# DNIESTER NOTEBOOK: Unsolved Problems in the Theory of Rings and Modules*

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## Translators' Introduction

The Dniester Notebook (Dnestrovskaya Tetrad) is a collection of problems in algebra, especially the theory of rings (both associative and nonassociative) and modules, which was first published in the Soviet Union in 1969. The second and third editions of 1976 and 1983 expanded the list of problems, and included comments on the current status of each problem together with bibliographical references, especially when a solution or a counter-example had been discovered. The fourth Russian edition of 1993 (edited by V. T. Filippov, V. K. Kharchenko and I. P. Shestakov) was the last; this is the edition which we have translated for the present English version.

The problems in the Dniester Notebook originate primarily from the Novosibirsk school of algebra and logic, which was founded by the mathematician and logician A. I. Malcev. The ring theory branch of this school was developed by the algebraist A. I. Shirshov. These problems have had a considerable influence on research in algebra in the countries of the former Soviet Union. They cover a wide range of topics, with a special emphasis on research directions that are characteristic of the "Russian School": associative rings and algebras, nonassociative rings and algebras, polynomial identities, free algebras, varieties of rings and algebras, group rings, topological and ordered rings, embedding of rings into fields and rings of fractions, and the theory of modules. Nonassociative rings

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receive as much attention as associative rings, and there is a notable emphasis on problems with connections to universal algebra and mathematical logic.

Since the publication of the fourth edition in 1993, many problems which were mentioned as unsolved have in fact been solved, partially or completely. However we have decided to go ahead with the publication of this translation, the first English version of the Dniester Notebook, for three major reasons.

First, there are many mathematicians working in areas related to the problems in the Notebook who do not read Russian. We hope that this English version will make it easier for them to appreciate the significant Russian work in these areas.

Second, even though some parts of the Notebook are somewhat out-of-date, it is still very stimulating to read as a source of research ideas. There are many contemporary areas of research, some of which did not even exist at the original publication date, which are closely related to the problems in the Notebook. We hope that reading the current version will inspire further research in those areas.

Third, we plan to prepare a fifth edition of the Dniester Notebook, which will be bilingual in Russian and English. We hope that the publication of the fourth edition in English will facilitate the collection on a worldwide basis of information on the current status of the problems, and of new problems to be included in the fifth edition. We would appreciate it very much if readers of this translation would send any comments on old problems or suggestions for new problems to


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With the influx of many mathematicians from the former Soviet Union to the West during the last two decades, the significance of the Dniester Notebook to Western mathematicians has never been greater. We believe that this is an opportune moment to make this important work easily accessible to the English-speaking world.

Murray R. Bremner and Mikhail V. Kochetov

# Preface

In September 1968 in Kishinev, at the First All-Union Symposium on the Theory of Rings and Modules, it was resolved to publish a collection of open problems in the theory of rings and modules, and as a result the "Dniester Notebook" appeared in 1969. Since then it has been republished twice, in 1976 and 1982. The first and both subsequent editions were quickly sold out, and to a certain extent promoted the development of research in ring theory in the USSR. Of the 326 problems in the third edition, at present more than one-third have been solved.

In the present collection we offer the reader the fourth edition of the “Dniester Notebook”, which consists of three parts. The first two parts are reproduced from the third edition with small editorial changes. The comments on the problems have been updated and extended. As before, the problems which have been completely solved are marked by an asterisk; a small circle indicates those problems on which progress has been made. The third part of the collection consists of new problems.

The compilers thank everyone who has taken part in the preparation of this fourth edition.

# 1 Part One

**1.1.** ◦ (A. A. Albert, reported by K. A. Zhevlakov) Let $A$ be a finite dimensional commutative power-associative nilalgebra over a field of characteristic $\neq 2$. Is $A$ solvable? Remark: It is known that such an algebra is not necessarily nilpotent: there exists a solvable but not nilpotent finite dimensional commutative power-associative nilalgebra over any field of characteristic $\neq 2$ (D. Suttles [164]).

**1.2.** ∗ (S. Amitsur [78]) Is the Jacobson radical of a finitely generated associative algebra over any field necessarily a nilalgebra? Remark: The answer is No (K. I. Beidar [13]).

**1.3.** (A. Z. Ananiyn) Find necessary and sufficient conditions for the existence of a faithful representation of associative PI algebra of $n \times n$ matrices over an associative commutative algebra.

**1.4.** (A. Z. Ananiyn) Is it true that the variety $M$ of associative algebras over a field $k$ of characteristic 0 is a matricial variety if and only if each algebra $A$ in $M$ satisfies the identities

$$[x_1, x_2, \cdots, x_n]z_1z_2 \cdots z_n[y_1, \cdots, y_n] = 0,$$
$$[z_1, z_2][z_3, z_4] \cdots [z_{2n-1}, z_{2n}] = 0?$$

**1.5.** (V. A. Andrunakievich) It is known that in any associative ring $R$ the sum of all right nilideals $\Sigma(R)$ coincides with the sum of all left nilideals. Is the quotient ring $R/\Sigma(R)$ a ring without one-sided nilideals, where $\Sigma(R)$ is the sum of all one-sided nilideals?

**1.6.** (V. A. Andrunakievich) By transfinite induction using the ideal $\Sigma(R)$ (see the previous problem) we construct the ideal $N$ analogous to the Baer radical as the sum of all nilpotent ideals in the class of associative rings. The radical $N$ is pronilpotent. Is the radical $N$ special, that is, is any associative ring without one-sided nilideals homomorphic to the (ordinally) first ring without one-sided nilideals?

**1.7.** ∗ (V. A. Andrunakievich, submitted by L. A. Bokut) Find necessary and sufficient conditions to embed an associative ring in a radical ring (in the sense of Jacobson). Remark: Such conditions have been found (A. I. Valitskas [179]). In the same paper it is shown that these conditions are not equivalent to a finite system of quasi-identities.

**1.8.** $*$ (V. A. Andrunakievich, Yu. M. Ryabukhin) Find necessary and sufficient conditions for an algebra over any associative commutative ring with identity to be decomposable into the direct sum of simple algebras. (The corresponding question for division algebras is solved.) Remark: A ring $R$ is isomorphic to a direct sum of rings without proper ideals if and only if the following two conditions are both satisfied: (a) $R$ satisfies the minimum condition on principal ideals; (b) $R$ has no large ideals. (An ideal is called large if it has nontrivial intersection with every nonzero ideal). Indeed, let $E$ be the ring of endomorphisms of the additive group of $R$, and let $T$ be the subring of $E$ generated by the identity element of $E$ and all left and right multiplications by the elements of $R$. Consider $R$ as a unital (right) module over $T$. It is clear that $R$ is isomorphic to a direct sum of rings without proper ideals if and only if this module is semisimple. Note that if a unital module $M$ is semisimple then every finitely generated submodule $N$ is isomorphic to a finite direct sum of simple modules and therefore has finite length. In particular, every cyclic submodule $N$ satisfies the minimum condition on submodules. It follows that $M$ satisfies the minimum condition on cyclic submodules. Now if $H$ is a submodule of $M$ and $M \neq H$ then $H$ has a complement in $M$; that is, there exists a submodule $H'$ of $M$ such that $H \oplus H' = M$. Therefore, $H$ is not large. Thus $M$ satisfies the minimum condition on cyclic submodules and does not have proper large submodules. Conversely, let $M$ be a module that does not have proper large submodules and satisfies the minimum condition on cyclic submodules. Let $S$ be the socle of $M$ (the sum of all simple submodules). Assume that $S \neq M$. Then $S$ is not a large submodule and therefore there exists a nonzero submodule $G$ of $M$ such that $G \cap S = (0)$. Denote by $P$ the minimal element of the set of all nonzero cyclic submodules of $G$. It is clear that $P$ is simple and that $P \subseteq G$. Therefore $P \cap S = (0)$. This contradicts the inclusion $P \subseteq S$ which holds by definition of the socle. Thus a unital module is semisimple if and only if it satisfies the minimum condition on cyclic submodules and has no proper large submodules. From this and from the fact that the direct sum of arbitrary rings is semiprime if and only if every summand is semiprime, it follows that the ring $R$ is isomorphic to a direct sum of simple rings if and only if $R$ is semiprime and satisfies conditions (a) and (b). (I. V. Lvov).

**1.9.** $*$ (V. I. Arnautov) An associative commutative ring $R$ is called weakly Boolean if for any element $x \in R$ there exists a natural number $n(x) > 1$ such that $x^{n(x)} = x$. (Boolean algebras correspond to the case $n(x) = 2$ for all $x$.) Is there any weakly Boolean (or Boolean) ring on which it is possible to define a topology which makes the ring into a connected topological ring? Remark: The answer is Yes (V. I. Arnautov, M. I. Ursul [8]).

**1.10.** (V. I. Arnautov) Does there exist a "non-weakenable" topology on the ring $\mathbb{Z}$ of integers in which $\mathbb{Z}$ does not contain closed ideals?

**1.11.** (V. I. Arnautov) Is it possible to embed any topological field $F$ into a connected field? This is true if $F$ is given the discrete topology.

**1.12.** $*$ (V. I. Arnautov) The ring $R$ is called hereditarily linearly compact if any closed subring in $R$ is linearly compact. Is the direct product, with the Tikhonov topology, of hereditarily linearly compact rings $R_i$ also hereditarily linearly compact? This is true if the Jacobson radical of every $R_i$ is a bounded set. Remark: The answer is Yes (M. I. Ursul [176]).

**1.13.** (V. I. Arnautov) Must a complete topological associative ring $R$, in which every closed commutative subring is compact, be compact?

**1.14.** (B. E. Barbaumov) Does there exist a division algebra, infinite dimensional over its center, in which all proper subalgebras are PI algebras?

**1.15.** (A. A. Bovdi) If the crossed product $(G, K, \rho, \sigma)$ is a division ring, then $G$ is a periodic group and $K$ is a division ring. Is the group $G$ locally finite?

**1.16.** $*$ (L. A. Bokut) Is it possible to embed every solvable Lie algebra of countable dimension into a solvable Lie algebra with two generators? Remark: The answer is Yes (G. P. Kukin [98]).

**1.17.** $\circ$ (L. A. Bokut) Let $R$ be an associative algebra over the field $P$, and let $F$ be the free associative algebra over $P$ on the countable set of generators $X = \{x_i\}$. Let $R * F$ be the free product of the algebras $R$ and $F$. By an equation over $R$ we mean an expression $f = 0$ where $f \in R * F$, $f \notin R$. We will call the algebra $R$ algebraically closed if any equation over $R$ has a solution in $R$. Do there exist algebraically closed associative algebras? Remark: A positive solution for equations in one variable is obtained by M. G. Makar-Limanov [106].

**1.18.** (L. A. Bokut) For which varieties $M$ of rings (resp. algebras) is the groupoid $\Gamma_M$ of subvarieties free? When is $\Gamma_M$ a free semigroup?

**1.19.** $\circ$ (L. A. Bokut) Describe (in terms of identities) varieties of rings (resp. algebras) with a distributive lattice of subvarieties. Remark: For associative algebras over a field of characteristic 0 the description has been obtained by A. Z. Ananyin and A. R. Kemer [4], and for right alternative algebras by V. D. Martirosyan [109].

**1.20.** $*$ (L. A. Bokut) Is a ring, which is the sum of three nilpotent subrings, also nilpotent? Remark: Not always (L. A. Bokut [25]). The nilpotency of an associative ring that is the sum of two nilpotent subrings has been proved by O. Kegel [81].

**1.21.** (L. A. Bokut) Do there exist two semigroup algebras $F_1(S)$ and $F_2(S)$ without zero-divisors (here $S$ is a semigroup and $F_1$ and $F_2$ are fields) such that one of them can be embedded in a division ring but the other cannot?

**1.22.** $*$ (L. A. Bokut) Is it possible to embed any recursively defined associative algebra (that is, finitely generated with recursively enumerable defining relations) over a prime field into a finitely defined associative algebra? The same question for Lie algebras. Remark: The answer is Yes (V. Ya. Belyaev [16] for associative algebras, and G. P. Kukin [101] for Lie algebras). In G. P. Kukin [102] the following general result is obtained for Lie algebras and groups: Every recursively presented Lie algebra (resp. group) in a variety $\mathcal{M}$ is embeddable into a finitely presented Lie algebra (resp. group) in the variety $\mathcal{M}\mathcal{A}^2$ (here $\mathcal{A}$ denotes the Abelian variety).

**1.23.** ∘ (L. A. Bokut) Describe the identities which hold in all $n$-dimensional associative algebras (with fixed $n$). Remark: A finite basis of identities in the variety generated by $n$-dimensional unital algebras ($n \leq 18$) over a field of characteristic 0 has been found by S. A. Pikhtilkov [132].

**1.24.** (L. A. Bokut) Describe Lie algebras for which the universal enveloping algebra has a classical ring of fractions.

**1.25.** ∘ (L. A. Bokut) Describe varieties of associative (resp. Lie) algebras which are not decomposable into a product. Remark: A series of results on this problem for Lie algebras has been obtained by M. V. Zaicev [185].

**1.26.** ∗ (L. A. Bokut) Find the generators of the group of automorphisms of the free algebra of rank 2 in the variety $\operatorname{Var} M_n(k)$ where $k$ is a field. Remark: These have been found (G. Bergman, preprint).

**1.27.** (A. T. Gainov) Is it possible to define by a finite number of identities the variety of power-commutative algebras over a field of characteristic 0?

**1.28.** ∗ (A. T. Gainov) Describe all finite dimensional simple binary-Lie algebras over an algebraically closed field of characteristic 0. Remark: These have been described (A. N. Grishkov [63]).

**1.29.** ∗ (N. Jacobson) We say that a Jordan ring $J$ has no zero-divisors if for any $a, b \in J$ the equation $aU_b \equiv 2(ab)b - ab^2 = 0$ implies either $a = 0$ or $b = 0$. Two elements $a, b$ are said to have a common multiple if the quadratic ideals $JU_a$ and $JU_b$ satisfy $JU_a \cap JU_b \neq (0)$. Suppose that in a Jordan ring $J$ without zero divisors, any two nonzero elements have a common multiple. Is $J$ embeddable in a Jordan division ring? Remark: The answer is Yes (E. I. Zelmanov [189]).

**1.30.** ∗ (N. Jacobson) Find necessary and sufficient conditions on a finite dimensional Lie algebra for its universal enveloping algebra to be primitive. Remark: These have been found (A. Ooms [125]).

**1.31.** (N. Jacobson, reported by G. P. Kukin) Let $L$ be a Lie $p$-algebra with a periodic $p$-operation. Is it true that $L$ has zero multiplication?

**1.32.** (V. P. Elizarov) Find necessary and sufficient conditions for a division ring $T$ to be a left or right (classical) ring of quotients of a proper subring.

**1.33.** ∗ (K. A. Zhevlakov) Let $A$ be a finitely generated associative ring, and $B$ a locally nilpotent ideal. Does $B$ contain nilpotent ideals? Remark: Not necessarily (E. I. Zelmanov [190]).

**1.34.** $*$ (K. A. Zhevlakov) Let $A$ be a finitely generated associative algebra satisfying an identity. Is every algebraic ideal of $A$ finite dimensional? Remark: Not necessarily (Yu. N. Malcev [107]).

**1.35.** (K. A. Zhevlakov) If an associative algebra contains a nonzero algebraic right ideal, must it also contain a nonzero algebraic two-sided ideal?

**1.36.** $*$ (K. A. Zhevlakov) Let $A$ be an associative ring and let $A^{\langle + \rangle}$ be the special Jordan ring generated by some set of generators of $A$. Suppose that $A^{\langle + \rangle}$ satisfies some (associative) identity. Must $A$ satisfy some identity? The same question if $A$ is finitely generated. Remark: If $A$ is finitely generated then the answer is Yes; in general, No (I. P. Shestakov [149]).

**1.37.** $*$ (K. A. Zhevlakov) Let $A$ be an associative ring and let $A^{\langle + \rangle}$ be the special Jordan ring generated by some set of generators of $A$. Let $J(X)$ denote the quasiregular radical of the ring $X$. Is it true that $J(A^{\langle + \rangle}) = A^{\langle + \rangle} \cap J(A)$? Remark: The answer is Yes (E. I. Zelmanov [198]).

**1.38.** $*$ (K. A. Zhevlakov) Is it true that every minimal ideal of a Jordan ring either is a simple ring or has zero multiplication? Remark: The answer is Yes (V. G. Skosyrskii [158]).

**1.39.** (K. A. Zhevlakov) Let $I$ be a locally nilpotent ideal in a Jordan ring $J$, and suppose that $J$ satisfies the minimum condition on ideals contained in $I$. Is it true that $I$ is nilpotent?[1] Remark: The answer is Yes (V. G. Skosyrskii [158]).

**1.40.** (K. A. Zhevlakov) In a Jordan algebra $J$ the least ideal for which the quotient is a special Jordan algebra will be called the specializer of $J$. Describe generators of the specializer of the free Jordan algebra on three generators.

**1.41.** $*$ (K. A. Zhevlakov) Is it always possible (at least over a field of characteristic 0) to express a Jordan algebra as a direct sum (of vector spaces) of its specializer and a special Jordan algebra? Remark: Not always. As an example we can take the free nilpotent Jordan algebra $A$ of index 9 on 3 generators. Let $S(A)$ be the specializer of $A$. The quotient algebra $A/S(A)$ is isomorphic to the free nilpotent special Jordan algebra of index 9. Suppose that $A$ contains a subalgebra $B$ isomorphic to $A/S(A)$ such that $B \cap S(A) = (0)$. If $x, y, z$ are the generators of $B$ then $x, y, z$ are linearly independent modulo $A^2$ and by nilpotency of $A$ they generate $A$. Therefore $B = A$ and $S(A) = (0)$. But $A$ is not special [61] so $S(A) \neq (0)$. This contradiction shows that $A$ cannot be decomposed into the sum of the specializer and a special algebra. (I. P. Shestakov).

**1.42.** $*$ (K. A. Zhevlakov) Is the locally nilpotent radical of a Jordan ring always ideally-hereditary? Remark: The answer is Yes (A. M. Slinko [160]).

[1] This problem is incomplete in the original text. The translation is from the third edition; $A$ and $B$ have been changed to $J$ and $I$ respectively. (Translators)

**1.43.** $*$ (K. A. Zhevlakov) Let $J$ be an algebraic Jordan algebra with the maximal condition on subalgebras. Must $J$ be finite dimensional? Remark: The answer is Yes (E. I. Zelmanov, unpublished, and A. V. Chekhonadskikh [32]).

**1.44.** $*$ (K. A. Zhevlakov) Do there exist solvable prime Jordan rings? Remark: The answer is No (E. I. Zelmanov and Yu. A. Medvedev [121]).

**1.45.** $*$ (K. A. Zhevlakov) Do there exist nil elements in the free Jordan algebra with $n$ generators ($n \geq 3$)? Remark: The answer is Yes (Yu. A. Medvedev [117]).

**1.46.** $*$ (K. A. Zhevlakov) Describe nil elements in free alternative rings. Remark: These have been completely described (I. P. Shestakov [146]).

**1.47.** $*$ (K. A. Zhevlakov) Is it true that the additive group of a free alternative ring on any number of generators is torsion free? Remark: The answer is No (S. V. Pchelintsev [128]).

**1.48.** (K. A. Zhevlakov) (a) Describe trivial ideals of the free alternative ring on $n$ generators.
(b) $\circ$ Is the free alternative ring on 3 generators semiprime?
Remark: (b) The free alternative algebra on 3 generators over a field of characteristic $\neq 2, 3$ is semiprime (A. V. Iltyakov [70]).

**1.49.** (K. A. Zhevlakov) Find elements that generate the quasi-regular radical of a free alternative ring as a $T$-ideal.

**1.50.** $*$ (K. A. Zhevlakov) Describe identities satisfied by the quasi-regular radical of a free alternative ring. In particular, is it nilpotent or solvable? It is known to be locally nilpotent. Remark: The nilpotency has been proved in the case of finitely many generators, and also for the free alternative algebra over a field of characteristic 0 with any number of generators (I. P. Shestakov [149], E. I. Zelmanov and I. P. Shestakov [199]). In the general case the quasi-regular radical is not solvable (S. V. Pchelintsev [128]).

**1.51.** $\circ$ (K. A. Zhevlakov) Does a free alternative ring have nonzero ideals contained in its commutative center? Remark: The answer is Yes for the free alternative algebra of characteristic $\neq 2, 3$ with a finite number $k \geq 5$ of free generators (V. T. Filippov [48]).

**1.52.** $*$ (K. A. Zhevlakov) Let $A$ be an alternative ring. Let $Z(A)$, $N(A)$ and $D(A)$, respectively, be the commutative center, the associative center, and the ideal generated by all the associators. It is known (G. V. Dorofeev [37]) that $(N(A) \cap D^2(A))^2 \subset Z(A)$. Is it true that $N(A) \cap D^2(A) \subset Z(A)$? Remark: The answer is No (E. Kleinfeld [88]).

**1.53.** $*$ (K. A. Zhevlakov) Let $A$ be an alternative ring, $I$ an ideal of $A$, and $H$ an ideal of $I$ such that in $A$ the ideal $I$ is generated by $H$. Is the quotient ring $B = I/H$ nilpotent or solvable? Remark: If the ring of operators contains 1/6 or if $A$ is finitely generated, then $B$ is nilpotent (S. V. Pchelintsev [127], I. P. Shestakov [147]); in the general case the answer is No (S. V. Pchelintsev [129]).

**1.54.** $*$ (K. A. Zhevlakov) Is every nil subring of a Noetherian alternative ring nilpotent? Remark: If the ring of operators contains 1/3 then the answer is Yes (Yu. A. Medvedev [116]).

**1.55.** (K. A. Zhevlakov) Find a basis of identities for the Cayley-Dickson algebra

(a) $*$ over a finite field;
(b) over a field of characteristic 0;
(c) over an infinite field of prime characteristic.

Remark: (a) A basis was found by I. M. Isaev [74].

**1.56.** $*$ (K. A. Zhevlakov) Let $\sigma$ be an arbitrary radical in the class of alternative rings. Are the following statements equivalent: $a \in \sigma(A)$, and $R_a \in \sigma(A^*)$ (where $A^*$ is the ring of right multiplications of $A$)? Remark: The equivalence has been proved by A. M. Slinko and I. P. Shestakov [162] for the quasi-regular radical, by V. G. Skosyrskii [158] for the locally nilpotent and locally finite radicals. In general these inclusions are not equivalent. Indeed, let $s_1$ and $s_2$ be the upper radicals for the class of alternative algebras defined respectively by the class of all simple associative rings and the class containing only the Cayley-Dickson algebra $\mathbf{C}$. Then $s_1(\mathbf{C}) = \mathbf{C}$, $s_1(\mathbf{C}^*) = (0)$, $s_2(\mathbf{C}) = (0)$, $s_2(\mathbf{C}^*) = \mathbf{C}^*$, and so for every $a \neq 0$ in $\mathbf{C}$ we have

$$a \in s_1(\mathbf{C}),\ R_a \notin s_1(\mathbf{C}^*), \qquad a \notin s_2(\mathbf{C}),\ R_a \in s_2(\mathbf{C}^*).$$

(I. P. Shestakov).

**1.57.** $\circ$ (K. A. Zhevlakov) Describe simple non-alternative right alternative rings. Is it true that every simple right alternative ring with a non-trivial idempotent is alternative? Remark: An example of a simple non-alternative right alternative ring has been constructed by I. M. Mikheev [122]. Any simple right alternative ring which is not nil (in particular, a ring with a nonzero idempotent) is alternative (V. G. Skosyrskii [156], Ts. Dashdorzh [35]).

**1.58.** $*$ (K. A. Zhevlakov) Can every finite dimensional right alternative algebra over a "good" field be expressed as a direct sum (of vector spaces) of its nil radical and a semisimple subalgebra? Remark: The answer is No (A. Thedy [169], I. P. Shestakov, unpublished, see [169], p. 428).

**1.59.** (K. A. Zhevlakov) For right alternative rings, do there exist polynomials that take on values only in the right (resp. left) associative center? In the alternative center?

**1.60.** (K. A. Zhevlakov) Does every right alternative ring on two generators have a finite normal series with associative quotients?

**1.61.** (K. A. Zhevlakov) Let $A$ be an Engel Lie algebra, $A^*$ its multiplication algebra, and $L(X)$ the locally nilpotent radical of the algebra $X$. Are the statements $a \in L(A)$ and $R_a \in L(A^*)$ equivalent?

**1.62.** (A. E. Zalessky) Let $G = SL(n,\mathbb{Z})$ where $\mathbb{Z}$ is the ring of integers and $n \geq 3$. Let $P(G)$ be the group algebra of $G$ over a field $P$. Does the maximal condition on two-sided ideals hold in $P(G)$?

**1.63.** $*$ (A. E. Zalessky) Let $P(A)$ be the group algebra over a field $P$ of a finitely generated torsion-free Abelian group $A$. Let $G$ be the group of automorphisms of $A$; $G$ acts on $P(A)$ in a natural way. Let $J$ be an ideal of $P(G)$ of infinite index, and $H \subseteq G$ the subgroup stabilizing this ideal: $H = \{ h \in G \mid h(J) \subseteq J \}$. Is it true that in this case $H$ has a subgroup $H_0$ of finite index such that $A$ has a subgroup $A_0$ of infinite index that is invariant under $H_0$? (This is equivalent to the statement that $H_0$ can block-triangularized in $GL(n,\mathbb{Z})$ where $\mathbb{Z}$ is the ring of integers and $n$ is the rank of $A$.) Remark: The answer is Yes (G. Bergman [18]).

**1.64.** (A. E. Zalessky) Do there exist non-isomorphic finitely generated nilpotent groups whose group algebras over some field have isomorphic division rings of quotients? Remark: The answer is No (D. Farkas, A. Schofield, R. Snider, J. Stafford [40]).

**1.65.** (I. Kaplansky) Does there exist a torsion-free group whose group algebra has zero-divisors?

**1.66.** (I. Kaplansky) Must the group algebra of an arbitrary group $G$ over a field $k$ of characteristic 0 be semisimple in the sense of Jacobson?

**1.67.** (I. Kaplansky, reported by A. A. Bovdi) If the augmentation ideal of the group algebra $KG$ is a nilideal then $K$ is a field of characteristic $p$ and $G$ is a $p$-group. Must $G$ also be locally finite?

**1.68.** (H. Köthe) Is it true that in any associative ring a sum of two left nilideals is a left nilideal?

**1.69.** (A. I. Kokorin) Develop a theory of totally ordered skew fields analogous to the Artin-Schreier theory of totally ordered fields.

**1.70.** (A. I. Kokorin) Is it always possible to embed a totally ordered skew field into another totally ordered skew field whose set of positive elements is a divisible multiplicative group?

**1.71.** $*$ (A. T. Kolotov) Let $d$ be a derivation of the free associative algebra $k\langle X\rangle$. Must the kernel of $d$ be a free algebra? Remark: The answer is No. Let $F = k\langle x,y,z\rangle$ be the free associative algebra and $d$ the derivation of $F$ defined on generators by $d(x) = xyx + x$, $d(y) = -yxy - y$, $d(z) = -x$. Then $\ker d$ coincides with the subalgebra $G = \mathrm{alg}\langle p,q,r,s\rangle$ that has a single defining relation $pq = rs$ where $p = xyz + x + z$, $q = yx + 1$, $r = xy + 1$, $s = zyx + x + z$. The subalgebra is not free. (G. Bergman).

**1.72.** $*$ (A. T. Kolotov) Is it true that the union of any increasing chain of free subalgebras in an arbitrary free associative algebra is also free? Remark:

The answer is No. Let $F = k\langle s, x_w, y_w, z_w\rangle$ be the free associative algebra with generators indexed by $w$ ranging over the free semigroup $\langle p, q, b, d\rangle$. Define inductively the elements $s_w \in F$ by setting

1) $s_1 = s$,
2) $s_{wp} = s_w(x_w y_w + 1),\ s_{wb} = z_w y_w + 1,\ s_{wq} = s_w(x_w y_w z_w + x_w + z_w),$
$s_{wd} = y_w$.

For every $n \geq 0$ set $S_n = \{\, s_w : |w| = n \,\}$. Then $S_n$ is a family of free generators of a free subalgebra $G_n$ and $G_1 \subset G_2 \subset \cdots$, but the algebra $G = \bigcup_n G_n$ is not free. (G. Bergman).

**1.73.** $*$ (A. T. Kolotov) Is it true that the intersection of two free subalgebras of an arbitrary free associative algebra is again free? Remark: The answer is No. Let $F = k\langle x, y_1, y_2, y_3, z\rangle$, $G_1 = \mathrm{alg}\langle x, y_1, y_3, y_2 z, z\rangle$, $G_2 = \mathrm{alg}\langle x, x y_1 y_2 - y_3, y_2, z\rangle$. Then $G_1$ and $G_2$ are free subalgebras but $G_1 \cap G_2$ is not free. (G. Bergman).

**1.74.** (P. M. Cohn, reported by L. A. Bokut) Is every automorphism of an arbitrary free associative algebra tame (that is, a product of elementary automorphisms)?

**1.75.** (V. M. Kopytov) Describe the real Lie algebras that admit a topology in which the Campbell-Hausdorff series converges for any two elements from some neighborhood of zero. Are all such Lie algebras residually finite dimensional?

**1.76.** $*$ (V. M. Kopytov) Is the free Lie product of ordered Lie algebras again ordered? Remark: The answer is Yes (S. A. Agalakov, L. S. Shtern [2]).

**1.77.** $\circ$ (E. G. Koshevoy) Describe complete subalgebras of free unital associative algebras. A subalgebra $A \subset k\langle X\rangle$ is called complete if $f(a) \in A$ implies $a \in A$ for any nonconstant polynomial $f(t) \in k[t]$. Remark: An example of a complete subalgebra in the free algebra on 3 generators can be found in the work of E. G. Koshevoy [90].

**1.78.** $\circ$ (E. N. Kuzmin) Is every Malcev algebra that satisfies the $n$-th Engel condition locally nilpotent? Remark: The answer is Yes, if the characteristic of the ground field is $p \neq 2$ (V. T. Filippov [43], E. I. Zelmanov [197]).

**1.79.** $*$ (E. N. Kuzmin) Is every Malcev algebra over a field of characteristic 0 that satisfies the $n$-th Engel condition solvable? Remark: The answer is Yes (V. T. Filippov [42], E. I. Zelmanov [195]).

**1.80.** $*$ (E. N. Kuzmin) Can every finite dimensional Malcev algebra $A$ over a field of characteristic 0 be expressed as a direct sum (of vector spaces) of its radical and a semisimple subalgebra? Are the semisimple components of this decomposition conjugate by automorphisms of $A$? (Analogue of Levi-Malcev theorem for Lie algebras.) Remark: The answer is Yes. This decomposition has been obtained independently by A. N. Grishkov [62], R. Carlsson [29] and E. N. Kuzmin [104]. The conjugacy of semisimple factors is proved by R. Carlsson [28].

**1.81.** ∘ (E. N. Kuzmin) Does an arbitrary Malcev algebra over a field of characteristic $\neq 2, 3$ have a representation as a subalgebra of $A^{(-)}$ where $A^{(-)}$ is the minus algebra of some alternative algebra $A$? (Analogue of the Poincaré-Birkhoff-Witt theorem for Lie algebras.) Remark: There exists a representation for the ideal (in an arbitrary Malcev algebra) generated by all Jacobians (V. T. Filippov [49]).

**1.82.** ∗ (E. N. Kuzmin) Let $G$ be a local analytic Moufang loop. Is $G$ locally isomorphic to an analytic Moufang loop? Remark: The answer is Yes (F. S. Kerdman [84]).

**1.83.** (E. N. Kuzmin) Does there exist an invariant integral (that is, a Haar integral) on compact Moufang loops?

**1.84.** ∗ (E. N. Kuzmin) Let $G$ be a simply connected Moufang loop whose tangent algebra is a solvable Malcev algebra. Is the topological space $G$ homeomorphic to a Euclidean space $\mathbb{R}^n$? Remark: The answer is Yes (F. S. Kerdman [84]).

**1.85.** (G. P. Kukin) Is it true that the minimal number of generators of the free product of two algebras is equal to the sum of the minimal numbers of generators of the factors?

**1.86.** ∗ (G. P. Kukin) Is the membership problem for the free product of Lie algebras decidable if it is decidable for both factors? Remark: The answer is No (U. U. Umirbaev [174]).

**1.87.** (G. P. Kukin) Is the isomorphism problem decidable for Lie algebras with one defining relation?

**1.88.** (G. P. Kukin) Is it true that every two decompositions of a Lie algebra into a free product have isomorphic refinements?

**1.89.** ∗ (V. N. Latyshev) Must the Jacobson radical of a finitely generated PI algebra of characteristic 0 be nilpotent? Remark: The answer is Yes (Yu. P. Razmyslov [141, 142], A. R. Kemer [82]). Later, A. Braun proved the nilpotency of the radical in any finitely generated PI algebra over a Noetherian ring [27].

**1.90.** ∗ (V. N. Latyshev) If a PI algebra has a classical ring of quotients must this ring of quotients also be a PI algebra? Remark: The answer is Yes (K. I. Beidar [12]).

**1.91.** (V. N. Latyshev) What are the necessary and sufficient conditions that a semigroup must satisfy so that its semigroup algebra will be a PI-algebra?

**1.92.** (V. N. Latyshev) Let $A$ be an associative algebra with a finite number of generators and relations. If $A$ is a nilalgebra must it be nilpotent?

**1.93.** ∘ (I. V. Lvov) Does there exist an infinite critical associative (resp. nonassociative) ring? A ring is called critical if it does not lie in the variety generated by its proper quotient rings. Remark: The answer is Yes in the nonassociative case (Yu. M. Ryabukhin, R. S. Florya [143]).

**1.94.** (I. V. Lvov) Find all critical finite associative commutative rings.

**1.95.** ∘ (I. V. Lvov) Must the variety generated by a finite right alternative (resp. Jordan, Malcev, binary-Lie) ring have a finite basis of identities? Remark: The answer is No in the right alternative case (I. M. Isaev [76]), and Yes in the Jordan and Malcev cases (Yu. A. Medvedev [113, 115]).

**1.96.** ∗ (I. V. Lvov) Is it true that every minimal variety of rings is generated by a finite ring? Remark: The answer is No (Yu. M. Ryabukhin, R. S. Florya [143]).

**1.97.** (I. V. Lvov) Is it true that an associative algebra of dimension greater than one over the field of rational numbers, all of whose proper subalgebras are nilpotent, is also nilpotent? This is true for algebras over fields satisfying the Brauer condition; for instance over finite or algebraically closed fields.

**1.98.** (I. V. Lvov) Let $f$ be a multilinear polynomial over a field $k$. Is the set of values of $f$ on the matrix algebra $M_n(k)$ a vector space?

**1.99.** ∗ (I. V. Lvov, V. A. Parfyonov) Is every radical (in the sense of Kurosh) on the class of Lie algebras characteristic? A Lie subalgebra is called characteristic if it is invariant under all derivations. Remark: The answer is No (Yu. A. Kuczynski [95]).

**1.100.** ∗ (K. McCrimmon, reported by K. A. Zhevlakov) Is it true that the quasi-regular radical of a Jordan ring is equal to the intersection of the maximal modular quadratic ideals? Remark: The answer is Yes (L. Hogben, K. McCrimmon [68]).

**1.101.** ∗ (K. McCrimmon, reported by K. A. Zhevlakov) Is it true that in a Jordan ring $A$ with minimum condition on quadratic ideals the quasi-regular radical $J(A)$ is nilpotent? Remark: The answer is Yes (E. I. Zelmanov [187]).

**1.102.** (A. I. Malcev, reported by A. A. Bovdi, L. A. Bokut and D. M. Smirnov) Is it possible to embed the group algebra of a right ordered group into a division ring?

**1.103.** ∗ (A. I. Malcev, reported by L. A. Bokut) Find necessary and sufficient conditions for the embeddability of an associative ring into a division ring. Remark: Such conditions have been found by P. M. Cohn [34].

**1.104.** (A. I. Malcev, reported by L. A. Bokut) Do there exist two associative rings with isomorphic multiplicative semigroups one of which is embeddable in a division ring and the other is not?

**1.105.** ∗ (A. I. Malcev) Do there exist varieties of Lie algebras that are not finitely axiomatizable? Remark: They exist over a field of characteristic $p > 0$ (V. S. Drensky [38], M. Vaughan-Lee [183]).

**1.106.** (A. I. Malcev) Does there exist a finitely axiomatizable variety of rings whose set of identities is not recursive?

**1.107.** (A. I. Malcev) What is the structure of the groupoid of the following quasi-varieties:
(a) all rings;
(b) all associative rings?

**1.108.** $*$ (A. I. Malcev, reported by E. N. Kuzmin) Is every finite dimensional real Malcev algebra the tangent algebra of some locally analytic Moufang loop? Remark: The answer is Yes (E. N. Kuzmin [103]).

**1.109.** $*$ (Yu. N. Malcev) Find a basis of identities for the algebra of upper triangular matrices over a field of characteristic $p > 0$. Remark: A basis has been found by S. V. Polin [135] and P. I. Siderov [152].

**1.110.** $*$ (Yu. N. Malcev) Let $R$ be an associative algebra with no nilideals which is a radical extension of some PI subalgebra $A$ (that is, for any $x \in R$ there exists $n(x)$ such that $x^{n(x)} \in A$). Must $R$ be a PI algebra? Remark: The answer is Yes (E. I. Zelmanov [188]).

**1.111.** $*$ (Yu. N. Malcev) Let $R$ be an associative algebra which is an $H$-extension of some PI subalgebra $A$ (that is, for any $x \in R$ there exists $n(x) > 1$ such that $x^{n(x)} - x \in A$). Must $R$ be a PI algebra? Remark: The answer is Yes (M. Chacron [30]).

**1.112.** (R. E. Roomeldi) Describe minimal ideals of right alternative rings. Is it true that they are either simple as rings or solvable (resp. right nilpotent)?

**1.113.** (Yu. M. Ryabukhin) Find necessary and sufficient conditions for an algebra $R$ over an arbitrary associative commutative unital ring $k$ to be decomposable into a subdirect product of algebras with unique left and right division.

**1.114.** (Yu. M. Ryabukhin) Let $F$ be an arbitrary field. Do there exist
(a) an associative nilalgebra $A$ of at most countable dimension such that every countable dimensional nilalgebra is a homomorphic image of $A$;
(b) $\circ$ an algebraic algebra with the analogous property with respect to algebraic algebras?
Remark: (b) The answer is No if the ground field is uncountable (G. P. Chekanu [31]).

**1.115.** $*$ (Yu. M. Ryabukhin, I. V. Lvov) Let $S$ be a class of algebras over a fixed field $F$ closed under homomorphic images. If $S$ is not radical in the sense of Kurosh then is it true that the chain of Kurosh classes

$$S = S_0 \subseteq S_1 \subseteq S_2 \subseteq \cdots \subseteq S_\alpha \subseteq \cdots$$

formed by constructing the lower radical does not stabilize? This is true if the class $S$ is closed not only under homomorphic images but also under ideals. Remark: The answer is Yes (K. I. Beidar [14]).

**1.116.** $*$ (L. A. Skornyakov) Over which rings is every left module (resp. every finitely generated left module) decomposable into a direct sum of distributive modules (that is, modules with distributive lattices of submodules)? Does there exist a non-Artinian ring with this property? Remark: For the first question, descriptions of such rings are given in the works of A. A. Tuganbaev [171] and K. R. Fuller [57]. For the second question, the answer is No.

**1.117.** $\circ$ (L. A. Skornyakov) Over which rings is every finitely generated left module decomposable into a direct sum of uniserial modules? Remark: Such rings have been described by G. Ivanov [77].

**1.118.** $\circ$ (L. A. Skornyakov) Which rings (resp. algebras) are projective in the category of rings (resp. algebras over a fixed field)? Do there exist projective rings other than free rings? Remark: The ring $P$ is projective if and only if $P \cong S$ with $S \oplus K \cong F$ (direct sum of Abelian groups) where $F$ is a free ring, $K$ is an ideal, and $S$ is a subring. The solution of this problem probably depends on the bicategory in which we work. Therefore it is useful to take into account the fact that the collection of such bicategories is not a set (S. V. Polin [134]). (L. A. Skornyakov).

**1.119.** $*$ (L. A. Skornyakov) Does there exist a ring $A$ which is not left Noetherian and such that every module, which is injective in the category of finitely generated left $A$-modules, is injective? Remark: The negative answer can be extracted from the results of V. S. Ramamurthi and K. N. Rangaswamy [138]. Indeed, let $Q$ be a module that is injective in the category of finitely generated left modules over an arbitrary ring $A$. Then $Q$ is finitely generated and injective with respect to natural embeddings of finitely generated left ideals of $A$, which implies the injectivity of $Q$ (Theorem 3.1 and Corollary 3.4(i) in [138]). Therefore every non-Noetherian ring, whose only finitely generated injective module is zero, gives the desired example. From results of E. Matlis [110] (Theorem 4) and D. Gill [60] it follows that every non-Noetherian almost maximal commutative uniserial ring is an example. More or less the same considerations were articulated by C. U. Jensen (private correspondence, 1969). (L. A. Skornyakov).

**1.120.** $\circ$ (L. A. Skornyakov) Describe all the rings whose left ideals are homomorphic images of injective modules. Remark: For the commutative case the answer is known (L. A. Skornyakov [154]).

**1.121.** $*$ (L. A. Skornyakov) Must a ring, over which every module has a decomposition complementing direct summands, be a generalized uniserial ring? Remark: The answer is No (K. R. Fuller [56]).

**1.122.** $*$ (L. A. Skornyakov, reported by L. A. Bokut) Do there exist free (with respect to $T$-homomorphisms) associative division rings? Remark: The answer is Yes. Let $R$ be an arbitrary semifir. Then there exists a universal $R$-division ring $U$ that contains $R$. (Every $R$-division ring is a specialization, or a $T$-homomorphic image, of the $R$-division ring $U$.) In particular, if $R = k\langle X\rangle$, the free algebra on an infinite set $X$ of generators, over the prime field of characteristic $p \geq 0$, then the universal $R$-division ring $U$ is a "free" division ring in the class of division rings of characteristic $p$ and cardinality $\leq |X|$. (See P. M. Cohn [34]). The first proof of the existence of a universal division ring was given by S. Amitsur [3]. J. Lewin [105] proved that the division subring generated by $k\langle X\rangle$ in the division ring of Malcev-Neumann (containing $k\langle X\rangle$) is the universal $k\langle X\rangle$-division ring. (L. A. Bokut).

**1.123.** $*$ (M. Slater, reported by K. A. Zhevlakov) Does there exist a prime alternative ring that is neither associative nor Cayley-Dickson? Remark: The answer is Yes (S. V. Pchelintsev [129]).

**1.124.** $*$ (M. Slater, reported by K. A. Zhevlakov) Let $A$ be a free alternative ring, $D$ the ideal generated by the associators of $A$, and $U$ a maximal ideal of $A$ lying in the associative center. Must $U \cap D$ be nonzero? Is it true that in a free alternative ring every trivial ideal is contained in $U \cap D$? Remark: The answer to the first question is Yes, to the second No (V. T. Filippov [47, 45]).

**1.125.** (M. Slater, reported by K. A. Zhevlakov) Let $M$ be the ideal of a free alternative ring $A$ generated by the set $[N, A]$ where $N$ is the associative center of $A$. Is it true that $M \subseteq N$? (This is equivalent to the statement $[n, t](x, y, z) = 0$ for all $x, y, z, t \in A$ and $n \in N$.) This statement is true for rings with three generators.

**1.126.** (A. M. Slinko) What is the minimal possible dimension of a non-special Jordan algebra?

**1.127.** (A. M. Slinko) Is every ideal of a semiprime Jordan ring itself semiprime? This condition is necessary and sufficient for the class of Jordan rings to have the lower nilradical.

**1.128.** $*$ (A. M. Slinko) It is known that in a special Jordan algebra $J$ every absolute zero divisor (that is, an element $b$ such that $aU_b = 2(ab)b - ab^2 = 0$ for all $a \in J$) generates a locally nilpotent ideal (A. M. Slinko [161]). Is this true for arbitrary Jordan algebras? Remark: The answer is Yes (E. I. Zelmanov [191]).

**1.129.** $\circ$ (A. M. Slinko) Does every variety of solvable alternative (resp. Jordan) algebras have a finite basis of identities? Remark: In the case of alternative algebras the answer is Yes if the characteristic is not 2 or 3 (U. U. Umirbaev [172]), and No over a field of characteristic 2 (Yu. A. Medvedev [114]). In the case of Jordan algebras the answer is Yes for algebras of solvability index 2 (Yu. A. Medvedev [112]).

**1.130.** (A. M. Slinko, I. P. Shestakov) Find a system of relations that defines right representations of alternative algebras. Does there exist a finite system of relations?

**1.131.** $\circ$ (A. M. Slinko, I. P. Shestakov) Let $A$ be an alternative PI algebra. Is the universal associative algebra $\mathcal{R}(A)$ for alternative representations of $A$ also PI? Remark: The answer is Yes for finitely generated algebras (I. P. Shestakov [149]).

**1.132.** (A. M. Slinko, I. P. Shestakov) Let **C** be a Cayley-Dickson algebra. It is known (A. M. Slinko, I. P. Shestakov [162]) that the map $\rho\colon x \to L_x$ is a right-alternative right representation of **C**. Is $\rho$ an alternative right representation of **C**?

**1.133.** $*$ (D. M. Smirnov) What is the cardinality of the set of minimal varieties of rings? Remark: The cardinality is that of the continuum (Yu. M. Ryabukhin, R. S. Florya [143]).

**1.134.** (D. M. Smirnov) If a group $G$ is Hopf must the group ring $\mathbb{Z}(G)$ also be Hopf?

**1.135.** (D. M. Smirnov, A. A. Bovdi) Can the group ring $\mathbb{Z}(G)$ of a torsion-free group contain invertible elements other than $\pm g$, $g \in G$?

**1.136.** (E. A. Sumenkov) Does the universal enveloping algebra of an arbitrary PI Lie algebra satisfy the Ore condition?

**1.137.** $*$ (V. T. Filippov) Does a free Malcev algebra have trivial ideals? Remark: The answer is Yes (I. P. Shestakov [148]).

**1.138.** $*$ (V. T. Filippov) Does the simple 7-dimensional non-Lie Malcev algebra over a field of characteristic 0 have a finite basis of identities? Remark: The answer is Yes (A. V. Iltyakov [71]).

**1.139.** (V. T. Filippov) Let $A$ be a free Malcev algebra and $J(A)$ the ideal generated by the Jacobians. Does the variety generated by $J(A)$ have a finite basis of identities?

**1.140.** (I. Fleischer, reported by V. I. Arnautov) Does there exist a topological field that is not locally bounded whose topology cannot be weakened?

**1.141.** $\circ$ (P. A. Freidman) Describe right Hamiltonian rings (that is, rings such that every subring is a right ideal). Remark: Right Hamiltonian rings have been described (P. A. Freidman [50], V. I. Andriyanov, P. A. Freidman [5]). A description of periodic rings and torsion-free rings has been announced (O. D. Artemovich [9]).

**1.142.** $\circ$ (P. A. Freidman) Describe the rings whose lattice of subrings is modular. Remark: These rings have been described in the case of prime characteristic (P. A. Freidman, Yu. G. Shmalakov [52, 53]) and in the case of torsion-free nil rings (P. A. Freidman [51]).

**1.143.** $*$ (P. A. Freidman) Must a ring in which all proper subrings are nilpotent also be nilpotent? Remark: The answer is No. An obvious example is the field with $p$ elements ($p$ prime). The answer is Yes if we assume that the ring is nil (I. L. Khmelnitsky [86]).

**1.144.** $*$ (P. A. Freidman) A subring $Q$ of a ring $K$ is called a meta-ideal of finite index if $Q$ is a member a finite normal series

$$Q = A_0 \lhd A_1 \lhd \cdots \lhd A_n = K,$$

where $A_i$ is a two-sided ideal in $A_{i+1}$ (Baer). Must a nilpotent $p$-nilring $K$, in which all subrings are meta-ideals of finite index with uniformly bounded indices, be nilpotent? Remark: The answer is Yes, even without the assumption that the indices are uniformly bounded (I. L. Khmelnitsky [87]).

**1.145.** $*$ (V. K. Kharchenko) Let $L$ be a Lie algebra that admits an automorphism of finite order such that the fixed elements are in the center of $L$. Must $L$ be solvable? Remark: The answer is No (A. I. Belov, A. G. Gein [15]).

**1.146.** (I. Herstein) Must the Jacobson radical of a left and right Noetherian associative ring be generalized nilpotent?

**1.147.** (I. Herstein, reported by Yu. N. Malcev) Let $R$ be an associative ring, without nilideals, that satisfies the condition $\forall x, y \in R, \exists n = n(x, y)$ such that $[x, y]^n = 0$. Must $R$ be commutative?

**1.148.** (P. Hall, reported by A. A. Bovdi) If the group ring $K(G)$ satisfies the maximal condition on right ideals then $G$ is Noetherian and $K$ satisfies the maximal condition on right ideals. Is the converse true? It is true for solvable groups (P. Hall [66]).

**1.149.** $\circ$ (I. P. Shestakov) Is it true that the center of the free alternative ring on three generators is equal to the intersection of the associative center and the associator ideal? If not, the free alternative ring on three generators is not semiprime. Remark: For the free alternative algebra on 3 generators over a field of characteristic $\neq 2, 3$ the answer is Yes (A. V. Iltyakov [70]).

**1.150.** $*$ (I. P. Shestakov) Is it true that every simple exceptional Jordan algebra is finite dimensional over its center? The answer is not known even in the case of Jordan division algebras (N. Jacobson [79]). Remark: The answer is Yes (E. I. Zelmanov [193]).

**1.151.** $*$ (I. P. Shestakov) Is every solvable subring of a finitely generated alternative (resp. Jordan) ring nilpotent? Remark: For alternative algebras over a field of characteristic $\neq 2, 3$ the answer is Yes (I. P. Shestakov [148], V. T. Filippov [46]). For Jordan algebras the answer is No (I. P. Shestakov [148]).

**1.152.** (I. P. Shestakov) Must a right alternative nilalgebra over an associative commutative ring $\Phi$ with the maximal condition on $\Phi$-subalgebras be right nilpotent?

**1.153.** $*$ (A. I. Shirshov) Describe subalgebras of a free product of Lie algebras. Remark: These have been described (G. P. Kukin [96, 97]).

**1.154.** $*$ (A. I. Shirshov) Is the word problem decidable in the class of all Lie algebras? Remark: The answer is No (L. A. Bokut [24]). An explicit example has been constructed by G. P. Kukin [99].

**1.155.** $*$ (A. I. Shirshov) Is the word problem decidable in the class of all Lie algebras which are solvable of a fixed index? Remark: The answer is No for solvability index $\geq 3$ (G. P. Kukin [100], see also O. G. Kharlampovich [85]).

**1.156.** $*$ (A. I. Shirshov) Must a Jordan nil ring of bounded index be locally nilpotent? Remark: The answer is Yes (E. I. Zelmanov [191]).

**1.157.** $\circ$ (A. I. Shirshov) Must a Jordan nil ring of index $n$ in characteristic 0 or $p > n$ be solvable? Remark: The answer is Yes for algebras over a field of characteristic 0 (E. I. Zelmanov [196]).

**1.158.** $*$ (A. I. Shirshov) Does there exist a natural number $n$ such that every Jordan algebra of at most countable dimension embeds in a Jordan algebra with $n$ generators? Remark: For special Jordan algebras $n = 2$ (A. I. Shirshov [150]). In the general case there is no such number $n$ (E. I. Zelmanov [192]).

**1.159.** $\circ$ (A. I. Shirshov) Let $Alt_n$ be the variety of alternative rings generated by the free alternative ring on $n$ generators. Does the chain

$$Alt_1 \subseteq Alt_2 \subseteq Alt_3 \subseteq \cdots$$

stabilize? The same question for Jordan, right alternative, Malcev and binary-Lie rings. It is known that in the class of $(-1, 1)$ rings this chain does not stabilize (S. V. Pchelintsev [126]). Remark: For alternative and Malcev rings it does not stabilize (I. P. Shestakov [148]).

**1.160.** (A. I. Shirshov) Construct a basis of the free alternative (resp. right alternative, Jordan, Malcev, binary-Lie) algebra on $n$ generators.

**1.161.** $*$ (A. I. Shirshov) Must a right alternative nil ring of bounded index be locally nilpotent? Remark: The answer is No (G. V. Dorofeev [36]).

**1.162.** (A. I. Shirshov) Is it true that every finitely generated right alternative nil ring of bounded index is solvable?

**1.163.** $*$ (A. I. Shirshov, A. T. Gainov) Does the variety of binary-Lie algebras of characteristic 2 have a finite basis of identities? Remark: The answer is Yes if the ground field has more than 3 elements (A. T. Gainov [58]).

**1.164.** $*$ (W. Specht [163]) Is it true that every variety of associative (unital) algebras over a field of characteristic 0 has a finite basis of identities? Remark: The answer is Yes (A. R. Kemer [83]).

**1.165.** $*$ (Reported by V. I. Arnautov) Does there exist an infinite ring that admits only the discrete topology? Remark: The answer is Yes (V. I. Arnautov [7]). An associative commutative ring always admits a non-discrete topology (V. I. Arnautov [6]).

**1.166.** $\circ$ (Reported by L. A. Bokut) Is the freeness theorem true for associative algebras with one relation? Remark: References and some partial results on this problem and 1.168 can be found in V. N. Gerasimov [59].

**1.167.** (Problem of Keller, reported by L. A. Bokut) Let $f\colon x_i \to f_i$ $(1 \le i \le n)$ be an endomorphism of the polynomial algebra $F[x_1, x_2, \dots, x_n]$ where $n \ge 2$ and $F$ is a field of characteristic 0. Suppose that the Jacobian

$$\det\left(\frac{\partial f_i}{\partial x_j}\right)$$

is equal to 1. Must $f$ be an automorphism?

**1.168.** ∘ (Reported by L. A. Bokut) Is the word problem decidable for associative algebras with a single relation? Remark: References and some partial results on this problem and 1.166 can be found in V. N. Gerasimov [59].

**1.169.** (Reported by L. A. Bokut) Does there exist a group such that its group ring does not have zero divisors but is not embeddable into a division ring?

**1.170.** (Reported by L. A. Bokut and A. R. Kemer) Let $R$ be an associative ring without nilideals that satisfies the condition $\forall x, y \in R, \exists n = n(x, y) \ge 2$ such that $(xy)^n = x^n y^n$. Must $R$ be commutative?

**1.171.** (Reported by L. A. Bokut) Does there exist an infinite associative division ring which is finitely generated as a ring?

**1.172.** (Reported by K. A. Zhevlakov) Does there exist a simple associative nil ring?

**1.173.** (Reported by K. A. Zhevlakov and V. N. Latyshev) Does there exist an algebraic, but not locally finite, associative division algebra?

**1.174.** ∗ (Reported by E. N. Kuzmin) Is it true that every algebraic Lie algebra of bounded degree over a field of characteristic 0 must be locally finite? Remark: The answer is Yes (E. I. Zelmanov [194]).

**1.175.** ∘ (Reported by E. N. Kuzmin and A. I. Shirshov) Must a Lie ring of characteristic 0 or $p > n$ satisfying the $n$-th Engel condition be nilpotent? Remark: The answer is Yes in the case of characteristic 0 (E. I. Zelmanov [195]). If $p = n + 2$ then the answer is No (Yu. P. Razmyslov [139]).

**1.176.** (Reported by G. P. Kukin) Is the membership problem decidable in a Lie algebra with a single defining relation?

**1.177.** ∘ (Reported by I. V. Lvov and Yu. N. Malcev) Is the variety of associative algebras generated by a full matrix algebra finitely based or Specht

(a) ∗ over a field of characteristic 0;

(b) over a field of characteristic $p$?

Remark: (a) The answer is Yes. Moreover, every variety of associative algebras over a field of characteristic 0 is Specht (A. R. Kemer [83]).

**1.178.** ∗ (Reported by Yu. N. Malcev) An algebra $A$ over a field $F$ is said to have type $M_k$ if $A$ satisfies all identities of the matrix algebra $M_k(F)$ and only those. Let the matrix algebra $M_n(R)$ over an algebra $R$ have type $M_k$. Does it follow that $R$ has type $M_t$ for some $t$? Remark: The answer is No. It is easy to see that this is not true for a free algebra in the variety defined by the identities $[x_1, x_2]x_3 = x_1[x_2, x_3] = 0$ over an infinite field (I. I. Benediktovich).

**1.179.** (Reported by V. A. Parfyonov) Describe all Schreier varieties of nonassociative algebras. Do there exist Schreier varieties other than the known ones: the variety of all algebras, $\varepsilon$-algebras, Lie algebras, and algebras with zero multiplication?

**1.180.** $*$ (Reported by A. A. Nikitin and S. V. Pchelintsev) Do there exist nonassociative prime $(-1, 1)$-rings without elements of order 6 in the additive group? Remark: The answer is Yes (S. V. Pchelintsev [129]).

**1.181.** (Reported by A. I. Shirshov) Is the isomorphism problem decidable in the class of nonassociative algebras over a "good" field, for instance, over the field of rational numbers?

# 2 Part Two

**2.1.** (S. Amitsur) Find the conditions for embeddability of an algebra over a field into an algebra that is a finitely generated module over a commutative ring.

**2.2.** (A. Z. Ananyin, L. A. Bokut, I. V. Lvov) A variety is called locally residually finite if every finitely generated ring (resp. algebra) can be approximated by finite rings (resp. finite dimensional algebras). Describe (in terms of identities) locally residually finite varieties of
(a) associative rings;
(b) associative algebras over a finite field.

**2.3.** (V. I. Arnautov) Does there exist an infinite associative ring that admits only the discrete topology?

**2.4.** (V. A. Artamonov) Let $k$ be a principal ideal domain and $V$ a variety of linear $k$-algebras defined by multilinear identities. Is it true that every retract of a $V$-free algebra is again a $V$-free algebra?

**2.5.** (V. A. Artamonov) Let $k$ be a commutative associative unital ring and $G$ an almost polycyclic group without torsion.
(a) Compute the kernel of the natural epimorphism of $K_0(kG)$ onto $K_0(k)$.
(b) If all projective modules over the group algebra $kG$ are free, must $G$ be commutative?

**2.6.** (Yu. A. Bahturin) Prove that for multilinear monomials $[x_1, \cdots, x_n]_\sigma = w_\sigma$ and $[x_1, \cdots, x_n]_\tau = w_\tau$ (with bracket structures $\sigma$ and $\tau$), the Lie algebra identities $w_\sigma = 0$ and $w_\tau = 0$ are equivalent if and only if $w_\sigma$ and $w_\tau$ are equal as elements of the free commutative nonassociative groupoid on $x_1, \cdots, x_n$.

**2.7.** (Yu. A. Bahturin) Find examples of varieties $U$, $V$ of Lie algebras over a finite field that have a finite basis of identities such that one of the varieties $UV$, $U \cup V$, $[U, V]$ does not have a finite basis of identities.

**2.8.** (Yu. A. Bahturin) Find an example of a variety $V$ of Lie algebras that has a finite basis of identities but for some natural number $n$ the variety $V^{(n)}$ (defined by all the identities of $V$ in $n$ variables) does not have a finite basis of identities.

**2.9.** (Yu. A. Bahturin) Find a basis of identities for the Lie algebra $W_n$ of derivations of the ring of polynomials in $n$ variables over a field of characteristic 0.

**2.10.** $*$ (Yu. A. Bahturin) Find a basis of identities for the full matrix Lie algebra $gl(2, k)$ over a finite field $k$ of characteristic $\neq 2$. Remark: A basis has been found (K. N. Semenov [145]). For an infinite field of positive characteristic see S. Yu. Vasilovsky [182].

**2.11.** $*$ (Yu. A. Bahturin) Prove that a Lie algebra $L$ whose derived algebra $L' = [L, L]$ is nilpotent of index $c$ ($c < p$) over a field of characteristic $p > 0$ has a finite basis of identities. Remark: This has been proved by A. N. Krasilnikov [94].

**2.12.** (Yu. A. Bahturin) Describe solvable special varieties of Lie algebras (that is, varieties generated by a special Lie algebra) over a field of characteristic 0.

**2.13.** $*$ (Yu. A. Bahturin) Is it true that a central extension of a special Lie algebra is again special (that is, embeddable into an associative PI algebra)? In the case of characteristic 0 this question is equivalent to Latyshev's problem (2.64) (S. A. Pikhtilkov [133]). Remark: The answer is No (Yu. V. Billig [20], see also Yu. A. Bahturin, A. I. Kostrikin [11]).

**2.14.** (Yu. A. Bahturin) Is it true that free algebras of finite rank in an arbitrary variety over a finite field are residually finite or at least Hopf?

**2.15.** (Yu. A. Bahturin, G. P. Kukin) Describe Hopf (resp. locally Hopf) varieties of Lie algebras.

**2.16.** $\circ$ (Yu. A. Bahturin, L. A. Bokut) Describe in terms of identities locally residually finite varieties of Lie algebras
(a) $*$ over a field of characteristic 0;
(b) over a finite field.
Remark: (a) They have been described by M. V. Zaicev [186].

**2.17.** (K. I. Beidar) Must a finitely generated domain be semisimple in the sense of Jacobson?

**2.18.** (L. A. Bokut) A variety is called Higman if every recursively presented algebra of this variety is embeddable into a finitely presented algebra. Are the following varieties of rings (algebras over a prime field) Higman:

(a) alternative;
(b) Jordan;
(c) Malcev;
(d) binary-Lie;
(e) solvable Lie algebras (resp. groups) of index $n \geq 3$?

**2.19.** (L. A. Bokut) Is the problem of existence of a solution for an equation in a free associative (resp. Lie) algebra over an algebraically closed field decidable?

**2.20.** (L. A. Bokut) Find the axiomatic rank of the class of associative rings that are embeddable into division rings.

**2.21.** (L. A. Bokut) Is the class of associative rings that are embeddable into division rings definable by an independent system of quasi-identities?

**2.22.** (L. A. Bokut) For a given $p \geq 0$ construct a non-invertible ring of characteristic $p$ whose multiplicative semigroup of nonzero elements is embeddable into a group. An associative ring is called invertible if all nonzero elements are invertible in some ring extension. So far the only known example is in the case $p = 2$ (L. A. Bokut [22, 23]).

**2.23.** (L. A. Bokut) Is an arbitrary finitely generated associative (resp. Lie) algebra with a recursive basis over a prime field embeddable into a simple finitely presented associative (resp. Lie) algebra?

**2.24.** (L. A. Bokut) Is the problem of equality decidable in the following classes of rings? In a class of rings, the problem of equality is the question of the existence of an algorithm to decide the truth of a quasi-identity in that class:
(a) finite Lie;
(b) finite alternative;
(c) finite Jordan;
(d) finite binary-Lie;
(e) free associative algebras;
(f) free Lie algebras.

**2.25.** (L. A. Bokut) A variety is called Magnus if the word problem is decidable for algebras with one relation. Determine whether the following varieties of algebras are Magnus:
(a) the variety $M(n)$ generated by the full matrix algebra $M_n$ of order $n$ over a field of characteristic 0;
(b) the variety $S(n)$ defined by the standard identity

$$\sum_{\sigma \in S_n} (-1)^{\sigma} x_{\sigma(1)} x_{\sigma(2)} \cdots x_{\sigma(n)} = 0.$$

**2.26.** (L. A. Bokut) Is the problem of equality decidable in the varieties $M(n)$ and $S(n)$?

**2.27.** (L. A. Bokut, I. P. Shestakov) Is the variety of alternative algebras Magnus?

**2.28.** (L. A. Bokut. I. V. Lvov) Must every relatively free algebra in a variety of associative algebras over a finite field be residually finite? [Compare 2.14.]

**2.29.** $*$ (Björk, reported by V. N. Gerasimov) Suppose that a division ring is finitely generated as a right module over a subring. Must this subring also be a division ring? Remark: The answer is No (G. Bergman [19]).

**2.30.** (P. Gabriel, reported by Yu. A. Drozd) Prove (or disprove) that for any natural number $n$ there exist only finitely many (up to isomorphism) associative algebras of dimension $n$ over an algebraically closed field $K$ that have only finitely many non-isomorphic indecomposable representations.

**2.31.** $*$ (V. N. Gerasimov) Suppose that $R$ is a radical ring that satisfies a non-trivial identity with the signature $\langle +, \cdot, ' \rangle$ where $'$ is the quasi-inverse. Must $R$ satisfy a polynomial identity? Remark: The answer is Yes (A. I. Valitskas [180]).

**2.32.** (A. G. Gein, A. Yu. Olshanski) Do there exist infinite dimensional simple Lie algebras over a field such that every proper subalgebra is one-dimensional?

**2.33.** (A. N. Grishkov) Describe semisimple finite dimensional binary-Lie algebras over a field of characteristic $p > 3$.

**2.34.** (A. N. Grishkov) Assume that the annihilator of every non-central element of a Lie algebra, which is nilpotent of index 2 over an algebraically closed field, is finite dimensional modulo the center. Prove that the algebra is residually finite dimensional.

**2.35.** $*$ (A. N. Grishkov) Must a finite dimensional solvable binary-Lie algebra over a field of characteristic $p > 3$ have an Abelian ideal? Remark: The answer is Yes (A. N. Grishkov [65]).

**2.36.** (K. R. Goodearl) A ring $R$ is called invertibly regular if for every $a \in R$ the equation $axa = a$ has an invertible solution. Must a regular ring whose homomorphic images are directly finite (see 2.141) be invertibly regular?

**2.37.** (K. R. Goodearl) Let $A$ and $B$ be finitely generated projective right modules over an invertibly regular ring. If $A^n$ is isomorphic to $B^n$ must $A$ and $B$ be isomorphic? If $A^n$ is isomorphic to a direct summand of $B^n$ then must $A$ be isomorphic to a direct summand of $B$?

**2.38.** (A. Jategaonkar, reported by A. A. Tuganbaev) Must every ideal of a prime ring, all of whose right ideals are principal, be a product of prime ideals?

**2.39.** (V. P. Elizarov) For a prime $p$ describe nilpotent rings of order $p^4$.

**2.40.** (K. A. Zhevlakov, reported by I. P. Shestakov) Must the locally nilpotent (or even anti-simple) radical of a weakly Noetherian associative (resp. alternative, Jordan) algebra be nilpotent?

**2.41.** (V. N. Zhelyabin) Must any two inertial subalgebras of a finite dimensional Jordan algebra over a local Hensel ring be conjugate?

**2.42.** (A. E. Zalesski) Is it true that the left annihilator of every element of a group algebra over a field is finitely generated as a left ideal?

**2.43.** $*$ (A. E. Zalesski) Is it true that every idempotent of a group algebra over a field is conjugate by an automorphism to an idempotent whose support subgroup is finite? Remark: The answer is No (D. P. Farkas, Z. S. Marciniak [39]).

**2.44.** (A. E. Zalesski, D. Passman) Find necessary and sufficient conditions for the group algebra of a locally finite group (over a field of nonzero characteristic) to be semisimple.

**2.45.** (I. Kaplansky, reported by A. A. Tuganbaev) Describe the rings in which every one-sided ideal is two-sided and over which every finitely generated module can be decomposed as a direct sum of cyclic modules.

**2.46.** (O. V. Kaptsov) Let $\mathbb{R}$ be the field of real numbers. Consider the commutative differential ring $\mathbb{R}[u_i]$ in infinitely many variables $u_i$, $i \geq 0$. The derivation $d$ acts on the $u_i$ as follows: $d(u_i) = u_{i+1}$. Define a Lie algebra structure on $\mathbb{R}[u_i]$ by

$$[f, g] = \sum_{i=0}^{\infty} (f_i d^i g - g_i d^i f), \text{ where } f_i = \frac{\partial f}{\partial u_i},\ g_i = \frac{\partial g}{\partial u_i}, \text{ for any } f, g \in \mathbb{R}[u_i].$$

Is it true that if $[f, g] = 0$ ($f \neq \lambda g$, $\lambda \in \mathbb{R}$) and $f_k \neq 0$, $g_m \neq 0$ for some $k, m > 1$, then the centralizer of $g$ is infinite dimensional? For instance, if $g = u_3 + u_0 u_1$ (the right side of the Korteweg-deVries equation $u_t = u_{xxx} - u u_x$), this conjecture holds. A positive answer would allow us to approach a solution of the following well-known problem: Describe the set of elements $g$ that have an infinite dimensional centralizer.

**2.47.** (A. V. Kaptsov) Let $\mathbb{R}[u_i]$ be the ring defined in the previous problem. Define a new multiplication by

$$f * g = \sum_{i=0}^{\infty} \big(f_i d^i g + (-d)^i (f g_i)\big).$$

Is it true that if $f * g = 0$ where $f = f(u_0, \ldots, u_n)$, $g = g(u_0, \cdots, u_m)$ and $f_n \neq 0$, $g_m \neq 0$ for some $n > m > 1$, then the subspace $H$ of all $h$ such that $h * g = 0$ is infinite dimensional? Is it possible to prove that if $H$ is infinite dimensional then so is the centralizer of $g$ (in the sense of the previous problem)?

**2.48.** (H. Köthe, reported by A. A. Tuganbaev) Describe the rings over which every right and left module is a direct sum of cyclic modules.

**2.49.** (L. A. Koifman) Let $R$ be a left hereditary ring and $P(R)$ its prime radical. Is it true that $P(R)$ is nilpotent and the quotient ring $R/P(R)$ is also left hereditary? If $R$ does not have an infinite set of orthogonal idempotents then this is true (Yu. A. Drozd).

**2.50.** (A. T. Kolotov) Let $F$ be a free associative algebra of finite rank, $A$ a finitely generated subalgebra of $F$, and $I$ an ideal of $F$ such that $I \subset A$ and $F/I$ is a nilalgebra. Is it true that $\operatorname{codim} A < \infty$?

**2.51.** $*$ (A. T. Kolotov) Does there exist an algorithm that decides, for any finite family of elements of a free associative algebra, whether this family is algebraically dependent? Remark: The answer is No (U. U. Umirbaev [175]).

**2.52.** (A. T. Kolotov, I. V. Lvov) Let $k$ be a field, and let $D$ consist of the pairs $(F, A)$ where $F$ is a free associative $k$-algebra and $A$ is a subalgebra of $F$. Let $D_0 \subset D$ consist of those pairs in which $A$ is free. Can $D_0$ be defined axiomatically in $D$ if we add to the signature the predicate that defines the subalgebra?

**2.53.** (P. M. Cohn) Must a retract of a free associative algebra also be free? (This is a special case of 2.4.)

**2.54.** (A. I. Kostrikin) Can every finite dimensional complex simple Lie algebra be decomposed into a direct sum of Cartan subalgebras which are pairwise orthogonal with respect to the Killing form? One of the conjectured negative examples is the Lie algebra of type $A_5$. (See A. I. Kostrikin, I. A. Kostrikin, V. A. Ufnarovskii [92].)

**2.55.** $*$ (A. I. Kostrikin) Do there exist finite dimensional simple Lie algebras over a field of characteristic $p > 5$ such that $(\operatorname{ad} x)^{p-1} \neq 0$ for all $x \neq 0$? The conjectured answer is negative. Remark: The answer is No (A. A. Premet [136]).

**2.56.** $*$ (A. I. Kostrikin, I. R. Shafarevich) Prove that every finite dimensional simple Lie $p$-algebra over an algebraically closed field of characteristic $p > 5$ is isomorphic to one of the algebras of classical or Cartan type. (See A. I. Kostrikin, I. R. Shafarevich [93].) Remark: The conjecture is true for characteristic $> 7$ (R. E. Block, R. L. Wilson [21]).

**2.57.** (A. I. Kostrikin) What are the maximal subalgebras of simple Lie algebras of classical type over an algebraically closed field of characteristic $p > 0$?

**2.58.** (E. N. Kuzmin) Must a binary-Lie algebra that has a regular automorphism of finite order be solvable?

**2.59.** $*$ (E. N. Kuzmin) Is there a connection between Moufang loops of prime exponent $p$ and Malcev algebras of characteristic $p$ analogous to the connection between Lie groups and Lie algebras? Remark: The answer is Yes (A. N. Grishkov [64]).

**2.60.** (G. P. Kukin) Describe the varieties of Lie algebras in which every finitely presented algebra (resp. finitely presented algebra with decidable word problem) is residually finite dimensional. Here finite presentability can be understood in the absolute or relative sense.

**2.61.** $*$ (G. P. Kukin) It can be shown that a free Lie algebra of characteristic $p > 0$ is residually finite with respect to inclusion into a finitely generated subalgebra. Is this true for Lie algebras of characteristic 0? Remark: The answer is Yes (U. U. Umirbaev [173]).

**2.62.** (G. P. Kukin) Is the problem of conjugacy by an automorphism for finitely generated subalgebras of a free algebra (resp. free Lie algebra) decidable?

**2.63.** $*$ (G. P. Kukin) Must every finitely generated subalgebra of a free solvable Lie algebra be finitely separated? Remark: The answer is No (S. A. Agalakov [1]).

**2.64.** $*$ (V. N. Latyshev) Is it true that a homomorphic image of a special Lie algebra is again a special Lie algebra (that is, embeddable into an associative PI algebra)? Remark: The answer is No (Yu. V. Billig [20]).

**2.65.** (I. V. Lvov) Is it true that every PI ring is a homomorphic image of a PI ring with torsion-free additive group?

**2.66.** (I. V. Lvov) Does there exist a nonzero PI ring that coincides with its derived Lie algebra?

**2.67.** (I. V. Lvov) Suppose that a ring satisfies an identity of degree $d$ with coprime coefficients. Is it true that this ring satisfies a multilinear identity of degree $d$ with some coefficient equal to 1?

**2.68.** (I. V. Lvov) (a) Does there exist a simple infinite dimensional finitely presented algebra $R$ over an arbitrary field $k$ of positive characteristic?
(b) The same question with additional assumptions: $R$ is Noetherian without zero divisors and has finite Gelfand-Kirillov dimension (see W. Borho, H. Kraft [26]). In the case of a field $k$ of characteristic 0, Weyl algebras are examples.

**2.69.** (I. V. Lvov) Does every finitely generated algebra (over a field) with finite Gelfand-Kirillov dimension have a greatest nilpotent ideal?

**2.70.** (I. V. Lvov) Is it true that every (right) primitive ring is a subdirect product of subdirectly indecomposable (right) primitive rings?

**2.71.** (I. V. Lvov) Is the class of residually finite rings (resp. groups) axiomatizable in the language $L_{\infty,\infty}$?

**2.72.** (I. V. Lvov) Is it true that the class of free associative algebras over a fixed field $k$ is not axiomatizable in the language $L_{\infty,\infty}$?

**2.73.** (I. V. Lvov) Is the class of subdirect products of division rings axiomatizable?

**2.74.** (I. V. Lvov) Does every algebra (over a field) without zero divisors and with the maximal condition on subalgebras satisfy a polynomial identity?

**2.75.** (I. V. Lvov) Must every finitely generated nilalgebra (over a field) with finite Gelfand-Kirillov dimension be nilpotent?

**2.76.** (I. V. Lvov) Let $A$ be a Noetherian alternative algebra. Is the algebra of formal power series $A[[x]]$ also Noetherian?

**2.77.** (I. V. Lvov) (a) Must the Gelfand-Kirillov dimension of a finitely generated Noetherian PI algebra be an integer? (It is finite by Shirshov's height theorem.) The same question for reduced-free (not necessarily Noetherian) algebras.
(b) Describe varieties of algebras over an infinite field in which all finitely generated algebras have integral Gelfand-Kirillov dimension.

**2.78.** (I. V. Lvov) Is it true that two free associative algebras (over a field) of finite ranks $m$, $n$ ($m > n \geq 2$) are elementarily equivalent?

**2.79.** (A. I. Malcev, reported by A. N. Grishkov) Prove that every analytic alternative local loop is locally isomorphic to an analytic alternative loop.

**2.80.** $*$ (Yu. N. Malcev) Let $R$ be a critical unital ring. Is it true that the matrix ring $M_n(R)$ is also critical? Remark: The answer is Yes (Yu. N. Malcev [108]).

**2.81.** $\circ$ (Yu. N. Malcev) Describe varieties of rings whose lattice (of subvarieties) is not distributive but the lattice of every proper subvariety is distributive. Remark: A complete description has not yet been obtained. Significant progress on this problem has been announced by M. V. Volkov [184].

**2.82.** (Yu. N. Malcev) Let $\mathcal{M}$ be the variety of associative rings satisfying the identity $x^3 = x^n$ for some $n \geq 4$. Does $\mathcal{M}$ satisfy the minimum condition on subvarieties?

**2.83.** (Yu. N. Malcev) Describe the critical rings in the variety of rings satisfying the identity $x^3 = x^n$ for some $n \geq 4$.

**2.84.** (E. Matlis, reported by A. A. Tuganbaev) Must a direct summand of a direct sum of indecomposable injective modules also be a direct sum of indecomposable injective modules?

**2.85.** (Yu. A. Medvedev) Must a variety $\mathcal{M}$ of alternative algebras be solvable if every associative algebra in $\mathcal{M}$ is nilpotent?

**2.86.** $\circ$ (Yu. A. Medvedev) Let $A$ be an alternative (resp. Jordan) ring, $G$ a finite group of automorphisms, $A^G$ the subalgebra of fixed elements. Must $A$ be solvable if $A^G$ is solvable and $A$ has no $|G|$-torsion? Remark: If $A$ is an algebra over a field of characteristic 0 then the answer is Yes (A. P. Semenov [144]).

**2.87.** (Yu. A. Medvedev) Does every finite alternative (resp. Jordan) ring have a finite basis of quasi-identities?

**2.88.** $*$ (S. Montgomery, V. K. Kharchenko) Consider the free associative algebra $F$ of rank $n$ over a field $k$ as the tensor algebra of the $n$-dimensional space $V$. For which linear groups $G \subseteq GL(V)$ is the subalgebra of invariants of $F$ with respect to $G$ finitely generated? Remark: A description of such groups has been obtained (A. I. Koryukin [89]).

**2.89.** (Yu. A. Ryabukhin, R. A. Florya) Does there exist in some variety a simple free ring with characteristic $p \geq 3$?

**2.90.** (L. A. Skornyakov) Describe the rings over which all finitely presented modules are injective.

**2.91.** (A. M. Slinko) What is the minimal possible dimension of a nilpotent exceptional Jordan algebra?

**2.92.** $\circ$ (A. M. Slinko) Does every variety of solvable alternative algebras over a field of characteristic $\neq 2$ have a finite basis of identities? (This is a more precise version of 1.129.) Remark: The answer is Yes over a field of characteristic $\neq 2, 3$ (U. U. Umirbaev [172]).

**2.93.** $\circ$ (A. M. Slinko) Must every nilideal of a Jordan algebra with the minimum condition on annihilators be nilpotent? Remark: The solvability of nil subalgebras of Jordan algebras with minimum condition on annihilators has been proved by A. V. Chekhonadskikh [33].

**2.94.** (A. M. Slinko) Describe maximal special varieties of Jordan algebras.

**2.95.** $\circ$ (A. M. Slinko) Must the variety generated by the Jordan algebra of a bilinear form be special? Remark: The answer is Yes in the case of a field of characteristic 0 (S. R. Sverchkov [165]) and in the case of a finite field (I. M. Isaev [75]).

**2.96.** (A. M. Slinko) Find a basis of weak identities of the pair $(F_2, H(F_2))$. Do they all follow from the standard identity $S_4(x_1, x_2, x_3, x_4)$?

**2.97.** (A. M. Slinko) If a homogeneous variety of algebras has a locally nilpotent radical, must it also have a locally finite dimensional radical?

**2.98.** $*$ (G. F. Smit) Must a right alternative nilalgebra with the minimum condition on right ideals be right nilpotent? Remark: The answer is Yes (V. G. Skosyrskii [157]).

**2.99.** (G. F. Smit) Must a one-sided nilideal of a Noetherian $(-1, 1)$ ring be nilpotent?

**2.100.** (D. A. Suprunenko) Must a torsion group of matrices over a division ring be locally finite? The modular case is especially interesting.

**2.101.** (A. Thedy, reported by I. P. Shestakov) Is it true that every finite dimensional right alternative algebra has an isotope which splits over its radical?

**2.102.** (A. A. Tuganbaev) A module is called weakly injective if every endomorphism of every submodule can be extended to an endomorphism of the whole module. Describe the rings over which all cyclic modules are weakly injective.

**2.103.** (A. A. Tuganbaev) Must a weakly injective module with an essential socle be quasi-injective?

**2.104.** (A. A. Tuganbaev) Can every right Noetherian ring with a distributive lattice of right ideals be decomposed as a direct sum of a right Artinian ring and a semiprime ring?

**2.105.** (V. T. Filippov) Let $A$ be the free Malcev algebra of countable rank, and let $\mathcal{M}_{[n]} = \mathrm{Var}(A^n)$. Does the chain of varieties

$$\mathcal{M}_{[1]} \subset \mathcal{M}_{[2]} \subseteq \mathcal{M}_{[3]} \subseteq \cdots \subseteq \mathcal{M}_{[n]} \subseteq \cdots$$

stabilize after a finite number of steps?

**2.106.** (V. T. Filippov) Does the free binary-Lie algebra contain nonzero nilpotent ideals?

**2.107.** (V. T. Filippov) Is the associative center of a free Moufang loop nontrivial?

**2.108.** (V. T. Filippov) Let $A$ be a free Malcev algebra over a field $F$ of characteristic 0, and $C_7$ the simple 7-dimensional non-Lie Malcev algebra over $F$. Is the ideal of identities of $C_7$ a Lie ideal in $A$?

**2.109.** (V. T. Filippov) An algebra is called assocyclic if it satisfies the identity $(x, y, z) = (z, x, y)$ where $(x, y, z) = (xy)z - x(yz)$. It is easy to show that the minus algebra of such an algebra is binary-Lie. Is every binary-Lie algebra over a field of characteristic $\neq 2, 3$ embeddable into the minus algebra of a suitable assocyclic algebra?

**2.110.** (V. T. Filippov) Are the varieties generated by the free Malcev algebras (over a field of characteristic $\neq 2, 3$) of ranks 3 and 4 distinct?

**2.111.** (V. T. Filippov) Is the ideal of the free alternative algebra over a field $F$ of characteristic 0, generated by the identities of the split Cayley-Dickson algebra over $F$, associative?

**2.112.** (V. T. Filippov) Describe the class of finite dimensional Malcev algebras over a field of characteristic 0 that have a faithful (not necessarily finite dimensional) representation.

**2.113.** (V. T. Filippov) Describe the center of the algebra of right multiplications of the free Malcev algebra.

**2.114.** $*$ (Fischer, reported by L. A. Skornyakov) Is the ring of matrices over an invertibly regular ring also invertibly regular (that is, a ring in which for every $a$ the equation $axa = a$ has an invertible solution)? Remark: The answer is Yes (M. Henriksen [67]).

**2.115.** (V. K. Kharchenko) Is the subalgebra of constants (that is, invariants) for a finite dimensional Lie $p$-algebra of derivations of the free associative algebra (over a field of characteristic $p$) also free?

**2.116.** (V. K. Kharchenko) Is the subalgebra of invariants for a finite group of automorphisms of the free associative algebra also free? The answer is unknown also for infinite groups.

**2.117.** (V. K. Kharchenko) Let $L$ be a Lie algebra that admits an automorphism of order 2 all of whose fixed elements are in the center of $L$. Must $L$ be solvable?

**2.118.** $*$ (V. K. Kharchenko) Must the restricted enveloping algebra of a Lie $p$-algebra with a polynomial identity also satisfy a polynomial identity? Remark: The answer is No (V. M. Petrogradsky [131]).

**2.119.** $*$ (V. K. Kharchenko) Can every associative algebra over a field of characteristic 0 that satisfies the identity $x^n = 0$ be represented by $n \times n$ matrices over a commutative ring? Remark: The answer is Yes (C. Procesi [137]).

**2.120.** $\circ$ (I. P. Shestakov) Describe the ideal of identities of the free alternative algebra on 3 generators. Remark: This ideal coincides with the radical over a field of characteristic $\neq 2, 3$ (A. V. Iltyakov [70]). Over a field of characteristic 0 it is finitely generated as a T-ideal (A. V. Iltyakov [72]) and nilpotent (E. I. Zelmanov and I. P. Shestakov [199]).

**2.121.** (I. P. Shestakov) Describe the center and the associative center of a free alternative algebra as completely characteristic subalgebras. Are they finitely generated?

**2.122.** (I. P. Shestakov) Does the anti-simple radical of an associative (resp. Jordan) algebra coincide with the intersection of the kernels of all irreducible birepresentations of this algebra?

**2.123.** (I. P. Shestakov) Is every finitely generated associative (resp. special Jordan) PI algebra embeddable into a 2-generated PI algebra?

**2.124.** $\circ$ (I. P. Shestakov) Describe the following varieties of alternative and Jordan algebras (resp. rings):
(a) almost nilpotent;
(b) almost Cross;
(c) $\circ$ locally residually finite;
(d) $\circ$ locally Noetherian (resp. weakly Noetherian);
(e) Hopf;
(f) alternative almost associative;

(g) Jordan almost special;
(h) Jordan distributive.
Remark: (c,d) Locally residually finite and locally (weakly) Noetherian varieties of alternative algebras have been described (S. V. Pchelintsev [130]).

**2.125.** $*$ (I. P. Shestakov) Does the free Jordan algebra on three or more generators contain Albert subrings? Remark: The answer is Yes (Yu. A. Medvedev [120]).

**2.126.** $*$ (I. P. Shestakov) Find a basis of identities of the Jordan algebra of a bilinear form over an infinite field. Does this algebra generate a Specht variety? Remark: A finite basis of identities has been found by S. Yu. Vasilovsky [181]. Over a field of characteristic 0 the unitary Specht property has been proved (S. Yu. Vasilovsky [181], A. V. Iltaykov [71], P. E. Koshlukov [91]).

**2.127.** $\circ$ (I. P. Shestakov) Are the varieties of Jordan algebras generated by
(a) $*$ the algebra $F_n^+$;
(b) $*$ the algebra $H(F_n)$;
(c) $*$ the algebra $H_3(\mathbf{C})$;
finitely based or Specht? Describe the trace identities that hold in these algebras. Do they have a finite basis? Remark: It has been shown that every finitely generated Jordan PI algebra over a field of characteristic 0 is Specht (A. Ya. Vais, E. I. Zelmanov [178]).

**2.128.** $*$ (I. P. Shestakov) Suppose that a special Jordan algebra $J$ satisfies an identity that does not hold in a Jordan algebra of a bilinear form on an infinite dimensional space. Must $J$ have an enveloping associative PI algebra? Remark: The answer is No (S. V. Pchelintsev [129]).

**2.129.** $*$ (I. P. Shestakov) Let $J$ be a finitely generated Jordan PI algebra. Is its universal multiplicative enveloping algebra $\mathcal{R}(J)$ also a PI algebra? Remark: The answer is Yes (Yu. A. Medvedev [119]).

**2.130.** $*$ (I. P. Shestakov) Let $J$ be a Jordan algebra, $I \triangleleft J$, $H \triangleleft I$. Suppose that the ideal $I$ is generated in $J$ by the set $H$. Must the quotient algebra $I/H$ be solvable or nilpotent? Remark: The answer is No (Yu. A. Medvedev [118], S. V. Pchelintsev [129]).

**2.131.** $*$ (I. P. Shestakov) Let $A$ be a simple right alternative ring such that $A^{(+)}$ is a simple Jordan ring. Must $A$ be alternative? Remark: The answer is Yes (V. G. Skosyrskii [156]).

**2.132.** (I. P. Shestakov) Describe finite dimensional irreducible right alternative bimodules over the matrix algebra $M_2(F)$. Is their number finite (up to isomorphism)?

**2.133.** $*$ (I. P. Shestakov) Describe noncommutative Jordan division algebras, at least in the finite dimensional case. Remark: Strictly prime algebras of characteristic $\neq 2, 3$ have been described by V. G. Skosyrskii [159].

**2.134.** (I. P. Shestakov) Does there exist a simple infinite dimensional non-commutative Jordan algebra, with the identity $([x, y], y, y) = 0$, that is neither alternative nor Jordan?

**2.135.** $*$ (A. I. Shirshov) Must the variety of algebras generated by a finite dimensional associative (resp. Lie) algebra over a field of characteristic 0 have a finite basis of identities? Remark: The answer is Yes (A. R. Kemer [83], A. V. Iltyakov [73]).

**2.136.** (A. L. Shmelkin) Do there exist infinite dimensional Noetherian Lie algebras (that is, satisfying the maximal condition on subalgebras) that can be approximated by nilpotent Lie algebras? The analogous question for groups: Does there exist a non-nilpotent group that is approximable by nilpotent torsion-free groups?

**2.137.** (Reported by V. A. Artamonov) Let $k$ be a principal ideal domain and $G$ an almost polycyclic group without torsion. Is it true that every projective module over the group algebra $kG$ is a direct sum of a free module and a one-sided ideal?

**2.138.** $\circ$ (Reported by Yu. A. Bahturin) Is it true that a variety of Lie algebras over a field $k$ of characteristic 0 that does not contain the algebra $sl(2, k)$ is (locally) solvable? Remark: The answer is Yes for special Lie algebras (A. Ya. Vais [177]) and also for some other varieties (S. P. Mishchenko [123]).

**2.139.** (Reported by Yu. A. Bahturin) Describe finite dimensional simple Lie algebras (over an arbitrary field) such that all proper subalgebras are nilpotent (or even Abelian).

**2.140.** (Reported by A. L. Voronov) Let $G$ be a polycyclic group. Is it true that the algebra $kG$ is primitive if and only if the field $k$ is not absolute and $\Delta(G) = 1$?

**2.141.** (Reported by K. R. Goodearl and L. A. Skornyakov) A ring is called directly finite if $xy = 1$ implies $yx = 1$. Is the ring of matrices over a regular directly finite ring also directly finite?

**2.142.** (Reported by Yu. A. Drozd) Let $A$ be a finite dimensional Lie algebra, $U$ its universal enveloping algebra, and $P$ a finitely generated projective $U$-module. Must $P$ be a free module?

**2.143.** (Reported by I. V. Lvov) Does every finitely generated PI ring satisfy all the identities of a ring of $n \times n$ matrices over the integers?

**2.144.** (Reported by Yu. M. Ryabukhin) Describe in terms of identities the varieties of commutative associative algebras over a finite field.

**2.145.** (Reported by A. I. Kostrikin) Find a formula for the dimensions of the irreducible $p$-modules of the classical Lie algebras over a field of characteristic $p > 0$.

# 3 Part Three

**3.1.** (T. Anderson) Let $\mathcal{M}$ be a variety of power-associative algebras whose finite dimensional solvable algebras are nilpotent. Must the nilalgebras of $\mathcal{M}$ be solvable?

**3.2.** (V. I. Arnautov) Is every ring topology of a ring (resp. division ring) $R$ a greatest lower bound of some family of maximal ring topologies of $R$ in the lattice of all topologies?

**3.3.** (V. I. Arnautov) Is there a ring in which one can build maximal ring topologies, without assuming the Continuum Hypothesis, such that the corresponding topological ring is not complete?

**3.4.** (V. I. Arnautov) Is there a ring which is complete with respect to any maximal ring topology?

**3.5.** (V. I. Arnautov, A. V. Mikhalev) Is it possible to embed an arbitrary topological group into the multiplicative semigroup of a topological ring?

**3.6.** (V. I. Arnautov, A. V. Mikhalev) Is it true that for any topological ring $(R, \tau_0)$ and any discrete monoid $G$ the topology $\tau_0$ can be extended to a ring topology on the semigroup ring $RG$?

**3.7.** (V. I. Arnautov, I. V. Protasov) Is it true that for an arbitrary ring there exists a ring topology for which all endomorphisms of the ring are continuous?

**3.8.** (A well-known problem reported by V. I. Arnautov) Can any ring topology of a division ring be weakened to a ring topology in which the inverse operation is a continuous function?

**3.9.** (V. A. Artamonov) Let $B$ be an associative left Noetherian algebra of Krull dimension $d$, let $H$ be a commutative and cocommutative Hopf algebra, and let $A = B\#_t H$ be a crossed product. Suppose that $P$ is a finitely generated projective module of rank $> d$. If $P$ is stably extended from $B$, then is $P$ extended from $B$?

**3.10.** (L. A. Bokut) Is an arbitrary finitely generated associative (resp. Lie) algebra with a recursive basis embeddable in a finitely definable associative (resp. Lie) algebra?

**3.11.** (L. A. Bokut) How many non-isomorphic algebraically closed Lie algebras of a given cardinality are there?

**3.12.** (L. A. Bokut, V. N. Gerasimov) Is an arbitrary free associative algebra embeddable in an algebraically closed associative algebra (that is, an algebra in which any non-trivial generalized polynomial in one variable has a root)?

**3.13.** (L. A. Bokut, V. N. Gerasimov) Is it true that the class of associative rings embeddable into division rings cannot be defined by an independent system of quasi-identities?

**3.14.** (L. A. Bokut, V. N. Gerasimov) Is it true that the class of semigroups embeddable into groups cannot be defined by an independent system of axioms?

**3.15.** (L. A. Bokut, M. V. Sapir) Describe all varieties of algebras over a field of characteristic 0 in which for every finitely definable algebra the word problem is solvable.

**3.16.** (L. A. Bokut, M. V. Sapir) Describe all varieties of algebras over a field of characteristic 0 in which every finitely definable algebra is residually finite dimensional.

**3.17.** (L. A. Bokut, M. V. Sapir) Describe all varieties of algebras over a field of characteristic 0 in which every finitely definable algebra is representable.

**3.18.** (N. A. Vavilov, A. V. Mikhalev) This and the following problem are related to the attempt to extend some results in algebraic $K$-theory from commutative rings to PI rings. Positive answers are known in the case when the ring is a finitely generated module over its center (A. A. Suslin, M. S. Tulenbaev). Let $R$ be a unital PI ring. Consider, in the group $GL_n(R)$ of all invertible $n \times n$ matrices over $R$, the subgroup of elementary matrices $E_n(R)$ generated by the transvections

$$t_{ij}(r) = I + rE_{ij}, \quad i \neq j, \quad 1 \leq i, j \leq n, \quad r \in R.$$

Is $E_n(R)$ a normal subgroup of $GL_n(R)$?

**3.19.** (N. A. Vavilov, A. V. Mikhalev) Let $R$ be a unital PI ring, and let $St_n(R)$ be its $n$-th Steinberg group, that is, the group generated by formal transvections over $R$: the elements $u_{ij}(a)$ with defining relations

$$\begin{aligned} [u_{ij}(a), u_{jk}(b)] &= u_{ik}(ab), \quad i \neq j, j \neq k, k \neq i, \\ [u_{ij}(a), u_{kl}(b)] &= 1, \quad j \neq k, i \neq l, \\ u_{ij}(a)u_{ij}(b) &= u_{ij}(a+b). \end{aligned}$$

There exists a group homomorphism of $St_n(R)$ onto $E_n(R)$ (see 3.18) sending $u_{ij}(r)$ to $t_{ij}(r)$. Let $K_{2,n}(R)$ be its kernel:

$$1 \longrightarrow K_{2,n}(R) \longrightarrow St_n(R) \longrightarrow E_n(R) \longrightarrow 1.$$

Is it true that $K_{2,n}(R)$ is contained in the center of the group $St_n(R)$ for sufficiently large $n$ (for instance, for $n \geq 5$)?

**3.20.** (Yu. M. Vazhenin) What are the $SA$-critical theories of a free associative ring? The lists of all $SA$-critical theories of the ring of integers and of the absolutely free (nonassociative) ring are known.

**3.21.** (Yu. M. Vazhenin) Of the rings defined by one relation in the following varieties:

(a) alternative rings;

(b) Jordan rings;
(c) associative rings;
which have a decidable elementary theory?

**3.22.** (Yu. M. Vazhenin, I. P. Shestakov) What are the $SA$-critical theories of a free Jordan ring?

**3.23.** (Yu. M. Vazhenin, I. P. Shestakov) What are the $SA$-critical theories of the variety of all Jordan rings?

**3.24.** (A. T. Gainov) Let $M_n(\Phi)$ be the matrix algebra over a field $\Phi$ of characteristic not 2. We call a subspace $V$ of the algebra $M_n(\Phi)$ a space of anticommuting matrices (SAM for short) if $x^2 = 0$ for all $x \in V$. Two SAMs $V$ and $W$ in $M_n(\Phi)$ are called equivalent if $\phi(V) = W$ for some automorphism or anti-automorphism $\phi$ of $M_n(\Phi)$. Find all inequivalent maximal (with respect to inclusion) SAMs of the algebra $M_n(\Phi)$.

**3.25.** (A. T. Gainov) Let $\Phi$ be a field of characteristic not 2, and assume $\Phi^2 \neq \Phi$. We will call a subspace $V$ of the algebra $M_n(\Phi)$ a space of anticommuting antisymmetric (resp. symmetric) matrices (SAAM for short, resp. SASM) if $x^2 = 0$ and $x^t = -x$ (resp. $x^t = x$) for all $x \in V$. Two SAAMs (resp. SASMs) $V$ and $W$ in $M_n(\Phi)$ are called equivalent if $W = qVq^t$ for some orthogonal matrix $q \in M_n(\Phi)$. Find all inequivalent maximal (with respect to inclusion) SAAMs (resp. SASMs) of the algebra $M_n(\Phi)$.

**3.26.** (A. T. Gainov) Describe all finite dimensional simple anticommutative algebras $A$ over an infinite field of characteristic not 2 that satisfy the condition that any element $a \in A$ lies in some two-dimensional subalgebra.

**3.27.** (A. G. Gein) An element $a$ of a Lie algebra $L$ is called ad-pure if any finite dimensional $\mathrm{ad}\,a$-invariant subspace of the algebra $L$ lies in the kernel of the operator $\mathrm{ad}\,a$. Is there a simple Lie algebra all of whose elements are ad-pure?

**3.28.** (A. G. Gein) Does there exist
(a) an infinite dimensional Lie algebra all of whose proper subalgebras are finite dimensional;
(b) an infinitely generated Lie algebra all of whose proper subalgebras are finite dimensional;
(c) an infinitely generated Lie algebra all of whose proper subalgebras are finitely generated?

**3.29.** (A. V. Grishin) Let $\mathcal{F}$ be a countably generated free algebra over a field of characteristic 0 from a variety of finite base rank, $\mathcal{F}(d)$ a $d$-generated subalgebra of $\mathcal{F}$. We say that a subspace $V$ of $\mathcal{F}(d)$ is a $T$-space if $V = \overline{V} \cap \mathcal{F}(d)$ where $\overline{V}$ is the subspace of $\mathcal{F}$ spanned by all possible substitutions into the polynomials in $V$ of elements in $\mathcal{F}$. Does any $T$-space have a finite base? A positive answer is known in the case of the variety of associative algebras. In particular, this result would imply that the variety is Specht. It is interesting to consider also the cases of alternative, Jordan and $(-1, 1)$ algebras.

**3.30.** (A. V. Grishin) Find an upper bound for the nilpotency index of the radical of the free (associative) algebra satisfying the standard identity of degree $n$.

**3.31.** (A. V. Grishin) Find an upper bound for the dimension of the (finite dimensional) algebra of least dimension that generates the variety of associative algebras defined by the standard identity of degree $n$. (Such an algebra exists by the results of the author and A. R. Kemer.)

**3.32.** (A. V. Grishin) Let $\mathcal{M}(n)$ be the variety generated by all $n$-dimensional algebras, and $\mathcal{A}$ the variety of all associative algebras. Is it true that $\mathcal{M}(n) \cap \mathcal{A}$ can be defined by the Capelli identity of order $n+1$:

$$\sum_{\sigma \in S_{n+1}} y_0 x_{\sigma(1)} y_1 x_{\sigma(2)} \cdots y_n x_{\sigma(n+1)} y_{n+1} = 0 \ ?$$

**3.33.** (A. V. Grishin) If a variety is Specht then it is a sum of indecomposable subvarieties. Investigate the question of the uniqueness of such a decomposition in the associative or nearly associative case.

**3.34.** (A. N. Grishkov) Describe finite dimensional Malcev algebras (resp. binary Lie algebras) to which there correspond algebraic Moufang loops (resp. alternative loops).

**3.35.** (A. N. Grishkov) Describe all simple algebraic Bol loops.

**3.36.** (V. N. Zhelyabin) Is a countably categorical alternative (resp. Jordan) nilring solvable?

**3.37.** (A. E. Zalessky) Describe the two-sided ideals of the group ring of the finitary symmetric group over a field of prime characteristic. (The finitary symmetric group consists of all permutations of an infinite set which only move a finite number of elements.) The description is known over a field of characteristic 0.

**3.38.** (A. E. Zalessky) Let $P$ be a field of characteristic $p > 0$, and $A$ an associative algebra over $P$ graded by a finite Abelian group of order $k$. Assume that the zero component $A_0$ is commutative. Is it true that $A$ satisfies the standard identity of degree $kp$? This is the case for the matrix algebra $M_k(P)$.

**3.39.** (E. I. Zelmanov) Let $F_{2,m}$ be the free 2-generated associative ring with identity $x^m = 0$. Is it true that the nilpotency index of $F_{2,m}$ grows exponentially as a function of $m$?

**3.40.** (E. I. Zelmanov) Is it true that the nilpotency index of the $m$-generated $(p-1)$-Engel Lie algebra over a field of characteristic $p > 0$ grows linearly as a function of $m$ and exponentially as a function of $p$?

**3.41.** (E. I. Zelmanov) Let $L$ be a $(p-1)$-Engel Lie algebra over a field of characteristic $p > 0$. Is it true that an arbitrary element of $L$ generates a nilpotent ideal?

**3.42.** (A. V. Iltyakov) Let $A$ be a finitely generated alternative (resp. Jordan) PI algebra. Does there exist a finite dimensional alternative (resp. Jordan) algebra $B$ whose ideal of identities $T(B)$ is contained the ideal of identities $T(A)$ of the algebra $A$?

**3.43.** (I. Kaplansky, reported by A. E. Zalessky) Let $H$ be a group, $P$ a field, and $A(PH)$ the augmentation ideal of the group ring $PH$. Describe all the groups $H$ for which $A(PH)$ is a simple ring (at least for $P = \mathbb{C}$, the field of complex numbers).

**3.44.** (I. Kaplansky, M. Henriksen, reported by A. A. Tuganbaev) Let $M$ be a $2 \times 2$ matrix with entries from a commutative Bezout domain $A$. Is it true that there always exist invertible $2 \times 2$ matrices $C$ and $D$ such that $CMD$ is a diagonal matrix?

**3.45.** (I. Kaplansky, reported by K. A. Pavlov) Is it true that there are only finitely many (up to isomorphism) Hopf algebras of a given dimension?

**3.46.** (A. R. Kemer) Does the algebra of $2 \times 2$ matrices over an infinite field of positive characteristic have a finite basis of identities?

**3.47.** (G. P. Kukin) Prove that a Lie algebra has cohomological dimension $\leq 2$ if and only if its module of relations is free.

**3.48.** (G. P. Kukin) Prove that the elementary (resp. universal) theory of a free Lie algebra over a field $F$ is decidable if and only if the elementary (resp. univeral) theory of $F$ is decidable.

**3.49.** (I. V. Lvov) At the present time there is no reasonable conjecture about the structure of the automorphism group of a free PI algebra. A question in the negative direction: Let $\mathcal{M}$ be a variety of PI algebras strictly containing the variety of commutative algebras, and $A$ a free algebra (in countably many generators). Is it true that the automorphism group of the algebra $A$ is not generated by tame automorphisms? If $A$ has a non-trivial center, then the answer is Yes (G. Bergman).

**3.50.** (I. V. Lvov) Let $A$ be a free PI ring. Does there always exist an epimorphism $B \to A$ where $B$ is a free PI ring without additive torsion? If yes (or in those cases when the answer is yes) then what is the “minimal” $B$ with this property? The analogous question under the assumption that $A$ has prime characteristic $p > 0$.

**3.51.** (I. V. Lvov, Yu. N. Malcev) Is a free PI ring residually finite? Equivalently, is every variety of PI rings generated by its finite rings?

**3.52.** (Yu. N. Malcev) Is a finite local (associative) unital ring necessarily representable?

**3.53.** (Yu. N. Malcev) Describe the minimal non-Engel varieties of associative rings. In particular, are they Cross varieties?

**3.54.** (S. P. Mishchenko) Describe the non-solvable varieties of Lie algebras that have almost polynomial growth. (An example of such a variety is $\mathrm{Var}(sl_2)$.)

**3.55.** (S. P. Mishchenko) Describe the solvable varieties of Lie algebras that have exponential growth. (An example is $AN_2$.)

**3.56.** (S. P. Mishchenko) Does the identity $x_0(\bar{x}_1\hat{y}_1)\cdots(\bar{x}_m\hat{y}_m) = 0$ (for some $m$) follow from the standard Lie identity? Here bars and hats denote skew-symmetrization in the corresponding group of variables.

**3.57.** (S. P. Mishchenko) Is there a variety of Lie algebras over a field of characteristic 0 with a distributive lattice of subvarieties and whose basis of identities is not limited to degree 6?

**3.58.** (S. Montgomery) Let $R$ be an associative ring with a derivation $d$, and let $S = R[x; d]$ be the Ore extension. Is it true that if $R$ has no nonzero nilideals then $S$ is semisimple? If $d = 0$ then it is true by a well-known theorem of Amitsur. See some partial results in J. Bergen, S. Montgomery, D. S. Passman [17].

**3.59.** (V. M. Petrogradsky) Suppose that a Lie $p$-algebra has no elements algebraic with respect to the $p$-mapping. Is it true that its restricted enveloping algebra has no zero-divisors?

**3.60.** (V. M. Petrogradsky) Let $R$ be a PI subalgebra of the restricted enveloping algebra of a Lie $p$-algebra, and $n$ the minimal number such that $R$ satisfies a power of the standard identity $S_{2n}$. Is it true that $n = p^k$?

**3.61.** (S. V. Pchelintsev) Is the ideal of a finitely generated binary-$(-1, 1)$ algebra generated by the alternators nilpotent or solvable?

**3.62.** (S. V. Pchelintsev) Do there exist simple non-alternative right alternative Malcev-admissible algebras?

**3.63.** (S. V. Pchelintsev) Is it true that the additive group of the free alternative ring on three generators is torsion-free?

**3.64.** (S. V. Pchelintsev) Is it true that every prime non-associative $(-1, 1)$ algebra over a field of characteristic 0 generates the variety of all strictly $(-1, 1)$ algebras?

**3.65.** (S. V. Pchelintsev) Is the variety of alternative algebras over a field of characteristic 0 decomposable into a union of proper subvarieties?

**3.66.** (G. E. Puninsky) Let $R$ be a uniserial ring without zero divisors. Is it true that every purely injective module over $R$ contains an indecomposable direct summand?

**3.67.** (Yu. P. Razmyslov, reported by S. P. Mishchenko) Prove that the variety of algebras with the standard identity has exponential growth.

**3.68.** (D. A. Rumynin) Let $k$ be an absolute algebraic field, and $H$ a Hopf algebra over $k$. Is it true that every irreducible $H$-module is finite dimensional?

**3.69.** (D. A. Rumynin) Describe all finite dimensional semisimple Hopf algebras.

**3.70.** (Yu. M. Ryabukhin) Is it true that every reduced-free quasi-regular algebra (that is, an algebra with an additional unary operation $x \mapsto x^*$ that provides the adjoint group) is generalized nilpotent?

**3.71.** (L. V. Sabinin) Develop the structure theory of finite dimensional Bol algebras of characteristic 0.

**3.72.** (M. V. Sapir) Let $k$ be a field of characteristic 0, and let $R$ be a $k$-algebra. Do there exist an extension field $F(R) \supset k$, and an algebra $A(R) \supset R$ finite dimensional over $F(R)$, such that every family of elements of $R$, which becomes linearly dependent upon some embedding of the $k$-algebra $R$ into an algebra finite dimensional over some field extension of $k$, is linearly dependent in $A(R)$? It is not even clear whether, for any two finite subsets $U_1, U_2 \subset R$, which are linearly dependent in algebras $A_1, A_2 \supset R$ finite dimensional over extension fields $F_1, F_2 \supset k$, it is possible to make $U_1, U_2$ simultaneously linearly dependent in some algebra $A \supset R$ finite dimensional over some extension field $F \supset k$.

**3.73.** (M. V. Sapir) Is it true that in a variety of associative algebras (over a constructive field of characteristic 0) the word problem is decidable if and only if the variety does not contain the variety defined by the identities

$$x[y,z][t,u]v = 0, \quad x[y,z,t]u = 0?$$

**3.74.** (A. I. Sozutov) Describe all finite dimensional simple Lie algebras with a monomial basis.

**3.75.** (A. A. Tuganbaev) Describe all rings over which every right module is a distributive left module over its endomorphism ring. (A module is called distributive if its submodule lattice is distributive.)

**3.76.** (A. A. Tuganbaev) Describe all right distributive monoid rings.

**3.77.** (A. A. Tuganbaev) Does every left and right distributive ring have a classical ring of quotients?

**3.78.** (A. A. Tuganbaev) Is every right distributive ring, which is integral over its center, also left distributive?

**3.79.** (A. A. Tuganbaev) Describe all the rings over which every left module is isomorphic to a submodule of a direct sum of uniserial modules.

**3.80.** (A. A. Tuganbaev) Let $M$ be a maximal right ideal of a right distributive ring $A$, and let $T = A \setminus M$. Does there always exist a ring $Q$ and a ring homomorphism $f\colon A \to Q$ such that the elements $f(T)$ are invertible in $Q$, and such that

$$\ker f = \{\, a \in A \,|\, \exists t \in T,\, at = 0 \,\}, \qquad Q = \{\, f(a)f(t)^{-1} \,|\, a \in A, t \in T \,\}?$$

**3.81.** (A. A. Tuganbaev) Let $A$ be a right distributive ring without nonzero nilpotent elements. Are all right ideals of $A$ flat?

**3.82.** (A. A. Tuganbaev) Is every left and right distributive domain a semi-hereditary ring?

**3.83.** (A. A. Tuganbaev) Describe all the rings over which the ring of formal power series in one variable has weak global dimension one.

**3.84.** (V. T. Filippov) Let $A$ be the free Malcev algebra over a field of characteristic 0, let $\mathrm{Var}(A^2)$ be the variety generated by the square of $A$, and let $\mathcal{M}_3$ be the variety generated by the free Malcev algebra on three generators. Is it true that $\mathrm{Var}(A^2) = \mathcal{M}_3$?

**3.85.** (V. T. Filippov) Does there exist a trivial characteristic ideal, not lying in the Lie center, in the free countably generated Malcev algebra of characteristic $\neq 2, 3$?

**3.86.** (V. T. Filippov) Does there exist a trivial characteristic ideal, not lying in the associative center, in the free countably generated alternative algebra of characteristic $\neq 2, 3$?

**3.87.** (V. T. Filippov) Does there exist a simple non-Malcev binary-Lie algebra of characteristic 0?

**3.88.** (V. T. Filippov) Classify simple finite dimensional $n$-Lie algebras over an algebraically closed field of characteristic 0.

**3.89.** (V. T. Filippov) Is it true that in any non-solvable finite dimensional $n$-Lie algebra over an algebraically closed field of characteristic 0 there exists an $(n+1)$-dimensional simple subalgebra?

**3.90.** (V. T. Filippov) Do there exist non-Lie simple finite dimensional Sagle algebras over a field of characteristic 0? A Sagle algebra is an anticommutative algebra satisfying the identity

$$J(x,y,z)t = J(xt,y,z) + J(x,yt,z) + J(x,y,zt),$$

where $J(x,y,z) = (xy)z + (zx)y + (yz)x$.

**3.91.** (J. Faulkner) An Abelian group $A$ together with mappings $j_a$ (defined for each $0 \neq a \in A$) from the set $A \cup \{\infty\}$ to itself is called a Hua system if the following conditions are satisfied:

(HS1) $j_a^2 = id,\ j_a(a) = a,\ j_a(-a) = -a,\ j_a(0) = \infty$

(HS2) $(s_a j_a)^3 = id$ where $s_a(b) = a - b,\ s_a(\infty) = \infty$

(HS3) $j_a j_b \in \mathrm{End}\, A$

Every quadratic Jordan division ring is a Hua system if we set $j_a(b) = U_a(b^{-1})$. Can every Hua system be obtained from a quadratic Jordan division ring in this way? (See J. R. Faulkner [41] for references and a survey of known results.)

**3.92.** (P. A. Freidman) Is it true that every associative nil ring all of whose proper subrings have an annihilator series also has an annihilator series?

**3.93.** (V. K. Kharchenko) Let an associative unital ring satisfy an essential polynomial identity with automorphisms and skew derivations. Will it be a PI ring? (An identity is called essential if the two-sided ideal generated by all values of its generalized monomials contains the unit element.)

**3.94.** (V. K. Kharchenko) Let $R$ be a prime ring with generalized centroid $C$, and $B$ a quasi-Frobenius finite dimensional $C$-subalgebra of $RC$. Is $R$ necessarily a PI ring if the centralizer of $B$ in $R$ is a PI ring?

**3.95.** (V. K. Kharchenko) Describe the identities with skew derivations and automorphisms of an arbitrary prime ring.

**3.96.** (V. K. Kharchenko) Develop Galois theory in the class of prime rings for reduced finite groups which have a quasi-Frobenius group algebra. At the present time such a theory has been developed for groups which have a semisimple group algebra, and it is also known that a reduced finite group with a quasi-Frobenius group algebra is a Galois group.

**3.97.** (V. K. Kharchenko) Let a Hopf algebra $H$ act on an associative unital algebra $R$, and suppose that $R$ satisfies an essential multilinear generalized identity with operators from $H$. Is $R$ necessarily a PI algebra? A multilinear generalized identity is called essential if the two-sided ideal generated by the values of all generalized monomials contains the unit element. A generalized monomial is the sum of all the monomials having a fixed order of variables.

**3.98.** (I. P. Shestakov) Do there exist exceptional prime noncommutative alternative algebras (that is, algebras other than associative or Cayley-Dickson rings)?

**3.99.** (I. P. Shestakov) Compute (or at least find an upper bound for) the nilpotency index of the radical of the free alternative algebra over a field of characteristic 0.

**3.100.** (I. P. Shestakov) Describe all simple finite dimensional superalgebras for the following classes of algebras:
(a) noncommutative Jordan (that are not super-anticommutative);
(b) right alternative;
(c) structurable;
(d) binary-Lie.

**3.101.** (I. P. Shestakov) Describe all finite dimensional irreducible superbimodules for the following classes of algebras:

(a) alternative;
(b) Jordan;
(c) Malcev;
(d) structurable.

**3.102.** (I. P. Shestakov) Describe all simple finite dimensional Jordan superpairs and triple supersystems.

**3.103.** (I. P. Shestakov) Do there exist finite dimensional central simple algebras over a field of characteristic 0 that do not have a finite basis of identities?

**3.104.** (I. P. Shestakov) Let $A$ be a finite dimensional central simple algebra over a field $F$, let $F_k(A)$ be the free algebra of rank $k$ in the variety generated by $A$, and let $\Gamma_k$ be the field of quotients of the centroid of $F_k(A)$. Is $\Gamma_k$ always a purely transcendental extension of $F$? If $A = M_n(F)$ then this is the well-known problem on the center of the ring of generic matrices, which has been solved positively only for $n \leq 4$.

**3.105.** (I. P. Shestakov) Is it true that every nilpotent (not necessarily associative) algebra is representable (that is, embeddable in a finite dimensional algebra over some extension of the ground field)?

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