Using Vedic Mathematics to Make Sense of the Finger Algorithm

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Abstract

Vedic mathematics, based on sutras from ancient Hindu texts, was rediscovered in the early twentieth century by Bharati Krishna Tirthaji (Stemn & Collins, 2001). Vedic mathematics deals mainly with mentally carrying out tedious arithmetical operations. While studying a particular Vedic algorithm and its connections with standard arithmetic operations, the authors discovered connections to a nonstandard multiplication algorithm, the finger algorithm, found in a mathematics textbook for preservice elementary teachers. These connections provided justification for why the finger algorithm works. This paper discusses a justification of the Vedic algorithm and how it led to justification of the finger algorithm.

Using Vedic mathematics to make sense of the finger algorithm

"Why in the world does that work?" That was the question asked in my mathematics for elementary and middle grades teachers course in response to seeing a finger algorithm for multiplication for the first time. My answer was, "I have no idea." While this answer was accepted by students in the class, I could not let it go. When I was unable to answer my students' question about why the finger algorithm works, I challenged myself to figure it out. In this article, we will explain the finger algorithm and why it works through an exploration of Vedic mathematics.

Vedic mathematics was rediscovered in the early twentieth century by Swami Bharati Krishna Tirthaji and is based on sutras from ancient Hindu texts known as the Vedas (Stemn & Collins, 2001; Tirthaji, 1965). Vedic mathematics and our use of it may be viewed as ethnomathematics. D'Ambrosio (1985), described ethnomathematics as the field which bridges anthropologists and historians of mathematics. According to D'Ambrosio (1985), building this bridge is "an important step towards recognizing that different modes of thoughts may lead to different forms of mathematics" (p. 195). In the early twentieth century, Tirthaji used his knowledge of the Vedas, which included areas of thought presumably unrelated to mathematics, to develop Vedic mathematics (Tirthaji, 1965). Vedic mathematics deals mainly with mentally carrying out tedious arithmetical operations (Tirthaji, 1965). Tirthaji's work created an example of the bridge described by D'Ambrosio. Our explanation of the relationship between the finger algorithm and Vedic mathematics also describes how a mode of thinking very different from ours led us to deeper mathematical understanding.

My exploration of Vedic mathematics was not a solitary one. At the same time that I was grappling with understanding why the finger algorithm works mathematically, I was studying Vedic mathematics and its connections with standard arithmetic operations with one of my students (who was not in the mathematics for elementary and middle grades teachers course), Emily Book (a pseudonym). The synchronicity of discussing the finger algorithm in class and the study of Vedic mathematics was imperative to our progress in understanding why the finger algorithm works.

The Finger Algorithm

The finger algorithm is introduced in a problem in the exercises in the text *Mathematics for elementary teachers: A contemporary approach* by Gary L. Musser, William F. Burger, and Blake E. Peterson (2004). The problem is:

> The use of finger numbers and systems of finger computations has been widespread through the years. One such system for multiplication uses the finger positions shown for computing the products of numbers from 6 through 10.


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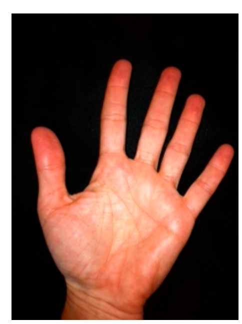
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The two numbers to be multiplied are each represented on a different hand. The sum of the raised fingers is the number of tens, and the product of the closed fingers is the number of ones. For the example below, 1+3=4 fingers raised and 4X2=8 fingers down yields 6X8=48 (Musser, Burger, and Peterson, 2004, p. 180-181).

6x8

When we discussed this problem in class, we spent a few minutes just talking about the representation of the numbers. For this algorithm, first think of representing a number between 6 and 10 using both hands. For the numbers 6 through 10, one hand always has all five fingers raised. The finger algorithm representation of a number is obtained by taking away the hand that has all five fingers raised, leaving a hand with between 1 and 5 fingers raised. Because the finger algorithm is used to determine the product of two numbers between 6 and 10, two hands will be used in the algorithm, each hand representing a factor in the product.

To reiterate and clarify the algorithm, let's look at another example. To determine the product of 8 and 7, one hand will have three fingers raised (representing 8), and one hand will have two fingers raised (representing 7).

8x7

The sum of the raised fingers is the number of tens in the product. In the 8x7 example, there are a total of five fingers raised, representing 5 tens, or 50. The number of ones in the product is given by the product of the closed fingers on each hand. In our example, there are two closed fingers on the hand representing 8 and three closed fingers on the hand representing 7. Since 2 times 3 is 6, there are 6 ones in the product of 8 times 7. Adding the tens to the ones, 50+6, we get the product of 56.

Figuring it out

One day, as Emily and I studied a particular Vedic algorithm for the multiplication of integers based on the Nikhilam sutra, I mentioned the finger algorithm and the fact that I did not have an answer to my students' inquiries about why it works. While our conversation ended there, Emily's thinking did not. At our next meeting, Emily shared some insights into the finger algorithm based on her understanding of the Nikhilam approach. As she thought about the finger algorithm, she kept coming back to two aspects of the algorithm that she recognized from the Nikhilam approach. First, in the finger algorithm, numbers are represented on one hand by taking five away. Also, in the algorithm, the "down" fingers are complements from ten. These are both also aspects of the Nikhilam approach. In the rest of this paper, we describe the Nikhilam approach and its relation to the finger algorithm.

The Nikhilam algorithm

Let's consider the above example 8x7 using Vedic mathematics. To use the Nikhilam algorithm, express the multiplication vertically.

$$\begin{array}{r} 8 \\ \times\ 7 \\ \hline \end{array}$$

Next, take the complement of each number from ten and write it as shown below.

$$\begin{array}{rl} 8 & 2 \leftarrow (10-8) \\ \times\ 7 & 3 \leftarrow (10-7) \\ \hline \end{array}$$

At this point there are two columns with which we work. We begin with the column on the right. Multiplying these two numbers together yields the number of ones in the product. Since 2x3=6, there are 6 ones in the product of 8x7. Place that product directly under the 2 and 3.

$$\begin{array}{rl} 8 & 2 \leftarrow (10-8) \\ \times\ 7 & 3 \leftarrow (10-7) \\ \hline & 6 \end{array}$$

To get the number of tens in the product, we turn our attention to the numbers in the column on the left. Subtract diagonally 8-3 (or 7-2; both differences will always be the same. This

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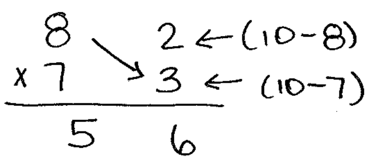
difference is the number of tens in the product. Since 8-3=5, there are 5 tens in the product of 8x7. Place that difference directly under the 8 and 7.

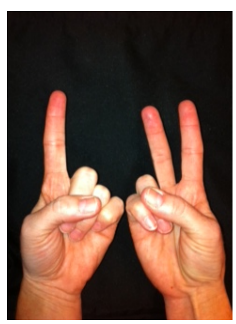


Since 5 tens and 6 ones gives 56, we get 8x7=56.

Regrouping

We typically do not think about regrouping when multiplying two one-digit numbers. However, regrouping can be seen using the finger algorithm, as well as the Nikhilam algorithm, with certain products. Let's consider an example using the finger algorithm where regrouping occurs.

Consider 6x7.



6x7

One hand has 1 finger raised (representing 6) and the other hand has 2 fingers raised (representing 7), so 1 + 2 gives the number of tens in the product of 6 times 7. So, there are 3 tens, or 30, so far. There are 4 fingers closed on one hand (representing 6) and 3 fingers closed

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on the other hand (representing 7). Multiplying 4 by 3 gives the number of ones in the product 6 X 7. So, there are 12 ones. Adding the number of tens to the number of ones, 30 + 12, gives a final product of 42. Regrouping occurs because there are more than 9 ones obtained, thus creating the need to regroup 10 ones as a ten.

We can also see regrouping in the Nikhilam algorithm with the product 6x7. Again, begin by writing the multiplication vertically.

$$\begin{array}{r} 6 \\ \times 7 \\ \hline \end{array}$$

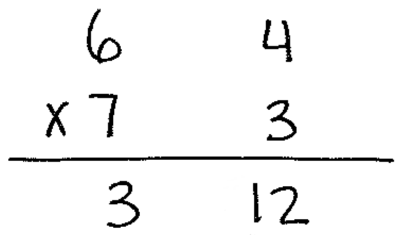
Then, taking complements from ten we get the following.

$$\begin{array}{rr} 6 & 4 \\ \times 7 & 3 \\ \hline \end{array}$$

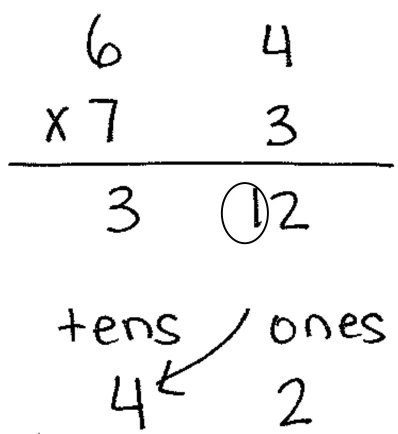
Multiplying 4x3 we get 12 ones.

$$\begin{array}{rr} 6 & 4 \\ \times 7 & 3 \\ \hline & 12 \end{array}$$

Subtracting diagonally 6-3 (or 7-4) we get 3 tens.



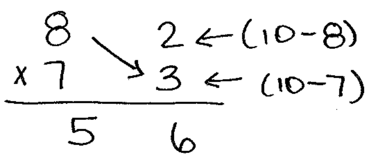
The product of 6x7 is 3 tens and 12 ones, or 42.



The 10 ones in the 12 are regrouped to form a ten, leaving 2 ones. The 1 ten from the regrouping is added to the 3 tens. This gives 4 tens and 2 ones for a total of 42.

Justifying the Nikhilam algorithm

During our exploration of the Nikhilam algorithm, Emily sensed a connection to the finger algorithm. In order to explore this connection further, we answered the question of why the Nikhilam algorithm works. First, we considered a specific example. Let’s look again at the example 8x7.



Remember, in the multiplication of 8x7, the number of tens in the product came from 8-3 (illustrated by the diagonal arrow) and the number of ones in the product came from (2)(3) (the product of the complements from 10). Considering this, we can write the algorithm as a single expression, 10[8-(10-7)]+(10-8)(10-7), where [8-(10-7)] is the number of tens and (10-8)(10-7) is the number of ones.

To generalize, we substitute x and y into this expression and simplify. From this, we see how the Vedic algorithm does indeed yield the product xy.

$10[x-(10-y)]+(10-x)(10-y)$

$= 10[x-10+y]+(10-x)(10-y)$

$= 10x-100+10y+100-10x-10y+xy$

$= xy$

Relating the Nikhilam algorithm to the finger algorithm

Once we justified the Nikhilam algorithm, we were able to uncover the connections between the Nikhilam algorithm and the finger algorithm. We did this by manipulating the expression 10[8-(10-7)]+(10-8)(10-7) in order to make sense of the finger algorithm as a result of the Vedic algorithm. We also tried to relate our manipulations to the earlier connections Emily had made between the finger algorithm and the Nikhilam approach. Emily had focused her connection-making on two important aspects of the finger algorithm: the "up" fingers being a

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representation of a number by taking five away and the "down" fingers being a representation of a number's complement from ten. Consider the following manipulations with these two aspects in mind.

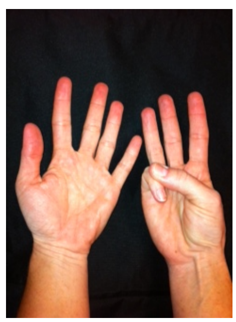
10[8-(10-7)]+(10-8)(10-7) [the Nikhilam algorithm written as a single expression]

= 10(8+7-10)+(10-8)(10-7) [distributing the negative sign]

= 10(8+7-5-5)+(10-8)(10-7) [rewriting -10 as -5-5]

= 10((8-5)+(7-5))+(10-8)(10-7) [associativity]

At this point, I could see what Emily sensed all along. The Nikhilam algorithm really illustrates the finger algorithm. First, we address the meaning of the expression ((8-5)+(7-5)). Traditionally, you use both hands to represent the number 8, with 8 of the ten fingers raised.



If you take away the hand with all five fingers raised, you are left with a hand with three fingers raised and two fingers down.

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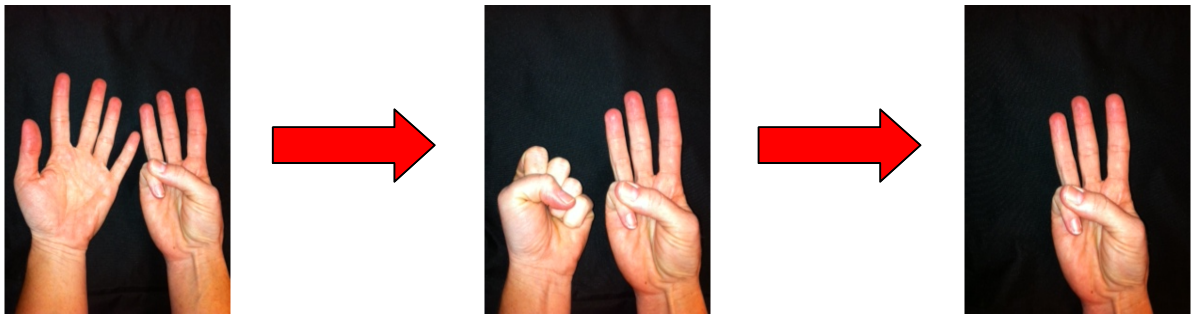

This one hand with three fingers raised is the finger algorithm representation of the number 8. So, one way to count the "up" fingers in the finger algorithm representation of the number 8 is 8-5 (all 8 fingers take away the full hand of 5 fingers). So, the (8-5) represents the 3 raised fingers on one hand of the finger algorithm. The (8-5) can be seen clearly in the expression 10((8-5)+(7-5))+(10-8)(10-7). Similarly, (7-5) represents the 2 raised fingers on the other hand of the finger algorithm. So, ((8-5)+(7-5)) is the total number of raised fingers in the finger algorithm, which, as previously described, gives the number of tens in the product. So, it makes sense to multiply ((8-5)+(7-5)) by 10: 10((8-5)+(7-5))+(10-8)(10-7).

Again, the traditional way of representing the number 8 using fingers, is to raise 8 of the ten fingers. This leaves 10-8 or 2 (complement of 10) fingers down on the hand with three fingers up. Similarly, the representation of 7 will leave 10-7 or 3 (complement of 10) fingers down on the hand with two fingers up. As previously described, the product of the down fingers gives the number of ones in the product. This can be seen clearly in the expression 10((8-5)+(7-5))+(10-8)(10-7). Finally simplifying the expression 10((8-5)+(7-5))+(10-8)(10-7), we see the final product of 56.

10((8-5)+(7-5))+(10-8)(10-7)

= 10(3+2)+(2)(3) [sum of up fingers multiplied by ten plus the product of the down fingers]

= 10(5)+(2)(3)

= 50+6 = 56

My answer to "why in the world does that work?"

When showing the finger algorithm as an example of a nonstandard algorithm for multiplication, the question of why it works has unfortunately gone unanswered in my class. But, after exploring the finger algorithm in the context of its connections to the Nikhilam algorithm, I now can give my students the opportunity to explore and justify two nonstandard multiplication algorithms, while making mathematical connections between them. Being able to provide students with a way for them to answer the question, "why in the world does that work?" is much more satisfying not only for them, but for me as their teacher. My students and I were not the only people to benefit from this mathematical exploration. Emily's contribution to the investigation of the finger algorithm through her understanding of the Vedic algorithm was invaluable. She made necessary connections that brought clarity not only to my understanding of both algorithms, but her own understanding was deepened as well. Being able to apply her knowledge of one algorithm to justify another solidified her understanding even more.

Why study nonstandard algorithms?

One of the emphases in my Mathematics for Elementary and Middle Grades Teachers course is the understanding of different nonstandard algorithms for arithmetic operations. Prior thinking in mathematics education supported the idea that children memorize basic math facts

(Adding it up, 2001). More recent research has shown that children actually progress through a series of more and more advanced and abstract methods for determining the answers to arithmetic problems (Adding it up, 2001). While not all children will progress through the same series of methods, all children will use intermediate and temporary procedures (Adding it up, 2001). Nonstandard algorithms may be the intermediate and temporary procedures that serve as a bridge between first exposure to arithmetic calculations and quick recall of basic math facts. It is this power of nonstandard algorithms that motivates me to share algorithms such as the finger and Nikhilam algorithms with my preservice elementary and middle grades teachers. The fact that one of these nonstandard algorithms is also a non-Western algorithm is also beneficial to my students as preservice teachers. Culture manifests itself through, among other things, jargons, codes, symbols, and ways of reasoning. (D'Ambrosio, 1985). Certainly Vedic mathematics has jargon, symbols and ways of reasoning that reflect a culture different from the culture most of my students have experienced, especially in a mathematics classroom. Providing them with exposure to a culturally different way of viewing mathematics, while deepening their mathematical understanding, is an exciting opportunity for me. My hope is that they take with them the mathematics that they have learned as well as a cultural awareness prompted by connecting an algorithm introduced in a traditional Western textbook to a non-Western, nontraditional multiplication algorithm.

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