E7. TURBOMACHINERY

E7.1 An axial flow compressor for a jet engine is operating on a test stand under standard sea level atmospheric conditions. The pressure ratio provided by the compressor is $p_3/p_1 = 10$ and it processes a mass flow rate of 45.3kg/s. The dimensions of the compressor are shown in Figure E7.1. Assuming that all processes are isentropic determine (a) the force F experienced by the load cell on the test stand, (b) the power P required to drive the compressor, and (c) the temperature leaving the compressor T_3.

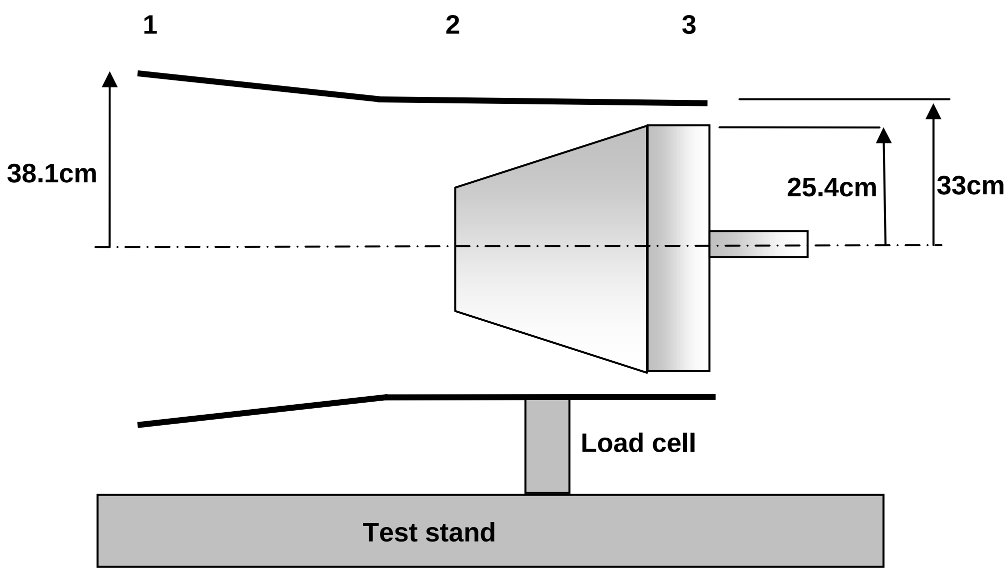


Figure E7.1 Schematic diagram of axial flow compressor on test stand

Solution

(a) The areas $A_1 = \pi r_1^2 = 0.456\text{m}^2$ and $A_3 = \pi(r_{3a}^2 - r_{3,b}^2) = 0.139\text{m}^2$. Sea level 1976 U.S. Standard Atmosphere has $p_0 = 101.3\text{kPa}$, $T_0 = 288\text{K}$. For isentropic flow $p/p_0 = (\rho/\rho_0)^\gamma = (T/T_0)^{\gamma/(\gamma-1)}$

Mass flow: $\rho AV = 45.3\text{kg/s}$ = constant. The mass flow entering station 1 is then

$$\dot{m} = \rho_1 A_1 V_1 = \left(1.225\frac{kg}{m^3}\right)\frac{\rho_1}{\rho_0}\left(0.456m^2\right)V_1 = 45.3kg/s \qquad \text{(E7.1)}$$

The flow throughout the system is isentropic so we may rewrite Equation (E7.1) as

$$\dot{m} = \left(0.559\frac{kg}{m}\right)\left(\frac{p_1}{p_0}\right)^{\frac{1}{\gamma}} V_1 = \left(0.559\frac{kg}{m}\right)\left(\frac{97}{101.3}\right)^{\frac{1}{1.4}} V_1 = 45.3\frac{kg}{s} \qquad \text{(E7.2)}$$

The velocity at station 1 is then V_1=83.5m/s and likewise, the velocity at station 3 is

$$V_3 = \left(\frac{\rho_1}{\rho_3}\right)\left(\frac{A_1}{A_3}\right)V_1 = \left(\frac{p_1}{p_3}\right)^{\frac{1}{\gamma}}\left(\frac{0.456}{0.139}\right)V_1 = \left(273\frac{m}{s}\right)\left(\frac{p_1}{p_3}\right)^{\frac{1}{\gamma}} = \left(273\frac{m}{s}\right)\left(0.1\right)^{0.714} = 52.7\frac{m}{s}$$

Applying the conservation of momentum to the fluid in a control volume between stations 1 and 3 yields

$$p_1 A_1 + \left(-\dot{m}_1\right)V_1 + F - p_3 A_3 + \dot{m}_3 V_3 = 0$$

Then solving for F yields

$$F = p_3 A_3 - p_1 A_1 + \dot{m}\left(V_1 - V_3\right)$$

$$F = \left(970kPa\right)\left(0.139m^2\right) - \left(97kPa\right)\left(0.456m^2\right) + \left(45.3\frac{kg}{s}\right)\left[\left(83.5\frac{m}{s}\right)^2 - \left(52.7\frac{m}{s}\right)^2\right]$$

$$F = 90.6kN + 19kN = 109.6kN$$

This force *on* the fluid is acting to the right so the force on the load cell is F_{cell}=109.6kN to the left; therefore it is a *thrust* force.

(b) The energy equation for adiabatic flow through the compressor is

$$h_1 + \frac{1}{2}V_1^2 + \frac{P}{m} = h_3 + \frac{1}{2}V_3^2$$

The power added to the fluid is then

$$\frac{P}{m} = \left(h_3 - h_1\right) + \frac{1}{2}\left(V_3^2 - V_1^2\right) = c_p T_1\left(\frac{T_3}{T_1} - 1\right) + \frac{1}{2}\left[\left(52.7\frac{m}{s}\right)^2 - \left(83\frac{m}{s}\right)^2\right]$$

$$\frac{P}{m} = c_p T_0 \frac{T_1}{T_0}\left[\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] - 4195\frac{m^2}{s^2} = c_p T_0\left(\frac{p_1}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\left[\left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] - 4195\frac{m^2}{s^2}$$

$$\frac{P}{m} = \left(1\frac{kJ}{kg-K}\right)\left(288K\right)\left(\frac{97}{101}\right)^{0.286}\left[\left(10\right)^{0.286} - 1\right] - \left(4195\frac{m^2}{s^2}\right)\left(\frac{kJ/kg}{10^3 m^2/s^2}\right) = 261\frac{kJ}{kg}$$

Using the mass flow from Equation (E7.1) we find P = 11.82MW

(c) The temperature after isentropic compression is

$$T_3 = T_0\left(\frac{T_1}{T_0}\right)\left(\frac{T_3}{T_1}\right) = T_0\left(\frac{p_1}{p_0}\right)^{0.286}\left(\frac{p_3}{p_1}\right)^{0.286} = (288K)(0.96)^{0.286}(10)^{0.286} = 550K$$

Thus T_3=550K

E7.2 A turbojet engine is operated under standard sea level conditions and the following measurements are made at the compressor exit: stagnation temperature equals 216C and stagnation pressure equals 482kPa. The airflow through the compressor is determined to be 27.2kg/s. Determine: (a) the work input required per unit mass of air, (b) the power required, (c) the adiabatic compression efficiency of the compressor, and (d) the torque exerted on the fluid at a rotational speed of 12,000rpm.

Solution

(a) The engine is being run up on the ground so M_0=0 and therefore $p_{t,2}$=101.3kPa and $T_{t,2}$=288K. Assuming a constant specific heat for air, c_p=1.0kJ/kg-K, the work required by the compressor is

$$W_c = c_p(T_{t,3} - T_{t,2}) = (1.0\text{kJ/kg-K})(489\text{K} - 288\text{K}) = 201\text{kJ/kg}$$

(b) The power required is

$$P = \dot{m}W_c = 27.2\text{kg/s}(201\text{kJ/kg}) = 5.47\text{MW}$$

(c) Assuming that for standard sea level air γ=1.4 the adiabatic efficiency of compression is

$$\eta_c = [(p_{t,3}/p_{t,2})^{(k-1)/k} - 1][(T_{t,3}/T_{t,2}) - 1]^{-1} = [(482/101.3)^{2/7} - 1][(489/288) - 1]^{-1} = 0.805$$

(d) The torque exerted on the fluid is

$$T_q = P/\omega = (5.47\text{x}10^6\text{N-m/s})/[2\pi(12000\text{rpm})/60\text{s}^{-1}] = 4.35\text{kN-m}$$

E7.3 A compressor operates with a stagnation pressure ratio of 4 and an inlet stagnation temperature of 5C. If the exit stagnation temperature is 171C, what is the adiabatic efficiency of compression?

Solution

Assuming a constant value for γ =1.4, the adiabatic efficiency of compression may be written as

$\eta_c = [(p_{t,3}/p_{t,2})^{(k-1)/k} - 1][(T_{t,3}/T_{t,2}) - 1]^{-1} = [(4)^{2/7} - 1][(444/278) - 1]^{-1} = 0.814$

E7.4 Axial flow compressors are characterized by flow passages whose radial coordinates change little over the length of the machine while centrifugal compressors have blade passages that start near the axis of rotation and increase substantially in radius by the time the exit of the machine is reached. Some idea of the difference is afforded by the accompanying photograph from NASA in the text showing examples of two such machines side by side. Consider standard sea level air entering a centrifugal flow compressor with the following conditions: r_2 =6in, c_2 = 300ft/s, and α_2 =70°. The air leaves the rotor with r_3 =18in, c_3 =1200ft/s, and α_3 =25°. The rotational speed of the compressor is 9000rpm and it processes 32.2lb/s of air. Carry out the following: (a) draw the combined entrance and exit velocity diagrams to scale, (b) determine the entrance and exit blade angles β_2 and β_3, (c) determine the torque required to drive the compressor, (d) determine the horsepower required, (e) find the ideal pressure head, (f) determine the contribution to the pressure rise provided by the external, internal, and centrifugal effects, (g) determine the whirl velocity Δc_u, (h) determine the ideal pressure ratio of the compressor

Solution

(a)The velocity triangles for the centrifugal compressor are shown in Figure E7.2.

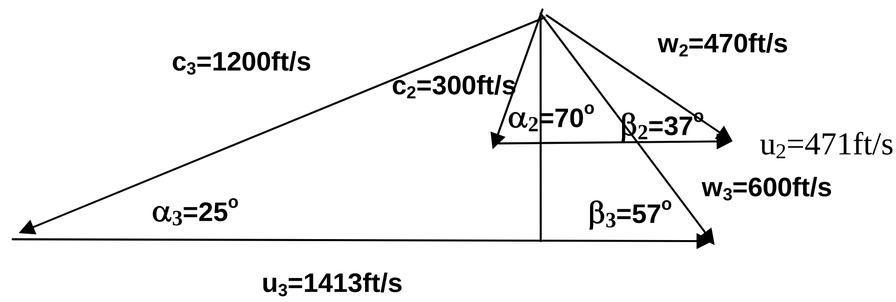


Figure E7.2 Velocity diagrams for the centrifugal compressor of exercise E7.4

(b) From the diagram we can find $c_{2,a} = c_2\sin\alpha_2$ = (300ft/s)sin70 = 281.9ft/s and $c_{3,a}$ = $c_3\sin\alpha_3$ = (1200ft/s)sin25 = 507.1ft/s.

Similarly, $c_{2,u} = c_2\cos\alpha_2$ = (300ft/s)cos70 = 102.6ft/s and $c_{3,u} = c_3\cos\alpha_2$ = (1200ft/s)cos25 = 1088ft/s.

Now $u_2 = (2\pi N/60)r_2 = (942.5\text{s}^{-1})(0.5\text{ft})$ = 471.2ft/s and $u_3 = (2\pi N/60)r_3 = (942.5\text{s}^{-1})(1.5\text{ft})$ =1414ft/s.

The relative speeds are
$w_2=[(u_2-c_{2,u})^2+c_{2,a}^2]^{1/2}=[(471.2-102.6)^2+281.9^2]$=464.0ft/s and
$w_3=[(u_3-c_{3,u})^2+c_{3,a}^2]^{1/2}=[(1414-1088)^2+507.1^2]$=602.8ft/s

The rotor blade angles are
$\beta_2 = \arcsin(c_{2,a}/w_2) = \arcsin(281.9/464) = 37.41^\circ$ and
$\beta_3 = \arcsin(c_{3,a}/w_3) = \arcsin(507.1/602.8) = 57.27^\circ$

(c)The required torque $T_q = \dot{m}\,(r_3c_{3,u}-r_2c_{2,u})$ = [(32.2lb/s)/(32.2ft/s^2)][(1.5ft)(1088ft/s)-(0.5ft)(102.6ft/s)] = 1581ft-lbs

(d) The horsepower required is $P = T_q\omega$ = (1581ft-lbs)(942.5s^{-1})/(550ft-lb/s/hp) = 2709hp (e)The ideal pressure head is the work done per unit weight of working fluid and is given by $\mathbf{W_c} = (1/2g)[(c_3^2-c_2^2)+(u_3^2-u_2^2)+(w_2^2-w_3^2)] = (1/32.2\text{ft/s}^2)[(135.0\text{x}10^4+177.7\text{x}10^4-14.87\text{x}10^4)\text{ft}^2/\text{s}^2] = 46{,}250$ft-lb/lb (=24.09psia for air at standard conditions)

(f)Pressure head due to external effect = $(1/2g)(c_3^2-c_2^2) = 20{,}960$ft; internal effect = $(1/2g)(w_2^2-w_3^2) = -2{,}309$ft; centrifugal effect = $(1/2g)(u_3^2-u_2^2) = 27{,}590$ft

(g)Whirl velocity $\Delta c_u = c_{3,u} - c_{2,u} = 1088-102.6 = 985.4$ft/s

(h) Ideal pressure ratio $p_{t,3}/p_{t,2} = [(\eta_c W_c/c_p T_{t,2})+1]^{\gamma/(\gamma-1)}$

$p_{t,3}/p_{t,2} = [\{(1)(46250\text{ft})/(0.24\text{Btu/lb-R})(778\text{ft-lb/Btu}*535\text{R})\}+1]^{7/2} = 3.78$

(Note: $T_{t2}=T_2+c_2^2/2c_p$)

E7.5 Repeat problem 4, changing only the entrance radius to $r_2=r_3=18$in, so that the machine now acts as an axial flow compressor.

Solution

(a) The velocity triangles for the axial compressor are shown below

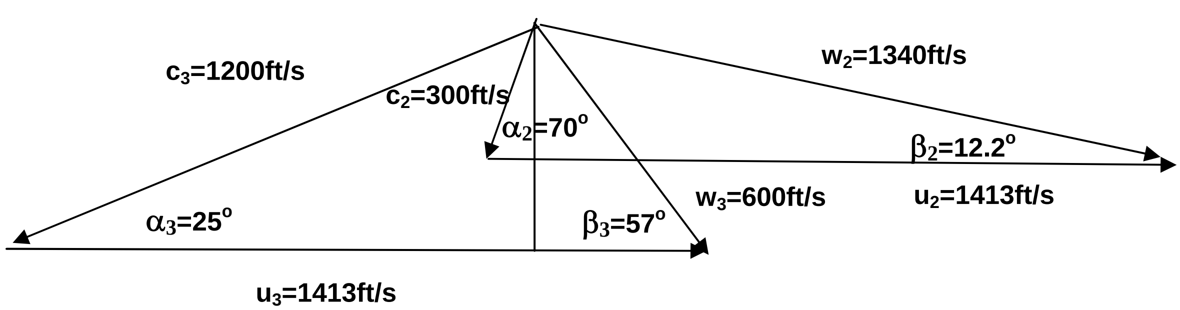


Figure E7.3 Velocity diagram for the axial compressor of exercise E7.5

(b) From the diagram we can find $c_{2,a} = c_2\sin\alpha_2$ = (300ft/s)sin70 = 281.9ft/s and $c_{3,a}$ = $c_3\sin\alpha_3$ = (1200ft/s)sin25 = 507.1ft/s.

Similarly, $c_{2,u} = c_2\cos\alpha_2$ = (300ft/s)cos70 = 102.6ft/s and $c_{3\,u} = c_3\cos\alpha_2$ = (1200ft/s)cos25 = 1088ft/s.

Now $u_2 = (2\pi N/60)r_2 = (942.5\text{s}^{-1})(1.5\text{ft})$ = 1414ft/s and $u_3 = (2\pi N/60)r_3 = (942.5\text{s}^{-1})(1.5\text{ft})$ = 1414ft/s.

The relative speeds are

$w_2 = [(u_2-c_{2,u})^2+c_{2,a}^2]^{1/2} = [(1414-102.6)^2+281.9^2]^{1/2}$ =1341ft/s and

$w_3 = [(u_3-c_{3,u})^2+c_{3,a}^2]^{1/2} = [(1414-1088)^2+507.1^2]^{1/2}$ = 602.8ft/s

The rotor blade angles are

$\beta_2 = \arcsin(c_{2,a}/w_2) = \arcsin(281.9/1341) = 12.14^o$ and

$\beta_3 = \arcsin(c_{3,a}/w_3) = \arcsin(507.1/602.8) = 57.27^o$

(c)The required torque $T_q = \dot{m}\,(r_3c_{3,u}-r_2c_{2,u}) = [(32.2\text{lb/s})/(32.2\text{ft/s}^2)][(1.5\text{ft})(1088\text{ft/s})-(1.5\text{ft})(102.6\text{ft/s})]$ =1478ft-lbs

(d) The horsepower required is $P = T_q\omega = (1478\text{ft-lbs})(942.5\text{s}^{-1})/(550\text{ft-lb/s/hp})$ = 2533hp

(e) The ideal pressure head is the work done per unit weight of working fluid and is given by $W_c = (1/2g)[(c_3^2-c_2^2)+ (u_3^2-u_2^2)+ (w_2^2-w_3^2)] = (1/32.2\text{ft/s}^2)[(135.0\text{x}10^4+0 +143.5\text{x}10^4)$ ft^2/s^2 = 43,240ft-lb/lb (=22.52psia for air at standard temperature and pressure)

(f) Pressure head due to external effect = $(1/2g)(c_3^2-c_2^2)$ = 20,960ft; internal effect = $(1/2g)(w_2^2-w_3^2)$ = 22,280ft; centrifugal effect = $(1/2g)(u_3^2-u_2^2)$ = 0 ft

(g) Whirl velocity $\Delta c_u = c_{3,u} - c_{2,u}$ = 1088-102.6 = 985.4ft/s

(h) Ideal pressure ratio $p_{t,3}/p_{t,2} = [(\eta_c W_c/c_p T_{t,2})+1]^{\gamma/(\gamma-1)}$ = [{(1)(43240ft)/(0.24Btu/lb-R)(778ft-lb/Btu)(535R)}+1]$^{7/2}$ = 3.52 (Note: $T_{t2}=T_2+c_2^2/2c_p$)

E6. A compressor rotor like that in Figure E7.4 is operated at 10,000rpm on a test stand where the atmospheric conditions are those of a standard sea level day. The compressor has an axial inlet with a hub diameter of 13.35cm and an eye diameter of 25.40cm. The compressor has a radial exhaust and the rotor tip diameter is 50.80cm. If the average inlet Mach number is 0.7 determine: (a) the mass flow through the compressor, (b) the power required to drive the compressor, (c) the total temperature at the compressor exit, (d) the portion of the required power due to the centrifugal effect, (e) the overall pressure ratio of the machine, if the adiabatic efficiency of compression η_c=85%.

Solution

(a) The mass flow through the compressor $(\rho A c_a)_2$. The Mach number at the inlet is M_2=0.7 so $c_{2a}=a_2M_2=0.7(\gamma RT_2)^{1/2}$=0.7(1.4)(287m^2/s^2-K)$(T_2)^{1/2}$.

1. The compressor is operating on a laboratory test stand so the stagnation pressure and temperature entering the compressor assumes its standard sea level values. Thus $T_2 = T_{t,2}[1+0.5(k-1)M_2^2]$ and thus $T_2 = T_{t,2}[1+0.5(\gamma-1)M_2^2]^{-1}$ = (289K)[1+0.2(0.49)]$^{-1}$ and T_2 = 263K.
Similarly $p_2 = p_{t,2}[1+0.5(\gamma-1)M_2^2]^{-\gamma/(\gamma-1)}$ = (101kPa)(0.721) = 72.8kPa
2. Then c_{2a} = 0.7(1.4)(287m^2/s^2-K)(263K)$^{1/2}$ = 227m/s
3. The density is obtained from the equation of state $\rho = p/RT$ so that ρ_2 = (72,800N/m^2)/(287 m^2/s^2-K)(263K) = 0.964kg/m^3
4. The flow area is $A_2 = \pi(r_{eye}^2 - r_{hub}^2)$ = 3.14[(25.4cm^2)-(13.35cm^2)]10^{-4}/4 = 0.0366m^2

Then the mass flow is $\dot{m}$ = 0.964kg/m^3(0.0366m^2)(227m/s) =8.01kg/s

(b) The power required is $P = \dot{m}\left(u_3c_{3u} - u_2c_{2u}\right)$. However, c_{2u}=0 because the flow enters the compressor axially and $u_3 = c_{3u}$ because the flow leaves the impeller radially. The linear speed of rotation at the exit is $u_3 = (2\pi N/60)\, r_3 =$ (2)(3.14)(10,000/60)[(0.5)(0.508m)] = 265m/s. Therefore the power $P = \dot{m}u_3^2 =$ (8.01kg/s) $(265\text{m/s})^2$ = 563kW

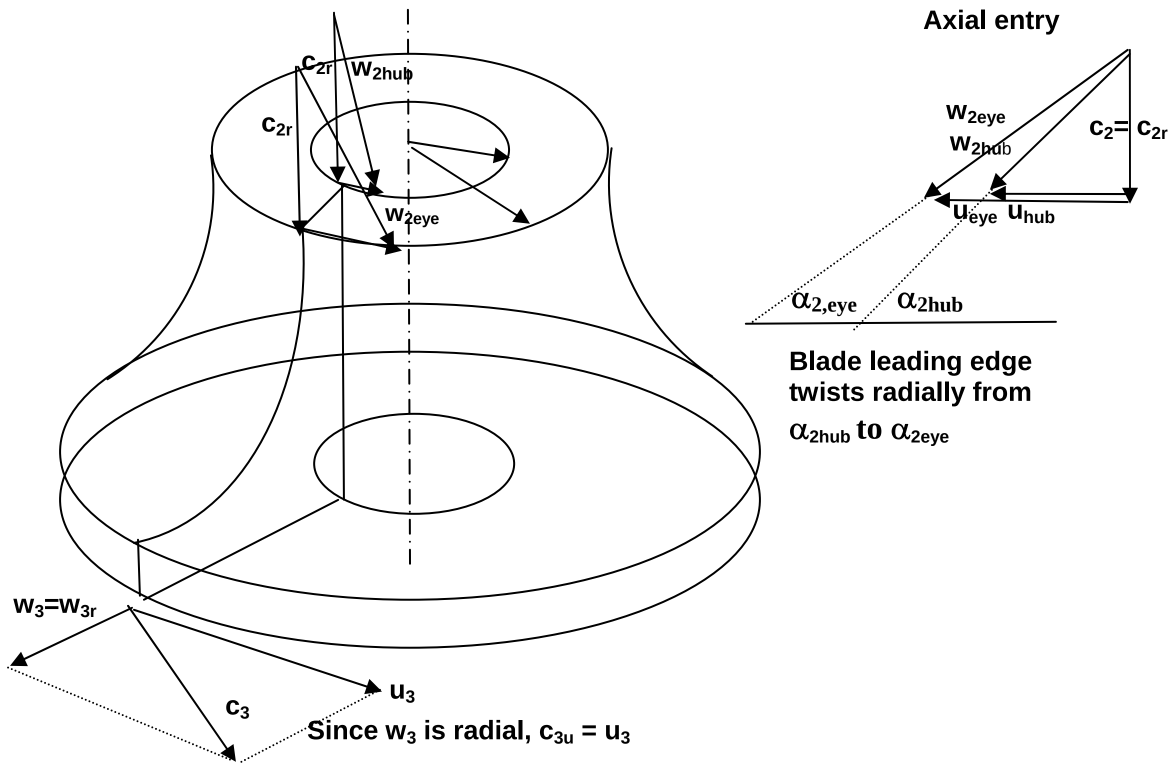


Figure E7.4 Centrifugal compressor showing hub and eye details

(c) The stagnation temperature at the exit may be found from $W_c=c_p(T_{t,3} - T_{t,2})$ or $T_{t,3}=T_{t,2}+W_c/c_p$ = 289K+(563,000N-m/s)$[(8.01\text{kg/s})(1000\text{N-m/kg-K})]^{-1}$.
Then $T_{t,3}$ = 289K+70.3K = 359K.

(d) At the exit u_3= 265m/s as already calculated. At the eye station of the exit $u_2 =$ $(2\pi N/60)r_2$ = (2)(3.14)(10,000/60)[(0.5)(0.254m)] =133m/s. The linear speed of rotation decreases as the hub is approached so we'll use the eye value just calculated to give a conservative estimate of the fraction of the power arising from the centrifugal effect. Then

$$\frac{P_{centrifugal}}{P}=\frac{\frac{1}{2}\dot{m}\left(u_3^2-u_2^2\right)}{P}=\frac{\frac{1}{2}\left(8.01\frac{kg}{s}\right)\left[\left(265\frac{m}{s}\right)^2-\left(133\frac{m}{s}\right)^2\right]}{563\frac{kJ}{s}}=0.374$$

(e) The pressure ratio of the machine is $p_{t,3}/p_{t,2}=\{1+\eta_c[(T_{t,3}/T_{t,2})-1]\}^{\gamma/(\gamma-1)}$

$p_{t,3}/p_{t,2}=\{1+0.85[(359/289)-1]\}^{7/2}=1.93$

E7. Carry out exercise E7.6 for the case where instead of the engine being operated on a test stand it is operating in an aircraft flying at M_0=0.7 at sea level where the atmospheric conditions are those of a standard day.

Solution

(a) The mass flow through the compressor is $(\rho A c_a)_2$. The Mach number at the inlet is M_2 = 0.7 so $c_{2a}=a_2M_2=0.7(\gamma RT_2)^{1/2}=0.7[(1.4)(287\text{m}^2/\text{s}^2\text{-K})(T_2)]^{1/2}$.

(i)The compressor is operating in flight so the static pressure and the static temperature entering the compressor assume their standard sea level values. Thus T_2 = 288K and p_2 =101.3kPa. At M_2 = 0.7, $T_{t,2}$ = 316K and $p_{t,2}$ =140kPa

(ii) Then $c_{2a}=0.7[(1.4)(287\text{m}^2/\text{s}^2\text{-K})(288\text{K})]^{1/2}$=238m/s and a=340m/s

(iii) The density is the standard sea level value ρ_2=1.225kg/m^3

(iv) The flow area is $A_2=\pi(r_{eye}^2-r_{hub}^2)=3.14[(25.4\text{cm}^2)-(13.35\text{cm}^2)]10^{-4}/4$ = 0.0366m^2

Then the mass flow is 1.225kg/m^3(0.0366m^2)(238m/s) =10.7kg/s

(b) The power required is $P=\dot{m}\left(u_3c_{3u}-u_2c_{2u}\right)$. However, c_{2u}=0 because the flow enters the compressor axially and $u_3=c_{3u}$ because the flow leaves the impeller radially. The linear speed of rotation at the exit is $u_3=(2\pi N/60)r_3$ = (2)(3.14)(10,000/60)[0.5)(0.508m)] = 265m/s. Therefore the power $P=\dot{m}u_3^2$ = (10.7kg/s)(265m/s)^2 = 571kW

(c) The stagnation temperature at the exit may be found from $W_c = c_p(T_{t,3} - T_{t,2})$ or $T_{t,3} = T_{t,2}+W_c/c_p$ = 316K+(751,000N-m/s)[(10.7kg/s)(1000N-m/kg-K)]$^{-1}$.
Then $T_{t,3}$ = 316K+70.2K =386K.

(d) At the exit u_3= 265m/s as already calculated. At the eye station of the exit $u_2 = (2\pi N/60)r_2$ = (2)(3.14)(10,000/60)[(0.5)(0.254m)] =133m/s. The linear speed of rotation decreases as the hub is approached so we'll use the eye value just calculated to give a conservative estimate of the fraction of the power arising from the centrifugal effect. Then

$$\frac{P_{centrifugal}}{P} = \frac{\frac{1}{2}\dot{m}\left(u_3^2 - u_2^2\right)}{P} = \frac{\frac{1}{2}\left(10.7\frac{kg}{s}\right)\left[\left(265\frac{m}{s}\right)^2 - \left(133\frac{m}{s}\right)^2\right]}{751\frac{kJ}{s}} = 0.374$$

(e) The pressure ratio of the machine is $p_{t,3}/p_{t,2}=\{1+\eta_c[(T_{t,3}/T_{t,2}) -1]\}^{\gamma/(\gamma-1)}$
$p_{t,3}/p_{t,2}=\{1+0.85[(386/316) -1]\}^{7/2}=1.83$

8. A radial bladed centrifugal compressor designed with zero pre-whirl (axial entry) for a turbojet engine is shown in Figure E7.5 and has the following data:

Slip angle =14^{o}
Impeller tip radius = 26.3cm
Impeller eye radius = 15cm
Impeller hub radius = 7cm
Impeller tip width = 4.4cm
Number of vanes = 29
Diffuser inner radius = 30.5cm
Diffuser outer radius = 40.6cm
N =16,750rpm

The compressor operates with an axial inlet Mach number of 0.8 at maximum rpm at the impeller inlet as the aircraft moves through the atmosphere at standard sea level conditions.

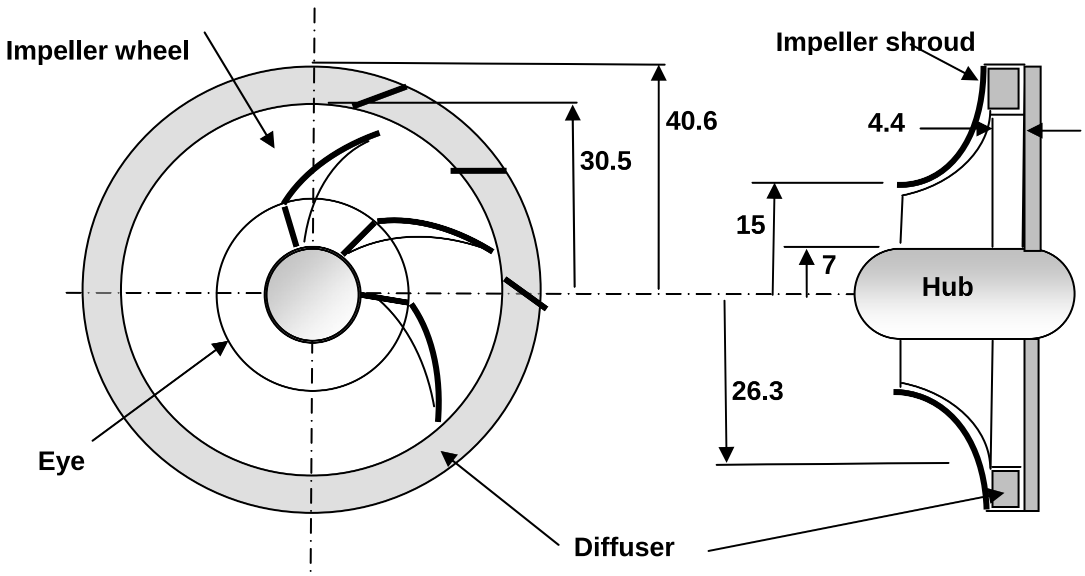


Figure E7.5 Schematic diagram of impeller wheel

(a) Determine the linear speed of rotation at the impeller tip, eye, and hub

(b) Determine the inlet flow area allowing for 5% blockage by vane thickness

(c) Sketch the inlet blade design showing proper diameters and blade angles at the hub, eye, and at the station midway between the two.

(d) Calculate the airflow rate

(e) If $\tau = 0.94$ draw the exit velocity diagram to scale and indicate all magnitudes

(f) If $\eta_c = 80\%$ determine p_3, $p_{t,3}$, T_3, and $T_{t,3}$

(g) What is the entrance angle for the fixed diffuser vanes? (Note that angular momentum of the fluid is conserved in the annulus between the impeller tip and the diffuser entrance

(h) What is the diffuser entrance Mach number?

(i) What is the pressure ratio of the compressor $p_{t,3}/p_{t,2}$?

(j) What is the torque and power required to drive the impeller?

Solution

(a) The linear speed of rotation $u = \omega r = 2\pi Nr/60 = 1754r$, therefore

$u_{tip} = 461\text{m/s} \quad u_{eye} = 263\text{m/s} \quad u_{hub} = 123\text{m/s}$

(b) The flow area $A_2 = \pi(r_{eye}^2 - r_{hub}^2) - A_{blades}$. Thus $A_2 = 0.0525\text{m}^2$

(c) Blade design is shown in Figure E7.7. For clarity the velocity diagram for the point midway between the hub and eye is omitted. It should be clear how to construct it. Note that $c_{2a} = M_{2a}a = 0.8[1.4(287\text{m}^2/\text{s}^2\text{-K})(288\text{K})]^{1/2} = 272\text{m/s}$

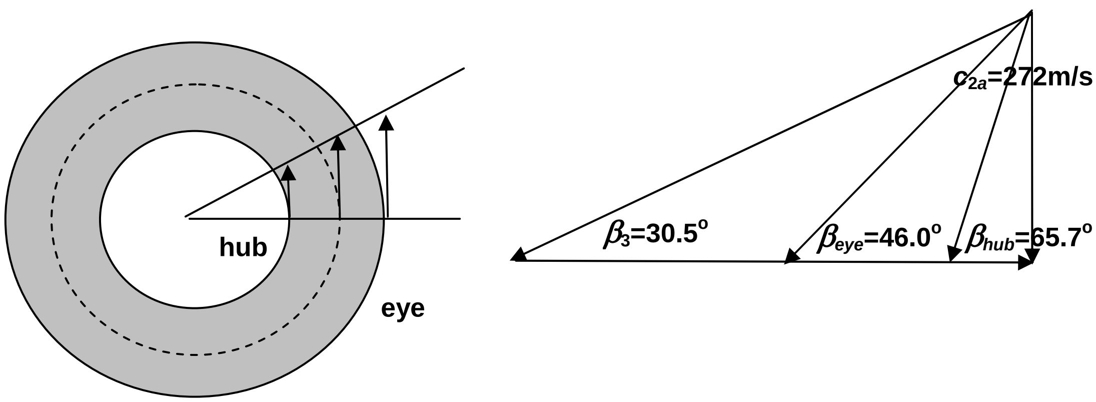


Figure E7.7 Blade angles at hub, eye, and tip

(d) Airflow rate = $\rho c_{2a}A_2 = (1.23\text{kg/m}^3)(272\text{m/s})(0.0525\text{m}^2) = 17.6\text{kg/s}$

(e) The slip coefficient $\tau = c_{3u}/u_3 = 0.94$. From part (a) $u_3 = 461\text{m/s}$ so $c_{3u} = 433\text{m/s}$. The velocity diagram appears in Figure E7.8. The velocity triangle yields $[w_{3r} / (u_3 - c_{3u})] = \tan 76^\circ = 4.01$ and $u_3 - c_{3u} = u_3 - 0.94\,u_3 = 0.06\,u_3$. Therefore $w_{3r} = 0.06(461\text{m/s})(4.01) = 111\text{m/s}$ and $c_3 = [c_u^2 + w_r^2]^{1/2} = 447\text{m/s}$. Then $w_3 = w_{3r}/\sin 76^\circ = 114\text{m/s}$

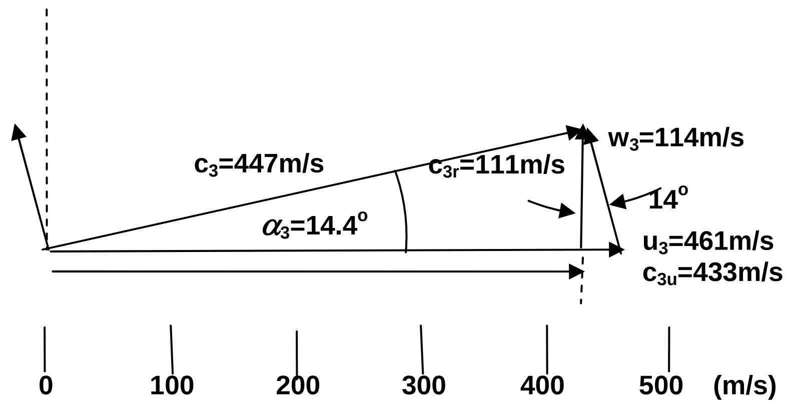


Figure E7.8 Velocity diagram for $\tau = 0.94$

(f) The work done per unit mass

$$W_c = \dot{m}\left(u_3 c_{3u} - u_2 c_{2u}\right)$$

Because the flow enters the compressor axially, c_{2u}=0 and W_c= (u_3c_{3u}) = (461m/s)(433m/s) = 200,000m^2/s^2. But the work per unit mass is also given by W_c = $c_p(T_{t,3} - T_{t,2})$ = (1kg/kJ-K)($T_{t,3} - T_{t,2}$). Since the compressor is moving through the air at M_0=0.8, $T_{t,2} = T_2[1+0.5(\gamma-1)M_0^2]$ = 288K[(1+0.2(0.64)], so $T_{t,2}$=325K. Then $T_{t,3}$ = [(2x10^{5}m^2/s^2)/(1000J/kg-K)]+325K and $T_{t,3}$=525K**.**

The total pressure ratio is given by $p_{t,3}/p_{t,2} = \{1+\eta_c[(T_{t,3}/T_{t,2}) -1]\}^{\gamma/(\gamma-1)}$ so that $p_{t,3}/p_{t,2}$ = $\{1+0.8[(525/325) -1]\}^{7/2}$ = 4.06. The stagnation pressure entering the compressor is $p_{t,2}$ = $p_2[1+0.5(\gamma-1)M_2^2]^{\gamma/(\gamma-1)}$ = (101kPa)$[1+(0.2)(0.64]^{7/2}$ = (101kPa)(1.52), so $p_{t,2}$=154kPa and thus $p_{t,3}$=625kPa.

The static temperature may be found from the adiabatic energy equation for a streamline leaving the compressor: $c_pT_3+c_3^2/2 = c_pT_{t,3}$. Then T_3= 525K- (447m/s)^2/(2)(10^{3}J/kg-K) and T_3=425K. Therefore the sound speed $a_3 = (\gamma RT_3)^{1/2}$ = [1.40(287m^2/s^2-K)(425K)$^{1/2}$ = 413m/s. The Mach number $M_3 = c_3/a_3$ = (447m/s)/(413m/s) =1.08. Then the static pressure is given by $p_3 = p_{t,3}[1+0.5(\gamma-1)M_3^2]^{-\gamma/(\gamma-1)}$ = (625kPa)$[1+0.2\ (1.08)^2]^{-7/2}$ and p_3 =

299kPa**.** For reference, the density $\rho_3 = p_3/RT_3 = (299{,}000\text{N/m}^2)/(287\text{J/kg-K})(425\text{K})$ and $\rho_3 = 2.45\text{kg/m}^3$.

(g) The flow leaves the impeller with angular momentum per unit mass of r_3c_{3u} = (0.263m)(433m/s) = 114m^2/s. Since no work is done on the fluid as it proceeds to the diffuser entrance, the angular momentum per unit mass remains constant and the tangential component of absolute velocity at the diffuser entrance is $c_{u,ent}=c_{3u}(r_3/r_{ent})$. This becomes $c_{u,ent}$ = (433m/s)(0.263m/0.305m) =373m/s. The conservation of mass requires that $(\rho Ac_r)_3 = (\rho Ac_r)_{\text{ent}}$ and

$$c_{r,ent} = c_{r3}(2\pi hr_3/2\pi hr_{ent})(\rho_3/\rho_{ent}) = 111\text{m/s}(26.3\text{m}/30.5\text{m})\ (\rho_3/\rho_{ent}).$$

Thus $c_{r,ent}$=95.7m/s(ρ_3/ρ_{ent}), assuming that the height of the passage, h, is constant across the gap. We know that $c_{ent}^2 = c_{u,ent}^2[1+(c_{r,ent}/c_{u,ent})^2] = c_{u,ent}^2[1+0.0658(\rho_3/\rho_{ent})]$. Since we expect the density to change little across the gap, and the ratio of radial to tangential absolute velocity is small, a reasonable approximation is the density is constant and c_{ent} $\underline{\sim}$1.03 $c_{u,ent}$ =1.03(373m/s)=384m/s. There is essentially no difference in the diffuser blade angle (keeping accuracy of 3 significant figures), as can be seen in the results shown in Figure E7.9.

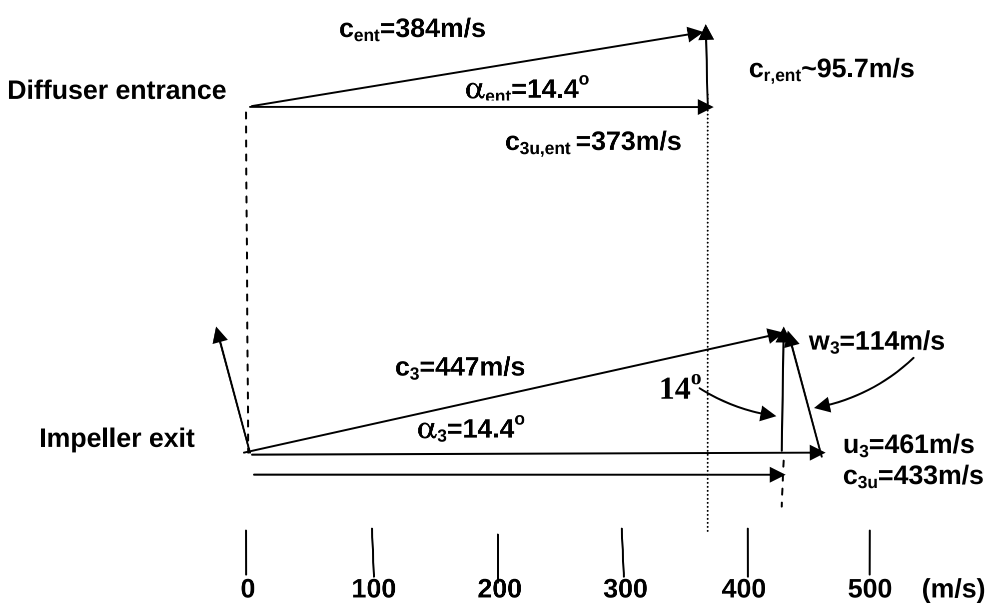


Figure E7.9 Velocity diagram for impeller exit and diffuser entrance

(h) The energy equation across the gap, assuming c_p = constant, is $c_pT_3 + c_3^2/2 = c_pT_{ent} + c_{ent}^2/2$ so that $T_{ent} = T_3 + (c_3^2 - c_{ent}^2)/2c_p$ = 425K+(447^2-384^2)/2(1030m^2/s^2-K) and T_{ent} = 450K and the speed of sound is a_{ent} = [1.4(287m^2/s^2-K)(450K)]$^{1/2}$ = 425m/s. The Mach number entering the diffuser is thus $M_{ent} = c_{ent} / a_{ent}$ = 384m/s/425m/s, or M_{ent} = 0.904.

(i) The pressure ratio across the machine is $p_{t,3}/p_{t,2}$ since there is assumed to be no friction in the diffuser. From part (f) $p_{t,3}/p_{t,2}$=625kPa/154kPa = 4.06.

(j) The power is $P = \dot{m}u_3c_{3u}$ = (17.7kg/s)(461m/s)(433m/s) = 3.53MW and the torque is $T_q = P/\omega$ = (3.53x10^{6}N-m/s)/[2π(16,750)/60s^{-1}] = 20.12kN-m

(k) The mass flow at the gap is $\rho_3(0.95)(2\pi r_3 h)c_{3r}$ =17.6kg/s so the gap height h = 0.0412m = 4.12cm

E7.9 Redo exercise E7.8 at a reduced rotational speed N=6,750rpm. Discuss the major differences between the two cases.

Solution

(a) The linear speed of rotation u=ωr=2πNr/60=707r, therefore

u_{tip}=185m/s u_{eye}=106m/s u_{hub}=49.5m/s

(b) The flow area $A_2=\pi(r_{eye}^2-r_{hub}^2)-A_{blades}$. Thus A_2=0.0525m^2

(c) Blade design is shown in Figure E7.10. For clarity the velocity diagram for the point midway between the hub and eye is omitted. It should be clear how to construct it. Note that $c_{2a}=M_{2a}a$ =0.8[1.4(287m^2/s^2-K)(288K)]$^{1/2}$ = 272m/s

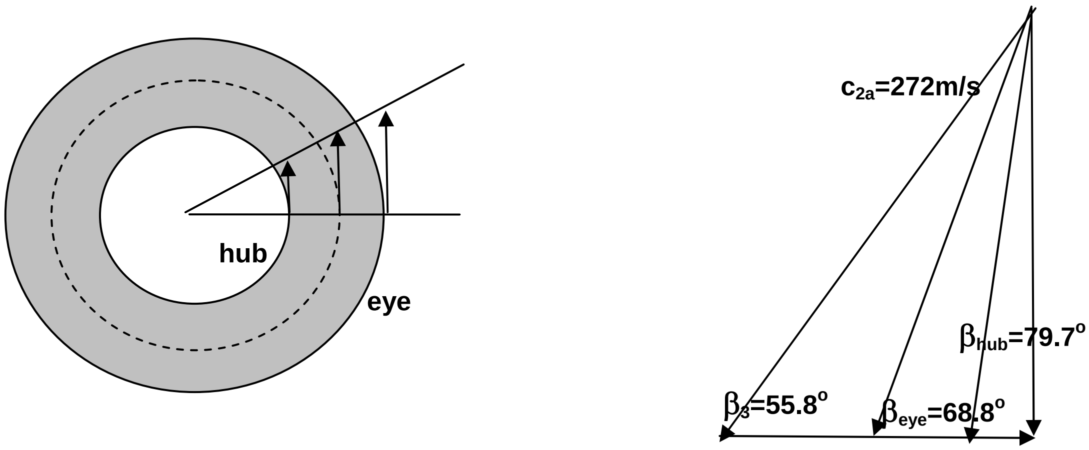


Figure E7.10 Blade angles at hub, eye, and tip

(d) Airflow rate $\rho c_{2a}A_2$=(1.23kg/m^3)(272m/s)(0.0525m^2) =17.6kg/s

(e) The slip coefficient $\tau = c_{3u}/u_3 = 0.94$. From part (a) u_3=185m/s so c_{3u}=174m/s. The velocity diagram appears in Figure E7.11.

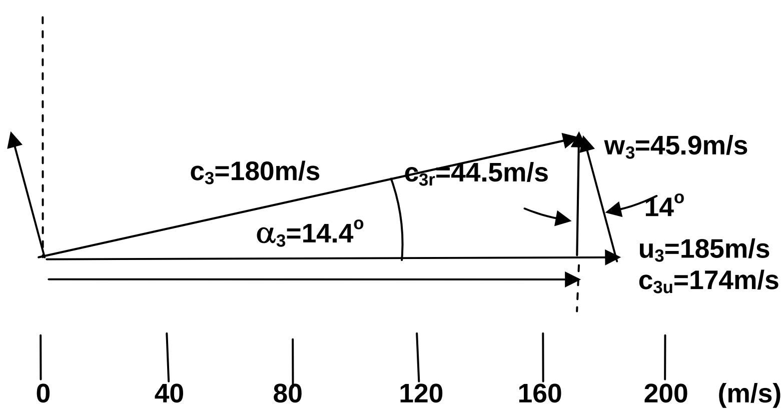


Figure E7.11 Velocity diagram for $\tau = 0.94$

The velocity triangle yields $[w_{3r}/(u_3\text{-}c_{3u})] = \tan 76^o = 4.01$ and $u_3\text{-}c_{3u} = u_3\text{-}0.94\,u_3 = 0.06\,u_3$. Therefore $w_{3r} = 0.06$(185m/s)(4.01) = 44.5m/s and $c_3 = [c_u^{\,2}+w_r^{\,2}]^{1/2}$ =180m/s. Then $w_3 = w_{3r}/\sin 76^o$ = 45.94m/s

(f) The work done per unit mass

$$W_c = \dot{m}\left(u_3 c_{3u} - u_2 c_{2u}\right)$$

Because the flow enters the compressor axially, c_{2u}=0 and W_c= (u_3c_{3u}) = (185m/s)(174m/s)=32,200m^2/s^2. But the work per unit mass is also given by $W_c = c_p(T_{t,3} - T_{t,2})$ = (1kg/kJ-K)($T_{t,3} - T_{t,2}$). Since the compressor is moving through the air at M_0=0.8, $T_{t,2} = T_2[1+0.5(\gamma-1)M_0^2]$ = 288K[(1+0.2(0.64)], so $T_{t,2}$=325K. Then $T_{t,3}$ = [(3.22x10^{4}m^2/s^2)/(1000J/kg-K)]+325K =358K.

The total pressure ratio is given by $p_{t,3}/p_{t,2} =\{1+\eta_c[(T_{t,3}/T_{t,2}) -1]\}^{\gamma/(\gamma-1)}$ so that $p_{t,3}/p_{t,2}$ = $\{1+0.8[(358/325) -1]\}^{7/2}$ = 1.31. The stagnation pressure entering the compressor is $p_{t,2} = p_2[1+0.5(\gamma-1)M_2^2]^{\gamma/(\gamma-1)}$ = (101kPa)$[1+(0.2)(0.64]^{7/2}$ = (101kPa)(1.52), so $p_{t,2}$=154kPa and thus $p_{t,3}$=202kPa.

The static temperature may be found from the adiabatic energy equation for a streamline leaving the compressor: $c_pT_3+c_3^2/2 = c_pT_{t,3}$. Then T_3= 358K- (180m/s)^2/(2)(10^{3}J/kg-K) and T_3=342K. Therefore the sound speed $a_3 = (\gamma RT_3)^{1/2}$ = [1.40(287m^2/s^2-K)(342K)]$^{1/2}$ = 371m/s. The Mach number $M_3 = c_3/a_3$ = (180m/s)/(371m/s) =0.486. Then the static pressure is given by $p_3 = p_{t,3}[1+0.5(\gamma-1)M_3^2]^{-\gamma/(\gamma-1)}$ = (202kPa)$[1+0.2\,(0.486)^2]^{-7/2}$ and p_3 = 172kPa**.** For reference, the density $\rho_3 = p_3/RT_3$ = (172,000N/m^2)/(287J/kg-K)(342K) and ρ_3 = 1.75kg/m^3.

(g) The flow leaves the impeller with angular momentum per unit mass of r_3c_{3u} = (0.263m)(174m/s) = 45.8m^2/s. Since no work is done on the fluid as it proceeds to the diffuser entrance, the angular momentum per unit mass remains constant and the tangential component of absolute velocity at the diffuser entrance is $c_{u,ent}=c_{3u}(r_3/r_{ent})$. This becomes $c_{u,ent}$ = (174m/s)(0.263m/0.305m) =150m/s. The conservation of mass requires that $(\rho Ac_r)_3 = (\rho Ac_r)_{\mathrm{ent}}$ and

$$c_{r,ent} = c_{r3}(2\pi hr_3/2\pi hr_{ent})(\rho_3/\rho_{ent}) = 44.5\text{m/s}(26.3\text{m}/30.5\text{m})\ (\rho_3/\rho_{ent}).$$

Thus $c_{r,ent}$=38.4m/s(ρ_3/ρ_{ent}), assuming that the height of the passage, h, is constant across the gap. We know that $c_{ent}^2 = c_{u,ent}^2[1+(c_{r,ent}/c_{u,ent})^2] = c_{u,ent}^2[1+0.0655(\rho_3/\rho_{ent})]$. Since we expect the density to change little across the gap, and the ratio of radial to tangential absolute velocity is small, a reasonable approximation is the density is constant and c_{ent} $\underset{\sim}{}$1.03 $c_{u,ent}$ =1.03(150m/s)=155m/s. There is essentially no difference in the diffuser blade angle (keeping accuracy of 3 significant figures), as can be seen in the results shown in Figure E7.12.

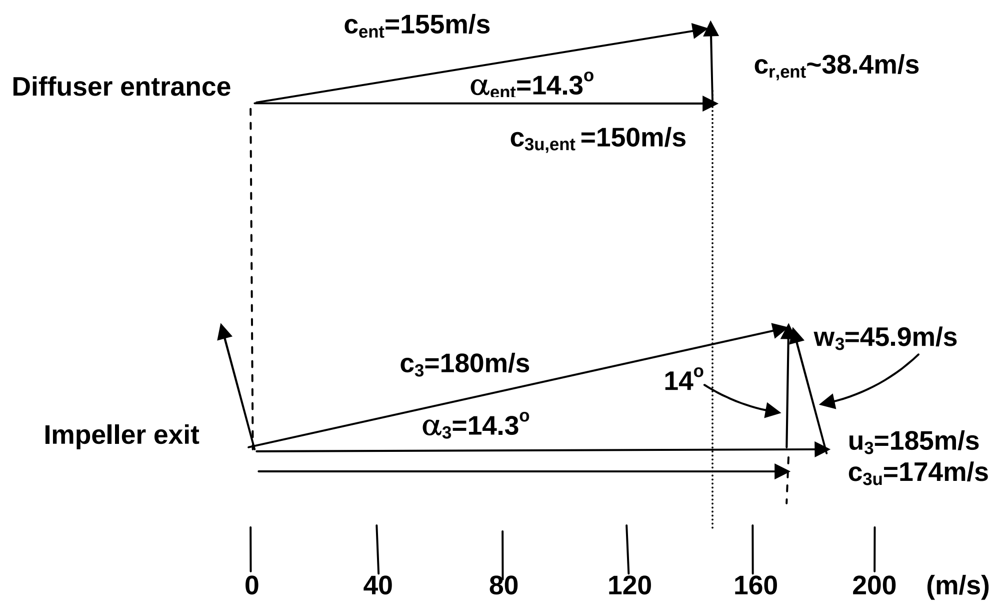


Figure E7.12 Velocity diagram for impeller exit and diffuser entrance

(h) The energy equation across the gap, assuming c_p = constant, is $c_pT_3 + c_3^2/2 = c_pT_{ent} + c_{ent}^2/2$ so that $T_{ent} = T_3 + (c_3^2 - c_{ent}^2)/2c_p$ = 342K+(180^2-155^2)/2(1030m^2/s^2-K) and T_{ent} = 346K and the speed of sound is a_{ent} = [1.4(287m^2/s^2-K)(346K)]$^{1/2}$ = 373m/s. The Mach number entering the diffuser is thus $M_{ent} = c_{ent} / a_{ent}$ = 155m/s/373m/s, or M_{ent} = 0.416.

(i) The pressure ratio across the machine is $p_{t,3}/p_{t,2}$ since there is assumed to be no friction in the diffuser. From part (f) $p_{t,3}/p_{t,2}$=202kPa/154kPa = 1.31.

(j) The power is $P = \dot{m}u_3c_{3u}$ = (17.7kg/s)(185m/s)(174m/s) = 0.57MW and the torque is $T_q = P/\omega$ = (0.57x10^{6}N-m/s)/[2π(6,750)/60s^{-1}] = 806N-m

(k) The mass flow at the gap is $\rho_3(0.95)(2\pi r_3 h)c_{3r}$ =17.6kg/s so the gap height h = 0.142m = 14.2cm

E7.10 Consider a staged axial flow compressor with the general configuration shown in the Figure E7.13. Only the rotor of the first stage is shown and it accepts air from the inlet guide vanes with the following conditions: p_2=96.2kPa, T_2=289K. The rotor mean line has r=30.5cm, α_2=60^o, β_2=80^o, and u_2=427m/s.

(a) Draw the velocity diagram of the entry to the rotor and find w_2, c_2, c_{2a} and then find p_{t2}, T_{t2}, and M_2, where the last 3 items are based upon the absolute velocity c_2.

(b) Determine N, the rotational speed of the machine in rpm.

(c) If the stagnation pressure ratio across the rotor is1.15 and η_c=85%, find c_{3u}-c_{2u}.

(d) Draw the exit velocity diagram to scale and find w_3, c_3, β_3, α_3, assuming c_{2a}=c_{3a}.

(e) Find the blade angle of the first stator passage at the median radius r of the rotor.

(f) Determine the airflow rate for the compressor if the blade height is 12.7cm assuming that 5% of the flow passage is blocked by blade leading edges.

(g) Calculate the required torque, power, and work per unit mass for this first stage.

(h) Determine the power required to drive a compressor using of 13 stages like the first stage.

(i) Calculate the static pressure rise achieved in the one stage shown.

(j) Determine the percent reaction of the one stage shown

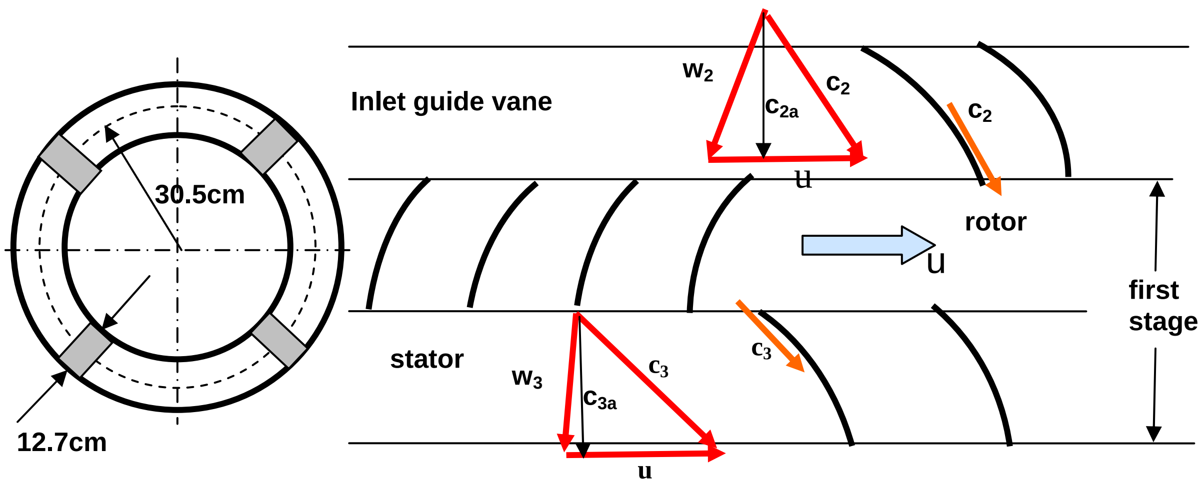


Figure E7.13 Schematic diagram of a staged axial flow compressor

Solution

(a) A velocity diagram may be drawn to scale using the data given and Figure E7.10 as a guide. Velocities from velocity diagram: w_2 = 575m/s, c_2 = 654m/s, c_{2a} = 566m/s. From the given value of the temperature T_2 = 289K the sound speed is found to be a_2= 341m/s. Then from the definition of the Mach number $M_2 = c_2/a_2$ and M_2 =1.92. The given pressure p_2 = 96.2kPa may be used to find the stagnation pressure from $p_{t,2} = p_2[1+0.2M_2^2]^{3.5}$ yielding $p_{t,2}$ = 664kPa. Similarly, the stagnation temperature may be found from $T_{t,2} = T_2[1+0.2M_2^2]$ = 502K.

(b) The rotational speed: $\omega=u/r$, $N=60\omega/2\pi$ so that N=13,370rpm

(c) For the given value of pressure ratio and efficiency $W_c = (c_pT_{t,2}/\eta_c)[(p_{t,3}/p_{t,,2})^{0.286}-1]$ = 24.1kJ/kg = $u(c_{3u}-c_{2u})$, therefore $(c_{3u}-c_{2u})$ = 56.4m/s

(d) Again from the velocity diagram: w_3 = 568m/s, c_3 = 684m/s, $\beta_3 = 85^o$, $\alpha_3 = 55.8^o$

(e) The blade angle of stator is $\alpha_3=55.8^o$

(f) Airflow rate:

$\rho_2 A_2 c_{2a}$=(1.16kg/m^3)π[(0.305m+0.0635m)^2–(0.305m–0.0635m)^2](0.95)(566m/s)

= 152kg/s

(g)The torque $T_q = \dot{m}r\left(c_{3u} - c_{2u}\right)$=(152kg/s)(0.305m)(56.4m/s)=2,610N-m, the power P_c= ωT_q = 3.66MW and the work per unit mass of fluid $W_c = \dfrac{P}{\dot{m}}$= 24.1kJ/kg

(h) To find the power required for 13 identical stages of pressure ratio =1.15 we calculate the overall pressure ratio to be $p_{t,3}$ / $p_{t,2}$ = (1.15)^{13} = 6.15. Then using Equation 7.65 we find the overall efficiency to be η_c=0.0812 so the overall work is

$$W_c = \frac{c_p T_{t,2}}{\eta_c}\left[\left(\frac{p_{t,3}}{p_{t,2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] = \frac{\left(1\dfrac{kg}{kJ-K}\right)\left(502K\right)}{0.812}\left[\left(6.15\right)^{0.286} - 1\right] = 421\frac{kJ}{kg}$$

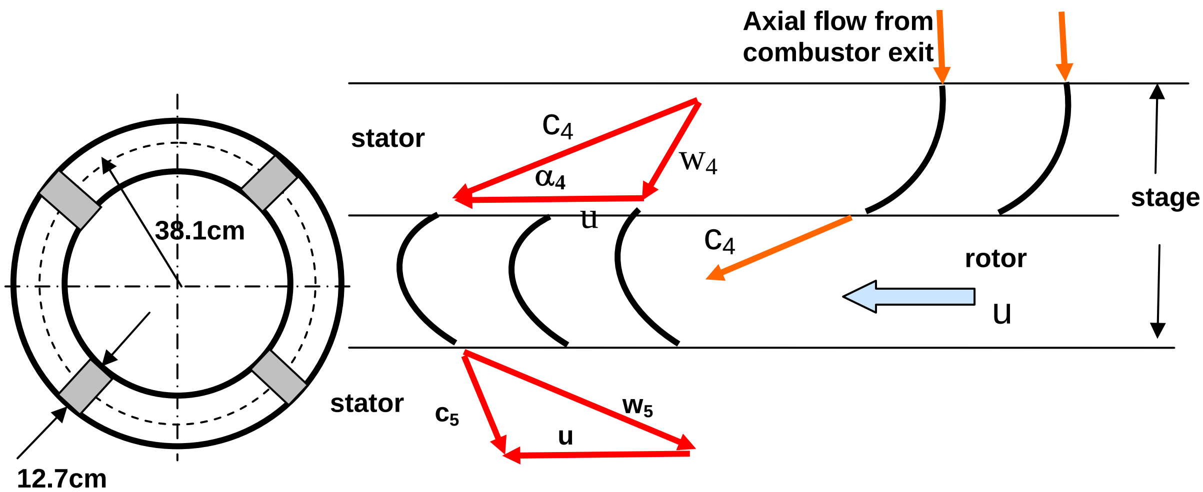
Then the overall power is P_c = (152kg/s)(421kJ/kg) = 64MW

(i) The stagnation pressure rise in the stage: p_{t3}-p_{t2} = p_{t2}[(p_{t3}/$p_{t,2}$)-1] = 664kPa[1.15-1] = 99.6kPa.

Static pressure rise across stage: p_3-p_2. The work result may be used to find T_{t3} = 526K and the energy equation along a streamline yields T_3 = 292K leading to M_3 = 1.97, showing that there is little change in the Mach number across the rotor. Then p_3 = 98.1kPa and therefore the static pressure rise is p_3-p_2 = 1.95kPa.

(j)Thus the degree of reaction is r =1.95/99.6 or r = 2%. An incompressible analysis would suggest that $r_{\text{inc}} = (w_2^2 - w_3^2)/[(c_3^2 - c_2^2) + (w_2^2 - w_3^2)]$ = 16.6%. Although the density is almost constant in this problem, the flow must still be considered compressible because of the high Mach number and therefore compressible flow relations must be used. For example, the stagnation pressure calculated for incompressible flow relations would be $p_{t,2,inc} = p_2 + 0.5\rho_2 c_2^2$ = 96.2kPa+248kPa =344kPa whereas the compressible flow result is 664kPa.

E7.11 A 50% reaction stage of an axial flow turbine shown in Figure E7.14 has blades 12.7cm in height and a linear speed of rotation u=366m/s at the mean line of the rotor where the diameter is d=76.2cm. The stagnation temperature and stagnation pressure entering the stator from the combustor are T_t=1144K and p_t=687kPa. The angle for the stator exit absolute velocity is α_4=25°.



FigureE7.14 A 50% reaction turbine stage with α_4=25°

(a) Draw 2 possible combined velocity diagrams for this stage. The diagrams should be neat, executed with a straight edge or by computer, of reasonable size, clearly marked, and shown to scale. The combined velocity diagram is one which shows the inlet and outlet velocities referenced to a single u velocity. For example, the general velocity diagrams shown in the figure may be superimposed to form a combined velocity diagram.
(b) Calculate the work per unit mass W for the case where the turbine is extracting the maximum work per unit of entering kinetic energy $W/(c_4^2/2)$
(c) Determine the blade efficiency η_b, which is defined as $\eta_b=u/c_4$, for the case of maximum work described in part (c) and draw the corresponding combined velocity diagram.
(d) Plot the distribution of stagnation and static values of pressure and temperature, as well as the Mach number, through the stage as a function of x, starting from the entrance to the stator blades and ending at the exit of the rotor blades.

(e) If the blades block 5% of the flow area determine the mass flow rate through the turbine

(f) Determine the power developed by the turbine

Solution

(a) Since only u and α_4 are given there are a wide variety of possible velocity diagrams for a 50% reaction axial turbine stage and these are shown in Figure E7.15. Note that the work done per unit mass by flow turning, as expressed by the product of u and the difference in u-components of velocity $\Delta c_u = c_{4u} - c_{5u}$ is also given by

$$W = u\Delta c_u = u(c_{4u} - c_{5u}) = (1/2)[(c_4^2 - c_5^2) - (w_4^2 - w_5^2)] \qquad \text{(E7.1)}$$

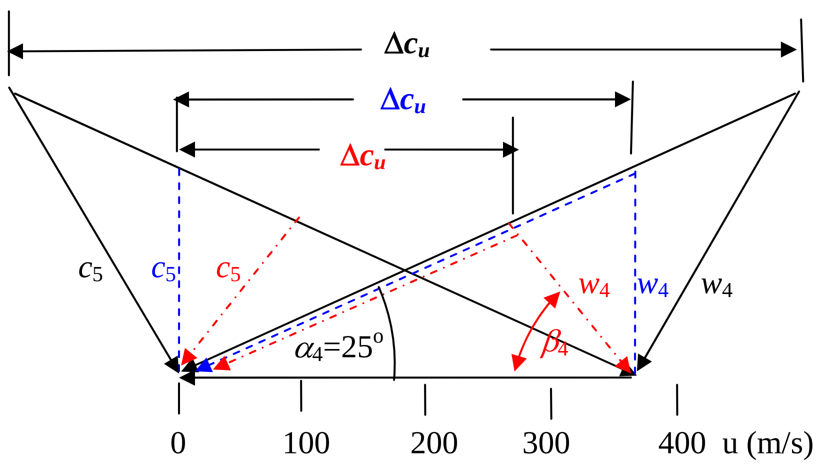


Figure E7.15 Possible velocity diagrams for fixed values u=366m/s and α_4=25°. Values of β_4 are shown in red for $\beta_4<\pi/2$, in blue for $\beta_4=\pi/2$, and in black for $\beta_4>\pi/2$, along with the corresponding velocities c_4 and w_4.

(b) Then the work extracted, for a 50% reaction axial turbine, where $c_4=w_5$ and $c_5=w_4$, is given by either of the two following equalities:

$$W = (c_4^2 - w_4^2) = u(c_4\cos\alpha_4 - w_4\cos\beta_4) \qquad \text{(E7.2)}$$

Note that $\cos\beta_4 = -\cos(\pi - \beta_4)$ so that the work extracted continually grows as β_4 increases, and that $w_4\cos\beta_4 = u\text{-}c_4\cos\alpha_4$ so that Equation (E7.2) may be written as

$$W = u(2c_4\cos\alpha_4\text{-}u) = c_4^2[2(u/c_4)\cos\alpha_4 - (u/c_4)^2] \qquad \text{(E7.3)}$$

Then Equation (E7.3) may be expressed non-dimensionally as

$$W/c_4^2 = 2\eta_b\cos\alpha_4 - \eta_b^2 \qquad \text{(E7.4)}$$

This ratio of the work to twice the kinetic energy entering the rotor is expressed as a function of what may be called the blade efficiency $\eta_b = u/c_4$ and it has a maximum value when $\eta_b = \cos\alpha_4$ and therefore when

$$u = c_4\cos\alpha_4 \qquad \text{(E7.5)}$$

That is, when the absolute velocity leaving the blade is purely axial, as shown by the blue lines in Figure E7.15, the blade efficiency is a maximum. Under this maximum condition the work extracted per unit mass is

$$W_{max.\ blade\ eff.} = c_4^2\cos^2\alpha_4 = u^2 \qquad \text{(E7.6)}$$

The velocity c_4=404m/s and the axial velocity component $c_{4a}=c_4\sin 25$=171m/s. For fixed values of u and α_4 the work extracted per unit mass continues to grow linearly with c_4, and therefore with the mass flow, which is proportional to the axial component of velocity $c_{4a}=c_4\sin\alpha_4$. If we choose to consider the case of the maximum blade efficiency, then the work extracted is

$$W_{max.\ blade\ eff.} = (366\text{m/s})^2 = 134\text{kJ/kg} \qquad \text{(E7.7)}$$

(c) To determine the pressure and temperature profiles through the blade passages we must consider the flow to be compressible. The variation of the stagnation temperature and the stagnation pressure (red and blue solid lines, respectively) and the static temperature and static pressure (red and blue dashed lines, respectively) are shown in Figure E7.16 2.

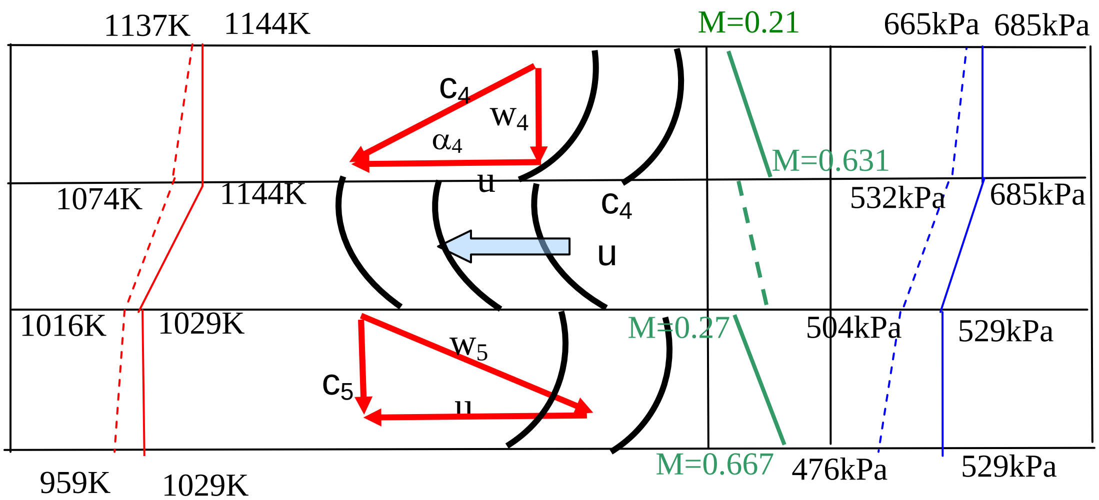


Figure E7.16 The variation of the stagnation temperature and the stagnation pressure (red and blue solid lines, respectively) and the static temperature and static pressure (red and blue dashed lines, respectively) through the turbine passages. Also shown is the variation of the absolute Mach number entering and leaving the rotor (solid green lines) and the relative Mach number entering and leaving the passage (dashed green line)

We assume that there are no friction losses and no heat transfer through the passages so that the stagnation pressure and temperature are constant through the stator passages. We further assume that c_p = 1.16kJ/kg-K and γ = 4/3 and that both are constant throughout the passages. We are given that the stagnation temperature entering the rotor is $T_{t,4}$ =1144K and in the rotor passage the work extracted per unit mass, which we have calculated already as 134kJ/kg, is given by

$$W = c_p(T_{t,4}-T_{t,5}) = (1.16\text{kJ/kg-K})(1144\text{K}-T_{t,5}) = 134\text{kJ/kg}$$

We find then that the stagnation temperature leaving the rotor is $T_{t,5}$=1029K. The stagnation temperature variation through the passages is shown in Figure E7.16. The static temperature (and the corresponding sound speeds) entering and leaving the rotor may be found by applying the energy equation along a streamline for adiabatic flow as follows:

$$T_4 = T_{t4} - c_4^2/2c_p = 1144\text{K} - (404\text{m/s})^2/[2(1160\text{m}^2/\text{s}^2\text{-K})] = 1074\text{K}$$

$$a_4 = (\gamma R T_4)^{1/2} = (1.33*287\text{m}^2\text{s}^2\text{-K}*1074\text{K}) = 640\text{m/s}$$

$$M_4 = (404\text{m/s})/(640\text{m/s}) = 0.631$$

$$M_4' = (171\text{m/s})/(640\text{m/s}) = 0.267 \text{ (relative to rotor at rotor exit)}$$

$$T_5 = T_{t5} - c_5^2/2c_p = 1029\text{K} - (171\text{m/s})^2/[2(1160\text{m}^2/\text{s}^2\text{-K})] = 1016\text{K}$$

$$a_5 = (\gamma R T_5)^{1/2} = (1.33*287\text{m}^2\text{s}^2\text{-K}*1016\text{K}) = 622\text{m/s}$$

$$M_5 = (171\text{m/s})/(622\text{m/s}) = 0.275$$

$$M_5' = (404\text{m/s})/(622\text{m/s}) = 0.649 \text{ (relative to rotor at rotor exit)}$$

Knowing the sound speed at any point we may find the corresponding Mach numbers from $M=c/a$. Then the isentropic relations between static and stagnation properties may be used to find the static pressures and densities. At the entrance to the rotor the stagnation pressure is given as 685kPa and the absolute Mach number has been calculated to be 0.631 so the static pressure at the rotor entrance is

$$p_4 = p_{t4}[1+0.5(\gamma-1)M_4^2]^{-\gamma/(\gamma-1)} = 685\text{kPa}[1+\text{M}_4^2/6]^{-4} = 532\text{kPa}$$

The density may be found from the equation of state

$$\rho_4 = p_4/RT_4 = (532\text{kPa})/[(0.287\text{kJ/kg-K})(1074\text{K})] = 1.73\text{kg/m}^3$$

Note that the conservation of mass requires

$$\rho_4 A c_{a4} = \rho_5 A c_{a5}$$

In other words the density is constant through the stage, as may be expected since we have assumed, *a priori*, that the axial component of velocity is unchanged through the stage. Then using the state equation to calculate the static pressure leaving the rotor we obtain

$$p_5 = \rho_5 R T_5 = (1.73\text{kg/m}^3)(0.287\text{kJ/kg-K})(1016\text{K}) = 504\text{kPa}$$

The stagnation pressure leaving the rotor is then given by the isentropic relation

$$p_{t5} = p_5\,[1+0.5(\gamma-1)M_5^2]^{\gamma/(\gamma-1)} = 504\text{kPa}[1+M_5^2/6]^4 = 529\text{kPa}$$

(d) The area of the turbine flow path is, accounting for 5% blockage due to blade width, given by

$$A = 0.95\pi(r_o^2 - r_i^2) = 0.95\pi[(0.381+0.0635)^2 - (0.381-0.0635)^2] = 0.304\text{m}^2$$

The mass flow through the turbine is then

$$\rho_4 A c_{a4} = \rho_4(0.304\text{m}^2)c_a = (1.73\text{kg/m}^3)(0.304\text{m}^2)(171\text{m/s}) = 85.3\text{ kg/s}$$

(e) The power developed by the turbine is

$$P = (134\text{kJ/kg})(85.3\text{kg/s}) = 11.45\text{MW}$$

Note: To find the flow conditions at the entrance to the inlet guide vanes one may make use of the fact that one knows the mass flow, the area, and the stagnation conditions at that station so that using

$$\dot{m} = p_t A\sqrt{\frac{\gamma}{RT_t}}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Solving this equation yields M=0.21. Then using the usual isentropic flow equations we find p=665kPa and T=1137K

E7.12 A turbine blade receives hot gas from a stator with an outlet angle of 70° as shown in the diagram. The blade is designed to have 3° incidence at the root when the linear speed of rotation at the root is 213.4 m/s. The absolute speed leaving the stator is 548.6m/s and the radius of the turbine disc at the root.

(a) Find the rotor blade angle β_4 for a blade that is untwisted over its entire length

(b) Find the incidence at the tip for this untwisted blade

(c) Find the stator outlet angle α_4 and the rotor blade inlet angle β_4 at the root and at the tip under the following set of assumptions:

(i) The original conditions pertain only to the root station

(ii) There is free vortex flow in the gap between the stator and the rotor, which implies that uc_u = constant through the gap and, likewise, the axial component c_a=constant.

(iii) The incidence at all points along the leading edge of the blade is 3°

(iv) The rotor outlet absolute velocity c_4 is 231.6m/s and is in the axial direction

(v) The deviation is 5° at all points along the trailing edge of the stator blade

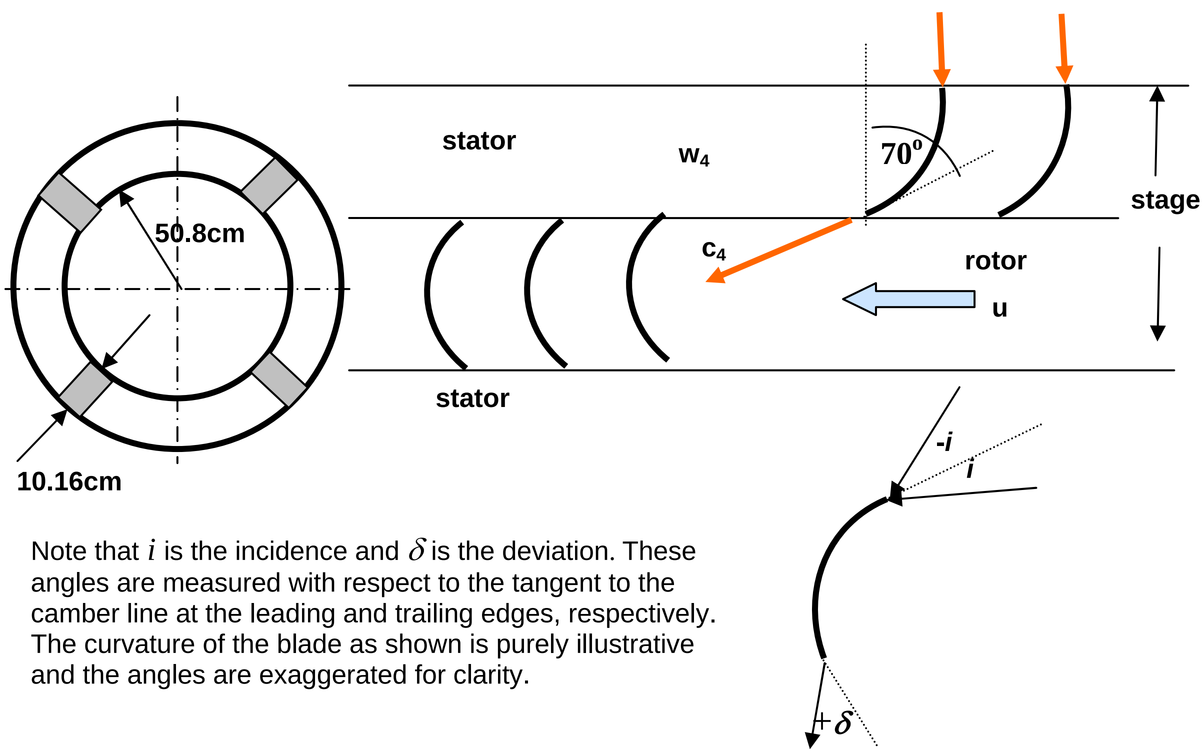


Figure E7.17 Sketch of an axial flow turbine passage

Solution

The solution is given in the details appearing on Figure E7.18.

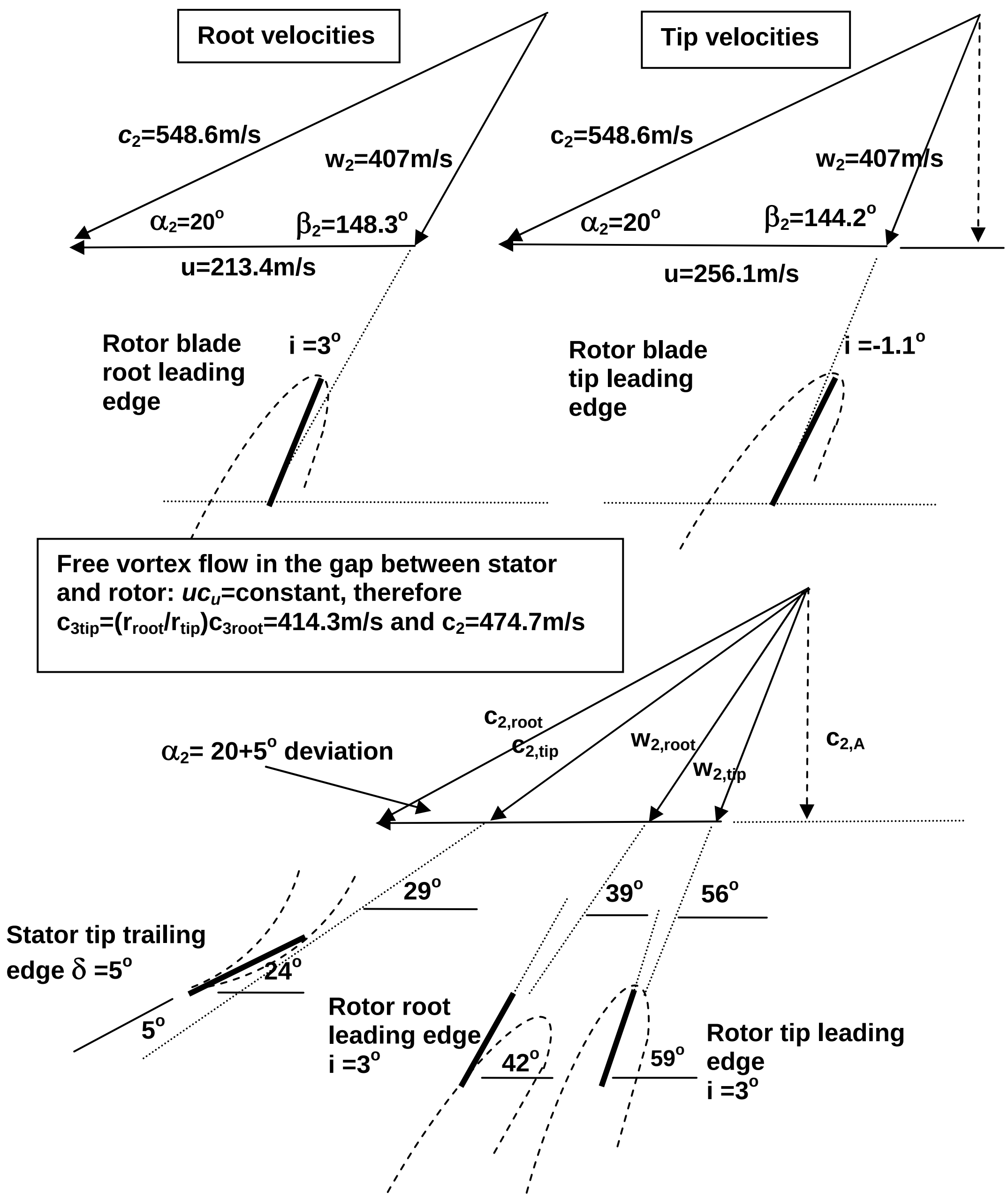


Figure E7.18 Diagram of blade properties for the axial turbine stage

E7.13 An axial flow turbine blade 10.2cm in height is attached to a 50.8cm radius turbine wheel as shown in Figure E7.19. The blade is designed to receive air from a stator that has an outlet angle α_4=20°. When the linear speed of rotation at the root of the blade (that is, at the point where the blade is attached to the wheel) is 213m/s the angle of incidence of the chord line of the blade there is i=3°. The absolute velocity from the stator outlet is constant at c_4=549m/s.

Consulting the sketch below and assuming incompressible flow:

(a) Find the stagger angle γ of the blade chord at the root

(b) If the blade is untwisted throughout its height find the angle of incidence at the tip of the blade under the given conditions

(c) Find the work performed per unit mass of fluid processed if the rotor blade provides an exit absolute velocity that is purely axial.

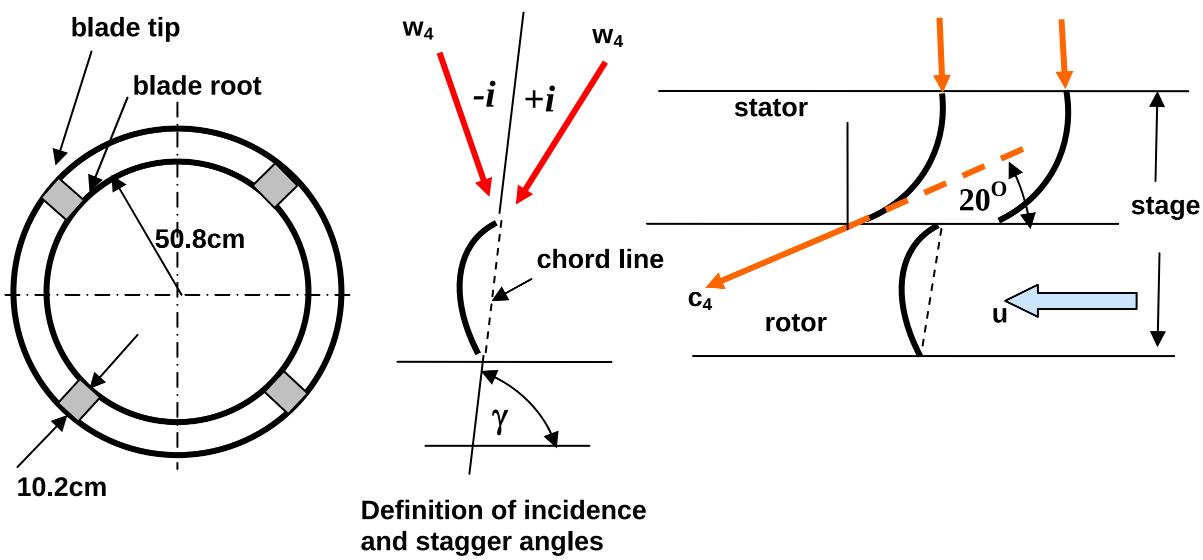


Figure E7.19 Schematic diagram of axial flow turbine with blade details

Solution

(a) The axial velocity component is $c_a = c_4 \sin\alpha_4$ = 549m/s(sin20) =188m/s

The component c_{4u}=$c_4\cos\alpha_4$=549cos20=516m/s

At the blade root the angle $\pi - \beta_{4,root} = \arctan[c_a/(c_{4u}\text{-}u_{\text{root}})] = \arctan[188/(516\text{-}213)] = 31.8^{\text{o}}$. The stagger angle at the root is $\gamma = (\pi - \beta_4) + i = 31.8+3 = 34.8^{\text{o}}$

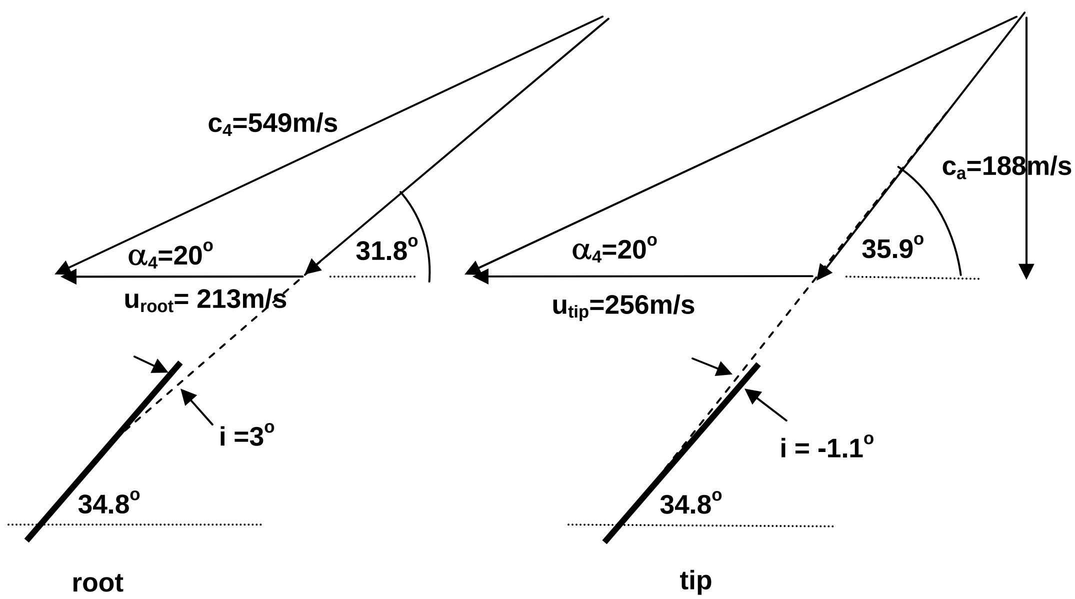


Figure E7.20 Velocity diagrams constructed for the turbine blade

(b) The linear speed of rotation at the tip is $u_{tip} = u_{root}\,(r_{tip} / r_{root}) =$
(213m/s)(50.8+10.2)/50.8 = 256m/s

Then the angle $\pi - \beta_{4,tip} = \arctan[c_a/(c_{4u}\text{-}u_{tip})] = \arctan[188/(516\text{-}256)] = 35.9^{\text{o}}$

The incidence at the tip for an untwisted blade is $i_{tip} = \gamma - (\pi - \beta_{4,\text{tip}}) = 34.8\text{-}35.9 =$ $\mathbf{-1.1^{o}}$

(c) The work per unit mass is $W = u_{mean}\Delta c_u = 0.5(u_{tip}+u_{root})(c_{4u}\text{-}c_{5u}) = 0.5(213+256)(516\text{-}0)$
$W = 121{,}000\text{m}^2/\text{s}^2$ or the work per unit weight is W/g=12,334m

E7.14 An axial flow compressor stage is designed for 50% reaction, a rotational speed of 13,760rpm, and a mean radius of 25.4cm. Carry out all analyses at the mean radius location and you may assume the flow is incompressible in this problem.

(a) If the rotor blade relative velocity exits at an angle $\beta_3 = 70^o$ and c_a=152m/s, draw the velocity diagram for the entrance and the exit of the rotor at the mean radius approximately to scale and determine all appropriate angles and velocities and note them directly on the diagram.

(b) If the airflow rate is 36.2kg/s find the torque and the power required and the ideal total head produced.

Solution

i. The stage is a 50% (symmetric) stage so $\beta_3 = \alpha_2$ and $\beta_2 = \alpha_3$ while $w_3 = c_2$ and $w_2 = c_3$.

ii. The rotational speed is $\omega = 2\pi N/60 = 2\pi(13760)/60 = 1441 s^{-1}$

iii. The linear speed of rotation at the mean radius is $u = \omega r_m$= (1441)(0.254m) = 366m/s

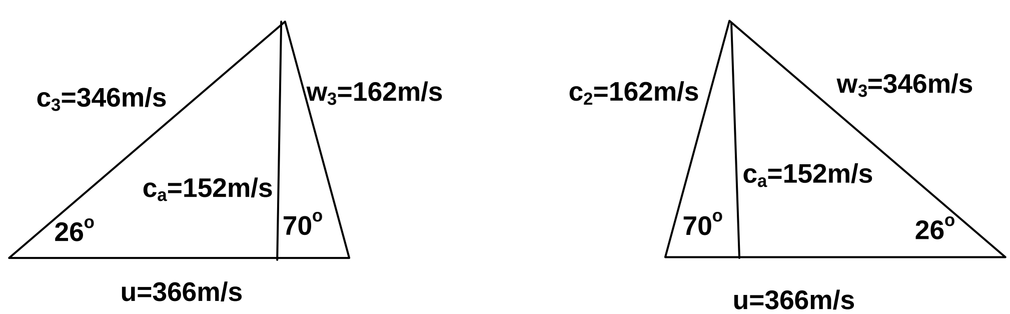
iv. The axial velocity $c_a = w_3 \sin 70$ or w_3 = (152m/s)$(\sin 70)^{-1}$=162m/s and thus c_2 =162m/s

v. The velocity component $c_{3u} = u - w_3 \cos 70$ = 366m/s-(162m/s)(0.3420) = 311m/s

vi. The absolute velocity $c_3 = (c_{3u}^2 + c_{3a}^2)^{1/2} = (311^2+152^2)^{1/2}$ = 346m/s

vii. The velocity component $c_{2u} = c_2 \cos 70$ = 162m/s(cos70) = (162m/s)(0.3420) =55.4m/s

(a)



(b)

(i) The torque is $T_q = \dot{m}\, r_m(c_{3u}-c_{2u})$ = (36.2kg/s)(.254m)(311m/s – 55.4m/s) = 2350N-m)

(ii) The power required is $P = T_q\omega$ = (2350N-m)(1441s^{-1}) = 3387kW

(iii) The ideal total heat is $P/mg=(3387000\text{N-m/s})/(36.2\text{kg/s})(9.81\text{m/s}^2)=9537\text{m}$