Control System Design - PID Control

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Control System Design - PID Control

Theme: The most common controller.

Introduction

- PID control is widely used in all areas where control is applied (solves $\approx 90\%$ of all control problems)
- A PID controller is more than meets the eye
- The tuning adventure (Tore+KJ)

  Telemetric, Eurotherm 1979
  Adaptive control and auto-tuning
  STU, patents, NAF (Sune Larsson) SDM20
  Satt Control, Alfa Laval Automation, ABB
  Fisher Control, Emerson 1979–
  Research and the PID books 1988, 1995, 2006, ?
  Interactive Learning Modules Guzman, Dormido
  http://aer.ual.es/ilm/

- Revival of PID Control - publications, conferences
- Technology transitions

  Pneumatic, mechanical,electric, electronic, computer

- Modeling: the FOTD model $P(s) = \frac{K}{1+sT} e^{-sL}$
- To PID or not to PID - that is the question

Predictions about PID Control

- 1982: The ASEA Novatune Team 1982 (Novatune is a useful general digital control law with adaptation):

  PID Control will soon be obsolete

- 1989: Conference on Model Predictive Control:

  Using a PI controller is like driving a car only looking at the rear view mirror: It will soon be replaced by Model Predictive Control.

- 2002: Desborough and Miller (Honeywell):

  Based on a survey of over 11 000 controllers in the refining, chemicals and pulp and paper industries, 98% of regulatory controllers utilise PID feedback

- Similar studies in Japan and Germany

PID is here to stay!

Typical Scenarios

- Process control
  - Standard distributed control system for 500-10000 loops
  - One control room, commissioning, tuning, operations, upgrading handeled by operators and instrument engineers
  - Loops are tuned and retuned at installation and during operation
  - Automatic tuning
- Equipment manufacturers
  - Automotive systems: emissions, cruise control, antiskid, ...
  - Motor drives, robots and motion control
  - Dedicated equipment for air conditioning
  - Controllers may be tuned based on models or by bumptests and empirical rules
  - Installation tuning and upgrading very different for different applications
- Tasks: regulation, command signal following

Entech Experience & Protuner Experiences

Bill Bialkowsk Entech - Canadian consulting company for pulp and paper industry *Average paper mill has 3000-5000 loops, 97% use PI the remaining 3% are PID, MPC, adaptive etc.*

- 50% works well, 25% ineffective, 25% dysfunctional

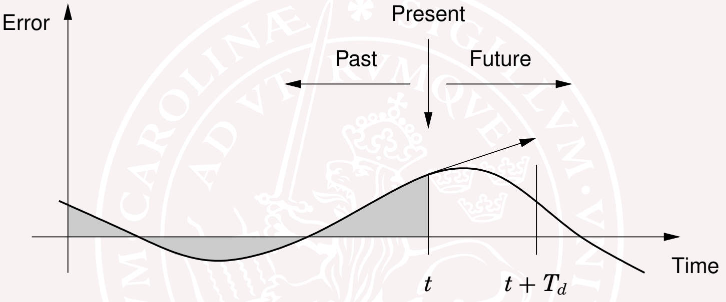
Major reasons why they don't work well

- Poor system design 20%
- Problems with valve, positioners, actuators 30%
- Bad tuning 30%

Process Performance is not as good as you think. D. Ender, Control Engineering 1993.

- More than 30% of installed controllers operate in manual
- More than 30% of the loops increase short term variability
- About 25% of the loops use default settings
- About 30% of the loops have equipment problems

PID versus More Advanced Controllers



$$u(t) = k_p e + k_i \int_0^t e(\tau)d\tau + k_d \frac{de}{dt}, \qquad T_d = k_d/k_p$$

- PID predicts by linear extrapolation, T_d prediction horizon
- Advanced controllers predict using a mathematical model

The Amazing Property of Integral Action

Consider a PI controller

$$u = ke + k_i \int_0^t e(\tau) d\tau$$

Assume that all signals converge to constant values $e(t) \to e_0$, $u(t) \to u_0$ and that $\int_0^t (e(\tau) - e_0) d\tau$ converges, then e_0 must be zero.

Proof: Assume $e_0 \neq 0$, then

$$u(t) = ke_0 + k_i \int_0^t e(\tau) d\tau = ke_0 + k_i \int_0^t \big(e(\tau) - e_0\big) d\tau + k_i e_0 t$$

The left hand side converges to a constant and the left hand side does not converge to a constant unless $e_0 = 0$, futhermore

$$u(\infty) = k_i \int_0^\infty \big(e(\tau) - e_0\big) d\tau$$

A controller with integral action will always give the correct steady state provided that a steady state exists. It *adapts* to changing disturbances. Integral action is sometimes even called *adaptive*.

Interactive Learning Modules

A series of interactive learning tools for PID control has been developed by Tore and KJ in collaboration with Control Groups in Spain (Jose-Luis Guzman Almeria, Sebastian Dormido Madrid), Yves Piquet (creator of Sysquake, a highly interactive version of Matlab). Executable modules for PC, Mac and Linux are available for free download from

http: www. http://aer.ual.es

PID Basics, PID Loop Shaping and PID Windup

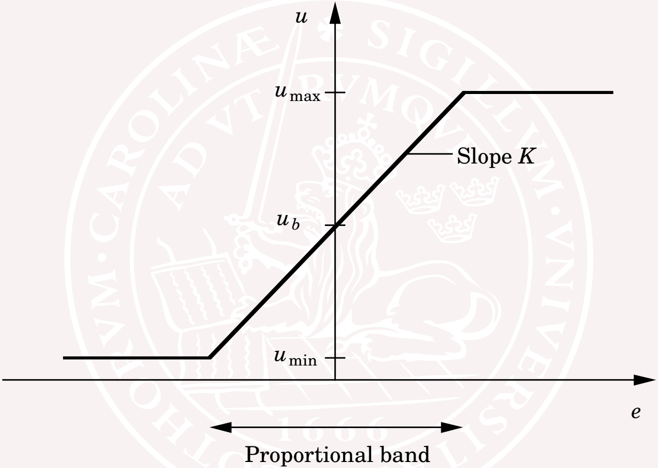
ILM Demo

Control System Design - PID Control

1. Introduction
2. The Controller
3. Performance and Robustness
4. Tuning Rules
5. Relay Auto-tuning
6. Limitations of PID Control
7. Summary

Theme: The most common controller.

Static Characteristics



P: Controller $u = Ke + u_b$, $\quad K$ gain, $\quad u_b$ bias or reset

A PID Algorithm

A PID controller is much more than

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}$$

We have to consider

- Filter for measurement noise
- Set point weigthing
- Actuator limitations:
- Rate limitations
- Integrator Windup
- Mode switches
- Bumpless parameter changes
- Computer implementation

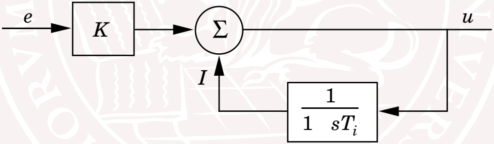
Dealing with these issues is a good introduction to practical aspects of any control algorithm.

Integral Action or Reset

It was noticed early that proportional control gives steady state error. A bias term u_b called **reset** was introduced to eliminate steady state errors.

$$u = k_p e + u_b$$

Bias was adjusted manually and then replaced by the following way to adjust bias automatically. (Filter out low frequency component of u and add it by positive feedback.)



A simple calculation gives $U(s) = k\left(1 + \frac{1}{sT_i}\right)$.

Voilá a PI controller!

Derivative Action

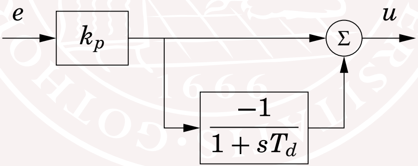
A derivative is the limit

$$\frac{dy}{dt} \approx \frac{y(t) - y(t-T)}{T}, \qquad sY(s) \approx \frac{1 - e^{-sT}}{T} Y(s)$$

Approximate the time delay by a low pass filter

$$e^{-sT} \approx \frac{1}{1+sT}, \qquad sY(s) \approx \frac{1}{T}\Big(1 - \frac{1}{1+sT}\Big) Y(s) = \frac{s}{1+sT} Y(s)$$

Block diagram



Is this how the body does it?

Parallel and Series Form PID

Parallel or non-interactive form:

$$C_{fb}(s) = k_p\left(1 + \frac{1}{sT_i} + sT_d\right) = \frac{k_p}{sT_i}(1 + sT_i + s^2T_iT_d)$$

with independent gain parametrization

$$C_{fb}(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}$$

Series form or interactive form:

$$\tilde{C}_{fb}(s) = \tilde{k}_p\left(1 + \frac{1}{s\tilde{T}_i}\right)(1 + s\tilde{T}_d) = \frac{\tilde{k}_p}{s\tilde{T}_i}\left(1 + s(\tilde{T}_i + \tilde{T}_d) + s^2\tilde{T}_i\tilde{T}_d\right)$$

Relations between coefficients

$$k_p = \tilde{k}_p\frac{\tilde{T}_i + \tilde{T}_d}{\tilde{T}_i}, \qquad T_i = \tilde{T}_i + \tilde{T}_d, \qquad T_d = \frac{\tilde{T}_i\tilde{T}_d}{\tilde{T}_i + \tilde{T}_d}$$

Parallel form is more general. Equivalence only if $T_i \geq 4T_d$.

Filtering

Filter only derivative part (absolute essential)

$$C_{fb}(s) = k\Big(1 + \frac{1}{sT_i} + \frac{sT_d}{1+sT_f}\Big) = k_p + \frac{k_i}{s} + \frac{k_d s}{1+sT_f}$$

Filter the measured signal (several advantages)

- Better noise attenuation and robustness due to high frequency roll-off
- Process dynamics can be augmented by filter and design can be made for an ideal PID

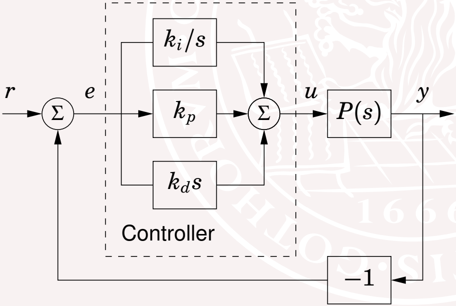
$$C_{fb}(s) = \frac{k_d s^2 + k_p s + k_i}{s(1+sT_f)} = k_i \frac{1 + sT_i + s^2 T_i T_d}{s(1+sT_f)}$$

$$C_{fb}(s) = \frac{k_d s^2 + k_p s + k_i}{s(1+sT_f + s^2 T_f^2/2)} = k_i \frac{1 + sT_i + s^2 T_i T_d}{s(1+sT_f + s^2 T_f^2/2)}$$

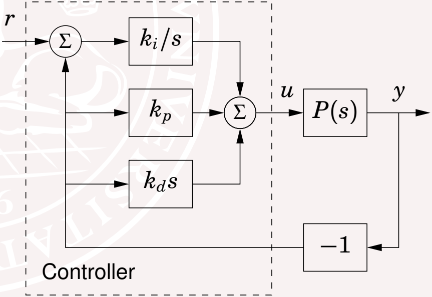
2DOF in PID Controllers

A 2DOF structure makes set-point response independent of disturbance response. Set-point weighting "Poor man's" 2DOF, allows a moderate adjustment of set point response through parameters b and c. Comment on practical controllers.

$$U(s) = k_p(bR(s) - Y(s)) + \frac{k_i}{s}(R(s) - Y(s)) + k_d s(cR(s) - Y(s))$$

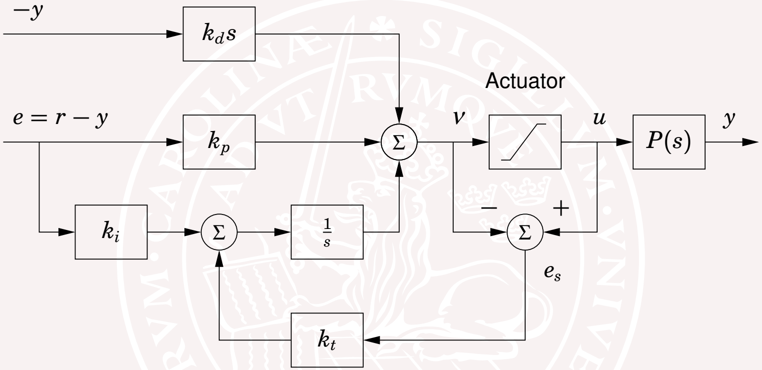


$b = 1 = 1$



$b = c = 0$

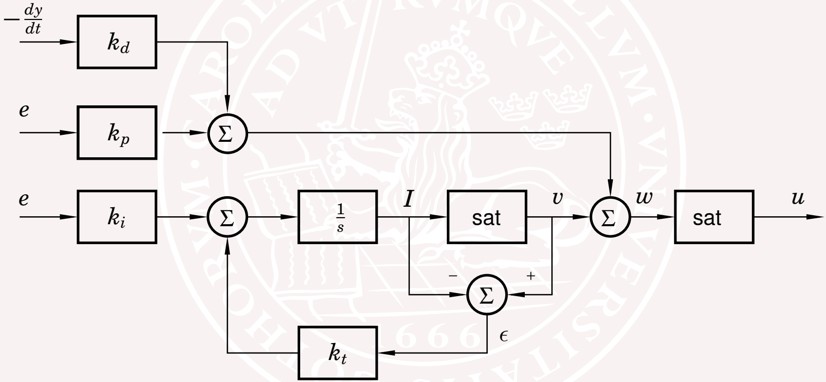
Avoiding Windup



A local feedback loop keeps integrator output close to the actuator limits. The gain k_t or the time constant $T_t = 1/k_t$ determines how quickly the integrator is reset. Intuitive Explanation - Cherchez l'erreur! Useful to replace k_t by a general transfer function.

Dow Chemical Version of Anti-windup

Many process industries (also in Sweden) had their own control departments and they developed their own systems based on standard computers. Dow, Monsanto and Billerud were good examples.



The integrator is reset based on its output and not based on the nominal control signal as in previous scheme.

The Proportional Band

The proportional band is the range of the error signal where the controller (actuator) does not saturate.

$$u = K(by_{sp} - y) + I - KT_d \frac{dy}{dt}.$$

Solving for the predicted process output

$$y_p = y + T_d \frac{dy}{dt},$$

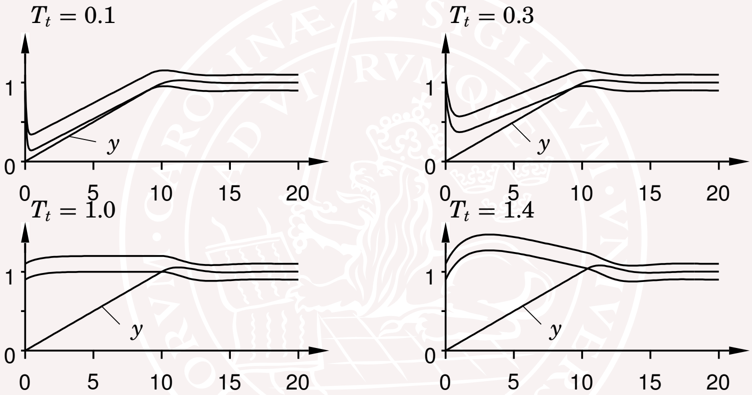
gives the proportional band (y_l, y_h) (also PB=100/K) as

$$y_l = by_{sp} + \frac{I - u_{\max}}{K} \qquad y_h = by_{sp} + \frac{I - u_{\min}}{K},$$

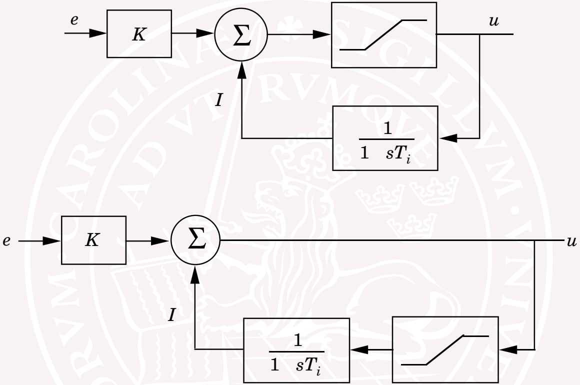
where $u_{\min}, u_{\max}$ are the values of the control signal for which the actuator saturates.

Anti-windup changes the proportional band.

Anti-windup and Proportional Band



Anti-windup in Series Implementation



- These schemes are natural for pneumatic controllers
- Have been used by Foxboro (Invensys) for a long time
- Tracking time constant $T_t = T_i$

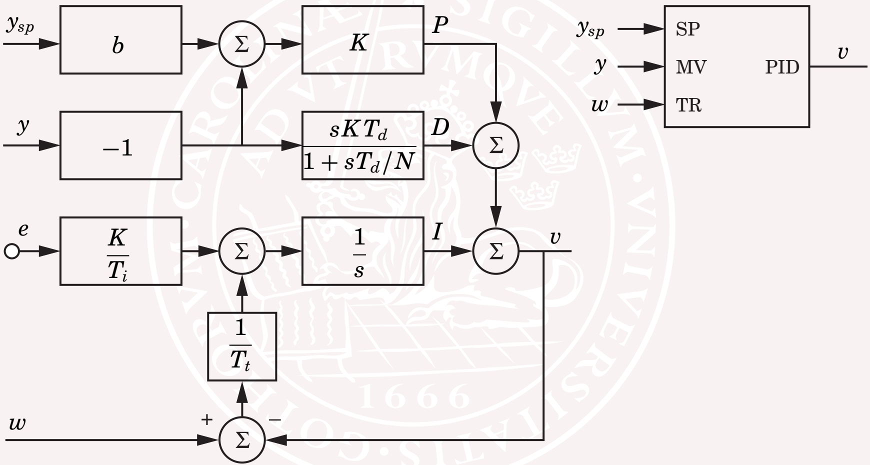
Manual and Automatic Control

- Most controllers have several modes

  Manual/automatic

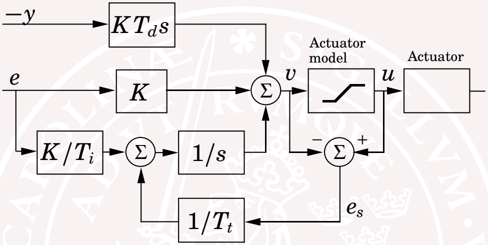
- In manual control the controllers output is adjusted manually by an operator often by increase/decrease buttons
- Mode switching is an important issue
- Switching transients should be avoided
- Easy to do if the same integrator is used for manual and automatic control

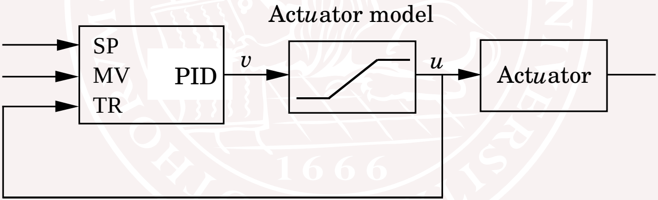
PID Controller with Tracking Mode



No tracking if $w = v$!

Anti-windup for Controller with Tracking Mode





- Notice that there is no tracking effect if $u = v$!
- The tracking input can be used in many other ways

Control System Design - PID Control

Theme: The most common controller.

Requirements

Disturbances

- Effect of feedback on disturbances
- Attenuate effects of load disturbances
- Moderate measurement noise injection

Robustness

- Reduce effects of process variations
- Reduce effects of modeling errors

Command signal response

- Follow command signals
- Architectures with two degrees of freedom (2DOF)

Tune for Load Disturbances

G. Shinskey Intech Letters 1993: *"The user should not test the loop using set-point changes if the set point is to remain constant most of the time. To tune for fast recovery from load changes, a load disturbance should be simulated by stepping the controller output in manual, and then transferring to auto. For lag-dominant processes, the two responses are markedly different."*

For typical process control problems

- Tune k_p, k_i, and k_d for load disturbances, filtering for measurement noise and β, and γ for set-points

$$u(t) = k_p(\beta r(t) - y(t)) + k_i \int_0^t (r(\tau) - y(\tau)) d\tau + k_d\left(\gamma \frac{dr}{dt} - \frac{dy_f}{dt}\right)$$

- The literature is often very misleading!

Motion control is different

Performance

Disturbance reduction by feedback

$$Y_{cl} = SY_{ol} = \frac{1}{1+PC} Y_{ol}$$

Load disturbance attenuation (typically low frequencies)

$$G_{yd} = \frac{P}{1+PC} \approx \frac{s}{k_i}, \qquad -G_{ud} = \frac{PC}{1+PC}$$

Measurement noise injection (typically high frequencies)

$$G_{xn} = \frac{PC}{1+PC}, \qquad -G_{un} = \frac{C}{1+PC} \approx C = G_f(k_p + \frac{k_i}{s} + k_d s)$$

Command signal following

$$G_{xr} = \frac{PG_f(\gamma k_d s^2 + \beta k_p s + k_i)}{s + PG_f(k_d s^2 + k_p s + k_i)}, G_{ur} = \frac{G_f(\gamma k_d s^2 + \beta k_p s + k_i)}{s + PG_f(k_d s^2 + k_p s + k_i)}$$

Criteria IE and IAE

Traditionally the criteria

$$IE = \int_0^\infty e(t)dt, \qquad IAE = \int_0^\infty |e(t)|dt, \qquad IE2 = \int_0^\infty e^2(t)dt$$

$$ITAE = \int_0^\infty t\,|e(t)|dt, \qquad QE = \int_0^\infty (e^2(t) + \rho u^2(t))dt$$

where e is the error for a unit step in the set point or the load disturbance have often been used to evaluate PID controllers

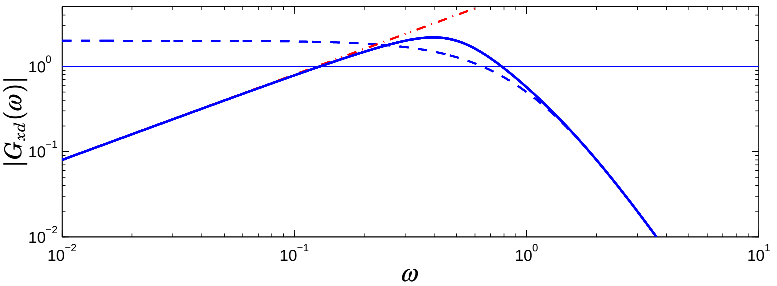
Notice that for a step u_0 in the load disturbance we have

$$u(\infty) = k_i \int_0^\infty e(t)dt$$

For a unit step disturbance we have $u(\infty) = 1$ and hence $IE = 1/k_i$. If the responses are well damped we have $IE \approx IAE$ and integral gain is then a measure of load disturbance attenuation.

Load Disturbance Attenuation

$P = 2(s+1)^{-4}$ PI: $k_p = 0.5$, $k_i = 0.25$

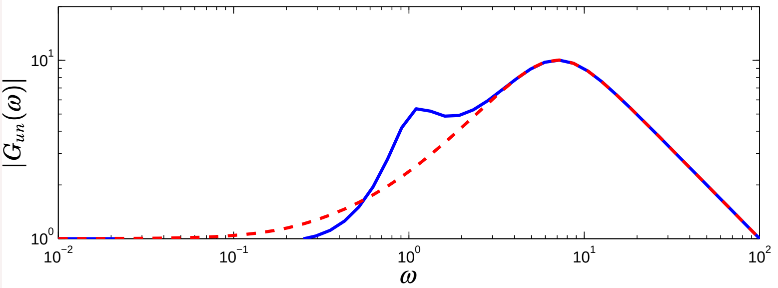


Approximations for low (red dashed) and high frequencies (blue dashed)

$$\frac{P}{1+PC} \approx \frac{1}{C} \approx \frac{s}{k_i}, \qquad \frac{P}{1+PC} \approx P$$

Measurement Noise Injection

$P = (s+1)^{-4}$ PID: $k_p = 1$, $k_i = 0.2$, $k_d = 1$, $T_d = 1$ $T_f = 0.2$



First order filter (dashed), second order filter (full)

$$-G_{un} = CS = \frac{k_d s^2 + k_p s + k_i}{s(1 + sT_f + (sT_f)^2/2)} \times \frac{s}{s + Kk_i}$$

Peaks of G_{un} at ω_{ms} and at $\omega \approx \sqrt{2}/T_f$

Robustness

Gain and phase margins g_m and φ_m

Maximum sensitivities $M_s = \max_\omega |S(i\omega)|, M_t = \max_\omega |T(i\omega)|$

$$H = \frac{1}{1+PC}\begin{pmatrix} 1 & P \\ C & PC \end{pmatrix} = \begin{pmatrix} \frac{1}{1+PC} & \frac{P}{1+PC} \\ \frac{C}{1+PC} & \frac{PC}{1+PC} \end{pmatrix}$$

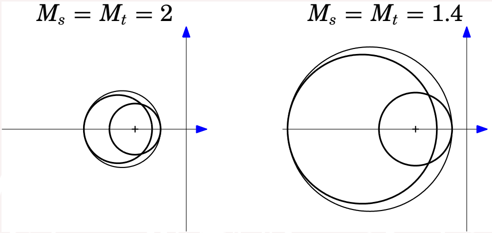
Dimensions! For SISO systems the $\mathcal{H}_\infty$ norm of G_s is

$$\gamma^2 = \max \frac{(1+|P|^2)(1+|C|^2)}{|1+PC|^2}$$

With scaling of process and controller

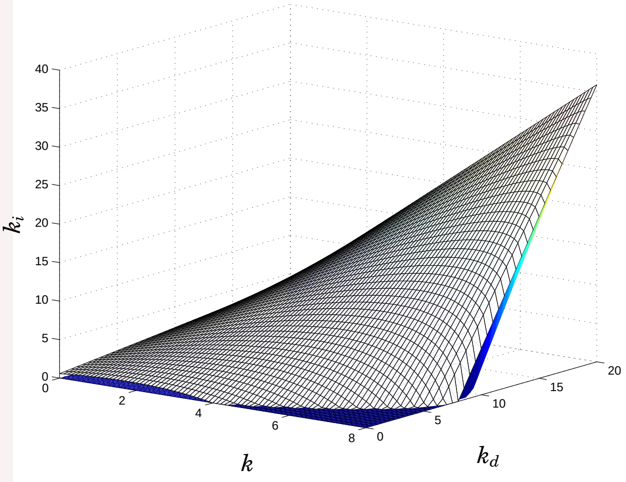
$$\gamma = \max \frac{1+|PC|}{|1+PC|} = \max\Big(\Big|\frac{1}{1+PC}\Big| + \Big|\frac{PC}{1+PC}\Big|\Big)$$

Circles



Contour	Center	Radius
M_s	-1	$1/M_s$
M_t	$-\frac{M_t^2}{M_t^2 - 1}$	$\frac{M_t}{M_t^2 - 1}$
M_s, M_t	$-\frac{M_s(2M_t - 1) - M_t + 1}{2M_s(M_t - 1)}$	$\frac{M_s + M_t - 1}{2M_s(M_t - 1)}$
$M_s = M_t = M$	$-\frac{2M^2 - 2M + 1}{2M(M - 1)}$	$\frac{2M - 1}{2M(M - 1)}$

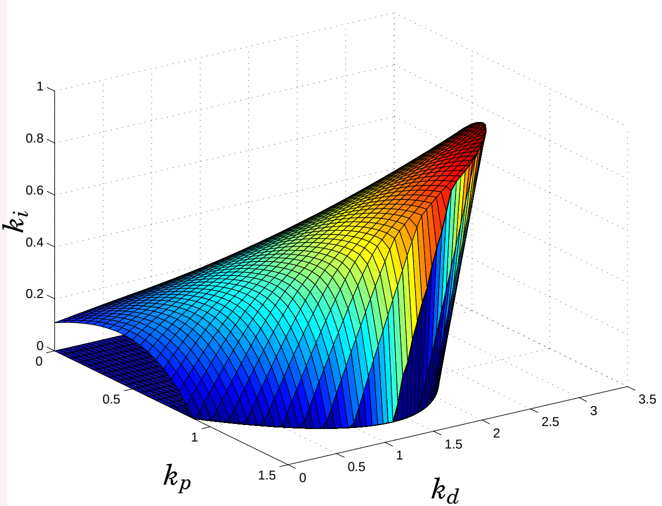
Stability Region for $P = (s+1)^{-4}$



Explains why derivative action is difficult

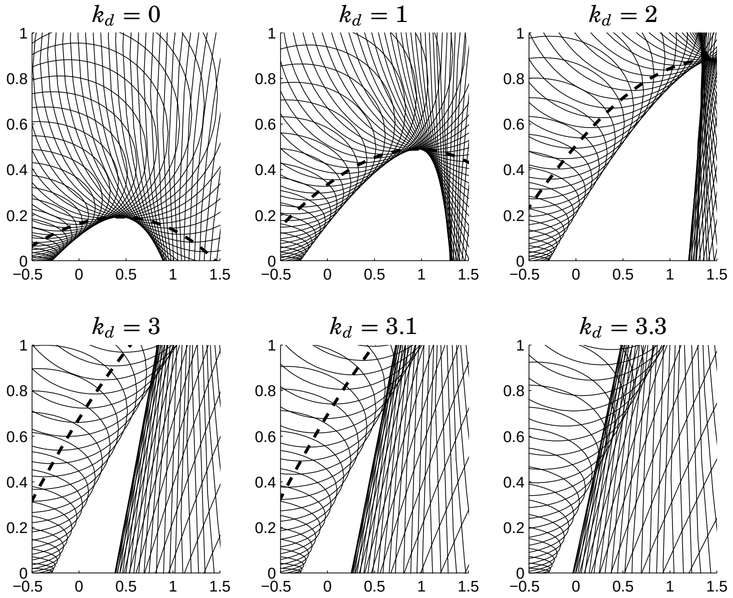
Don't fall off the edge!

Robustness Region for $P = (s+1)^{-4}$ & $M_s \leq 1.4$

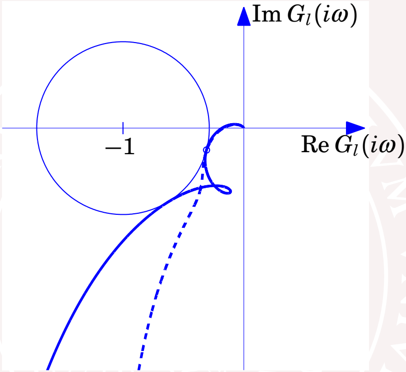


Compare with stability region

Projections on the $k_p - k_i$ plane



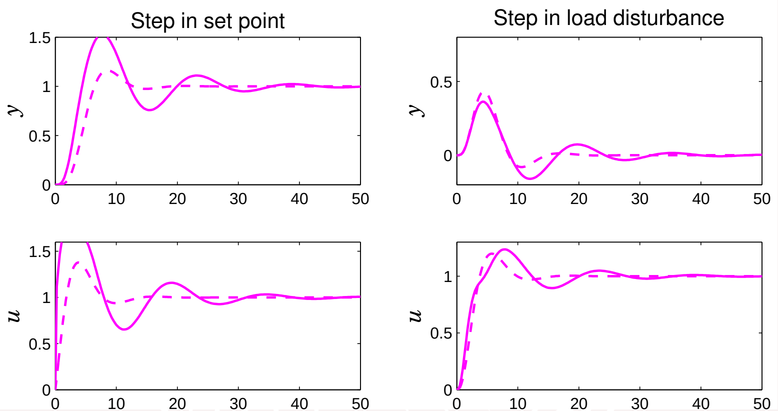
Edges Correspond to Cusps in the Nyquist Plot



Nyquist curve of the loop transfer function for PID control of the process $P(s) = 1/(s+1)^4$, with a controller having parameters $k_p = 0.925$, $k_i = 0.9$, and $k_d = 2.86$.

Cusps are avoided in this example by minimizing IAE instead (dashed curve) $k_p = 1.33$, $k_i = 0.63$, and $k_d = 1.78$

Time Responses



Process $P(s) = 1/(s+1)^4$, with controller having parameters $k_p = 0.925$, $k_i = 0.9$, and $k_d = 2.86$ (max k_i solid lines IAE=3.0) and $k_p = 1.33$, $k_i = 0.63$, and $k_d = 1.78$ (min IAE=2.2 dashed lines). Damping ratios of zeros $\zeta = 0.16$ and 0.37.

Tuning based on Optimization

A reasonable formulation of the design problem is to optimize performance subject to constraints on robustness and noise injection.

- Performance criteria IE or IAE for load disturbance attenuation
  - Small difference between IE and IAE for PI
  - Larger differences for PI because of derivative cliff
  - Necessary to use an *edge* constraint
- Robustness M_s and M_t
- Noise injection $\max |G_{un}(i\omega)|$ or $||G_{un}||_2$
- Maximize performance with noise attenuation and robustness as constraints (Shinskey: Minimize effect of load disturbances)
- Minimize noise injection with performance and robustness as constraints (Horowitz: minimize cost of control)
- Many efficient algorithms available
- Key issues: How to find the model

Control System Design - PID Control

Theme: The most common controller.

Tuning Rules

- When do you need rules?
- Why not model by physics or experiments and design a controller?
- Typical processes - essentially monotone - modeled by FOTD
- Ziegler-Nichols Tuning 1942 (for historical reasons)
- Lambda tuning - Common in pulp and paper industry
- SIMC - Skogestad: *Probably the best simple PID tuning rules in the world*
- Optimization, criteria and constraints
- AMIGO - Minimize IE, maiximze Integral gain subject to robustness constraint and edge constraint for PID
- MIAEO - Minimize IAE subject to robustness constraint (for local reasons and insight)
- How to get the models?

Ziegler-Nichols Tuning - Commissioning

Process control scenario: You have a controller with adjustable parameters and a process. How do you find suitable values of the controller parameters? Ziegler-Nichols idea was to tune controller based on simple experiments on the process

- The step response method - open loop experiment
  - Make an open loop step response (bump test)
  - Pick out features of the step response and determine parameters from a table
- The frequency response method - closed loop
  - Connect the controller change controller parameters, observe process behavior and adjust parmeters

The rules were developed by picking out typical process models, tuning controller by hand or simulation (MITs differential analyzer and pneumatic), and correlating controller parameters to process features

Assessment of Ziegler-Nichols Methods

Great simple idea: base tuning on simple process experiments,

- Published in 1942 in Trans. ASME 64 (1942) 759–768.
- Tremendously influential for establishing process control
- Slight modifications used extensively by controller manufacturers and process engineers
- The Million $ question: What structure (series or parallel) did they use?

BUT poor execution

- Uses too little process information: only 2 parameters

  Step response method: a, L

  Frequency response method: T_u, K_u
- Basic design principle quarter amplitude damping is not robust, gives closed loop systems with too high sensitivity ($M_s > 3$) and too poor damping ($\zeta \approx 0.2$)

Lambda Tuning

Process model and desired command response

$$P(s) = \frac{K_p}{1+sT}e^{-sL}. \qquad G_{yy_{sp}} = \frac{1}{1+sT_{cl}}e^{-sL}.$$

The controller becomes

$$C(s) = P^{-1}(s)\frac{G_{yy_{sp}}(s)}{1-G_{yy_{sp}}(s)} = \frac{1+sT}{K_p(1+sT_{cl}-e^{-sL})},$$

Cancellation of the process pole $s=-1/T$!! Approximations of e^{-sL} give PI and PID controllers, for example $e^{-sL} \approx 1 - sL$

$$C(s) = \frac{1+sT}{K_p(L+T_{cl})s} = \frac{T}{K_p(L+T_{cl})}\Big(1+\frac{1}{sT}\Big)$$

PI controller with the parameters

$$k_p = \frac{1}{K_p}\frac{T}{L+T_{cl}}, \quad k_i = \frac{1}{K_p(L+T_{cl})}, \quad T_i = T.$$

Closed loop response time $T_{cl} = \lambda_f T$ is a design parameter, common choices $\lambda_f = 3$ (robust tuning), $\lambda_f \le 1$ aggressive tuning.

Lambda Tuning - Gang of Four

$$S = \frac{s(L+T_{cl})}{s(L+T_{cl})+e^{-sL}} \approx \frac{s(L+T_{cl})}{1+sT_{cl}}$$

$$PS = \frac{sK_p(L+T_{cl})}{(s(L+T_{cl})+e^{-sL})(1+sT)}e^{-sL} \approx \frac{sK_p(L+T_{cl})}{(1+sT_{cl})(1+sT)}e^{-sL}$$

$$CS = \frac{s(T+T_{cl})(1+sT)}{(s(L+T_{cl})+e^{-sL})(1+sT)} \approx \frac{(L+T_{cl})(1+sT)}{K(L+T_{cl})(1+sT_{cl})}$$

$$T = \frac{e^{-sL}}{s(L+T_{cl})+e^{-sL}} \approx \frac{1}{1+sT_{cl}}e^{-sL}.$$

- Very nice to have a tuning parameter T_{cl} with good physical interpretation, see T
- Perhaps better to pick T_{cl} proportional to L
- Notice presence of canceled mode $s=-1/T$ in PS, very poor load disturbance response if $T_{cl} < T$

Skogestad SIMC

Process models

$$P_1(s) = \frac{K_p}{1+sT} e^{-sL}, \qquad P_2(s) = \frac{K_p}{(1+sT_1)(1+sT_2)} e^{-sL}.$$

Desired closed-loop transfer function

$$G_{yy_{sp}} = \frac{1}{1+sT_{cl}} e^{-sL}.$$

Hence

$$C(s) = \frac{1}{P} \times \frac{G_{yy_{sp}}}{1-G_{yy_{sp}}} = \frac{1+sT}{K_p(1+sT_{cl}-e^{-sL})} \approx \frac{1+sT}{sK_p(T_{cl}+L)}$$

typical choices of design parameter $T_{cl} = \lambda_f L$. Control law

$$k_p = \frac{1}{K_p}\frac{T}{L+T_{cl}}, \qquad T_i = \min(T, 4(T_{cl}+L)).$$

Fixes after lots of simulations SIMC++

$$k_p = \frac{1}{K_p}\frac{T+L/3}{L+T_{cl}}, \qquad T_i = \min(T{+}L/3, 4(T_{cl}{+}L)), \qquad T_{cl} = \lambda L.$$

Optimization Based Rules - MIGO

Some questions:

- What information is required to tune a PID controller?
- Two parameter models do not work well
- How about the FOTD model?
- Can we find good Ziegler-Nichols-type type tuning rules?

Towards a solution

- Pick a class of representative processes
- Pick a design criterion: Maximize integral gain subject to constraints on robustness M_s and M_t MIGO (M-constrained Integral Gain Optimization)
- Relate controller parameters to FOTD model $Ke^{-sL}/(1+sT)$

Results:

- Insight and simple tuning rules
- The importance of lag- and delay-dominance
- Rules for PI control, conservative rules for PID control

The Test Batch -

$$P_1(s) = \frac{e^{-s}}{1+sT}, \qquad P_2(s) = \frac{e^{-s}}{(1+sT)^2}$$

$$P_3(s) = \frac{1}{(s+1)(1+sT)^2}, \qquad P_4(s) = \frac{1}{(s+1)^n}$$

$$P_5(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}$$

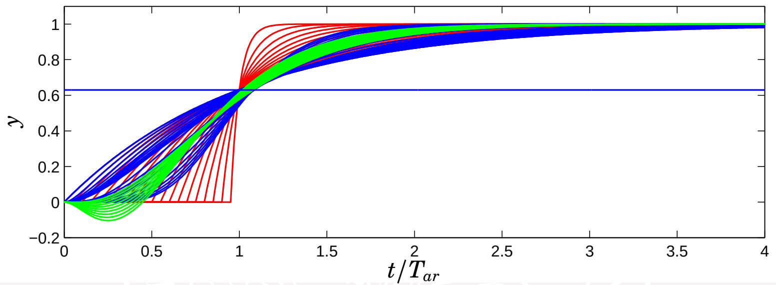
$$P_6(s) = \frac{1}{s(1+sT_1)} e^{-sL_1}, \quad T_1 + L_1 = 1$$

$$P_7(s) = \frac{T}{(1+sT)(1+sT_1)} e^{-sL_1}, \qquad T_1 + L_1 = 1$$

$$P_8(s) = \frac{1-\alpha s}{(s+1)^3}$$

$$P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$$

Essentially Monotone Step Responses



Step responses for test batch mormalized by the average residence time $T_{ar} = \int tg(t)dt / \int g(t)dt = -P'(0)$. Empirical criterion for monotonicity

$$a = \frac{\int_0^\infty e(t)dt}{\int_0^\infty |e(t)|dt}, \qquad \text{essentially positive if} \quad a > 0.8$$

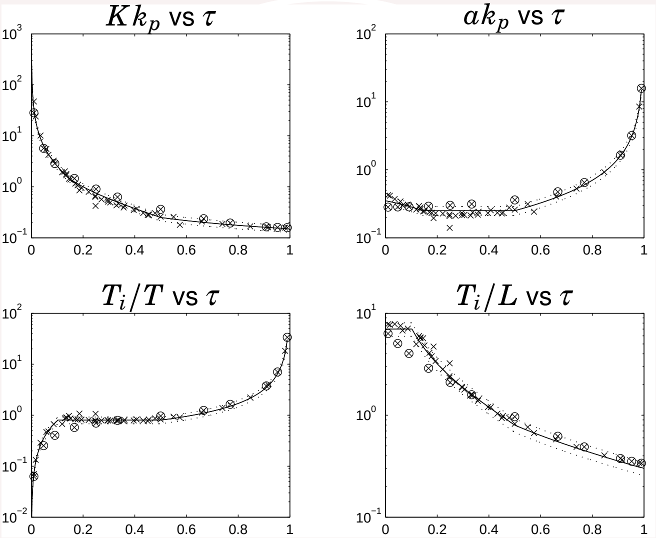
Positive systems is a research issue (Sontag)

The FOTD Model

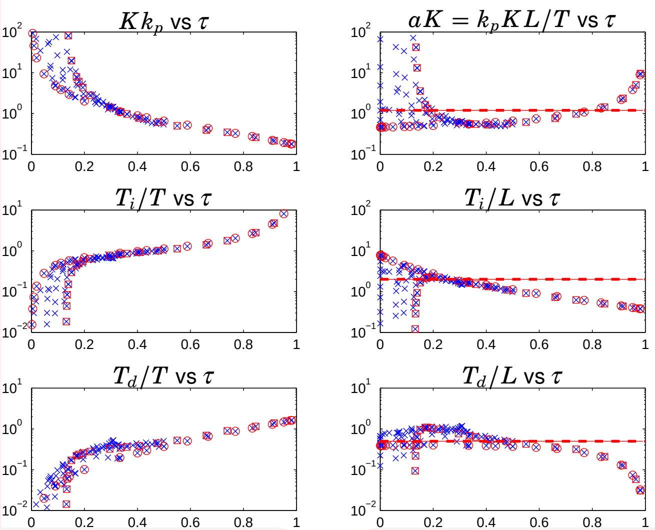
$$P(s) = \frac{K}{1 + sT} e^{-sL}$$

- L *apparent time delay*, T *apparent lag*
- Approximation of processes with (almost) monotone step responses
- Commonly used in process control and for PID tuning
- Performance limited by time delay $\omega_{gc} L < 1$. Useful to have a simple model that captures performance limitations
- Average residence time $T_{ar} = L + T$
- Delay ratio $\tau = L/T_{ar} = L/(L + T) \quad 0 \leq \tau \leq 1$ is useful to classify dynamics
  - Lag dominant: τ close to 0
  - Balanced: τ around 0.5
  - Delay dominant τ close to 1

PI Control $M = 1.4$



PID Control $M = 1.4$



What happens for small τ?

AMIGO Tuning Rules

PI Control, combined sensitivity $M = 1.4$

$$k_p = \frac{0.15}{K} + \left(0.35 - \frac{LT}{(L+T)^2}\right)\frac{T}{KL} \approx \frac{0.35T}{KL} \text{ small } \tau$$

$$T_i = 0.35L + \frac{13LT^2}{T^2 + 12LT + 7L^2} \approx 13.4L \text{ small } \tau,$$

PID Control, combined sensitivity $M = 1.4$ + edge constraint

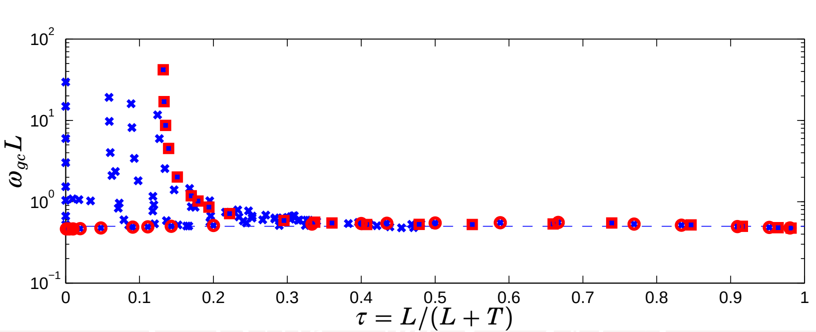
$$k_p = \frac{1}{K}\left(0.2 + 0.45\frac{T}{L}\right) \approx \frac{0.45T}{KL} \text{ small } \tau$$

$$T_i = \frac{0.4L + 0.8T}{L + 0.1T}L \approx 8L \text{ small } \tau,$$

$$T_d = \frac{0.5LT}{0.3L + T} \approx 0.5L \text{ small } \tau.$$

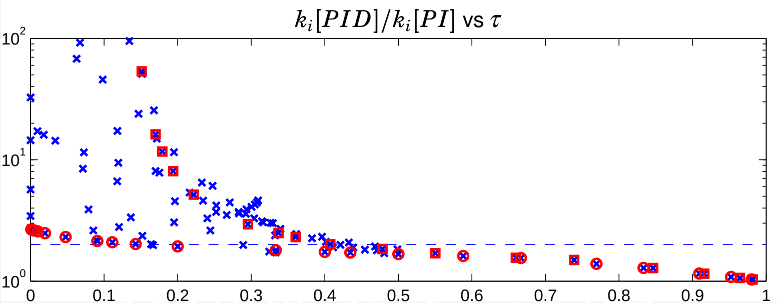
Maximum sensitivity is a good tuning variable

An Observation



- Compare with fundamental limit due to time delay $\omega_{sc}L < \frac{2(M_s-1)}{M_s} \approx 0.57$
- Close to limit for P_1 (red circles) for all τ
- Close to limit for whole batch for $\tau > 0.3$
- Reason for large variability for small τ is that the FOTD model overestimates L for lag dominated systems, high order dynamics approximated by time delay

Benefit of Derivative Action

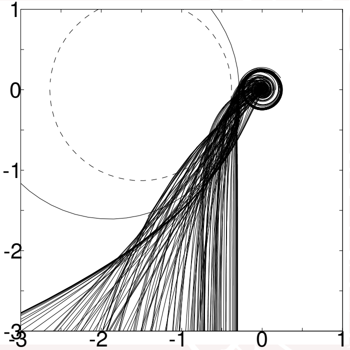


- Derivative action gives small benefits for processes with delay dominated dynamics (derivative is a poor predictor for systems which are dominated by time delay)
- Derivative action doubles performance for $\tau = 0.5$
- Significant may be possible for small τ, but better modeling may be required, notice difference between P_1 (red circles) and P_2 (red squares)
- Processes with small τ are easy to control and admit very high gains. In practice the admissible gains are limited by sensor noise. A PI controller will

Nyquist Plots for Testbatch

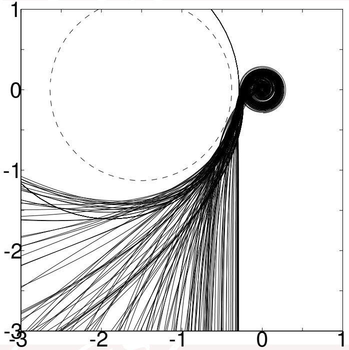
AMIGO

Approximate by $\frac{K_p e^{-sL}}{1+sT}$



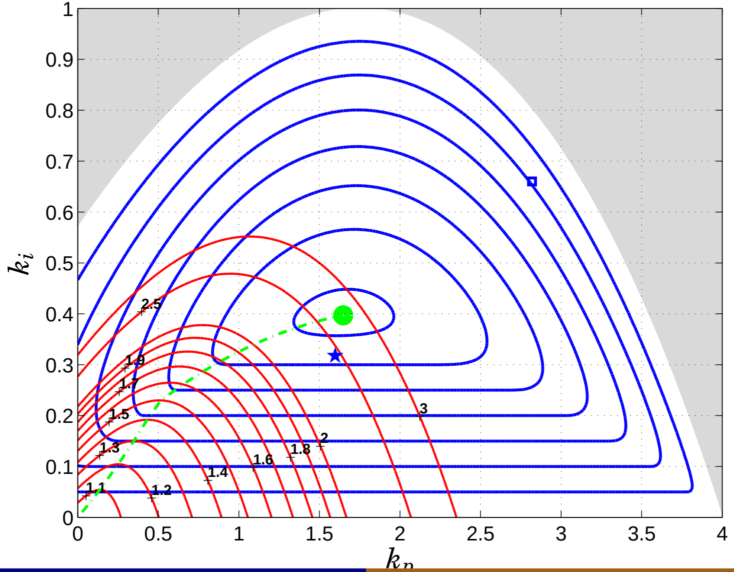
AMIGO++

Approximate by $\frac{K_p e^{-sL}}{(1+sT_1)(1+sT_2)}$

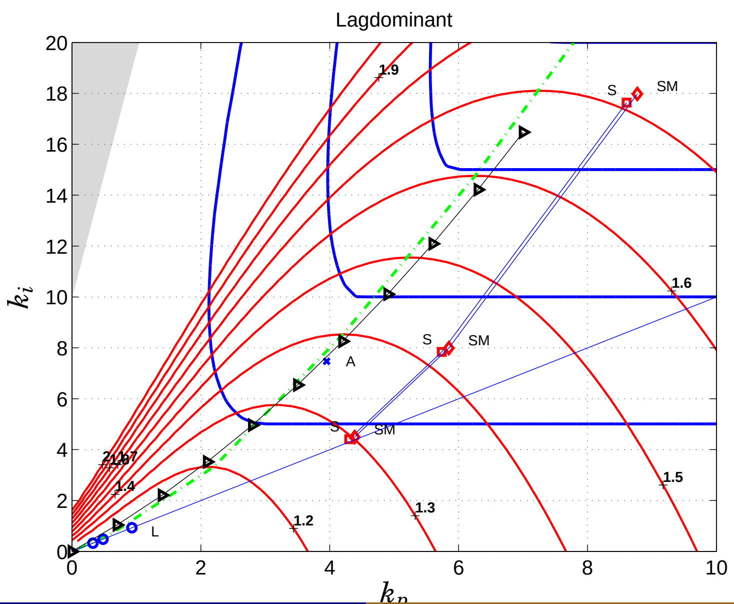


Worth while to model better for small τ

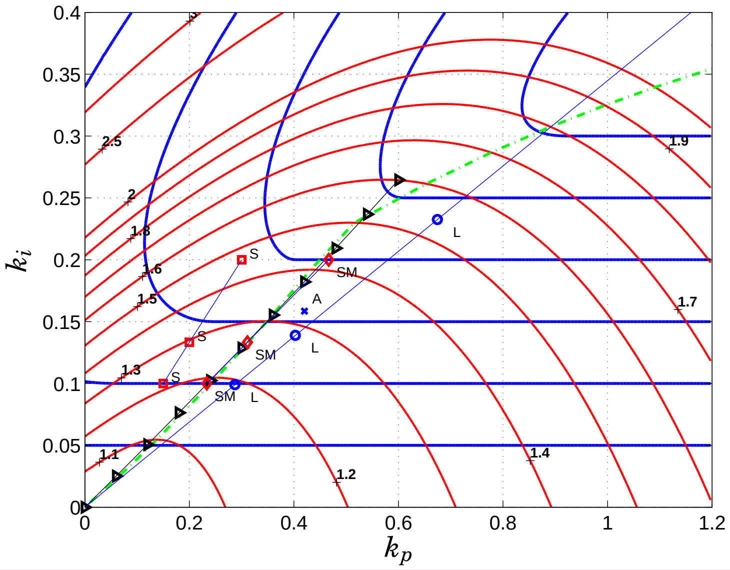
Level Curves Performance (blue) Robustness (red)



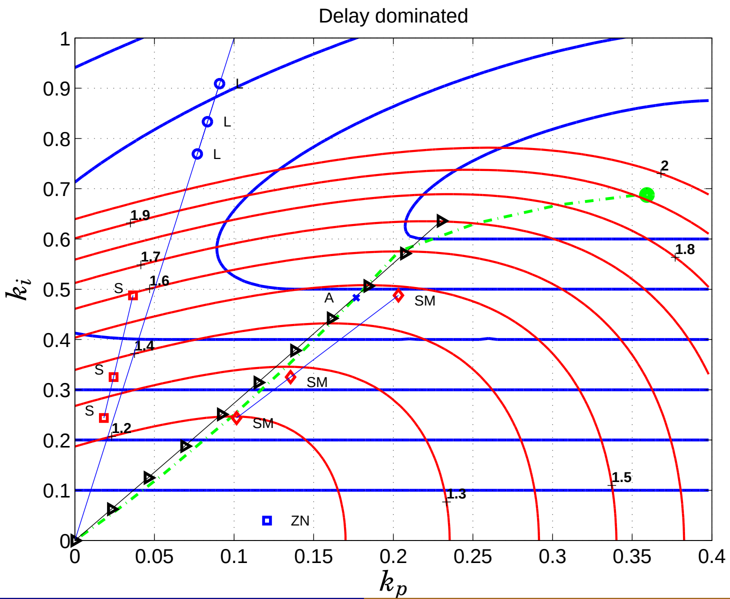
Level Curves - Lag Dominant Dynamics



Level Curves - Balanced Dynamics

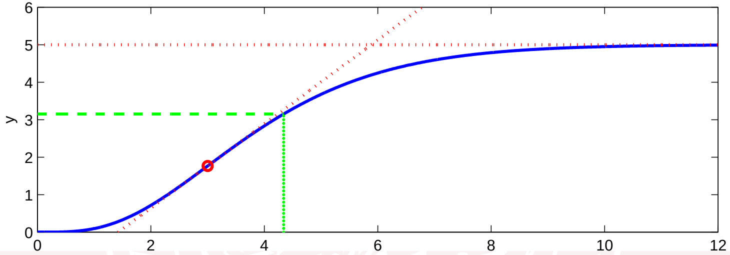


Level Curves - Delay Dominant Dynamics



How to Get the Models

Bump test



Relay feedback

Model reduction - Skogestads half rule

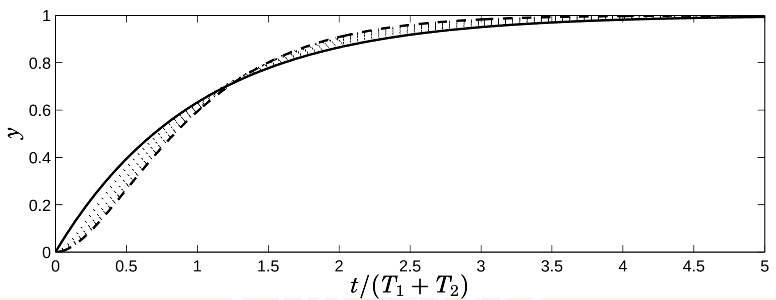
System identification

Modeling and control design should match

A Difficulty in Step Response Modeling

Normalized step responses for

$$P(s) = \frac{1}{(1+sT_1)(1+sT_2)}, \qquad T_1/T_2 = 0, 0.1, \ldots 1$$



Difficult to estimate T_1 and T_2

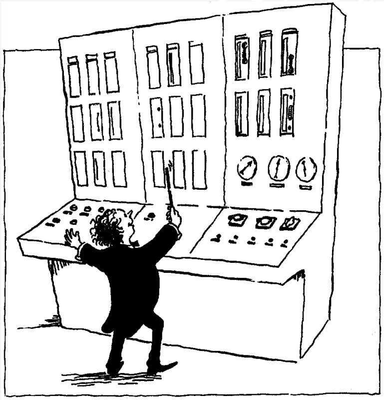
Summary

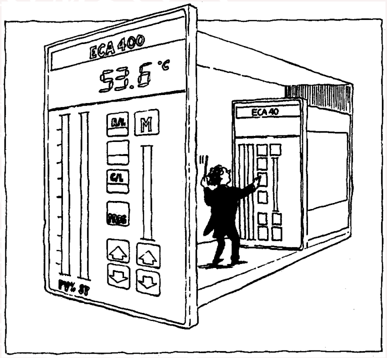
- Processes with essentially monotone step responses
- The FOTD model gives insight
- Realize difference between lag and delay dominated dynamics τ
- PI is sufficient for processes with delay dominated dynamics
- Advantage of derivative action increases with decreasing τ
- Derivative action doubles performance for $\tau = 0.4$
- Derivative action may give significant improvement for processes with lag dominated dynamics but more complex models may be useful
- Processes with small τ admit high controller gains and performance may be limited by noise injection, a PI controller may then be sufficient
- AMIGO and Skogestad SIMC+ are reasonable rules
- Modeling is essential

1. Introduction
2. The Controller
3. Performance and Robustness
4. Tuning Rules
5. Relay Auto-tuning
6. Limitations of PID Control
7. Summary

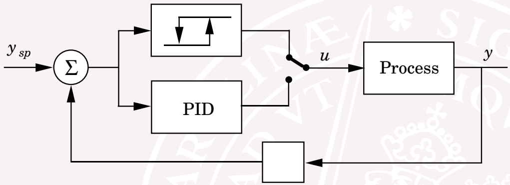
Theme: The most common controller.

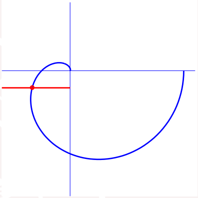
Relay Auto-tuning



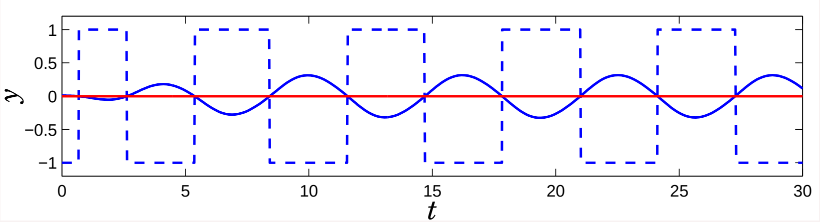


Relay Auto-tuning

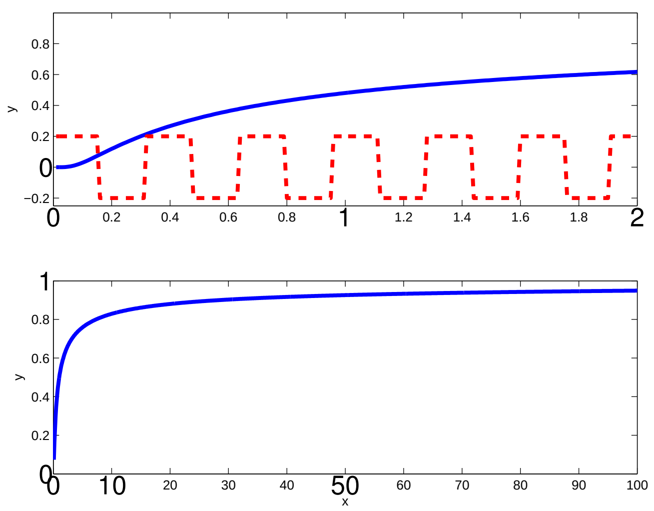




What happens when relay feedback is applied to a system with dynamics? Think about a thermostat?

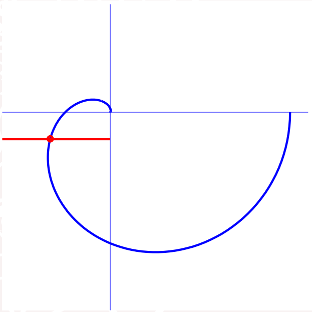


Short Experiment Time $G(s) = exp(-\sqrt{s})$



Practical Details

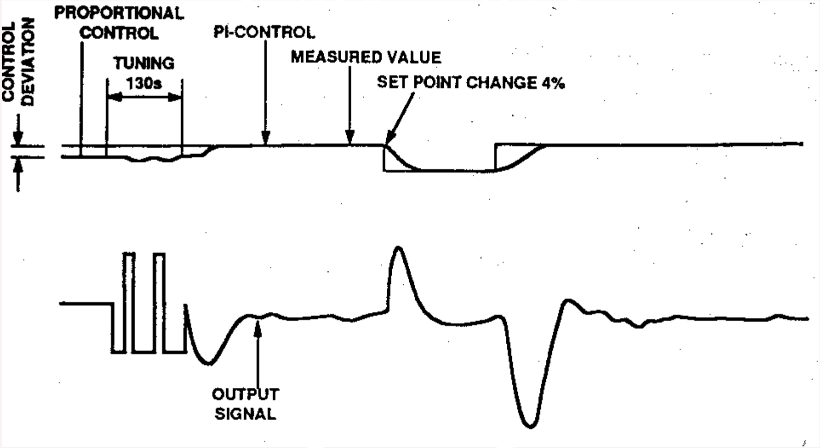
- Bring process to equilibrium
- Measure noise level
- Compute hysteresis width
- Initiate relay
- Monitor each half period
- Change relay amplitude automatically
- Check for steady state
- Compute controller parameters
- Resume PID control



The First Industrial Test 1982

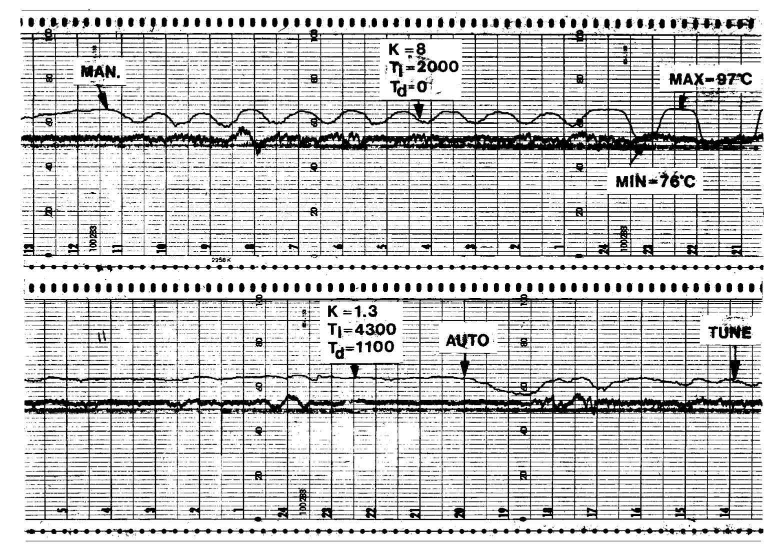
The Hardware

Automatic Tuning of a Level Controller



Notice negative controller gain - found by relay tuner

Temperature Control of Distillation Column



Commercial Autotuners

- One-button autotuning
- Automatic generation of gain schedules
- Adaptation of feedback gains
- Adaptation of feedforward gain
- Many versions
  - Single loop controllers
  - DCS systems
- Robust
- Excellent industrial experience
- Large numbers

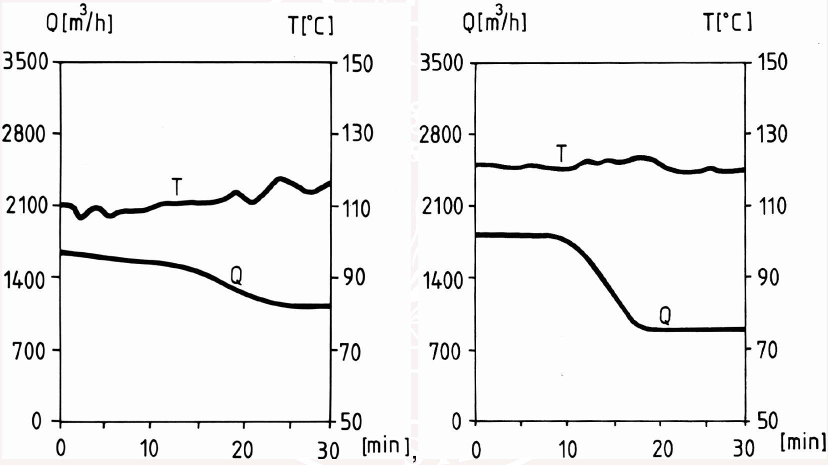
Automatic Generation of Gain-schedules

Igelsta 120 MW co-generation plant outside Stocholm. Heat exchanger with nonlinear valve.

An ordinary PID controller was replace with a PID controller having gain scheduling. Operating regions were set manually. The schedule was determined by relay auto-tuning.

Valve position	K	T_i	T_d
0.00-0.15	1.7	95	23
0.15-0.22	2.0	89	22
0.22-0.35	2.9	82	21
0.35-1.00	4.4	68	17

Results


Q[m³/h]
T[°C]
3500
2800
2100
1400
700
0
150
130
110
90
70
50
T
Q
0
10
20
30
[min]
Q[m³/h]
T[°C]
3500
2800
2100
1400
700
0
150
130
110
90
70
50
T
Q
0
10
20
30
[min]

Industrial Systems

Functions

- Automatic tuning AT
- Automatic generation of gain scheduling GC
- Adaptive feedback AFB and adaptive feedforward AFF

Sample of products

- NAF Controls SDM 20 - 1984 DCS AT, GS
- SattControl ECA 40 - 1986 SLC AT, GS
- Satt Control ECA 04 - 1988 SLC AT
- Alfa Laval Automation Alert 50 - 1988 DCS AT, GS
- Satt Control SattCon31 - 1988 PLC AT, GS
- Satt Control ECA 400 -1988 2LC AT, GS, AFB, AFF
- Fisher Control DPR 900 - 1988 SLC
- Satt Control SattLine - 1989 DCS AT, GS, AFB, AFF
- Emerson Delta V - 1999 DCS AT, GS, AFB, AF
- ABB 800xA - 2004 DCS AT, GS, AFB, AFF

Emerson Experience

- Tuner can be used by the production technicians on shift with complete control over what is going on.
- Operator is aware of the tuning process and has complete control.
- The user-friendly operator interface is consistend with other DCS applications so technicians are comfortable with it. It can be taught and become useful in less than half an hour.
- The single most important factor is that operators and technicians take ownership of control loop performance. This results in more loops being tuned, retuned or fine-tuned, tighter oprating conditions and more consistent operations, resulting in more consitent quality and lower costs.

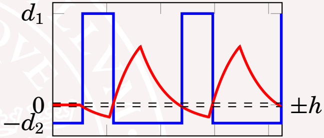
`McMillan, Wojsznis and Meyer Easy Tuner for DCS ISA'93`

Potential Improvements

Dramatic increases of computing power

Better modeling

- Asymmetric relay - better excitation
- Identification - dont wait for steady state
- Additonal test signal - chirp
- Assessment of several models

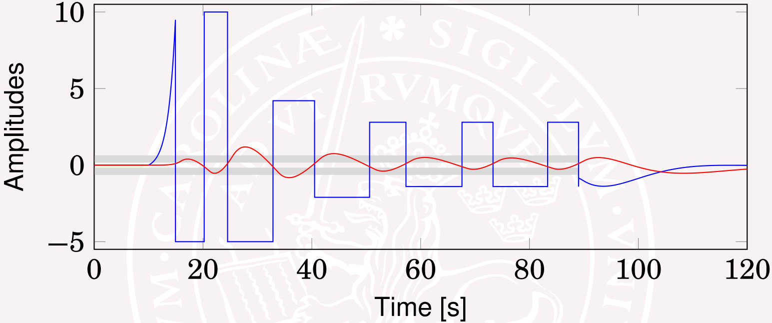


Improved control design

- Load disturbance attenuation: minimize IAE$= \int_0^\infty |s(t)|dt$
- Robustness: limit maximum sensitivities M_s, M_t
- Measurement noise injection: bound noise gain $||G_{un}||$
- Constrained optimization: efficient algorithms

Multivariable systems

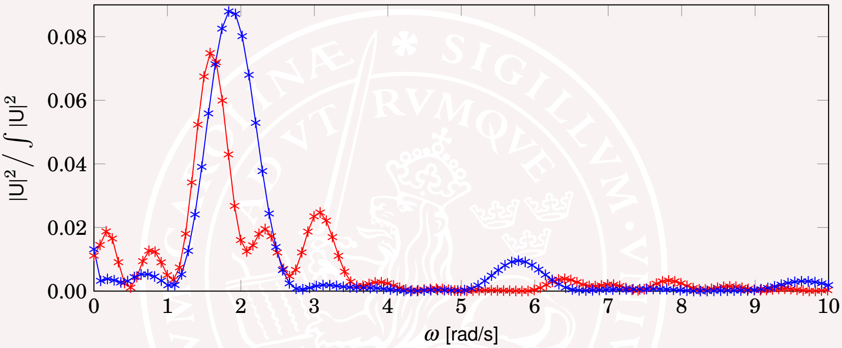
Initialization and Asymmetric Relay



- Better excitation
- The amplitudes are ramped up, and adjusted to get the desired process deviations.

`Figure from Josefin Berner`

Better Excitation with Asymmetric Relay



- Symmetric relay blue
- Asymmetric relay red

Figure from Josefin Berner

Typical Experiments

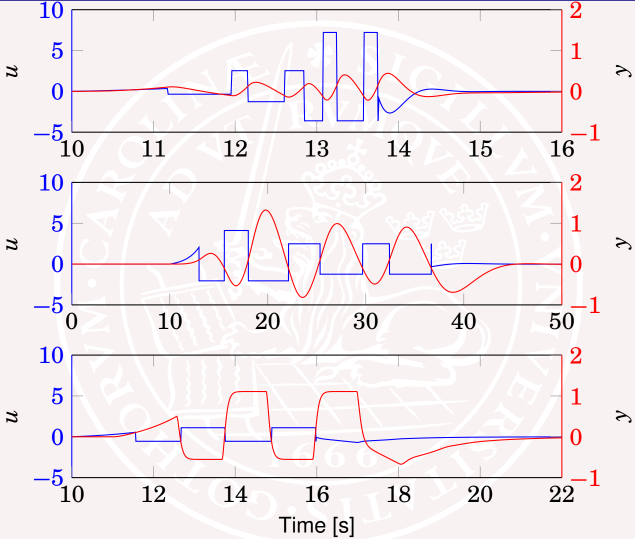


Figure from Josefin Berner

Models

Two parameter models

$$P(s) = \frac{b}{s+a}, \quad P(s) = K\, e^{-sL}$$

Three parameter models

$$P(s) = \frac{b}{s^2 + a_1 s + a_2}, \quad P(s) = \frac{b}{s+a}\, e^{-sL}, \quad P(s) = \frac{K}{1+sT}\, e^{-sL}$$

$$P(s) = \frac{K}{(1+sT)^2}\, e^{-sL}$$

Four parameter models

$$P(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}, \quad P(s) = \frac{b}{s^2 + a_1 s + a_2}\, e^{-sL}$$

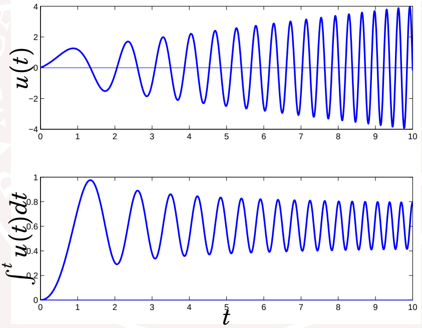
Five parameter model

$$P(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}\, e^{-sL}$$

The Chirp Signal

$$u(t) = (a + b\,t)\sin(c + d\,t)$$

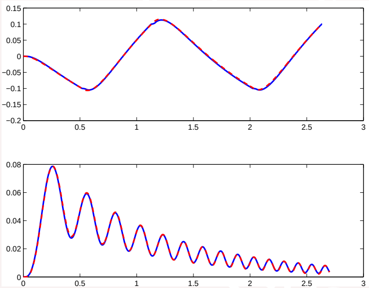
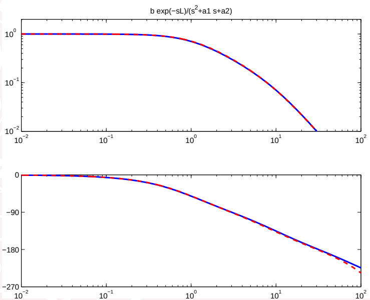
Frequency varies between a and $c + d\,t_{max}$ amplitude between a and $a + b\,t_{max}$



Notice both high and low frequency excitation

Asymmetric Relay with Chirp

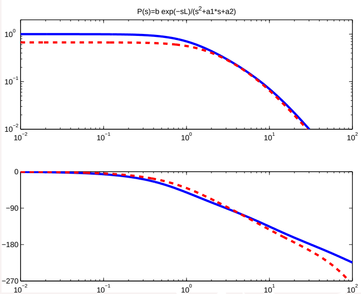
- Asymmetrical relay experiment combined chirp signal experiment
- Double experiment time. Constant amplitude,
  $L = 0.01, w = 15 * (1 + 0.5 * t), t_{max} = 2.7,$
  $0.15 \leq \omega L \leq 0.35$

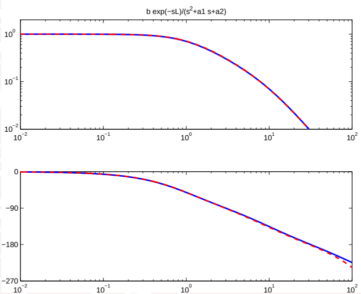
Parameters: $a_1 = 10.37 \pm 0.03, \qquad a_2 = 9.57 \pm 0.03,$
$b = 9.57 \pm 0.03, \qquad L = 0.0109 \pm 0.0002$

Effect of Chirp Experiment

Only relay



Relay and chirp



Properties of Relay Auto-tuning

- Safe for stable systems
- Close to industrial practice

  Easy to explain similar to Ziegler-Nichols tuning
- Little prior information. Relay amplitude
- One-button tuning
- Automatic generation of test signal

  Injects much energy at ω_{180} with no prior knowledge of ω_{180}

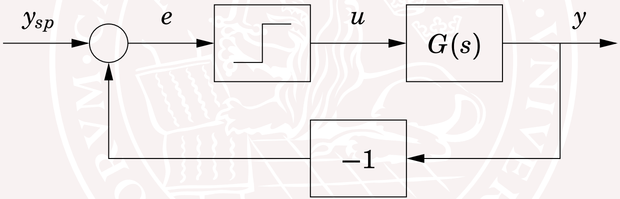
  Easy to modify for signal injection at other frequencies
- Good industrial experience for more than 25 years. Many patents are running out.
- Good for pre-tuning of adaptive controllers
- Still room for improvement

  Exploit advances in computing

  Exploit understanding of modeling and controller design

The Millon Dollar Question

Classify Systems where Relay Feedback Works



Characterize all transfer functions G that give a unique stable limit cycle

Theme: The most common controller.

Limitations of PID Control

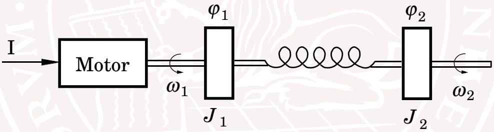
PID control is simple and useful but there are limitations

- Multivariable and strongly coupled systems
- Complicated dynamics
- Large parameter variations

  Adding gainscheduling and adaptation (later)
- Difficult compromises between load disturbance attenuation and measurement noise injection

Complicated Dynamics

- Any stable system can be controlled by an integrating controller if performance requirements are modest
- PI control and systems with first order dynamics
- PID control and systems with second order dynamics
- States are the variables required to account for storage of mass, energy and momentum



Transfer function (physical meaning of approximation)

$$P(s) = \frac{0.045s + 0.45}{s^2(s^2 + 0.1s + 1)} \approx \frac{0.45}{s^2}$$

PID Control

With an ideal PID controller and the approximate model the loop transfer function is

$$L(s) = \frac{0.45(k_d s^2 + k_p s + k_i)}{s^3}$$

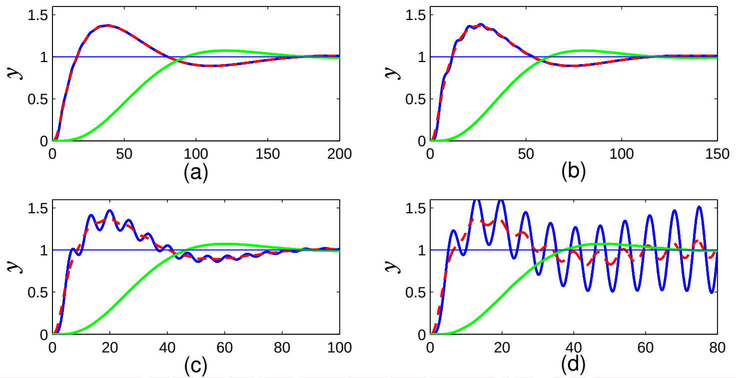
We will add high frequency roll-off later. Closed loop characteristic polynomial

$$s^3 + 0.45 k_d s^2 + 0.45 k_p s + 0.45 k_i = s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3$$

$$(s + \omega_c)(s^2 + \omega_c s + \omega_c^2), \qquad \text{Butterworth}$$

The approximation is valid if ω_c small (say $\omega_c < 0.1\omega_0$. Increasing ω_c leads to instability. The bandwidth and the performance $k_i = \omega_c^3/0.45$ are limited.

PID Control ...

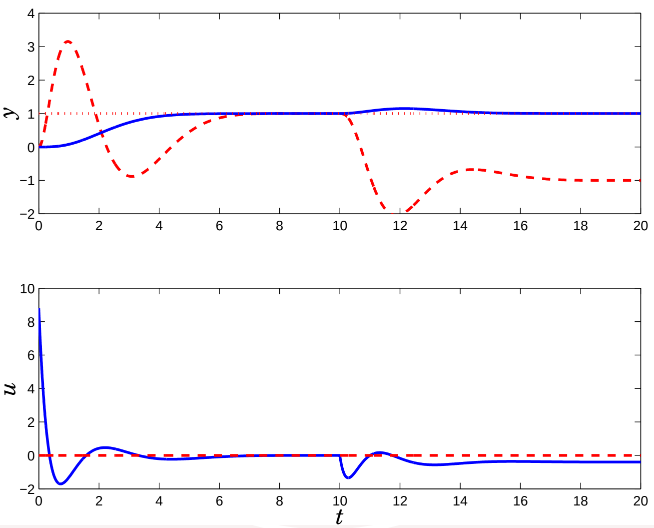


$\omega_c/\omega_0 =$ a) 0.04, b) 0.06, c) 0.08 d) 0.1

φ_1 blue, φ_2 red, setpoint weighting green

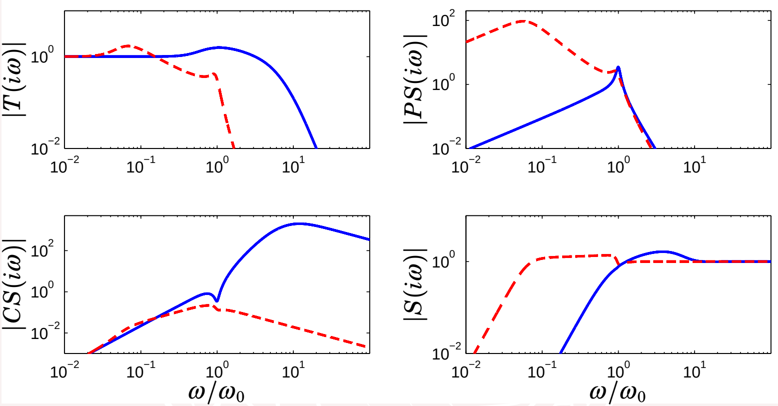
With low bandwidth controller the inertias move together

Observer and State Feedback



φ_1 blue, φ_2 red

Comparison PID SFB - GoF



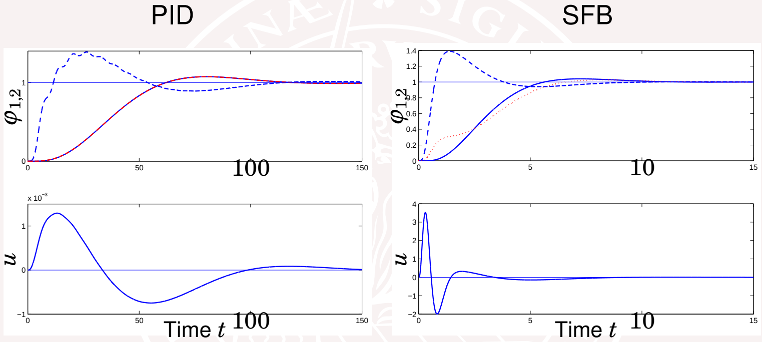
PID is designed for $\omega_c = 0.06\omega_0$

PID red dashed SFB blue

Notice orders of magnitude

SFB requires high quality low noise sensors

Comparison PID SFB Command Response

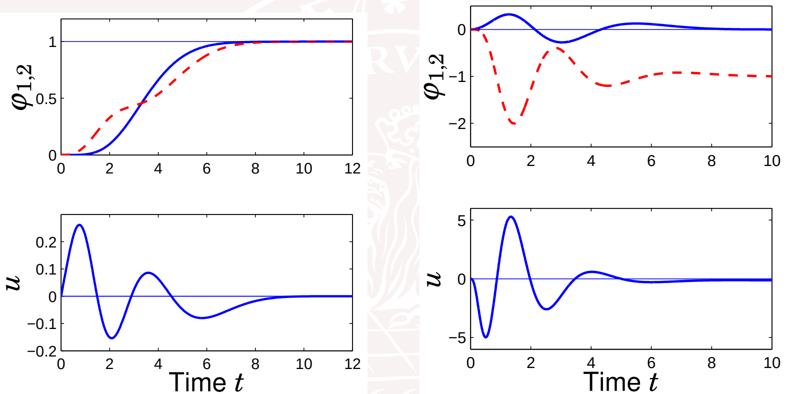


notice time scales and control signal amplitudes!

SFB gives ten times faster response

φ_1 red dotted, φ_2 blue solid, dashed without 2DOF

Set Point and Load Disturbance Response SFBI



φ_1 red dotted, φ_2 blue solid

Explain behavior of inertias!

Control System Design - PID Control

1. Introduction
2. The Controller
3. Performance and Robustness
4. Tuning Rules
5. Relay Auto-tuning
6. Limitations of PID Control
7. Summary

Theme: The most common controller.

Summary

- A simple and useful controller
- Much tradition and legacy
- Many things to consider: set point weighting, filtering, windup protection, mode switching and tracking modes
- Many versions, a reasonable choice

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s} G_f(s), \qquad G_f(s) = \frac{1}{1 + sT_f + s^2 T_f^2/(4\zeta_f^2)}$$

  Incorporate filter G_f in process, design ideal PID for PG_f
- Many design methods relative time delay τ is important to classify
- Good models can be obtained by relay feedback
- Next generation auto-tuners are not far away
- There are processes where PID can be outperformed significantly

Reading Suggestions

Åström and Hägglund Advanced PID Control. Instrument Society of America, Research Triangle Park. 2006. Second edition which contains oscillatory systems in preparation.