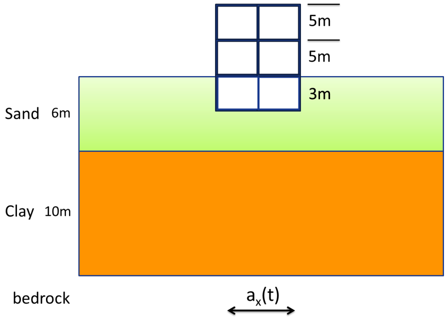
Dynamics.
Linear/Nonlinear analysis of soil structure interaction problems.
Constitutive aspects

August 2014

Scope of the lecture

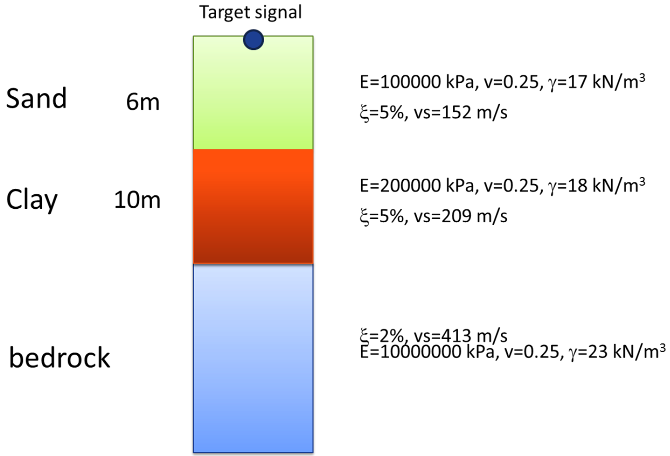
- Example of a building subject to the earthquake (using Domain Reduction Method (DRM))
  1. Free field motion as 1D wave propagation problem
     1. Basic requirements for mesh size and time step
     2. Mesh and BC for rigid base model
     3. Application of earthquake signal to the model (relative vs absolute)
     4. Signal processing (linear deconvolution, baseline correction, Butterworth filtering)
  2. Analysis of reduced model
     1. Setting reduced DRM model (exterior and boundary domain setting)
     2. Running DRM model
- Pure periodic BC model
- Application of nonlinear models to the problem

Problem: time history analysis of a building subject to the Loma Prieta earthquake



- To run this example we can use so-called Domain Reduction Method (DRM)
- It is the two step analysis that consists of
  1. Free field motion analysis
  2. Reduced model analysis (structure and small part of subsoil adjacent to it)

Free field motion: layered subsoil subject to Loma Prieta earthquake (1D) (single phase)



Free field motion: estimating mesh size and time step

- To trace wave propagation in the medium we need approximately 10 nodes per wavelength
- The mesh size depends on the maximum frequency $f_{\max}$ that is to be represented
- For typical seismic application $f_{\max}$ is limited up to 10 Hz
- Hence the maximum mesh size should be smaller than

$$h^e \leq \frac{\lambda}{10} = \frac{v_s}{10\ f_{\max}}$$

- Hence the time step limitation can be formulated as follows (in a single time step signal should not pass through more than one element and v is the maximum wave velocity)

$$\Delta t \leq \frac{h^e}{v}$$

Free field motion: estimating mesh size and time step

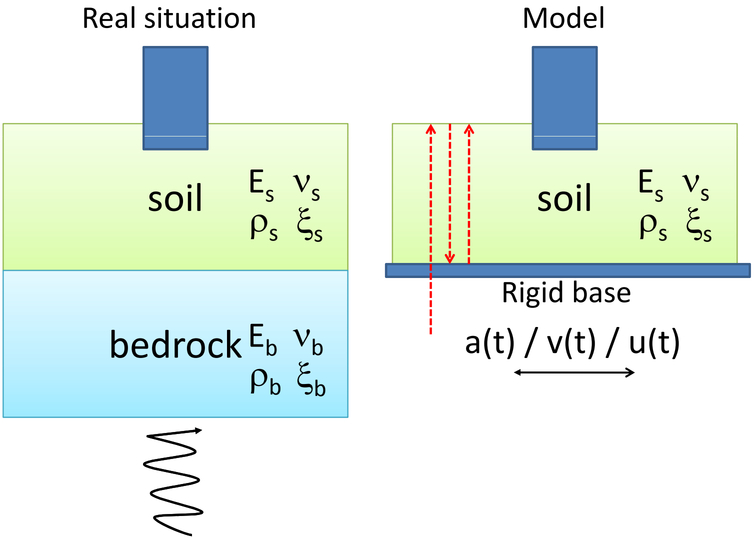
Material data

Layer	Depth	E	ν	γ	K_o^{insitu}	G	λ
	[m]	[kPa]	[-]	[kN/m^3	[-]	[kPa]	[kPa]
Sand	6	$1 \cdot 10^5$	0.25	17	0.5	40000	40000
Clay	10	$2 \cdot 10^5$	0.25	18	0.6	80000	80000
Bedrock	∞	$2 \cdot 10^6$	0.3	23	1.0	3846154	5769231

Layer	Depth	ρ	v_s	v_p	h^e	$\overline{h}^e$	Δt
	[m]	[kN s^2/m]	[m/s]	[m/s]	[m]	[m]	[s]
Sand	6	1.73	152	263	1.5	1.5	0.010
Clay	10	1.83	209	362	2.1	2.0	0.010
Bedrock	∞	2.34	1281	2396	12.8	12.0	0.009

Hence in sand layer we will use elements with $\overline{h}^e$=1.5 m, and for clay $\overline{h}^e$=2.0 m

General comments: Rigid base model

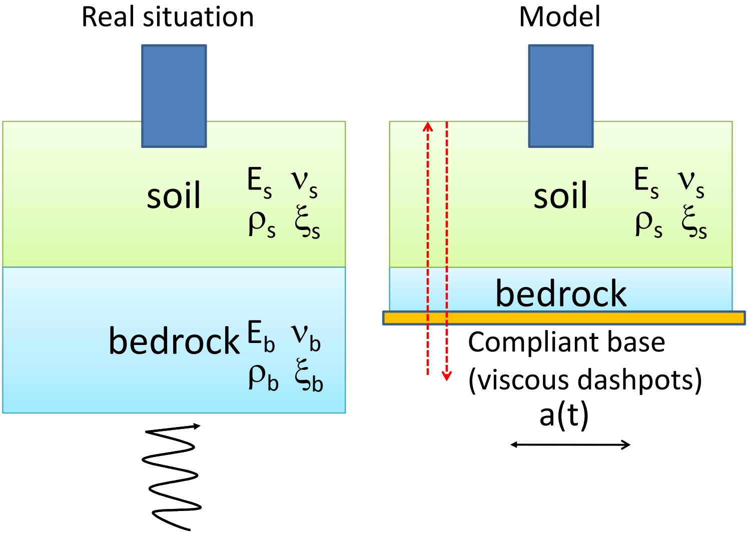


Impedance ratio $\alpha_z = \dfrac{\rho_s \; v_{ss}}{\rho_r \; v_{sr}} \rightarrow 0$ (complex for nonzero damping)

General comments: Rigid base model

- Nodes at the bottom are fixed
- (a) Motion is imposed by displacement/velocity/acceleration boundary conditions (absolute format)
  1. Output: absolute displacements/velocities/accelerations
- (b) Motion is imposed by application of global acceleration to the whole domain (relative format)
  1. Output: relative displacements/velocities/accelerations (with respect to the rigid base)

General comments: Compliant base model



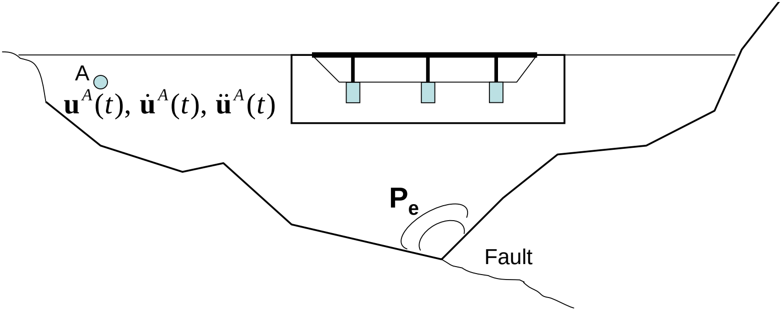
Impedance ratio $\alpha_z = \dfrac{\rho_s \; v_{ss}}{\rho_r \; v_{sr}} > 0$ (complex for nonzero damping)

Compliant base model: basic assumptions

- Nodes at the bottom are free to move
- Viscous dashpots are added to the base
- Motion can exclusively be applied through the acceleration record (seismic input)
    1. Output: absolute displacements/velocities/accelerations
- Accelerations are integrated to velocities via Newmark method ($a(t) \rightarrow v_{su}(t)$)
- Viscous shear tractions are computed and applied to the base
- The input $a(t)$ does not need to be compatible with $a(t)$ computed at the base (!)

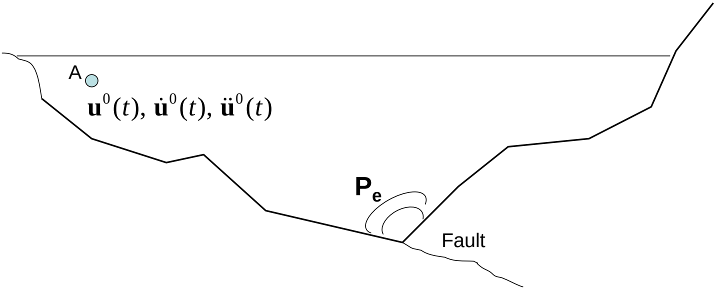
L.H. Mejia, E.M. Dawson. Earthquake deconvolution for FLAC. 2006 (available on the web).

General comments: Domain Reduction Method: General idea



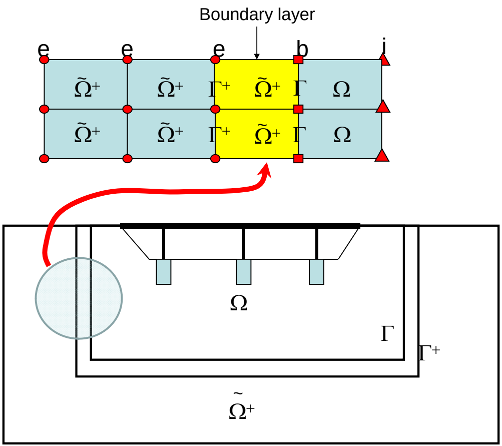
- Goal: **analyze computational model that concerns the structure and only a small adjacent part of subsoil**
- Single and two-phase formulations are supported
- This model with a large subsoil zone and source of load $\mathbf{P}_e(t)$ is decomposed into two models:
  - **Background model**
  - **Reduced model**

DRM: Background model



1. In the background model **structure is removed**
2. **Background model** yields $\rightarrow$ free field motion: $\mathbf{u}^0(t)$, $\dot{\mathbf{u}}^0(t)$, $\ddot{\mathbf{u}}^0(t)$

DRM: **Reduced model**



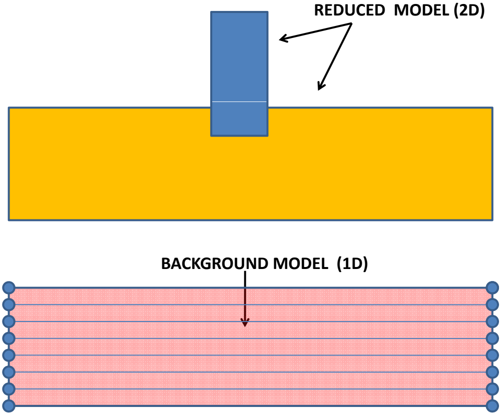
1. Viscous dampers are added to $\hat{\Gamma}^+$ to cancel wave reflections
2. Displ. decomposition in the exterior domain: $\mathbf{u}_e = \mathbf{u}_e^0 + \mathbf{w}_e$

DRM: Consequences

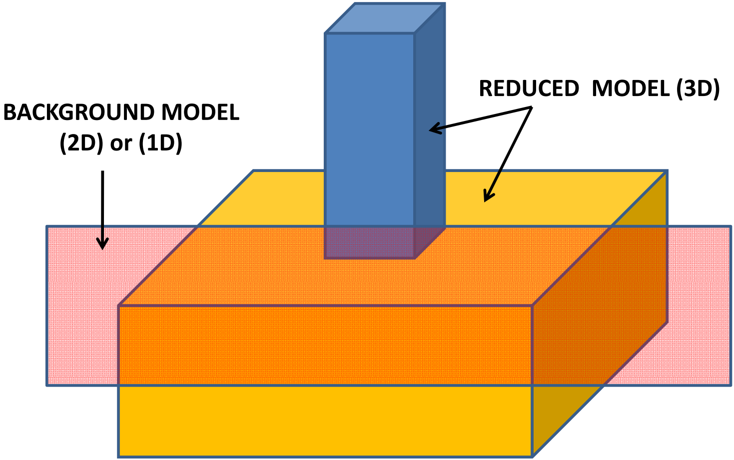
1. Continuum in the exterior and boundary layer must work in the elastic mode
2. ZSoil freezes stiffness moduli in these zones at the begining of the analysis
3. The interior domain can be analyzed with a suitable nonlinear soil model (HSs, Densification (DNS), HB)
4. For HSs, DNS, HB small strain stiffness is automaticaly taken into account
5. For MC we have to adjust stiffness moduli (after static analysis) by hands or by using load time function

DRM: Dimensions of background and reduced models

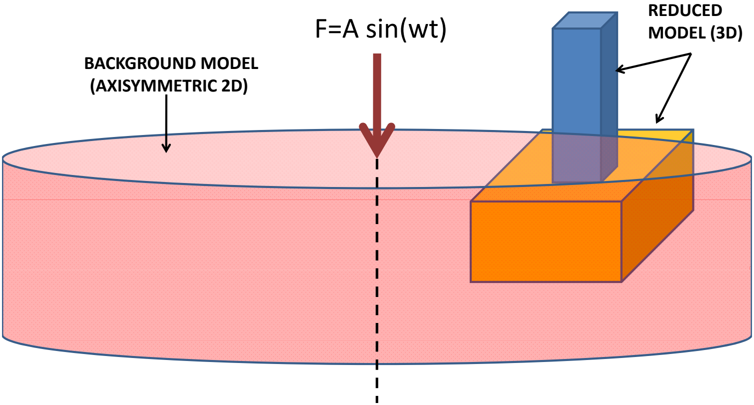
- Space dimension for background model can be lower than for the reduced model



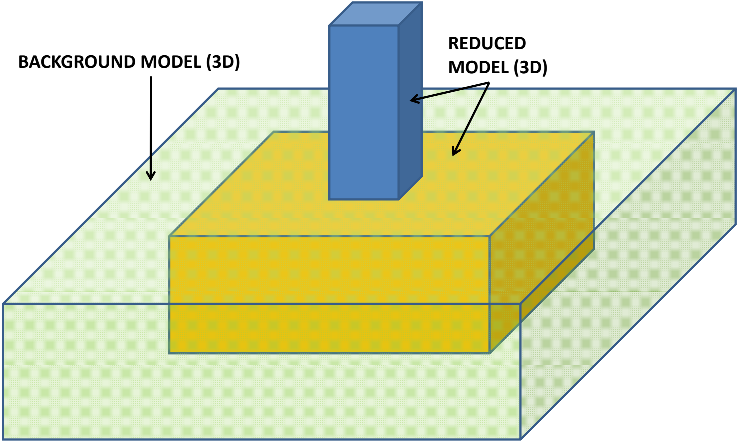
DRM: Background (2D) Reduced (3D)



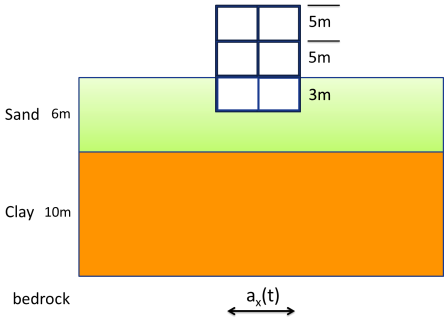
DRM: Background (2D-axisymmetric) Reduced (3D)



DRM: Background (3D) Reduced (3D)



Coming back to our problem

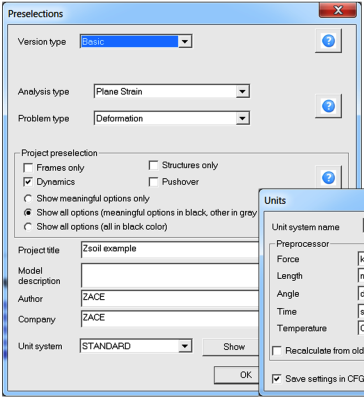


- We will use DRM method
- Rigid base model is analyzed

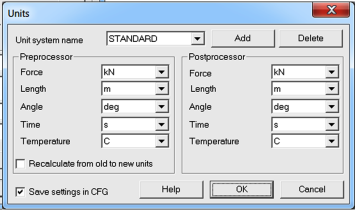
Free field motion: Model generation, step by step

1. **File/New** - select **Plane strain** model
2. Fill **Project preselection** form (set ON option ☒ **Dynamics**)
3. Under (**Control/Units**) set units system (use [s] for time unit)
4. **File/Save As** : shear-layer-RB-rel.inp

Project preselection

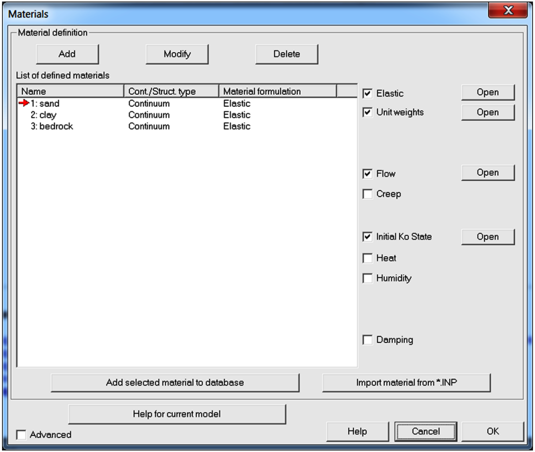


Units



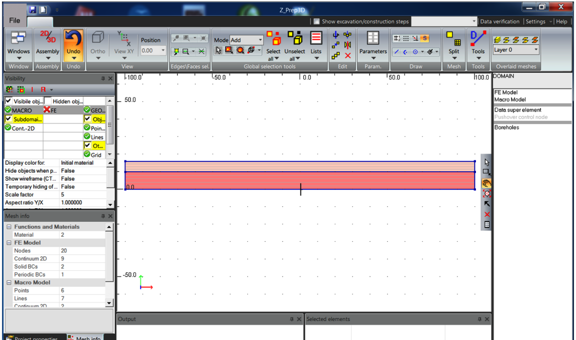
Free field motion: Setting material data

1. Add three continuum materials for sand, clay and bedrock via **Assembly/Materials** (material data is given in tabular form in slide 6)



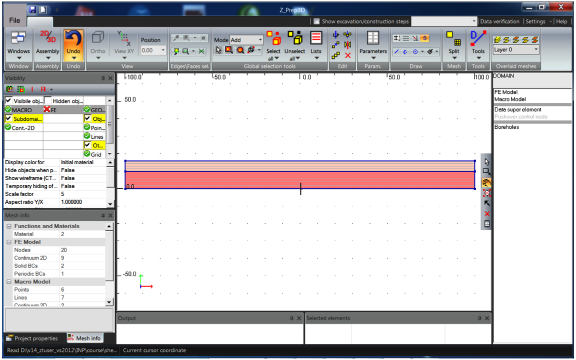
Free field motion: geometrical model

1. Use option draw rectangle from ribbon menu **Draw**
2. Draw 2 rectangles for sand and clay layers (so far we do not know how big the reduced model should be; hence let us assume $\pm$ 100m from the axis of the building)
3. Create 2 subdomains using **Macro-model/ Subdomains/ Create continuum inside contour**
4. Apply materials to subdomains using ribbon menu **Parameters/Update par.**



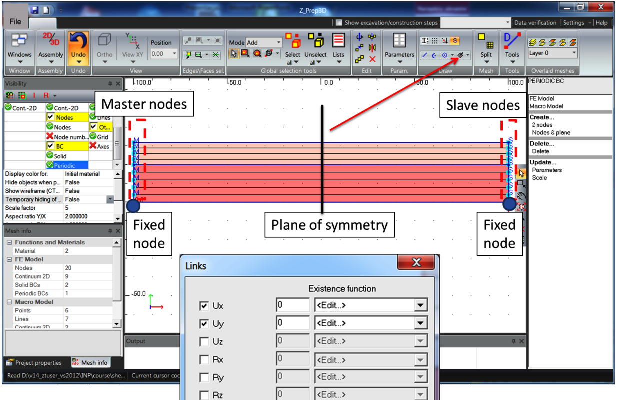
Free field motion: model discretization

1. Create virtual meshes
   (4 elements in sand layer, 5 elements in clay layer)
   (use options from ribbon menu **Mesh** or right menu
   Subdomain/Mesh/Create virtual mesh
2. Then convert virtual mesh to the real one



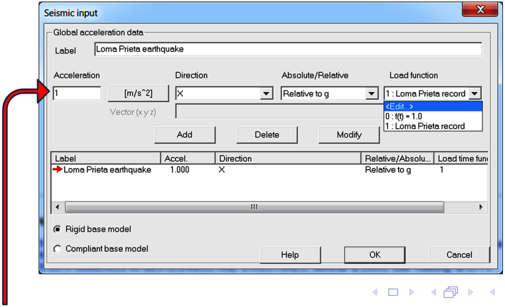
Free field motion: boundary conditions for rigid base model

1. Select 2 nodes at the base and fix them
2. Create an auxiliary vertical symmetry plane (at $x = 0.0$)
3. Select nodes at the left wall (except one at the base)
4. Set periodic BC (use FE model/Boundary conditions/Periodic BC/Nodes & Plane option)



Free field motion: Let us define seismic input (relative format)

1. Go to **Assembly/Seismic input** option
2. By using this option our results will be output as relative with respect to the rigid base
3. Set scaling factor to 1.0 (or another), define if a given signal is normalized (or not) by g value and edit the load time function $a(t)/g$ (or $a(t)/1\frac{m}{s^2}$) (click on *Edit..* in the combo box containing list of load time functions)



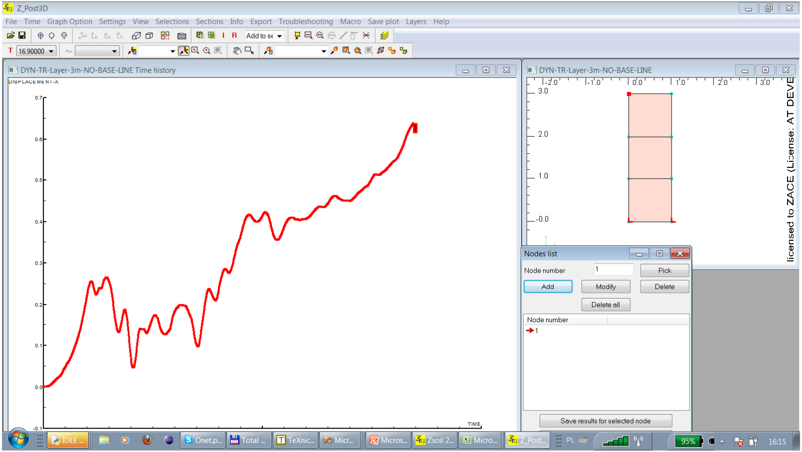
Scaling factor for acceleration record

Free field motion: Signal processing

Before we apply given earthquake record as a seismic input in our model the following operations must be carried out

1. Baseline correction and Butterworth filtering on the target earthquake record
2. Deconvolution of a given signal to the base (only linear deconvolution is handled in ZSoil)

Effect of baseline correction is well visible when earthquake signal is applied as an imposed acceleration (at the base)(absolute format)

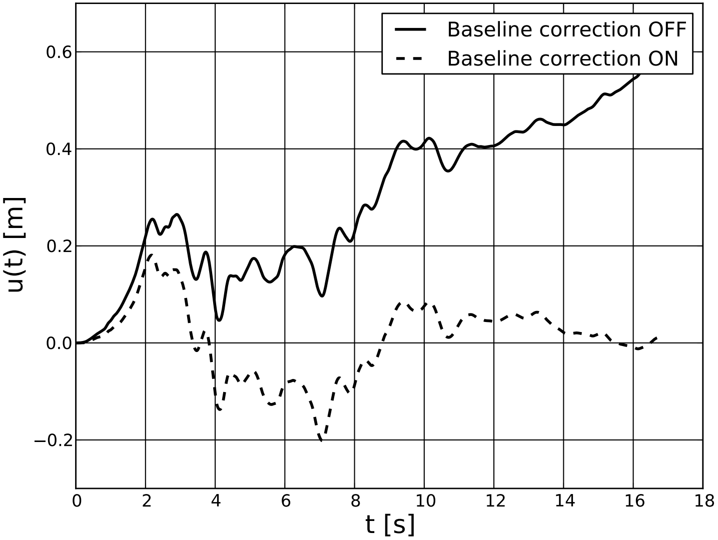


Baseline correction is inactive

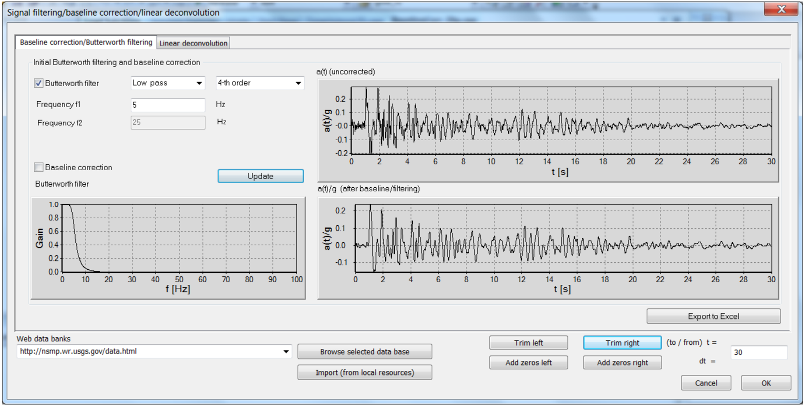
Signal processing: baseline correction

- Given set of points: $a_i(t_i)$
- Goal: remove trend line from $a_i(t_i)$ (qubic polynomial)
- Method: solve optimization problem: $\sum_i^n \{a_i - \tilde{a}(t_i)\}^2 \rightarrow MIN$
- Trend line equation: $\tilde{a}(t_i) = a_o + a_1 * t_i + a_2 * t_i^2 + a_3 * t_i^3$

Signal processing: baseline correction



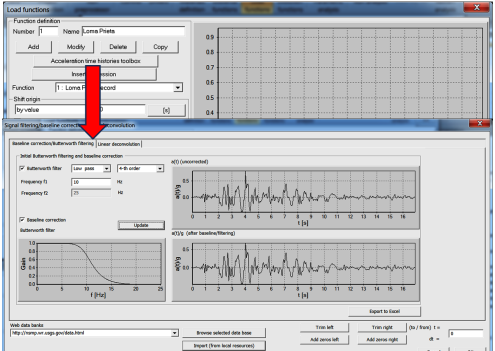
Signal processing: Butterworth filter



NB. In most cases we will apply low pass filter to cancel high frequencies (in earthquake engineering we usually use 10 Hz low pass BF filter)

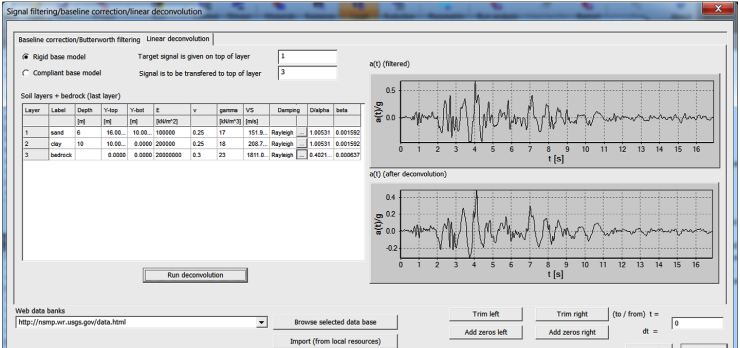
Free field motion: Signal processing: deconvolution

1. **Assembly/Load functions** - **Add** a function and label it as *Loma Prieta*
2. Press button **Acceleration time histories toolbox** and then in the foregoing dialog box **Import from local resources** record *LomaPrieta-18-10-1989-Station-Corralitos.ear*
3. Set ON 4-th order, 10 Hz low pass ☒ **Butterworth filtering**, set ON ☒ **Baseline correction** and **Update** the signal



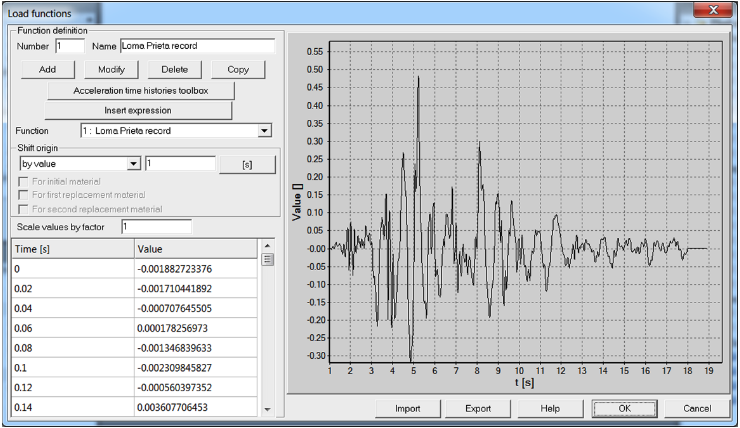
Free field motion: Signal processing: deconvolution

1. select tab **Linear deconvolution** and fill the form with data
2. declare that the signal is given on top of layer 1 (sand) and is to be transfered to top of layer 3 (bedrock)
3. use Rayleigh damping (for $f_1 = 2$ Hz $\xi_1 = 0.05$ and for $f_1 = 8$ Hz $\xi_1 = 0.05$) (use button **...** next to the combo box)
4. add quiet zone (**Add zeros left**) to the record at the beginning with $\Delta t = 6\text{m}/150\frac{m}{s} + 10\text{m}/208\frac{m}{s} \approx 0.1$ s
5. **Run deconvolution** and press **OK** to accept modifications



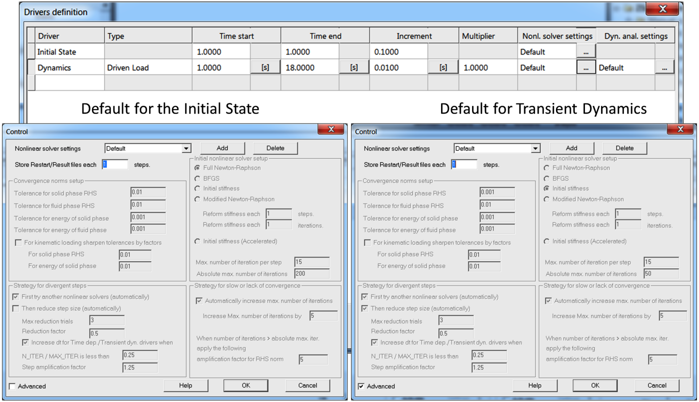
Free field motion: shifting load time function in time

- In order to be able to reuse this project as a free field motion for the reduced one we need to shift the dynamic driver in time (by 1s for instance)(will be explained later on why..?)

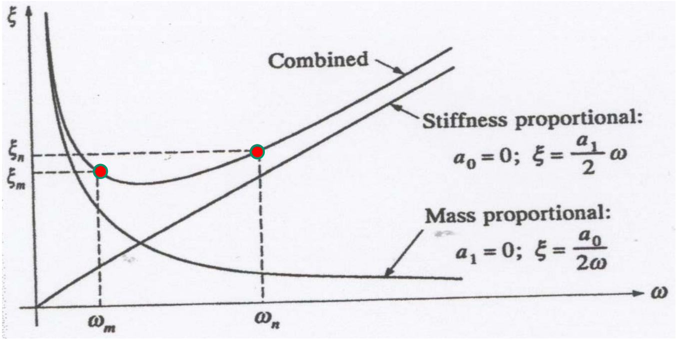


Free field motion: Drivers

1. Set drivers (**Control/Drivers**)
2. Note that Default control for each driver can be different (2014)



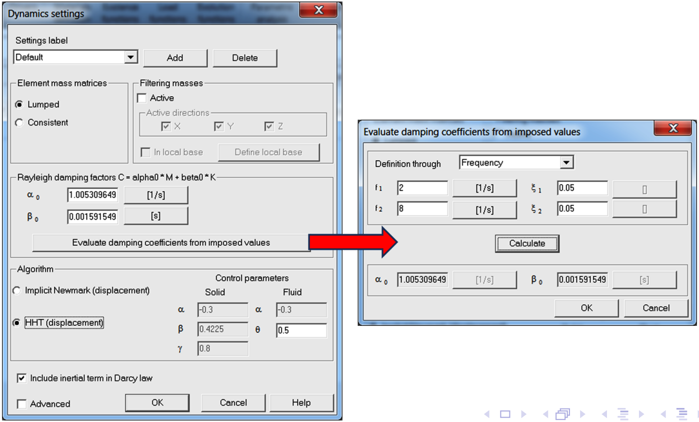
Specific settings: Rayleigh damping



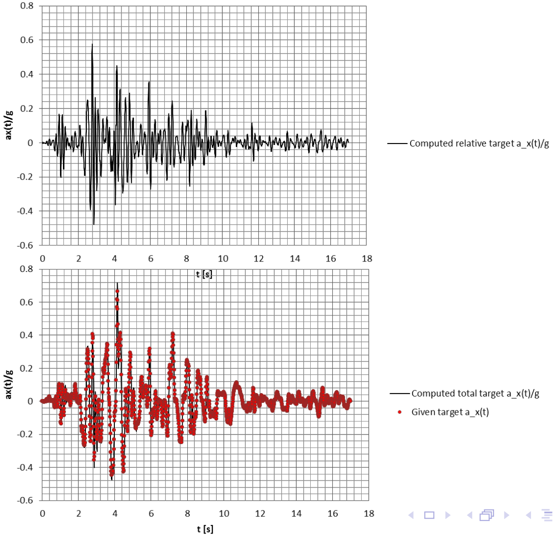
- α (or a_o) parameter applies to the mass (low frequency damping, may represent viscous damping)
- β (or a_1) parameter applies to the stiffness (high frequency damping, may represent hysteretic damping)

NB. If all materials have the same damping coefficients we can set them at the global level in settings for all dynamic drivers (will be done later on). We can do it also at the material level.

1. Set dynamic settings in **Control/Dynamics** menu or by pressing button `...` in column `Dynamic analysis settings` directly in the list of drivers
2. set up global Rayleigh damping coefficients, set ON ◎ **HHT** integration scheme

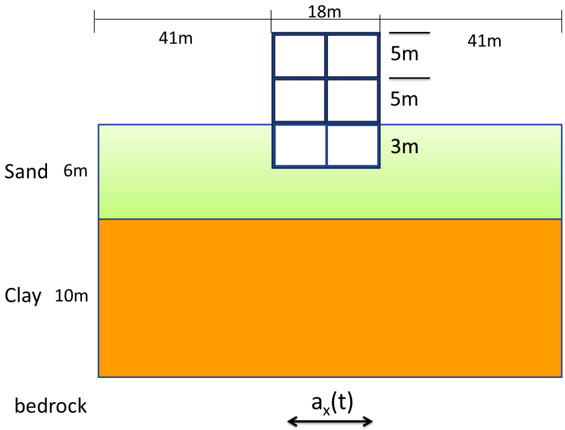


1. Displacements, velocities and accelerations are output as relative to the rigid base



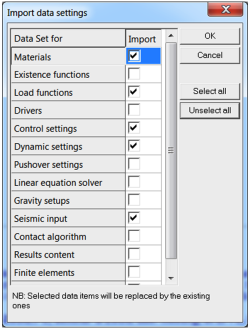
Reduced model: Geometry of the concrete frame

1. Concrete columns 60×60cm (spacing in Z direction: 6m)
2. Concrete beams 60×100cm (spacing in Z direction: 6m)
3. Foundation raft 70 cm



Reduced model: Creating new project

1. **File/New**
2. Select plane-strain
3. In **Project preselection** set ON ☒ **Dynamics**
4. **File/Save As** *reduced-RB-rel*
5. Import data from free field motion project (in main ZSoil menu use **File/Import data from another *.inp file**)



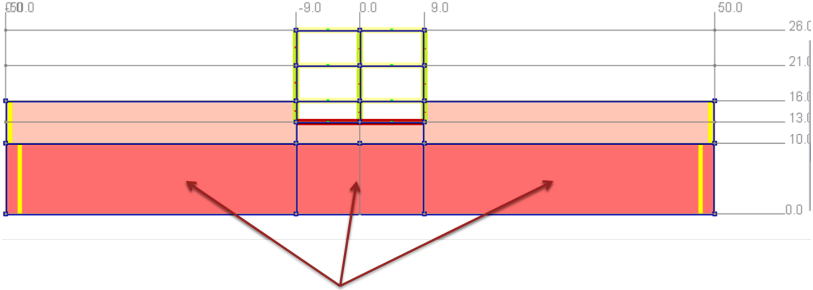
Reduced model: importing meaningful data from free field motion project

- Add 3 new materials for the raft (material 4), beams (5) and columns (6) (assume $E = 30000000$ kPa, $\nu = 0.2$, $\gamma = 24$ kN/m^3); in group ☒ **Main** select ◎ **Flexibility based formulation** (single beam per member is sufficient once mass lumping only at connections between beams and columns is a correct assumption)

NB. In ◎ **Flexibility based formulation** beam elements have 2 nodal points (as in standard) but stress resultants are output in 5 integration points (two of them are placed exactly at the endpoints); mass is lumped at two nodal points; these beam elements are mainly dedicated to structural aplications.

Reduced model: Creating macro-model

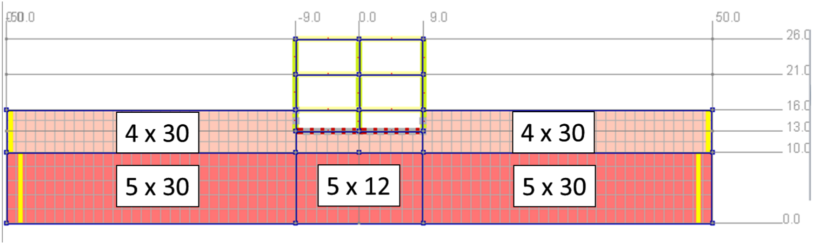
1. Let us go to the preprocessor
2. First we will create construction axes ($X = \{-50, -9, 0, 9, 50\}$, $Y = \{0, 10, 13, 16, 21, 26\}$ (here we have to be coherent with the free field motion geometry !) and then draw the geometry



3 subdomains (to have best quality meshes)

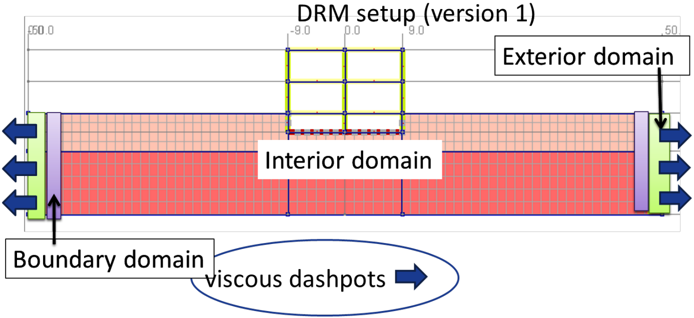
Reduced model: meshing (at the macro-level)

1. The optimal situation is when at the left and right walls we have mesh that is compatible with the one used for free field motion
2. Therefore in the vertical direction we will have 4 elements in sand layer and 5 elements in the clay layer
3. After meshing continuum select all beam macro-elements and create virtual mesh with split parameter equal to 1



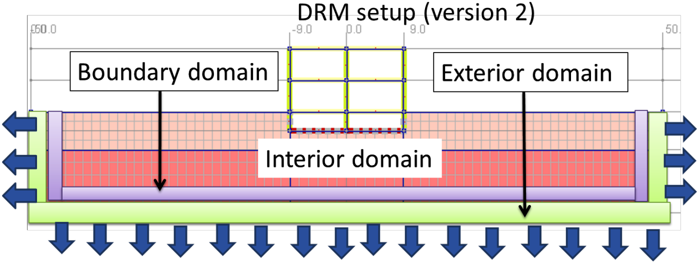
Reduced model: DRM setup (1)

1. Here interior domain is extended until the bedrock
2. Boundary and exterior layers are located at the left+right vertical walls (single column of elements in each)
3. On the exterior domain we have to add viscous dashpots
4. In our case setup (1) is optimal



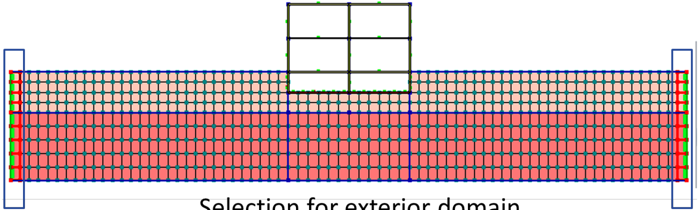
Reduced model: DRM setup (2)

1. Here interior domain is far from the bedrock
2. Boundary and exterior layers (still single layers of elements) are located around the structure
3. On the exterior domain we have to add viscous dashpots

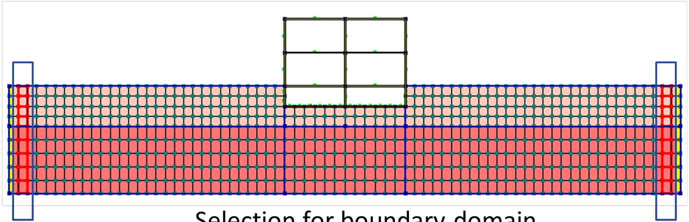


Reduced model: real mesh, DRM domains

1. Once we have FE mesh we can define DRM exterior and boundary domains **(FE model/DRM domains/Create..)**



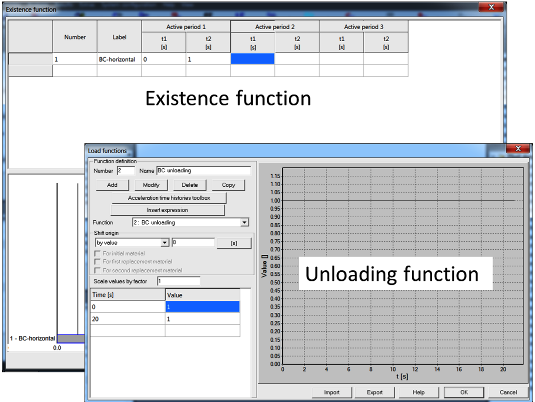
Selection for exterior domain



Selection for boundary domain

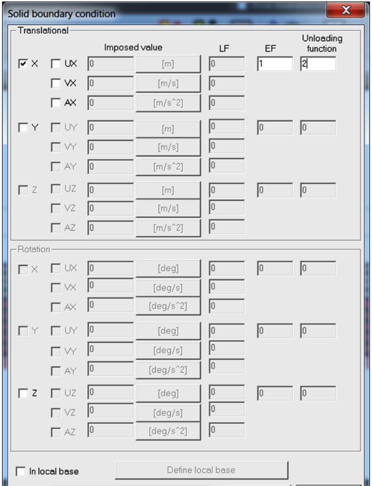
Reduced model: existence function and unloading function for horizontal fixities

1. Before starting dynamic analysis horizontal fixities must be released but static equilibrium must be preserved
2. To do that we need to define one existence function for these fixities in X-direction and one additonal load function that will be used as an unloading one for reactions in released fixities



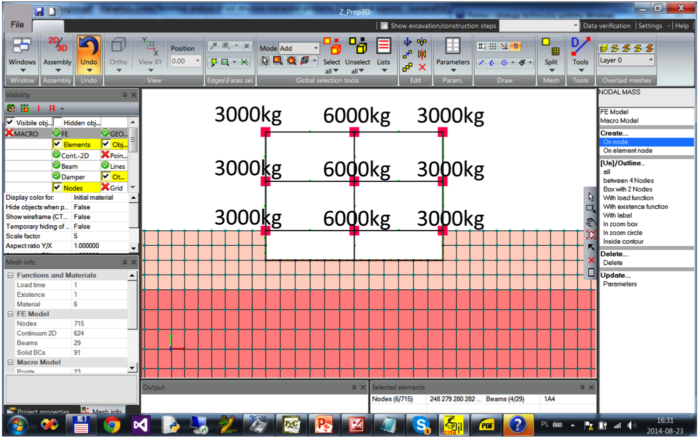
Reduced model: setting variable in time solid BC

1. First we apply standard BC (**FE model/Boundary conditions/Solid BC/On box**)
2. Then we can select nodes at both vertical walls (except those at the base) and define BC using option (in same menu **On node**)

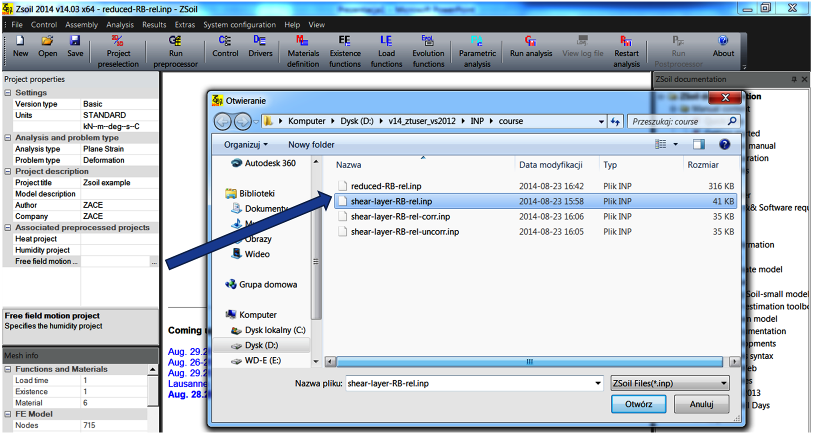


Reduced model: added masses

1. Remaining part of the structure add some mass to the system
2. Here we add 6000 kg at each nodal point in structure located on axis of symmetry and 3000 kg in the remaining one

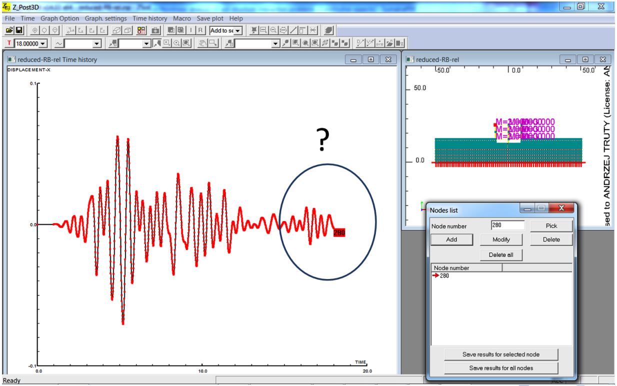


Reduced model: associate free field motion project with current one



Reduced model: computation and analysis of results

- Tracing time histories of relative displacements and accelerations of selected points
- Making envelopes in structures
- Observe parasitic effect of high displacement amplitude at end of shaking visible in displacement/acceleration time history plot for top frame node (this is an effect of rigid base assumption)



Reduced model: what about nonlinearities ?

- Main sources of nonlinearities
  1. In subsoil (plasticity, pore water pressure generation (in fully saturated media) that may lead to liquefaction in case of loose deposits)
  2. In structure (cracking)

NB. Once we use nonlinear models for subsoil deconvolution is no more unique (stiffness and damping are stress/strain dependent)

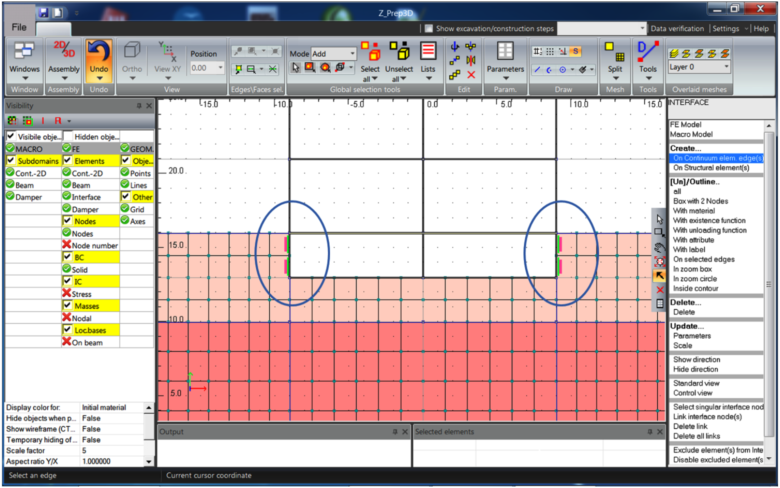
1. **M-C, H-B models**
   - here we have to define small strain E modulus that is larger than static one; to use different stiffness moduli within one computation we can apply a load time function to this parameter
   - these models have built in hysteresis effect but for larger strain values; therefore we have to be careful when setting damping parameters; using low frequency damping (to mass) is correct but stiffness proportional damping (if defined) is already included (partially or fully) in the elasto-plastic law; in this case we could define mass proportional damping parameter $\alpha = 4\pi f \xi$ (at $f = 2$Hz for instance) while keeping $\beta = 0.0$
2. **HSs, Densification models**
   - HSs and Densification models can be used for dynamic analyses but we need to activate ☒ **Small strain stiffness** option in both
   - In both cases mass proportional damping is correct while hysteretic damping is fully embedded in the constitutive theory

Let us try M-C model for subsoil

- Save data file for current reduced model as *reduced-RB-rel-MC.inp* (**File/Save As**)
- Switch elastic model for sand to M-C ($\phi = 30^o$, $c = 1$ kPa, $\psi = 0^o$), activate group ☒ **Damping** and set $\alpha = 1.256$ (5% damping at f =2 Hz), $\beta = 0.0$
- Switch elastic model for clay to M-C ($\phi = 25^o$, $c = 10$ kPa, $\psi = 0^o$), activate group ☒ **Damping** and set $\alpha = 1.256$ (5% damping at f =2 Hz), $\beta = 0.0$
- In the preprocessor add initial stresses to sand and clay layers (use option **FE model/Initial conditions/Initial stresses**)
- In the preprocessor add contact interface between wall and subsoil

DRM with M-C model: contact interface

Select continuum element edges adjacent to the concrete wall and then use option **FE model/Interface/On continuum element edges**



DRM with M-C model: analysis and results

- Here we can see that in some periods of shaking plastic zones are active near boundary layer
- What about periodic BC ?

M-C model with periodic BC

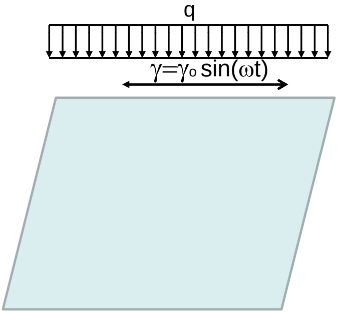
- Save data file for current reduced model as *periodic-RB-rel-MC.inp* (**File/Save As**)
- Go to preproceesor
  1. remove DRM domains
  2. delete horizontal fixities at left and right wall
  3. select nodes at the left wall (except one at the base)
  4. create auxiliary symmetry plane at $x = 0.0$ (use option from ribbon menu **Draw**)
  5. use option **FE model/Boundary conditions/Periodic BC/Nodes & Plane**
  6. dashpots will not play any role (so we can leave them)
  7. quit preprocessor
- Delete free field motion project from project description
- Run analysis

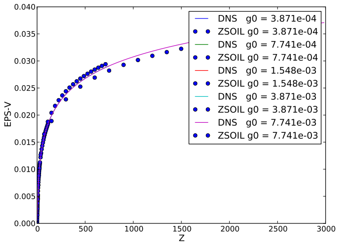
NB. Periodic BC can be used only for flat configurations

Let us use HSs and densification models

- For loose sand layer we will use Densification model by Sawicki
- For clay layer HSs model will be used

- Concept of common compaction curve (in simple shear)
- $\varepsilon^{acc} = e_o \; C_1 \; \ln\left(1 + C_2 \; z\right)$
- $z = \frac{1}{4} \; N \; \gamma_o^2$
- Only the 2 parameters: C_1 i C_2

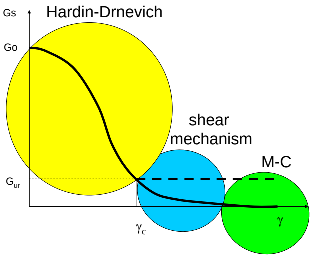


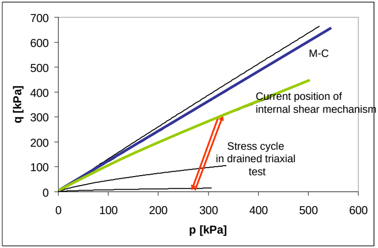


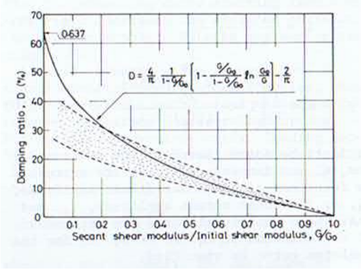
Global construction of densification model

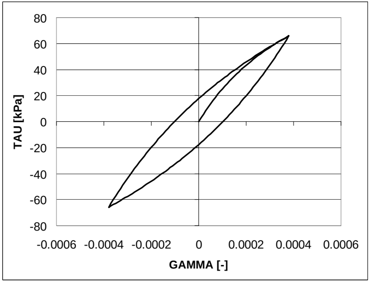
- Model is derived directly from HSs (Schanz, Vermeer)
- Deviatoric plastic flow rule is adopted ($\psi = 0^o$)
- Model is equiped with a current shear yield surface controlled in addition by an external M-C surface
- $E_{ur} = E_{ur}^{ref} \left(\frac{p}{\sigma^{ref}} \right)^m$
- $E_o = E_o^{ref} \left(\frac{p}{\sigma^{ref}} \right)^m$
- Elastic stiffness modulus E varies from $E_o \div E_{ur}$
- All volume changes are due to explicit densification mechanism

Global construction of densification model









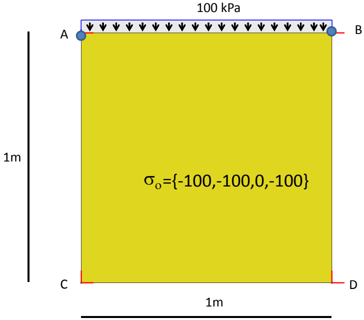
Customization of the model

Model is very flexible and can be used in different contexts (not only for loose deposits)

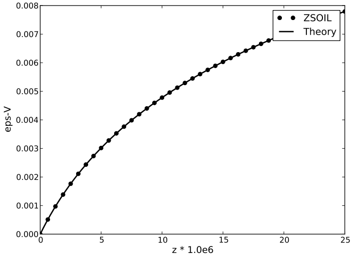
- **Standard Mohr-Coulomb**
  ($E(p)$ + <small-strain> + <densification>)

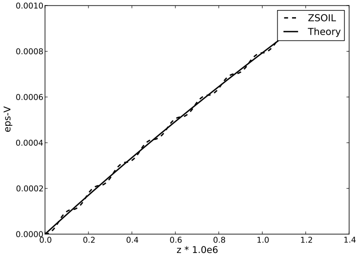
- **M-C + hardening**
  ($E(p)$ + <small-strain> + <densification>)

Simple shear test (static) (drained case)



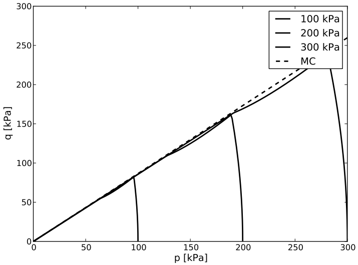
- $u_x^A = u_x^B = 10^{-3}\ sin(2*\pi*t)$
- t=0..100s, step 0.25s
- $u_y^A = u_y^B$
- $C_1 = 8.7 * 10^{-3}$,
  $C_2 = 2 * 10^5$
- m_{ij} - nonisotropic

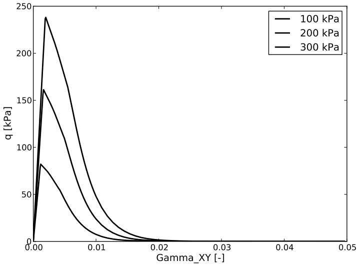




Monotonic undrained simple shear test

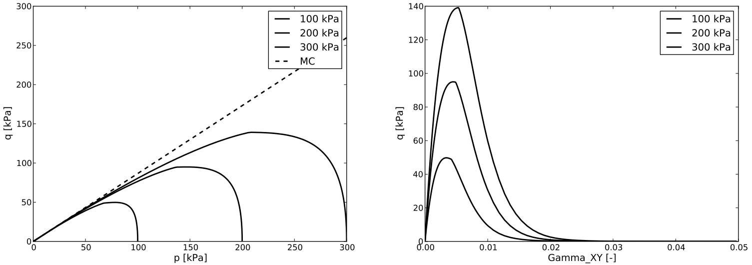
Internal shear mechanism is inactive (no notion of E_{50} is included)





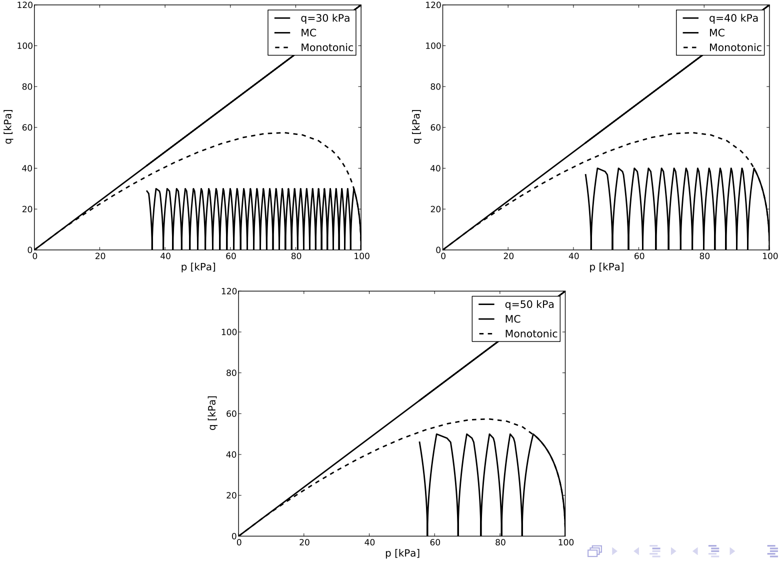
Monotonic undrained simple shear test

Internal shear mechanism is active (effect of E_{50} is included)



Cyclic undrained simple shear test

Internal shear mechanism is active (effect of E_{50} is included)

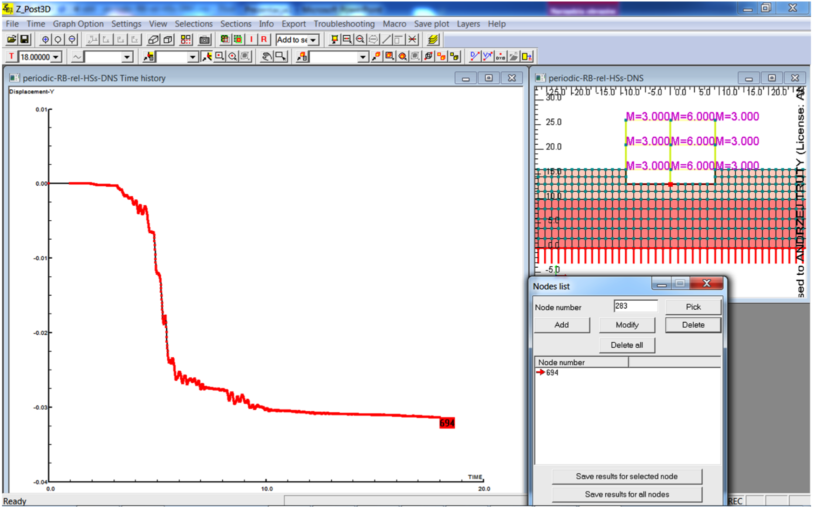


So let us use HSs and densification models

- Here we will use periodic BC so let us reuse data prepared for M-C model and make corrections for materials (save data as *periodic-RB-rel-HSs-DNS.inp*)
- For sand layer Densification model is used
  $E_o^{ref} = 200000$ kPa, $\nu = 0.25$, $m = 0.5$, $\gamma_{0.7} = 10^{-4}$, $E_{ur}^{ref} = 60000$ kPa, $E_{50}^{ref} = 20000$ kPa, $OCR = 1.0$, $K_o^{SR} = 0.5$, $C_1 = 8.7 \cdot 10^{-3}$, $C_2 = 2 \cdot 10^5$
- Assume $e_o = 0.57$ for sand (this is needed for densification law)
- For clay layer HSs model is used
  $E_o^{ref} = 240000$ kPa, $\nu = 0.25$, $m = 0.5$, $\gamma_{0.7} = 2 \cdot 10^{-4}$, $E_{ur}^{ref} = 80000$ kPa, $E_{50}^{ref} = 30000$ kPa, $E_{oed} = 30000$ kPa , $OCR = 20.0$, $K_o^{SR} = K_o^{NC} = 0.577$
- Assume $e_o = 0.48$ for clay
- Let's run and analyze permanent settlement of the building

Permanent settlements

Densification model allows description of permanent settlements



Permanent horizontal movement

Densification model allows description of permanent settlements