this method does not use the intermediate calculation of v.

7.15: Of the many ways to find energy in a spring in terms of the force and the distance, one way (which avoids the intermediate calculation of the spring constant) is to note that the energy is the product of the average force and the distance compressed or extended. a) $(1/2)(800\ \text{N})(0.200\ \text{m}) = 80.0\ \ \text{J}$. b) The potential energy is proportional to the square of the compression or extension; $(80.0\ \text{J})\,(0.050\ \text{m}/0.200\ \text{m})^2 = 5.0\ \text{J}$.

7.16: $U = \frac{1}{2}ky^2$, where y is the vertical distance the spring is stretched when the weight $w = mg$ is suspended. $y = \frac{mg}{k}$, and $k = \frac{F}{x}$, where x and F are the quantities that "calibrate" the spring. Combining,

$$U = \frac{1}{2}\frac{(mg)^2}{F/x} = \frac{1}{2}\frac{((60.0\,\text{kg})\,(9.80\,\text{m/s}^2))^2}{(720\,\text{N}/0.150\,\text{m})} = 36.0\ \text{J}$$

7.17: a) Solving Eq. (7.9) for $x, x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\,\text{J})}{(1600\,\text{N/m})}} = 0.063\,\text{m}.$

b) Denote the initial height of the book as h and the maximum compression of the spring by x. The final and initial kinetic energies are zero, and the book is initially a height $x + h$ above the point where the spring is maximally compressed. Equating initial and final potential energies, $\frac{1}{2}kx^2 = mg(x+h)$. This is a quadratic in x, the solution to which is

$$\begin{aligned} x &= \frac{mg}{k}\left[1 \pm \sqrt{1 + \frac{2kh}{mg}}\right] \\ &= \frac{(1.20\,\text{kg})(9.80\,\text{m/s}^2)}{(1600\,\text{N/m})}\left[1 \pm \sqrt{1 + \frac{2(1600\,\text{N/m})\,(0.80\,\text{m})}{(1.20\,\text{kg})\,(9.80\,\text{m/s}^2)}}\right] \\ &= 0.116\,\text{m}, -0.101\,\text{m}. \end{aligned}$$

The second (negative) root is not unphysical, but represents an extension rather than a compression of the spring. To two figures, the compression is 0.12 m.

7.18: a) In going from rest in the slingshot's pocket to rest at the maximum height, the potential energy stored in the rubber band is converted to gravitational potential energy; $U = mgy = (10 \times 10^{-3}\,\text{kg})(9.80\,\text{m/s}^2)\,(22.0\,\text{m}) = 2.16\,\text{J}.$

b) Because gravitational potential energy is proportional to mass, the larger pebble rises only 8.8 m.

c) The lack of air resistance and no deformation of the rubber band are two possible assumptions.

7.19: The initial kinetic energy and the kinetic energy of the brick at its greatest height are both zero. Equating initial and final potential energies, $\frac{1}{2}kx^2 = mgh$, where h is the greatest height. Solving for h,

$$h = \frac{kx^2}{2mg} = \frac{(1800\,\text{N/m})\,(0.15\,\text{m})^2}{2(1.20\,\text{kg})\,(9.80\,\text{m/s}^2)} = 1.7\,\text{m}.$$

7.20: As in Example 7.8, $K_1 = 0$ and $U_1 = 0.0250\text{ J}$. For $v_2 = 0.20\text{ m/s}$, $K_2 = 0.0040\text{ J}$, so $U_2 = 0.0210\text{ J} = \frac{1}{2}kx^2$, so $x = \pm\sqrt{\frac{2(0.0210\text{ J})}{5.00\text{ N/m}}} = \pm 0.092\text{ m}$. In the absence of friction, the glider will go through the equilibrium position and pass through $x = -0.092\text{ m}$ with the same speed, on the opposite side of the equilibrium position.

7.21: a) In this situation, $U_2 = 0$ when $x = 0$, so $K_2 = 0.0250\text{ J}$ and $v_2 = \sqrt{\frac{2(0.0250\text{ J})}{0.200\text{ kg}}} = 0.500\text{ m/s}$. b) If $v_2 = 2.50\text{ m/s}$, $K_2 = (1/2)(0.200\text{ kg})(2.50\text{ m/s})^2 = 0.625\text{ J} = U_1$, so $x_1 = \sqrt{\frac{2(0.625\text{ J})}{5.00\text{ N/m}}} = 0.500\text{ m}$. Or, because the speed is 5 times that of part (a), the kinetic energy is 25 times that of part (a), and the initial extension is $5 \times 0.100\text{ m} = 0.500\text{ m}$.

7.22: a) The work done by friction is $W_{\text{other}} = -\mu_k mg\Delta x = -(0.05)(0.200\text{ kg})(9.80\text{ m/s}^2)(0.020\text{ m}) = -0.00196\text{ J}$, so $K_2 = 0.00704\text{ J}$ and $v_2 = \sqrt{\frac{2(0.00704\text{ J})}{0.200\text{ kg}}} = 0.27\text{ m/s}$. b) In this case $W_{\text{other}} = -0.0098\text{ J}$, so $K_2 = 0.0250\text{ J} - 0.0098\text{ J} = 0.0152\text{ J}$, and $v_2 = \sqrt{\frac{2(0.0152\text{ J})}{0.200\text{ kg}}} = 0.39\text{ m/s}$.
c) In this case, $K_2 = 0$, $U_2 = 0$, so $U_1 + W_{\text{other}} = 0 = 0.0250\text{ J} - \mu_k(0.200\text{ kg})(9.80\text{ m/s}^2) \times (0.100\text{ m})$, or $\mu_k = 0.13$.

7.23: a) In this case, $K_1 = 625{,}000\text{ J}$ as before, $W_{\text{other}} = -17{,}000\text{ J}$ and

$$\begin{aligned} U_2 &= (1/2)ky_2^2 + mgy_2 \\ &= (1/2)(1.41\times10^5\text{ N/m})(-1.00\text{ m})^2 + (2000\text{ kg})(9.80\text{ m/s}^2)(-1.00) \\ &= 50{,}900\text{ J}. \end{aligned}$$

The kinetic energy is then $K_2 = 625{,}000\text{ J} - 50{,}900\text{ J} - 17{,}000\text{ J} = 557{,}100\text{ J}$, corresponding to a speed $v_2 = 23.6\text{ m/s}$. b) The elevator is moving down, so the friction force is up (tending to stop the elevator, which is the idea). The net upward force is then $-mg + f - kx = -(2000\text{ kg})(9.80\text{ m/s}^2) + 17{,}000\text{ N} - (1.41\times10^5\text{ N/m})(-1.00\text{ m}) = 138{,}40($
for an upward acceleration of 69.2 m/s^2.

7.24: From $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, the relations between m, v, k and x are

$$kx^2 = mv^2, \quad kx = 5mg.$$

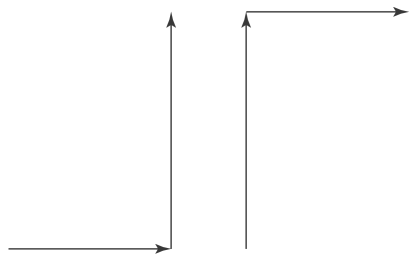
Dividing the first by the second gives $x = \frac{v^2}{5g}$, and substituting this into the second gives $k = 25\frac{mg^2}{v^2}$, so a) & b),

$$x = \frac{(2.50\ \text{m/s})^2}{5(9.80\ \text{m/s}^2)} = 0.128\ \text{m},$$

$$k = 25\frac{(1160\ \text{kg})(9.80\ \text{m/s}^2)^2}{(2.50\ \text{m/s})^2} = 4.46\times10^5\ \text{N/m}.$$

7.25: a) Gravity does negative work, $-(0.75\,\text{kg})(9.80\ \text{m/s}^2)(16\,\text{m}) = -118\,\text{J}$. b) Gravity does 118 J of positive work. c) Zero d) Conservative; gravity does no net work on any complete round trip.

7.26: a) & b) $-(0.050\,\text{kg})(9.80\ \text{m/s}^2)(5.0\,\text{m}) = -2.5\,\text{J}$.



(a) (b)

c) Gravity is conservative, as the work done to go from one point to another is path-independent.

7.27: a) The displacement is in the y-direction, and since $\vec{F}$ has no y-component, the work is zero.

b)

$$\int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} = -12\int_{x_1}^{x_2} x^2 dx = -\frac{12\ \text{N/m}^2}{3}(x_2^3 - x_1^3) = -0.104\,\text{J}.$$

c) The negative of the answer to part (b), $0.104\,\text{m}^3$ d) The work is independent of path, and the force is conservative. The corresponding potential energy is $U = \frac{(12\ \text{N/m}^2)x^3}{3} = (4\ \text{N/m}^2)x^3$.

7.28: a) From (0, 0) to (0, L), $x = 0$ and so $\vec{\boldsymbol{F}} = \mathbf{0}$,, and the work is zero. From (0, L) to (L, L), $\vec{\boldsymbol{F}}$ and $d\vec{\boldsymbol{l}}$ are perpendicular, so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0$. and the net work along this path is zero. b) From (0, 0) to (L, 0), $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = 0$. From ($L$, 0) to ($L$, L), the work is that found in the example, $W_2 = CL^2$, so the total work along the path is CL^2. c) Along the diagonal path, $x = y$, and so $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = Cy\,dy$; integrating from 0 to L gives $\frac{CL^2}{2}$. (It is not a coincidence that this is the average to the answers to parts (a) and (b).) d) The work depends on path, and the field is not conservative.

7.29: a) When the book moves to the left, the friction force is to the right, and the work is $-(1.2\ \text{N})(3.0\ \text{m}) = -3.6\ \text{J}$. b) The friction force is now to the left, and the work is again $-3.6\ \text{J}$. c) $-7.2\ \text{J}$. d) The net work done by friction for the round trip is not zero, and friction is not a conservative force.

7.30: The friction force has magnitude $\mu_k mg = (0.20)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 58.8\ \text{N}$. a) For each part of the move, friction does $-(58.8\ \text{N})(10.6\ \text{m}) = -623\ \text{J}$, so the total work done by friction is $-1.2\ \text{kN}$. b) $-(58.8\ \text{N})(15.0\ \text{m}) = -882\ \text{N}$.

7.31: The magnitude of the friction force on the book is

$$\mu_k mg = (0.25)(1.5\ \text{kg})(9.80\ \text{m/s}^2) = 3.68\ \text{N}.$$

a) The work done during each part of the motion is the same, and the total work done is $-2(3.68\ \text{N})(8.0\ \text{m}) = -59\ \text{J}$ (rounding to two places). b) The magnitude of the displacement is $\sqrt{2}(8.0\ \text{m})$, so the work done by friction is $-\sqrt{2}(8.0\ \text{m})(3.68\ \text{N}) = -42\ \text{N}$. c) The work is the same both coming and going, and the total work done is the same as in part (a), $-59\ \text{J}$. d) The work required to go from one point to another is not path independent, and the work required for a round trip is not zero, so friction is not a conservative force.

7.32: a) $\frac{1}{2}k(x_1^2 - x_2^2)$ b) $-\frac{1}{2}k(x_1^2 - x_2^2)$. The total work is zero; the spring force is conservative c) From x_1 to x_3, $W = -\frac{1}{2}k(x_3^2 - x_1^2)$. From x_3 to x_2, $W = \frac{1}{2}k(x_2^2 - x_3^2)$. The net work is $-\frac{1}{2}k(x_2^2 - x_1^2)$. This is the same as the result of part (a).

7.33: From Eq. (7.17), the force is

$$F_x = -\frac{dU}{dx} = C_6 \frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}.$$

The minus sign means that the force is attractive.

7.34: From Eq. (7.15), $F_x = -\frac{dU}{dx} = -4\alpha x^3 = -(4.8\ \text{J/m}^4)x^3,$ and so

$$F_x(-0.800\,\text{m}) = -(4.8\ \text{J/m}^4)(-0.80\,\text{m})^3 = 2.46\ \text{N}.$$

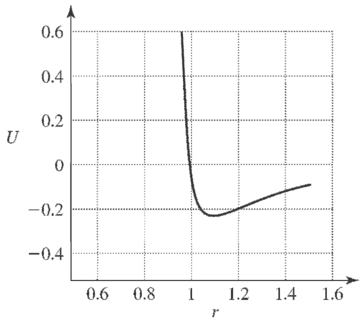
7.35: $\frac{\partial U}{\partial x} = 2kx + k'y,\ \frac{\partial U}{\partial y} = 2ky + k'x$ and $\frac{\partial U}{\partial z} = 0,$ so from Eq. (7.19),

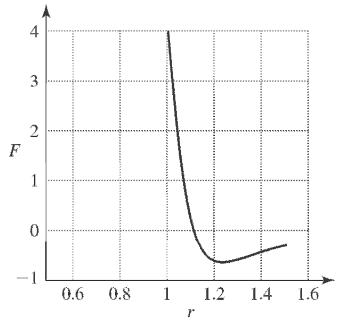
$$\vec{\boldsymbol{F}} = -(2kx + k'y)\hat{\boldsymbol{i}} - (2ky + k'x)\hat{\boldsymbol{j}}.$$

7.36: From Eq. (7.19), $\vec{\boldsymbol{F}} = -\frac{\partial U}{\partial x}\hat{\boldsymbol{i}} - \frac{\partial U}{\partial y}\hat{\boldsymbol{j}},$ since U has no z-dependence. $\frac{\partial U}{\partial x} = \frac{-2\alpha}{x^3}$ and $\frac{\partial U}{\partial y} = \frac{-2\alpha}{y^3}$, so

$$\vec{\boldsymbol{F}} = -\alpha\left(\frac{-2}{x^3}\hat{\boldsymbol{i}} + \frac{-2}{y^3}\hat{\boldsymbol{j}}\right).$$

7.37: a) $F_r = -\frac{\partial U}{\partial r} = 12\frac{a}{r^{13}} - 6\frac{b}{r^7}$.





b) Setting $F_r = 0$ and solving for r gives $r_{\text{min}} = (2a/b)^{1/6}$. This is the minimum of potential energy, so the equilibrium is stable.

c)

$$\begin{aligned} U(r_{\text{min}}) &= \frac{a}{r_{\text{min}}^{12}} - \frac{b}{r_{\text{min}}^{6}} \\ &= \frac{a}{((2a/b)^{1/6})^{12}} - \frac{b}{((2a/b)^{1/6})^{6}} \\ &= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = -\frac{b^2}{4a}. \end{aligned}$$

To separate the particles means to remove them to zero potential energy, and requires the negative of this, or $E_0 = b^2/4a$. d) The expressions for E_0 and r_{min} in terms of a and b are

$$E_0 = \frac{b^2}{4a} \quad r_{\text{min}}^6 = \frac{2a}{b}.$$

Multiplying the first by the second and solving for b gives $b = 2E_0 r_{\text{min}}^6$, and substituting this into the first and solving for a gives $a = E_0 r_{\text{min}}^{12}$. Using the given numbers,

$$a = (1.54 \times 10^{-18}\ \text{J})(1.13 \times 10^{-10}\ \text{m})^{12} = 6.68 \times 10^{-138}\ \text{J} \cdot \text{m}^{12}$$

$$b = 2(1.54 \times 10^{-18}\ \text{J})(1.13 \times 10^{-10}\ \text{m})^{6} = 6.41 \times 10^{-78}\ \text{J} \cdot \text{m}^{6}.$$

(Note: the numerical value for a might not be within the range of standard calculators, and the powers of ten may have to be handled seperately.)

7.38: a) Considering only forces in the x-direction, $F_x = -\frac{dU}{dx}$, and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d. b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable. c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

7.39: a) At constant speed, the upward force of the three ropes must balance the force, so the tension in each is one-third of the man's weight. The tension in the rope is the force he exerts, or $(70.0\,\text{kg})(9.80\ \text{m/s}^2)/3 = 229\,\text{N}$. b) The man has risen 1.20 m, and so the increase in his potential energy is $(70.0\,\text{kg})(9.80\ \text{m/s}^2)(1.20\,\text{m}) = 823\,\text{J}$. In moving up a given distance, the total length of the rope between the pulleys and the platform changes by three times this distance, so the length of rope that passes through the man's hands is $3 \times 1.20\,\text{m} = 3.60\,\text{m}$, and $(229\,\text{N})(3.6\,\text{m}) = 824\,\text{J}$.

7.40: First find the acceleration**:**

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(3.00\ \text{m/s})^2}{2(1.20\,\text{m})} = 3.75\ \text{m/s}^2$$

Then, choosing motion in the direction of the more massive block as positive**:**

$$F_{\text{net}} = Mg - mg = (M + m)a = Ma + ma$$

$$M(g - a) = m(g + a)$$

$$\frac{M}{m} = \frac{g + a}{g - a} = \frac{(9.80 + 3.75)\ \text{m/s}^2}{(9.80 - 3.75)\ \text{m/s}^2} = 2.24$$

$$M = 2.24\,\text{m}$$

$$\text{Since } M + m = 15.0\,\text{kg}:$$

$$2.24m + m = 15.0\,\text{kg}$$

$$m = 4.63\,\text{kg}$$

$$M = 15.0\,\text{kg} - 4.63\,\text{kg} = 10.4\,\text{kg}$$

7.41: a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = U_2 = K_2 = 0$

$W_{\text{other}} = W_f = -\mu_{\text{k}} mgs,$ with $s = 280\,\text{ft} = 85.3\,\text{m}$

The work-energy expression gives $\frac{1}{2}mv_1^2 - \mu_{\text{k}} mgs = 0$

$v_1 = \sqrt{2\mu_{\text{k}} gs} = 22.4\ \text{m/s} = 50\,\text{mph};$ the driver was speeding.

a) 15 mph over speed limit so \$150 ticket.

7.42: a) Equating the potential energy stored in the spring to the block's kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\ \text{N/m}}{2.00\ \text{kg}}}(0.220\ \text{m}) = 3.11\ \text{m/s}.$$

b) Using energy methods directly, the initial potential energy of the spring is the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or

$$L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\ \text{N/m})(0.220\ \text{m})^2}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0°} = 0.821\ \text{m}.$$

7.43: The initial and final kinetic energies are both zero, so the work done by the spring is the negative of the work done by friction, or $\frac{1}{2}kx^2 = \mu_k mgl$, where l is the distance the block moves. Solving for μ_k,

$$\mu_k = \frac{(1/2)kx^2}{mgl} = \frac{(1/2)(100\ \text{N/m})(0.20\ \text{m})^2}{(0.50\ \text{kg})(9.80\ \text{m/s}^2)(1.00\ \text{m})} = 0.41.$$

7.44: Work done by friction against the crate brings it to a halt:

$$f_k x = \text{potential energy of compressed spring}$$

$$f_k = \frac{360\ \text{J}}{5.60\ \text{m}} = 64.29\ \text{N}$$

The friction force working over a 2.00-m distance does work $f_k x = (-64.29\ \text{N})(2.00\ \text{m}) = -128.6\ \text{J}$. The kinetic energy of the crate at this point is thus $360\ \text{J} - 128.6\ \text{J} = 231.4\ \text{J}$, and its speed is found from

$$\frac{mv^2}{2} = 231.4\ \text{J}$$

$$v^2 = \frac{2(231.4\ \text{J})}{50.0\ \text{kg}} = 9.256\ \text{m}^2/\text{s}^2$$

$$v = 3.04\ \text{m/s}$$

7.45: a) $mgh = (0.650\ \text{kg})(9.80\ \text{m/s}^2)(2.50\ \text{m}) = 15.9\ \text{J}$

b) The second height is $0.75(2.50\ \text{m}) = 1.875\ \text{m}$, so second $mgh = 11.9\ \text{J}$; loses $15.9\ \text{J} - 11.9\ \text{J} = 4.0\ \text{J}$ on first bounce. This energy is converted to thermal energy.

a) The third height is $0.75(1.875\ \text{m}) = 1.40\ \text{m},$, so third $mgh = 8.9\ \text{J}$; loses $11.9\ \text{J} - 8.9\ \text{J} = 3.0\ \text{J}$ on second bounce.

7.46: a) $U_A - U_B = mg(h-2R) = \frac{1}{2}mv_A^2$. From previous considerations, the speed at the top must be at least $\sqrt{gR}$. Thus,

$$mg(h-2R) > \frac{1}{2}mgR, \text{ or } h > \frac{5}{2}R.$$

b) $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80\ \text{m/s}^2)(20.0\ \text{m})} = 31.3\ \text{m/s}.$$

The radial acceleration is $a_{\text{rad}} = \frac{v_C^2}{R} = 49.0\ \text{m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80\ \text{m/s}^2$.

7.47: a) Use work-energy relation to find the kinetic energy of the wood as it enters the rough bottom: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4\ \text{J}$.

Now apply work-energy relation to the motion along the rough bottom:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4\ \text{J}$

$78.4\ \text{J} - \mu_k mgs = 0$; solving for s gives $s = 20.0\ \text{m}$.

The wood stops after traveling 20.0 m along the rough bottom.

b) Friction does $-78.4\ \text{J}$ of work.

7.48: (a)

$$KE_{\text{Bottom}} + W_{\text{f}} = PE_{\text{Top}}$$

$$\frac{1}{2}mv_0^2 - \mu_{\text{k}} mg\cos\theta\, d = mgh$$

$$d = h/\sin\theta$$

$$\frac{1}{2}v_0^2 - \mu_{\text{k}} g\cos\theta \frac{h}{\sin\theta} = gh$$

$$\frac{1}{2}(15\ \text{m/s})^2 - (0.20)(9.8\ \text{m/s}^2)\frac{\cos 40°}{\sin 40°}h = (9.8\ \text{m/s}^2)h$$

$$h = 9.3\ \text{m}$$

(b) Compare maximum static friction force to the weight component down the plane.

$$f_{\text{s}} = \mu_{\text{s}} mg\cos\theta = (0.75)(28\ \text{kg})(9.8\ \text{m/s}^2)\cos 40°$$
$$= 158\ \text{N}$$

$$mg\sin\theta = (28\ \text{kg})(9.8\ \text{m/s}^2)(\sin 40°) = 176\ \text{N} > f_{\text{s}}$$

so the rock will slide down.

(c) Use same procedure as (a), with $h = 9.3\ \text{m}$

$$PE_{\text{Top}} + W_{\text{f}} = KE_{\text{Bottom}}$$

$$mgh - \mu_{\text{k}} mg\cos\theta \frac{h}{\sin\theta} = \frac{1}{2}mv_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{2gh - 2\mu_{\text{k}} gh\cos\theta/\sin\theta} = 11.8\ \text{m/s}$$

7.49: a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Let point 1 be point A and point 2 be point B. Take $y = 0$ at point B.

$mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\ \text{m}$ and $v_1 = 10.0\ \text{m/s}$

$v_2 = \sqrt{v_1^2 + 2gh} = 22.2\ \text{m/s}$

b) Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at B and point 2 where the spring has its maximum compression x.

$U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\ \text{m/s}$

$W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\ \text{m} + x$

The work-energy relation gives $K_1 + W_{\text{other}} = 0$.

$\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\ \text{m}$.

b) When the spring is compressed $x = 16.4\ \text{m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\ \text{N}$. The maximum possible static friction force is

$$\max f_s = \mu_s mg = (0.80)(15.0\ \text{kg})(9.80\ \text{m/s}^2) = 118\ \text{N}.$$

The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.50: First get speed at the top of the hill for the block to clear the pit.

$$y = \frac{1}{2}gt^2$$

$$20\ \text{m} = \frac{1}{2}(9.8\ \text{m/s}^2)t^2$$

$$t = 2.0\ \text{s}$$

$$v_{\text{Top}}t = 40\ \text{m} \rightarrow v_{\text{Top}} = \frac{40\ \text{m}}{20\ \text{s}} = 20\ \text{m/s}$$

Energy conservation:

$$KE_{\text{Bottom}} = PE_{\text{Top}} + KE_{\text{Top}}$$

$$\frac{1}{2}mv_{\text{B}}^2 = mgh + \frac{1}{2}mv_{\text{T}}^2$$

$$\begin{aligned} v_{\text{B}} &= \sqrt{v_{\text{T}}^2 + 2gh} \\ &= \sqrt{(20\ \text{m/s})^2 + 2(9.8\ \text{m/s}^2)(70\ \text{m})} \\ &= 42\ \text{m/s} \end{aligned}$$

7.51: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y = 0$ at point 2. $y_1 = 41.0\ \text{m}$. $W_{\text{other}} = -\frac{1}{2}kx^2$, where $x = 11.0\ \text{m}$ is the amount the cord is stretched at point 2.The cord does negative work.

$K_1 = K_2 = U_2 = 0$, so $mgy_1 - \frac{1}{2}kx^2 = 0$ and $k = 631\ \text{N/m}$.

Now apply $F = kx$ to the test pulls:

$F = kx$ so $x = F/k = 0.602\ \text{m}$.

7.52: For the skier to be moving at no more than $30.0\ \text{m/s}$; his kinetic energy at the bottom of the ramp can be no bigger than

$$\frac{mv^2}{2} = \frac{(85.0\ \text{kg})(30.0\ \text{m/s})^2}{2} = 38{,}250\ \text{J}$$

Friction does $-4000\ \text{J}$ of work on him during his run, which means his combined PE and KE at the top of the ramp must be no more than $38{,}250\ \text{J} + 4000\ \text{J} = 42{,}250\ \text{J}$. His KE at the top is

$$\frac{mv^2}{2} = \frac{(85.0\ \text{kg})(2.0\ \text{m/s})^2}{2} = 170\ \text{J}$$

His PE at the top should thus be no more than $42{,}250\ \text{J} - 170\ \text{J} = 42{,}080\ \text{J}$, which gives a height above the bottom of the ramp of

$$h = \frac{42{,}080\ \text{J}}{mg} = \frac{42{,}080\ \text{J}}{(85.0\ \text{kg})(9.80\ \text{m/s}^2)} = 50.5\ \text{m}.$$

7.53: The net work done during the trip down the barrel is the sum of the energy stored in the spring, the (negative) work done by friction and the (negative) work done by gravity. Using $\frac{1}{2}kx^2 = \frac{1}{2}(F^2/k)$, the performer's kinetic energy at the top of the barrel is

$$K = \frac{1}{2}\frac{(4400\ \text{N})^2}{1100\ \text{N/m}} - (40\ \text{N})(4.0\ \text{m}) - (60\ \text{kg})(9.80\ \text{m/s}^2)(2.5\ \text{m}) = 7.17 \times 10^3\ \text{J},$$

and his speed is $\sqrt{\frac{2(7.17\times10^3\ \text{J})}{60\ \text{kg}}} = 15.5\ \text{m/s}$.

7.54: To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. The work-energy theorem requires that the energy stored in the spring is equal to the sum of the work done by friction, the work done by gravity and the initial kinetic energy, or

$$\frac{1}{2}kx_0^2 = (w\sin\theta - f)L + \frac{1}{2}mv^2,$$

where L is the total length traveled down the ramp and v is the speed at the top of the ramp. With the given parameters, $\frac{1}{2}kx_0^2 = 248\ \text{J}$ and $kx_0 = 1.10\times10^3\ \text{N}$. Solving for k gives $k = 2440\ \text{N/m}$.

7.55: The potential energy has decreased by $(12.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) - (4.0\ \text{kg})\times(9.80\ \text{m/s}^2)(2.00\ \text{m}) = 156.8\ \text{J}$. The kinetic energy of the masses is then $\frac{1}{2}(m_1+m_2)v^2 = (8.0\ \text{kg})v^2 = 156.8\ \text{J}$, so the common speed is $v = \sqrt{\frac{(156.8\ \text{J})}{8.0\ \text{kg}}} = 4.43\ \text{m/s}$, or $4.4\ \text{m/s}$ to two figures.

7.56: a) The energy stored may be found directly from

$$\frac{1}{2}ky_2^2 = K_1 + W_{\text{other}} - mgy_2 = 625{,}000\ \text{J} - 51{,}000\ \text{J} - (-58{,}000\ \text{J}) = 6.33\times10^5\ \text{J}.$$

b) Denote the upward distance from point 2 by h. The kinetic energy at point 2 and at the height h are both zero, so the energy found in part (a) is equal to the negative of the work done by gravity and friction,
$-(mg + f)h = -((2000\ \text{kg})(9.80\ \text{m/s}^2) + 17{,}000\ \text{N})h = (36{,}600\ \text{N})h,$ so
$h = \frac{6.33\times10^5\ \text{J}}{3.66\times10^4\ \text{J}} = 17.3\ \text{m}.$ c) The net work done on the elevator between the highest point of the rebound and the point where it next reaches the spring is
$(mg - f)(h - 3.00\ \text{m}) = 3.72\times10^4\ \text{J}.$ Note that on the way down, friction does negative work. The speed of the elevator is then $\sqrt{\frac{2(3.72\times10^4\ \text{J})}{2000\ \text{kg}}} = 6.10\ \text{m/s}$. d) When the elevator next comes to rest, the total work done by the spring, friction, and gravity must be the negative of the kinetic energy K_3 found in part (c), or

$$K_3 = 3.72\times10^4\ \text{J} = -(mg - f)x_3 + \frac{1}{2}kx_3^2 = -(2{,}600\ \text{N})x_3 + (7.03\times10^4\ \text{N/m})x_3^2.$$

(In this calculation, the value of k was recalculated to obtain better precision.) This is a quadratic in x_3, the positive solution to which is

$$\begin{aligned} x_3 &= \frac{1}{2(7.03\times10^4\ \text{N/m})} \\ &\quad\times\left[2.60\times10^3\ \text{N} + \sqrt{(2.60\times10^3\ \text{N})^2 + 4(7.03\times10^4\ \text{N/m})(3.72\times10^4\ \text{J})}\right] \\ &= 0.746\ \text{m}, \end{aligned}$$

corresponding to a force of $1.05\times10^5\ \text{N}$ and a stored energy of $3.91\times10^4\ \text{J}$. It should be noted that different ways of rounding the numbers in the intermediate calculations may give different answers.

7.57: The two design conditions are expressed algebraically as $ky = f + mg = 3.66 \times 10^4 \text{ N}$ (the condition that the elevator remains at rest when the spring is compressed a distance *y*; *y* will be taken as positive) and $\frac{1}{2}mv^2 + mgy - fy = \frac{1}{2}kx^2$ (the condition that the change in energy is the work $W_{\text{other}} = -fy$). Eliminating *y* in favor of *k* by $y = \frac{3.66 \times 10^4 \text{ N}}{k}$ leads to

$$\frac{1}{2}\frac{(3.66 \times 10^4 \text{ N})^2}{k} + \frac{(1.70 \times 10^4 \text{ N})(3.66 \times 10^4 \text{ N})}{k}$$
$$= 62.5 \times 10^4 \text{ J} + \frac{(1.96 \times 10^4 \text{ N})(3.66 \times 10^4 \text{ N})}{k}.$$

This is actually not hard to solve for $k = 919 \text{ N/m}$, and the corresponding *x* is 39.8 m. This is a very weak spring constant, and would require a space below the operating range of the elevator about four floors deep, which is not reasonable. b) At the lowest point, the spring exerts an upward force of magnitude $f + mg$. Just before the elevator stops, however, the friction force is also directed upward, so the net force is $(f + mg) + f - mg = 2f$, and the upward acceleration is $\frac{2f}{m} = 17.0 \text{ m/s}^2$.

7.58: One mass rises while the other falls, so the net loss of potential energy is

$$(0.5000 \text{ kg} - 0.2000 \text{ kg})(9.80 \text{ m/s}^2)(0.400 \text{ m}) = 1.176 \text{ J}.$$

This is the sum of the kinetic energies of the animals. If the animals are equidistant from the center, they have the same speed, so the kinetic energy of the combination is $\frac{1}{2}m_{\text{tot}}v^2$, and

$$v = \sqrt{\frac{2(1.176 \text{ J})}{(0.7000 \text{ kg})}} = 1.83 \text{ m/s}.$$

7.59: a) The kinetic energy of the potato is the work done by gravity (or the potential energy lost), $\frac{1}{2}mv^2 = mgl$, or $v = \sqrt{2gl} = \sqrt{2(9.80 \text{ m/s}^2)(2.50 \text{ m})} = 7.00 \text{ m/s}$.

b)

$$T - mg = m\frac{v^2}{l} = 2mg,$$

so $T = 3mg = 3(0.100 \text{ kg})(9.80 \text{ m/s}^2) = 2.94 \text{ N}.$

7.60: a) The change in total energy is the work done by the air,

$$(K_2 + U_2) - (K_1 + U_1) = m\left(\frac{1}{2}(v_2^2 - v_1^2) + gy_2\right)$$

$$= (0.145\ \text{kg})\left(\begin{aligned}&(1/2)\,((18.6\ \text{m/s})^2 - (30.0\ \text{m/s})^2 \\ &\quad - (40.0\ \text{m/s})^2) + (9.80\ \text{m/s}^2)(53.6\ \text{m})\end{aligned}\right)$$

$$= -80.0\ \text{J}.$$

b) Similarly,

$$(K_3 + U_3) - (K_2 + U_2) = (0.145\ \text{kg})\left(\begin{aligned}&(1/2)((11.9\ \text{m/s})^2 + (-28.7\ \text{m/s})^2 \\ &\quad - (18.6\ \text{m/s})^2) - (9.80\ \text{m/s}^2)(53.6\ \text{m})\end{aligned}\right)$$

$$= -31.3\ \text{J}.$$

c) The ball is moving slower on the way down, and does not go as far (in the x-direction), and so the work done by the air is smaller in magnitude.

7.61: a) For a friction force f, the total work done sliding down the pole is $mgd - fd$. This is given as being equal to mgh, and solving for f gives

$$f = mg\frac{(d-h)}{d} = mg\left(1 - \frac{h}{d}\right).$$

When $h = d$, $f = 0$, as expected, and when $h = 0$, $f = mg$; there is no net force on the fireman. b) $(75\ \text{kg})(9.80\ \text{m/s}^2)(1 - \frac{1.0\ \text{m}}{2.5\ \text{m}}) = 441\ \text{N}$. c) The net work done is $(mg - f)(d - y)$, and this must be equal to $\frac{1}{2}mv^2$. Using the above expression for f,

$$\frac{1}{2}mv^2 = (mg - f)(d - y)$$

$$= mg\left(\frac{h}{d}\right)(d - y)$$

$$= mgh\left(1 - \frac{y}{d}\right),$$

from which $v = \sqrt{2gh(1 - y/d)}$. When $y = 0$ $v = \sqrt{2gh}$, which is the original condition. When $y = d$, $v = 0$; the fireman is at the top of the pole.

7.62: a) The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction,
$K_1 = mgh - W_F = (60.0\ \text{kg})(9.8\ \text{N/kg})(65.0\ \text{m}) - 10.500\ \text{J}$, or
$K_1 = 38{,}200\ \text{J} - 10{,}500\ \text{J} = 27{,}720\ \text{J}$. Then $v_1 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(27{,}720\ \text{J})}{60\ \text{kg}}} = 30.4\ \text{m/s}$.

b) $K_2 + K_1 - (W_F + W_A) = 27{,}720\ \text{J} - (\mu_k mgd + f_{\text{air}}d)$, $K_2 = 27{,}720\ \text{J} - [(.2)(588\ \text{N}) \times (82\ \text{m}) + (160\ \text{N})(82\ \text{m})]$, , or $K_2 = 27{,}720\ \text{J} - 22{,}763\ \text{J} = 4957\ \text{J}$. Then,

$$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\ \text{J})}{60\ \text{kg}}} = 12.85\ \text{m/s} \approx 12.9\ \text{m/s}.$$

c) Use the Work-Energy Theorem to find the force. $W = \Delta KE$,
$F = KE/d = (4957\ \text{J})/(2.5\ \text{m}) = 1983\ \text{N} \approx 2000\ \text{N}$.

7.63: The skier is subject to both gravity and a normal force; it is the normal force that causes her to go in a circle, and when she leaves the hill, the normal force vanishes. The vanishing of the normal force is the condition that determines when she will leave the hill. As the normal force approaches zero, the necessary (inward) radial force is the radial component of gravity, or $mv^2/R = mg\cos\alpha$, where R is the radius of the snowball. The speed is found from conservation of energy; at an angle α, she has descended a vertical distance $R(1-\cos\alpha)$, so $\frac{1}{2}mv^2 = mgR(1-\cos\alpha)$, or $v^2 = 2gR(1-\cos\alpha)$. Using this in the previous relation gives $2(1-\cos\alpha) = \cos\alpha$, or $\alpha = \arccos\left(\frac{2}{3}\right) = 48.2°$. This result does not depend on the skier's mass, the radius of the snowball, or g.

7.64: If the speed of the rock at the top is v_t, then conservation of energy gives the speed v_b from $\frac{1}{2}mv_b^2 = \frac{1}{2}mv_t^2 + mg(2R)$, R being the radius of the circle, and so $v_b^2 = v_t^2 + 4gR$. The tension at the top and bottom are found from $T_t + mg = \frac{mv_t^2}{R}$ and $T_b - mg = \frac{mv_b^2}{R}$, so $T_b - T_t = \frac{m}{R}(v_b^2 - v_t^2) + 2mg = 6mg = 6w$.

7.65: a) The magnitude of the work done by friction is the kinetic energy of the package at point B, or $\mu_{\text{k}} mgL = \frac{1}{2} mv_B^2$, or

$$\mu_{\text{k}} = \frac{(1/2)v_B^2}{gL} = \frac{(1/2)(4.80 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(3.00 \text{ m})} = 0.392.$$

b)

$$\begin{aligned} W_{\text{other}} &= K_B - U_A \\ &= \tfrac{1}{2}(0.200 \text{ kg})(4.80 \text{ m/s})^2 - (0.200 \text{ kg})(9.80 \text{ m/s}^2)(1.60 \text{ m}) \\ &= -0.832 \text{ J.} \end{aligned}$$

Equivalently, since $K_A = K_B = 0,\ U_A + W_{AB} + W_{BC} = 0$, or

$$W_{AB} = -U_A - W_{BC} = mg(-(1.60 \text{ m}) - (0.300)(-3.00 \text{ m})) = -0.832 \text{ J.}$$

7.66: Denote the distance the truck moves up the ramp by x. $K_1 = \frac{1}{2} mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_{\text{r}} mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for x,

$$x = \frac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_{\text{r}}\cos\beta)} = \frac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_{\text{r}}\cos\beta}.$$

7.67: a) Taking $U(0) = 0$,

$$U(x) = \int_0^x F_x dx = \frac{\alpha}{2}x^2 + \frac{\beta}{3}x^3 = (30.0 \text{ N/m})x^2 + (6.00 \text{ N/m}^2)x^3.$$

b)

$$\begin{aligned} K_2 &= U_1 - U_2 \\ &= ((30.0 \text{ N/m})(1.00 \text{ m})^2 + (6.00 \text{ N/m}^2)(1.00 \text{ m})^3) \\ &\quad - ((30.0 \text{ N/m})(0.50 \text{ m})^2 + (6.00 \text{ N/m}^2)(0.50 \text{ m})^3) \\ &= 27.75 \text{ J,} \end{aligned}$$

and so $v_2 = \sqrt{\frac{2(27.75 \text{ J})}{0.900 \text{ kg}}} = 7.85 \text{ m/s}$.

7.68: The force increases both the gravitational potential energy of the block and the potential energy of the spring. If the block is moved slowly, the kinetic energy can be taken as constant, so the work done by the force is the increase in potential energy, $\Delta U = mga\sin\theta + \frac{1}{2}k(a\theta)^2$.

7.69: With $U_2 = 0,\ K_1 = 0,\ K_2 = \frac{1}{2}mv_2^2 = U_1 = \frac{1}{2}kx^2 + mgh$, and solving for v_2,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\ \text{N/m})(0.045\ \text{m})^2}{(0.150\ \text{kg})} + 2(9.80\ \text{m/s}^2)(1.20\ \text{m})} = 7.01\ \text{m/s}$$

7.70: a) In this problem, use of algebra avoids the intermediate calculation of the spring constant *k*. If the original height is *h* and the maximum compression of the spring is *d*, then $mg(h+d) = \frac{1}{2}kd^2$. The speed needed is when the spring is compressed $\frac{d}{2}$, and from conservation of energy, $mg(h+d/2) - \frac{1}{2}k(d/2)^2 = \frac{1}{2}mv^2$. Substituting for *k* in terms of $h+d$,

$$mg\left(h + \frac{d}{2}\right) - \frac{mg(h+d)}{4} = \frac{1}{2}mv^2,$$

which simplifies to

$$v^2 = 2g\left(\frac{3}{4}h + \frac{1}{4}d\right).$$

Insertion of numerical values gives $v = 6.14\ \text{m/s}$. b) If the spring is compressed a distance *x*, $\frac{1}{2}kx^2 = mgx$, or $x = \frac{2mg}{k}$. Using the expression from part (a) that gives *k* in terms of *h* and *d*,

$$x = (2mg)\frac{d^2}{2mg(h+d)} = \frac{d^2}{h+d} = 0.0210\ \text{m}.$$

7.71: The first condition, that the maximum height above the release point is *h*, is expressed as $\frac{1}{2}kx^2 = mgh$. The magnitude of the acceleration is largest when the spring is compressed to a distance *x*; at this point the net upward force is $kx - mg = ma$, so the second condition is expressed as $x = (m/k)(g+a)$. a) Substituting the second expression into the first gives

$$\frac{1}{2}k\left(\frac{m}{k}\right)^2(g+a)^2 = mgh, \quad \text{or} \quad k = \frac{m(g+a)^2}{2gh}.$$

b) Substituting this into the expression for *x* gives $x = \frac{2gh}{g+a}$.

7.72: Following the hint, the force constant k is found from $w = mg = kd$, or $k = \frac{mg}{d}$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}\frac{mg}{d}y^2$. Equating these,

$$\frac{1}{2}\frac{mg}{d}y^2 = mgy, \quad \text{or} \quad y = 2d.$$

9.73: a)

$$\begin{aligned}\Delta a_{\text{rad}} &= \omega^2 r - \omega_0^2 r = (\omega^2 - \omega_0^2)r \\ &= [\omega - \omega_0][\omega + \omega_0]r \\ &= \left[\frac{\omega - \omega_0}{t}\right][(\omega + \omega_0)t]r \\ &= [\alpha][2(\theta - \theta_0)r.\end{aligned}$$

b) From the above,

$$\alpha r = \frac{\Delta a_{\text{rad}}}{2\Delta\theta} = \frac{(85.0\ \text{m/s}^2 - 25.0\ \text{m/s}^2)}{2(15.0\ \text{rad})} = 2.00\ \text{m/s}^2.$$

c) Similar to the derivation of part (a),

$$\Delta K = \frac{1}{2}\omega^2 I - \frac{1}{2}\omega_0{}^2 I = \frac{1}{2}[\alpha][2\Delta\theta]I = I\alpha\Delta\theta.$$

d) Using the result of part (c),

$$I = \frac{\Delta K}{\alpha\Delta\theta} = \frac{(45.0\ \text{J} - 20.0\ \text{J})}{((2.00\ \text{m/s}^2)/(0.250\ \text{m}))(15.0\ \text{rad})} = 0.208\ \text{kg}\cdot\text{m}^2.$$

7.74: a) From either energy or force considerations, the speed before the block hits the spring is

$$\begin{aligned} v &= \sqrt{2gL(\sin\theta - \mu_k \cos\theta)} \\ &= \sqrt{2(9.80\ \text{m/s}^2)(4.00\ \text{m})(\sin 53.1° - (0.20)\cos 53.1°)} \\ &= 7.30\ \text{m/s}. \end{aligned}$$

b) This does require energy considerations; the combined work done by gravity and friction is $mg(L+d)(\sin\theta - \mu_k \cos\theta)$, and the potential energy of the spring is $\frac{1}{2}kd^2$, where *d* is the maximum compression of the spring. This is a quadratic in *d*, which can be written as

$$d^2 \frac{k}{2mg(\sin\theta - \mu_k \cos\theta)} - d - L = 0.$$

The factor multiplying d^2 is $4.504\ \text{m}^{-1}$, and use of the quadratic formula gives $d = 1.06\ \text{m}$. c) The easy thing to do here is to recognize that the presence of the spring determines *d*, but at the end of the motion the spring has no potential energy, and the distance below the starting point is determined solely by how much energy has been lost to friction. If the block ends up a distance *y* below the starting point, then the block has moved a distance $L+d$ down the incline and $L+d-y$ up the incline. The magnitude of the friction force is the same in both directions, $\mu_k mg\cos\theta$, and so the work done by friction is $-\mu_k(2L+2d-y)mg\cos\theta$. This must be equal to the change in gravitational potential energy, which is $-mgy\sin\theta$. Equating these and solving for *y* gives

$$y = (L+d)\frac{2\mu_k \cos\theta}{\sin\theta + \mu_k \cos\theta} = (L+d)\frac{2\mu_k}{\tan\theta + \mu_k}.$$

Using the value of *d* found in part (b) and the given values for μ_k and θ gives $y = 1.32\ \text{m}$.

7.75: a) $K_B = W_{\text{other}} - U_B = (20.0\ \text{N})(0.25\ \text{m}) - (1/2)(40.0\ \text{N/m})(.25\ \text{m})^2 = 3.75\ \text{J}$, so $v_B = \sqrt{\frac{2(3.75\ \text{J})}{0.500\ \text{kg}}} = 3.87\ \text{m/s}$, or $3.9\ \text{m/s}$ to two figures. b) At this point (point *C*), $K_C = 0$, and so $U_C = W_{\text{other}}$ and $x_c = -\sqrt{\frac{2(5.00\ \text{J})}{40.0\ \text{N/m}}} = -0.50\ \text{m}$ (the minus sign denotes a displacement to the left in Fig. (7.65)), which is 0.10 m from the wall.

7.76: The kinetic energy K' after moving up the ramp the distance *s* will be the energy initially stored in the spring, plus the (negative) work done by gravity and friction, or

$$K' = \frac{1}{2}kx^2 - mg(\sin\alpha + \mu_k \cos\alpha)s.$$

Minimizing the speed is equivalent to minimizing K', and differentiating the above expression with respect to α and setting $\frac{dK'}{d\alpha} = 0$ gives

$$0 = -mgs(\cos\alpha - \mu_k \sin\alpha),$$

or $\tan\alpha = \frac{1}{\mu_k}$, $\alpha = \arctan\left(\frac{1}{\mu_k}\right)$. Pushing the box straight up $(\alpha = 90°)$ maximizes the vertical displacement *h*, but not $s = h/\sin\alpha$.

7.77: Let $x_1 = 0.18\ \text{m}$, $x_2 = 0.71\ \text{m}$. The spring constants (assumed identical) are then known in terms of the unknown weight *w*, $4kx_1 = w$. The speed of the brother at a given height *h* above the point of maximum compression is then found from

$$\frac{1}{2}(4k)x_2^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 + mgh,$$

or

$$v^2 = \frac{(4k)g}{w}x_2^2 - 2gh = g\left(\frac{x_2^2}{x_1} - 2h\right),$$

so $v = \sqrt{(9.80\ \text{m/s}^2)((0.71\ \text{m})^2/(0.18\ \text{m}) - 2(0.90\ \text{m}))} = 3.13\ \text{m/s}$, or $3.1\ \text{m/s}$ to two figures. b) Setting $v = 0$ and solving for *h*,

$$h = \frac{2kx_2^2}{mg} = \frac{x_2^2}{2x_1} = 1.40\ \text{m},$$

or 1.4 m to two figures. c) No; the distance x_1 will be different, and the ratio $\frac{x_2^2}{x_1} = \frac{(x_1 + 0.53\ \text{m})^2}{x_1} = x_1\left(1 + \frac{0.53\ \text{m}}{x_1}\right)^2$ will be different. Note that on a small planet, with lower *g*, x_1 will be smaller and *h* will be larger.

7.78: a) $a_x = d^2x/dt^2 = -\omega_0^2 x, \quad F_x = ma_x = -m\,\omega_0^2 x$

$a_y = d^2y/dt^2 = -\omega_0^2 y = -\omega_0^2 y, \quad F_y = ma_y = -m\,\omega_0^2 y$

b) $U = -\left[\int F_x dx + \int F_y dy\right] = m\omega_0^2\left[\int xdx + \int ydy\right] = \frac{1}{2}m\omega_0^2(x^2 + y^2)$

c)

$$v_x = dx/dt = -x_0\omega_0 \sin \omega_0 t = -x_0\omega_0(y/y_0)$$
$$v_y = dy/dt = +y_0\omega_0 \cos \omega_0 t = +y_0\omega_0(x/x_0)$$

(i) When $x = x_0$ and $y = 0$, $v_x = 0$ and $v_y = y_0\omega_0$

$$K = \frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2}my_0^2\omega_0^2,\ U = \frac{1}{2}\omega_0^2 mx_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

(ii) When $x = 0$ and $y = y_0$, $v_x = -x_0\omega_0$ and $v_y = 0$

$$K = \frac{1}{2}\omega_0^2 mx_0^2,\ U = \frac{1}{2}m\omega_0^2 y_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

Note that the total energy is the same.

7.79: a) The mechanical energy increase of the car is

$K_2 - K_1 = \frac{1}{2}(1500 \text{ kg})(37 \text{ m/s})^2 = 1.027 \times 10^6$ J.

Let α be the number of gallons of gasoline consumed.

$\alpha(1.3 \times 10^8 \text{ J})(0.15) = 1.027 \times 10^6$ J

$\alpha = 0.053$ gallons

b) $(1.00 \text{ gallons})/\alpha = 19$ accelerations

7.80: (a) Stored energy $= mgh = (\rho V)gh = \rho A(1\ \text{m})gh$

$$= (1000\ \text{kg/m}^3)(3.0\times10^6\ \text{m}^2)(1\ \text{m})(9.8\,\tfrac{\text{m}}{\text{s}^2})(150\ \text{m})$$

$$= 4.4\times10^{12}\ \text{J}.$$

(b) 90% of the stored energy is converted to electrical energy, so

$$(0.90)(mgh) = 1000\ \text{kW h}$$

$$(0.90)\rho V\,gh = 1000\ \text{kW h}$$

$$V = \frac{(1000\ \text{kW h})(\frac{3600\ \text{s}}{1\ \text{h}})}{(0.90)(1000\ \text{kg/m}^3)(150\ \text{m})(9.8\ \text{m/s}^2)}$$

$$= 2.7\times10^3\ \text{m}^3$$

Change in level of the lake**:**

$$A\Delta h = V_{\text{water}}$$

$$\Delta h = \frac{V}{A} = \frac{2.7\times10^3\ \text{m}^3}{3.0\times10^6\ \text{m}^2} = 9.0\times10^{-4}\ \text{m}$$

7.81: The potential energy of a horizontal layer of thickness dy, area A, and height y is $dU = (dm)gy$. Let ρ be the density of water.

$dm = \rho dV = \rho A\,dy$, so $dU = \rho Agy\,dy$.

The total potential energy U is

$U = \int_0^h dU = \rho Ag\int_0^h ydy = \frac{1}{2}\rho Ah^2.$

$A = 3.0\times10^6\ \text{m}^2$ and $h = 150\ \text{m}$, so $U = 3.3\times10^{14}\ \text{J} = 9.2\times10^7\ \text{kWh}.$

7.82: a) Yes; rather than considering arbitrary paths, consider that

$$\vec{F} = -\left[\frac{\partial}{\partial y}\left(-\frac{Cy^3}{3}\right)\right]\hat{j}.$$

b) No; consider the same path as in Example 7.13 (the field is not the same). For this force, $\vec{F} = \mathbf{0}$ along Leg 1, $\vec{F}\cdot d\vec{l} = 0$ along legs 2 and 4, but $\vec{F}\cdot d\vec{l} \neq 0$ along Leg 3.

7.83: a) Along this line, $x = y$, so $\vec{F} \cdot d\vec{l} = -\alpha y^3 dy$, and

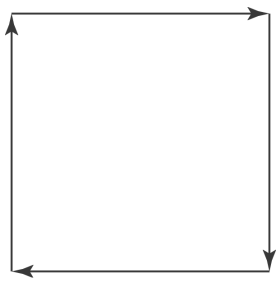
$$\int_{y_1}^{y_2} F_y dy = -\frac{\alpha}{4}(y_2^4 - y_1^4) = -50.6 \text{ J}.$$

b) Along the first leg, $dy = 0$ and so $\vec{F} \cdot d\vec{l} = 0$. Along the second leg, $x = 3.00 \text{ m}$, so $F_y = -(7.50 \text{ N/m}^2)y^2$, and

$$\int_{y_1}^{y_2} F_y dy = -(7.5/3 \text{ N/m}^2)(y_2^3 - y_1^3) = -67.5 \text{ J}.$$

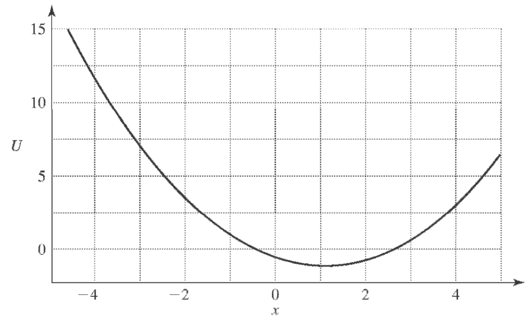
c) The work done depends on the path, and the force is not conservative.

7.84: a)



b) (1): $x = 0$ along this leg, so $\vec{F} = \mathbf{0}$ and $W = 0$. (2)**:** Along this leg, $y = 1.50 \text{ m}$, so $\vec{F} \cdot d\vec{l} = (3.00 \text{ N/m})xdx$, and $W = (1.50 \text{ N/m})((1.50 \text{ m})^2 - 0) = 3.38 \text{ J}$ (3) $\vec{F} \cdot d\vec{l} = 0$, so $W = 0$ (4) $y = 0$, so $\vec{F} = \mathbf{0}$ and $W = 0$. The work done in moving around the closed path is 3.38 J. c) The work done in moving around a closed path is not zero, and the force is not conservative.

7.85: a) For the given proposed potential $U(x)$, $-\frac{dU}{dx} = -kx + F$, so this is a possible potential function. For this potential, $U(0) = -F^2/2k$, not zero. Setting the zero of potential is equivalent to adding a constant to the potential; any additive constant will not change the derivative, and will correspond to the same force. b) At equilibrium, the force is zero; solving $-kx + F = 0$ for x gives $x_0 = F/k$. $U(x_0) = -F^2/k$, and this is a minimum of U, and hence a stable point.
c)



d) No; $F_{\text{tot}} = 0$ at only one point, and this is a stable point. e) The extreme values of x correspond to zero velocity, hence zero kinetic energy, so $U(x_{\pm}) = E$, where $x_{\pm}$ are the extreme points of the motion. Rather than solve a quadratic, note that $\frac{1}{2}k(x - F/k)^2 - F^2/k$, so $U(x_{\pm}) = E$ becomes

$$\frac{1}{2}k\left(x_{\pm} - \frac{F}{k}\right)^2 - F/k = \frac{F^2}{k}$$

$$x_{\pm} - \frac{F}{k} = \pm 2\frac{F}{k},$$

$$x_{+} = 3\frac{F}{k} \quad x_{-} = -\frac{F}{k}.$$

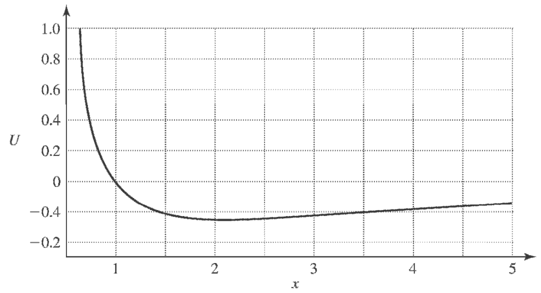
f) The maximum kinetic energy occurs when $U(x)$ is a minimum, the point $x_0 = F/k$ found in part (b). At this point $K = E - U = (F^2/k) - (-F^2/k) = 2F^2/k$, so $v = 2F/\sqrt{mk}$.

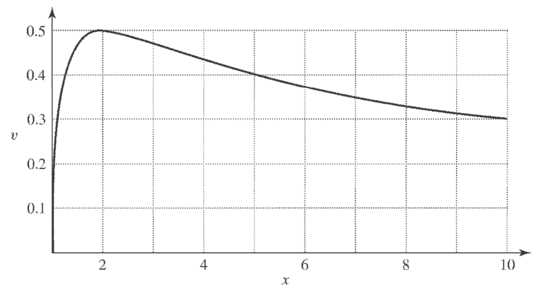
7.86: a) The slope of the *U vs*. *x* curve is negative at point *A*, so F_x is positive (Eq. (7.17)). b) The slope of the curve at point *B* is positive, so the force is negative. c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m. d) The curve at point *C* looks pretty close to flat, so the force is zero. e) The object had zero kinetic energy at point *A*, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m. f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m. g) The only potential maximum, and hence the only point of unstable equilibrium, is at point *C*.

7.87: a) Eliminating β in favor of α and $x_0 (\beta = \alpha / x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \frac{\alpha}{x_0^2}(1-1) = 0$. $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive).





b)

$$v(x) = \sqrt{-\frac{2}{m}U} = \sqrt{\left(\frac{2\alpha}{mx_0^2}\right)\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2\right)}.$$

The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$.

c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\frac{dU}{dx} = 0$, or

$$\frac{dU}{dx} = \frac{\alpha}{x_0}3\left[-2\left(\frac{x_0}{x}\right)^3 + \left(\frac{x_0}{x}\right)^2\right] = 0,$$

which has the solution $x = 2x_0$. $U(2x_0) = -\frac{\alpha}{4x_0^2}$, so $v = \sqrt{\frac{\alpha}{2mx_0^2}}$. d) The maximum speed occurs at a point where $\frac{dU}{dx} = 0$, and from Eq. (7.15), the force at this point is zero. e) $x_1 = 3x_0$, and $U(3x_0) = -\frac{2}{9}\frac{\alpha}{x_0^2}$; $v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} =$ $\sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - 2/9\right)}$. The particle is confined to the region where $U(x) < U(x_1)$. The

Capítulo 8

8.1: a) $(10{,}000\ \text{kg})(12.0\ \text{m/s}) = 1.20\times10^{5}\ \text{kg}\cdot\text{m/s}$.

b) (i) Five times the speed, $60.0\ \text{m/s}$. (ii) $\sqrt{5}\ (12.0\ \text{m/s}) = 26.8\ \text{m/s}$.

8.2: See Exercise 8.3 (a); the iceboats have the same kinetic energy, so the boat with the larger mass has the larger magnitude of momentum by a factor of $\sqrt{(2m)/(m)} = \sqrt{2}$.

8.3: a)
$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}.$$

b) From the result of part (a), for the same kinetic energy, $\dfrac{p_1^2}{m_1} = \dfrac{p_2^2}{m_2}$, so the larger mass baseball has the greater momentum; $\left(p_{\text{bird}}/p_{\text{ball}}\right) = \sqrt{0.040/0.145} = 0.525$. From the result of part (b), for the same momentum $K_1m_1 = K_2m_2$, so $K_1w_1 = K_2w_2$; the woman, with the smaller weight, has the larger kinetic energy. $\left(K_{\text{man}}/K_{\text{woman}}\right) = 450/700 = 0.643$.

8.4: From Eq. (8.2),

$$p_x = mv_x = (0.420\ \text{kg})(4.50\ \text{m/s})\cos 20.0° = 1.78\ \text{kg m/s}$$
$$p_y = mv_y = (0.420\ \text{kg})(4.50\ \text{m/s})\sin 20.0° = 0.646\ \text{kg m/s}.$$

8.5: The y-component of the total momentum is

$$(0.145\ \text{kg})(1.30\ \text{m/s}) + (0.0570\ \text{kg})(-7.80\ \text{m/s}) = -0.256\ \text{kg}\cdot\text{m/s}.$$

This quantity is negative, so the total momentum of the system is in the $-y$-direction.

8.6: From Eq. (8.2), $p_y = -(0.145\ \text{kg})(7.00\ \text{m/s}) = -1.015\ \text{kg}\cdot\text{m/s}$, and $p_x = (0.045\ \text{kg})(9.00\ \text{m/s}) = 0.405\ \text{kg}\cdot\text{m/s}$, so the total momentum has magnitude

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(-0.405\ \text{kg}\cdot\text{m/s})^2 + (-1.015\ \text{kg}\cdot\text{m/s})^2} = 1.09\ \text{kg}\cdot\text{m/s},$$

and is at an angle arctan $\left(\frac{-1.015}{+.405}\right) = -68°$, using the value of the arctangent function in the fourth quadrant $(p_x > 0, \text{p}_\text{y} < 0)$.

8.7: $\frac{\Delta p}{\Delta t} = \frac{(0.0450\,\text{kg})(25.0\,\text{m/s})}{2.00\times10^{-3}\,\text{s}} = 563\,\text{N}$. The weight of the ball is less than half a newton, so the weight is not significant while the ball and club are in contact.

8.8: a) The magnitude of the velocity has changed by

$$(45.0\,\text{m/s}) - (-55.0\,\text{m/s}) = 100.0\,\text{m/s},$$

and so the magnitude of the change of momentum is $(0.145\,\text{kg})\,(100.0\,\text{m/s}) = 14.500\,\text{kg m/s}$, to three figures. This is also the magnitude of the impulse. b) From Eq. (8.8), the magnitude of the average applied force is $\frac{14.500\,\text{kg.m/s}}{2.00\times10^{-3}\,\text{s}} = 7.25\times10^{3}\,\text{N}$.

8.9: a) Considering the $+x$-components, $p_2 = p_1 + J = (0.16\,\text{kg})(3.00\,\text{m/s}) + (25.0\,\text{N}) \times (0.05\,\text{s}) = 1.73\,\text{kg}\cdot\text{m/s}$, and the velocity is $10.8\,\text{m/s}$ in the $+x$-direction. b) $p_2 = 0.48\,\text{kg}\cdot\text{m/s} + (-12.0\,\text{N})(0.05\,\text{s}) = -0.12\,\text{kg}\cdot\text{m/s}$, and the velocity is $+0.75\,\text{m/s}$ in the $-x$-direction.

8.10: a) $\vec{F}\ t = (1.04\times10^{5}\,\text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$. b) $(1.04\times10^{5}\,\text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$. c) $\frac{(1.04\times10^{5}\,\text{kg.}\,\text{m/s})}{(95{,}000\,\text{kg})}\hat{\boldsymbol{j}} = (1.10\,\text{m/s})\hat{\boldsymbol{j}}$. d) The initial velocity of the shuttle is not known; the change in the square of the speed is not the square of the change of the speed.

8.11: a) With $t_1 = 0$,

$$J_x = \int_0^{t_2} F_x\, dt = (0.80\times10^7\ \text{N/s})t_2^2 - (2.00\times10^9\ \text{N/s}^2)t_2^3,$$

which is $18.8\,\text{kg}\cdot\text{m/s}$, and so the impulse delivered between *t=0* and $t_2 = 2.50\times10^{-3}\,\text{s}$ is $(18.8\,\text{kg}\cdot\text{m/s})\hat{\boldsymbol{i}}.$ b) $J_y = -(0.145\,\text{kg})\,(9.80\,\text{m/s}^2)\,(2.50\times10^{-3}\text{s})$, and the impulse is $(-3.55\times10^{-3}\ \text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$ c) $\dfrac{J_x}{t_2} = 7.52\times10^3\,\text{N}$, so the average force is $(7.52\times10^3\,\text{N})\hat{\boldsymbol{i}}.$

d)
$$\begin{aligned}\vec{\boldsymbol{p}}_2 &= \vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{j}}\\ &= -(0.145\text{kg})(40.0\hat{\boldsymbol{i}} + 5.0\hat{\boldsymbol{j}})\text{m/s} + (18.8\hat{\boldsymbol{i}} - 3.55\times10^{-3}\hat{\boldsymbol{j}})\\ &= (13.0\ \text{kg.m/s})\hat{\boldsymbol{i}} - (0.73\ \text{kg.m/s})\hat{\boldsymbol{j}}.\end{aligned}$$

The velocity is the momentum divided by the mass, or (89.7 m/s) $\hat{\boldsymbol{i}} - (5.0\text{m/s})\hat{\boldsymbol{j}}.$

8.12: The change in the ball's momentum in the *x*-direction (taken to be positive to the right) is $(0.145\ \text{kg})\,(-(65.0\ \text{m/s})\cos\ 30^\circ - 50.0\text{m/s}) = -15.41\,\text{kg}\cdot\text{m/s}$, so the *x*-component of the average force is

$$\frac{-15.41\,\text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\text{s}} = -8.81\times10^3\,\text{N},$$

and the *y*-component of the force is

$$\frac{(0.145\text{kg})(65.0\,\text{m/s})\sin 30^\circ}{(1.75\times10^{-3}\text{s})} = 2.7\times10^3\,\text{N}.$$

8.13: a)
$$J = \int_{t_1}^{t_2} F dt = A(t_2 - t_1) + \frac{B}{3}(t_2^3 - t_1^3),$$

or $J = At_2 + (B/3)t_2^3$ if $\text{t}_1 = 0.$ b) $v = \dfrac{p}{m} = \dfrac{J}{m} = \dfrac{A}{m}t_2 + \dfrac{B}{3m}t_2^3.$

8.14: The impluse imparted to the player is opposite in direction but of the same magnitude as that imparted to the puck, so the player's speed is $\frac{(0.16\,\text{kg})\,(20.0\,\text{m/s})}{(75.0\,\text{kg})} = 4.27\,\text{cm/s}$, in the direction opposite to the puck's.

8.15: a) You and the snowball now share the momentum of the snowball when thrown so your speed is $\frac{(0.400\,\text{kg})\,(10.0\,\text{m/s})}{(70.0\,\text{kg}+0.400\,\text{kg})} = 5.68\,\text{cm/s}$. b) The change in the snowball's momentum is $(0.400\,\text{kg})\,(18.0\,\text{m/s}) = 7.20\,\text{kg}\cdot\text{m/s})$, so your speed is $\frac{7.20\,\text{kg}\cdot\text{m/s}}{70.0\,\text{kg}} = 10.3\,\text{cm/s}$.

8.16: a) The final momentum is

$$(0.250\,\text{kg})(-0.120\,\text{m/s}) + (0.350)(0.650\,\text{m/s}) = 0.1975\,\text{kg}\cdot\text{m/s},$$

taking positive directions to the right. a) Before the collision, puck B was at rest, so all of the momentum is due to puck A's motion, and

$$v_{A1} = \frac{p}{m_A} = \frac{0.1975\,\text{kg}\cdot\text{m/s}}{0.250\,\text{kg}} = 0.790\,\text{m/s}.$$

b) $$\Delta K = K_2 - K_1 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_A v_{A1}^2$$

$$= \frac{1}{2}(0.250\,\text{kg})\,(-0.120\,\text{m/s})^2 + \frac{1}{2}(0.350\,\text{kg})(0.650\,\text{m/s})^2$$

$$-\frac{1}{2}(0.250\,\text{kg})(-0.7900\,\text{m/s})^2$$

$$= -0.0023\,\text{J}$$

8.17: The change in velocity is the negative of the change in Gretzky's momentum, divided by the defender's mass, or

$$\begin{aligned} v_{B2} &= v_{B1} - \frac{m_A}{m_B}(v_{A2} - v_{A1}) \\ &= -5.00 \text{ m/s} - \frac{756 \text{ N}}{900 \text{ N}}(1.50 \text{ m/s} - 13.0 \text{ m/s}) \\ &= 4.66 \text{ m/s}. \end{aligned}$$

Positive velocities are in Gretzky's original direction of motion, so the defender has changed direction.

b) $$\begin{aligned} K_2 - K_1 &= \frac{1}{2} m_A (v_{A2}^2 - v_{A1}^2) + \frac{1}{2} m_B (v_{B2}^2 - v_{B1}^2) \\ &= \frac{1}{2(9.80 \text{ m/s}^2)} \begin{bmatrix} (756 \text{ N})((1.50 \text{ m/s})^2 - (13.0 \text{ m/s})^2) \\ + (900 \text{ N})((4.66 \text{ m/s})^2 - (-5.00 \text{ m/s})^2) \end{bmatrix} \\ &= -6.58 \text{ kJ}. \end{aligned}$$

8.18: Take the direction of the bullet's motion to be the positive direction. The total momentum of the bullet, rifle, and gas must be zero, so

$$(0.00720 \text{ kg})(601 \text{ m/s} - 1.85 \text{ m/s}) + (2.80 \text{ kg})(-1.85 \text{ m/s}) + p_{\text{gas}} = 0,$$

and $p_{\text{gas}} = 0.866 \text{ kg} \cdot \text{m/s}$. Note that the speed of the bullet is found by subtracting the speed of the rifle from the speed of the bullet relative to the rifle.

8.19: a) See Exercise 8.21; $v_A = \left(\frac{3.00 \text{ kg}}{1.00 \text{ kg}}\right)(0.800 \text{ m/s}) = 3.60 \text{ m/s}$.

b) $(1/2)(1.00 \text{ kg})(3.60 \text{ m/s})^2 + (1/2)(3.00 \text{ kg})(1.200 \text{ m/s})^2 = 8.64 \text{ J}$.

8.20: In the absence of friction, the horizontal component of the hat-plus-adversary system is conserved, and the recoil speed is

$$\frac{(4.50 \text{ kg})(22.0 \text{ m/s}) \cos 36.9°}{(120 \text{ kg})} = 0.66 \text{ m/s}.$$

8.21: a) Taking v_A and v_B to be magnitudes, conservation of momentum is expressed as $m_A v_A = m_B v_B$, so $v_B = \frac{m_A}{m_B} v_A$.

b) $$\frac{K_A}{K_B} = \frac{(1/2) m_A v_A^2}{(1/2) m_B v_B^2} = \frac{m_A v_A^2}{m_B ((m_A / m_B) v_A)^2} = \frac{m_B}{m_A}.$$

(This result may be obtained using the result of Exercise 8.3.)

8.22:

$$^{214}\text{Po decay: } {}^{214}\text{Po} \rightarrow {}^{4}\alpha + {}^{210}\text{X}$$

$$Set v_\alpha : KE_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$$

$$v_\alpha = \sqrt{\frac{2KE_\alpha}{m_\alpha}}$$

$$= \sqrt{\frac{2(1.23 \times 10^{-12}\ \text{J})}{6.65 \times 10^{-27}\ \text{kg}}} = 1.92 \times 10^7\ \text{m/s}$$

Momentum conservation:

$$0 = m_\alpha v_\alpha - m_\text{x} v_\text{x}$$

$$v_x = \frac{m_\alpha v_\alpha}{m_x} = \frac{m_\alpha v_\alpha}{210 m_\text{p}}$$

$$= \frac{(6.65 \times 10^{-27}\ \text{kg})(1.92 \times 10^7\ \text{m/s})}{(210)(1.67 \times 10^{-27}\ \text{kg})}$$

$$= 3.65 \times 10^5\ \text{m/s}$$

8.23: Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object A be you and object B be the rock.

$$0 = -m_A v_A + m_B v_B \cos 35.0°$$

$$v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 2.11\,\text{m/s}$$

8.24: Let Rebecca's original direction of motion be the x-direction. a) From conservation of the x-component of momentum,

$$(45.0\,\text{kg})(13.0\,\text{m/s}) = (45.0\,\text{kg})(8.0\,\text{m/s})\cos 53.1° + (65.0\,\text{kg})v_x,$$

So $v_x = 5.67\,\text{m/s}$. If Rebecca's final motion is taken to have a positive y-component, then

$$v_y = -\frac{(45.0\,\text{kg})(8.0\,\text{m/s})\sin 53.1°}{(65.0\,\text{kg})} = -4.43\,\text{m/s}.$$

Daniel's final speed is

$$\sqrt{v_x^2 + v_y^2} = \sqrt{(5.67\,\text{m/s})^2 + (-4.43\,\text{m/s})^2 = 7.20\,\text{m/s}},$$

and his direction is $\arctan\left(\frac{-4.42}{5.67}\right) = -38°$ from the x-axis, which is $91.1°$ from the direction of Rebecca's final motion.

b) $\Delta K = \frac{1}{2}(45.0\,\text{kg})\,(8.0\,\text{m/s})^2 + \frac{1}{2}(65.0\,\text{kg})\,(7.195\,\text{m/s})^2 - \frac{1}{2}(45.0)\,(13.0\,\text{m/s})^2$
$= -680\,\text{J}.$

Note that an extra figure was kept in the intermediate calculation.

8.25: $(m_{\text{Kim}} + m_{\text{Ken}})(3.00\ \text{m/s}) = m_{\text{Kim}}(4.00\ \text{m/s}) + m_{\text{Ken}}(2.25\ \text{m/s})$, so

$$\frac{m_{\text{Kim}}}{m_{\text{Ken}}} = \frac{(3.00\ \text{m/s}) - (2.25\ \text{m/s})}{(4.00\ \text{m/s}) - (3.00\ \text{m/s})} = 0.750,$$

and Kim weighs $(0.750)(700\ \text{N}) = 525\ \text{N}.$

8.26: The original momentum is $(24{,}000\ \text{kg})(4.00\ \text{m/s}) = 9.60 \times 10^4\ \text{kg} \cdot \text{m/s}$, the final mass is $24{,}000\ \text{kg} + 3000\ \text{kg} = 27{,}000\ \text{kg}$, and so the final speed is

$$\frac{9.60 \times 10^4\ \text{kg} \cdot \text{m/s}}{2.70 \times 10^4\ \text{kg}} = 3.56\ \text{m/s}.$$

8.27: Denote the final speeds as v_A and v_B and the initial speed of puck A as v_0, and omit the common mass. Then, the condition for conservation of momentum is

$$\begin{aligned} v_0 &= v_A \cos 30.0° + v_B \cos 45.0° \\ 0 &= v_A \sin 30.0° - v_B \sin 45.0°. \end{aligned}$$

The $45.0°$ angle simplifies the algebra, in that $\sin 45.0° = \cos 45.0°$, and so the v_B terms cancel when the equations are added, giving

$$v_A = \frac{v_0}{\cos 30.0° + \sin 30.0°} = 29.3\ \text{m/s}$$

From the second equation, $v_B = \frac{v_A}{\sqrt{2}} = 20.7\ \text{m/s}$. b) Again neglecting the common mass,

$$\frac{K_2}{K_1} = \frac{(1/2)(v_A^2 + v_B^2)}{(1/2)v_0^2} = \frac{(29.3\ \text{m/s})^2 + (20.7\ \text{m/s})^2}{(40.0\ \text{m/s})^2} = 0.804,$$

so 19.6% of the original energy is dissipated.

8.28: a) From $m_1v_1 + m_2v_2 = m_1v + m_2v = (m_1 + m_2)v$, $v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$. Taking positive velocities to the right, $v_1 = -3.00.\text{m/s}$ and $v_2 = 1.20\ \text{m/s}$, so $v = -1.60\ \text{m/s}$.

b) $$\Delta K = \frac{1}{2}(0.500\ \text{kg} + 0.250\ \text{kg})(-1.60\ \text{m/s})^2$$
$$-\frac{1}{2}(0.500\ \text{kg})(-3.00\ \text{m/s})^2 - \frac{1}{2}(0.250\ \text{kg})(1.20\ \text{m/s})^2$$
$$= -1.47\ \text{J}.$$

8.29: For the truck, $M = 6320\ \text{kg}$, and $V = 10\ \text{m/s}$, for the car, $m = 1050\ \text{kg}$ and $v = -15\ \text{m/s}$ (the negative sign indicates a westbound direction).

a) Conservation of momentum requires $(M + m)v' = MV + mv$, or

$$v' = \frac{(6320\ \text{kg})(10\ \text{m/s}) + (1050\ \text{kg})(-15\ \text{m/s})}{(6320\ \text{kg} + 1050\ \text{kg})} = 6.4\ \text{m/s eastbound.}$$

b) $$V = \frac{-mv}{M} = \frac{-(1050\ \text{kg})(-15\ \text{m/s})}{6320\ \text{kg}} = 2.5\ \text{m/s}.$$

c) $\Delta KE = -281\ \text{kJ}$ for part (a) and $\Delta KE = -138\ \text{kJ}$ for part (b).

8.30: Take north to be the x-direction and east to be the y-direction (these choices are arbitrary). Then, the final momentum is the same as the intial momentum (for a sufficiently muddy field), and the velocity components are

$$v_x = \frac{(110\ \text{kg})(8.8\ \text{m/s})}{(195\ \text{kg})} = 5.0\ \text{m/s}$$
$$v_y = \frac{(85\ \text{kg})(7.2\ \text{m/s})}{(195\ \text{kg})} = 3.1\ \text{m/s}.$$

The magnitude of the velocity is then $\sqrt{(5.0\ \text{m/s})^2 + (3.1\ \text{m/s})^2} = 5.9\ \text{m/s}$, at an angle or arctan $\left(\frac{3.1}{5.0}\right) = 32°$ east of north.

8.31: Use conservation of the horizontal component of momentum to find the velocity of the combined object after the collision. Let $+x$ be south.

$$\mathrm{P}_x \text{ is constant gives}$$
$$(0.250\ \text{kg})(0.200\ \text{m/s}) - (0.150\ \text{kg})(0.600\ \text{s}) = (0.400\ \text{kg})v_{2x}$$
$$v_{2x} = -10.0\ \text{cm/s}\ (v_2 = 10.0\ \text{cm/s, north})$$
$$K_1 = \tfrac{1}{2}(0.250\ \text{kg})(0.200\ \text{s})^2 + \tfrac{1}{2}(0.150\ \text{kg})(0.600\ \text{s})^2 = 0.0320\ \text{J}$$
$$K_2 = \tfrac{1}{2}(0.400\ \text{kg})(0.100\ \text{s})^2 = 0.0020\ \text{J}$$
$$\Delta K = K_2 - K_1 = -0.0300\ \text{J}$$

Kinetic energy is converted to thermal energy due to work done by nonconservative forces during the collision.

8.32: (a) Momentum conservation tells us that both cars have the same change in momentum, but the smaller car has a greater velocity change because it has a smaller mass.

$$M\Delta V = m\Delta v$$
$$\Delta v\ (\text{small car}) = \frac{M}{m}\Delta V\ (\text{large car})$$
$$= \frac{3000\ \text{kg}}{1200\ \text{kg}}\Delta V = 2.5\Delta \text{V}\ (\text{large car})$$

(b) The occupants of the small car experience 2.5 times the velocity change of those in the large car, so they also experience 2.5 times the acceleration. Therefore they feel 2.5 times the force, which causes whiplash and other serious injuries.

8.33: Take east to be the x-direction and north to be the y-direction (again, these choices are arbitrary). The components of the common velocity after the collision are

$$v_x = \frac{(1400\ \text{kg})\,(-35.0\ \text{km/h})}{(4200\ \text{kg})} = -11.67\ \text{km/h}$$
$$v_\text{y} = \frac{(2800\ \text{kg})\,(-50.0\ \text{km/h})}{(4200\ \text{kg})} = -33.33\ \text{km/h}.$$

The velocity has magnitude $\sqrt{(-11.67\ \text{km/h})^2 + (-33.33\ \text{km/h})^2} = 35.3\ \text{km/h}$ and is at a direction arctan $\left(\frac{-33.33}{-11.67}\right) = 70.7°$ south of west.

8.34: The initial momentum of the car must be the x-component of the final momentum as the truck had no intial x-component of momentum, so

$$
\begin{aligned}
v_{\text{car}} &= \frac{p_x}{m_{\text{car}}} = \frac{(m_{\text{car}} + m_{\text{truck}})v\cos\theta}{m_{\text{car}}} \\
&= \frac{2850\ \text{kg}}{950\ \text{kg}}(16.0\ \text{m/s})\cos(90° - 24°) \\
&= 19.5\ \text{m}/s.
\end{aligned}
$$

Similarly, $v_{\text{truck}} = \dfrac{2850}{1900}(16.0\ \text{m/s})\sin 66° = 21.9\ \text{m/s}.$

8.35: The speed of the block immediately after being struck by the bullet may be found from either force or energy considerations. Either way, the distance s is related to the speed v_{block} by $v^2 = 2\mu_{\text{k}}\text{gs}.$ The speed of the bullet is then

$$
\begin{aligned}
v_{\text{bullet}} &= \frac{m_{\text{block}} + m_{\text{bullet}}}{m_{\text{bullet}}}\sqrt{2\mu_{\text{k}}\text{gs}} \\
&= \frac{1.205\ \text{kg}}{5.00\times10^{-3}\ \text{kg}}\sqrt{2(0.20)(9.80\ \text{m}/s^2)(0.230\ \text{m})} \\
&= 229\ \text{m/s},
\end{aligned}
$$

or $2.3\times10^2\ \text{m/s}$ to two places.

8.36: a) The final speed of the bullet-block combination is

$$V = \frac{12.0\times10^{-3}\ \text{kg}}{6.012\ \text{kg}}(380\ \ \text{m/s}) = 0.758\ \text{m/s}.$$

Energy is conserved after the collision, so $(m + M)gy = \frac{1}{2}(m + M)V^2$, and

$$y = \frac{1}{2}\frac{V^2}{g} = \frac{1}{2}\frac{(0.758\ \text{m/s})^2}{(9.80\ \text{m/s}^2)} = 0.0293\ \text{m} = 2.93\ \text{cm}.$$

b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{2}(12.0\times10^{-3}\ \text{kg})(380\ \text{m/s})^2 = 866\ \text{J}.$

c) From part a), $K_2 = \frac{1}{2}(6.012\ \text{kg})(0.758\ \text{m/s})^2 = 1.73\ \text{J}.$

8.37: Let $+y$ be north and $+x$ be south. Let v_{S1} and v_{A1} be the speeds of Sam and of Abigail before the collision. $m_S = 80.0\ \text{kg}, m_A = 50.0\ \text{kg}, v_{S2} = 6.00\ \text{m/s}, v_{A2} = 9.00\ \text{m/s}$.

P_x is constant gives

$$m_S v_{S1} = m_S v_{S2} \cos 37.0° + m_A v_{A2} \cos 23.0°$$

$$v_{S1} = 9.67\ \text{m/s (Sam)}$$

P_y is constant gives

$$m_A v_{A1} = m_S v_{S2} \sin 37.0° - m_A v_{A2} \sin 23.0°$$

$$v_{A1} = 2.26\ \text{m/s (Abigail)}$$

$$\text{b) } K_1 = \tfrac{1}{2} m_S v_{S1}^2 + \tfrac{1}{2} m_A v_{A1}^2 = 4101\ \text{J}$$

$$K_2 = \tfrac{1}{2} m_S v_{S2}^2 + \tfrac{1}{2} m_A v_{A2}^2 = 3465\ \text{J}$$

$$\Delta K = K_2 - K_1 = -640\ \text{J}$$

8.38: (a) At maximum compression of the spring, $v_2 = v_{10} = V$. Momentum conservation gives $(2.00\ \text{kg})(2.00\ \text{m/s}) = (12.0\ \text{kg})V$

$$V = 0.333\ \text{m/s}$$

$$\text{Energy conservation}: \frac{1}{2} m_2 v_0^2 = \frac{1}{2}(m_2 + m_{10})V^2 + U_{\text{spr}}$$

$$\frac{1}{2}(2.00\ \text{kg})(2.00\ \text{m/s})^2 = \frac{1}{2}(12.0\ \text{kg})(0.333\ \text{m}/s)^2 + U_{\text{spr}}$$

$$U_{\text{spr}} = 3.33\ \text{J}$$

(b) The collision is elastic and Eqs. (8.24) and (8.25) may be used**:**

$$v_2 = -1.33\ \text{m/s},\ v_{10} = +0.67\ \text{m/s}$$

8.39: In the notation of Example 8.10, with the smaller glider denoted as A, conservation of momentum gives $(1.50)v_{A2} + (3.00)v_{B2} = -5.40\ \text{m/s}$. The relative velocity has switched direction, so $v_{A2} - v_{B2} = -3.00\ \text{m/s}$. Multiplying the second of these relations by (3.00) and adding to the first gives $(4.50)v_{A2} = -14.4\ \text{m/s}$, or $v_{A2} = -3.20\ \text{m/s}$, with the minus sign indicating a velocity to the left. This may be substituted into either relation to obtain $v_{B2} = -0.20\ \text{m/s}$; or, multiplying the second relation by (1.50) and subtracting from the first gives $(4.50)v_{B2} = -0.90\ \text{m/s}$, which is the same result.

8.40: a) In the notation of Example 8.10, with the large marble (originally moving to the right) denoted as $A, (3.00)v_{A2} + (1.00)v_{B2} = 0.200\ \text{m/s}$. The relative velocity has switched direction, so $v_{A2} - v_{B2} = -0.600\ \text{m}/s$. Adding these eliminates v_{B2} to give $(4.00)v_{A2} = -0.400\ \text{m/s}$, or $v_{A2} = -0.100\ \text{m/s}$, with the minus sign indicating a final velocity to the left. This may be substituted into either of the two relations to obtain $v_{B2} = 0.500\ \text{m/s}$; or, the second of the above relations may be multiplied by 3.00 and subtracted from the first to give $(4.00)v_{B2} = 2.00\ \text{m/s}$, the same result.

b) $\Delta P_A = -0.009\ \text{kg}\cdot\text{m/s}, \Delta P_B = 0.009\ \text{kg}\cdot\text{m/s}$

c) $\Delta K_A = -4.5\times10^{-4}, \Delta K_B = 4.5\times10^{-4}.$

Because the collision is elastic, the numbers have the same magnitude.

8.41: Algebraically, $v_{B2} = \sqrt{20}\ \text{m/s}$. This substitution and the cancellation of common factors and units allow the equations in α and β to be reduced to

$$2 = \cos\alpha + \sqrt{1.8}\cos\beta$$
$$0 = \sin\alpha - \sqrt{1.8}\ \sin\beta.$$

Solving for $\cos\alpha$ and $\sin\ \alpha$, squaring and adding gives

$$\left(2 - \sqrt{1.8}\cos\beta\right)^2 + \left(\sqrt{1.8}.\sin\beta\right)^2 = 1.$$

Minor algebra leads to $\cos\ \beta = \frac{1.2}{\sqrt{1.8}}$, or $\beta = 26.57°$. Substitution of this result into the first of the above relations gives $\cos\ \alpha = \frac{4}{5}$, and $\alpha = 36.87°$.

8.42: a) Using Eq. (8.24), $\frac{v_A}{V} = \frac{1\,\text{u} - 2\,\text{u}}{1\,\text{u} + 2\,\text{u}} = \frac{1}{3}$. b) The kinetic energy is proportional to the square of the speed, so $\frac{K_A}{K} = \frac{1}{9}$ c) The magnitude of the speed is reduced by a factor of $\frac{1}{3}$ after each collision, so after N collisions, the speed is $\left(\frac{1}{3}\right)^N$ of its original value. To find N, consider

$$\left(\frac{1}{3}\right)^N = \frac{1}{59{,}000} \quad \text{or}$$
$$3^N = 59{,}000$$
$$N\ln(3) = \ln(59{,}000)$$
$$N = \frac{\ln(59{,}000)}{\ln(3)} = 10.$$

to the nearest integer. Of course, using the logarithm in any base gives the same result.

8.43: a) In Eq. (8.24), let $m_A = m$ and $m_B = M$. Solving for M gives

$$M = m\frac{v - v_A}{v + v_A}$$

In this case, $v = 1.50 \times 10^7$ m/s, and $v_A = -1.20 \times 10^7$ m/s, with the minus sign indicating a rebound. Then, $M = m\frac{1.50+1.20}{1.50+)-1.20)} = 9m.$ Either Eq. (8.25) may be used to find $v_B = \frac{v}{5} = 3.00 \times 10^6$ m/s, or Eq. (8.23), which gives $v_B = (1.50 \times 10^7 \text{ m/s}) + (-1.20 \times 10^7 \text{ m/s})$, the same result.

8.44: From Eq. (8.28),

$$x_{\text{cm}} = \frac{(0.30\text{ kg})(0.20\text{ m}) + (0.40\text{ kg})(0.10\text{ m}) + (0.20\text{ kg})(-0.30\text{ m})}{(0.90\text{ kg})} = +0.044\text{ m},$$

$$y_{\text{cm}} = \frac{(0.30\text{ kg})(0.30\text{ m}) + (0.40\text{ kg})(-0.40\text{ m}) + (0.20\text{ kg})(0.60\text{ m})}{(0.90\text{ kg})} = 0.056\text{ m}.$$

8.45: Measured from the center of the sun,

$$\frac{(1.99 \times 10^{30}\text{ kg})(0) + (1.90 \times 10^{27}\text{ kg})(7.78 \times 10^{11}\text{ m})}{1.99 \times 10^{30}\text{ kg} + 1.90 \times 10^{27}\text{ kg}} = 7.42 \times 10^8\text{ m}.$$

The center of mass of the system lies outside the sun.

8.46: a) Measured from the rear car, the position of the center of mass is, from Eq. (8.28), $\frac{(1800\text{ kg})(40.0\text{ m})}{(1200\text{ kg} + 1800\text{ kg})} = 24.0\text{ m}$, which is 16.0 m behind the leading car.

b) $(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s}.$

c) From Eq. (8.30),

$$v_{\text{cm}} = \frac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{(1200\text{ kg} + 1800\text{ kg})} = 16.8\text{ m/s}.$$

d) $(1200\text{ kg} + 1800\text{ kg})(16.8\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s}.$

8.47: a) With $x_1 = 0$ in Eq. (8.28),

$$m_1 = m_2((x_2 / x_{\text{cm}}) - 1) = (0.10\ \text{kg})((8.0\ \text{m})/(2.0\ \text{m}) - 1) = 0.30\ \text{kg}.$$

b) $\vec{\boldsymbol{P}} = M\vec{\boldsymbol{v}}_{cm} = (0.40\ \text{kg})(5.0\ \text{m/s})\,\hat{\boldsymbol{i}} = (2.0\ \text{kg}\cdot\text{m/s})\,\hat{\boldsymbol{i}}$. c) In Eq. (8.32), $\vec{\boldsymbol{v}}_2 = 0$, so $\vec{\boldsymbol{v}}_1 = \vec{\boldsymbol{P}}/(0.30\ \text{kg}) = (6.7\ \text{m/s})\hat{\boldsymbol{i}}$.

8.48: As in Example 8.15, the center of mass remains at rest, so there is zero net momentum, and the magnitudes of the speeds are related by $m_1 v_1 = m_2 v_2$, or $v_2 = (m_1 / m_2) v_1 = (60.0\ \text{kg} / 90.0\ \text{kg})(0.70\ \text{m/s}) = 0.47\ \text{m/s}$.

8.49: See Exercise 8.47(a); with $y_1 = 0$, Eq. (8.28) gives $m_1 = m_2((y_2 / y_{\text{cm}}) - 1) =$ $(0.50\ \text{kg})((6.0\ \text{m})/(2.4\ \text{m}) - 1) = 0.75\ \text{kg}$, so the total mass of the system is 1.25 kg.

b) $\vec{\boldsymbol{a}}_{\text{cm}} = \frac{d}{dt}\vec{\boldsymbol{v}}_{\text{cm}} = (1.50\ \text{m/s}^3)\, t\hat{\boldsymbol{i}}$.

c) $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}_{\text{cm}} = (1.25\ \text{kg})\,(1.50\ \text{m/s}^3)\,(3.0\ \text{s})\hat{\boldsymbol{i}} = (5.63\ \ \text{N})\hat{\boldsymbol{i}}$.

8.50: $\boldsymbol{p}_z = 0$, so $F_z = 0$. The x-component of force is

$$F_x = \frac{dp_x}{dt} = (-1.50\ \text{N/s})t.$$

$$F_y = \frac{dp_y}{dt} = 0.25\ \text{N}$$

8.51: a) From Eq. (8.38), $F = (1600\ \text{m/s})(0.0500\ \text{kg/s}) = 80.0\ \ \text{N}$. b) The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the fuel in a direction with a component perpendicular to the rocket's velocity, and braked by ejecting in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

8.52: It turns out to be more convenient to do part (b) first; the thrust is the force that accelerates the astronaut and MMU, $F = ma = (70\ \text{kg} + 110\ \text{kg})(0.029\ \text{m/s}^2) = 5.22\ \text{N}$.
a) Solving Eq. (8.38) for $|dm|$,

$$|dm| = \frac{F\,dt}{v_{\text{ex}}} = \frac{(5.22\ \text{N})(5.0\ \text{s})}{490\ \text{m/s}} = 53\ \text{gm}.$$

8.53: Solving for the magnitude of *dm* in Eq. (8.39),

$$|dm| = \frac{ma}{v_{\text{ex}}}dt = \frac{(6000\ \text{kg})(25.0\ \text{m/s}^2)}{(2000\ \text{m/s})}(1\ \text{s}) = 75.0\ \text{kg}.$$

8.54: Solving Eq. (8.34) for v_{ex} and taking the magnitude to find the exhaust speed, $v_{ex} = a\frac{m}{dm/dt} = \left(15.0\ \text{m/s}^2\right)\ (160\ \text{s}) = 2.4\ \text{km/s}$. In this form, the quantity $\frac{m}{dm/dt}$ is approximated by $\frac{m}{\Delta m/\Delta t} = \frac{m}{\Delta m}\Delta t = 160\ \text{s}$.

8.55: a) The average thrust is the impulse divided by the time, so the ratio of the average thrust to the maximum thrust is $\frac{(10.0\ \text{N}\cdot\text{s})}{(13.3\ \text{N})(1.70\ \text{s})} = 0.442$. b) Using the average force in Eq. (8.38), $v_{\text{ex}} = \frac{F\,dt}{|dm|} = \frac{10.0\ \text{N}\cdot\text{s}}{0.0125\ \text{kg}} = 800\ \text{m/s}$. c) Using the result of part (b) in Eq. (8.40), $v = (800\,\text{m/s})\ln(0.0258/0.0133) = 530\,\text{m/s}$.

8.56: Solving Eq. (8.4) for the ratio $\frac{m_0}{m}$, with $v_0 = 0$,

$$\frac{m_0}{m} = \exp\left(\frac{v}{v_{\text{ex}}}\right) = \exp\left(\frac{8.00\ \text{km/s}}{2.10\ \text{km/s}}\right) = 45.1.$$

8.57: Solving Eq. (8.40) for $\frac{m}{m_0}$, the fraction of the original rocket mass that is not fuel,

$$\frac{m}{m_0} = \exp\left(-\frac{v}{v_{\text{ex}}}\right).$$

a) For $v = 1.00\times10^{-3}c = 3.00\times10^5\ \text{m/s}, \exp(-(3.00\times10^5\ \text{m/s}/(2000\,\text{m/s})) = 7.2\times10^{-66}$.
b) For $v = 3000\ \text{m/s}, \exp(-(3000\ \text{m/s})/(2000\ \text{m/s})) = 0.22$.

8.58: a) The speed of the ball before and after the collision with the plate are found from the heights. The impulse is the mass times the sum of the speeds,

$J = m(v_1 + v_2) = m(\sqrt{2gy_1} + \sqrt{2gy_2}) = (0.040\text{kg})\sqrt{2(9.80\text{m/s}^2}\left(\sqrt{2.00\text{m}} + \sqrt{1.60\text{m}}\right) = 0.4$

b) $\frac{J}{\Delta t} = (0.47\ \text{N}\cdot\text{s}/2.00\times10^{-3}\text{s}) = 237\ \ \text{N}.$

8.59: $\vec{\boldsymbol{p}} = \int \vec{\boldsymbol{F}}\, dt = (\alpha t^3/3)\,\hat{\boldsymbol{j}} + (\beta t + \gamma t^2/2)\hat{\boldsymbol{j}} = (8.33\ \text{N/s}^2\ \text{t}^3)\hat{\boldsymbol{i}} + (30.0\ \text{N}t + 2.5\ \text{N/s}t^2)\,\hat{\boldsymbol{j}}$

After 0.500 s, $\vec{\boldsymbol{p}} = (1.04\text{kg}\cdot\text{m/s})\hat{\boldsymbol{i}} + (15.63\text{kg}\cdot\text{m/s})\hat{\boldsymbol{j}}$, and the velocity is

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{p}}/m = (0.52\text{m/s})\hat{\boldsymbol{i}} + (7.82\text{m/s})\hat{\boldsymbol{j}}.$$

8.60: a)

$$J_x = F_x t = (-380\ \text{N})\,(3.00\times10^{-3}\text{s}) = -1.14\ \ \text{N}\cdot\text{s}.$$

$$J_y = F_y t = (110\ \text{N})\,(3.00\times10^{-3}\text{s}) = 0.33\ \ \text{N}\cdot\text{s}.$$

b)

$$v_{2x} = v_{1x} + J_x/m = (20.0\ \text{m/s}) + \frac{(-1.14\text{N.s})}{(0.560\ \text{N})/(9.80\ \text{m/s}^2)} = 0.05\text{m/s}$$

$$v_{2y} = v_{1y} + J_y/m = (-4.0\text{m/s}) + \frac{(0.33\ \text{N.s})}{((0.560\ \text{N})/(9.80\text{m/s}^2))} = 1.78\text{m/s}.$$

8.61: The total momentum of the final combination is the same as the initial momentum; for the speed to be one-fifth of the original speed, the mass must be five times the original mass, or 15 cars.

8.62: The momentum of the convertible must be the south component of the total momentum, so

$$v_{\text{con}} = \frac{(800\ \text{kg}\cdot\text{m/s})\cos 60.0^\circ}{(1500\ \text{kg})} = 2.67\ \text{m/s}.$$

Similarly, the speed of the station wagon is

$$v_{\text{sw}} = \frac{(800\text{kg}\cdot\text{m/s})\sin 60.0^\circ}{(2000\ \text{kg})} = 3.46\ \text{m/s}.$$

8.63: The total momentum must be zero, and the velocity vectors must be three vectors of the same magnitude that sum to zero, and hence must form the sides of an equilateral triangle. One puck will move $60°$ north of east and the other will move $60°$ south of east.

8.64: a) $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{tot} v_x$, therefore

$$v_{Cx} = \frac{(0.100\ \text{kg})(0.50\ \text{m/s}) - (0.020\ \text{kg})(-1.50\ \text{m/s}) - (0.030\ \text{kg})(-0.50\ \text{m/s})\cos 60°}{0.050\text{kg}}$$

$$v_{Cx} = 1.75\ \text{m/s}$$

Similarly,

$$v_{Cy} = \frac{(0.100\text{kg})(0\ \text{m/s}) - (0.020\text{kg})(0\ \text{m/s}) - (0.030\ \text{kg})(-0.50\ \text{m/s})\sin 60°}{0.050\ \text{kg}}$$

$$v_{Cy} = 0.26\ \text{m/s}$$

b)

$$\Delta K = \tfrac{1}{2}(0.100\ \text{kg})(0.5\ \text{m/s})^2 - \tfrac{1}{2}(0.020\ \text{kg})(1.50\ \text{m/s})^2 - \tfrac{1}{2}(0.030\ \text{kg})(-0.50\ \text{m/s})^2$$
$$- \tfrac{1}{2}(0.050\ \text{kg}) \times [(1.75\ \text{m/s})^2 + (0.26\ \text{m/s})^2] = -0.092\ \text{J}$$

8.65: a) To throw the mass sideways, a sideways force must be exerted on the mass, and hence a sideways force is exerted on the car. The car is given to remain on track, so some other force (the tracks on the car) act to give a net horizontal force of zero on the car, which continues at $5.00\ \text{m/s}$ east.

b) If the mass is thrown with backward with a speed of $5.00\ \text{m/s}$ relative to the initial motion of the car, the mass is at rest relative to the ground, and has zero momentum. The speed of the car is then $(5.00\ \text{m/s})\frac{(200\ \text{kg})}{(175\ \text{kg})} = 5.71\ \text{m/s}$, and the car is still moving east.

c) The combined momentum of the mass and car must be same before and after the mass hits the car, so the speed is $\frac{(200\ \text{kg})(5.00\ \text{m/s}) + (25.0\ \text{kg})(-6.00\ \text{m/s})}{(225\ \text{kg})} = 3.78\ \text{m/s}$, with the car still moving east.

8.66: The total mass of the car is changing, but the speed of the sand as it leaves the car is the same as the speed of the car, so there is no change in the velocity of either the car or the sand (the sand acquires a downward velocity after it leaves the car, and is stopped on the tracks *after* it leaves the car). Another way of regarding the situation is that v_{ex} in Equations (8.37), (8.38) and (8.39) is zero, and the car does not accelerate. In any event, the speed of the car remains constant at 15.0 m/s. In Exercise 8.24, the rain is given as falling vertically, so its velocity relative to the car as it hits the car is not zero.

8.67: a) The ratio of the kinetic energy of the Nash to that of the Packard is $\frac{m_N v_N^2}{m_P v_P^2} = \frac{(840\ \mathrm{kg})(9\ \mathrm{m/s})^2}{(1620\ \mathrm{kg})(5\ \mathrm{m/s})^2} = 1.68.$ b) The ratio of the momentum of the Nash to that of the Packard is $\frac{m_N v_N}{m_P v_P} = \frac{(840\ \mathrm{kg})(9\ \mathrm{m/s})}{(1620\ \mathrm{kg})(5\ \mathrm{m/s})} = 0.933,$ therefore the Packard has the greater magnitude of momentum. c) The force necessary to stop an object with momentum P in time t is $F = -P/t$. Since the Packard has the greater momentum, it will require the greater force to stop it. The ratio is the same since the time is the same, therefore $F_N / F_P =$ 0.933. d) By the work-kinetic energy theorem, $F = \frac{\Delta k}{d}$. Therefore, since the Nash has the greater kinetic energy, it will require the greater force to stop it in a given distance. Since the distance is the same, the ratio of the forces is the same as that of the kinetic energies, $F_N / F_P =$ 1.68.

8.68: The recoil force is the momentum delivered to each bullet times the rate at which the bullets are fired,

$$F_{\mathrm{ave}} = (7.45 \times 10^{-3}\ \mathrm{kg})\,(293\ \mathrm{m/s})\left(\frac{1000\ \mathrm{bullets/min}}{60\ \mathrm{s/\ min}}\right) = 36.4\ \mathrm{N}.$$

8.69: (This problem involves solving a quadratic. The method presented here formulates the answer in terms of the parameters, and avoids intermediate calculations, including that of the spring constant.)

Let the mass of the frame be M and the mass putty be m. Denote the distance that the frame streteches the spring by x_0, the height above the frame from which the putty is dropped as h, and the maximum distance the frame moves from its initial position (with the frame attached) as d.

The collision between the putty and the frame is completely inelastic, and the common speed after the collision is $v_0 = \sqrt{2gh}\,\frac{m}{m+M}$. After the collision, energy is conserved, so that

$$\frac{1}{2}(m+M)v_0^2 + (m+M)gd = \frac{1}{2}k((d+x_0)^2 - x_0^2 \text{ , or}$$

$$\frac{1}{2}\frac{m^2}{m+M}(2gh) + (m+M)gd = \frac{1}{2}\frac{mg}{x_0}((d+x_0)^2 - x_0^2 \text{ ,}$$

where the above expression for v_0, and $k = mg/x_0$ have been used. In this form, it is seen that a factor of g cancels from all terms. After performing the algebra, the quadratic for d becomes

$$d^2 - d\left(2x_0\frac{m}{M}\right) - 2hx_0\frac{m^2}{m+M} = 0,$$

which has as its positive root

$$d = x_0\left[\left(\frac{m}{M}\right) + \sqrt{\left(\frac{m}{M}\right)^2 + 2\frac{h}{x_0}\left(\frac{m^2}{M(m+M)}\right)}\right].$$

For this situation, $m = 4/3\ M$ and $h/x_0 = 6$, so

$$d = 0.232 \text{ m.}$$

8.70: a) After impact, the block-bullet combination has a total mass of 1.00 kg, and the speed V of the block is found from $\frac{1}{2}M_{\text{total}}V^2 = \frac{1}{2}kX^2$, or $V = \sqrt{\frac{k}{m}}X$. The spring constant k is determined from the calibration; $k = \frac{0.75\ \text{N}}{2.50\ \times 10^{-3}\ \text{m}} = 300\ \text{N/m}$. Combining,

$$V = \sqrt{\frac{300\ \text{N/m}}{1.00\ \text{kg}}}\left(15.0 \times 10^{-2}\ \text{m}\right) = 2.60\ \text{m/s}.$$

b) Although this is not a pendulum, the analysis of the inelastic collision is the same;

$$v = \frac{M_{\text{total}}}{m}V = \frac{1.00\ \text{Kg}}{8.0 \times 10^{-3}\ \text{Kg}}(2.60\ \text{m/s}) = 325\ \text{m/s}.$$

8.71: a) Take the original direction of the bullet's motion to be the x-direction, and the direction of recoil to be the y-direction. The components of the stone's velocity after impact are then

$$v_x = \left(\frac{6.00 \times 10^{-3}\ \text{Kg}}{0.100\ \text{Kg}}\right)(350\ \text{m/s}) = 21.0\ \text{m/s},$$

$$v_y = -\left(\frac{6.00 \times 10^{-3}\ \text{Kg}}{0.100\ \text{Kg}}\right)(250\ \text{m/s}) = 15.0\ \text{m/s},$$

and the stone's speed is $\sqrt{(21.0\ \text{m/s})^2 + (15.0\ \text{m/s})^2} = 25.8\ \text{m/s}$, at an angle of arctan $\left(\frac{15.0}{21.0}\right) = 35.5°$. b) $K_1 = \frac{1}{2}\left(6.00 \times 10^{-3}\ \text{kg}\right)(350\ \text{m/s})^2 = 368\ \text{J}$
$K_2 = \frac{1}{2}\left(6.00 \times 10^{-3}\ \text{kg}\right)(250\ \text{m/s})^2 + \frac{1}{2}(0.100\ \text{kg})\left(25.8\ \text{m}^2/\text{s}^2\right) = 221\ \text{J}$, so the collision is not perfectly elastic.

8.72: a) The stuntman's speed before the collision is $v_{0s} = \sqrt{2gy} = 9.9\ \mathrm{m/s}$. The speed after the collision is

$$v = \frac{m_s}{m_s + m_v} v_{0s} = \frac{80.0\ \mathrm{kg}}{0.100\ \mathrm{kg}}(9.9\ \mathrm{m/s}) = 5.3\ \mathrm{m/s}.$$

b) Momentum is not conserved during the slide. From the work-energy theorem, the distance x is found from $\frac{1}{2} m_{\mathrm{total}} v^2 = \mu_k m_{\mathrm{total}} gx$, or

$$x = \frac{v^2}{2\mu_{kg}} = \frac{(5.28\ \mathrm{m/s})^2}{2(0.25)(9.80\ \mathrm{m/s^2})} = 5.7\ \mathrm{m}.$$

Note that an extra figure was needed for V in part (b) to avoid roundoff error.

8.73: Let v be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass m.

Conservation of energy says $\frac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$

This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision. Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$

Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

$$\tfrac{1}{2}(2m)v_2^2 = (2m)gy_3$$

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = R/4$$

Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

8.74: Collision: Momentum conservation gives

$$mv_0 = mv_1 + (3m)v_3$$
$$v_0 = v_1 + 3v_3 \qquad (1)$$

Energy Conservation**:**

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_3^2$$
$$v_0^2 = v_1^2 + 3v_3^2 \qquad (2)$$

Solve (1) and (2) for $v_3 : v_3 = 2.50\ \text{m/s}$

Energy conservation after collision:

$$\frac{1}{2}(3m)v_3^2 = (3m)gh = (3m)gl(1-\cos\theta)$$

Solve for $\theta : \theta = 68.8°$

8.75: First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\Sigma\vec{F} = m\vec{a}$ to the object at the top of the circular loop, where the object has speed v_3.

$$T + mg = m\frac{v_3^2}{R}$$

The minimum speed v_3 for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg},$ where $R = 3.50$ m.

Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at the point 2. Only gravity does work, so

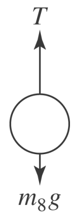
$$K_2 + U_2 = K_3 + U_3$$
$$\tfrac{1}{2}m_{\text{tot}}v_2^2 = \tfrac{1}{2}m_{\text{tot}}v_3^2 + m_{\text{tot}}g(2R)$$

Use $v_3 = \sqrt{Rg}$ and solve for $v_2 : v_2 = \sqrt{5gR} = 13.1\ \text{m/s}$

Now apply conservation of momentum to the collision between the dart and the sphere. Let v_1 be the speed of the dart before the collision.

$$(5.00\ \text{kg})v_1 = (25.0\ \text{kg})(13.1\ \text{m/s})$$
$$v_1 = 65.5\ \text{m/s}$$

8.76: Just after the collision: $\sum F = ma$



$$T - m_8 g = m_8 \frac{v_8^2}{R}$$

$$1600\text{N} - (8.00\text{ kg})(9.80\text{ m/s}^2) = (8.00\text{ kg})\frac{v_8^2}{1.35\text{ m}}$$

$$v_8 = 16.0\text{ m/s}$$

Energy and momentum are conserved during the elastic collision.

$$m_2 v_0 = m_2 v_2 + m_8 v_8$$

$$(2.00\text{ kg})v_0 = (2.00\text{ kg})v_2 + (8.00\text{ kg})(16.0\text{ m/s})$$

$$v_0 = v_2 + 64.0\text{ m/s} \qquad (1)$$

$$\frac{1}{2}m_2 v_0^2 = \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_8 v_8^2$$

$$(2.00\text{ kg})v_0^2 = (2.00\text{ kg})v_2^2 + (8.00\text{ kg})(16.0\text{ m/s})^2$$

$$v_0^2 = v_2^2 + 1024\text{ m}^2/\text{s}^2 \qquad (2)$$

Solve (1) and (2) for $v_0 : v_0 = 40.0\text{ m/s}$

8.77: a) The coefficient of friction, from either force or energy consideration, is $\mu_k = v^2/2gs$, where v is the speed of the block after the bullet passes through. The speed of the block is determined from the momentum lost by the bullet, $(4.00 \times 10^{-3}\text{ kg})(280\text{ m/s}) = 1.12\text{ kg}\cdot\text{m/s}$, and so the coefficient of kinetic friction is

$$\mu_k = \frac{((1.12\text{ kg}\cdot\text{m/s})/(0.80\text{ kg}))^2}{2(9.80\text{ m/s}^2)(0.45\text{m})} = 0.22.$$

b) $\frac{1}{2}(4.00 \times 10^{-3}\text{ kg})((400\text{ m/s})^2 - (120\text{ m/s})^2) = 291\text{ J}$. c) From the calculation of the momentum in part (a), the block's initial kinetic energy was $\frac{p^2}{2m} = \frac{(1.12\text{ kg}\cdot\text{m/s})^2}{2(0.80\text{ kg})} = 0.784\text{ J}$.

8.78: The speed of the block after the bullet has passed through (but before the block has begun to rise; this assumes a large force applied over a short time, a situation characteristic of bullets) is

$$V = \sqrt{2gy} = \sqrt{2(9.80\ \text{m/s}^2)(0.45\times 10^{-2}\ \text{m})} = 0.297\ \text{m/s}.$$

The final speed v of the bullet is then

$$\begin{aligned} v &= \frac{p}{m} = \frac{mv_0 - MV}{m} = v_0 - \frac{M}{m}V \\ &= 450\ \text{m/s} - \frac{1.00\ \text{kg}}{5.00\times 10^{-3}\ \text{kg}}(0.297\ \text{m/s}) = 390.6\ \text{m/s}, \end{aligned}$$

or 390 m/s to two figures.

8.79: a) Using the notation of Eq. (8.24),

$$\begin{aligned} K_0 - K_2 &= \frac{1}{2}mv^2 - \frac{1}{2}mv_A^2 \\ &= \frac{1}{2}mv^2\left(1 - \left(\frac{m-M}{m+M}\right)^2\right) \\ &= K_0\left(\frac{(m+M)^2 - (m-M)^2}{(m+M)^2}\right) \\ &= K_0\left(\frac{4mM}{(m+M)^2}\right). \end{aligned}$$

b) Of the many ways to do this calculation, the most direct way is to differentiate the expression of part (a) with respect to M and set equal to zero;

$$\begin{aligned} 0 &= (4m\mathrm{K}_0)\frac{d}{dM}\left(\frac{M}{(m+M)^2}\right), \text{ or} \\ 0 &= \frac{1}{(m+M)^2} - \frac{2M}{(m+M)^3} \\ 0 &= (m+M) - 2M \\ m &= M. \end{aligned}$$

c) From Eq.(8.24), with $m_A = m_B = m, v_A = 0;$ the neutron has lost all of its kinetic energy.

8.80: a) From the derivation in Sec. 8.4 of the text we have

$$V_A = \frac{M_A - M_B}{M_A + M_B} V_0 \text{ and } V_B = \frac{2M_A}{M_A + M_B}$$

The ratio of the kinetic energies of the two particles after the collision is

$$\frac{\frac{1}{2}M_A V_A^2}{\frac{1}{2}M_B V_B^2} = \frac{M_A}{M_B}\left(\frac{V_A}{V_B}\right)^2 = \frac{M_A}{M_B}\left(\frac{M_A - M_B}{2M_A}\right)^2 = \frac{(M_A - M_B)^2}{4M_A M_B}$$

or $KE_A = KE_B \dfrac{(M_A - M_B)^2}{4M_A M_B}$

b) i) For $M_A = M_B, KE_A = 0$; i.e., the two objects simply exchange kinetic energies.

ii) For $M_A = 5M_B$,

$$\frac{KE_A}{KE_B} = \frac{(4M_B)^2}{4(5M_B)(M_B)} = \frac{4}{5}$$

i.e., M_A gets 4/9 or 44% of the total.

c) We want

$$\frac{KE_A}{KE_B} = 1 = \frac{(M_A - M_B)^2}{4M_A M_B} = \frac{M_A^2 - 2M_A M_B + M_B^2}{4M_A M_B}$$

which reduces to

$$M_A^2 - 6M_A M_B + M_B^2 = 0,$$

from which, using the quadratic formula, we get the two possibilities $M_A = 5.83\,M_B$ and $M_A = 0.172\,M_B$.

8.81: a) Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y = 0$ at point 2, so $y_1 = 4.00$ m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2$$
$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2$$
$$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35 \text{ m/s}$$

b) In the collision between the package and the cart momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let A be the package and B be the cart.

P_x is constant gives

$$m_a v_{A1x} + m_B v_{B1x} = (m_A + M_B)v_{2x}$$
$$v_{B1x} = -5.00 \text{ m/s}$$

$v_{A1x} = (3.00 \text{ m/s})\cos 37.0^o$ (The horizontal velocity of the package is constant during its free-fall.)

Solving for v_{2x} gives $v_{2x} = -3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

8.82: Even though one of the masses is not known, the analysis of Section (8.4) leading to Eq. (8.26) is still valid, and $v_{\text{red}} = 0.200 \text{ m/s} + 0.050 \text{ m/s} = 0.250 \text{ m/s}$. b) The mass m_{red} may be found from either energy or momentum considerations. From momentum conservation,

$$m_{\text{red}} = \frac{(0.040 \text{ kg})(0.200 \text{ m/s} - 0.050 \text{ m/s})}{(0.250 \text{ m/s})} = 0.024 \text{ kg}.$$

As a check, note that

$$K_1 = \tfrac{1}{2}(0.040 \text{ kg})(0.200 \text{ m/s})^2 = 8.0\times10^{-4} \text{ J, and}$$
$$K_2 = \tfrac{1}{2}(0.040 \text{ kg})(0.050 \text{ m/s})^2 + \tfrac{1}{2}(0.024 \text{ kg})(0.250 \text{ m/s})^2 = 8.0\times10^{-4} \text{ J},$$

so $K_1 = K_2$, as it must for a perfectly elastic collision.

8.83: a) In terms of the primed coordinates,

$$\begin{aligned} v_A^2 &= (\vec{v}'_A + \vec{v}_{\rm cm}) \cdot (\vec{v}'_A + \vec{v}_{\rm cm}) \\ &= \vec{v}'_A \cdot \vec{v}'_A + \vec{v}_{\rm cm} \cdot \vec{v}_{\rm cm} + 2\vec{v}'_A \cdot \vec{v}_{\rm cm} \\ &= {v'_A}^2 + {v_{\rm cm}}^2 + 2\vec{v}'_A \cdot \vec{v}_{\rm cm}, \end{aligned}$$

with a similar expression for v_B^2. The total kinetic energy is then

$$\begin{aligned} K &= \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \\ &= \frac{1}{2}m_A\left({v'_A}^2 + {v_{\rm cm}}^2 + 2\vec{v}'_A \cdot \vec{v}_{\rm cm}\right) + \frac{1}{2}m_B\left({v'_B}^2 + {v_{\rm cm}}^2 + 2\vec{v}'_B \cdot \vec{v}_{\rm cm}\right) \\ &= \frac{1}{2}(m_A + m_B)v_{\rm cm}^2 + \frac{1}{2}\left(m_A {\vec{v}'_A}{}^2 + m_B {\vec{v}'_B}{}^2\right) \\ &\quad + 2\left[m_A \vec{v}'_A . \vec{v}_{\rm cm} + m_B \vec{v}'_B . \vec{v}_{\rm cm}\right]. \end{aligned}$$

The last term in brackets can be expressed as

$$2(m_A \vec{v}'_A + m_B\, \vec{v}'_B) \cdot \vec{v}_{\rm cm},$$

and the term

$$\begin{aligned} m_A \vec{v}'_A + m_B \vec{v}'_B &= m_A \vec{v}'_A + m_B \vec{v}'_B - (m_A + \mathrm{m}_B)\vec{v}_{\rm cm} \\ &= 0, \end{aligned}$$

and so the term in square brackets in the expression for the kinetic energy vanishes, showing the desired result. b) In any collision for which other forces may be neglected the velocity of the center of mass does not change, and the $\frac{1}{2}Mv_{\rm cm}^2$ in the kinetic energy will not change. The other terms can be zero (for a perfectly inelastic collision, which is not likely), but never negative, so the minimum possible kinetic energy is $\frac{1}{2}Mv_{\rm cm}^2$.

8.84: a) The relative speed of approach before the collision is the relative speed at which the balls separate after the collision. Before the collision, they are approaching with relative speed $2v$, and so after the collision they are receding with speed $2v$. In the limit that the larger ball has the much larger mass, its speed after the collision will be unchanged (the limit as $m_{\rm A} >> m_{\rm B}$ in Eq. (8.24)), and so the small ball will move upward with speed $3v$. b) With three times the speed, the ball will rebound to a height time times greater than the initial height.

8.85: a) If the crate had final speed v, J&J have speed $4.00\ \text{m/s} - v$ relative to the ice, and so $(15.0\,\text{kg})v = (120.0\,\text{kg})(4.00\,\text{m/s} - v)$. Solving for v, $v = \frac{(120.0\,\text{kg})\,(4.00\,\text{m/s})}{(135.0\,\text{kg})} = 3.56\,\text{m/s}$.

b) After Jack jumps, the speed of the crate is $\frac{(75.0\,\text{kg})}{(135.0\,\text{kg})}(4.00\,\text{m/s}) = 2.222\,\text{m/s}$, and the momentum of Jill and the crate is $133.3\,\text{kg}\cdot\text{m/s}$. After Jill jumps, the crate has a speed v and Jill has speed $4.00\ \text{m/s} - v$, and so

$133.3\,\text{kg}\cdot\text{m/s} = (15.0\,\text{kg})v - (45.0\,\text{kg})(4.00\,\text{m/s} - v)$, and solving for v gives $v = 5.22\,\text{m/s}$. c) Repeating the calculation for part (b) with Jill jumping first gives a final speed of $4.67\,\text{m/s}$.

8.86: (a) For momentum to be conserved, the two fragments must depart in opposite directions. We can thus write

$$M_A V_A = -M_B V_B$$

Since $M_A = M - M_B$, we have

$$(M - M_B) V_A = -M_B V_B$$

$$\frac{V_A}{V_B} = \frac{-M_B}{M - M_B}$$

Then for the ratio of the kinetic energies

$$\frac{KE_A}{KE_B} = \frac{\frac{1}{2} M_A V_A^2}{\frac{1}{2} M_B V_B^2} = \frac{M_A}{M_B} \frac{M_B^2}{(M - M_B)^2} = \frac{M_B}{M_A}$$

The ratio of the KE's is simply the inverse ratio of the masses.

From the two equations

$$KE_A = \frac{M_B}{M_A} KE_B \text{ and } KE_A + KE_B = Q$$

We can solve for KE_B to find

$$KE_A = Q - KE_B = Q\left(1 - \frac{M_A}{M_A + M_B}\right)$$

$$KE_B = \frac{Q}{1 + \frac{M_B}{M_A}} = \frac{QM_A}{M_A + M_B}$$

(b) If $M_B = 4M_A$, then M_A will get 4 times as much KE as M_B, or 80% of Q for M_A and 20% for M_B.

8.87: Let the proton be moving in the $+x$-direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$-direction after the collision; let its speed be v_e.

P_x is constant gives $0 = -m_e v_e + m_p v_p$.

$v_e = (m_p/m_e)v - p$

The total kinetic energy after decay is $K_{\text{tot}} = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2$. Using the momentum equation to replace v_e gives $K_{\text{tot}} = \frac{1}{2}m_p v_p^2\,(1 + m_p/m_e)$.

$$\text{Thus}\,\frac{K_p}{K_{\text{tot}}} = \frac{1}{1+m_{\text{p}}/m_e} = \frac{1}{1836} = 5.44\times10^{-4} = 0.0544\ \%$$

8.88: The ratios that appear in Eq. (8.42) are $\frac{0.0176}{1.0176}$ and $\frac{1}{1.0176}$, so the kinetic energies are a) $\frac{0.0176}{1.0176}(6.54\times10^{-13}\,\text{J}) = 1.13\times10^{-14}$ J and b) $\frac{1}{1.0176}(6.54\times10^{-13}\,\text{J}) = 6.43\times10^{-13}\,\text{J}$.
Note that the energies do not add to 6.54×10^{-13} J exactly, due to roundoff.

8.89: The "missing momentum" is

$$5.60\times10^{-22}\ \text{kg}\cdot\text{m/s} - (3.50\times10^{-25}\,\text{kg})(1.14\times10^{3}\ \text{m/s}) = 1.61\times10^{-22}\,\text{kg}\cdot\text{m/s}\ .$$

Since the electron has momentum to the right, the neutrino's momentum must be to the left.

8.90: a) For the x - and y - directions, respectively, and m as the common mass of a proton,

$$mv_{A1} = mv_{A2}\cos\alpha + mv_2\cos\beta$$
$$0 = mv_{A2}\sin\alpha - mv_{B2}\sin\beta$$

or

$$v_{A1} = v_{A2}\cos\alpha + v_{B2}\cos\beta$$
$$0 = v_{A2}\sin\alpha - v_{B2}\sin\beta .$$

b) After minor algebra,

$$v_{A1}^2 = v_{A2}^2 + v_{B2}^2 + 2v_{A2}v_{B2}(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$
$$= v_{A2}^2 + v_{B2}^2 + 2v_{A2}v_{B2}\cos(\alpha+\beta).$$

c) For a perfectly elastic collision,

$$\frac{1}{2}mv_{A1}^2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2 \text{ or } v_{A1}^2 = v_{A2}^2 + v_{B2}^2.$$

Substitution into the above result gives $\cos(\alpha+\beta) = 0$. d) The only positive angle with zero cosine is $\frac{\pi}{2}(90°)$.

8.91: See Problem 8.90. Puck B moves at an angle $65.0°$(i.e. $90° - 25° = 65°$) from the original direction of puck A's motion, and from conservation of momentum in the y-direction, $v_{B2} = 0.466v_{A2}$. Substituting this into the expression for conservation of momentum in the x – direction, $v_{A2} = v_{A1}/(\cos 25.0° + 0.466\cos 65°) = 13.6$ m/s, and so $v_{B2} = 6.34$ m/s.

As an alternative, a coordinate system may be used with axes along the final directions of motion (from Problem 8.90, these directions are known to be perpendicular). The initial direction of the puck's motion is $25.0°$ from the final direction, so $v_{A2} = v_{A1}\cos 25.0°$and $v_{B2} = v_{A1}\cos 65.0$, giving the same results.

8.92: Since mass is proportional to weight, the given weights may be used in determining velocities from conservation of momentum. Taking the positive direction to the left,

$$v = \frac{(800\ \text{N})(5.00\ \text{m/s})\cos 30.0^\circ - (600\ \text{N})(7.00\ \text{m/s})\cos 36.9^\circ}{1000\ \text{N}} = 0.105\ \text{m/s}$$

8.93: a) From symmetry, the center of mass is on the vertical axis, a distance $(L/2)\cos(\alpha/2)$ from the apex. b) The center of mass is on the (vertical) axis of symmetry, a distance $2(L/2)/3 = L/3$ from the center of the bottom of the $\sqcup$. c) Using the wire frame as a coordinate system, the coordinates of the center of mass are equal, and each is equal to $(L/2)/2 = L/4$. The distance of this point from the corner is $\left(1/\sqrt{8}\right)L = (0.353)L.$ This may also be found from consideration of the situation of part (a), with $\alpha = 45°$ d) By symmetry, the center of mass is in the center of the equilateral triangle, a distance $(L/2)(\tan 60°) = L/\sqrt{12} = (0.289)L$ above the center of the base.

8.94: The trick here is to notice that the final configuration is the same as if the canoe (assumed symmetrical) has been rotated about its center of mass. Intially, the center of mass is a distance $\frac{(45.0\ \text{kg})(1.5\ \text{m})}{(105\ \text{kg})} = 0.643\ \text{m}$ from the center of the canoe, so in rotating about this point the center of the canoe would move $2 \times 0.643\ \text{m} = 1.29\ \text{m}$.

8.95: Neglecting friction, the total momentum is zero, and your speed will be one-fifth of the slab's speed, or $0.40\ \text{m/s}$.

8.96: The trick here is to realize that the center of mass will continue to move in the original parabolic trajectory, "landing" at the position of the original range of the projectile. Since the explosion takes place at the highest point of the trajectory, and one fragment is given to have zero speed after the explosion, neither fragment has a vertical component of velocity immediately after the explosion, and the second fragment has *twice* the velocity the projectile had before the explosion. a) The fragments land at positions symmetric about the original target point. Since one lands at $\frac{1}{2}R$, the other lands at

$$\frac{3}{2}R = \frac{3}{2}\frac{v_0^2}{g}\sin 2\alpha_0 = \frac{3}{2}\frac{(80\,\mathrm{m/s})^2}{(9.80\,\mathrm{m/s^2})}\sin 120^\circ = 848\,\mathrm{m}.$$

b) In terms of the mass m of the original fragment and the speed v before the explosion, $K_1 = \frac{1}{2}mv^2$ and $K_2 = \frac{1}{2}\frac{m}{2}(2v)^2 = mv^2$, so $\Delta K = mv^2 - \frac{1}{2}mv^2 = \frac{1}{2}mv^2$. The speed v is related to v_0 by $v = v_0\cos\alpha_0$, so

$$\Delta K = \frac{1}{2}mv_0^2\cos^2\alpha_0 = \frac{1}{2}(20.0\text{ kg})(80\text{ m/s})\cos\,60.0^\circ)^2 = 1.60\times 10^4\text{ J}.$$

8.97: Apply conservation of energy to the explosion. Just before the explosion the sheel is at its maximum height and has zero kinetic energy. Let A be the piece with mass 1.40 kg and B be the piece with mass 0.28 kg. Let v_A and v_B be the speeds of the two pieces immediately after the collision.

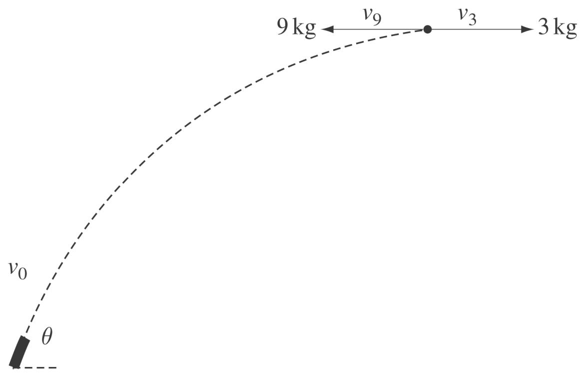
$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = 860\ \text{J}$$

Since the two fragments reach the ground at the same time, their velocitues just after the explosion must be horizontal. The initial momentum of the shell before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_A v_A = m_B v_B$.

Use this to eliminate v_A in the first equation and solve for v_B :
$\tfrac{1}{2}m_B v_B^2(1 + m_B/m_A) = 860$ J and $v_B = 71.6$ m/s.
Then $v_A = (m_B/m_A)v_B = 14.3$ m/s.

b) Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.
$v_{0y} = 0, a_y = +9.80\ \text{m/s}^2, y - y_0 = 80.0\ \text{m}, t = ?$
$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives t = 4.04 s.
During this time the horizontal distance each piece moves is
$x_A = v_A t = 57.8$ m and $x_B = v_B t = 289.1$ m.
They move in opposite directions, so they are $x_A + x_B = 347$ m apart when they land.

8.98: The two fragments are 3.00 kg and 9.00 kg. Time to reach maximum height = time to fall back to the ground.



$$v_9 = v_0 \sin\theta - gt$$
$$0 = (150\ \text{m/s})\sin 55.0^\circ - 9.8\ \text{m/s}^2\ t$$
$$t = 12.5\ \text{s}.$$

The heavier fragment travels back to its starting point, so it reversed its velocity. $v_x = v_0 \cos\theta = (150\ \text{m/s})\cos 55^\circ = 86.0\ \text{m/s}$ to the left after the explosion; this is v_9. No get v_3 using momentum conversation.

$$Mv_0 = m_3 v_3 + m_9 v_9$$
$$(12\ \text{kg})(86.0\ \text{m/s}) = (3.00\ \text{kg})v_3 + (9.00\text{kg})(-86.0\ \text{m/s})$$
$$v_3 = 602\ \text{m/s}$$

$$x_3 = x_{\text{Before explosion}} + x_{\text{After explosion}} = (86.0\ \text{m/s})(12.5\ \text{s}) + (602\ \text{m/s})(12.5)$$

$x_3 = 8600\text{m}$ from where it was launched

Energy released = Energy after explosion – Energy before explosion

$$= \frac{1}{2}\text{m}_3 v_3^2 + \frac{1}{2}\text{m}_9 v_9^2 - \frac{1}{2}(m_3 + m_9)v_9^2$$
$$= \frac{1}{2}(3.00\ \text{kg})(602\ \text{m/s})^2 + \frac{1}{2}(9.00\ \text{kg})(86.0\ \text{m/s})^2$$
$$- \frac{1}{2}(12.0\ \text{kg})(86.0\ \text{m/s})^2$$
$$= 5.33 \times 10^5\ \text{J}$$

8.99: The information is not sufficient to use conservation of energy. Denote the emitted neutron that moves in the $+y$ - direction by the subscript 1 and the emitted neutron that moves in the $-y$-direction by the subscript 2. Using conservation of momentum in the x- and y-directions, neglecting the common factor of the mass of neutron,

$$v_0 = (2v_0/3)\cos 10° + v_1 \cos 45° + v_2 \cos 30°$$
$$0 = (2v_0/3)\sin 10° + v_1 \sin 45° - v_2 \sin 30°.$$

With $\sin 45° = \cos 45°$, these two relations may be subtracted to eliminate v_1, and rearrangement gives

$$v_0(1-(2/3)\cos 10° + (2/3)\sin 10°) = v_2(\cos 30° + \sin 30°),$$

from which $v_2 = 1.01\times10^3\ \mathrm{m/s}$ or $1.0\times10^3\ \mathrm{m/s}$ to two figures. Substitution of this into either of the momentum relations gives $v_1 = 221\ \mathrm{m/s}$. All that is known is that there is no z - component of momentum, and so only the ratio of the speeds can be determined. The ratio is the inverse of the ratio of the masses, so $v_{\mathrm{Kr}} = (1.5)v_{\mathrm{Ba}}$.

8.100: a) With block B initially at rest, $v_{\mathrm{cm}} = \frac{m_A}{m_A+m_B} v_{A1}$. b) Since there is no net external force, the center of mass moves with constant velocity, and so a frame that moves with the center of mass is an inertial reference frame. c) The velocities have only x - components, and the x -components are
$u_{A1} = v_{A1} - v_{\mathrm{cm}} = \frac{m_B}{m_A+m_B} v_{A1}, u_{B1} = -v_{\mathrm{cm}} = -\frac{m_A}{m_A+m_B} u_{A1}$. Then, $P_{\mathrm{cm}} = m_A u_{A1} + m_B u_{B1} = 0$.
d) Since there is zero momentum in the center-of-mass frame before the collision, there can be no momentum after the collision; the momentum of each block after the collision must be reversed in direction. The only way to conserve kinetic energy is if the

momentum of each has the same magnitude so in the center-of-mass frame, the blocks change direction but have the same speeds. Symbolically, $u_{A2} = -u_{A1}, u_{B2} = -u_{B1}$. e) The velocities all have only x –components; these components are
$u_{A1} = \frac{0.200}{0.600} 6.00\ \mathrm{m/s} = 2.00\ \mathrm{m/s}, u_{B1} = -\frac{0.400}{0.600} 6.00\ \mathrm{m/s} = -4.00\ \mathrm{m/s}, u_{A2} = -2.00\ \mathrm{m/s}, u_{B2} = 4.$
and $v_{A2} = +2.00\ \mathrm{m/s}, v_{B2} = 8.00\ \mathrm{m/s}$. and Equation (8.24) predicts $v_{A2} = +\frac{1}{3} v_{A1}$ and Eq. (8.25) predicts $v_{B2} = \frac{4}{3} u_{A1}$, which are in agreement with the above.

8.101: a) If the objects stick together, their relative speed is zero and $\in = 0$. b) From Eq. (8.27), the relative speeds are the same, and $\in = 1$. c) Neglecting air resistance, the speeds before and after the collision are $\sqrt{2gh}$ and $\sqrt{2gH_1}$, and $\in = \frac{\sqrt{2gH_1}}{\sqrt{2gh}} = \sqrt{H_1/h}$. d) From part (c), $H_1 = \in^2 h = (0.85)^2 (1.2\,\text{m}) = 0.87\,\text{m}$. e) $H_{k+1} = H_k \in^2$, and by induction $H_n = \in^{2n} h$. f) $(1.2\,\text{m})(0.85)^{16} = 8.9\,\text{cm}$.

8.102: a) The decrease in potential energy $(-\Delta < 0)$ means that the kinetic energy increases. In the center of mass frame of two hydrogen atoms, the net momentum is necessarily zero and after the atoms combine and have a common velocity, that velocity must have zero magnitude, a situation precluded by the necessarily positive kinetic energy. b) The initial momentum is zero before the collision, and must be zero after the collision. Denote the common initial speed as v_0, the final speed of the hydrogen atom as v, the final speed of the hydrogen molecule as V, the common mass of the hydrogen atoms as m and the mass of the hydrogen molecules as $2m$. After the collision, the two particles must be moving in opposite directions, and so to conserve momentum, $v = 2V$. From conservation of energy,

$$\frac{1}{2}(2m)V^2 - \Delta + \frac{1}{2}mv^2 = 3\tfrac{1}{2}mv_0^2$$
$$mV^2 - \Delta + 2mV^2 = \tfrac{3}{2}mv_0^2$$
$$V^2 = \frac{v_0^2}{2} + \frac{\Delta}{3m},$$

from which $V = 1.203 \times 10^4$ m/s, or 1.20×10^4 m/s to two figures and the hydrogen atom speed is $v = 2.41 \times 10^4$ m/s.

8.103: a) The wagon, after coming down the hill, will have speed $\sqrt{2gL\sin\alpha} = 10$ m/s. After the "collision", the speed is $\left(\frac{300\,\text{kg}}{435\,\text{kg}}\right)(10\,\text{m/s}) = 6.9\,\text{m/s}$, and in the 5.0 s, the wagon will not reach the edge. b) The "collision" is completely inelastic, and kinetic energy is not conserved. The change in kinetic energy is
$\tfrac{1}{2}(435\,\text{kg})(6.9\,\text{m/s})^2 - \tfrac{1}{2}(300\,\text{kg})(10\,\text{m/s})^2 = -4769\,\text{J}$, so about 4800 J is lost.

8.104: a) Including the extra force, Eq. (8.37) becomes

$$m\frac{dv}{dt}=-v_{\text{ex}}\frac{dm}{dt}-mg,$$

where the positive direction is taken upwards (usually a sign of good planning). b) Diving by a factor of the mass m,

$$a=\frac{dv}{dt}=-\frac{v_{\text{ex}}}{m}\frac{dm}{dt}-g.$$

c) $20\ \text{m/s}^2-9.80\ \text{m/s}^2=10.2\ \text{m/s}^2$. d) $3327\ \text{m/s}\ -(9.80\ \text{m/s}^2)(90\ )=2.45\ \text{km/s}$, which is about three-fourths the speed found in Example 8.17.

8.105: a) From Eq. (8.40), $v=v_{\text{ex}}\ln\left(\frac{13{,}000\ \text{kg}}{3{,}300\ \text{kg}}\right)=(1.37)v_{\text{ex}}$.

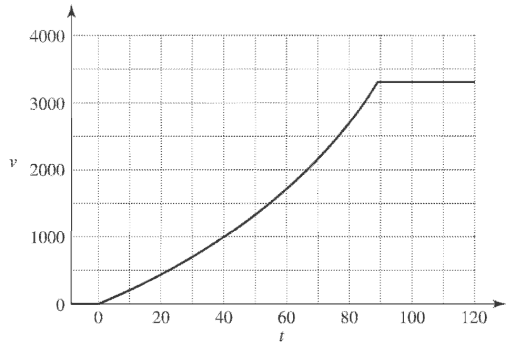
b) $v_{\text{ex}}\ln(13{,}000/4{,}000)=(1.18)v_{\text{ex}}$.

c) $(1.18)v_{\text{ex}}+v_{\text{ex}}\text{In}(1000/300)=(2.38)v_{\text{ex}}$. d) Setting the result of part (c) equal to 7.00 km/s and solving for v_{ex} gives $v_{\text{ex}}=2.94$ km/s.

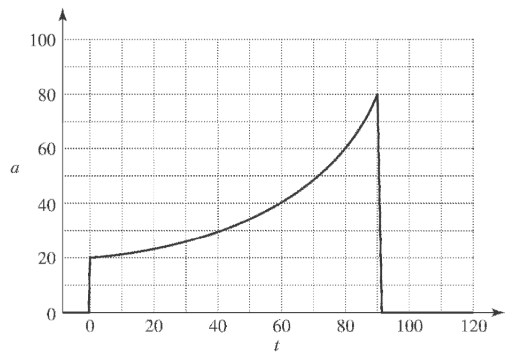
8.106: a) There are two contribution to $F_{\text{net}}, F_{\text{net}}=v_{\text{ex}}\,|\,dm/dt\,|-v\,|\,dm/dt\,|$, or $F_{\text{net}}=(v_{\text{ex}}-v)|dm/dt|$.

b) $F_{\text{net}}/\,|\,dm/dt\,|=(1300\ \text{N})/(150\ \text{kg/s})=8.66\ \text{m/s}=31$ km/h. This equal to $v_{\text{ex}}-v$.

8.107: a) For $t < 0$ the rocket is at rest. For $0 \le t \le 90$ s, Eq. (8.40) is valid, and $v(t) = (2400\text{ m/s})\ln(1/(1-(t/120\text{ s})))$. At $t = 90$ s, this speed is 3.33 km/s, and this is also the speed for $t > 90$ s.



b) The acceleration is zero for $t < 0$ and $t > 90$ s. For $0 \le t \le 90$ s, Eq. (8.39) gives, with $\frac{dm}{dt} = -m_0/120\text{ s}$, $a = \frac{20\text{ m/s}^2}{(1-(t/120\text{ s}))}$.



c) The maximum acceleration occurs at the latest time of firing, $t = 90$ s, at which time the acceleration is, from the result of part (a), $\frac{20\text{ m/s}^2}{(1-90/120)} = 80\text{ m/s}^2$, and so the astronaut is subject to a force of 6.0 kN, about eight times her weight on earth.

8.108: The impulse applied to the cake is $J = \mu_{k1} mgt = mv$, where m is the mass of the cake and v is its speed after the impulse is applied. The distance d that the cake moves during this time is then $d = \frac{1}{2}\mu_{k1} gt^2$. While sliding on the table, the cake must lose its kinetic energy to friction, or $\mu_{k2} mg(r-d) = \frac{1}{2}mv^2$. Simplification and substitution for v gives $r - d = \frac{1}{2} g \frac{\mu_{k1}^2}{\mu_{k2}} t^2$, substituting for d in terms of t^2 gives

$$r = \frac{1}{2}gt^2\left(\mu_{k1} + \frac{\mu_{k1}^2}{\mu_{k2}}\right) = \frac{1}{2}gt^2\frac{\mu_{k1}}{\mu_{k2}}(\mu_{k1} + \mu_{k2}),$$

which gives t = 0.59 s.

8.109: a) Noting than $dm = \frac{M}{L}dx$ avoids the intermediate variable ρ. Then,

$$x_{cm} = \frac{1}{M}\int_0^L x\frac{M}{L}dx = \frac{L}{2}.$$

b) In this case, the mass M may be found in terms of ρ and L, specifically by using $dm = \rho A dx = \alpha A dx$ to find that $M = \alpha A \int x dx = \alpha A L^2/2$. Then,

$$x_{\text{cm}} = \frac{2}{\alpha AL^2}\int_0^L \alpha A x^2 dx = \frac{2}{\alpha AL^2}\frac{L^3}{3} = \frac{2L}{3}.$$

8.110: By symmetry, $x_{cm} = 0$. Using plane polar coordinates leads to an easier integration, and using the Theorem of Pappus $\left(2\pi y_{\text{cm}}\left(\frac{\pi a^2}{2}\right) = \frac{4}{3}\pi a^3\right)$ is easiest of all, but the method of Problem 8.109 involves Cartesian coordinates.

For the x-coordinate, $dm = \rho t\sqrt{a^2 - x^2}dx$, which is an even function of x, so $\int x\,dx = 0$. For the y-coordinate, $dm = \rho t 2\sqrt{a^2 - y^2}dy$, and the range of integration is from 0 to a, so

$$y_{\text{cm}} = \frac{2\rho t}{M}\int_0^a y\sqrt{a^2 - y^2},dy.$$

Making the substitutions $M = \frac{1}{2}\rho\pi a^2 t,\ u = a^2 - y^2,\ du = -2y,$ and

$$y_{\text{cm}} = \frac{-2}{\pi a^2}\int_{a^2}^{o} u^{\frac{1}{2}}du = \frac{-4}{3\pi a^2}\left[u^{\frac{3}{2}}\right]_{a^2}^{0} = \frac{4a}{3\pi}.$$

8.111: a) The tension in the rope at the point where it is suspended from the table is $T = (\lambda x)g$, where x is the length of rope over the edge, hanging vertically. In raising the rope a distance $-dx$, the work done is $(\lambda g)\,x\,(-dx)(dx$ is negative). The total work done is then

$$-\int_{l/4}^{0} (\lambda g)\,x\,dx = (\lambda g)\frac{x^2}{2}\Big|_0^{l/4} = \frac{\lambda g l^2}{32}.$$

b) The center of mass of the hanging piece is initially a distance $l/8$ below the top of the table, and the hanging weight is $(\lambda g)(l/4)$, so the work required to raise the rope is $(\lambda g)(l/4)(l/8) = \lambda g l^2/32$, as before.

8.112: a) For constant acceleration a, the downward velocity is $v = at$ and the distance x that the drop has fallen is $x = \frac{1}{2}at^2$. Substitution into the differential equation gives

$$\frac{1}{2}at^2 g = \frac{1}{2}at^2 a + (at)^2 = \frac{3}{2}a^2t^2,$$

the non-zero solution of which is $a = \frac{g}{3}$.

b) $$\frac{1}{2}at^2 = \frac{1}{2}\left(\frac{9.80\ \text{m/s}^2}{3}\right)(3.00\ \text{s})^2 = 14.7\ \text{m}.$$

c) $kx = (2.00\ \text{g/m})(14.7\ \text{m}) = 29.4\ \text{g}.$

Capítulo 9

9.1: a) $$\frac{1.50\ \text{m}}{2.50\ \text{m}} = 0.60\ \text{rad} = 34.4°.$$

b) $$\frac{(14.0\ \text{cm})}{(128°)(\pi\ \text{rad}/180°)} = 6.27\ \text{cm}.$$

c) $$(1.50\ \text{m})(0.70\ \text{rad}) = 1.05\ \text{m}.$$

9.2: a) $$\left(1900\frac{\text{rev}}{\text{min}}\right)\times\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 199\ \text{rad/s}.$$

b) $$(35° \times \pi\ \text{rad}/180°)/(199\ \text{rad/s}) = 3.07 \times 10^{-3}\ \text{s}.$$

9.3: a) $\alpha_z = \dfrac{d\omega_z}{dt} = (12.0\ \text{rad/s}^3)\,t$, so at $t = 3.5\ \text{s}$, $\alpha = 42\ \text{rad/s}^2$. The angular acceleration is proportional to the time, so the average angular acceleration between any two times is the arithmetic average of the angular accelerations. b) $\omega_z = (6.0\ \text{rad/s}^3)t^2$, so at $t = 3.5\ \text{s}$, $\omega_z = 73.5\ \text{rad/s}$. The angular velocity is not linear function of time, so the average angular velocity is not the arithmetic average or the angular velocity at the midpoint of the interval.

9.4: a) $$\alpha_z(\text{t}) = \tfrac{d\omega_z}{dt} = -2\beta t = (-1.60\ \text{rad/s}^3)t.$$

b) $$\alpha_z(3.0\ \text{s}) = (-1.60\ \text{rad/s}^3)(3.0\ \text{s}) = -4.80\ \text{rad/s}^2.$$

$$\alpha_{\text{av}-z} = \frac{\omega(3.0\ \text{s}) - \omega(0)}{3.0\ \text{s}} = \frac{-2.20\ \text{rad/s} - 5.00\ \text{rad/s}}{3.0\ \text{s}} = -2.40\ \text{rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0\ \text{s}$.

9.5: a) $\omega_z = \gamma + 3\beta t^2 = (0.400\ \text{rad/s}) + (0.036\ \text{rad/s}^3)t^2$ b) At $t = 0$, $\omega_z = \gamma =$ $0.400\ \text{rad/s}$. c) At $t = 5.00\ \text{s}$, $\omega_z = 1.3\ \text{rad/s}$, $\theta = 3.50$ rad, so $\omega_{\text{av}-z} = \frac{3.50\ \text{rad}}{5.00\ \text{s}} = 0.70\ \text{rad/s}$. The acceleration is not constant, but increasing, so the angular velocity is larger than the average angular velocity.

9.6: $\omega_z = (250\ \text{rad/s}) - (40.0\ \text{rad/s}^2)t - (4.50\ \text{rad/s}^3)t^2$, $\alpha_z = -(40.0\ \text{rad/s}^2) - (9.00\ \text{rad/s}^3)t$. a) Setting $\omega_z = 0$ results in a quadratic in t; the only positive time at which $\omega_z = 0$ is $t = 4.23\ \text{s}$. b) At $t = 4.23\ \text{s}$, $\alpha_z = -78.1\ \text{rad/s}^2$.
c) At $t = 4.23\ \text{s}$, $\theta = 586\ \text{rad} = 93.3\ \text{rev}$.
d) At $t = 0$, $\omega_z = 250\ \text{rad/s}$. e) $\omega_{\text{av}-z} = \frac{586\ \text{rad}}{4.23\ \text{s}} = 138\ \text{rad/s}$.

9.7: a) $\omega_z = \frac{d\theta}{dt} = 2bt - 3ct^2$ and $\alpha_z = \frac{dw_z}{dt} = 2b - 6ct$. b) Setting $\alpha_z = 0, t = \frac{b}{3c}$.

9.8: (a) The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{8.00\ \text{rad/s} - (-6.00\ \text{rad/s})}{7.00\ \text{s}} = 2.00\ \text{rad/s}^2$$

Thus it takes 3.00 seconds for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 4.00 s its speed is increasing from $0\ \text{rad/s}$ to $+8.00\ \text{rad/s}$.

(c) We have

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ &= 0 + (-6.00\ \text{rad/s})(7.00\ \text{s}) + \tfrac{1}{2}(2.00\ \text{rad/s}^2)(7.00\ \text{s})^2 \\ &= -42.0\ \text{rad} + 49.0\ \text{rad} = +7.00\ \text{rad}.\end{aligned}$$

Alternatively, the average angular velocity is

$$\frac{-6.00\ \text{rad/s} + 8.00\ \text{rad/s}}{2} = 1.00\ \text{rad/s}$$

Which leads to displacement of 7.00 rad after 7.00 s.

9.9: a) $\omega - \theta_0 = 200\ \text{rev}$, $\omega_0 = 500\ \text{rev/min} = 8.333\ \text{rev/s}$, $t = 30.0\ \text{s}$, $\omega = ?$

$$\theta - \theta_0 = \left(\frac{\omega_0 + \omega}{2}\right)t \text{ gives } \omega = 5.00\ \text{rev/s} = 300\ \text{rpm}$$

b) Use the information in part (a) to find α:

$\omega = \omega_0 + \alpha t$ gives $\alpha = -0.1111\ \text{rev/s}^2$

Then $\omega = 0, \alpha = -0.1111\ \text{rev/s}^2, \omega_0 = 8.333\ \text{rev/s}, t = ?$

$\omega = \omega_0 + \alpha t$ gives $t = 75.0$ and

$$\theta - \theta_0 = \left(\frac{\omega_0 + \omega}{2}\right)t \text{ gives } \theta - \theta_0 = 312\ \text{rev}$$

9.10: a) $\omega_z = \omega_{0z} + \alpha_z t = 1.50 \text{ rad/s} + (0.300 \text{ rad/s}^2)(2.50 \text{ s}) = 2.25 \text{ rad/s}$.

b) $\theta = \omega_{0z} t + 1/2\, \alpha_z t^2 = (1.50 \text{ rad/s})(2.50 \text{ s}) + \frac{1}{2}(0.300 \text{ rad/s}^2)(2.50 \text{ s})^2 = 4.69 \text{ rad}$.

9.11: a) $$\frac{(200 \text{ rev/min} - 500 \text{ rev/min}) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right)}{(4.00 \text{ s})} = -1.25 \frac{\text{rev}}{\text{s}^2}.$$

The number of revolutions is the average angular velocity, 350 rev/min, times the time interval of 0.067 min, or 23.33 rev. b) The angular velocity will decrease by another 200 rev/min in a time $\frac{200 \text{ rev/min}}{60 \text{ s/min}} \cdot \frac{1}{1.25 \text{ rev/s}^2} = 2.67 \text{ s}$.

9.12: a) Solving Eq. (9.7) for t gives $t = \frac{\omega_z - \omega_{0z}}{\alpha_z}$.

Rewriting Eq. (9.11) as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\alpha_z t)$ and substituting for t gives

$$\begin{aligned}
\theta - \theta_0 &= \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \frac{1}{2}(\omega_z - \omega_{0z})\right) \\
&= \frac{1}{\alpha}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) \\
&= \frac{1}{2\alpha}(\omega_z^2 - \omega^2{}_{0z}),
\end{aligned}$$

which when rearranged gives Eq. (9.12).

b) $\alpha_z = (1/2)(1/\Delta\theta)(\omega_z^2 - \omega_{0z}^2) = (1/2)(1/(7.00 \text{ rad}))((16.0 \text{ rad/s})^2 - (12.0 \text{ rad/s})^2) =$ 8 rad/s^2.

9.13: a) From Eq. (9.7), with $\omega_{0z} = 0, t = \frac{\omega_z}{\alpha_z} = \frac{36.0 \text{ rad/s}}{1.50 \text{ rad/s}^2} = 24.0 \text{ s}$.

b) From Eq. (9.12), with $\omega_{0z} = 0,\ \theta - \theta_0 = \frac{(36.0 \text{ rad/s})^2}{2(1.50 \text{ rad})/\text{s}^2} = 432 \text{ rad} = 68.8 \text{ rev}$.

9.14: a) The average angular velocity is $\frac{162 \text{ rad}}{4.00 \text{ s}} = 40.5 \text{ rad/s}$, and so the initial angular velocity is $2\omega_{\text{av}-z} - \omega_{2z} = \omega_{0z}, \omega_{0z} = -27 \text{ rad/s}$.

b) $$\alpha_z = \frac{\Delta\omega_z}{\Delta t} = \frac{108 \text{ rad/s} - (-27 \text{ rad/s})}{4.00 \text{ s}} = 33.8 \text{ rad/s}^2.$$

9.15: From Eq. (9.11),

$$\omega_{0z} = \frac{\theta - \theta_0}{t} - \frac{\alpha_z t}{2} = \frac{60.0\,\text{rad}}{4.00\,\text{s}} - \frac{(2.25\,\text{rad/s}^2)(4.00\,\text{s})}{2} = 10.5\,\text{rad/s}.$$

9.16: From Eq. (9.7), with $\omega_{0z} = 0, \alpha_z = \frac{\omega_z}{t} = \frac{140\,\text{rad/s}}{6.00\,\text{s}} = 23.33\,\text{rad/s}^2$. The angle is most easily found from $\theta = \omega_{\text{av-z}} t = (70\,\text{rad/s})(6.00\,\text{s}) = 420\,\text{rad}$.

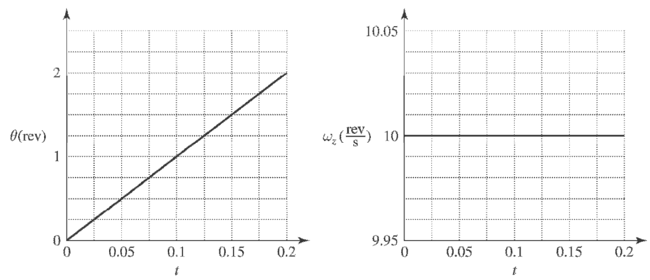
9.17: From Eq. (9.12), with $\omega_z = 0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.0 rev.

9.18: The following table gives the revolutions and the angle θ through which the wheel has rotated for each instant in time and each of the three situations:

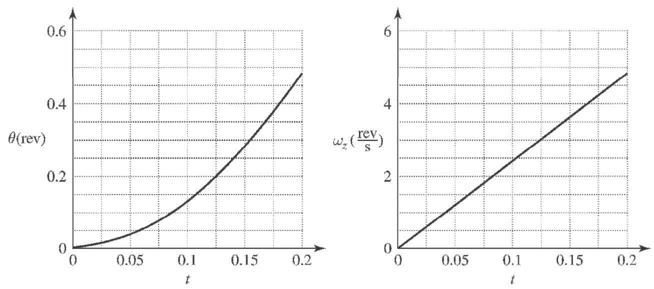
	(a)		(b)		(c)	
t	rev's	θ	rev's	θ	rev's	θ
0.05	0.50	180	0.03	11.3	0.44	158
0.10	1.00	360	0.13	45	0.75	270
0.15	1.50	540	0.28	101	0.94	338
0.20	2.00	720	0.50	180	1.00	360

The θ and ω_z graphs are as follows:

a)



b)



c)

9.19: a) Before the circuit breaker trips, the angle through which the wheel turned was $(24.0\,\text{rad/s})\,(2.00\,\text{s}) + (30.0\,\text{rad/s}^2)\,(2.00\,\text{s})^2/2 = 108\,\text{rad}$, so the total angle is $108\,\text{rad} + 432\,\text{rad} = 540\,\text{rad}$. b) The angular velocity when the circuit breaker trips is $(24.0\,\text{rad/s}) + (30.0\,\text{rad/s}^2)(2.00\,\text{s}) = 84\,\text{rad/s}$, so the average angular velocity while the wheel is slowing is $42.0\,\text{rad/s}$, and the time to slow to a stop is $\frac{432\,\text{rad}}{42.0\,\text{rad/s}} = 10.3\,\text{s}$, so the time when the wheel stops is $12.3\,\text{s}$. c) Of the many ways to find the angular acceleration, the most direct is to use the intermediate calculation of part (b) to find that while slowing down $\Delta\omega_z = -84\,\text{rad/s}$ so $\alpha_z = \frac{-84\,\text{rad/s}}{10.3\,\text{s}} = -8.17\,\text{rad/s}^2$.

9.20: a) Equation (9.7) is solved for $\omega_{0z} = \omega_z - \alpha_z t$, which gives $\omega_{z\text{-ave}} = \omega_z - \frac{\alpha_z}{2}t$, or $\theta - \theta_0 = \omega_z t - \frac{1}{2}\alpha_z t^2$. b) $2\left(\frac{\omega_z}{t} - \frac{\Delta\theta}{t^2}\right) = -0.125\,\text{rad/s}^2$. c) $\omega_z - \alpha_z t = 5.5\,\text{rad/s}$.

9.21: The horizontal component of velocity is $r\omega$, so the magnitude of the velocity is

a) 47.1 m/s

b) $$\sqrt{\left((5.0\,\text{m})(90\,\text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2 + (4.0\,\text{m/s})^2} = 47.3\,\text{m/s}.$$

9.22: a) $\frac{1.25\,\text{m/s}}{25.0\times10^{-3}\,\text{m}} = 50.0\ \text{rad/s}$, $\frac{1.25\,\text{m}}{58.0\times10^{-3}} = 21.55\ \text{rad/s}$, or $21.6\ \text{rad/s}$ to three figures.

b) $(1.25\ \text{m/s})\ (74.0\ \text{min})\ (60\ \text{s/min}) = 5.55\ \text{km}$.

c) $\alpha_z = \frac{50.0\,\text{rad/s} - 21.55\,\text{rad/s}}{(74.0\,\text{min})\,(60\,\text{s/min})} = 6.41\times10^{-3}\ \text{rad/s}^2$.

9.23: a) $\omega^2 r = (6.00\,\text{rad/s})^2\ (0.500\,\text{m}) = 18\ \text{m/s}^2$.

b) $v = \omega r = (6.00\,\text{rad/s})\ (0.500\,\text{m}) = 3.00\,\text{m/s}$, and $\frac{v^2}{r} = \frac{(3.00\,\text{m/s})^2}{(0.500\,\text{m})} = 18\,\text{m/s}^2$.

9.24: From $a_{\text{rad}} = \omega^2 r$,

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{400{,}000 \times 9.80 \text{ m/s}^2}{2.50 \times 10^{-2} \text{ m}}} = 1.25 \times 10^4 \text{ rad/s},$$

which is $(1.25 \times 10^4 \text{ rad/s}) \left(\frac{1 \text{ rev}/2\pi \text{ rad}}{1 \text{ min}/60 \text{ s}}\right) = 1.20 \times 10^5 \text{ rev/min}$.

9.25: a) $a_{\text{rad}} = 0, a_{\tan} = \alpha r = (0.600 \text{ rad/s}^2)(0.300 \text{ m}) = 0.180 \text{ m/s}^2$ and so $a = 0.180 \text{ m/s}^2$.
b) $\theta = \frac{\pi}{3}$ rad, so $a_{\text{rad}} = \omega^2 r = 2(0.600 \text{ rad/s}^2)(\pi/3 \text{ rad})(0.300 \text{ m}) = 0.377 \text{ m/s}^2$.

The tangential acceleration is still 0.180 m/s^2, and so on

$$a = \sqrt{(0.180 \text{ m/s}^2)^2 + (0.377 \text{ m/s}^2)^2} = 0.418 \text{ m/s}^2.$$

c) For an angle of 120°, $a_{\text{rad}} = 0.754 \text{ m/s}^2$, and $a = 0.775 \text{ m/s}^2$, since $a_{\tan}$ is still 0.180 m/s^2

9.26: a) $\omega_z = \omega_{0z} + \alpha_z t = 0.250 \text{ rev/s} + (0.900 \text{ rev/s}^2)(0.200 \text{ s}) = 0.430 \text{ rev/s}$
(note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians). b) $\omega_{\text{av}-z}\Delta t = (0.340 \text{ rev/s})\ (0.2\text{s}) = 0.068 \text{ rev}$. c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750 \text{ m}}{2}\right)(0.430 \text{ rev/s} \times 2\pi \text{ rad/rev}) = 1.01 \text{ m/s}.$$

d) Combining equations (9.14) and (9.15),

$$a = \sqrt{a^2{}_{\text{rad}} + a^2{}_{\tan}} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}$$
$$= [((0.430 \text{ rev/s} \times 2\pi \text{ rad/rev})^2 (0.375 \text{ m}))^2 + ((0.900 \text{ rev/s}^2 \times 2\pi \text{ rad/rev})(0.375 \text{ m}))^2]^{\frac{1}{2}}$$
$$= 3.46 \text{ m/s}^2.$$

9.27:
$$r = \frac{a_{\text{rad}}}{\omega^2} = \frac{(3000)(9.80 \text{ m/s}^2)}{\left((5000 \text{ rev/min})\left(\frac{\pi}{30} \frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2} = 10.7 \text{ cm},$$

so the diameter is more than 12.7 cm, contrary to the claim.

9.28: a) Combining Equations (9.13) and (9.15),

$$a_{\text{rad}} = \omega^2 r = \omega^2 \left(\frac{v}{\omega}\right) = \omega v.$$

b) From the result of part (a), $\omega = \frac{a_{\text{rad}}}{v} = \frac{0.500\ \text{m/s}}{2.00\ \text{m/s}} = 0.250\ \text{rad/s}$.

9.29: a) $\omega r = (1250\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(\frac{12.7\times10^{-3}\ \text{m}}{2}\right) = 0.831\ \text{m/s}$.

b) $\frac{v^2}{r} = \frac{(0.831\ \text{m/s})^2}{(12.7\times10^{-3}\ \text{m})/2} = 109\ \text{m/s}^2$.

9.30: a) $\alpha = \frac{a_{\tan}}{r} = \frac{-10.0\ \text{m/s}^2}{0.200\ \text{m}} = -50.0\ \text{rad/s}^2$ b) At $t = 3.00\ \text{s}$, $v = 50.0\ \text{m/s}$ and $\omega = \frac{v}{r} = \frac{50.0\ \text{m/s}}{0.200} = 250\ \text{rad/s}$ and at $t = 0$, $v = 50.0\ \text{m/s} + (-10.0\ \text{m/s}^2)$ $(0 - 3.00\ \text{s}) = 80.0\ \text{m/s}$, so $\omega = 400\ \text{rad/s}$. c) $\omega_{\text{ave}}t = (325\ \text{rad/s})(3.00\ \text{s})$ $= 975\ \text{rad} = 155\ \text{rev}$. d) $v = \sqrt{a_{\text{rad}}r} = \sqrt{(9.80\ \text{m/s}^2)(0.200\ \text{m})} = 1.40\ \text{m/s}$. This speed will be reached at time $\frac{50.0\ \text{m/s} - 1.40\ \text{m/s}}{10.0\ \text{m/s}} = 4.86\ \text{s}$ after $t = 3.00\ \text{s}$, or at $t = 7.86\ \text{s}$. (There are many equivalent ways to do this calculation.)

9.31: (a) For a given radius and mass, the force is proportional to the square of the angular velocity; $\left(\frac{640\ \text{rev/min}}{423\ \text{rev/min}}\right)^2 = 2.29$ (note that conversion to rad/s is not necessary for this part). b) For a given radius, the tangential speed is proportional to the angular velocity; $\frac{640}{423} = 1.51$ (again conversion of the units of angular speed is not necessary).

c) $(640\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(\frac{0.470\ \text{m}}{2}\right) = 15.75\ \text{m/s}$, or $15.7\ \text{m/s}$ to three figures, and $a_{\text{rad}} = \frac{v^2}{r} = \frac{(15.75\ \text{m/s})^2}{(0.470\ \text{m}/2)} = 1.06\times10^3\ \text{m/s}^2 = 108\ g$.

9.32: (a)

$$v_{\text{T}} = R\omega$$

$$2.00\ \text{cm/s} = R\left(\frac{7.5\ rev}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ rad}{1\ \text{rev}}\right)$$

$$R = 2.55\ \text{cm}$$

$$D = 2R = 5.09\ \text{cm}$$

b)

$$a_{\text{T}} = R\alpha$$

$$\alpha = \frac{a_{\text{T}}}{R} = \frac{0.400\ \text{m/s}^2}{0.0255\ \text{m}} = 15.7\ \text{rad/s}^2$$

9.33: The angular velocity of the rear wheel is $\omega_{\text{r}} = \frac{v_{\text{r}}}{r} = \frac{5.00\ \text{m/s}}{0.330\ \text{m}} = 15.15\ \text{rad/s}.$

The angular velocity of the front wheel is $\omega_{\text{f}} = 0.600\ \text{rev/s} = 3.77\ \text{rad/s}$

Points on the chain all move at the same speed, so $r_{\text{r}}\omega_{\text{r}} = r_{\text{f}}\omega_{\text{f}}$

$r_{\text{r}} = r_{\text{r}}(\omega_{\text{f}}/\omega_{\text{r}}) = 2.99\ \text{cm}$

9.34: The distances of the masses from the axis are $\frac{L}{4}, \frac{L}{4}$ and $\frac{3L}{4}$, and so from Eq. (9.16), the moment of inertia is

$$I = m\left(\frac{L}{4}\right)^2 + m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = \frac{11}{16}mL^2.$$

9.35: The moment of inertia of the cylinder is $M\frac{L^2}{12}$ and that of each cap is $m\frac{L^2}{4}$, so the moment of inertia of the combination is $\left(\frac{M}{12} + \frac{m}{2}\right)L^2$.

9.36: Since the rod is 500 times as long as it is wide, it can be considered slender.

a) From Table $(9.2(\text{a}))$,

$$I = \frac{1}{12}ML^2 = \frac{1}{12}(0.042\ \text{kg})(1.50\ \text{m})^2 = 7.88\times10^{-3}\text{kg}\cdot\text{m}^2.$$

b) From Table $(9.2(\text{b}))$,

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(0.042\ \text{kg})(1.50\ \text{m})^2 = 3.15\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

c) For this slender rod, the moment of inertia about the axis is obtained by considering it as a solid cylinder, and from Table $(9.2(\text{f}))$,

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.042\ \text{kg})\ (1.5\times10^{-3}\ \text{m})^2 = 4.73\times10^{-8}\ \text{kg}\cdot\text{m}^2.$$

9.37: a) For each mass, the square of the distance from the axis is $2(0.200\text{ m})^2 = 8.00\times10^{-2}\text{ m}^2$, and the moment of inertia is $4(0.200\text{ kg})\,(0.800\times10^{-2}\text{ m}^2) = 6.40\times10^{-2}\text{ kg}\cdot\text{m}^2$. b) Each sphere is 0.200 m from the axis, so the moment of inertia is $4(0.200\text{ kg})(0.200\text{ m})^2 = 3.20\times10^{-2}\text{ kg}\cdot\text{m}^2$.

a) The two masses through which the axis passes do not contribute to the moment of inertia. $I = 2(0.2\text{ kg})\left(0.2\sqrt{2}\text{ m}\right)^2 = 0.032\text{ kg}\cdot\text{m}^2$.

9.38: (a) $I = I_{\text{bar}} + I_{\text{balls}} = \dfrac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2$

$$= \frac{1}{12}(4.00\text{ kg})(2.00\text{ m})^2 + 2(0.500\text{ kg})(1.00\text{ m})^2 = 2.33\text{ kg}\cdot\text{m}^2$$

(b) $I = \dfrac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2$

$$= \frac{1}{3}(4.00\text{ kg})(2.00\text{ m})^2 + (0.500\text{ kg})(2.00\text{ m})^2 = 7.33\text{ kg}\cdot\text{m}^2$$

c) $I = 0$ because all masses are on the axis

(d) $I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2$
$$= (5.00\text{ kg})(0.500\text{ m})^2 = 1.25\text{ kg}\cdot\text{m}^2$$

9.39: $I = I_{\text{d}} + I_{\text{r}}$ (d = disk, r = ring)

disk: $m_{\text{d}} = (3.00\text{ g/cm}^3)\pi r_{\text{d}}^2 = 23.56\text{ kg}$

$$I_{\text{d}} = \frac{1}{2}m_{\text{d}}r_{\text{d}}^2 = 2.945\text{ kg}\cdot\text{m}^2$$

ring: $m_{\text{r}} = (2.00\text{ g/cm}^3)\,\pi\,(r_2^2 - r_1^2) = 15.08\text{ kg}$ $(r_1 = 50.0\text{ cm}, r_2 = 70.0\text{ cm})$

$$I_{\text{r}} = \frac{1}{2}m_{\text{r}}(r_1^2 + r_2^2) = 5.580\text{ kg}\cdot\text{m}^2$$

$$I = I_{\text{d}} + I_{\text{r}} = 8.52\text{ kg}\cdot\text{m}^2$$

9.40: a) In the expression of Eq. (9.16), each term will have the mass multiplied by f^3 and the distance multiplied by f, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$. b) $(2.5)(48)^5 = 6.37 \times 10^8$.

9.41: Each of the eight spokes may be treated as a slender rod about an axis through an end, so the moment of inertia of the combination is

$$\begin{aligned} I &= m_{\text{rim}} R^2 + 8\left(\frac{m_{\text{spoke}}}{3}\right)R^2 \\ &= \left[(1.40\ \text{kg}) + \frac{8}{3}(0.20\ \text{kg})\right](0.300\ \text{m})^2 \\ &= 0.193\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

9.42: a) From Eq. (9.17), with *I* from Table (9.2(a)),

$$K = \frac{1}{2}\frac{1}{12}mL^2\omega^2 = \frac{1}{24}(117\ \text{kg})(2.08\ \text{m})^2(2400\frac{\text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad/rev}}{60\ \text{s/min}})^2 = 1.3 \times 10^6\ \text{J}.$$

b) From $mgy = K$,

$$y = \frac{K}{mg} = \frac{(1.3 \times 10^6\ \text{J})}{(117\ \text{kg})(9.80\ \text{m/s}^2)} = 1.16 \times 10^3\ \ \text{m} = 1.16\ \text{km}.$$

9.43: a) The units of moment of inertia are $[\text{kg}][\text{m}^2]$ and the units of ω are equivalent to $[\text{s}^{-1}]$ and so the product $\frac{1}{2}I\omega^2$ has units equivalent to $[\text{kg}\cdot\text{m}\cdot\text{s}^{-2}] = [\text{kg}\cdot(\text{m/s})^2]$, which are the units of Joules. A radian is a ratio of distances and is therefore unitless.

b) $K = \pi^2 I\omega^2/1800$, when is in rev/min.

9.44: Solving Eq. (9.17) for *I*,

$$I = \frac{2K}{\omega^2} = \frac{2(0.025\text{J})}{(45\ \text{rev/min} \times \frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}})^2} = 2.25 \times 10^{-3}\ \ \text{kg}\cdot\text{m}^2.$$

9.45: From Eq. (9.17), $K_2 - K = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$, and solving for *I*,

$$\begin{aligned} I &= 2\frac{(K_2 - K_1)}{(\omega_2^2 - \omega_1^2)} \\ &= 2\frac{(-500\ \text{J})}{((520\ \text{rev/min})^2 - (650\ \text{rev/min})^2)\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)^2} \\ &= 0.600\ \text{kg}\cdot\text{m}^2. \end{aligned}$$

9.46: The work done on the cylinder is *PL*, where *L* is the length of the rope. Combining Equations (9.17), (9.13) and the expression for *I* from Table (9.2(g)),

$$PL = \frac{1}{2}\frac{\omega}{g}v^2, \text{ or } P = \frac{1}{2}\frac{\omega}{g}\frac{v^2}{L} = \frac{(40.0\ \text{N})(6.00\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 14.7\ \text{N}.$$

9.47: Expressing ω in terms of $\alpha_{\text{rad}}, \omega^2 = \frac{a_{\text{rad}}}{R}$. Combining with $I = \frac{1}{2}MR^2$, Eq. (9.17) becomes $K = \frac{1}{2}\frac{1}{2}MRa_{\text{rad}} = \frac{(70.0\ \text{kg})(1.20\ \text{m})(3500\ \text{m/s})^2}{4} = 7.35\times10^4$ J.

9.48: a) With $I = MR^2$, with expression for *v* is

$$v = \sqrt{\frac{2gh}{1 + M/m}}.$$

b) This expression is smaller than that for the solid cylinder; more of the cylinder's mass is concentrated at its edge, so for a given speed, the kinetic energy of the cylinder is larger. A larger fraction of the potential energy is converted to the kinetic energy of the cylinder, and so less is available for the falling mass.

9.49: a) $\omega = \frac{2\pi}{T}$, so Eq. (9.17) becomes $K = 2\pi^2 I/T^2$.

b) Differentiating the expression found in part (a) with respect to T,

$$\frac{dK}{dt} = (-4\pi^2 I/T^3)\tfrac{dT}{dt}.$$

c) $2\pi^2 (8.0\ \text{kg}\cdot\text{m}^2)/(1.5\ \text{s})^2 = 70.2$ J, or 70 to two figures.

d) $(-4\pi^2 (8.0\ \text{kg}\cdot\text{m}^2)/(1.5\ \text{s})^3)(0.0060) = -0.56$ W.

9.50: The center of mass has fallen half of the length of the rope, so the change in gravitational potential energy is

$$-\frac{1}{2}mgL = -\frac{1}{2}(3.00\ \text{kg})(9.80\ \text{m/s}^2)(10.0\ \text{m}) = -147\ \text{J}.$$

9.51: $(120\ \text{kg})(9.80\ \text{m/s}^2)(0.700\ \text{m}) = 823\ \text{J}.$

9.52: In Eq; (9.19), $I_{\text{cm}} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

9.53: $\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$, so $d^2 = \frac{4}{15}R^2$, and the axis comes nearest to the center of the sphere at a distance $d = (2/\sqrt{15})R = (0.516)R$.

9.54: Using the parallel-axis theorem to find the moment of inertia of a thin rod about an axis through its end and perpendicular to the rod,

$$I_p = I_{\text{cm}} + Md^2 = \frac{M}{12}L^2 + M\left(\frac{L}{2}\right)^2 = \frac{M}{3}L^2.$$

9.55: $Ip = I_{\text{cm}} + md^2$, so $I = \frac{1}{12}M\left(a^2 + b^2\right) + M\left((\frac{a}{2})^2 + (\frac{b}{2})^2\right)$, which gives

$$I = \frac{1}{12}M\left(a^2 + b^2\right) + \frac{1}{4}\text{M}\left(a^2 + b^2\right), \text{ or } I = \frac{1}{3}M\left(a^2 + b^2\right)$$

9.56: a) $I = \frac{1}{12}Ma^2$ b) $I = \frac{1}{12}Mb^2$

9.57: In Eq. (9.19), $I_{cm} = \frac{M}{12}L^2$ and $d = (L/2 - h)$, so

$$Ip = M\left[\frac{1}{12}L^2 + \left(\frac{L}{2} - h\right)^2\right]$$

$$= M\left[\frac{1}{12}L^2 + \frac{1}{4}L^2 - Lh + h^2\right]$$

$$= M\left[\frac{1}{3}L^2 - Lh + h^2\right],$$

which is the same as found in Example 9.12.

9.58: The analysis is identical to that of Example 9.13, with the lower limit in the integral being zero and the upper limit being R, and the mass $M = \pi L\rho R^2$. The result is $I = \frac{1}{2}MR^2$, as given in Table $(9.2(\text{f}))$.

9.59: With $dm = \frac{M}{L}dx$

$$I = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{M}{3}L^2.$$

9.60: For this case, $dm = \gamma\, dx$.

a) $M = \int dm = \int_0^L \gamma x\, dx = \gamma \frac{x^2}{2}\Big|_0^L = \frac{\gamma L^2}{2}$

b) $I = \int_0^L x^2(\gamma x)dx = \gamma \frac{x^4}{4}\Big|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2}L^2.$

This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater further away from the axis than nearer the axis.

c) $I = \int_0^L (L-x)^2 \gamma x dx$

$$= \gamma \int_0^L (L^2 x - 2Lx^2 + x^3)dx$$

$$= \gamma \left(L^2 \frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4} \right)\Bigg|_0^L$$

$$= \gamma \frac{L^4}{12}$$

$$= \frac{M}{6}L^2.$$

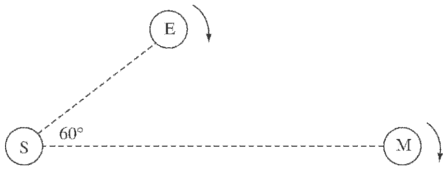
This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

9.61: a) For a clockwise rotation, $\vec{\omega}$ will be out of the page. b) The upward direction crossed into the radial direction is, by the right-hand rule, counterclockwise. $\vec{\omega}$ and $\vec{r}$ are perpendicular, so the magnitude of $\vec{\omega} \times \vec{r}$ is $\omega r = v$. c) Geometrically, $\vec{\omega}$ is perpendicular to $\vec{v}$, and so $\vec{\omega} \times \vec{v}$ has magnitude $\omega v = a_{\text{rad}}$, and from the right-hand rule, the upward direction crossed into the counterclockwise direction is inward, the direction of $\vec{a}_{\text{rad}}$. Algebraically,

$$\vec{a}_{\text{rad}} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$= \vec{\omega}(\vec{\omega} \cdot \vec{r}) - \vec{r}(\vec{\omega} \cdot \vec{\omega})$$
$$= -\omega^2 \vec{r},$$

where the fact that $\vec{\omega}$ and $\vec{r}$ are perpendicular has been used to eliminate their dot product.

9.62:



For planetary alignment, earth must go through $60°$ more than Mars:

$$\theta_{\mathrm{E}} = \theta_{\mathrm{M}} + 60°$$
$$w_{\mathrm{E}} t = \omega_{\mathrm{M}} t + 60°$$
$$t = \frac{60°}{\omega_{\mathrm{E}} - \omega_{\mathrm{M}}}$$
$$w_{\mathrm{E}} = \frac{360°}{1\mathrm{yr}} \text{ and } w_{\mathrm{M}} = \frac{360°}{1.9\mathrm{yr}}$$
$$t = \frac{60°}{\frac{360°}{1\mathrm{yr}} - \frac{360°}{1.9\mathrm{yr}}} = 0.352 yr\left(\frac{365\mathrm{d}}{1\mathrm{yr}}\right) = 128\,\mathrm{d}$$

9.63: a) $v = 60\,\mathrm{mph} = 26.82\ \mathrm{m/s}$

$r = 12\,\mathrm{in.} = 0.3048\,\mathrm{m}$

$\omega = \frac{v}{r} = 88.0\,\mathrm{rad/s} = 14.0\ \mathrm{rev/s} = 840\,\mathrm{rpm}$

b) same ω as in part (a) since speedometer reads same

$r = 15\,\mathrm{in.} = 0.381\,\mathrm{m}$

$v = r\omega = (0.381\,\mathrm{m})(88.0\,\mathrm{rad/s}) = 33.5\ \mathrm{m/s} = 75\ \mathrm{mph}$

c) $v = 50\,\mathrm{mph} = 22.35\,\mathrm{m/s}$

$r = 10\,\mathrm{in.} = 0.254\,\mathrm{m}$

$\omega = \frac{v}{r} = 88.0\ \mathrm{rad/s}$;. this is the same as for 60 mph with correct tires, so speedometer read 60 mph.

9.64: a) For constant angular acceleration $\theta = \frac{\omega^2}{2\alpha}$, and so $a_{\mathrm{rad}} = \omega^2 r = 2\alpha\theta r.$

b) Denoting the angle that the acceleration vector makes with the radial direction as β, and using Equations (9.14) and (9.15),

$$\tan\beta = \frac{a_{\tan}}{a_{\mathrm{rad}}} = \frac{\alpha\,\mathrm{r}}{\omega^2 r} = \frac{\alpha\,\mathrm{r}}{2\,\alpha\theta\,r} = \frac{1}{2\theta},$$

so $\theta = \frac{1}{2\tan\beta} = \frac{1}{2\tan 36.9°} = 0.666\,\mathrm{rad}.$

9.65: a) $\omega_z = \dfrac{d\theta}{dt} = 2\gamma t - 3\beta t^2 = (6.40\ \text{rad/s}^2)t - (1.50\ \text{rad/s}^3)t^2.$

b) $\alpha_z = \dfrac{dw_z}{dt} = 2\gamma - 6\beta\, t = (6.40\ \text{rad/s}^2) - (3.00\ \text{rad/s}^3)t.$

c) An extreme of angular velocity occurs when $\alpha_z = 0$, which occurs at $t = \frac{\gamma}{3\beta} = \frac{3.20\ \text{rad/s}^2}{1.50\ \text{rad/s}^3} = 2.13$ s, and at this time

$$\omega_z = (2\gamma)(\gamma/3\beta) - (3\beta)(\gamma/3\beta)^2 = \gamma^2/3\beta = \frac{(3.20\ \text{rad/s}^2)^2}{3(0.500\ \text{rad/s}^3)} = 6.83\text{rad/s}.$$

9.66: a) By successively integrating Equations (9.5) and (9.3),

$$\omega_z = \gamma t - \frac{\beta}{2}t^2 = (1.80\ \text{rad/s}^2)t - (0.125\ \text{rad/s}^3)t^2,$$

$$\theta = \frac{\gamma}{2}t^2 - \frac{\beta}{6}t^3 = (0.90\ \text{rad/s}^2)t^2 - (0.042\ \text{rad/s}^3)t^3.$$

b) The maximum positive angular velocity occurs when $\alpha_z = 0$, $t = \dfrac{\gamma}{\beta}$, the angular velocity at this time is

$$\omega_z = \gamma\left(\frac{\gamma}{\beta}\right) - \frac{\beta}{2}\left(\frac{\gamma}{\beta}\right)^2 = \frac{1}{2}\frac{\gamma^2}{\beta} = \frac{1}{2}\frac{(1.80\ \text{rad/s}^2)^2}{(0.25\ \text{rad/s}^3)} = 6.48\ \text{rad/s}.$$

The maximum angular displacement occurs when $\omega_z = 0$, at time $t = \dfrac{2\gamma}{\beta}$ ($t = 0$ is an inflection point, and $\theta(0)$ is not a maximum) and the angular displacement at this time is

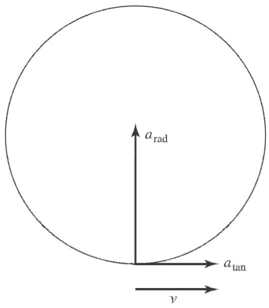
$$\theta = \frac{\gamma}{2}\left(\frac{2\gamma}{\beta}\right)^2 - \frac{\beta}{6}\left(\frac{2\gamma}{\beta}\right)^3 = \frac{2}{3}\frac{\gamma^3}{\beta^2} = \frac{2}{3}\frac{(1.80\ \text{rad/s}^2)^3}{(0.25\ \text{rad/s}^3)^2} = 62.2\ \text{rad}.$$

9.67: a) The scale factor is 20.0, so the actual speed of the car would be $35\ \text{km/h} = 9.72$ m
b) $(1/2)mv^2 = 8.51\ \text{J}$. c) $\omega = \sqrt{\frac{2K}{I}} = 652\ \text{rad/s}.$

9.68: a) $\alpha = \frac{a_{\tan}}{r} = \frac{3.00\,\text{m/s}^2}{60.0\,\text{m}} = 0.050\,\text{rad/s}^2$. b) $\alpha t = (0.05\,\text{rad/s}^2)(6.00s) = 0.300\,\text{rad/s}$.

c) $a_{\text{rad}} = \omega^2 r = (0.300\,\text{rad/s})^2\,(60.0\,\text{m}) = 5.40\,\text{m/s}^2$.

d)



e) $a = \sqrt{a^2{}_{\text{rad}} + a^2{}_{\tan}} = \sqrt{(5.40\,\text{m/s}^2)^2 + (3.00\,\text{m/s}^2)^2} = 6.18\,\text{m/s}^2$,
and the magnitude of the force is $F = ma = (1240\,\text{kg})(6.18\,\text{m/s}^2) = 7.66\,\text{kN}$.

f) $\arctan\left(\frac{a_{\text{rad}}}{a_{\tan}}\right) = \arctan\left(\frac{5.40}{3.00}\right) = 60.9°$.

9.69: a) Expressing angular frequencies in units of revolutions per minute may be accomodated by changing the units of the dynamic quantities; specifically,

$$\begin{aligned}\omega_2 &= \sqrt{\omega_1^2 + \frac{2W}{I}} \\ &= \sqrt{(300\,\text{rev/min})^2 + \left(\frac{2(-4000\,\text{J})}{16.0\,\text{kg}\cdot\text{m}^2}\right)\Big/\left(\frac{\pi}{30}\,\frac{\text{rad/s}}{\text{rev/min}}\right)^2} \\ &= 211\,\text{rev/min}.\end{aligned}$$

b) At the initial speed, the 4000 J will be recovered; if this is to be done is 5.00 s, the power must be $\frac{4000\,\text{J}}{5.00\,\text{s}} = 800\,\text{W}$.

9.70: a) The angular acceleration will be zero when the speed is a maximum, which is at the bottom of the circle. The speed, from energy considerations, is $v = \sqrt{2gh} = \sqrt{2gR(1-\cos\beta)}$, where β is the angle from the vertical at release, and

$$\omega = \frac{v}{R} = \sqrt{\frac{2g}{R}(1-\cos\beta)} = \sqrt{\frac{2(9.80\ \mathrm{m/s^2})}{(2.50\ \mathrm{m})}(1-\cos 36.9°)} = 1.25\ \mathrm{rad/s}.$$

b) α will again be 0 when the meatball again passes through the lowest point.

c) a_{rad} is directed toward the center, and $a_{\mathrm{rad}} = \omega^2 R$, $a_{\mathrm{rad}} = (1.25\ \mathrm{rad/s^2})(2.50\ \mathrm{m}) = 3.93$

d) $a_{\mathrm{rad}} = \omega^2 R = (2g/R)(1-\cos\beta)R = (2g)(1-\cos\beta)$, independent of R.

9.71: a) $(60.0\ \mathrm{rev/s})(2\pi\ \mathrm{rad/rev})(0.45\times10^{-2}\ \mathrm{m}) = 1.696\ \mathrm{m/s}$.

b) $\omega = \frac{v}{r} = \frac{1.696\ \mathrm{m/s}}{2.00\times10^{-2}\ \mathrm{m}} = 84.8\ \mathrm{rad/s}$.

9.72: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade.

a) $$(2(3450\ \mathrm{rev/min}))\left(\frac{\pi}{30}\frac{\mathrm{rad/s}}{\mathrm{rev/min}}\right)\left(\frac{0.208\ \mathrm{m}}{2}\right) = 75.1\ \mathrm{m/s}.$$

b) $$a_{\mathrm{rad}} = \omega^2 r = \left(2(3450\ \mathrm{rev/min})\left(\frac{\pi}{30}\frac{\mathrm{rad/s}}{\mathrm{rev/min}}\right)\right)^2\left(\frac{0.208\ \mathrm{m}}{2}\right) = 5.43\times10^4\ \mathrm{m/s^2},$$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

9.73: a)

$$\Delta a_{\text{rad}} = \omega^2 r - \omega_0^2 r = (\omega^2 - \omega_0^2) r$$
$$= [\omega - \omega_0][\omega + \omega_0] r$$
$$= \left[\frac{\omega - \omega_0}{t}\right][(\omega + \omega_0) t] r$$
$$= [\alpha][2(\theta - \theta_0) r.$$

b) From the above,

$$\alpha r = \frac{\Delta a_{\text{rad}}}{2\Delta\theta} = \frac{(85.0\ \text{m/s}^2 - 25.0\ \text{m/s}^2)}{2(15.0\ \text{rad})} = 2.00\ \text{m/s}^2.$$

c) Similar to the derivation of part (a),

$$\Delta K = \frac{1}{2}\omega^2 I - \frac{1}{2}\omega_0{}^2 I = \frac{1}{2}[\alpha][2\Delta\theta] I = I\alpha\Delta\theta.$$

d) Using the result of part (c),

$$I = \frac{\Delta K}{\alpha\Delta\theta} = \frac{(45.0\ \text{J} - 20.0\ \text{J})}{((2.00\ \text{m/s}^2)/(0.250\ \text{m}))(15.0\ \text{rad})} = 0.208\ \text{kg}\cdot\text{m}^2.$$

9.74: $I = I_{\text{wood}} + I_{\text{lead}}$

$$= \frac{2}{5} m_{\text{w}} R^2 + \frac{2}{3} m_{\text{L}} R^2$$
$$m_{\text{w}} = \rho_{\text{w}} V_{\text{w}} = \rho_{\text{w}} \frac{4}{3}\pi R^3$$
$$m_{\text{L}} = \sigma_{\text{L}} A_{\text{L}} = \sigma_{\text{L}} 4\pi R^2$$
$$I = \frac{2}{5}\left(\rho_{\text{w}} \frac{4}{3}\pi R^3\right) R^2 + \frac{2}{3}(\sigma_{\text{L}} 4\pi R^2) R^2$$
$$= \frac{8}{3}\pi R^4 \left(\frac{\rho_{\text{w}} R}{5} + \sigma_{\text{L}}\right)$$
$$= \frac{8\pi}{3}(0.20\ \text{m})^4 \left[\frac{(800\ \text{kg/m}^3)(0.20\ \text{m})}{5} + 20\ \text{kg/m}^2\right]$$
$$= 0.70\ \text{kgm}^2$$

9.75: I approximate my body as a vertical cylinder with mass 80 kg, length 1.7 m, and diameter 0.30 m (radius 0.15 m)

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(80\ \text{kg})\,(0.15\ \text{m})^2 = 0.9\ \text{kg}\cdot\text{m}^2$$

9.76: Treat the V like two thin 0.160 kg bars, each 25 cm long.

$$\text{I} = 2\left(\frac{1}{3}mL^2\right) = 2\left(\frac{1}{3}\right)(0.160\ \text{kg})(0.250\ \text{m})^2$$
$$= 6.67\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

9.77: a) $\omega = 90.0\ \text{rpm} = 9.425\ \text{rad/s}$

$$K = \frac{1}{2}I\omega^2 \ \text{ so } \ I = \frac{2K}{\omega^2} = \frac{2(10.0\times10^6\ \text{J})}{(9.425\ \text{rad/s})^2} = 2.252\times10^5\ \text{kg}\cdot\text{m}^2$$

$m = \rho V = \rho\pi R^2 t(\rho = 7800\ \text{kg/m}^3$ is the density of iron and t=0.100 m is the thickness of the flywheel)

$$I = \frac{1}{2}mR^2 = \frac{1}{2}\rho\pi t R^4$$

$$R = (2I/\rho\pi t)^{1/4} = 3.68\ \text{m; diameter} = 7.36\ \text{m}$$

b) $a_c = R\omega^2 = 327\ \text{m/s}^2$

9.78: Quantitatively, from Table (9.2), $I_A = \frac{1}{2}mR^2, I_B = mR^2$ and $I_C = \frac{2}{3}mR^2$ · a) Object A has the smallest moment of inertia because, of the three objects, its mass is the most concentrated near its axis. b) Conversely, object B's mass is concentrated and farthest from its axis. c) Because $I_{\text{sphere}} = 2/5\,mR^2$, the sphere would replace the disk as having the smallest moment of inertia.

9.79: a) See Exercise 9.50.

$$K = \frac{2\pi^2 I}{T^2} = \frac{2\pi^2(0.3308)(5.97\times10^{24}\ \text{kg})(6.38\times10^{6}\,\text{m})^2}{(86{,}164\ \text{s})^2} = 2.14\times10^{29}\ \text{J}.$$

b) $$\frac{1}{2}M\left(\frac{2\pi R}{T}\right)^2 = \frac{2\pi^2(5.97\times10^{24}\ \text{kg})(1.50\times10^{11}\ \text{m})^2}{(3.156\times10^{7}\ \text{s})^2} = 2.66\times10^{33}\ \text{J}.$$

c) Since the Earth's moment on inertia is less than that of a uniform sphere, more of the Earth's mass must be concentrated near its center.

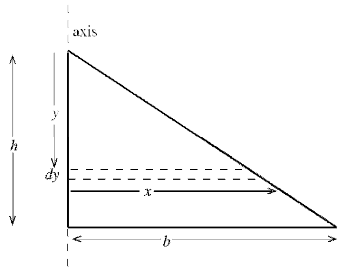
9.80: Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR. The kinetic energy is

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\ \omega^2 \cdot$$

Using $I = \frac{1}{2}mR^2$ and solving for ω,

$$\omega^2 = \frac{4}{3}\frac{g}{R}, \quad \text{and} \quad \omega = \sqrt{\frac{4}{3}\frac{g}{R}}.$$

9.81: a)



Consider a small strip of width dy and a distance y below the top of the triangle. The length of the strip is $x = (y/h)b.$

The strip has area $x\ dy$ and the area of the sign is $\frac{1}{2}bh,$ so the mass of the strip is

$$dm = M\left(\frac{xdy}{\frac{1}{2}bh}\right) = M\left(\frac{yb}{h}\right)\left(\frac{2\,dy}{bh}\right) = \left(\frac{2M}{h^2}\right)y\ dy$$

$$dI = \tfrac{1}{3}(dm)x^2 = \frac{2Mb^2}{3h^4}y^3\ dy$$

$$I = \int_0^h dI = \frac{2Mb^2}{3h^4}\int_0^h y^3 dy = \frac{2Mb^2}{3h^4}\left(\frac{1}{4}y^4\ |_0^h\right) = \frac{1}{6}Mb^2$$

b) $I = \frac{1}{6}Mb^2 = 2.304\ \mathrm{kg \cdot m^2}$

$\omega = 2.00\ \mathrm{rev/s} = 4.00\pi\ \mathrm{rad/s}$

$K = \frac{1}{2}I\omega^2 = 182\ \mathrm{J}$

9.82: (a) The kinetic energy of the falling mass after 2.00 m is $KE = \frac{1}{2}mv^2 = \frac{1}{2}(8.00\ \text{kg})(5.00\ \text{m/s})^2 = 100\ \text{J}$. The change in its potential energy while falling is $mgh = (8.00\ \text{kg})(9.8\ \text{m/s}^2)(2.00\ \text{m}) = 156.8\ \text{J}$

The wheel must have the "missing" 56.8 J in the form of rotational KE. Since its outer rim is moving at the same speed as the falling mass, $5.00\ \text{m/s}$:

$$v = r\omega$$

$$\omega = \frac{v}{r} = \frac{5.00\text{m/s}}{0.370\text{m}} = 13.51\ \text{rad/s}$$

$KE = \dfrac{1}{2}I\omega^2$; therefore

$$I = \frac{2KE}{\omega^2} = \frac{2(56.8\ \text{J})}{(13.51\ \text{rad/s})^2} = 0.6224\ \text{kg}\cdot\text{m}^2 \text{ or } 0.622\ \text{kg}\cdot\text{m}^2$$

(b) The wheel's mass is $280\ \text{N}/9.8\ \text{m/s}^2 = 28.6$ kg. The wheel with the largest possible moment of inertia would have all this mass concentrated in its rim. Its moment of inertia would be

$$I = MR^2 = (28.6\text{kg})(0.370\text{m})^2 = 3.92\ \text{kg}\cdot\text{m}^2$$

The boss's wheel is physically impossible.

9.83: a) $(0.160\ \text{kg})(-0.500\ \text{m})(9.80\ \text{m/s}^2) = -0.784\ \text{J}$. b) The kinetic energy of the stick is 0.784 J, and so the angular velocity is

$$\omega = \sqrt{\frac{2k}{I}} = \sqrt{\frac{2k}{ML^2/3}} = \sqrt{\frac{2(0.784\ \text{J})}{(0.160\ \text{kg})(1.00\ \text{m})^2/3}} = 5.42\ \text{rad/s}.$$

This result may also be found by using the algebraic form for the kinetic energy, $K = MgL/2$, from which $\omega = \sqrt{3g/L}$, giving the same result. Note that ω is independent of the mass.

c) $v = \omega L = (5.42\ \text{rad/s})(1.00\ \text{m}) = 5.42\ \text{m/s}$

d) $\sqrt{2gL} = 4.43\ \text{m/s}$; This is $\sqrt{2/3}$ of the result of part (c).

9.84: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle). When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω, the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). From conservation of energy, using $I = (1/2)MR^2$ for a uniform cylinder,

$$\left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega_0^2 = \left(\frac{M}{4} + \frac{m}{2}\right) R^2 \omega^2 - mg\pi R.$$

Solving for ω gives

$$\omega = \sqrt{\omega_0^2 + \frac{(4\pi mg/R)}{(M + 2m)}},$$

and the speed of any part of the rope is $v = \omega R$.

9.85: In descending a distance d, gravity has done work $m_B gd$ and friction has done work $-\mu_K m_A gd$, and so the total kinetic energy of the system is $gd(m_B - \mu_K m_A)$. In terms of the speed v of the blocks, the kinetic energy is

$$K = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(m_A + m_B + I/R^2)v^2,$$

where $\omega = v/R$, and condition that the rope not slip, have been used. Setting the kinetic energy equal to the work done and solving for the speed v,

$$v = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{(m_A + m_B + I/R^2)}}.$$

9.86: The gravitational potential energy which has become kinetic energy is $K = (4.00\ \text{kg} - 2.00\ \text{kg})(9.80\ \text{m/s}^2)(5.00\ \text{m}) = 98.0\ \text{J}$. In terms of the common speed v of the blocks, the kinetic energy of the system is

$$K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$= v^2\frac{1}{2}\left(4.00\,\text{kg} + 2.00\,\text{kg} + \frac{(0.480\,\text{kg}\cdot\text{m}^2)}{(0.160\,\text{m})^2}\right) = v^2(12.4\,\text{kg}).$$

Solving for v gives $v = \sqrt{\frac{98.0\,\text{J}}{12.4\,\text{kg}}} = 2.81\,\text{m/s}.$

9.87: The moment of inertia of the hoop about the nail is $2MR^2$ (see Exercise 9.52), and the initial potential energy with respect to the center of the loop when its center is directly below the nail is $gR\ (1-\cos\beta)$. From the work-energy theorem,

$$K = \frac{1}{2}I\omega^2 = M\omega^2R^2 = MgR(1-\ \cos\beta),$$

from which $\omega = \sqrt{(g/R)(1-\cos\beta)}.$

9.88: a) $K = \dfrac{1}{2}I\omega^2$

$$= \frac{1}{2}\left(\frac{1}{2}(1000\,\text{kg})(0.90\,\text{m})^2\right)\left(3000\,\text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)^2$$

$$= 2.00\times10^7\ \text{J}.$$

b) $$\frac{K}{P_{\text{ave}}} = \frac{2.00\times10^7\,\text{J}}{1.86\times10^4\ \text{W}} = 1075\ \text{s},$$

which is about 18 min.

9.89: a) $$\frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80\,\text{kg})(2.50\times10^{-2}\ \text{m})^2 + (1.60\,\text{kg})(5.00\times10^{-2}\ \text{m})^2)$$

$$= 2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

b) See Example 9.9. In this case, $\omega = v/R_1$, and so the expression for v becomes

$$v = \sqrt{\frac{2gh}{1+(I/mR^2)}}$$

$$= \sqrt{\frac{2(9.80\,\text{m/s}^2)(2.00\,\text{m})}{(1+((2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2)/(1.50\,\text{kg})(0.025\,\text{m})^2))}} = 3.40\,\text{m/s}.$$

c) The same calculation, with R_2 instead of R_1 gives $v = 4.95\ \text{m/s}$. This does make sense, because for a given total energy, the disk combination will have a larger fraction of the kinetic energy with the string of the larger radius, and with this larger fraction, the disk combination must be moving faster.

9.90: a) In the case that no energy is lost, the rebound height h' is related to the speed v by $h' = \frac{v^2}{2g}$, and with the form for h given in Example 9.9, $h' = \frac{h}{1+M/2m}$. b) Considering the system as a whole, some of the initial potential energy of the mass went into the kinetic energy of the cylinder. Considering the mass alone, the tension in the string did work on the mass, so its total energy is not conserved.

9.91: We can use $K(\text{cylinder}) = 250\ \text{J}$ to find ω for the cylinder and v for the mass.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(10.0\ \text{kg})(0.150\ \text{m})^2 = 0.1125\ \text{kg}\cdot\text{m}^2$$

$K = \tfrac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 66.67\ \text{rad/s}$

$v = R\omega = 10.0\ \text{m/s}$

Use conservation of energy $K_1 + U_1 = K_2 + U_2$. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends. $K_1 = U_2 = 0$ so $U_1 = K_2$.

$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$, where $m = 12.0\ \text{kg}$

For the cylinder, $I = \tfrac{1}{2}MR^2$ and $\omega = v/R$, so $\tfrac{1}{2}I\omega^2 = \tfrac{1}{4}Mv^2$.

$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{4}Mv^2$

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 7.23\ \text{m}$$

9.92: Energy conservation: Loss of PE of box equals gain in KE of system.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2$$
$$+\frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2$$

$$\omega_{\text{pulley}} = \frac{v_{\text{Box}}}{r_{\text{p}}} \text{ and } \omega_{\text{cylinder}} = \frac{v_{\text{Box}}}{r_{\text{cylinder}}}$$

$$m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{2}\left(\frac{1}{2}m_{\text{P}}r_p^2\right)\left(\frac{v_B}{r_p}\right)^2$$
$$+\frac{1}{2}\left(\frac{1}{2}m_{\text{C}}r_{\text{C}}^2\right)\left(\frac{v_{\text{B}}}{r_{\text{C}}}\right)^2$$

$$m_{\text{B}}gh = \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{P}}v_{\text{B}}^2 + \frac{1}{4}m_{\text{C}}v_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{\frac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{p}} + \frac{1}{4}m_{\text{C}}}}$$

$$= \sqrt{\frac{(3.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 3.68\text{ m/s}$$

9.93: a) The initial moment of inertia is $I_0 = \frac{1}{2}MR^2$. The piece punched has a mass of $\frac{M}{16}$ and a moment of inertia with respect to the axis of the original disk of

$$\frac{M}{16}\left[\frac{1}{2}\left(\frac{R}{4}\right)^2 + \left(\frac{R}{2}\right)^2\right] = \frac{9}{512}MR^2.$$

The moment of inertia of the remaining piece is then

$$I = \frac{1}{2}MR^2 - \frac{9}{512}MR^2 = \frac{247}{512}MR^2.$$

b) $I = \frac{1}{2}MR^2 + M(R/2)^2 - \frac{1}{2}(M/16)(R/4)^2 = \frac{383}{512}MR^2.$

9.94: a) From the parallel-axis theorem, the moment of inertia is

$I_P = (2/5)MR^2 + ML^2$, and

$$\frac{I_P}{ML^2} = \left(1 + \left(\frac{2}{5}\right)\left(\frac{R}{L}\right)^2\right).$$

If $R = (0.05)L$, the difference is $(2/5)(0.05)^2 = 0.001$. b) $(I_{\text{rod}}/ML^2) = (m_{\text{rod}}/3M)$, which is 0.33% when $m_{\text{rod}} = (0.01)M$.

9.95: a) With respect to O, each element r_i^2 in Eq. (9.17) is $x_i^2 + y_i^2$, and so

$$I_O = \sum_i m_i r_i^2 = \sum_i m_i (x_i^2 + y_i^2) = \sum_i m_i x_i^2 + \sum_i m_i y_i^2 = I_x + I_y.$$

b) Two perpendicular axes, both perpendicular to the washer's axis, will have the same moment of inertia about those axes, and the perpendicular-axis theorem predicts that they will sum to the moment of inertia about the washer axis, which is $\frac{M}{2}(R_1^2 + R_2^2)$, and so $I_x = I_y = \frac{M}{4}(R_1^2 + R_2^2)$.

c) From Table (9.2), $I = \frac{1}{12}m(L^2 + L^2) = \frac{1}{6}mL^2$.
Since $I_0 = I_x + I_y$, and $I_x = I_y$, both I_x and I_y must be $\frac{1}{12}\, mL^2$.

9.96: Each side has length a and mass $\frac{M}{4}$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\frac{M}{4}a^2 = \frac{Ma^2}{48}$. The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48} + \frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4 \times \frac{Ma^2}{12} = \frac{Ma^2}{3}$.

9.97: Introduce the auxiliary variable L, the length of the cylinder, and consider thin cylindrical shells of thickness dr and radius r; the cross-sectional area of such a shell is $2\pi\, r\, dr$, and the mass of shell is $dm = 2\pi\, rL\rho\, dr = 2\pi\, \alpha\, Lr^2 dr$. The total mass of the cylinder is then

$$M = \int dm = 2\pi\, L\alpha \int_0^R r^2 dr = 2\pi\, L\alpha \frac{R^3}{3}$$

and the moment of inertia is

$$I = \int r^2 dm = 2\pi\, L\alpha \int_o^R r^4 dr = 2\pi\, L\alpha \frac{R^5}{5} = \frac{3}{5} MR^2.$$

b) This is less than the moment of inertia if all the mass were concentrated at the edge, as with a thin shell with $I = MR^2$, and is greater than that for a uniform cylinder with $I = \frac{1}{2} MR^2$, as expected.

9.98: a) From Exercise 9.49, the rate of energy loss is $\frac{4\pi^2 I}{T^3} \frac{dT}{dt}$; solving for the moment of inertia I in terms of the power P,

$$I = \frac{PT^3}{4\pi} \frac{1}{dT/dt} = \frac{(5\times 10^{31}\ \text{W})(0.0331\,\text{s})^3}{4\pi^2} \frac{1\,\text{s}}{4.22\times 10^{-13}\ \text{s}} = 1.09\times 10^{38}\ \text{kg}\cdot\text{m}^2.$$

b) $R = \sqrt{\dfrac{5I}{2M}} = \sqrt{\dfrac{5(1.08\times 10^{38}\ \text{kg}\cdot\text{m}^2)}{2(1.4)(1.99\times 10^{30}\ \text{kg})}} = 9.9\times 10^3\ \text{m},$ about 10 km.

c) $$\frac{2\pi R}{T} = \frac{2\pi(9.9\times 10^3\,\text{m})}{(0.0331\,\text{s})} = 1.9\times 10^6\ \text{m/s} = 6.3\times 10^{-3}\ \text{c}.$$

d) $$\frac{M}{V} = \frac{M}{(4\pi/3)R^3} = 6.9\times 10^{17}\ \text{kg/m}^3,$$

which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

9.99: a) Following the hint, the moment of inertia of a uniform sphere in terms of the mass density is $I = \frac{2}{5}MR^2 = \frac{8\pi}{15}\rho R^5$, and so the difference in the moments of inertia of two spheres with the same density ρ but different radii R_2 and R_1 is $I = \rho(8\pi/15)(R_2^5 - R_1^5)$.

b) A rather tedious calculation, summing the product of the densities times the difference in the cubes of the radii that bound the regions and multiplying by $4\pi/3$, gives $M = 5.97 \times 10^{24}$ kg. c) A similar calculation, summing the product of the densities times the difference in the fifth powers of the radii that bound the regions and multiplying by $8\pi/15$, gives $I = 8.02 \times 10^{22}\ \text{kg} \cdot \text{m}^2 = 0.334MR^2$.

9.100: Following the procedure used in Example 9.14 (and using z as the coordinate along the vertical axis) $r(z) = z\frac{R}{h}$, $dm = \pi\rho\frac{R^2}{h^2}z^2\,dz$ and $dI = \frac{\pi\rho}{2}\frac{R^4}{h^4}z^4\,dz$. Then,

$$I = \int dI = \frac{\pi\rho}{2}\frac{R^4}{h}\int_0^h z^4\,dz = \frac{\pi\rho}{10}\frac{R^4}{h^4}\left[z^5\right]_0^h = \frac{1}{10}\pi\rho R^4 h.$$

The volume of a right circular cone is $V = \frac{1}{3}\pi R^2 h$, the mass is $\frac{1}{3}\pi R^2 h$and so

$$I = \frac{3}{10}\left(\frac{\pi\rho R^2 h}{3}\right)R^2 = \frac{3}{10}MR^2.$$

9.101: a) $ds = r\,d\theta = r_0 d\theta + \beta\theta\,d\theta$, so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. b) Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$$\theta(t) = \frac{1}{\beta} = \left[\sqrt{r_0^{\ 2} + 2\beta vt} - r_0\right].$$

(The negative solution would be going backwards, to values of r smaller than r_0 .)

c) Differentiating,

$$\omega_z(t) = \frac{d\theta}{dt} = \frac{v}{\sqrt{r_0^2 + 2\beta vt}},$$

$$\alpha_z = \frac{d\omega}{dt} = -\frac{\beta v^2}{\left(r_0^2 = 2\beta vt\right)^{3/2}}.$$

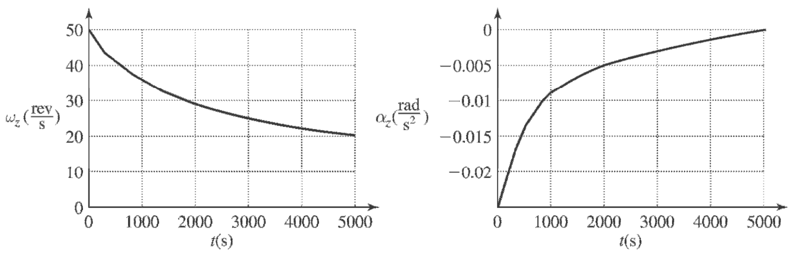
The angular acceleration α_z is not constant. d) $r_0 = 25.0\ \text{mm}$; It is crucial that θ is measured in radians, so $\beta = (1.55\ \mu\text{m/rev})(1\ \text{rev}/2\pi\ \text{rad}) = 0.247\ \mu\text{m/rad}$. The total angle turned in 74.0 min = 4440 s is

$$\theta = \frac{1}{2.47\times10^{-7}\,\text{m/rad}}\left[\sqrt{\begin{aligned}&2\left(2.47\times10^{-7}\,\text{m/rad}\right)(1.25\ \text{m/s})(4440\ \text{s})\\&+\left(25.0\times10^{-3}\,\text{m}\right)^2\end{aligned}} - 25.0\times10^{-3}\ \text{m}\right]$$

$$= 1.337\times10^5\ \text{rad}$$

which is 2.13 × 10^4 rev.

e)



Capítulo 10

10.1: Equation (10.2) or Eq. (10.3) is used for all parts.

a) $(4.00\ \text{m})(10.0\ \text{N})\sin 90° = 40.00\ \text{N}\cdot\text{m}$, out of the page.

b) $(4.00\ \text{m})(10.0\ \text{N})\sin 120° = 34.6\ \text{N}\cdot\text{m}$, out of the page.

c) $(4.00\ \text{m})(10.0\ \text{N})\sin 30° = 20.0\ \text{N}\cdot\text{m}$, out of the page.

d) $(2.00\ \text{m})(10.00\ \text{N})\sin 60° = 17.3\ \text{N}\cdot\text{m}$, into the page.

e) The force is applied at the origin, so $\tau = 0$.

f) $(4.00\ \text{m})(10.0\ \text{N})\sin 180° = 0$.

10.2:

$$\tau_1 = -(8.00\ \text{N})(5.00\ \text{m}) = -40.0\ \text{N}\cdot\text{m},$$
$$\tau_2 = (12.0\ \text{N})(2.00\ \text{m})\sin 30° = 12.0\ \text{N}\cdot\text{m},$$

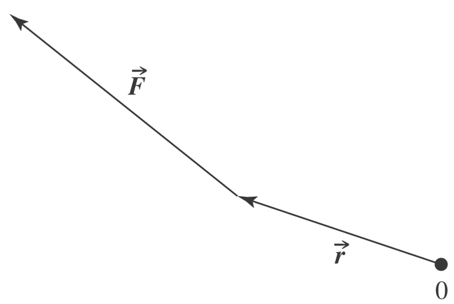
where positive torques are taken counterclockwise, so the net torque is $-28.0\ \text{N}\cdot\text{m}$, with the minus sign indicating a clockwise torque, or a torque into the page.

10.3: Taking positive torques to be counterclockwise (out of the page), $\tau_1 = -(0.090\ \text{m})\times(180.0\ \text{N}) = -1.62\ \text{N}\cdot\text{m}$, $\tau_2 = (0.09\ \text{m})(26.0\ \text{N}) = 2.34\ \text{N}\cdot\text{m}$, $\tau_3 = \left(\sqrt{2}\right)(0.090\ \text{m})(14.0\ \text{N}) = 1.78\ \text{N}\cdot\text{m}$, so the net torque is $2.50\ \text{N}\cdot\text{m}$, with the direction counterclockwise (out of the page). Note that for τ_3 the applied force is perpendicular to the lever arm.

10.4:

$$\tau_1 + \tau_2 = -F_1R + F_2R = (F_2 - F_1)R$$
$$= (5.30\ \text{N} - 7.50\ \text{N})(0.330\ \text{m}) = -0.726\ \text{N}\cdot\text{m}.$$

10.5: a)



b) Into the plane of the page.

c) $\vec{r}\times\vec{F} = [(-0.450\ \text{m})\hat{i} + (0.150\ \text{m})\hat{j}]\times[(-5.00\ \text{n})\hat{i} + (4.00\ \text{N})\hat{i}]$

$= (-0.450\ \text{m})(4.00\ \text{N}) - (0.150\ \text{m})(-5.00\ \text{N})\hat{k}$

$= (-1.05\ \text{N}\cdot\text{m})\hat{k}$

10.6: (a)

$$\tau_A = (50\text{ N})(\sin 60°)(0.2\text{ m}) = 8.7\text{ N}\cdot\text{m, CCW}$$
$$\tau_B = 0$$
$$\tau_C = (50\text{ N})(\sin 30°)(0.2\text{ m}) = 5\text{ N}\cdot\text{m, CW}$$
$$\tau_D = (50\text{ N})(0.2\text{ m}) = 10\text{ N}\cdot\text{m, CW}$$

(b) $\sum\tau = 8.7\text{ N}\cdot\text{m} - 5\text{ N}\cdot\text{m} - 10\text{ N}\cdot\text{m}$
$= -6.3\text{ N}\cdot\text{m, CW}$

10.7: $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40\text{ kg}$, $m = 2.00\text{ kg}$

$I = 0.600\text{ kg}\cdot\text{m}^2$

$\omega_0 = 75.0\text{ rpm} = 7.854\text{ rad/s}$; $\omega = 50.0\text{ rpm} = 5.236\text{ rad/s}$; $t = 30.0\text{ s}$, $\alpha = ?$

$\omega = \omega_0 + \alpha t$ gives $\alpha = -0.08726\text{ rad/s}^2$;

$\Sigma\tau = I\alpha,\ \tau_f = I\alpha = -0.0524\text{ N}\cdot\text{m}$

10.8: a) $\tau = I\alpha = I\dfrac{\Delta\omega}{\Delta t} = \left(2.50\text{ kg}\cdot\text{m}^2\right)\dfrac{\left(400\text{ rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)}{(8.00\text{ s})} = 13.1\text{ N}\cdot\text{m}.$

b) $\dfrac{1}{2}I\omega^2 = \dfrac{1}{2}(2.50\text{ kg}\cdot\text{m}^2)\left(400\text{ rev/min}\times\dfrac{2\pi}{60}\dfrac{\text{rad/s}}{\text{rev/min}}\right)^2 = 2.19\times10^3\text{ J}.$

10.9: $v = \sqrt{2as} = \sqrt{2\left(0.36\text{ m/s}^2\right)(2.0\text{ m})} = 1.2\text{ m/s}$, the same as that found in Example 9-8.

10.10:

$$\alpha = \frac{\tau}{I} = \frac{FR}{I} = \frac{(40.0\text{ N})(0.250\text{ m})}{\left(5.0\text{ kg}\cdot\text{m}^2\right)} = 2.00\text{ rad/s}^2.$$

10.11: a)

$$n = Mg + T = g\left[M + \frac{m}{1 + 2m/M}\right] = g\left[\frac{M + 3m}{1 + 2m/M}\right]$$

b) This is less than the total weight; the suspended mass is accelerating down, so the tension is less than *mg*. c) As long as the cable remains taut, the velocity of the mass does not affect the acceleration, and the tension and normal force are unchanged.

10.12: a) The cylinder does not move, so the net force must be zero. The cable exerts a horizontal force to the right, and gravity exerts a downward force, so the normal force must exert a force up and to the left, as shown in Fig. (10.9).

b) $n = \sqrt{(9.0\ \text{N})^2 + ((50\ \text{kg})(9.80\ \text{m/s}^2))^2}$ 490 N, at an angle of arctan $\left(\frac{9.0}{490}\right) = 1.1°$ from the vertical (the weight is much larger than the applied force F).

10.13: $\mu_\text{k} = \dfrac{f}{n} = \dfrac{\tau/R}{n} = \dfrac{I\alpha}{Rn} = \dfrac{MR(\omega_0/t)}{2n}$

$$= \frac{(50.0\ \text{kg})(0.260\ \text{m})(850\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)}{2(7.50\ \text{s})(160\ \text{N})} = 0.482.$$

10.14: (a) Falling stone: $g = \frac{1}{2}at^2$

$$12.6\ \text{m} = \tfrac{1}{2}a(3.00\ \text{s})^2$$

$$a = 2.80\ \text{m/s}^2$$

$$\text{Stone}: \Sigma F = ma : mg - T = ma (1)$$

$$\text{Pulley}: \Sigma \tau = I\alpha : TR = \tfrac{1}{2}MR^2\alpha = \tfrac{1}{2}MR^2\left(\tfrac{a}{\text{R}}\right)$$

$$T = \tfrac{1}{2}Ma (2)$$

Solve (1) and (2):

$$M = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\ \text{kg}}{2}\right)\left(\frac{2.80\ \text{m/s}^2}{9.80\ \text{m/s}^2 - 2.80\ \text{m/s}^2}\right)$$

$$M = 2.00\ \text{kg}$$

(b)From (2):

$$T = \frac{1}{2}Ma = \frac{1}{2}(10.0\ \text{kg})(2.80\ \text{m/s}^2)$$

$$T = 14.0\ \text{N}$$

10.15: $I = \frac{1}{2}mR^2 = \frac{1}{2}(8.25\ \text{kg})(0.0750\ \text{m})^2 = 0.02320\ \text{kg}\cdot\text{m}^2$

$\omega_0 = 220\ \text{rpm} = 23.04\ \text{rad/s};\ \omega = 0;\ \theta - \theta_0 = 5.25\ \text{rev} = 33.0\ \text{rad},\ \alpha = ?$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ gives $\alpha = -8.046\ \text{rad/s}^2$

$\sum\tau = I\alpha$

$\sum\tau = \tau_f = -f_k R = -\mu_k nR$

$-\mu_k nR = I\alpha$ so $n = \dfrac{I\alpha}{\mu_k R} = 7.47\ \text{N}$

10.16: This is the same situtation as in Example 10.3. a) $T = mg/(1 + 2m/M) = 42.0\ \text{N}$. b) $v = \sqrt{2\ gh/(1 + M/2m} = 11.8\ \text{m/s}$. c) There are many ways to find the time of fall. Rather than make the intermediate calculation of the acceleration, the time is the distance divided by the average speed, or $h/(v/2) = 1.69\ \text{s}$. d) The normal force in Fig. (10.10(b)) is the sum of the tension found in part (a) and the weight of the windlass, a total 159.6 N (keeping extra figures in part (a)).

10.17: See Example 10.4. In this case, the moment of inertia I is unknown, so $a_1 = (m_2 g)/(m_1 + m_2 + (I/R^2))$. a) $a_1 = 2(1.20\ \text{m})/(0.80\ \text{s})^2 = 3.75\ \text{m/s}^2$, so $T_1\ m_1 a_1 = 7.50\ \text{N}$ and $T_2 = m_2(g - a_1) = 18.2\ \text{N}$.

b) The torque on the pulley is $(T_2 - T_1)R = 0.803\ \text{N}\cdot\text{m}$, and the angular acceleration is $\alpha = a_1/R = 50\ \text{rad/s}^2$, so $I = \tau/\alpha = 0.016\ \text{kg}\cdot\text{m}^2$.

10.18:
$$\alpha = \frac{\tau}{I} = \frac{Fl}{\frac{1}{3}Ml^2} = \frac{3F}{Ml}.$$

10.19: The acceleration of the mass is related to the tension by $Ma_{\text{cm}} = Mg - T$, and the angular acceleration is related to the torque by $I\alpha = \tau = TR$, or $a_{\text{cm}} = T/M$, where $\alpha = a_{\text{cm}}/RSt$ and $I = MR^2$ have been used.
a) Solving these for T gives $T = Mg/2 = 0.882\ \text{N}$. b) Substituting the expression for T into either of the above relations gives $a_{\text{cm}} = g/2$, from which $t = \sqrt{2h/a_{\text{cm}}} = \sqrt{4h/g} = 0.553\ \text{s}$. c) $\omega = v_{\text{cm}}/R = a_{\text{cm}}t/R = 33.9\ \text{rad/s}$.

10.20: See Example 10.6 and Exercise 10.21. In this case, $K_2 = Mv_{\text{cm}}^2$ and $v_{\text{cm}} = \sqrt{gh}, \omega = v_{\text{cm}}/R = 33.9\ \text{rad/s}$.

10.21: From Eq. (10.11), the fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{\mathrm{cm}}\omega^2}{(1/2)Mv_{\mathrm{cm}}^2+(1/2)I_{\mathrm{cm}}\omega^2}=\frac{1}{1+(M/I_{\mathrm{cm}})/v_{\mathrm{cm}}^2/\omega^2}=\frac{1}{1+\frac{MR^2}{I_{\mathrm{cm}}}},$$

where $v_{\mathrm{cm}}=R\omega$ for an object that is rolling without slipping has been used.

a) $I_{\mathrm{cm}}=(1/2)MR^2$, so the above ratio is $1/3$. b) $\mathrm{I}=(2/5)\mathrm{MR}^2$, so the above ratio is $2/7$. c) $I=2/3\,MR^2$, so the ratio is $2/5$. d) $I=5/8\,\mathrm{MR}^2$, so the ratio is $5/13$.

10.22: a) The acceleration down the slope is $a=g\sin\theta-\frac{f}{M}$, the torque about the center of the shell is

$$\tau=Rf=I\alpha=I\frac{a}{R}=\frac{2}{3}MR^2\frac{a}{R}=\frac{2}{3}MRa,$$

so $\frac{f}{M}=\frac{2}{3}a$. Solving these relations a for f and simultaneously gives $\frac{5}{3}a=g\sin\theta$, or

$$a=\frac{3}{5}g\sin\theta=\frac{3}{5}(9.80\,\mathrm{m/s^2})\sin 38.0^\circ=3.62\,\mathrm{m/s^2},$$

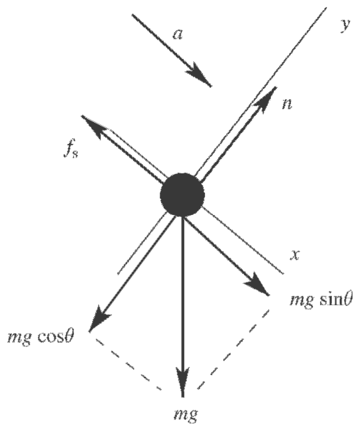
$$f=\frac{2}{3}Ma=\frac{2}{3}(2.00\,\mathrm{kg})(3.62\,\mathrm{m/s^2})=4.83\ \mathrm{N}.$$

The normal force is $Mg\cos\theta$, and since $f\le\mu_{\mathrm{s}}n$,

$$\mu_{\mathrm{s}}\ge\frac{f}{n}=\frac{\frac{2}{3}Ma}{Mg\cos\theta}=\frac{2}{3}\frac{a}{g\cos\theta}=\frac{2}{3}\frac{\frac{3}{5}g\sin\theta}{g\cos\theta}=\frac{2}{5}\tan\theta=0.313.$$

b) $a=3.62\,\mathrm{m/s^2}$ since it does not depend on the mass. The frictional force, however, is twice as large, 9.65 N, since it does depend on the mass. The minimum value of μ_{s} also does not change.

10.23:



$n = mg\cos\alpha$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (eq.1)

n and mg act at the center of the ball and provide no torque.

$\sum\tau = \tau_f = \mu_s mg\cos\theta R; I = \frac{2}{5}mR^2$

$\sum\tau = I\alpha$ gives $\mu_s mg\cos\theta = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (eq.2)

We have two equations in the two unknowns a and μ_s. Solving gives

$a = \frac{5}{7}g\sin\theta$ and $\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613$

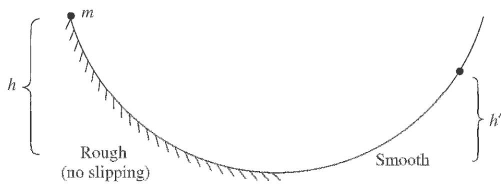
b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$.

$a = \frac{3}{5}g\sin\theta$ and $\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

c) There is no slipping at the point of contact.

10.24:



$v_{cm} = R\omega$ for no slipping

a) Get v at bottom:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$v = \sqrt{\frac{10}{7}gh}$$

Now use energy conservation. Rotational KE does not change

$$\frac{1}{2}mv^2 + KE_{\text{Rot}} = mgh' + KE_{\text{Rot}}$$

$$h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$$

(b) $mgh = mgh' \rightarrow h' = h$ With friction on both halves, all the PE gets converted back to PE. With one smooth side, some of the PE remains as rotational KE.

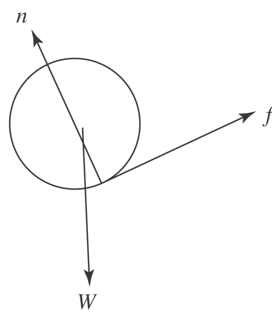
10.25: $wh - W_f = K_1 = (1/2)I_{\text{cm}}w^2{}_0 + \frac{1}{2}mv^2{}_{\text{cm}}$

Solving for h with $v_{\text{cm}} = Rw$

$$h = \frac{\frac{1}{2}\left(\frac{w}{9.80\ \text{m/s}^2}\right)[(0.800)(0.600\ \text{m})^2(25.0\ \text{rad}/s)^2 + (0.600\ \text{m})^2(25.0\ \text{rad/s})^2]}{w}$$

$$-\frac{3500\ \text{J}}{392\ \text{N}} = 11.7\ \text{m}.$$

10.26: a)



The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

b) The friction force results in an angular acceleration, related by $I\alpha = fR$. The equation of motion is $mg \sin \beta - f = ma_{\text{cm,}}$ and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$ (note that positive acceleration is taken to be *down* the incline, and relation between a_{cm} and α is correct for a friction force directed uphill). Combining,

$$mg \sin\beta = ma\left(1 + \frac{I}{mR^2}\right) = ma(7/5),$$

from which $a_{\text{cm}} = (5/7)g \sin\beta$. c) From either of the above relations between if f and a_{cm},

$$f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{7}mg \sin \beta \le \mu_s n = \mu_s mg \cos \beta,$$

from which $\mu_s \ge (2/7) \tan \beta$.

10.27: a) $\omega = \alpha\Delta t = (FR/I)\Delta t = ((18.0\ \text{N})(2.40\ \text{m})/(2100\ \text{kg}\cdot\text{m}^2))(15.0\ \text{s}) = 0.3086\ \text{rad/s}$, or 0.309 rad/s to three figures.
b) $W = K_2 = (1/2)I\omega^2 = (1/2)\times\ (2.00\ \text{kg}\cdot\text{m}^2)(0.3086\ \text{rad/s})^2 = 100\ \text{J}$.
c) From either $P = \tau\omega_{\text{ave}}$ or $P = W/\Delta t$, $P = 6.67$ W.

10.28: a) $$\tau = \frac{P}{\omega} = \frac{(175\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)} = 519\ \text{N}\cdot\text{m}.$$

b) $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi) = 3261\ \text{J}$.

10.29: a) $$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

$$= \frac{\left((1/2)(1.50\ \text{kg})(0.100\ \text{m})^2\right)(1200\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)}{2.5\ \text{s}}$$

$$= 0.377\ \text{N}\cdot\text{m}.$$

b) $$\omega_{\text{ave}}\Delta t = \frac{(600\ \text{rev/min})(2.5\ \text{s})}{60\ \text{s/min}} 25.0\ \text{rev} = 157\ \text{rad}.$$

c) $$\tau\Delta\theta = 59.2\ \text{J}.$$

d) $$K = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\left((1/2)(1.5\ \text{kg})(0.100\ \text{m})^2\right)\left((1200\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2$$

$$= 59.2\ \text{J},$$

the same as in part (c).

10.30: From Eq. (10.26), the power output is

$$P = \tau\omega = (4.30\ \text{N}\cdot\text{m})\left(4800\ \text{rev/min} \times \frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right) = 2161\ \text{W},$$

which is 2.9 hp.

10.31: a) With no load, the only torque to be overcome is friction in the bearings (neglecting air friction), and the bearing radius is small compared to the blade radius, so any frictional torque could be neglected.

b) $$F = \frac{\tau}{R} = \frac{P/\omega}{R} = \frac{(1.9\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)(0.086\ \text{m})} = 65.6\ \text{N}.$$

10.32: $I = \frac{1}{2}mL^2 = \frac{1}{2}(117\ \text{kg})(2.08\ \text{m})^2 = 42.2\ \text{kg}\cdot\text{m}^2$

a) $$\alpha = \frac{\tau}{I} = \frac{1950\ \text{N}\cdot\text{m}}{42.2\ \text{kg}\cdot\text{m}^2} = 46.2\ \text{rad/s}^2.$$

b) $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\ \text{rad/s}^2)(5.0\ \text{rev}\times 2\pi\ \text{rev})} = 53.9\ \text{rad/s}.$

c) From either $W = K = \dfrac{1}{2}\omega^2$ or Eq. (10.24),

$$W = \tau\theta = (1950\ \text{N.m})(5.00\ \text{rev}\times 2\pi\ \text{rad/rev}) = 6.13\times 10^4\ \text{J}.$$

d), e) The time may be found from the angular acceleration and the total angle, but the instantaneous power is also found from $P = \tau\omega = 105\ \text{kW}(141\ \text{hp})$. The average power is half of this, or $52.6\ \text{kW}$.

10.33: a) $\tau = P/\omega = (150\times 10^3\ \text{W})\Big/\left((400\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right) = 358\ \text{N}\cdot\text{m}.$

b) If the tension in the rope is $F, F = w$ and so $w = \tau/R = 1.79\times 10^3\ \text{N}.$

c) Assuming ideal efficiency, the rate at which the weight gains potential energy is the power output of the motor, or $wv = P$, so $v = P/w = 83.8\ \text{m/s}$. Equivalently, $v = \omega R.$

10.34: As a point, the woman's moment of inertia with respect to the disk axis is mR^2, and so the total angular momentum is

$$L = L_{\text{disk}} + L_{\text{woman}} = (I_{\text{disk}} + I_{\text{woman}})\omega = \left(\frac{1}{2}M + m\right)R^2\omega$$

$$= \left(\frac{1}{2}110\ \text{kg} + 50.0\ \text{kg}\right)(4.00\ \text{m})^2(0.500\ \text{rev/s}\times 2\pi\ \text{rad/rev})$$

$$= 5.28\times 10^3\ \text{kg}\cdot\text{m}^2/\text{s}.$$

10.35: a) $mvr\sin\varphi = 115\ \text{kg}\cdot\text{m}^2/\text{s}$, with a direction from the right hand rule of into the page.

b) $dL/dt = \tau = (2\ \text{kg})(9.8\ \text{N/kg})\cdot(8\ \text{m})\cdot\sin(90° - 36.9°) = 125\ \text{N}\cdot\text{m} = 125\ \text{kg}\cdot\text{m}^2/\text{s}^2$, out of the page.

10.36: For both parts, $L = I\omega$. Also, $\omega = v/r$, so $L = I(v/r)$.

a) $L = (mr^2)(v/r) = mvr$

$L = (5.97\times10^{24}\ \text{kg})(2.98\times10^{4}\ \text{m/s})(1.50\times10^{11}\ \text{m}) = 2.67\times10^{40}\ \text{kg}\cdot\text{m}^2/\text{s}$

b) $L = (2/5mr^2)(\omega)$

$L = (2/5)(5.97\times10^{24}\ \text{kg})(6.38\times10^{6}\ \text{m})^2(2\pi\ \text{rad}/(24.0\ \text{hr}\times 3600\ \text{s/hr}))$

$= 7.07\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$

10.37: The period of a second hand is one minute, so the angular momentum is

$$L = I\omega = \frac{M}{3}l^2\frac{2\pi}{T}$$

$$= \left(\frac{6.0\times10^{-3}\ \text{kg}}{3}\right)(15.0\times10^{-2}\ \text{m})^2\frac{2\pi}{60\ \text{s}} = 4.71\times10^{-6}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

10.38: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\ \text{rad}}{(30\ \text{d})(86{,}400\ \text{s/d}}\right)\left(\frac{7.0\times10^{5}\ \text{km}}{16\ \text{km}}\right)^2 = 4.6\times10^{3}\ \text{rad/s}.$$

10.39: a) The net force is due to the tension in the rope, which always acts in the radial direction, so the angular momentum with respect to the hole is constant.

b) $L_1 = m\omega_1 r^2{}_1, L_2 = m\omega_2 r_2{}^2$, and with $L_1 = L_2, \omega_2 = \omega_1(r_1/r_2)^2 = 7.00\ \text{rad}/s$.

c) $\Delta K = (1/2)m((\omega_2 r_2)^2 - (\omega_1 r_1)^2) = 1.03\times10^{-2}\ \text{J}$.

d) No other force does work, so $1.03\times10^{-2}\ \text{J}$ of work were done in pulling the cord.

10.40: The skater's initial moment of inertia is

$$I_1 = (0.400\,\text{kg}\cdot\text{m}^2) + \frac{1}{2}(8.00\,\text{kg})(1.80\,\text{m})^2 = 2.56\,\text{kg}\cdot\text{m}^2,$$

and her final moment of inertia is

$$I_2 = (0.400\,\text{kg}\cdot\text{m}^2) + (8.00\,\text{kg})(25\times10^{-2}\,\text{m}) = 0.9\,\text{kg}\cdot\text{m}^2.$$

Then from Eq. (10.33),

$$\omega_2 = \omega_1\frac{I_1}{I_2} = (0.40\,\text{rev/s})\frac{2.56\,\text{kg}\cdot\text{m}^2}{0.9\,\text{kg}\cdot\text{m}^2} = 1.14\,\text{rev/s}.$$

Note that conversion from rev/s to rad/s is not necessary.

10.41: If she had tucked, she would have made $(2)(3.6\,\text{kg}\cdot\text{m}^2)/18\,\text{kg}\cdot\text{m}^2) = 0.40$ rev in the last 1.0 s, so she would have made $(0.40\,\text{rev})(1.5/1.0) = 0.60$ rev in the total 1.5 s.

10.42: Let

$$I_1 = I_0 = 1200\text{kg}\cdot\text{m}^2,$$
$$I_2 = I_0 + mR^2 = 1200\text{kg}\cdot\text{m}^2 + (40.0\text{kg})(2.00\,\text{m})^2 = 1360\text{kg}\cdot\text{m}^2.$$

Then, from Eq. (10.33),

$$\omega_2 = \omega_1\frac{I_1}{I_2} = \left(\frac{2\pi\,\text{rad}}{6.00\,\text{s}}\right)\frac{1200\,\text{kg.m}^2}{1360\,\text{kg.m}^2} = 0.924\,\text{rad/s}.$$

10.43: a) From conservation of angular momentum,

$$\omega_2 = \omega_1 \frac{I_1}{I_0 + mR^2} = \omega_1 \frac{(1/2)MR^2}{(1/2)MR^2 + mR^2} = \omega_1 \frac{1}{1 + 2m/M}$$
$$= \frac{3.0 \text{ rad/s}}{1 + 2(70)/120} = 1.385 \text{ rad/s}$$

or 1.39 rad/s to three figures

b) $K_1 = (1/2)(1/2)(120 \text{ kg})(2.00 \text{ m})^2 (3.00 \text{ rad/s})^2 = 1.80 \text{ kJ}$, and $K_2 = (1/2)\left(I_0 + (70 \text{ kg})(2.00 \text{ m})^2\right)\omega_2^2 = 499 \text{ J}$. In changing the parachutist's horizontal component of velocity and slowing down the turntable, friction does negative work.

10.44: Let the width of the door be l;

$$\omega = \frac{L}{I} = \frac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}$$
$$= \frac{(0.500 \text{ kg})(12.0 \text{ m/s})(0.500 \text{ m})}{(1/3)(40.0 \text{ kg})(1.00 \text{ m})^2 + (0.500 \text{ kg})(0.500 \text{ m})^2} = 0.223 \text{ rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225 \text{ rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

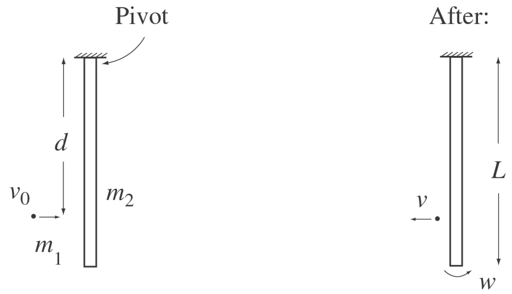
10.45: Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object A be the bug and object B be the bar.

Initially, all objects are at rest and $L_1 = 0$.

Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

$$L_2 = m_A v_A r - I_B \omega_B \text{ where } r = 1.00 \text{ m and } I_B = \tfrac{1}{3} m_B r^2$$
$$L_1 = L_2 \text{ gives } m_A v_A r = \tfrac{1}{3} m_B r^2 \omega_B$$
$$\omega_B = \frac{3 m_A v_A}{m_B r} = 0.120 \text{ rad/s}$$

10.46:



(a) Conservation of angular momentum:

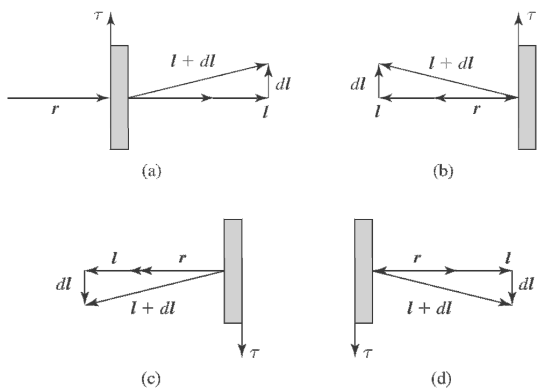
$$m_1 v_0 d = -m_1 v d + \tfrac{1}{3} m_2 L^2 \omega$$

$$(3.00\,\text{kg})(10.0\,\text{m/s})(1.50\,\text{m}) = -(3.00\,\text{kg})(6.00\,\text{m/s})(1.50\,\text{m}) + \frac{1}{3}\left(\frac{90.0\,\text{N}}{9.80\,\text{m/s}^2}\right)(2.00\,\text{m})^2 \omega$$

$$\omega = 5.88\,\text{rad/s}$$

(b) There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

10.47:



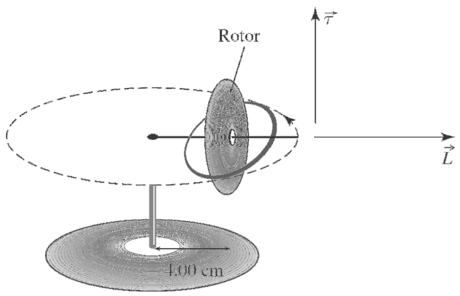
10.48: a) Since the gyroscope is precessing in a horizontal plane, there can be no net vertical force on the gyroscope, so the force that the pivot exerts must be equal in magnitude to the weight of the gyroscope,
$F = \omega = mg = (0.165\ \text{kg})(9.80\ \text{m/s}^2) = 1.617\ \text{N},\ 1.62\ \text{N}$ to three figures.

b) Solving Eq. (10.36) for ω,

$$\omega = \frac{\omega R}{I\Omega} = \frac{(1.617\ \text{N})(4.00\times10^{-2}\ \text{m})}{(1.20\times10^{-4}\ \text{kg}\cdot\text{m}^2)(2\pi)\text{rad}/2.20\ \text{s}} = 188.7\ \text{rad/s},$$

which is 1.80×10^3 rev/min. Note that in this and similar situations, since Ω appears in the denominator of the expression for ω, the conversion from rev/s and back to rev/min *must* be made.

c)



10.49: a) $\dfrac{K}{P} = \dfrac{(1/2)((1/2)MR^2)\omega^2}{P}$

$$= \frac{(1/2)\left((1/2)(60{,}000\ \text{kg})(2.00\ \text{m})^2\right)\left((500\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)\right)^2}{7.46\times10^4\ \text{W}}$$

$$= 2.21\times10^3\ \text{s},$$

or 36.8 min.

b) $\tau = I\Omega\omega$

$$= (1/2)(60{,}000\ \text{kg})(2.00\ \text{m})^2(500\ \text{rev/min})\left(\frac{\pi}{30}\frac{\text{rad/s}}{\text{rev/min}}\right)(1.00°/\text{s})\left(\frac{2\pi\ \text{rad}}{360°}\right)$$

$$= 1.10\times10^5\ \text{N}\cdot\text{m}.$$

10.50: Using Eq. (10.36) for all parts, a) halved b) doubled (assuming that the added weight is distributed in such a way that r and I are not changed) c) halved (assuming that w and r are not changed) d) doubled e) unchanged.

10.51: a) Solving Eq. (10.36) for $\tau, \tau = I\omega\,\Omega = (2/5)MR^2\omega\,\Omega$. Using $\omega = \frac{2\pi\ \text{rad}}{86{,}400\ \text{s}}$ and $\Omega = \frac{2\pi}{(26{,}000\ \text{y})(3.175\times10^7\ \text{s/y})}$ and the mass and radius of the earth from Appendix F, $\tau \sim 5.4\times10^{22}\ \text{N}\cdot\text{m}$.

10.52: a) The net torque must be

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = (1.86\ \text{kg}\cdot\text{m}^2)\frac{\left(120\ \text{rev/min}\times\dfrac{2\pi}{60}\dfrac{\text{rad/s}}{\text{rev/min}}\right)}{(9.00\ \text{s})} = 2.60\ \text{N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fr = \mu_\text{k}nr$ due to the knife. Combining,

$$\begin{aligned} F &= \frac{1}{R}(\tau + \tau_\text{f} + \mu_\text{k}nr) \\ &= \frac{1}{0.500\ \text{m}}\big((2.60\ \text{N}\cdot\text{m}) + (6.50\ \text{N}\cdot\text{m}) + (0.60)(160\ \ \text{N})(0.260\ \text{m})\big) \\ &= 68.1\ \text{N}. \end{aligned}$$

b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is $F' = \frac{1}{0.500\ \text{m}}(6.50\ \text{N}\cdot\text{m} + 24.96\ \text{N}\cdot\text{m}) = 62.9\ \text{N}$. c) The time t needed to come to a stop is found by taking the magnitudes in Eq. (10.27), with $\tau = \tau_\text{f}$ constant;

$$t = \frac{L}{\tau_\text{f}} = \frac{\omega I}{\tau_\text{f}} = \frac{\left(120\ \text{rev/min}\times\frac{2\pi}{60}\frac{\text{rad/s}}{\text{rev/min}}\right)\left(1.86\ \text{kg}\cdot\text{m}^2\right)}{(6.50\ \text{N}\cdot\text{m})} = 3.6\ \text{s}.$$

Note that this time can also be found as $t = (9.00\ \text{s})\frac{2.60\,\text{N}\cdot\text{m}}{6.50\text{N}\cdot\text{m}}$.

10.53: a) $I = \dfrac{\tau}{\alpha} = \dfrac{\tau\Delta t}{\Delta\omega} = \dfrac{(5.0\ \text{N}\cdot\text{m})(2.0\ \text{s})}{(100\ \text{rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)} = 0.955\ \text{kg}\cdot\text{m}^2.$

b) Rather than use the result of part (a), the magnitude of the torque is proportional to α and hence inversely proportional to $|\Delta t|$; equivalently, the magnitude of the change in angular momentum is the same and so the magnitude of the torque is again proportional to $1/|\Delta t|$. Either way, $\tau_\text{f} = (5.0\ \text{N}\cdot\text{m})\dfrac{2\ \text{s}}{125\ \text{s}} = 0.080\ \text{N}\cdot\text{m}$.

c) $\omega_\text{ave}\Delta t = (50.0\ \text{rev/min})(125\ \text{s})(1\ \text{min/60 s}) = 104.2\ \text{rev}$.

10.54: a) The moment of inertia is not given, so the angular acceleration must be found from kinematics;

$$\alpha = \frac{2\theta}{t^2} = \frac{2s}{rt^2} = \frac{2(5.00\ \text{m})}{(0.30\ \text{m})(2.00\ \text{s})^2} = 8.33\ \text{rad/s}^2.$$

b) $\alpha t = (8.33\ \text{rad/s}^2)(2.00\ \text{s}) = 16.67\ \text{rad/s}.$

c) The work done by the rope on the flywheel will be the final kinetic energy; $K = W = Fs = (40.0\ \text{N})(5.0\ \text{m}) = 200\ \text{J}.$

d)
$$I = \frac{2K}{\omega^2} = \frac{2(200\ \text{J})}{(16.67\ \text{rad/s})^2} = 1.44\ \text{kg}\cdot\text{m}^2.$$

10.55: a) $P = \tau\omega = \tau\alpha t = \tau\left(\frac{\tau}{I}\right)t = \tau^2\left(\frac{t}{I}\right).$

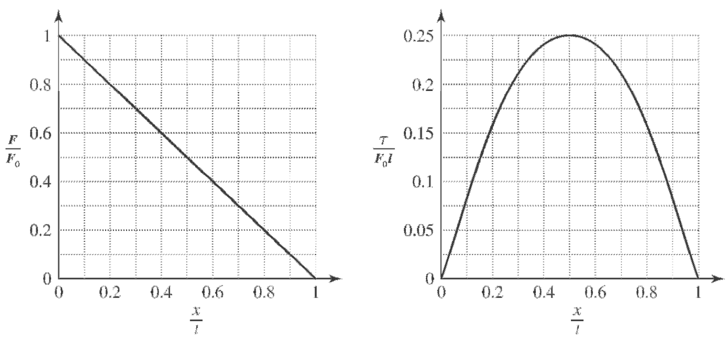
b) From the result of part (a), the power is $(500\ \text{W})\left(\frac{60.0}{20.0}\right)^2 = 4.50\ \text{kW}.$

c) $P = \tau\omega = \tau\sqrt{2\alpha\theta} = \tau\sqrt{2(\tau/I)\theta} = \tau^{3/2}\sqrt{2\theta/I}.$

d) From the result of part (c), the power is $(500\ \text{W})\left(\frac{6.00}{20.00}\right)^{3/2} = 2.6\ \text{kW}.$ e) No; the power is proportional to the time t or proportional to the square root of the angle.

10.56: a) From the right-hand rule, the direction of the torque is $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}},$ the $+z$ direction.

b), c)



d) The magnitude of the torque is $F_0(x - x^2/l),$ which has it maximum at $l/2.$ The torque at $x = l/2$ is $F_0 l/4.$

10.57: $$t^2 = \frac{2\theta}{\alpha} = \frac{2\theta}{(\tau/I)} = \frac{2\theta\, I}{\tau}.$$

The angle in radiants is $\pi/2$, the moment of inertia is

$$(1/3)((750\ \text{N})/(9.80\ \text{m}/\text{s}^2)(1.25\ \text{m}))^3 = 39.9\ \text{kg}\cdot\text{m}^2$$

and the torque is $(220\ \text{N})(1.25\ \text{m}) = 275\ \text{N}\cdot\text{m}$. Using these in the above expression gives $t^2 = 0.455\ \text{s}^2$, so $t = 0.675\ \text{s}$.

10.58: a) From geometric consideration, the lever arm and the sine of the angle between $\vec{F}$ and $\vec{r}$ are both maximum if the string is attached at the end of the rod. b) In terms of the distance x where the string is attached, the magnitude of the torque is $Fxh/\sqrt{x^2 + h^2}$. This function attains its maximum at the boundary, where $x = h$, so the string should be attached at the right end of the rod. c) As a function of x, l and h, the torque has magnitude

$$\tau = F\frac{xh}{\sqrt{(x - l/2)^2 + h^2}}.$$

This form shows that there are two aspects to increasing the torque; maximizing the lever arm l and maximizing sin ϕ. Differentiating τ with respect to x and setting equal to zero gives $x_{\max} = (l/2)(1 + (2h/l)^2)$. This will be the point at which to attach the string unless $2h > l$, in which case the string should be attached at the furthest point to the right, $x = l$.

10.59: a) A distance $L/4$ from the end with the clay.

b) In this case $I = (4/3)ML^2$ and the gravitational torque is $(3L/4)(2Mg)\sin\theta = (3Mg\,L/2)\sin\theta$, so $\alpha = (9g/8L)\sin\theta$.

c) In this case $I = (1/3)ML^2$ and the gravitational torque is $(L/4)(2Mg)\sin\theta = (Mg\,L/2)\sin\theta$, so $\alpha = (3g/2L)\sin\theta$. This is greater than in part (b).

d) The greater the angular acceleration of the upper end of the cue, the faster you would have to react to overcome deviations from the vertical.

10.60: In Fig. (10.22) and Eq. (10.22), with the angle θ measured from the vertical, $\sin\theta = \cos\theta$ in Eq. (10.2). The torque is then $\tau = FR\cos\theta$.

a) $$W = \int_0^{\pi/2} FR\cos\theta \, d\,\theta = FR$$

b) In Eq. (6.14), dl is the horizontal distance the point moves, and so $W = F\int dl = FR,$ the same as part (a). c) From $K_2 = W = (MR^2/4)\omega^2, \omega = \sqrt{4F/MR}.$ d) The torque, and hence the angular acceleration, is greatest when $\theta = 0,$ at which point $\alpha = (\tau/I) = 2F/MR$, and so the maximum tangential acceleration is $2F/M$. e) Using the value for ω found in part (c), $a_{\text{rad}} = \omega^2 R = 4F/M$.

10.61: The tension in the rope must be $m(g + a) = 530\ \text{N}$. The angular acceleration of the cylinder is $a/R = 3.2\ \text{rad/s}^2$, and so the net torque on the cylinder must be 9.28 $\text{N}\cdot\text{m}$. Thus, the torque supplied by the crank is $(530\ \text{N})(0.25\ \text{m}) + (9.28\ \text{N}\cdot\text{m}) = 141.8\ \text{N}\cdot\text{m},$ and the force applied to the crank handle is $\frac{141.8\ \text{N}\cdot\text{m}}{0.12\ \text{m}} = 1.2\ \text{kN}$ to two figures.

10.62: At the point of contact, the wall exerts a friction force f directed downward and a normal force n directed to the right. This is a situation where the net force on the roll is zero, but the net torque is *not* zero, so balancing torques would not be correct. Balancing vertical forces, $F_{\text{rod}}\cos\theta = f + w + F,$ and balacing horizontal forces $F_{\text{rod}}\sin\theta = n.$ With $f = \mu_k n,$ these equations become

$$F_{\text{rod}}\cos\theta = \mu_k n + F + w,$$
$$F_{\text{rod}}\sin\theta = n.$$

(a) Eliminating n and solving for F_{rod} gives

$$F_{\text{rod}} = \frac{\omega + F}{\cos\theta - \mu_k\sin\theta} = \frac{(16.0\ \text{kg})(9.80\ \text{m/s}^2) + (40.0\ \text{N})}{\cos 30° - (0.25)\sin 30°} = 266\ \text{N}.$$

b) With respect to the center of the roll, the rod and the normal force exert zero torque. The magnitude of the net torque is $(F - f)R,$ and $f = \mu_k n$ may be found insertion of the value found for F_{rod} into either of the above relations; *i.e.,* $f = \mu_k F_{\text{rod}}\sin\theta = 33.2\ \text{N}$. Then,

$$\alpha = \frac{\tau}{I} = \frac{(40.0\ \text{N} - 31.54\ \text{N})(18.0\times10^{-2}\ \text{m})}{(0.260\ \text{kg}\cdot\text{m}^2)} = 4.71\ \text{rad/s}^2.$$

10.63: The net torque on the pulley is TR, where T is the tension in the string, and $\alpha = TR/I$. The net force on the block down the ramp is $mg(\sin\beta - \mu_k \cos\beta) - T = ma$. The acceleration of the block and the angular acceleration of the pulley are related by $a = \alpha R$.

a) Multiplying the first of these relations by I/R and eliminating α in terms of a, and then adding to the second to eliminate T gives

$$a = mg\frac{(\sin\beta - \mu_k \cos\beta)}{m + I/R^2} = \frac{g(\sin\beta - \mu_k \cos\beta)}{(1 + I/mR^2)},$$

and substitution of numerical values given 1.12 m/s^2. b) Substitution of this result into either of the above expressions involving the tension gives $T = 14.0$ N.

10.64: For a tension T in the string, $mg - T = ma$ and $TR = I\alpha = I\frac{a}{R}$. Eliminating T and solving for a gives

$$a = g\frac{m}{m + I/R^2} = \frac{g}{1 + I/mR^2},$$

where m is the mass of the hanging weight, I is the moment of inertia of the disk combination $(I = 2.25\times10^{-3} \text{ kg}\cdot\text{m}^2 \text{ from Problem 9.89})$ and R is the radius of the disk to which the string is attached.

a) With $m = 1.50$ kg, $R = 2.50\times10^{-2}\,\text{m}, a = 2.88 \text{ m/s}^2$.

b) With $m = 1.50$ kg, $R = 5.00\times10^{-2}\,\text{m}, a = 6.13 \text{ m/s}^2$.

The acceleration is larger in case (b); with the string attached to the larger disk, the tension in the string is capable of applying a larger torque.

10.65: Taking the torque about the center of the roller, the net torque is $fR = \alpha I$, $I = MR^2$ for a hollow cylinder, and with $\alpha = a/R$, $f = Ma$ (note that this is a relation between magnitudes; the vectors $\vec{\boldsymbol{f}}$ and $\vec{\boldsymbol{a}}$ are in opposite directions). The net force is $F - f = Ma$, from which $F = 2Ma$ and so $a = F/2M$ and $f = F/2$.

10.66: The accelerations of blocks A and B will have the same magnitude a. Since the cord does not slip, the angular acceleration of the pulley will be $\alpha = \frac{a}{R}$. Denoting the tensions in the cord as T_A and T_B, the equations of motion are

$$m_A g - T_A = m_A a$$
$$T_B - m_B g = m_B a$$
$$T_A - T_B = \frac{I}{R^2} a,$$

where the last equation is obtained by dividing $\tau = I\alpha$ by R and substituting for α in terms of a.

Adding the three equations eliminates both tensions, with the result that

$$a = g \frac{m_A - m_B}{m_A + m_B + I/R^2}$$

Then,

$$\alpha = \frac{a}{R} = g \frac{m_A - m_B}{m_A R + m_B R + I/R}.$$

The tensions are then found from

$$T_A = m_A (g - a) = g \frac{2m_A m_B + m_A I/R^2}{m_A + m_B + I/R^2}$$
$$T_B = m_B (g + a) = g \frac{2m_B m_A + m_B I/R^2}{m_A + m_B + I/R^2}.$$

As a check, it can be shown that $(T_A - T_B)R = I\alpha.$

10.67: For the disk, $K = (3/4)Mv^2$ (see Example 10.6). From the work-energy theorem, $K_1 = MgL\sin\beta$, from which

$$L = \frac{3v^2}{4g\sin\beta} = \frac{3(2.50\ \mathrm{m/s})^2}{4(9.80\ \mathrm{m/s^2})\sin 30.0^\circ} = 0.957\ \mathrm{m}.$$

This same result may be obtained by an extension of the result of Exercise 10.26; for the disk, the acceleration is $(2/3)g\sin\beta$, leading to the same result.

b) Both the translational and rotational kinetic energy depend on the mass which cancels the mass dependence of the gravitational potential energy. Also, the moment of inertia is proportional to the square of the radius, which cancels the inverse dependence of the angular speed on the radius.

10.68: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\frac{a}{b}$. Dividing the second equation by b and adding to the first to eliminate T yields

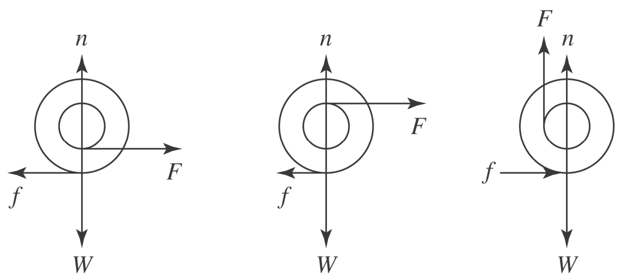
$$a = g\frac{2m}{(2m + I/b^2)} = g\frac{2}{2 + (R/b)^2}, \quad \alpha = g\frac{2}{2b + R^2/b},$$

where $I = 2\frac{1}{2}mR^2 = mR^2$ has been used for the moment of inertia of the yo-yo. The tension is found by substitution into either of the two equations; *e.g.,*

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

10.69: a) The distance the marble has fallen is $y = h - (2R - r) = h + r - 2R.$ The radius of the path of the center of mass of the marble is $R - r,$ so the condition that the ball stay on the track is $v^2 = g(R - r).$ The speed is determined from the work-energy theorem, $mgy = (1/2)mv^2 + (1/2)I\omega^2.$ At this point, it is crucial to know that even for the curved track, $\omega = v/r;$ this may be seen by considering the time T to move around the circle of radius $R - r$ at constant speed V is obtained from $2\pi\,(R - r) = Vt,$ during which time the marble rotates by an angle $2\pi\left(\frac{R}{r} - 1\right) = \omega\,T,$ from which $\omega = V/r.$ The work-energy theorem then states $mgy = (7/10)mv^2,$ and combining, canceling the factors of m and g leads to $(7/10)(R - r) = h + r - 2R,$ and solving for h gives $h = (27/10)R - (17/10)r.$ b) In the absence of friction, $mgy = (1/2)mv^2,$ and substitution of the expressions for y and v^2 in terms of the other parameters gives $(1/2)(R - r) = h - r - 2R,$ which is solved for $h = (5/2)R - (3/2)r.$

10.70: In the first case, $\vec{\boldsymbol{F}}$ and the friction force act in opposite directions, and the friction force causes a larger torque to tend to rotate the yo-yo to the right. The net force to the right is the difference $F - f,$ so the net force is to the right while the net torque causes a clockwise rotation. For the second case, both the torque and the friction force tend to turn the yo-yo clockwise, and the yo-yo moves to the right. In the third case, friction tends to move the yo-yo to the right, and since the applied force is vertical, the yo-yo moves to the right.



10.71: a) Because there is no vertical motion, the tension is just the weight of the hoop: $T = Mg = (0.180\ \text{kg})(9.8\ \text{N/kg}) = 1.76\ \text{N}$ b) Use $\tau = I\alpha$ to find $\alpha.$ The torque is $RT,$ so $\alpha = RT/I = RT/MR^2 = T/MR = Mg/MR,$
so $\alpha = g/R = (9.8\ \text{m/s}^2)/(0.08\ \text{m}) = 122.5\ \text{rad/s}^2$

c) $a = R\alpha = 9.8\ \text{m/s}^2$

d) T would be unchanged because the mass M is the same, α and a would be twice as great because I is now $\frac{1}{2}MR^2.$

10.72: (a) $\Sigma\tau = I\alpha$ and $a_{\mathrm{T}} = R\alpha$

$$PR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2\left(\frac{a_{\mathrm{T}}}{R}\right)$$

$$a_{\mathrm{T}} = \frac{2P}{M} = \frac{200\ \mathrm{N}}{4.00\ \mathrm{kg}} = 50\ \mathrm{m/s^2}$$

Distance the cable moves: $x = \frac{1}{2}at^2$

$$50\ \mathrm{m} = \frac{1}{2}\left(50\ \mathrm{m/s^2}\right)t^2 \rightarrow t = 1.41\ \mathrm{s}.$$

$$v = v_0 + at = 0 + \left(50\ \mathrm{m/s^2}\right)\left(1.41\ \mathrm{s}\right) = 70.5\ \mathrm{m/s}$$

(b) For a hoop, $I = MR^2$, which is twice as large as before, so α and a_{T} would be half as large. Therefore the time would be longer. For the speed, $v^2 = v_0^2 + 2ax$, in which x is the same, so v would be smaller since a is smaller

10.73: Find the speed v the marble needs at the edge of the pit to make it to the level ground on the other side. The marble must travel 36 m horizontally while falling vertically 20 m.

Use the vertical motion to find the time. Take $+\mathrm{y}$ to be downward.

$v_{0y} = 0,\ a_y = 9.80\ \mathrm{m/s^2},\ y - y_0 = 20\ \mathrm{m},\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.02\ \mathrm{s}$

Then $x - x_0 = v_{0x}t$ gives $v_{0x} = 17.82\ \mathrm{m/s}$.

Use conservation of energy, where point 1 is at the starting point and point 2 is at the edge of the pit, where $v = 17.82\ \mathrm{m/s}$. Take $y = 0$ at point 2, so $y_2 = 0$ and $y_1 = h$.

$K_1 + U_1 = K_2 + U_2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling without slipping means $\omega = v/r$. $I = \frac{2}{5}mr^2$, so $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$

$mgh = \frac{7}{10}mv^2$

$$h = \frac{7v^2}{10g} = \frac{7(17.82\ \mathrm{m/s})}{10(9.80\ \mathrm{m/s^2})} = 23\ \mathrm{m}$$

b) $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$, Independent of r.

c) All is the same, except there is no rotational kinetic energy term in $K : K = \frac{1}{2}mv^2$

$mgh = \frac{1}{2}mv^2$

$h = \dfrac{v^2}{2g} = 16\ \mathrm{m}$, 0.7 times smaller than the answer in part (a).

10.74: Break into 2 parts, the rough and smooth sections.

$Rough: mgh_1 = \frac{1}{2} mv_2 + \frac{1}{2} I\omega^2$

$$mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$v^2 = \frac{10}{7}gh_1$$

Smooth: Rotational *KE* does not change.

$$mgh_2 + \frac{1}{2}mv^2 + KE_{\text{Rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + KE_{\text{Rot}}$$

$$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{B}}^2$$

$$v_{\text{B}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2}$$

$$= \sqrt{\frac{10}{7}(9.80\ \text{m/s}^2)(25\ \text{m}) + 2(9.80\ \text{m/s}^2)(25\ \text{m})}$$

$$= 29.0\ \text{m/s}$$

10.75: a) Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0\ \text{m}$.

$$K_1 + U_1 = K_2 + U_2$$

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I\omega_1^2 = mgy_2 + \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I\omega_2^2$$

Rolling without slipping means $\omega = v/r$ and $\tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(v/r)^2 = \tfrac{1}{5}mv^2$

$$\tfrac{7}{10}mv_1^2 = mgy_2 + \tfrac{7}{10}mv_2^2$$

$$v_2 = \sqrt{v_1^2 - \tfrac{10}{7}gy_2} = 15.26\ \text{m/s}$$

Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward.

Use the vertical motion to find the time in the air:

$$v_{0y} = 0,\ a_y = 9.80\ \text{m/s}^2,\ y - y_0 = 28.0\ \text{m},\ t = ?$$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = 2.39\ \text{s}$

During this time the ball travels horizontally $x - x_0 = v_{0x}t = (15.26\ \text{m/s})(2.39\ \text{s}) = 36.5\ \text{m}$

Just before it lands, $v_y = v_{0y} + a_y t = 23.4\ \text{s}$ and $v_x = v_{0x} = 15.3\ \text{s}$ $v = \sqrt{v_x^2 + v_y^2} = 28.0\ \text{m/s}$

b) At the bottom of the hill, $\omega = v/r = (25.0\ \text{m/s})r$. The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3\ \text{s})r$. The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.76: (a) $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 (1)$

$$I = I_{\text{rim}} + I_{\text{spokes}} = M_r R^2 + 6\left(\frac{1}{3}m_s R^2\right)$$

Uniform density means: $m_r = \lambda 2\pi R$ and $m_s = \lambda R$. No slipping means that $\omega = v/R$. Also, $m = m_r + m_s = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$ substituting into (1) gives

$$2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6\left(\frac{1}{3}\pi RR^2\right)\right]\omega^2$$

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80\ \text{m/s}^2)(58.0\ \text{m})}{(0.210\ \text{m})^2(\pi+2)}} = 124\ \text{rad/s}$$

and $v = R\omega = 26.0\ \text{m/s}$

(b) Doubling the density would have no effect because it does not appear in the answer. $\omega\, \alpha\, \frac{1}{R}$, so doubling the diameter would double the radius which would reduce ω by half, but $v = R\omega$ would be unchanged.

10.77: a) The front wheel is turning at $\omega = 1.00\ \text{rev/s} = 2\pi\ \text{rad/s}$.

$v = r\omega = (0.330\ \text{m})(2\pi\ \text{rad/s}) = 2.07\ \text{s}$

b) $\omega = v/r = (2.07\ \text{m/s})/(0.655\ \text{m} = 3.16\ \text{rad/s} = 0.503\ \text{rev/s}$

c) $\omega = v/r = (2.07\ \text{m/s})/(0.220\ \text{m}) = 9.41\ \text{rad/s} = 1.50\ \text{rev/s}$

10.78: a) The kinetic energy of the ball when it leaves the tract (when it is still rolling without slipping) is $(7/10)mv^2$ and this must be the work done by gravity, W = mgh, so $v = \sqrt{10gh/7}$. The ball is in the air for a time $t = \sqrt{2y/g}$, so $x = vt = \sqrt{20hy/7}$.

b) The answer does not depend on g, so the result should be the same on the moon.

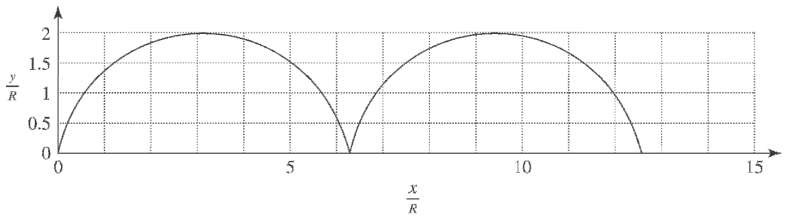
c) The presence of rolling friction would decrease the distance.

d) For the dollar coin, modeled as a uniform disc, $K = (3/4)mv^2$, and so $x = \sqrt{8hy/3}$.

10.79: a) $v=\sqrt{\dfrac{10K}{7m}}=\sqrt{\dfrac{(10)(0.800)(1/2)(400\ \text{N/m})(0.15\ \text{m})^2}{7(0.0590\ \text{kg})}}=9.34\ \text{m/s}.$

b) Twice the speed found in part (a), 18.7 m/s. c) If the ball is rolling without slipping, the speed of a point at the bottom of the ball is zero. d) Rather than use the intermediate calculation of the speed, the fraction of the initial energy that was converted to gravitational potential energy is $(0.800)(0.900)$, so $(0.720)(1/2)kx^2 = mgh$ and solving for h gives 5.60 m.

10.80: a)



b) R is the radius of the wheel (y varies from 0 to $2R$) and T is the period of the wheel's rotation.

c) Differentiating,

$$v_x=\frac{2\pi R}{T}\left[1-\cos\left(\frac{2\pi t}{T}\right)\right] \qquad a_x=\left(\frac{2\pi}{T}\right)^2 R\sin\left(\frac{2\pi t}{T}\right)$$

$$v_y=\frac{2\pi R}{T}\sin\left(\frac{2\pi t}{T}\right) \qquad a_y=\left(\frac{2\pi}{T}\right)^2 R\cos\left(\frac{2\pi t}{T}\right).$$

d) $v_x=v_y=0$ when $\left(\frac{2\pi t}{T}\right)=2\pi$ or any multiple of 2π, so the times are integer multiples of the period T. The acceleration components at these times are $a_x=0,\ a_y=\dfrac{4\pi^2R}{T^2}.$

e) $$\sqrt{a_x^2+a_y^2}=\left(\frac{2\pi}{T}\right)^2 R\sqrt{\cos^2\left(\frac{2\pi t}{T}\right)+\sin^2\left(\frac{2\pi t}{T}\right)}=\frac{4\pi^2R}{T^2},$$

independent of time. This is the magnitude of the radial acceleration for a point moving on a circle of radius R with constant angular velocity $\frac{2\pi}{T}$. For motion that consists of this circular motion superimposed on motion with constant velocity $(\vec{a}=0)$, the acceleration due to the circular motion will be the total acceleration.

10.81: For rolling without slipping, the kinetic energy is $(1/2)(m+I/R^2)v^2=(5/6)mv^2$; initially, this is 32.0 J and at the return to the bottom it is 8.0 J. Friction has done -24.0 J of work, -12.0 J each going up and down. The potential energy at the highest point was 20.0 J, so the height above the ground was $\frac{20.0\text{ J}}{(0.600\text{ kg})(9.80\text{ m/s}^2)}=3.40\text{ m}$.

10.82: Differentiating , and obtaining the answer to part (b),

$$\omega=\frac{d\theta}{dt}=3bt^2=3b\left(\frac{\theta}{b}\right)^{2/3}=3b^{1/3}\theta^{2/3},$$

$$\alpha=-\frac{d\omega}{dt}=6bt=6b\left(\frac{\theta}{b}\right)^{1/3}=6b^{2/3}\theta^{1/3}.$$

a) $$W=\int I_{\text{cm}}\alpha\,d\theta=6b^{2/3}I_{\text{cm}}\int\theta^{1/3}d\theta=\frac{9}{2}I_{\text{cm}}b^{2/3}\theta^{4/3}.$$

c) The kinetic energy is

$$K=\frac{1}{2}I_{\text{cm}}\omega^2=\frac{9}{2}I_{\text{cm}}b^{2/3}\theta^{4/3},$$

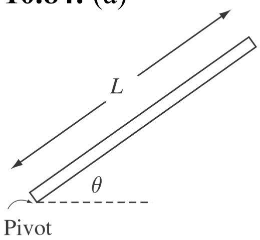
in agreement with Eq. (10.25); the total work done is the change in kinetic energy.

10.83: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed *v*, the pulley has angular velocity *v/R* and the cylinder has angular velocity *v/2R*, the total kinetic energy is

$$K=\frac{1}{2}\left[Mv^2+\frac{M(2R)^2}{2}(v/2R)^2+\frac{MR^2}{2}(v/R)^2+Mv^2\right]=\frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance y, $K=Mgy$, or $v^2=(2/3)gy$. For constant acceleration, $v^2=2ay$, and comparison of the two expressions gives $a=g/3$.

10.84: (a)



$$\Sigma\tau = I\alpha$$

$$mg\frac{1}{2}\cos\theta = \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}g\frac{\cos\theta}{\text{L}} = \left(\frac{3}{2}\right)\frac{(9.80\text{ m/s}^2)\cos 60°}{8.00\text{ m}} = 0.92\text{ rad/s}^2$$

(b) As the bridge lowers, θ changes, so α is not constant. Therefore Eq. (9.17) is not valid.

(c) Conservation of energy:

$$PE_\text{i} = KE_\text{f} \rightarrow mgh = \frac{1}{2}I\omega^2$$

$$mg\tfrac{L}{2}\sin\theta = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$\omega = \sqrt{\frac{3g\sin\theta}{L}}$$

$$= \sqrt{\frac{3(9.8\text{ m/s}^2)\sin 60°}{8.00\text{ m}}} = 1.78\text{ rad/s}$$

10.85: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34\text{ m/s}$.

Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

$L_1 = mvr = (5.00\text{ kg})(15.34\text{m/s})(2.00\text{ m}) = 153.4\text{ kg}\cdot\text{m}^2$

Immediately after the collsion the bar and both balls are rotating together.

$L_2 = I_\text{tot}\omega$

$I_\text{tot} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00\text{ kg})(4.00\text{ m})^2 + 2(5.00\text{ kg})(2.00\text{ m})^2 = 50.67\text{ kg}\cdot\text{m}^2$

$L_2 = L_1 = 153.4\text{ kg}\cdot\text{m}^2$

$\omega = L_2/I_\text{tot} = 3.027\text{ rad/s}$

Just after the collision the second ball has linear speed

$v = rw = (2.00\text{ m})(3.027\text{rad/s}) = 6.055\text{m/s}$ and is moving upward.

$\frac{1}{2}mv^2 = mgy$ gives y = 1.87 m for the height the second ball goes.

10.86: a) The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant. As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by Eq. (10.33),

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \left[\frac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2} \right] = \omega_1 \frac{5.00 \times 10^{-4}\ \text{kg} \cdot \text{m}^2}{2.00 \times 10^{-3}\ \text{kg} \cdot \text{m}^2} = \frac{\omega_1}{4},$$

ans so $\omega_2 = 7.5\ \text{rev/min}$. Note that conversion from rev/min to rad/s is not necessary.

b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 7.5 rev/min.

10.87: The intial angular momentum of the bullet is $(m/4)(v)(L/2)$, and the final moment of intertia of the rod and bullet is $(m/3)L^2 + (m/4)(L/2)^2 = (19/48)mL^2$. Setting the initial angular moment equal to ωI and solving for ω gives $\omega = \dfrac{mvL/8}{(19/48)mL^2} = \dfrac{6}{19} v/L.$

b) $$\frac{(1/2)I\omega^2}{(1/2)(m/4)v^2} = \frac{(19/48)mL^2((6/19)(v/L))^2}{(m/4)v^2} = \frac{3}{19}.$$

10.88: Assuming the blow to be concentrated at a point (or using a suitably chosen "average" point) at a distance r from the hinge, $\Sigma\tau_{\text{ave}} = rF_{\text{ave,}}$ and $\Delta L = rF_{\text{ave}}\Delta t = rJ$. The angular velocity ω is then

$$\omega = \frac{\Delta L}{I} = \frac{rF_{\text{ave}}\Delta t}{I} = \frac{(l/2)F_{\text{ave}}\Delta t}{\frac{1}{3}ml^2} = \frac{3}{2}\frac{F_{\text{ave}}\Delta t}{ml},$$

Where l is the width of the door. Substitution of the given numeral values gives $\omega = 0.514\ \text{rad/s}$.

10.89: a) The initial angular momentum is $L = mv(l/2)$ and the final moment of inertia is $I = I_0 + m(l/2)^2$, so

$$\omega = \frac{mv(l/2)}{(M/3)l^2 + m(l/2)^2} = 5.46\ \text{rad/s}.$$

b) $(M + m)gh = (1/2)\omega^2 I$, and after solving for h and substitution of numerical values, $h = 3.16 \times 10^{-2}$ m. c) Rather than recalculate the needed value of ω, note that ω will be proportional to v and hence h will be proportional to v^2; for the board to swing all the way over, $h = 0.250$ m. and so $v = (360\ \text{m/s})\sqrt{\frac{0.250\ \text{m}}{0.0316\ \text{m}}} = 1012\ \text{m/s}.$

10.90: Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$, or, using the fact that for a common mass the moment of inertia is proportional to the square of the radius, $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0 + \Delta R)^2(\omega_0 + \Delta\omega) \sim R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $\Delta\omega^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1\,\text{cm}.$$

10.91: The initial angular momentum is $L_1 = \omega_0 I_A$ and the initial kinetic energy is $K_1 = I_A\omega_0^2/2$. The final total moment of inertia is $4I_A$, so the final angular velocity is $(1/4)\omega_0$ and the final kinetic energy is $(1/2)4I_A(\omega_0/4)^2 = (1/4)K_1$. (This result may be obtained more directly from $K = L^2/I$. Thus, $\Delta K = -(3/4)K_1$ and $K_1 = -(4/3)(-2400\,\text{J})$ $= 3200\,\text{J}$.

10.92: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\dfrac{v^2}{r}$. The block's angular momentum with respect to the hole is $L = mvr$, so in terms of the angular momentum,

$$T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}.$$

The radius at which the string breaks can be related to the initial angular momentum by

$$r^3 = \frac{L^2}{mT_{\max}} = \frac{(mv_1r_1)^2}{mT_{\max}} = \frac{((0.250\,\text{kg})(4.00\,\text{m/s})(0.800\,\text{m}))^2}{(0.250\,\text{kg})(30.0\,\text{N})},$$

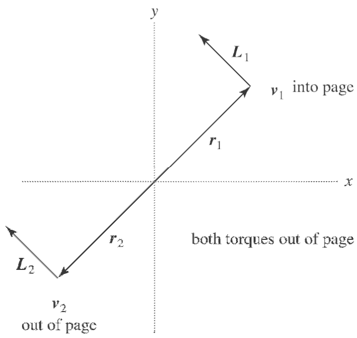
from which $r = 0.440\,\text{m}$.

10.93: The train's speed relative to the earth is $0.600\,\text{m/s} + \omega\,(0.475\,\text{m})$, so the total angular momentum is

$$((0.600\,\text{m/s}) + \omega(0.475\,\text{m}))(1.20\,\text{kg})(0.475\,\text{m}) + \omega(1/2)(7.00\,\text{kg})\left(\frac{1.00\,\text{m}}{2}\right)^2 = 0,$$

from which $\omega = -0.298\,\text{rad/s}$, with the minus sign indicating that the turntable moves clockwise, as expected.

10.94: a), g)



b) Using the vector product form for the angular momentum, $\vec{v}_1 = -\vec{v}_2$ and $\vec{r}_1 = -\vec{r}_2$, so

$$m\vec{r}_2 \times \vec{v}_2 = m\vec{r}_1 \times \vec{v}_1,$$

so the angular momenta are the same. c) Let $\vec{\omega} = \omega\hat{j}$. Then,

$$\vec{v}_1 = \vec{\omega} \times \vec{r}_1 = \omega\left(z\hat{i} - x\hat{k}\right), \text{ and}$$

$$\vec{L}_1 = m\vec{r}_1 \times \vec{v}_1 = m\omega\left((-xR)\hat{i} + \left(x^2 + y^2\right)\hat{j} + (xR)\hat{k}\right).$$

With $x^2 + y^2 = R^2$, the magnitude of $\vec{L}$ is $2m\omega R^2$, and $\vec{L}_1 \cdot \vec{\omega} = m\omega^2 R^2$, and so $\cos\theta = \frac{m\omega^2 R^2}{(2m\omega R^2)(\omega)} = \frac{1}{2}$, and $\theta = \frac{\pi}{6}$. This is true for $\vec{L}_2$ as well, so the total angular momentum makes an angle of $\frac{\pi}{6}$ with the $+y$-axis. d) From the intermediate calculation of part (c), $L_{y1} = m\omega R^2 = mvR$, so the total y-component of angular momentum is $L_y = 2mvR$. e) L_y is constant, so the net y-component of torque is zero. f) Each particle moves in a circle of radius R with speed v, and so is subject to an inward force of magnitude mv^2/R. The lever arm of this force is R, so the torque on each has magnitude mv^2. These forces are directed in opposite directions for the two particles, and the position vectors are opposite each other, so the torques have the same magnitude and direction, and the net torque has magnitude $2mv^2$.

10.95: a) The initial angular momentum with respect to the pivot is mvr, and the final total moment of inertia is $I + mr^2$, so the final angular velocity is $\omega = mvr/(mr^2 + I)$.

b) The kinetic energy after the collision is

$$K = \frac{1}{2}\omega^2(mr^2 + I) = (M + m)gh, \text{ or}$$

$$\omega = \sqrt{\frac{2(M + m)gh}{(mr^2 + I)}}.$$

c) Substitution of $I = Mr^2$ into either of the result of part (a) gives $\omega = \left(\frac{m}{m + M}\right)(v/r)$, and into the result of part (b), $\omega = \sqrt{2gh}(1/r)$, which are consistent with the forms for v.

10.96: The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(\mathrm{I} + mR^2)$, so

$$\omega_2 = \frac{I\omega_1 - mRv_1}{I + mR^2}$$

$$= \frac{(80\ \mathrm{kg\cdot m^2})(0.200\ \mathrm{rad/s}) - (55.0\ \mathrm{kg})(3.00\ \mathrm{m})(2.8\ \mathrm{m/s})}{(80\ \mathrm{kg\cdot m^2}) + (55.0\ \mathrm{kg})(3.00\ \mathrm{m})^2}$$

$$= -0.776\ \mathrm{rad/s},$$

where the minus sign indicates that the turntable has reversed its direction of motion *(i.e.,* the man had the larger magnitude of angular momentum initially).

10.97: From Eq. (10.36),

$$\Omega = \frac{\omega r}{\mathrm{I}\omega} = \frac{(50.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.040\ \mathrm{m})}{(0.085\ \mathrm{kg\cdot m^2})((6.0\ \mathrm{m/s})/(0.33\ \mathrm{m}))} = 12.7\ \mathrm{rad/s},$$

or 13 rad/s to two figures, which is quite large.

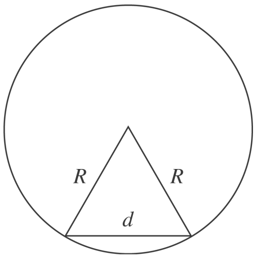
10.98: The velocity of the center of mass will change by $\Delta v_{\text{cm}} = \frac{J}{m}$, and the angular velocity will change by $\Delta\omega = \frac{J(x-x_{\text{cm}})}{I}\cdot$ The change is velocity of the end of the bat will then be $\Delta v_{\text{end}} = \Delta v_{\text{cm}} \_ \Delta\omega x_{\text{cm}} = \dfrac{J}{m} - \dfrac{J(x-x_{cm})x_{\text{cm}}}{I}\cdot$ Setting $\Delta v_{\text{end}} = 0$ allows cancellation of J, and gives $I = (x - x_{\text{cm}})x_{\text{cm}}m$, which when solved for x is

$$x = \frac{I}{x_{\text{cm}}m} + x_{\text{cm}} = \frac{(5.30\times10^{-2}\,\text{kg}\cdot\text{m}^2)}{(0.600\text{ m})(0.800\text{ kg})} + (0.600\text{ m}) = 0.710\text{ m}.$$

10.99: In Fig. (10.34(a)), if the vector $\vec{r}$, and hence the vector $\vec{L}$ are not horizontal but make an angle β with the horizontal, the torque will still be horizontal (the torque must be perpendicular to the vertical weight). The magnitude of the torque will be $\omega\, r \cos\beta$, and this torque will change the direction of the horizontal component of the angular momentum, which has magnitude $L\cos\beta$. Thus, the situation of Fig. (10.36) is reproduced, but with $\vec{L}_{\text{horiz}}$ instead of $\vec{L}$. Then, the expression found in Eq. (10.36) becomes

$$\Omega = \frac{d\phi}{dt} = \frac{\left|d\vec{L}\right| / \left|\vec{L}_{\text{horiz}}\right|}{dt} = \frac{\tau}{\left|\vec{L}_{\text{horiz}}\right|} = \frac{mgr\cos\beta}{L\cos\beta} = \frac{\omega r}{\mathrm{I}\omega}\cdot$$

10.100: a)



The distance from the center of the ball to the midpoint of the line joining the points where the ball is in contact with the rails is $\sqrt{R^2 - (d/2)^2}$, so $v_{\text{cm}} = \omega\sqrt{R^2 - d^2/4}.$ when $d = 0,$ this reduces to $v_{cm} = \omega R,$ the same as rolling on a flat surface. When $d = 2R,$ the rolling radius approaches zero, and $v_{cm} \to 0$ for any $\omega.$

b)

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

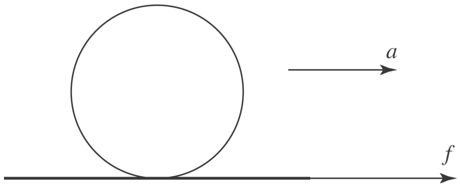
$$= \frac{1}{2}\left[mv_{\text{cm}}^2 + (2/5)mR^2\left(\frac{v_{\text{cm}}}{\sqrt{R^2 - (d^2/4)}}\right)^2\right]$$

c)

$$= \frac{mv_{\text{cm}}^2}{10}\left[5 + \frac{2}{(1 - d^2/4R^2)}\right].$$

Setting this equal to mgh and solving for v_{cm} gives the desired result. c) The denominator in the square root in the expression for v_{cm} is larger than for the case $d = 0,$ so v_{cm} is smaller. For a given speed, ω is large than the $d = 0$ case, so a larger fraction of the kinetic energy is rotational, and the translational kinetic energy, and hence v_{cm}, is smaller. d) Setting the expression in part (b) equal to 0.95 of that of the $d = 0$ case and solving for the ratio d/R gives $d/R = 1.05.$ Setting the ratio equal to 0.995 gives $d/R = 0.37.$

10.101: a)



The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\frac{fR}{I} = \frac{\mu_k MgR}{(1/2)MR^2} = \frac{2\mu_k g}{R}$. b) Setting $v = at = \omega R = (\omega_0 - \omega t)R$ and solving for t gives

$$t = \frac{R\omega_0}{a + R\alpha} = \frac{R\omega_0}{\mu_k g + 2\mu_k g} = \frac{R\omega_0}{3\mu_k g},$$

and

$$d = \frac{1}{2}at^2 = \frac{1}{2}(\mu_k g)\left(\frac{R\omega_0}{3\mu_k g}\right)^2 = \frac{R^2\omega_0^2}{18\mu_k g}.$$

c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(\text{at})^2$, so the change in kinetic energy is

$$\frac{3}{4}M\left(\mu_k g\frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = \frac{1}{6}MR^2\omega_0^2.$$

10.102: Denoting the upward forces that the hands exert as F_L and F_R, the conditions that F_L and F_R must satisfy are

$$F_L + F_R = w$$
$$F_L - F_R = \Omega\frac{I\omega}{r},$$

where the second equation is $\tau = \Omega L$, divided by *r*. These two equations can be solved for the forces by first adding and then subtracting, yielding

$$F_L = \frac{1}{2}\left(\omega + \Omega\frac{I\omega}{r}\right)$$
$$F_R = \frac{1}{2}\left(\omega - \Omega\frac{I\omega}{r}\right).$$

Using the values $\omega = mg = (8.00\text{ kg})(9.80\text{ m/s}^2) = 78.4\text{ N}$ and

$$\frac{I\omega}{r} = \frac{(8.00\text{ kg})(0.325\text{ m})^2(5.00\text{ rev/s} \times 2\pi\text{ rad/rev})}{(0.200\text{ m})} = 132.7\text{ kg}\cdot\text{m/s}$$

gives

$$F_L = 39.2\text{ N} + \Omega(66.4\text{ N}\cdot\text{s}),\ F_R = 39.2\text{ N} - \Omega(66.4\text{ N}\cdot\text{s}).$$

a) $\Omega = 0, F_L = F_R = 39.2\text{ N}.$

b) $\Omega = 0.05\text{ rev/s} = 0.314\text{ rad/s}, F_L = 60.0\text{ N}, F_R = 18.4\text{ N}.$

c) $\Omega = 0.3\text{ rev/s} = 1.89\text{ rad/s}, F_L = 165\text{ N}, F_R = -86.2\text{ N},$
with the minus sign indicating a downward force.

d) $F_R = 0$ gives $\Omega = \frac{39.2\text{ N}}{66.4\text{ N}\cdot\text{s}} = 0.575\text{ rad/s}$, which is 0.0916 rev/s.

10.103: a) See Problem 10.92; $T = mv_1^2 r_1^2/r^3$. b) $\vec{T}$ and $d\vec{r}$ are always antiparallel, so

$$W = -\int_{r1}^{r2} T\,dr = mv_1^2 r_1^2 \int_{r2}^{r1} \frac{dr}{r^3} = \frac{mv_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

c) $v_2 = v_1(r_1/r_2)$, so

$$\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{mv_1^2}{2}\left[\left(\frac{r_1}{r_2}\right)^2 - 1\right],$$

which is the same as the work found in part (b).

Capítulo 11

11.1: Take the origin to be at the center of the small ball; then,

$$x_{\text{cm}} = \frac{(1.00\ \text{kg})(0) + (2.00\ \text{kg})(0.580\ \text{m})}{3.00\ \text{kg}} = 0.387\ \text{m}$$

from the center of the small ball.

11.2: The calculation of Exercise 11.1 becomes

$$x_{\text{cm}} = \frac{(1.00\ \text{kg})(0) + (1.50\ \text{kg})(0.280\ \text{m}) + (2.00\ \text{kg})(0.580\ \text{m})}{4.50\ \text{kg}} = 0.351\ \text{m}$$

This result is smaller than the one obtained in Exercise 11.1.

11.3: In the notation of Example 11.1, take the origin to be the point S, and let the child's distance from this point be x. Then,

$$s_{\text{cm}} = \frac{M(-D/2) + mx}{M + m} = 0, \ \ x = \frac{MD}{2m} = 1.125\ \text{m},$$

which is $(L/2 - D/2)/2$, halfway between the point S and the end of the plank.

11.4: a) The force is applied at the center of mass, so the applied force must have the same magnitude as the weight of the door, or $300\ \text{N}$. In this case, the hinge exerts no force.

b) With respect to the hinge, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or $150\ \text{N}$. The hinge supplies an upward force of $300\ \text{N} - 150\ \text{N} = 150\ \text{N}$.

11.5: $F(8.0\ \text{m})\sin 40° = (2800\ \text{N})(10.0\ \text{m})$, so $F = 5.45\ \text{kN}$, keeping an extra figure.

11.6: The other person lifts with a force of $160\ \text{N} - 60\ \text{N} = 100\ \text{N}$. Taking torques about the point where the 60-N force is applied,

$$(100\ \text{N})x = (160\ \text{N})(1.50\ \text{m}), \ \text{or} \ x = (1.50\ \text{m})\left(\frac{160\ \text{N}}{100\ \text{N}}\right) = 2.40\ \text{m}.$$

11.7: If the board is taken to be massless, the weight of the motor is the sum of the applied forces, 1000 N. The motor is a distance $\frac{(2.00\ \text{m})(600\ \text{N})}{(1000\ \text{N})} = 1.200\ \text{m}$ from the end where the 400-N force is applied.

11.8: The weight of the motor is $400\text{ N}+600\text{ N}-200\text{ N}=800\text{ N}$. Of the myriad ways to do this problem, a sneaky way is to say that the lifters each exert 100 N to the lift the board, leaving 500 N and 300 N to the lift the motor. Then, the distance of the motor from the end where the 600-N force is applied is $\frac{(2.00\text{ m})(300\text{ N})}{(800\text{ N})}=0.75\text{ m}$ .The center of gravity is located at $\frac{(200\text{ N})(1.0\text{ m})+(800\text{ N})(0.75\text{ m})}{(1000\text{ N})}=0.80\text{ m}$ from the end where the 600 N force is applied.

11.9: The torque due to T_x is $-T_x h=-\frac{Lw}{D}\cot\theta\, h$, and the torque due to T_y is $T_y D=Lw$. The sum of these torques is $Lw(1-\frac{h}{D}\cot\theta)$. From Figure (11.9(b)), $h=D\tan\theta$, so the net torque due to the tension in the tendon is zero.

11.10: a) Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force. For the vertical forces to balance, $n_2=w_1+w_\text{m}=160\text{ N}+740\text{ N}=900\text{ N}$, and the maximum frictional forces is $\mu_\text{s}n_2=(0.40)(900\text{ N})=360\text{ N}$ (see Figure 11.7(b)). b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

$$(4.0\text{ m})n_1=(1.5\text{ m})(160\text{ N})+(1.0\text{ m})(3/5))(740\text{ N})=684\text{ N}\cdot\text{m},$$

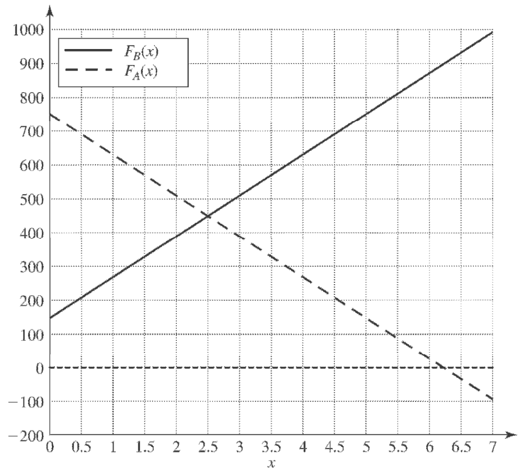
so $n_1=171.0\text{ N}$, keeping extra figures. This horizontal force about must be balanced by the frictional force, which must then be 170 N to two figures. c) Setting the frictional force, and hence n_1, equal to the maximum of 360 N and solving for the distance x along the ladder,

$$(4.0\text{ m})(360\text{ N})=(1.50\text{ m})(160\text{ N})+x(3/5)(740\text{ N}),$$

so $x=2.70$ m, or 2.7 m to two figures.

11.11: Take torques about the left end of the board in Figure (11.21). a) The force F at the support point is found from $F(1.00\text{ m})=+(280\text{ N})(1.50\text{ m})+(500\text{ N})(3.00\text{ m})$, or $F=1920\text{ N}$. b) The net force must be zero, so the force at the left end is $(1920\text{ N})-(500\text{ N})-(280\text{ N})=1140\text{ N}$, downward.

11.12: a)



b) $x = 6.25$ m when $F_A = 0$, which is 1.25 m beyond point B. c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900$ N. The distance that point B must be from the right end is then $\frac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50\text{ m}$.

11.13: In both cases, the tension in the vertical cable is the weight ω. a) Denote the length of the horizontal part of the cable by L. Taking torques about the pivot point, $TL\tan 30.0° = wL + w(L/2)$, from which $T = 2.60w$. The pivot exerts an upward vertical force of $2w$ and a horizontal force of $2.60w$, so the magnitude of this force is $3.28w$, directed $37.6°$ from the horizontal. b) Denote the length of the strut by L, and note that the angle between the diagonal part of the cable and the strut is $15.0°$. Taking torques about the pivot point, $TL\sin 15.0° = wL\sin 45.0° + (w/2)L\sin 45°$, so $T = 4.10w$. The horizontal force exerted by the pivot on the strut is then $T\cos 30.0° = 3.55\omega$ and the vertical force is $(2w) + T\sin 30° = 4.05w$, for a magnitude of $5.38w$, directed $48.8°$.

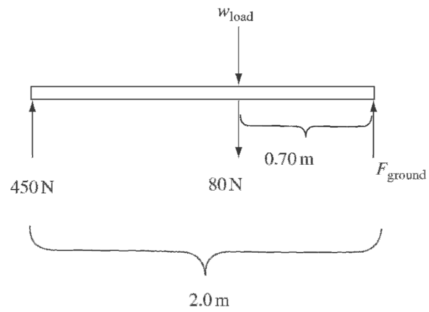
11.14: a) Taking torques about the pivot, and using the 3-4-5 geometry,

$$(4.00\text{ m})(3/5)T = (4.00\text{ m})(300\text{ N}) + (2.00\text{ m})(150\text{ N}),$$

so $T = 625$ N. b) The horizontal force must balance the horizontal component of the force exerted by the rope, or $T(4/5) = 500$ N. The vertical force is $300\text{ N} + 150\text{ N} - T(3/5) = 75\text{ N}$, upwards.

11.15: To find the horizontal force that one hinge exerts, take the torques about the other hinge; then, the vertical forces that the hinges exert have no torque. The horizontal force is found from $F_H(1.00\,\text{m}) = (280\,\text{N})(0.50\,\text{m})$, from which $F_H = 140\,\text{N}$. The top hinge exerts a force away from the door, and the bottom hinge exerts a force toward the door. Note that the magnitudes of the forces must be the same, since they are the only horizontal forces.

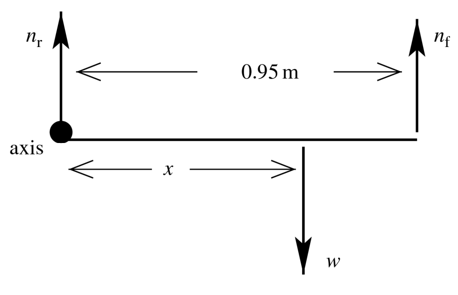
11.16: (a) Free body diagram of wheelbarrow:



$$\Sigma\tau_{\text{wheel}} = 0$$
$$-(450\text{ N})(2.0\text{ m}) + (80\text{ N})(0.70\text{ m}) + W_L(0.70\text{ m}) = 0$$
$$W_L = 1200\text{ N}$$

(b) From the ground.

11.17: Consider the forces on Clea.



$n_r = 89$ N, $n_f = 157$ N

$n_r + n_f = w$ so $w = 246$ N

11.18: a) Denote the length of the boom by L, and take torques about the pivot point. The tension in the guy wire is found from

$$TL\sin 60° = (5000\text{ N})\,L\cos 60.0° + (2600\text{ N})(0.35\,L)\cos 60.0°,$$

so $T = 3.14\text{ kN}$. The vertical force exerted on the boom by the pivot is the sum of the weights, 7.06 kN and the horizontal force is the tension, 3.14 kN. b) No; $\tan\left(\frac{\mathrm{F_v}}{\mathrm{F_H}}\right) \neq 0.$

11.19: To find the tension T_{L} in the left rope, take torques about the point where the rope at the right is connected to the bar. Then,
$T_{\mathrm{L}}\,(3.00\text{ m})\sin 150° = (240\text{ N})(1.50\text{ m}) + (90\text{ N})(0.50\text{ m})$, so $T_{\mathrm{L}} = 270\text{ N}$. The vertical component of the force that the rope at the end exerts must be
$(330\text{ N}) - (270\text{ N})\sin 150° = 195\text{ N}$, and the horizontal component of the force is
$-(270\text{ N})\cos 150°$, so the tension is the rope at the right is $T_{\mathrm{R}} = 304\text{ N}$. and $\theta = 39.9°$.

11.20: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point;
$T(3.00\text{ m}) = (1.00\text{ kN})(2.00\text{ m})\cos 25.0° + (5.00\text{ kN})(4.50\text{ m})\ \cos 25.0°$, or $T = 7.40\text{ kN}$. The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of T, $6.00\text{ kN} - T\cos 25.0° = 0.17\text{ kN}$. The horizontal force is $T\sin 25.0° = 3.13\text{ kN}$.

11.21: a) $F_1(3.00\text{ m}) - F_2(3.00\text{ m} + l) = (8.00\text{ N})(-l)$. This is given to have a magnitude of 6.40 N.m, so $l = 0.80\text{m}$. b) The net torque is clockwise, either by considering the figure or noting the torque found in part (a) was negative. c) About the point of contact of $\vec{F}_2$, the torque due to $\vec{F}_1$ is $-F_1 l$, and setting the magnitude of this torque to $6.40\text{ N}\cdot\text{m}$ gives $l = 0.80\text{ m}$, and the direction is again clockwise.

11.22: From Eq. (11.10),

$$Y = F\frac{l_0}{\Delta lA} = F\frac{(0.200\text{ m})}{(3.0\times 10^{-2}\text{ m})(50.0\times 10^{-4}\text{ m}^2)} = F(1333\text{ m}^{-2}).$$

Then, $F = 25.0\text{ N}$ corresponds to a Young's modulus of 3.3×10^4 Pa, and $F = 500\text{ N}$ corresponds to a Young's modulus of 6.7×10^5 Pa.

11.23: $$A=\frac{Fl_0}{Y\Delta l}=\frac{(400\ \text{N})(2.00\ \text{m})}{(20\times10^{10}\ \text{Pa})(0.25\times10^{-2}\ \text{m})}=1.60\times10^{-6}\ \text{m}^2,$$

and so $d=\sqrt{4A/\pi}=1.43\times10^{-3}$ m, or 1.4 mm to two figures.

11.24: a) The strain, from Eq. (11.12), is $\frac{\Delta l}{l_0}=\frac{F}{YA}$. For steel, using Y from Table (11.1) and $A=\pi\frac{d^2}{4}=1.77\times10^{-4}\ \text{m}^2$,

$$\frac{\Delta l}{l_0}=\frac{(4000\ \text{N})}{(2.0\times10^{11}\ \text{Pa})(1.77\times10^{-4}\ \text{m}^2)}=1.1\times10^{-4}.$$

Similarly, the strain for copper $(Y=1.10\times10^{11}\text{Pa})$ is 2.1×10^{-4}. b) Steel: $(1.1\times10^{-4})\times(0.750\ \text{m})=8.3\times10^{-5}\text{m}$. Copper: $(2.1\times10^{-4})(0.750\ \text{m})=1.6\times10^{-4}\text{m}\cdot$

11.25: From Eq. (11.10),

$$Y=\frac{(5000\ \text{N})(4.00\ \text{m})}{(0.50\times10^{-4}\text{m}^2)(0.20\times10^{-2}\ \text{m})}=2.0\times10^{11}\ \text{Pa}.$$

11.26: From Eq. (11.10),

$$Y=\frac{(65.0\ \text{kg})(9.80\ \text{m/s}^2)(45.0\ \text{m})}{(\pi(3.5\times10^{-3}\ \text{m})^2)(1.10\ \text{m})}=6.8\times10^8\ \text{Pa}.$$

11.27: a) The top wire is subject to a tension of $(16.0\ \text{kg})(9.80\ \text{m/s}^2)=157\ \text{N}$ and hence a tensile strain of $\frac{(157\ \text{N})}{(20\times10^{10}\ \text{Pa})(2.5\times10^{-7}\ \text{m}^2)}=3.14\times10^{-3}$, or 3.1×10^{-3} to two figures. The bottom wire is subject to a tension of 98.0 N, and a tensile strain of 1.96×10^{-3}, or 2.0×10^{-3} to two figures. b) $(3.14\times10^{-3})(0.500\ \text{m})=1.57\ \text{mm}$, $(1.96\times10^{-3})(0.500\ \text{m})=0.98\ \text{mm}$.

11.28: a) $\frac{(8000\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(12.5\times10^{-2}\ \text{m})^2}=1.6\times10^6\ \text{Pa}$. b) $\frac{1.6\times10^6\ \text{Pa}}{2.0\times10^{10}\ \text{Pa}}=0.8\times10^{-5}$.
c) $(0.8\times10^{-5})\times(2.50\ \text{m})=2\times10^{-5}\ \text{m}$.

11.29: $(2.8-1)(1.013\times10^5\ \text{Pa})(50.0\ \text{m}^2)=9.1\times10^6\ \text{N}$.

11.30: a) The volume would increase slightly. b) The volume change would be twice as great. c) The volume is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

11.31: a) $\frac{250\ \text{N}}{0.75\times10^{-4}\ \text{m}^2} = 3.33\times10^6\ \text{Pa}$. b) $(3.33\times10^6\ \text{Pa})(2)(200\times10^{-4}\ \text{m}^2) = 133\ \text{kN}$.

11.32: a) Solving Eq. (11.14) for the volume change,

$$\begin{aligned}\Delta V &= -kV\Delta P \\ &= -(45.8\times10^{-11}\ \text{Pa}^{-1})(1.00\ \text{m}^3)(1.16\times10^8\ \text{Pa} - 1.0\times10^5\ \text{Pa}) \\ &= -0.0531\ \text{m}^3.\end{aligned}$$

b) The mass of this amount of water not changed, but its volume has decreased to $1.000\ \text{m}^3 - 0.053\ \text{m}^3 = 0.947\ \text{m}^3$, and the density is now $\frac{1.03\times10^3\ \text{kg}}{0.947\ \text{m}^3} = 1.09\times10^3\ \text{kg/m}^3$.

11.33: $B = \dfrac{(600\ \text{cm}^3)(3.6\times10^6\ \text{Pa})}{(0.45\ \text{cm}^3)} = 4.8\times10^9\ \text{Pa}, \quad k = \dfrac{1}{B} = 2.1\times10^{-10}\ \text{Pa}^{-1}.$

11.34: a) Using Equation (11.17),

$$\text{Shear strain} = \frac{F_{\|}}{AS} = \frac{(9\times10^5\ \text{N})}{[(.10\ \text{m})(.005\ \text{m})][7.5\times10^{10}\ \text{Pa}]} = 2.4\times10^{-2}.$$

b) Using Equation (11.16), $x = \text{Shear stain}\cdot h = (.024)(.1\ \text{m}) = 2.4\times10^{-3}\ \text{m}.$

11.35: The area A in Eq. (11.17) has increased by a factor of 9, so the shear strain for the larger object would be $1/9$ that of the smaller.

11.36: Each rivet bears one-quarter of the force, so

$$\text{Shear stress} = \frac{F_{\|}}{A} = \frac{\frac{1}{4}(1.20\times10^4\ \text{N})}{\pi(.125\times10^{-2}\ \text{m})^2} = 6.11\times10^8\ \text{Pa}.$$

11.37: $\frac{F}{A} = \frac{(90.8\ \text{N})}{\pi(0.92\times10^{-3}\ \text{m})^2} = 3.41\times10^{7}\ \text{Pa}$, or $3.4\times10^{7}\ \text{Pa}$ to two figures.

11.38: a) $(1.6\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2) = 1.60\times10^{3}\ \text{N}$. b) If this were the case, the wire would stretch 6.4 mm.

c) $(6.5\times10^{-3})(20\times10^{10}\ \text{Pa})(5\times10^{-6}\ \text{m}^2) = 6.5\times10^{3}\ \text{N}$.

11.39: $$a = \frac{F_{\text{tot}}}{m} = \frac{(2.40\times10^{8}\ \text{Pa})(3.00\times10^{-4}\ \text{m}^2)/3}{(1200\ \text{kg})} - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2.$$

11.40: $A = \frac{350\ \text{N}}{4.7\times10^{8}\ \text{Pa}} = 7.45\times10^{-7}\ \text{m}^2$, so $d = \sqrt{4A/\pi} = 0.97\ \text{mm}$.

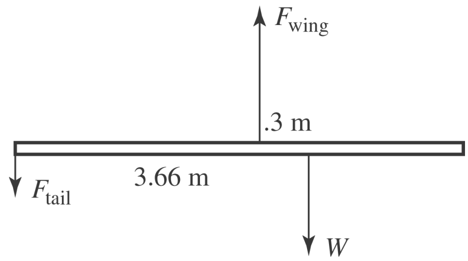
11.41: a) Take torques about the rear wheel, so that $f\omega d = \omega x_{\text{cm}}$, or $x_{\text{cm}} = fd$.
b) $(0.53)(2.46\ \text{m}) = 1.30\ \text{m}$ to three figures.

11.42: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0\ \text{N}) = (600\ \text{kg})(9.80\ \text{m/s}^2)(12.0\ \text{m}) + (200\ \text{kg})(9.80\ \text{m/s}^2)(6.0\ \text{m})$,
so $T = 6860\ \text{N}$. This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance x Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by x and the tension T by $T_{\text{max}} = 5.80\times10^{3}\ \text{N}$ and solve for x;

$$x = \frac{(5.80\times10^{3}\ \text{N})(12.0\ \text{m}) - (200\ \text{kg})(9.80\ \text{m/s}^2)(6.0\ \text{m})}{(600\ \text{kg})(9.80\ \text{m/s}^2)} = 9.84\ \text{m}.$$

11.43: For the airplane to remain in level flight, both $\sum F = 0$ and $\sum \tau = 0$.



Taking the clockwise direction as positive, and taking torques about the center of mass,

$$\text{Forces:} - F_{\text{tail}} - W + F_{\text{wing}} = 0$$

$$\text{Torques:} \ -(3.66\ \text{m})F_{\text{tail}} + (.3\ \text{m})F_{\text{wing}} = 0$$

A shortcut method is to write a second torque equation for torques about the tail, and solve for the $F_{\text{wing}}: -(3.66\ \text{m})(6700\ \text{N}) + (3.36\ \text{m})F_{\text{wing}} = 0.$ This gives $F_{\text{wing}} = 7300\ \text{N(up)}$, and $F_{\text{tail}} = 6700\ \text{N} - 7300\ \text{N} = -600\ \text{N(down)}.$

Note that the rear stabilizer provides a *downward* force, does not hold up the tail of the aircraft, but serves to counter the torque produced by the wing. Thus balance, along with weight, is a crucial factor in airplane loading.

11.44: The simplest way to do this is to consider the *changes* in the forces due to the extra weight of the box. Taking torques about the rear axle, the force on the front wheels is decreased by $3600\ \text{N}\frac{1.00\ \text{m}}{3.00\ \text{m}} = 1200\ \text{N},$ so the net force on the front wheels is $10{,}780\ \text{N} - 1200\ \text{N} = 9.58 \times 10^3\ \text{N}$ to three figures. The weight added to the rear wheels is then $3600\ \text{N} + 1200\ \text{N} = 4800\ \text{N},$ so the net force on the rear wheels is $8820\ \text{N} + 4800\ \text{N} = 1.36 \times 10^4\ \text{N},$ again to three figures.

b) Now we want a shift of $10{,}780\ \text{N}$ away from the front axle. Therefore, $W\frac{1.00\ \text{m}}{3.00\ \text{m}} = 10{,}780\ \text{N}$ and so $w = 32{,}340\ \text{N}.$

11.45: Take torques about the pivot point, which is 2.20 m from Karen and 1.65 m from Elwood. Then $w_{\text{Elwood}}(1.65\ \text{m}) = (420\ \text{N})(2.20\ \text{m}) + (240\ \text{N})(0.20\ \text{m}),$ so Elwood weighs 589 N. b) Equilibrium is neutral.

11.46: a) Denote the weight per unit length as α, so $w_1 = \alpha(10.0\,\text{cm})$, $w_2 = \alpha(8.0\,\text{cm})$, and $w_3 = \alpha l$.
The center of gravity is a distance x_{cm} to the right of point *O* where

$$x_{\text{cm}} = \frac{w_1(5.0\,\text{cm}) + w_2(9.5\,\text{cm}) + w_3(10.0\,\text{cm} - l/2)}{w_1 + w_2 + w_3}$$

$$= \frac{(10.0\,\text{cm})(5.0\,\text{cm}) + (8.0\,\text{cm})(9.5\,\text{cm}) + l(10.0\,\text{cm} - l/2)}{(10.0\,\text{cm}) + (8.0\,\text{cm}) + l}.$$

Setting $x_{\text{cm}} = 0$ gives a quadratic in l, which has as its positive root $l = 28.8\,\text{cm}$.

b) Changing the material from steel to copper would have no effect on the length l since the weight of each piece would change by the same amount.

11.47: Let $\vec{\boldsymbol{r}}_i' = \vec{\boldsymbol{r}}_i - \vec{\boldsymbol{R}}$, where $\vec{\boldsymbol{R}}$ is the vector from the point *O* to the point *P*. The torque for each force with respect to point *P* is then $\vec{\tau}_i' = \vec{\boldsymbol{r}}_i' \times \vec{\boldsymbol{F}}_i$, and so the net torque is

$$\begin{aligned}\sum \vec{\tau}_i &= \sum \left(\vec{\boldsymbol{r}}_i - \vec{\boldsymbol{R}}\right) \times \vec{\boldsymbol{F}}_i \\ &= \sum \vec{\boldsymbol{r}}_i \times \vec{\boldsymbol{F}}_i - \sum \vec{\boldsymbol{R}} \times \vec{\boldsymbol{F}}_i \\ &= \sum \vec{\boldsymbol{r}}_i \times \vec{\boldsymbol{F}}_i - \vec{\boldsymbol{R}} \times \sum \vec{\boldsymbol{F}}_i.\end{aligned}$$

In the last expression, the first term is the sum of the torques about point *O*, and the second term is given to be zero, so the net torques are the same.

11.48: From the figure (and from common sense), the force $\vec{\boldsymbol{F}}_1$ is directed along the length of the nail, and so has a moment arm of (0.0800 m) $\sin 60°$. The moment arm of $\vec{\boldsymbol{F}}_2$ is 0.300 m, so

$$F_2 = F_1 \frac{(0.0800\,\text{m})\ \sin 60°}{(0.300\,\text{m})} = (500\,\text{N})(0.231) = 116\,\text{N}.$$

11.49: The horizontal component of the force exerted on the bar by the hinge must balance the applied force $\vec{F}$, and so has magnitude 120.0 N and is to the left. Taking torques about point A, $(120.0\ \text{N})(4.00\ \text{m}) + F_{\text{V}}(3.00\ \text{m})$, so the vertical component is $-160\ \text{N}$, with the minus sign indicating a downward component, exerting a torque in a direction opposite that of the horizontal component. The force exerted by the bar on the hinge is equal in magnitude and opposite in direction to the force exerted by the hinge on the bar.

11.50: a) The tension in the string is $w_2 = 50\ \text{N}$, and the horizontal force on the bar must balance the horizontal component of the force that the string exerts on the bar, and is equal to $(50\ \text{N})\sin 37° = 30\ \text{N}$, to the left in the figure. The vertical force must be $(50\ \text{N})\cos 37° + 10\ \text{N} = 50\ \text{N}$, up. b) $\arctan\left(\dfrac{50\ \text{N}}{30\ \text{N}}\right) = 59°$. c) $\sqrt{(30\ \text{N})^2 + (50\ \text{N})^2} = 58\ \text{N}$.

d) Taking torques about (and measuring the distance from) the left end, $(50\ \text{N})x = (40\ \text{N})(5.0\ \text{m})$, so $x = 4.0\ \text{m}$, where only the vertical components of the forces exert torques.

11.51: a) Take torques about her hind feet. Her fore feet are 0.72 m from her hind feet, and so her fore feet together exert a force of $\frac{(190\ \text{N})(0.28\ \text{m})}{(0.72\ \text{m})} = 73.9\ \text{N}$, so each foot exerts a force of 36.9 N, keeping an extra figure. Each hind foot then exerts a force of 58.1 N. b) Again taking torques about the hind feet, the force exerted by the fore feet is $\frac{(190\ \text{N})(0.28\ \text{m}) + (25\ \text{N})(0.09\ \text{m})}{0.72\ \text{m}} = 105.1\ \text{N}$, so each fore foot exerts a force of 52.6 N and each hind foot exerts a force of 54.9 N.

11.52: a) Finding torques about the hinge, and using L as the length of the bridge and w_{T} and w_{B} for the weights of the truck and the raised section of the bridge,

$$TL\sin 70° = w_{\text{T}}\left(\tfrac{3}{4}L\right)\cos 30° + w_{\text{B}}\left(\tfrac{1}{2}\text{L}\right)\cos 30°,\ \text{so}$$

$$T = \frac{\left(\frac{3}{4}m_{\text{T}} + \frac{1}{2}m_{\text{B}}\right)(9.80\ \text{m/s}^2)\cos 30°}{\sin 70°} = 2.57\times 10^5\ \text{N}.$$

b) Horizontal: $T\cos(70° - 30°) = 1.97\times 10^5\ \text{N}$. Vertical: $w_{\text{T}} + w_{\text{B}} - T\sin 40°$
$= 2.46\times 10^5\ \text{N}$.

11.53: a) Take the torque exerted by $\vec{F}_2$ to be positive; the net torque is then $-F_1(x)\sin\phi + F_2(x+l)\sin\phi = Fl\sin\phi$, where F is the common magnitude of the forces. b) $\tau_1 = -(14.0\ \text{N})(3.0\ \text{m})\sin 37° = -25.3\ \text{N}\cdot\text{m}$, keeping an extra figure, and $\tau_2 = (14.0\ \text{N})(4.5\ \text{m})\sin 37° = 37.9\ \text{N}\cdot\text{m}$, and the net torque is $12.6\ \text{N}\cdot\text{m}$. About point P, $\tau_1 = (14.0\ \text{N})(3.0\ \text{m})(\sin 37°) = 25.3\ \text{N}\cdot\text{m}$, and $\tau_2 = (-14.0\ \text{N})(1.5\ \text{m})(\sin 37°) = -12.6\ \text{N}\cdot\text{m}$, and the net torque is $12.6\ \text{N}\cdot\text{m}$. The result of part (a) predicts $(14.0\ \text{N})(1.5\ \text{m})\sin 37°$, the same result.

11.54: a) Take torques about the pivot. The force that the ground exerts on the ladder is given to be vertical, and $F_V(6.0\ \text{m})\sin\theta = (250\ \text{N})(4.0\ \text{m})\sin\theta + (750\ \text{N})(1.50\ \text{m})\sin\theta$, so $F_V = 354\ \text{N}$. b) There are no other horizontal forces on the ladder, so the horizontal pivot force is zero. The vertical force that the pivot exerts on the ladder must be $(750\ \text{N}) + (250\ \text{N}) - (354\ \text{N}) = 646\ \text{N}$, up, so the ladder exerts a downward force of $646\ \text{N}$ on the pivot. c) The results in parts (a) and (b) are independent of θ.

11.55: a) $V = mg + w$ and $H = T$. To find the tension, take torques about the pivot point. Then, denoting the length of the strut by L,

$$T\left(\frac{2}{3}L\right)\sin\theta = w\left(\frac{2}{3}L\right)\cos\theta + mg\left(\frac{L}{6}\right)\cos\theta,\ \text{or}$$

$$T = \left(w + \frac{mg}{4}\right)\cot\theta.$$

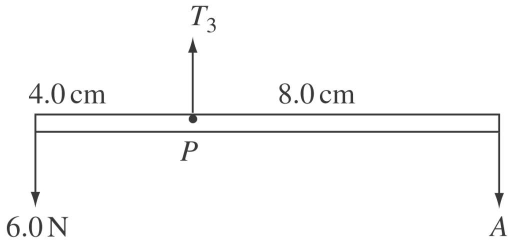
b) Solving the above for w, and using the maximum tension for T,

$$w = T\tan\theta - \frac{mg}{4} = (700\ \text{N})\tan 55.0° - (5.0\ \text{kg})(9.80\ \text{m/s}^2) = 951\ \text{N}.$$

c) Solving the expression obtained in part (a) for $\tan\theta$ and letting $\omega \to 0$, $\tan\theta = \frac{mg}{4T} = 0.700$, so $\theta = 4.00°$.

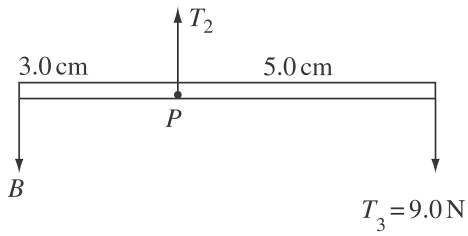
11.56: (a) and (b)

Lower rod:



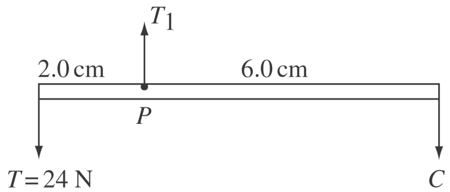
$$\Sigma\tau_p = 0 : (6.0\ \text{N})(4.0\ \text{cm}) = A(8.0\ \text{cm})$$
$$A = 3.0\ \text{N}$$
$$\Sigma F = 0 : T_3 = 6.0\ \text{N} + A = 6.0\ \text{N} + 3.0\ \text{N} = 9.0\ \text{N}$$

Middle rod:



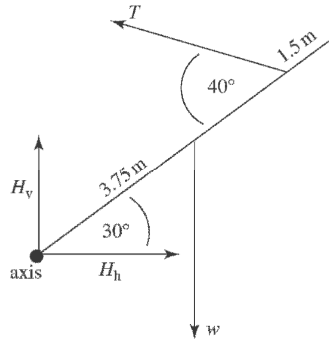
$$\Sigma\tau_p = 0 : B(3.0\ \text{cm}) = (9.0\ \text{N})(5.0\ \text{cm})$$
$$B = 15\ \text{N}$$
$$\Sigma F = 0 : T_2 = B + T_3 = 15\ \text{N} + 9.0\ \text{N} = 24\ \text{N}$$

Upper rod:



$$\Sigma\tau_p = 0 : (24\ \text{N})(2.0\ \text{cm}) = C(6.0\ \text{cm})$$
$$C = 8.0\ \text{N}$$
$$\Sigma F = 0 : T_1 = T_2 + C = 24\ \text{N} + 8.0\ \text{N} = 32\ \text{N}$$

11.57:

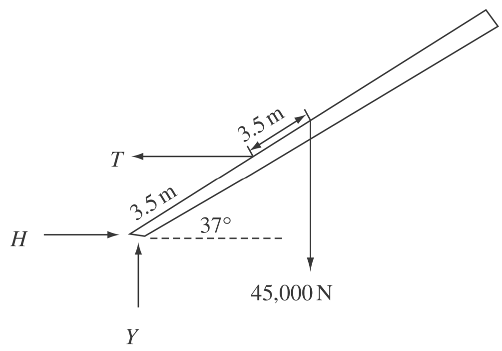


$\Sigma\tau = 0$, axis at hinge

$T(6.0\ \text{m})(\sin 40°) - w(3.75\ \text{m})(\cos 30°) = 0$

$$T = 760\ \text{N}$$

11.58: (a)



$$\Sigma\tau_{\text{Hinge}} = 0$$

$$T(3.5\ \text{m})\sin 37° = (45{,}000\ \text{N})(7.0\ \text{m})\cos 37°$$

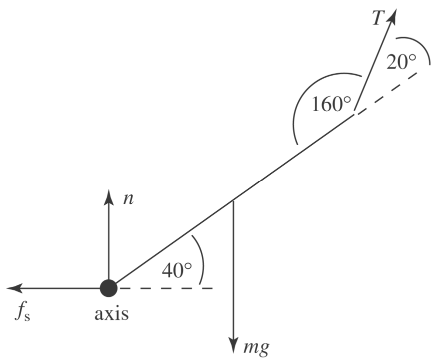
$$T = 120{,}000\ \text{N}$$

(b) $\sum F_x = 0 : H = T = 120{,}000\ \text{N}$

$\sum F_x = 0 : V = 45{,}000\ \text{N}$

The resultant force exerted by the hinge has magnitude $1.28 \times 10^5\ \text{N}$ and direction $20.6°$ above the horizontal.

11.59:



a) $\sum \tau = 0$, axis at lower end of beam
Let the length of the beam be L.

$$T(\sin 20°)L = -mg\left(\frac{L}{2}\right)\cos 40° = 0$$

$$T = \frac{\frac{1}{2}mg\cos 40°}{\sin 20°} = 2700\ \text{N}$$

b) Take $+y$ upward.

$\sum F_y = 0$ gives $n - w + T\sin 60° = 0$ so $n = 73.6\ \text{N}$
$\sum F_x = 0$ gives $f_s = T\cos 60° = 1372\ \text{N}$
$$f_s = \mu_s n, \mu_s = \frac{f_s}{n} = \frac{1372\ \text{N}}{73.6\ \text{N}} = 19$$

The floor must be very rough for the beam not to slip.

11.60: a) The center of mass of the beam is 1.0 m from the suspension point. Taking torques about the suspension point,

$$w(4.00\ \text{m}) + (140.0\ \text{N})(1.00\ \text{m}) = (100\ \text{N})(2.00\ \text{m})$$

(note that the common factor of $\sin\ 30°$ has been factored out), from which $w = 15.0\ \text{N}$.

b) In this case, a common factor of $\sin\ 45°$ would be factored out, and the result would be the same.

11.61: a) Taking torques about the hinged end of the pole $(200 \text{ N})(2.50 \text{ m}) + (600 \text{ N}) \times (5.00 \text{ m}) - T_y(5.00 \text{ m}) = 0$. Therefore the y-component of the tension is $T_y = 700 \text{ N}$. The x-component of the tension is then $T_x = \sqrt{(1000 \text{ N})^2 - (700 \text{ N})^2} = 714 \text{ N}$. The height above the pole that the wire must be attached is $(5.00 \text{ m})\frac{700}{714} = 4.90 \text{ m}$. b) The y-component of the tension remains 700 N and the x-component becomes $(714 \text{ N})\frac{4.90 \text{ m}}{4.40 \text{ m}} = 795 \text{ N}$, leading to a total tension of $\sqrt{(795 \text{ N})^2 + (700 \text{ N})^2} = 1059 \text{ N}$, an increase of 59 N.

11.62: A and B are straightforward, the tensions being the weights suspended; $T_A = (0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.353 \text{ N}, T_B = (0.0240 \text{ kg} + 0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.588$ N To find T_C and T_D, a trick making use of the right angle where the strings join is available; use a coordinate system with axes parallel to the strings. Then, $T_C = T_B \cos 36.9° = 0.470 \text{ N}, T_D = T_B \cos 53.1° = 0.353 \text{ N}$, To find T_E, take torques about the point where string F is attached;

$$\begin{aligned} T_E(1.000 \text{ m}) &= T_D \sin 36.9°(0.800 \text{ m}) + T_C \sin 53.1°(0.200 \text{ m}) \\ &\quad + (0.120 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m}) \\ &= 0.833 \text{ N} \cdot \text{m}, \end{aligned}$$

so $T_E = 0.833 \text{ N}. T_F$ may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180 \text{ kg})(9.80 \text{ m/s}^2) = 1.76 \text{ N}$, from which $T_F = 0.931 \text{ N}$.

11.63: a) The force will be vertical, and must support the weight of the sign, and is 300 N. Similarly, the torque must be that which balances the torque due to the sign's weight about the pivot, $(300 \text{ N})(0.75 \text{ m}) = 225 \text{ N} \cdot \text{m}$. b) The torque due to the wire must balance the torque due to the weight, again taking torques about the pivot. The minimum tension occurs when the wire is perpendicular to the lever arm, from one corner of the sign to the other. Thus, $T\sqrt{(1.50 \text{ m})^2 + (0.80 \text{ m})^2} = 225 \text{ N} \cdot \text{m}$, or $T = 132 \text{ N}$. The angle that the wire makes with the horizontal is $90° - \arctan\left(\frac{0.80}{1.50}\right) = 62.0°$. Thus, the vertical component of the force that the pivot exerts is $(300 \text{ N}) - (132 \text{ N}) \sin 62.0° = 183 \text{ N}$ and the horizontal force is $(132 \text{ N})\cos 62.0° = 62 \text{ N}$, for a magnitude of 193 N and an angle of $71°$ above the horizontal.

11.64: a) $\Delta w = -\sigma\,(\Delta l/l)\,w_0 = -(0.23)(9.0\times10^{-4})\sqrt{4(0.30\times10^{-4}\ \text{m}^2)/\pi} = 1.3\ \mu\text{m}.$

b)

$$F_\perp = AY\frac{\Delta l}{l} = AY\frac{1}{\sigma}\frac{\Delta w}{w}$$
$$= \frac{(2.1\times10^{11}\ \text{Pa})\,(\pi\,(2.0\times10^{-2}\ \text{m})^2)}{0.42}\ \frac{0.10\times10^{-3}\ \text{m}}{2.0\times10^{-2}\ \text{m}} = 3.1\times10^6\ \text{N},$$

where the Young's modulus for nickel has been used.

11.65: a) The tension in the horizontal part of the wire will be 240 N. Taking torques about the center of the disk, $(240\ \text{N})(0.250\ \text{m}) - w(1.00\ \text{m})) = 0$, or w = 60 N.
b) Balancing torques about the center of the disk in this case,
$(240\ \text{N})\,(0.250\ \text{m}) - ((60\ \text{N})(1.00\ \text{m}) + (20\ \text{N})(2.00\ \text{m}))\cos\theta = 0$, so $\theta = 53.1°$.

11.66: a) Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = \frac{w}{2}\cdot$ Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point),and noting that the moment arm of the normal force is $l\tan\theta, n\tan\theta = \frac{w}{2}\cdot$ Then, $\frac{f}{n} =\ \tan\theta < 0.40$, so $\theta <$ arctan (0.40) = 22°.

b) Taking torques as in part (a), and denoting the length of the meter stick as l,

$$fl = w\frac{l}{2} + w(l - x) \text{ and } nl\tan\theta = w\frac{l}{2} + wx.$$

In terms of the coefficient of friction μ_s,

$$\mu_s > \frac{f}{n} = \frac{\frac{l}{2} + (l - x)}{\frac{l}{2} + x}\tan\theta = \frac{3l - 2x}{l + 2x}\tan\theta.$$

Solving for x,

$$x > \frac{l}{2}\frac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2\ \text{cm}.$$

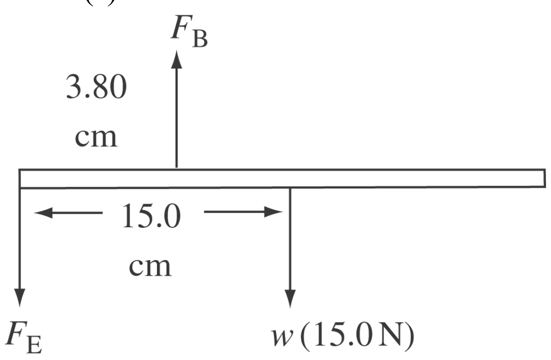
c) In the above expression, setting $x = 10$ cm and solving for μ_s gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

11.67: Consider torques around the point where the person on the bottom is lifting. The center of mass is displaced horizontally by a distance $(0.625\text{ m} - 0.25\text{ m})\sin 45°$ and the horizontal distance to the point where the upper person is lifting is $(1.25\text{ m})\sin 45°$, and so the upper lifts with a force of $w\frac{0.375\sin 45°}{1.25\sin 45°} = (0.300)w = 588\text{ N}$. The person on the bottom lifts with a force that is the difference between this force and the weight, 1.37 kN. The person above is lifting less.

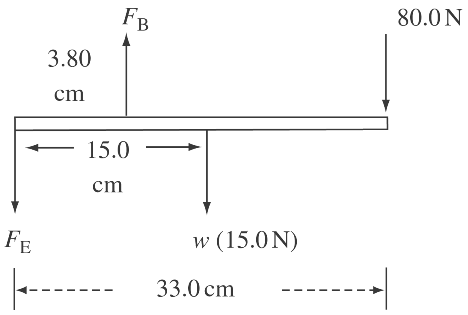
11.68:

(a)



$$\Sigma\tau_{\text{Elbow}} = 0$$
$$F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm})$$
$$F_B = 59.2\text{ N}$$

(b)



$$\Sigma\tau_E = 0$$
$$F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm}) + (80.0\text{ N})(33.0\text{ cm})$$
$$F_B = 754\text{ N}$$

11.69: a) The force diagram is given in Fig. 11.9.

$\Sigma\tau = 0$, axis at elbow

$wL - (T \sin\theta)D = 0$

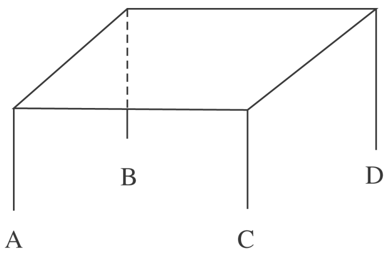
$\sin\theta = \dfrac{h}{\sqrt{h^2 + D^2}}$ so $w = T\dfrac{hD}{L\sqrt{h^2 + D^2}}$

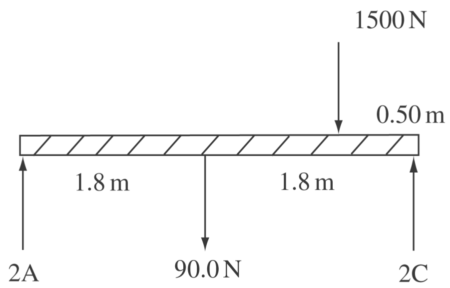
$w_{\max} = T_{\max}\dfrac{hD}{L\sqrt{h^2 + D^2}}$

b) $\dfrac{dw_{\max}}{dD} = \dfrac{T_{\max}h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive

c) The result of part (b) shows that $w_{\max}$ increases when D increases.

11.70:





By symmetry, A=B and C=D. Redraw the table as viewed from the AC side.

$\Sigma\tau$ (about right end) $= 0$:

$2A(3.6 \text{ m}) = (90.0 \text{ N})(1.8 \text{ m}) + (1500 \text{ N})(0.50 \text{ m})$

$A = 130 \text{ N} = B$

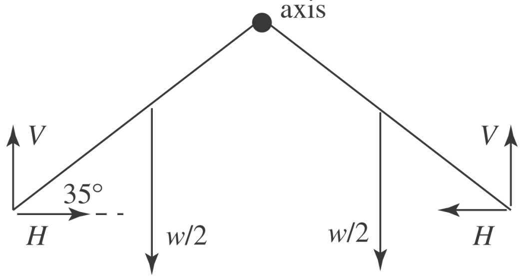
$\Sigma F = 0: A + B + C + D = 1590 \text{ N}$

Use $A = B = 130$ N and $C = D$

$C = D = 670$ N

By Newton's third law of motion, the forces A, B, C, and D on the table are the same as the forces the table exerts on the floor.

11.71: a) Consider the forces on the roof



V and H are the vertical and horizontal forces each wall exerts on the roof. $w = 20{,}000$ N is the total weight of the roof.

$2V = w$ so $V = w/2$

Apply $\Sigma\tau = 0$ to one half of the roof, with the axis along the line where the two halves join. Let each half have length L.

$(w/2)(L/2)(\cos 35.0°) + HL \sin 35.0° - VL \cos 35.0° = 0$

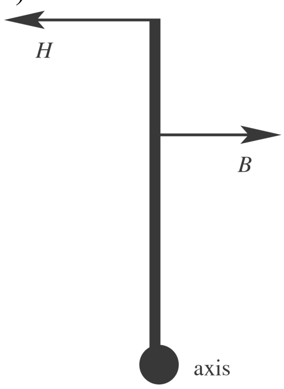
L divides out, and use $V = w/2$

$H \sin\ 35.0° = \frac{1}{4} w \cos\ 35.0°$

$$H = \frac{w}{4\tan 35.0°} = 7140\ \text{N}$$

By Newton's 3rd law, the roof exerts a horizontal, outward force on the wall. For torque about an axis at the lower end of the wall, at the ground, this force has a larger moment arm and hence larger torque the taller the walls.

b)



Consider the torques on one of the walls.

11.72: a) Take torques about the upper corner of the curb. The force $\vec{F}$ acts at a perpendicular distance $R-h$ and the weight acts at a perpendicular distance $\sqrt{R^2-(R-h)^2}=\sqrt{2Rh-h^2}$. Setting the torques equal for the minimum necessary force,

$$F = mg\frac{\sqrt{2Rh-h^2}}{R-h}.$$

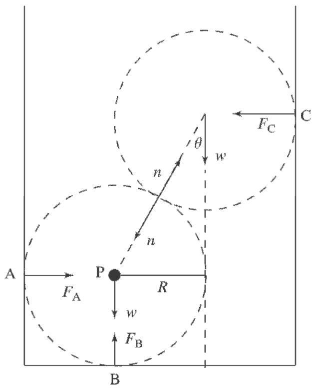
b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R-h$, so the minimum force is $(mg)\sqrt{2Rh-hv}$ /2*R*–*h*. c) Less force is required when the force is applied at the top of the wheel.

11.73: a) There are several ways to find the tension. Taking torques about point *B* (the force of the hinge at *A* is given as being vertical, and exerts no torque about *B*), the tension acts at distance $r=\sqrt{(4.00\text{ m})^2+(2.00\text{ m})^2}=4.47\text{ m}$ and at an angle of $\phi = 30° + \arctan\left(\frac{2.00}{4.00}\right) = 56.6°$. Setting $Tr\sin\varphi = (500\text{ N})(2.00\text{ m})$ and solving for T gives $T = 268\text{ N}$. b) The hinge at A is given as exerting no horizontal force, so taking torques about point D, the lever arm for the vertical force at point B is $(2.00\text{ m}) + (4.00\text{ m})\tan 30.0° = 4.31\text{ m}$, so the horizontal force at B is $\frac{(500\text{ N})(2.00\text{ m})}{4.31\text{ m}} = 232\text{ N}$. Using the result of part (a), however, $(268\text{ N})\cos 30.0° = 232\text{ N}$ In fact, finding the horizontal force at B first simplifies the calculation of the tension slightly. c) $(500\text{ N}) - (268\text{ N})\sin 30.0° = 366\text{ N}$. Equivalently, the result of part (b) could be used, taking torques about point C, to get the same result.

11.74: a) The center of gravity of top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ from the right edge of the upper block, so the overhang is $3L/4$.

b) Take the two-block combination from part (a), and place it on the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4)+(L/6)=11L/12$.
Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$. c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

11.75: a)



$$F_B = 2w = 1.47\text{ N}$$

$$\sin\theta = R/2R \text{ so } \theta = 30°$$

$$\tau = 0, \text{ axis at } P$$

$$F_C(2R\cos\theta) - wR = 0$$

$$F_C = \frac{mg}{2\cos 30°} = 0.424\text{ N}$$

$$F_A = F_C = 0.424\text{ N}$$

b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so

$$F_A = n\sin\theta$$

$$n = \frac{F_A}{\sin 30°} = 0.848\text{ N}$$

Could use instead that the vertical forces sum to zero

$$F_B - mg - n\cos\theta = 0$$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848\text{ N, which checks.}$$

11.76: (a) Writing an equation for the torque on the right-hand beam, using the hinge as an axis and taking counterclockwise rotation as positive:

$$F_{\text{wire}} L \sin\frac{\theta}{2} - F_c \frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$$

where θ is the angle between the beams, F_c is the force exerted by the cross bar, and w is the weight of one beam. The length drops out, and all other quantities except F_c are known, so

$$F_c = \frac{F_{\text{wire}} \sin\frac{\theta}{2} - \frac{1}{2} w \sin\frac{\theta}{2}}{\frac{1}{2}\cos\frac{\theta}{2}} = (2F_{\text{wire}} - w)\tan\frac{\theta}{2}$$

Therefore

$$F = 260 \tan\frac{53°}{2} = 130 \text{ N}$$

b) The cross bar is under compression, as can be seen by imagining the behavior of the two beams if the cross bar were removed. It is the cross bar that holds them apart.

c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmentry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the cross bar: 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

11.77: a) The angle at which the bale would slip is that for which $f = \mu_s N = \mu_s w \cos\beta = w \sin\beta$, or $\beta = \arctan(\mu_s) = 31.0°$. The angle at which the bale would tip is that for which the center of gravity is over the lower contact point, or $\arctan(\frac{0.25\text{ m})}{0.50\text{ m})} = 26.6°$, or 27° to two figures. The bale tips before it slips. b) The angle for tipping is unchanged, but the angle for slipping is $\arctan(0.40) = 21.8°$, or $22°$ to two figures. The bale now slips before it tips.

11.78: a) $F = f = \mu_k N = \mu_k mg = (0.35)(30.0\text{ kg})(9.80\text{ m/s}^2) = 103\text{ N}$

b) With respect to the forward edge of the bale, the lever arm of the weight is $\frac{0.250\text{ m}}{2} = 0.125\text{ m}$ and the lever arm h of the applied force is then $h = (0.125\text{ m})\frac{mg}{F} = (0.125\text{ m})\frac{1}{\mu_k} = \frac{0.125\text{ m}}{0.35} = 0.36\text{ m}$.

11.79: a) Take torques about the point where wheel B is in contact with the track. With respect to this point, the weight exerts a counterclockwise torque and the applied force and the force of wheel A both exert clockwise torques. Balancing torques, $F_A(2.00\text{ m}) + (F)(1.60\text{ m}) = (950\text{ N})(1.00\text{ m})$. Using $F = \mu_k w = 494\text{ N}, F_A = 80\text{ N, and } F_B = w - F_A = 870\text{ N}$. b) Again taking torques about the point where wheel B is in contact with the tract, and using $F = 494\text{ N}$ as in part (a), $(494\text{ N})\,h = (950\text{ N})(1.00\text{N})$, so $h = 1.92\text{ m}$.

11.80: a) The torque exerted by the cable about the left end is $TL\sin\theta$. For any angle θ, $\sin(180° - \theta) = \sin\theta$, so the tension T will be the same for either angle. The horizontal component of the force that the pivot exerts on the boom will be $T\cos\theta$ or $T\cos(180° - \theta) = -T\cos\theta$. b) From the result of part (a), $T \alpha \frac{1}{\sin\theta}$, and this becomes infinite as $\theta \to 0$ or $\to 180°$. Also, c), the tension is a minimum when $\sin\theta$ is a maximum, or $\theta = 90°$, a vertical string. d) There are no other horizontal forces, so for the boom to be in equilibrium, the pivot exerts zero horizontal force on the boom.

11.81: a) Taking torques about the contact point on the ground, $T(7.0\text{ m})\sin\theta = w(4.5\text{ m})\sin\theta$, so $T = (0.64)w = 3664\text{ N}$. The ground exerts a vertical force on the pole, of magnitude $w - T = 2052\text{ N}$. b) The factor of $\sin\theta$ appears in both terms of the equation representing the balancing of torques, and cancels.

11.82: a) Identifying x with Δl in Eq. (11.10), $k = YA/l_0$.
b) $(1/2)kx^2 = YAx^2/2l_0$.

11.83: a) At the bottom of the path the wire exerts a force equal in magnitude to the centripetal acceleration plus the weight,

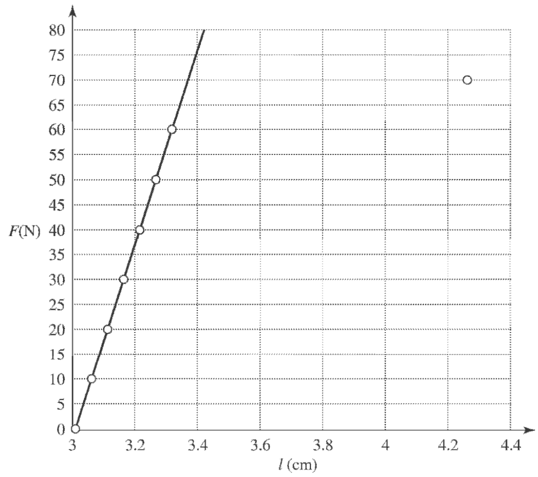
$$F = m(((2.00\text{ rev/s})(2\pi\text{ rad/rev}))^2(0.50\text{ m}) + 9.80\text{ m/s}^2) = 1.07\times10^3\text{ N}.$$

From Eq. (11.10), the elongation is

$$\frac{(1.07\times10^3\text{ N})(0.50\text{ m})}{(0.7\times10^{11}\text{ Pa})(0.014\times10^{-4}\text{ m}^2)} = 5.5\text{ mm}.$$

b) Using the same equations, at the top the force is 830 N, and the elongation is 0.0042 m.

11.84: a)



b) The ratio of the added force to the elongation, found from taking the slope of the graph, doing a least-squares fit to the linear part of the data, or from a casual glance at the data gives $\frac{F}{\Delta l} = 2.00 \times 10^4\ \text{N/m}$. From Eq. (11.10),

$$Y = \frac{F}{\Delta l}\frac{l_0}{A} = (2.00\times 10^4\ \text{N/m})\frac{(3.50\ \text{m})}{(\pi (0.35\times 10^{-3}\ \text{m})^2)} = 1.8\times 10^{11}\ \text{Pa}.$$

c) The total force at the proportional limit is $20.0\ \text{N} + 60\ \text{N} = 80\ \text{N}$, and the stress at this limit is $\frac{(80\ \text{N})}{\pi(0.35\times 10^{-3}\ \text{m})^2} = 2.1\times 10^8\ \text{Pa}$.

11.85: a) For the same stress, the tension in wire B must be two times in wire A, and so the weight must be suspended at a distance $(2/3)(1.05\ \text{m}) = 0.70\ \text{m}$ from wire A..

b) The product YA for wire B is $(4/3)$ that of wire B, so for the same strain, the tension in wire B must be $(4/3)$ that in wire A, and the weight must be 0.45 m from wire B.

11.86: a) Solving Eq. (11.10) for Δl and using the weight for F,

$$\Delta l = \frac{Fl_0}{YA} = \frac{(1900\ \text{N})(15.0\ \text{m})}{(2.0\times 10^{11}\ \text{Pa})(8.00\times 10^{-4}\ \text{m}^2)} = 1.8\times 10^{-4}\ \text{m}.$$

b) From Example 5.21, the force that each car exerts on the cable is $F = m\omega^2 l_0 = \frac{w}{g} w^2 l_0$, and so

$$\Delta l = \frac{Fl_0}{YA} = \frac{w\omega^2 l_0^2}{gYA} = \frac{(1900\ \text{N})(0.84\ \text{rad/s})^2(15.0\ \text{m})^2}{(9.80\ \text{m/s}^2)(2.0\times 10^{11}\ \text{Pa})(8.00\times 10^{-4}\ \text{m}^2)} = 1.9\times 10^{-4}\ \text{m}.$$

11.87: Use subscripts 1 to denote the copper and 2 to denote the steel. a) From Eq. (11.10), with $\Delta l_1 = \Delta l_2$ and $F_1 = F_2$,

$$L_2 = L_1\left(\frac{A_2Y_2}{A_1Y_1}\right) = (1.40\ \text{m})\left(\frac{(1.00\ \text{cm}^2)(21\times10^{10}\ \text{Pa})}{(2.00\ \text{cm}^2)(9\times10^{10}\ \text{Pa})}\right) = 1.63\ \text{m}.$$

b) For nickel, $\frac{F}{A_1} = 4.00\times10^8$ Pa and for brass, $\frac{F}{A_2} = 2.00\times10^8$ Pa. c) For nickel, $\frac{4.00\times10^8\ \text{Pa}}{21\times10^{10}\ \text{Pa}} = 1.9\times10^{-3}$ and for brass, $\frac{2.00\times10^8\ \text{Pa}}{9\times10^{10}\ \text{Pa}} = 2.2\times10^{-3}$.

11.88: a) $F_{\text{max}} = YA\left(\frac{\Delta l}{l_0}\right)_{\text{max}} = (1.4\times10^{10}\ \text{Pa})(3.0\times10^{-4}\ \text{m}^2)(0.010) = 4.2\times10^4\ \text{N}.$

b) Neglect the mass of the shins (actually the lower legs and feet) compared to the rest of the body. This allows the approximation that the compressive stress in the shin bones is uniform. The maximum height will be that for which the force exerted on each lower leg by the ground is F_{max} found in part (a), minus the person's weight. The impulse that the ground exerts is $J = (4.2\times10^4\ \text{N} - (70\ \text{kg})(9.80\ \text{m/s}^2))(0.030\ \text{s}) = 1.2\times10^3\ \text{kg}\cdot\text{m/s}$. The speed at the ground is $\sqrt{2gh}$, so $2J = m\sqrt{2gh}$ and solving for h,

$$h = \frac{1}{2g}\left(\frac{2J}{m}\right)^2 = 64\ \text{m},$$

but this is not recommended.

11.89: a) Two times as much, 0.36 mm, b) One-fourth (which is $(1/2)^2$) as much, 0.045 mm.c) The Young's modulus for copper is approximately one-half that for steel, so the wire would stretch about twice as much. $(0.18\ \text{mm})\frac{20\times10^{10}\ \text{Pa}}{11\times10^{10}\ \text{Pa}} = 0.33\ \text{mm}$.

11.90: Solving Eq. (11.14) for ΔV,

$$\Delta V = -kV_0\Delta P = -kV_0\frac{mg}{A}$$

$$= -(110\times10^{-11}\ \text{Pa}^{-1})(250\ \text{L})\frac{(1420\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(0.150\ \text{m})^2}$$

$$= -0.0541\ \text{L}.$$

The minus sign indicates that this is the volume by which the original hooch has shrunk, and is the extra volume that can be stored.

11.91: The normal component of the force is $F\cos\theta$ and the area (the intersection of the red plane and the bar in Figure (11.52)) is $A/\cos\theta$, so the normal stress is $(F/A)\cos^2\theta$

b) The tangential component of the force is $F\sin\theta$, so the shear stress is $(\mathrm{F/A})\sin\theta\cos\theta$.

c) $\cos^2\theta$ is a maximum when $\cos\theta = 1$, or $\theta = 0$. d) The shear stress can be expressed as $(F/2A)\sin(2\theta)$, which is maximized when $\sin(2\theta) = 1$, or $\theta = \dfrac{90°}{2} = 45°$. Differentiation of the original expression with respect to θ and setting the derivative equal to zero gives the same result.

11.92: a) Taking torques about the pivot, the tension T in the cable is related to the weight by $T\sin\theta\, l_0 = mgl_0/2$, so $T = \dfrac{mg}{2\sin\theta}$. The horizontal component of the force that the cable exerts on the rod, and hence the horizontal component of the force that the pivot exerts on the rod, is $\dfrac{mg}{2}\cot\theta$ and the stress is $\dfrac{mg}{2A}\cot\theta$.

b)

$$\Delta l = \frac{l_0 F}{AY} = \frac{mgl_0\cot\theta}{2AY}.$$

c) In terms of the density and length, $(m/A) = \rho l_0$, so the stress is $(\rho l_0 g/2)\cot\theta$ and the change in length is $(\rho l_0^2 g/2Y)\cot\theta$. d) Using the numerical values, the stress is 1.4×10^5 Pa and the change in length is 2.2×10^{-6} m. e) The stress is proportional to the length and the change in length is proportional to the square of the length, and so the quantities change by factors of 2 and 4.

11.93: a) Taking torques about the left edge of the left leg, the bookcase would tip when $F = \frac{(1500\ \text{N})(0.90\ \text{m})}{(1.80\ \text{m})} = 750\ \text{N}$, and would slip when $F = (\mu_s)(1500\ \text{N}) = 600\ \text{N}$, so the bookcase slides before tipping. b) If F is vertical, there will be no net horizontal force and the bookcase could not slide. Again taking torques about the left edge of the left leg, the force necessary to tip the case is $\frac{(1500\ \text{N})(0.90\ \text{m})}{(0.10\ \text{m})} = 13.5\ \text{kN}$.
c) To slide, the friction force is $f = \mu_s(w + F\cos\theta)$, and setting this equal to $F\sin\theta$ and solving for F gives

$$F = \frac{\mu_s w}{\sin\theta - \mu_s\cos\theta}.$$

To tip, the condition is that the normal force exerted by the right leg is zero, and taking torques about the left edge of the left leg,
$F\sin\theta\,(1.80\ \text{m}) + F\cos\theta(0.10\ \text{m}) = w(0.90\ \text{m})$, and solving for F gives

$$F = \frac{w}{(1/9)\cos\theta + 2\sin\theta}.$$

Setting the expression equal gives

$$\mu_s((1/9)\cos\theta + 2\sin\theta) = \sin\theta - \mu_s\cos\theta,$$

and solving for θ gives

$$\theta = \arctan\left(\frac{(10/9)\mu_s}{(1-2\mu_s)}\right) = 66°.$$

11.94: a) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $\frac{h}{2}$ and the lever arm of both the weight and the normal force is $h \tan\theta$, and so $F\frac{h}{2} = (n - w)h \tan\theta$. Taking torques about the upper point (where the rope is attached to the post), $f\,h = F\frac{h}{2}$. Using $f \le \mu_s n$ and solving for *F,*

$$F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\,nN)\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\,nN,$$

b) The above relations between F, n and f become

$$F\frac{3}{5}h = (n - w)h\tan\theta, f = \frac{2}{5}F,$$

and eliminating f and n and solving for F gives

$$F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1},$$

and substitution of numerical values gives 750 N to two figures. c) If the force is applied a distance y above the ground, the above relations become

$$Fy = (n - w)h\tan\theta, \; F(h - y) = fh,$$

which become, on eliminating n and f,

$$w \ge F\left[\frac{\left(1 - \frac{y}{h}\right)}{\mu_s} - \frac{\left(\frac{y}{h}\right)}{\tan\theta}\right].$$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of y is found by setting the term in square brackets equal to zero. Solving for y gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

11.95: Assume that the center of gravity of the loaded girder is at $L/2$, and that the cable is attached a distance x to the right of the pivot. The sine of the angle between the lever arm and the cable is then $h/\sqrt{h^2+((L/2)-x)^2}$, and the tension is obtained from balancing torques about the pivot;

$$T\left[\frac{hx}{\sqrt{h^2+((L/2)-x)^2}}\right]=wL/2,$$

where w is the total load (the exact value of w and the position of the center of gravity do not matter for the purposes of this problem). The minimum tension will occur when the term in square brackets is a maximum; differentiating and setting the derviative equal to zero gives a maximum, and hence a minimum tension, at $x_{\min}=(h^2/L)+(L/2)$. However, if $x_{\min}>L$, which occurs if $h>L/\sqrt{2}$, the cable must be attached at L, the furthest point to the right.

11.96: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless. a) Taking torques about the right end, $F_L(5.00\text{ m})=(480\text{ N})(3.40\text{ m})+(360\text{ N})(0.90\text{ m})$, so $F_L=391\text{ N}$. F_R may be found in a similar manner, or from $F_R=840\text{ N}-F_L=449\text{ N}$. b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\text{ m})=F_L(3.20\text{ m})-(480\text{ N})(1.60\text{ m})$, so $T=322.1\text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\text{ m})=F_R(1.80\text{ m})-(360\text{ N})(0.90\text{ m})$ gives the same result. c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, 480 N–391 N = 449 N–360 N = 89 N. The magnitude of the force at A is then

$$\sqrt{(322.1\text{ N})^2+(89\text{ N})^2}=334\text{ N}.$$

d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L'=F_L+(0.36)(800\text{ N})=679\text{ N}$, and $F_R'=F_R+(0.64)(800\text{ N})=961\text{ N}$. Using these forces in the form for the tension found in part (b) gives

$$T=\frac{F_L'(3.20\text{ m})-(480\text{ N})(1.60\text{ m})}{(1.50\text{ m})}=\frac{F_R'(1.80\text{ m})-(360\text{ N})(0.90\text{ m})}{(1.50\text{ m})}=936.53\text{ N},$$

which is 937 N to three figures.

11.97: The change in the volume of the oil is $= k_O v_O \Delta p$ and the change in the volume of the sodium is $= k_s v_s \Delta p.$ Setting the total volume change equal to Ax (x is positive) and using $\Delta p = F/A$,

$$Ax = (k_O V_O + k_s V_s)(F/A),$$

and solving for k_s gives

$$k_s = \left(\frac{A^2 x}{F} - k_O V_O\right)\frac{1}{V_s}.$$

11.98: a) For constant temperature $(\Delta T = 0)$,

$$\Delta(pV) = (\Delta p)V + p(\Delta V) = 0 \quad \text{and} \quad B = -\frac{(\Delta p)V}{(\Delta V)} = p.$$

b) In this situation,

$$(\Delta p)V^\gamma + \gamma p(\Delta V)V^{\gamma-1} = 0, \quad (\Delta p) + \gamma p\frac{\Delta V}{V} = 0,$$

and

$$B = -\frac{(\Delta p)V}{\Delta V} = \gamma p.$$

11.99: a) From Eq.(11.10), $\Delta l = \frac{(4.50\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m})}{(20\times10^{10}\ \text{Pa})(5.00\times10^{-7}\ \text{m}^2)} = 6.62\times10^{-4}$ m, or 0.66 mm to two figures. b) $(4.50\ \text{kg})(9.80\ \text{m/s}^2)(0.0500\times10^{-2}\ \text{m}) = 0.022\ \text{J}.$ c) The magnitude F will be vary with distance; the average force is $Y\,A(0.0250\ \text{cm}/l_0) = 16.7\ \text{N}$, and so the work done by the applied force is $(16.7\ \text{N})(0.0500\times10^{-2}\ \text{m}) = 8.35\times10^{-3}$ J. d) The wire is initially stretched a distance 6.62×10^{-4} m (the result of part (a)), and so the average elongation during the additional stretching is 9.12×10^{-4} m, and the average force the wire exerts is 60.8 N. The work done is negative, and equal to $-(60.8\ \text{N})(0.0500\times10^{-2}\ \text{m}) = -3.04\times10^{-2}$ J. e) See problem 11.82. The change in elastic potential energy is

$$\frac{(20\times10^{10}\ \text{Pa})(5.00\times10^{-7}\ \text{m}^2)}{2(1.50\ \text{m})}((11.62\times10^{-4}\ \text{m}^2) - (6.62\times10^{-4}\ \text{m})^2) = 3.04\times10^{-2}\ \text{J},$$

the negative of the result of part (d). (If more figures are kept in the intermediate calculations, the agreement is exact.)