NATIONAL TECHNICAL UNIVERSITY OF ATHENS
LABORATORY FOR EARTHQUAKE ENGINEERING

Displacement-Based Seismic Design

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Force-Based Seismic Design (codes)

- Although the structure is designed to yield during the design earthquake, only the elastic part of the response, up to yield, is examined. The analysis is based on the corresponding secant stiffness.
- The design loads are determined by dividing the seismic loads that would have been developed to the equivalent linear system by the behaviour factor, q.
- A unique value of q is considered for the whole structure, which does not reflect the real response.
- In order to satisfy the non-collapse criterion, capacity design rules and proper detailing are applied, which lead to safe and rather conservative design. However, the real deformation of the structure (displacements) is usually underestimated.

Displacement-Based Design (DBD)

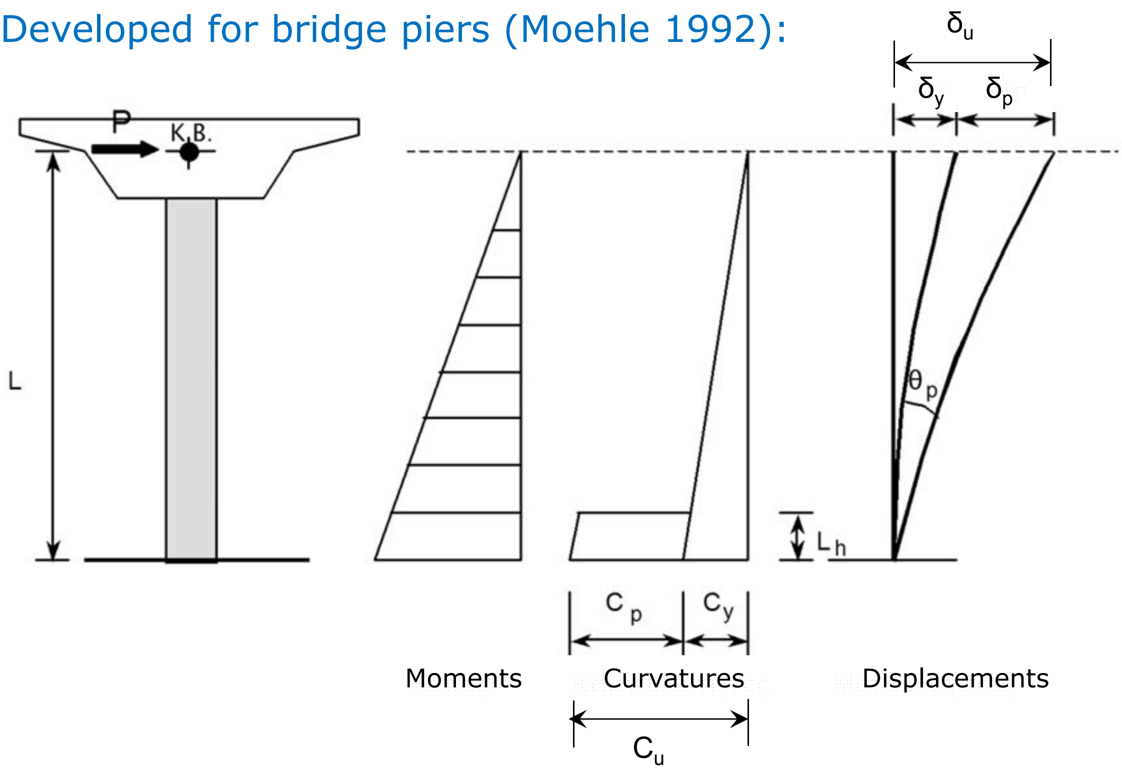
- The real deformation of each member of the structure is examined.
- Two approaches:
  - Method A

    Check of an already pre-designed structure and make improvements (increase dimensions of cross section) only to members that have problems.
  - Method B

    Design from the beginning the structure for a certain displacement (Direct Displacement-Based Design - DDBD). The design displacement is usually determined by serviceability or ultimate capacity considerations.

A1. SDOF Structures

Developed for bridge piers (Moehle 1992):



A1. SDOF Structures

$$\delta_u = \delta_y + (c_u - c_y) \cdot L_h \cdot \left(L - \frac{L_h}{2}\right)$$

where: $\delta_y = \dfrac{c_y \cdot L^2}{3}$ = yield displacement

c_y = yield curvature

L_h = length of plastic hinge

This relation yields to:

$$c_u = c_y + \frac{\delta_u - \delta_y}{L_h \cdot \left(L - \frac{L_h}{2}\right)}$$

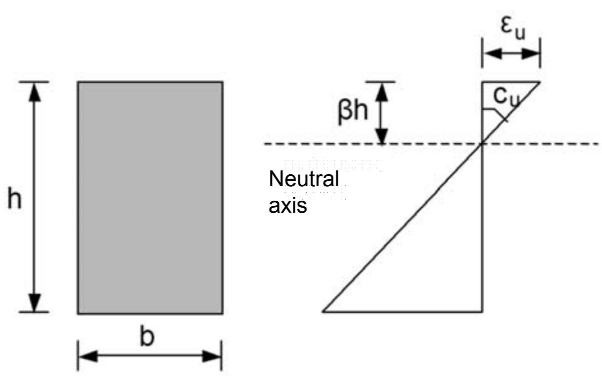
which relates the ultimate curvature c_u with the ultimate displacement δ_u.

A1. SDOF Structures

But c_u is directly related to critical strains of the cross section.

For example, the strain ε_u of the edge fiber can be written as:

$$c_u = \frac{\varepsilon_u}{\beta h}$$



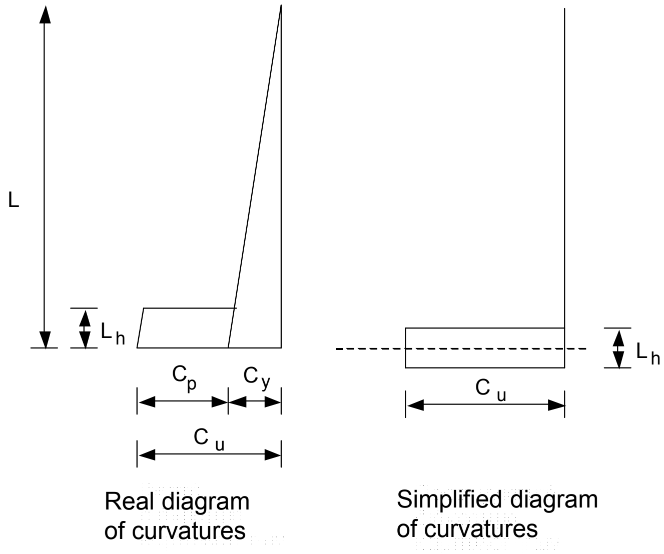
Thus, the ultimate displacement, δ_u, can be associated with the critical strains of the cross section of the plastic hinge.

A1. SDOF Structures

Simplification for:

- C_y small compared to $C_u \rightarrow C_y \approx 0$
- $L - L_h/2 \approx L$
- $L_h \approx h/2$

$$\frac{\delta_u}{L} = \frac{\varepsilon_u}{2\beta}$$



A1. SDOF Structures

Step 1

Calculate the effective period, T_{eff}, using the secant stiffness at the theoretical yield point.

Step 2

Calculate the ultimate displacement, δ_u, assuming that the equal displacement rule holds, i.e. from the elastic response spectrum for $T=T_{eff}$ and $\zeta=5\%$.

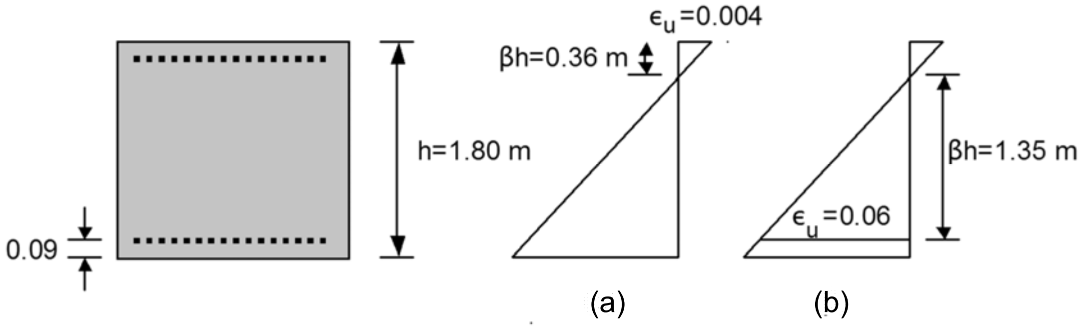
Step 3

Calculate the corresponding ultimate curvature, c_u.

Step 4

Calculate the corresponding critical strains, ε_u, and check if they are acceptable.

A1. SDOF Structures



a) If the critical check concerns the compression of the concrete:

Let maxε_u = 4‰. Also, βh = 0.36 m, β = 0.36/1.80 = 0.2.

$$\frac{\delta_u}{L} = \frac{0.004}{0.2 \times 2} = 0.01$$

b) If the critical check concerns the tension of the steel:

Let maxε_u = 6%. Also, βh = 1.35 m, β = 1.35/1.8 = 0.75.

$$\frac{\delta_u}{L} = \frac{0.06}{0.75 \times 2} = 0.04$$

A2. Multi-storey Buildings

Panagiotakos & Fardis (1996)

- Pre-dimensioning
  - Suggested to be based on the serviceability earthquake (can be taken equal to 40-50% of the design earthquake, depending on the importance) using the Force-Based design.
  - Elastic analysis (q=1) with simplified methods (e.g. lateral force method of EC8). However, the stiffness corresponding to the cracked sections must be used (can be taken equal to 25% of the uncracked).
  - The period can be determined from the Rayleigh quotient:

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^{n} m_i \delta_i^2}{\sum_{i=1}^{n} P_i \delta_i}} \quad \text{where} \quad P_i = V_o \frac{m_i \cdot z_i}{\sum_{j=1}^{n} m_j \cdot z_j}$$

and δ_i = static displacement of the i^{th} floor due to loads P_i.

A2. Multi-storey Buildings

Step 1

Calculate the required reinforcement of the beams (spans and supports) and of the columns and walls at their base (foundation level) for the most adverse combination:

- $1.35\ G + 1.5\ Q$ (non-seismic combination); or
- $G + \psi_2\ Q + E_s$ (seismic combination, E_s = serviceability earthquake).

Step 2

Calculate the required reinforcement of the columns and the walls at the rest of their height using the capacity design procedure.

Step 3

Calculate the required shear reinforcement using capacity design.

A2. Multi-storey Buildings

Step 4

Calculate more accurately the effective period, T_{eff}, using the secant stiffness at the yield point of each section, taking under consideration the actual reinforcement. Approximate formulas from the literature can be employed.

Step 5

Calculate the displacement δ_e of the equivalent SDOF system:

$$\delta_e = S_{de}(T_{eff}, \zeta = 5\%).$$

In the following, the subscript "e" denotes the equivalent SDOF system.

A2. Multi-storey Buildings

Step 6

Calculate the rotations at the end sections of the structural members (beams, columns):

- The displacement at the i[th] storey can be written in terms of the displacement of the equivalent linear system as:

  $\delta_i = \lambda_i \cdot \delta_e$

  where λ_i is an unknown coefficient. Similarly,

  $a_i = \lambda_i \cdot a_e$

A2. Multi-storey Buildings

- Basic assumptions

  1. The total seismic load of the multi-storey structure is equal to the one of the equivalent SDOF structure (i.e. the base shear V_0 is the same):

$$P_e = m_e a_e$$

$$P_e = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} m_i a_i = \sum_{i=1}^{n} m_i \lambda_i a_e = a_e \sum_{i=1}^{n} m_i \lambda_i$$

$$\Rightarrow \quad m_e = \sum_{i=1}^{n} m_i \lambda_i$$

Also,

$$P_i = m_i a_i = m_i \lambda_i a_e = m_i \lambda_i \frac{P_e}{m_e} = P_e \frac{m_i \lambda_i}{\sum_{j=1}^{n} m_j \lambda_j}$$

$$\lambda_i = \frac{\delta_i}{\delta_e}$$

$$\Rightarrow \quad P_i = P_e \frac{m_i \delta_i}{\sum_{j=1}^{n} m_j \delta_j}$$

A2. Multi-storey Buildings

2. The total work of the seismic loads is the same in the multi-storey and the equivalent SDOF structure:

$$P_e \delta_e = \sum_{i=1}^{n} P_i \delta_i \quad \Rightarrow \quad \delta_e = \frac{\sum_{i=1}^{n} P_i \delta_i}{P_e} \qquad \text{or}$$

$$\delta_e = \frac{\sum_{i=1}^{n} m_i \delta_i^2}{\sum_{i=1}^{n} m_i \delta_i}$$

(from the previous relation between P_i and P_e)

A2. Multi-storey Buildings

Let us define φ_i as the ratio of the displacement at the i^{th} storey over the top displacement:

$$\varphi_i = \frac{\delta_i}{\delta_{top}}$$

Then,

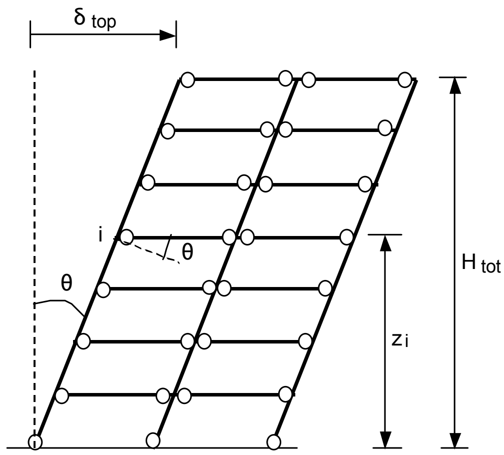
$$\delta_{top} = \frac{\sum_{i=1}^{n} m_i \varphi_i}{\sum_{i=1}^{n} m_i \varphi_i^2} \delta_e$$

- The coefficients φ_i denote the deformation of the structure at the maximum displacement and are not known a priori.
- If the expected plastic deformations are significant, the analysis can be performed considering the following two "extreme" cases:

A2. Multi-storey Buildings

Case 1

In typical building, and due to the capacity design that has been performed, plastic hinges are expected to form at the base of the columns and the walls and at the ends of the beams.



Then, assuming that the plastic deformation is significantly larger than the elastic one:

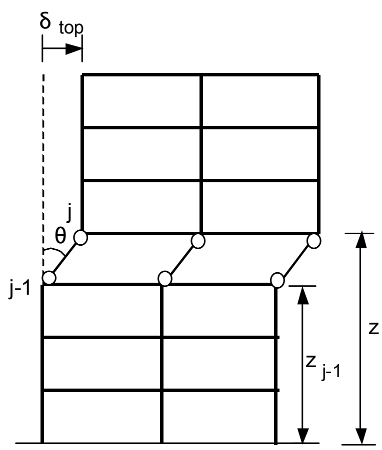
$$\varphi_i = \frac{z_i}{H_{tot}} \quad \Rightarrow \quad \frac{\delta_{top}}{H_{tot}} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i z_i^2} \delta_e$$

and $\theta = \dfrac{\delta_{top}}{H_{tot}}$

A2. Multi-storey Buildings

Case 2

In case that it is expected that a "soft storey" mechanism will be developed at the j^{th} floor in the ultimate deformation, it can be set:



$$\varphi_i = \begin{cases} 0 & \text{for } i = 1 \ldots (j-1) \\ 1 & \text{for } i = j \ldots n \end{cases}$$

In that case:

$\delta_{top} = \delta_e$ and

$$\theta = \frac{\delta_{top}}{z_j - z_{j-1}}$$

A2. Multi-storey Buildings

Intermediate case

For less extreme cases, it can be assumed that the relation of the storey displacements for inelastic response is similar to the one for elastic response. Then the coefficients φ_i can be assumed equal to the ones that correspond to the elastic displacements up to yield:

$$\varphi_i = \frac{\delta_i^{el}}{\delta_{top}^{el}}$$

The values of the 1st eigenmode can be used in that case as an approximation.

A2. Multi-storey Buildings

Inelastic storey displacements

After the values of φ_i have been determined with one of the above-mentioned methods, the inelastic storey displacements can be determined from the top displacement:

$$\delta_i = \delta_e \cdot \varphi_i \cdot \frac{\sum_{j=1}^{n} m_j \varphi_j}{\sum_{j=1}^{n} m_j \varphi_j^2}$$

and the storey drifts are:

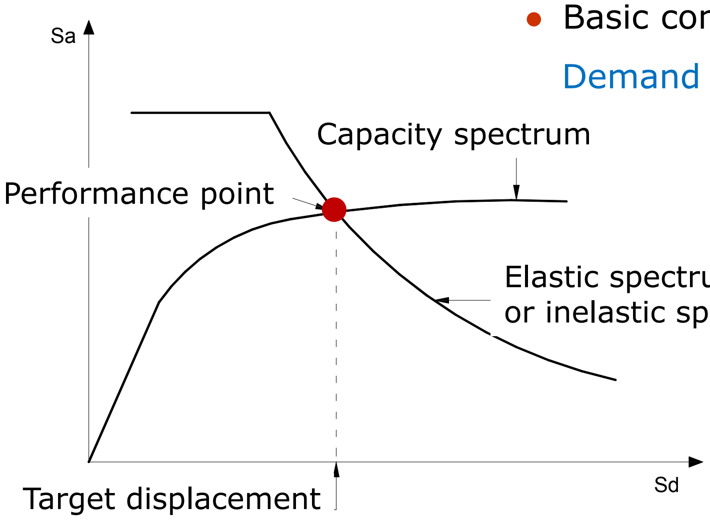
$$\theta_i = \frac{\delta_{i+1} - \delta_i}{z_{i+1} - z_i}$$

Step 6

Check that the above required rotations θ_i at the ends of the structural elements are within the allowable limits.

Use Pushover analysis

- Can be used for the determination of the maximum displacements of Method A and the plastic deformation at each cross section.
- The target displacement of the equivalent SDOF system is needed.



- Basic concept:

  Demand = Capacity

Equivalent SDOF system

- Storey distribution of seismic loads

$$F_i = V \frac{m_i \varphi_i}{\sum_j m_j \varphi_j}$$

where:
- ♦ V = base shear
- ♦ φ_i = assumed distribution of storey displacements with φ_{top}=1.

- Equivalence between forces and displacements

$$Q = \Gamma \cdot Q^*$$

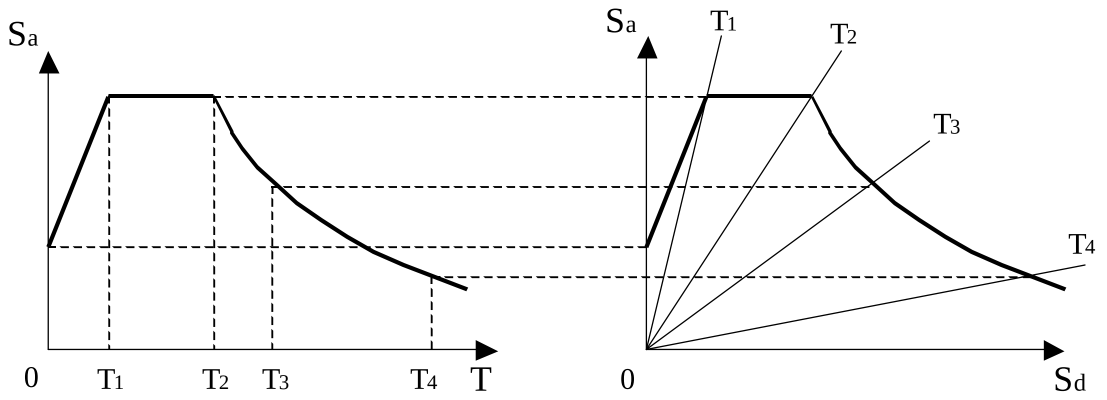
where:
- ♦ Q = force or displacement of the MDOF system
- ♦ Q^*= corresponding force or displacement of the SDOF system
- ♦ Γ = participation factor

$$\Gamma = \frac{\sum m_i \varphi_i}{\sum m_i \varphi_i^2}$$

Calculation of max displacements (ATC-40)

Step 1

Change elastic design spectrum to ADRS format



$$S_a = \frac{4\pi^2}{T^2} \cdot S_d$$

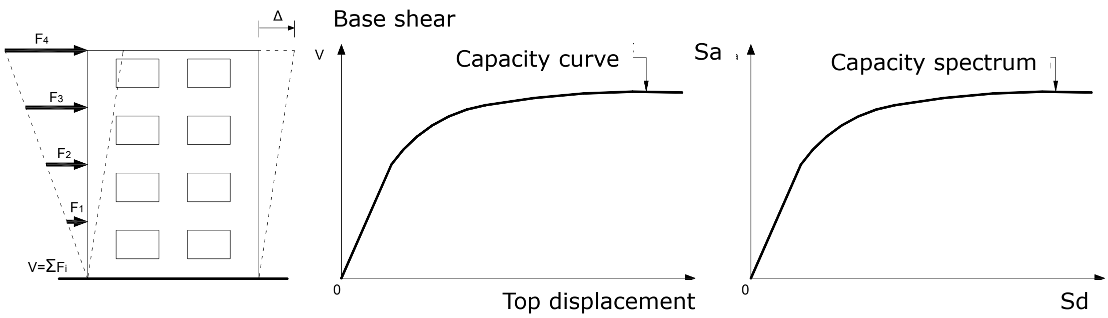
$$S_d = \frac{T^2}{4\pi^2} \cdot S_a$$

$$T = 2\pi\sqrt{\frac{S_d}{S_a}}$$

Calculation of max displacements (ATC-40)

Step 2

Calculate the capacity curve and change it to capacity spectrum



$$S_a = \frac{V}{\alpha \cdot m_{tot}}$$

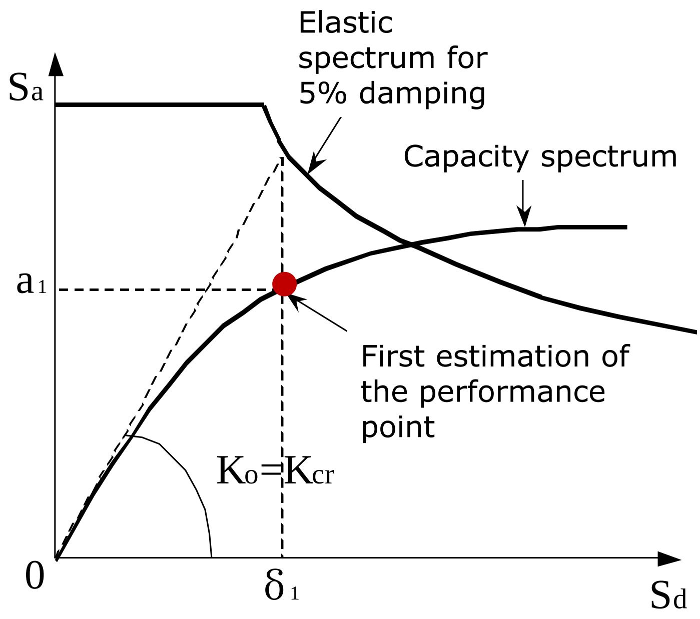
$$S_d = \frac{\Delta}{\Gamma}$$

- V = base shear
- Δ = top displacement
- m_{tot} = total mass of MDOF
- $\alpha = \frac{\left[\sum m_i \varphi_i\right]^2}{m_{tot} \cdot \sum m_i \varphi_i^2} = \frac{\Gamma \cdot \sum m_i \varphi_i}{m_{tot}} = \Gamma \frac{m^*}{m_{tot}}$
- $m^* = \sum m_i \varphi_i$

Calculation of max displacements (ATC-40)

Step 3

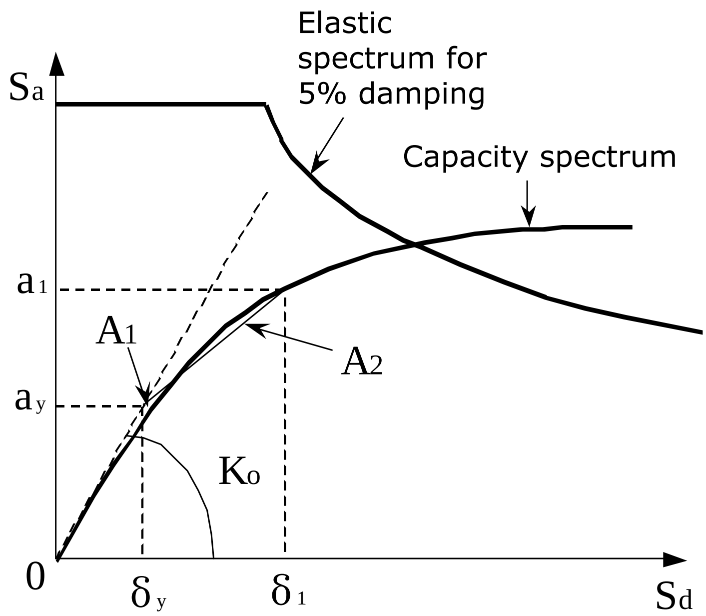
First estimation of the performance point



Calculation of max displacements (ATC-40)

Step 4

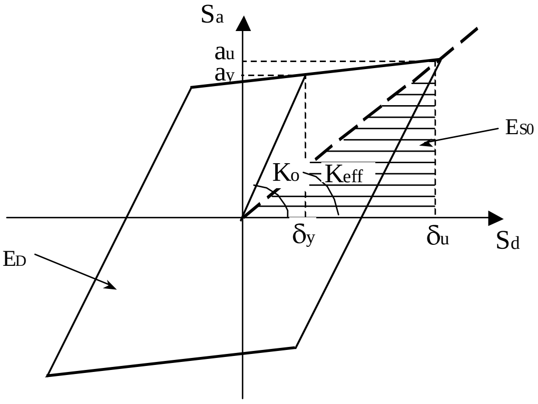
Bilinear representation of capacity spectrum



Calculation of max displacements (ATC-40)

Step 5

Calculation of effective damping



$$\zeta_{hyst} = \frac{1}{4\pi} \cdot \frac{E_D}{E_{S0}}$$

$$\zeta_{eff}\,(\%) = 5 + \frac{63.7 \cdot \kappa \cdot (a_y \cdot \delta_1 - \delta_y \cdot a_1)}{a_1 \cdot \delta_1}$$

Type of structure	ζ_{hyst} (%)	κ
A	≤ 16.25	1.00
A	> 16.25	$1.13 - \frac{0.51 \cdot (a_y \cdot \delta_u - \delta_y \cdot a_u)}{a_u \cdot \delta_u}$
B	< 25	0.67
B	> 25	$0.845 - \frac{0.446 \cdot (a_y \cdot \delta_u - \delta_y \cdot a_u)}{a_u \cdot \delta_u}$
C	All values	0.33

Earthquake duration	New structures of good seismic performance	Structures of medium seismic performance	Structures of poor seismic performance
Short (close to the epicenter)	A	B	C
Long (away from the epicenter)	B	C	C

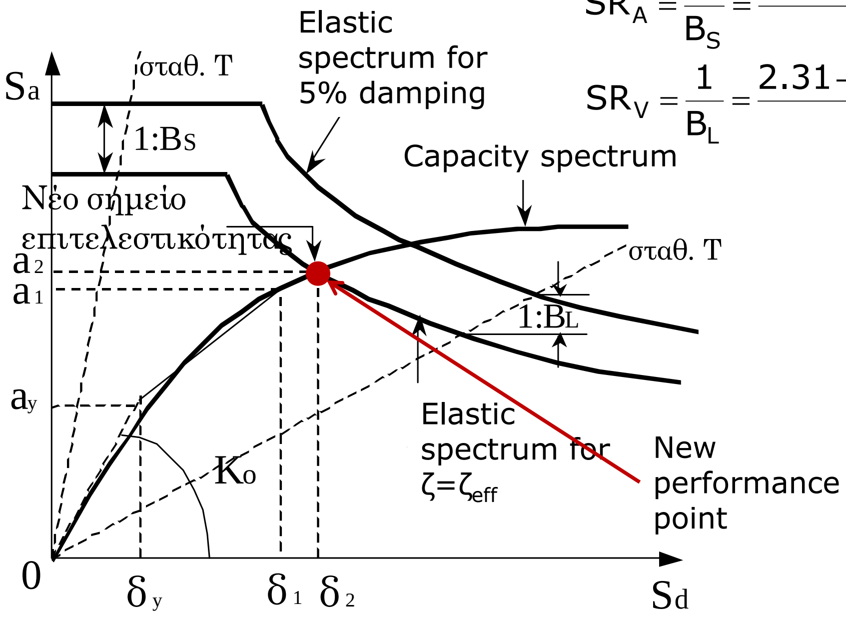
Calculation of max displacements (ATC-40)

Step 5 (cont.)

Design spectrum for ζ = ζ_{eff}

$$SR_A = \frac{1}{B_S} = \frac{3.21 - 0.68 \cdot \ln\zeta_{eff}}{2.12} \geq SR_{A,min}$$

$$SR_V = \frac{1}{B_L} = \frac{2.31 - 0.41 \cdot \ln\zeta_{eff}}{1.65} \geq SR_{V,min}$$



Calculation of max displacements (ATC-40)

Step 5 (cont.)

Minimum values of $SR_{A,min}$, $SR_{V,min}$

Type of structure	$SR_{A,min}$	$SR_{V,min}$
A	0.33	0.50
B	0.44	0.56
C	0.56	0.67

Step 6

Check convergence

- If $0.95 \cdot \delta_1 < \delta_2 < 1.05 \cdot \delta_1$ O.K.
- If not, repeat from step 5

Calculation of max displacements (ATC-40)

Step 7

Deformation of MDOF system and checks

- Calculate top displacement: $\Delta = \Gamma \cdot S_d$
- Perform pushover analysis up to top displacement equal to Δ and calculate the ultimate rotations at the joints.
- Check rotations of sections according to DBD.

Method B: DDB design

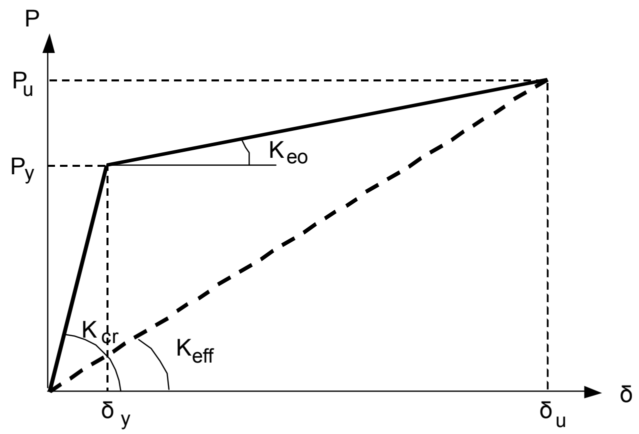
- The design is based on the target displacement, δ_u.
- The target displacement is defined by
  - Serviceability criteria; or
  - Ultimate capacity criteria.
- The "substitute structure" is used:
  - Effective stiffness at the maximum displacement
  - Effective damping considering the hysteretic energy dissipation.
- The design is not based on the displacement ductility.

B1. SDOF Structures

Developed for bridge piers (Kowalsky, Priestley & Macrae, 1995)

The method is based on the "substitute structure"

- Effective stiffness, K_{eff}
- Effective damping, ζ_{eff}
- Effective period, T_{eff}



B1. SDOF Structures

Step 1

Definition of the design parameters:

- m = mass
- L = height of pier
- f_c = concrete grade
- f_y = yield stress of reinforcement
- E = Young's modulus of elasticity
- δ_u = target displacement.
- An elastic displacement response spectrum must be available for various values of damping

B1. SDOF Structures

Step 2

Determination of the substitute structure:

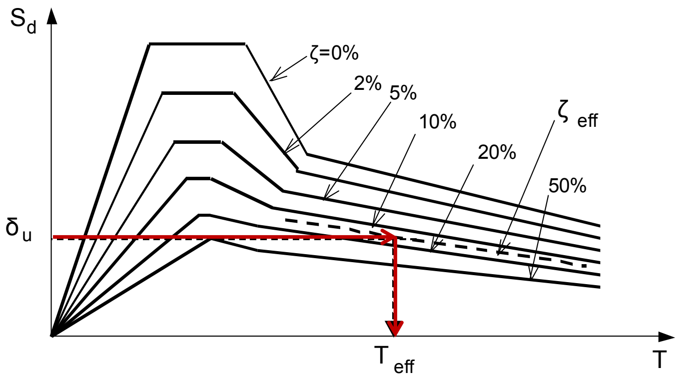
- Guess an initial value for the yield displacement, δ_y.
  This value is arbitrary and will be used as the first approximation. Suggestion: $\delta_y = 0.005 \cdot L$.
- Calculate the corresponding ductility $\mu = \delta_u / \delta_y$.
- Calculate the corresponding effective damping, ζ_{eff}.
- Effective damping consists of two terms:
  - the viscous damping, which is assumed equal to the one for elastic behaviour (5% for RC structures); and
  - the hysteretic damping, which can be estimated from the ductility using relations from the literature. Such a relation, based on the Takeda model, is suggested by the authors:

$$\zeta_{eff} = 0.05 + \frac{1 - \frac{0.95}{\sqrt{\mu}} - 0.05\sqrt{\mu}}{\pi}$$

B1. SDOF Structures

Step 2 (cont'd)

- Calculate the effective period of the substitute structure from the value of displacement spectrum that corresponds to $S_d=\delta_u$ and $\zeta=\zeta_{eff}$.



- Calculate the effective stiffness of the substitute structure:

$$K_{eff} = \frac{4\pi^2 m}{T_{eff}^2}$$

B1. SDOF Structures

Step 3

Calculate the design actions for the dimensioning of the pier.

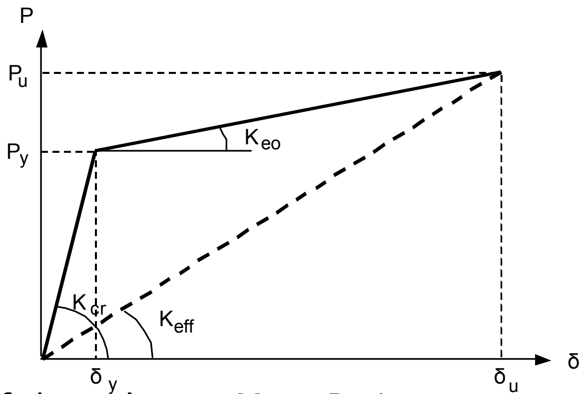
- Seismic force at maximum displacement: $P_u = K_{eff} \cdot \delta_u$.
- Seismic force to be used for the design of the pier: $P_d=P_y$:

$$P_u = P_d + r \cdot K_{cr} \cdot (\delta_u - \delta_y)$$

where $r=K_{eo}/K_{cr}$

$$\Rightarrow \quad P_u = P_d + r \cdot P_d \cdot (\mu - 1)$$

$$\Rightarrow \quad P_d = \frac{P_u}{r\mu - r + 1}$$



- Design moment at the base of the column: $M_d = P_d \cdot L$

B1. SDOF Structures

Step 4

- Choose the necessary cross section and calculate the required reinforcement at the base of the pier for the moment M_d and the axial load N.
- Calculate the elastic stiffness K_{cr} from the estimated moment of inertia of the cracked section, I_{cr}, using the actual reinforcement. Relations from the literature can be used

  For circular cross section:

$$\frac{I_{cr}}{I_g} = 0.21 + 12 \cdot \rho + \left[0.1 + 205 \cdot (0.05 - \rho)^2\right] \frac{N}{f_c \cdot A_g}$$

ρ = percentage of reinforcement

I_g = geometric moment of inertia

A_g = geometric area of section

N = axial force.

Then: $K_{cr} = \frac{3EI_{cr}}{L^3}$

B1. SDOF Structures

Step 5 (optional)

Check whether the selected section leads to reasonable results.

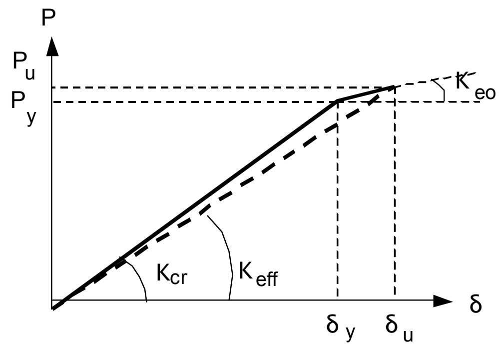
- "Elastic" period: $T_{cr} = 2\pi\sqrt{\frac{m}{K_{cr}}}$
- "Plastic" period: $T_{eo} = 2\pi\sqrt{\frac{m}{K_{eo}}}$ where $K_{eo} = r \cdot K_{cr}$

In general: $T_{cr} < T_{eff} < T_{eo}$

- If T_{eff} <u>is not</u> close to the limits, the design is correct and we proceed to the following step.
- If T_{eff} is close to T_{cr}, the response is close to the elastic. In this case, the design will lead to large amount of reinforcement and small required ductility.
- If T_{eff} is close to T_{eo}, the design will lead to small amount of reinforcement and large required ductility.

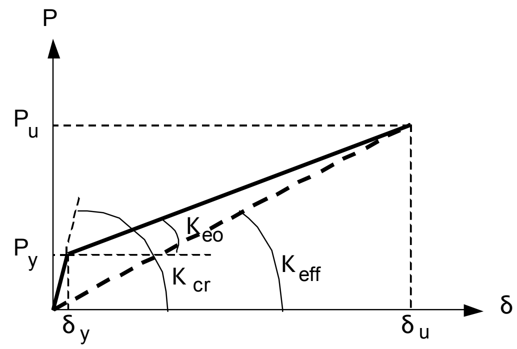
B1. SDOF Structures

Step 5 (cont'd)



T_{eff} close to T_{cr}

Suggested action:
Increase dimensions of cross section (decrease δ_y)



T_{eff} close to T_{eo}

Suggested action:
Decrease dimensions of cross section (increase δ_y)

B1. SDOF Structures

Step 6

Check convergence.

- Calculate new yield displacement: $\delta'_y = \dfrac{P_d}{K_{cr}}$
- If $\left|\delta'_y - \delta_y\right| \le \varepsilon \cdot \delta'_y$, where ε = required accuracy (e.g. ε=5%), stop iterations. Otherwise, repeat procedure from step 2 using δ_y' as the yield displacement.

Step 7

After convergence is achieved, calculate the horizontal reinforcement (stirrups) to guarantee capacity of the section to develop the required curvature ductility:

$$\mu_C = 1 + \frac{\mu_\Delta - 1}{3 \cdot (L_h / L) \cdot (1 - 0.5 L_h / L)}$$

where L_h = length of plastic hinge.

B1. SDOF Structures

Example

Step 1

- $m = 500$ Mgr
- $L = 5.0$ m
- $f_c = 40$ MPa
- $f_y = 400$ MPa
- $E = 31.62$ GPa
- $\delta_u/L = 3\% \Rightarrow \delta_u = 0.03 \times 5.0 = 0.15$ m.

B1. SDOF Structures

Step 2

- $\delta_y = 0.005 \times L = 0.025$ m
- $\mu = \delta_u / \delta_y = 0.150 / 0.025 = 6.0$
- $\zeta_{eff} = 0.05 + \dfrac{1 - \dfrac{0.95}{\sqrt{6.0}} - 0.05 \times \sqrt{6.0}}{\pi} = 0.206$
- Let $T_{eff} = 1.627$ sec, as derived from response spectrum for:

  $S_e = \delta_u = 0.15$ m and $\zeta = \zeta_{eff} = 0.206$
- $K_{eff} = \dfrac{4\pi^2 m}{T_{eff}^2} = \dfrac{4\pi^2 \times 500}{1.627^2} = 7457\ \text{KN/m}$

B1. SDOF Structures

Step 3

- $P_u = K_{eff} \times \delta_u = 1118$ KN
- $M_u = 1118 \times 5.0 = 5589$ KNm
- $P_d = \dfrac{P_u}{r\mu - r + 1} = \dfrac{1118}{0.05 \times 6.0 - 0.05 + 1} = 894.4\ \text{KN}$ (for r = 5%)
- $M_d = 894.4 \times 5.0 = 4472$ KNm

Step 4

- Circular section with diameter D = 1.1 m. Let N = m·g = 5000 KN
- Let ρ=1.76% for N = 5000 KN and M = 4472 KNm
- $\dfrac{I_{cr}}{I_g} = 0.21 + 12 \times 0.0176 + \left[0.1 + 205 \cdot (0.05 - 0.0176)^2\right] \dfrac{500 \times 10}{40 \cdot 10^3 \cdot \pi \cdot \dfrac{1.1^2}{4}} = 0.463$
- $I_g = \pi \cdot D^4/64 \Rightarrow I_{cr} = 0.033\ m^4 \Rightarrow I_{cr} = 0.033\ m^4$
- $K_{cr} = \dfrac{3 \cdot E \cdot I_{cr}}{L^3} = \dfrac{3 \times 31.62 \times 10^6 \times 0.033}{5.0^3} = 25233\ \text{KN/m}$

B1. SDOF Structures

Step 5

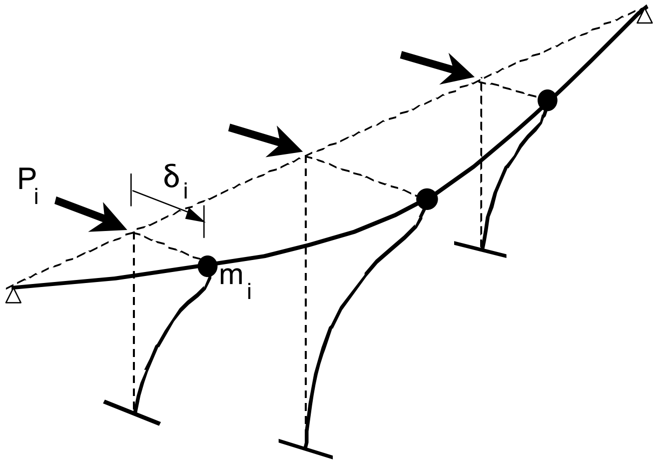
- $T_{cr} = 2\pi \sqrt{\dfrac{500}{25233}} = 0.884\ \text{sec}$
- $K_{eo} = 0.05 \times 25233 = 1262$ KN/m
- $T_{eo} = 2\pi \sqrt{\dfrac{500}{1262}} = 3.954\ \text{sec}$
- Since 0.884 < 1.627 < 3.954, we proceed to the following step.

Step 6

- $\delta'_y = \dfrac{P_d}{K_{cr}} = \dfrac{894.4}{25233} = 0.035\ \text{m}$
- Initial guess: δ_y = 0.025 m. No convergence achieved ⇒ repeat procedure.

B2. MDOF structures

Kalvi & Kingsley (1995) for bridges with many piers



The method starts with an initial guess for the displacements, which is improved through iterations.

B2. MDOF Structures

Equivalent DSOF system

- K_e = stiffness of equivalent SDOF
- ζ_e = damping of equivalent SDOF
- δ_e = displacement of equivalent SDOF
- P_e = seismic force of equivalent SDOF

Assume that the displacements, δ_i, of the MDOF system can be determined from the displacement of the equivalent SDOF, δ_e, through appropriate coefficients φ_i:

$$\delta_i = \varphi_i \cdot \delta_e$$

Assume that the accelerations follow the same distribution:

$$a_i = \varphi_i \cdot a_e$$

B2. MDOF Structures

Equivalent DSOF system (cont'd)

- Equal seismic force in the two systems:

$$P_e = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} m_i a_i = \sum_{i=1}^{n} m_i \varphi_i a_e = a_e \sum_{i=1}^{n} m_i \varphi_i$$

But, $P_e = m_e a_e$, thus $m_e = \sum_{i=1}^{n} m_i \varphi_i$

Also, $P_i = m_i a_i = m_i \varphi_i a_e = m_i \varphi_i \dfrac{P_e}{m_e} = P_e \dfrac{m_i \varphi_i}{\sum_{j=1}^{n} m_j \varphi_j}$

and $\varphi_i = \dfrac{\delta_i}{\delta_e}$

therefore

$$P_i = P_e \frac{m_i \delta_i}{\sum_{j=1}^{n} m_j \delta_j}$$

B2. MDOF Structures

Equivalent DSOF system (cont'd)

- Equal work of the seismic forces in the two systems:

$$P_e \delta_e = \sum_{i=1}^{n} P_i \delta_i \quad \Rightarrow \quad \delta_e = \frac{\sum_{i=1}^{n} P_i \delta_i}{P_e} = \frac{P_e \sum_{i=1}^{n} m_i \delta_i^2}{P_e \sum_{i=1}^{n} m_i \delta_i} \quad \Rightarrow \quad \delta_e = \frac{\sum_{i=1}^{n} m_i \delta_i^2}{\sum_{i=1}^{n} m_i \delta_i}$$

Properties of equivalent SDOF system

- Stiffness of equivalent SDOF system:

$$K_e = \frac{P_e}{\delta_e}$$

- The damping of the equivalent SDOF system, ζ_e, is calculated from the damping of each pier, ζ_i, which depends on the ductility μ_i that is developed at the pier and can be derived using relations from the literature.

B2. MDOF Structures

Step 1

- Define the target displacement $\delta_{i,u}$ of each pier.
- In order to have similar damage in all piers, we can assume same drifts: δ_i/H_i. Thus:

$$\delta_{i,u} = \text{drift} \times H_i$$

- Make an initial estimation of the yield displacements of the piers, $\delta_{i,y}$ and calculate the ductility for each pier:

$$\mu_i = \frac{\delta_{i,u}}{\delta_{i,y}}$$

- Calculate the equivalent damping, ζ_e, for each pier from the corresponding ductility (similarly to the SDOF systems).

B2. MDOF Structures

Step 2

- Derive the parameters of the equivalent SDOF structure:
  - Calculate the total effective damping, ζ_e, combining the damping, ζ_i, of the piers.
  - Calculate the displacement of the equivalent SDOF system, δ_e, from the displacements $\delta_{i,u}$ of the piers:

$$\delta_e = \frac{\sum_{i=1}^{n} m_i \delta_{i,u}^2}{\sum_{i=1}^{n} m_i \delta_{i,u}}$$

  - Calculate the period of the equivalent SDOF system, T_e.
  - Calculate the coefficients $\varphi_i = \delta_{i,u}/\delta_e$
  - Calculate the mass of the equivalent SDOF system:

$$m_e = \sum_{i=1}^{n} m_i \varphi_i$$

B2. MDOF Structures

Step 2

- Then:

$$K_e = \frac{4\pi^2 \cdot m_e}{T_e^2}$$

$$P_e = K_e \cdot \delta_e$$

and the forces P_i at the top of each pier can be derived:

$$P_i = P_e \frac{m_i \delta_i}{\sum_{j=1}^{n} m_j \delta_j}$$

B2. MDOF Structures

Step 3

- Static analysis of the system for the forces P_i and calculation of the displacements and the forces that are developed. The inelastic response must be considered, e.g. nonlinear static (pushover) analysis or elastic analysis with reduced stiffness for the piers.
- Calculate the accuracy ε_i of the obtained displacements δ_i of the piers, similarly to the SDOF systems.

Step 4: checks

- If satisfactory accuracy <u>is not</u> achieved, change dimensions of cross sections or reinforcement of piers and repeat procedure.
- If satisfactory accuracy <u>is</u> achieved, verify that the piers can bear the loads.

Comparison of the two methods

Method A (DBD)

- Check the capacity of a pre-designed structure to deform with the required plastic rotations at critical sections.
- Increase dimensions of cross sections that are insufficient.
- The analysis is based on the linear system that corresponds to the yield stiffness and 5% damping.

Method B (DDBD)

- Directly design the structure for the target displacement.
- The analysis is based on the substitute linear system that corresponds to the effective stiffness at the max displacement and the equivalent effective damping.

Problems in the application

- Accurate calculation of the real displacements is needed.
- The use of displacement design spectra is problematic, due to many uncertainties.
- The distribution of the deformation at the maximum displacement of MDOF systems is needed.
- The plastic rotation capacity of a section is not easy to be calculated (empirical formulas exist for simple cross sections only).
- Method B (DDBD) might not converge in some cases.