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MARKSCHEME

SPECIMEN

MATHEMATICS

Standard Level

Paper 1

DRAFT

11 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

If ***no*** *working shown, award* ***N*** *marks for* ***correct*** *answers. In this case, ignore mark breakdown (****M****,* ***A****,* ***R****).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

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4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

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7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

There are 2 types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

9 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded.

10 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

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SECTION A

1. (a) $\vec{AE} = \frac{1}{2}\vec{AD}$ ***A1***

attempt to find $\vec{AD}$ ***M1***

e.g. $\vec{AB} + \vec{BD}$, $\boldsymbol{u} + \boldsymbol{v}$

$\vec{AE} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v}) \left(= \frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v} \right)$ ***A1*** ***N2***

[3 marks]

(b) $\vec{ED} = \vec{AE} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})$ ***A1***

$\vec{DC} = 3\boldsymbol{v}$ ***A1***

attempt to find $\vec{EC}$ ***M1***

e.g. $\vec{ED} + \vec{DC}$, $\frac{1}{2}(\boldsymbol{u} + \boldsymbol{v}) + 3\boldsymbol{v}$

$\vec{EC} = \frac{1}{2}\boldsymbol{u} + \frac{7}{2}\boldsymbol{v} \left(= \frac{1}{2}(\boldsymbol{u} + 7\boldsymbol{v}) \right)$ ***A1*** ***N2***

[4 marks]

Total [7 marks]

2. (a) min value of r is -1, max value of r is 1 ***A1A1*** ***N2***

[2 marks]

(b) C ***A1*** ***N1***

[1 mark]

(c) linear, strong negative ***A1A1*** ***N2***

[2 marks]

Total [5 marks]

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3. (a) $4\ (\text{m s}^{-1})$ ***A1*** ***N1***

[1 mark]

(b) recognising that acceleration is the gradient ***M1***

e.g. $a(1.5) = \dfrac{4-0}{2-0}$

$a = 2\ (\text{m s}^{-2})$ ***A1*** ***N1***

[2 marks]

(c) recognizing area under curve ***M1***
e.g. trapezium, triangles, integration

correct substitution ***A1***
e.g. $\dfrac{1}{2}(3+6)4$, $\int_0^6 |v(t)|\,\mathrm{d}t$

distance $= 18$ (m) ***A1*** ***N2***

[3 marks]

Total [6 marks]

4. (a) (i) new mean is $20 + 10 = 30$ ***A1*** ***N1***

(ii) new sd is 6 ***A1*** ***N1***

[2 marks]

(b) (i) new mean is $20 \times 10 = 200$ ***A1*** ***N1***

(ii) **METHOD 1**

variance is 36 ***A1***
new variance is $36 \times 100 = 3600$ ***A1*** ***N2***

METHOD 2

new sd is 60 ***A1***
new variance is $60^2 = 3600$ ***A1*** ***N2***

[3 marks]

Total [5 marks]

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5. (a) attempt to use substitution or inspection ***M1***

e.g. $u = 1 + e^x$ so $\frac{du}{dx} = e^x$

correct working ***A1***

e.g. $\int \frac{du}{u} = \ln u$

$\ln(1 + e^x) + C$ ***A1*** ***N3***

[3 marks]

(b) **METHOD 1**

attempt to use substitution or inspection ***M1***

e.g. let $u = \sin 3x$

$\frac{du}{dx} = 3\cos 3x$ ***A1***

$\frac{1}{3}\int u\,du = \frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***

$\int \sin 3x \cos 3x dx = \frac{\sin^2 3x}{6} + C$ ***A1*** ***N2***

[4 marks]

METHOD 2

attempt to use substitution or inspection ***M1***

e.g. let $u = \cos 3x$

$\frac{du}{dx} = -3\sin 3x$ ***A1***

$-\frac{1}{3}\int u\,du = -\frac{1}{3} \times \frac{u^2}{2} + C$ ***A1***

$\int \sin 3x \cos 3x dx = \frac{\cos^2 3x}{6} + C$ ***A1*** ***N2***

[4 marks]

METHOD 3

recognizing double angle ***M1***

correct working ***A1***

e.g. $\frac{1}{2}\sin 6x$

$\int \sin 6x dx = \frac{-\cos 6x}{6} + C$ ***A1***

$\int \frac{1}{2}\sin 6x dx = -\frac{\cos 6x}{12} + C$ ***A1*** ***N2***

[4 marks]

Total [7 marks]

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6. (a) recognizing double angle ***M1***
e.g. $3\times 2\sin x\cos x$, $3\sin 2x$

$a=3,\ b=2$ ***A1A1*** ***N3***

[3 marks]

(b) substitution $3\sin 2x=\frac{3}{2}$ ***M1***

$\sin 2x=\frac{1}{2}$ ***A1***

finding the angle ***A1***
e.g. $\frac{\pi}{6},\ 2x=\frac{5\pi}{6}$

$x=\frac{5\pi}{12}$ ***A1*** ***N2***

Note: Award ***A0*** if other values are included.

[4 marks]

Total [7 marks]

7. (a) $f'(x)=-x^{-2}\ \left(\text{or } -\frac{1}{x^2}\right)$ ***A1*** ***N1***

$f''(x)=2x^{-3}\ \left(\text{or } \frac{2}{x^3}\right)$ ***A1*** ***N1***

$f'''(x)=-6x^{-4}\ \left(\text{or } -\frac{6}{x^4}\right)$ ***A1*** ***N1***

$f^{(4)}(x)=24x^{-5}\ \left(\text{or } \frac{24}{x^5}\right)$ ***A1*** ***N1***

[4 marks]

(b) $f^{(n)}(x)=\frac{(-1)^n n!}{x^{n+1}}$ or $(-1)^n n!\left(x^{-(n+1)}\right)$ ***A1A1A1*** ***N3***

[3 marks]

Total [7 marks]

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SECTION B

8. (a) $f(x) = 3(x^2 + 2x + 1) - 12$ ***A1***

$= 3x^2 + 6x + 3 - 12$ ***A1***

$= 3x^2 + 6x - 9$ ***AG*** ***N0***

[2 marks]

(b) (i) vertex is $(-1, -12)$ ***A1A1*** ***N2***

(ii) $y = -9$, or $(0, -9)$ ***A1*** ***N1***

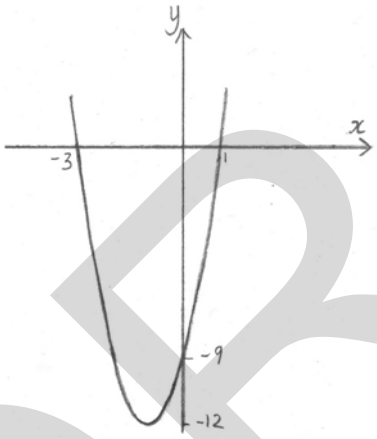
(iii) evidence of solving $f(x) = 0$ ***M1***
e.g. factorizing, formula

correct working ***A1***
e.g. $3(x+3)(x-1) = 0$, $x = \dfrac{-6 \pm \sqrt{36+108}}{6}$

$x = -3$, $x = 1$, or $(-3, 0), (1, 0)$ ***A1A1*** ***N2***

[7 marks]

(c)



A1A1A1 ***N3***

Note: Award ***A1*** for a parabola opening upward,
A1 for vertex in approximately correct position,
A1 for intercepts in approximately correct positions.
Scale and labelling not required.

[3 marks]

(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ ***A1A1A1*** ***N3***

[3 marks]

Total [15 marks]

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9. (a) (i) number of ways of getting $X = 6$ is 5 ***A1***

$\mathrm{P}(X = 6) = \frac{5}{36}$ ***A1*** ***N2***

(ii) number of ways of getting $X > 6$ is 21 ***A1***

$\mathrm{P}(X > 6) = \frac{21}{36} \left(= \frac{7}{12}\right)$ ***A1*** ***N2***

(iii) $\mathrm{P}(X = 7 \mid X > 6) = \frac{6}{21} \left(= \frac{2}{7}\right)$ ***A2*** ***N2***

[6 marks]

(b) attempt to find $\mathrm{P}(X < 6)$ ***M1***

e.g. $1 - \frac{5}{36} - \frac{21}{36}$

$\mathrm{P}(X < 6) = \frac{10}{36}$ ***A1***

fair game if $\mathrm{E}(W) = 0$ (may be seen anywhere) ***R1***

attempt to substitute into $\mathrm{E}(X)$ formula ***M1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right)$

correct substitution into $\mathrm{E}(W) = 0$ ***A1***

e.g. $3\left(\frac{5}{36}\right) + 1\left(\frac{21}{36}\right) - k\left(\frac{10}{36}\right) = 0$

work towards solving ***M1***

e.g. $15 + 21 - 10k = 0$

$36 = 10k$ ***A1***

$k = \frac{36}{10}$ $(= 3.6)$ ***A1*** ***N4***

[8 marks]

Total [14 marks]

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10. (a) $f'(x) = -\sin x + \sqrt{3}\cos x$ ***A1A1*** ***N2***

[2 marks]

(b) (i) at A, $f'(x) = 0$ ***R1***

correct working ***A1***

e.g. $\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$ ***A1***

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ ***A1***

attempt to substitute **their** x into $f(x)$ ***M1***

e.g. $\cos\left(\frac{4\pi}{3}\right) + \sqrt{3}\sin\left(\frac{4\pi}{3}\right)$

correct substitution ***A1***

e.g. $-\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$

correct working that clearly leads to -2 ***A1***

e.g. $-\frac{1}{2} - \frac{3}{2}$

$q = -2$ ***AG*** ***N0***

(ii) correct calculations to find $f'(x)$ either side of $x = \frac{4\pi}{3}$ ***A1A1***

e.g. $f'(\pi) = 0 - \sqrt{3},\ f'(2\pi) = 0 + \sqrt{3}$

$f'(x)$ changes sign from negative to positive ***R1***

so A is a minimum ***AG*** ***N0***

[10 marks]

(c) max when $x = \frac{\pi}{3}$ ***R1***

correctly substituting $x = \frac{\pi}{3}$ into $f(x)$ ***A1***

e.g. $\frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$

max value is 2 ***A1*** ***N1***

[3 marks]

(d) $r = 2,\ a = \frac{\pi}{3}$ ***A1A1*** ***N2***

[2 marks]

Total [17 marks]

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