

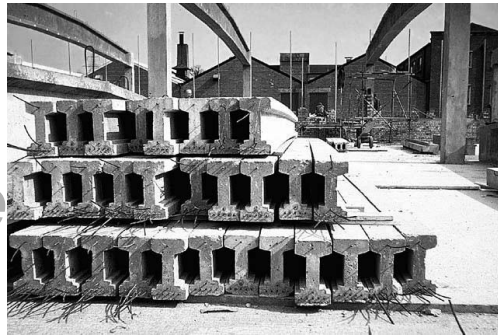
ELEMENTS OF ARCHITECTURAL STRUCTURES:  
FORM, BEHAVIOR, AND DESIGN

ARCH 614

DR. ANNE NICHOLS

SPRING 2014

lecture  
twenty one



<http://nisee.berkeley.edu/godden>

concrete construction:  
materials & beams

Concrete Beams 1  
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Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength =  $f'_c$



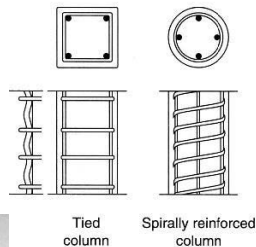
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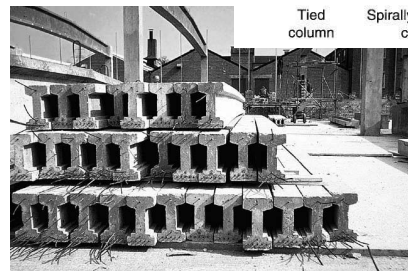
Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning



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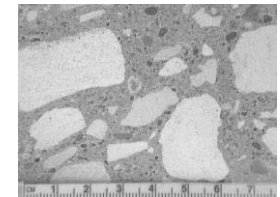
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Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
- hydration
- fire resistant
- creep & shrink



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## Reinforcement

- deformed steel bars (rebar)
  - Grade 40,  $F_y = 40$  ksi
  - Grade 60,  $F_y = 60$  ksi - most common
  - Grade 75,  $F_y = 75$  ksi
  - US customary in # of 1/8"  $\phi$
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...



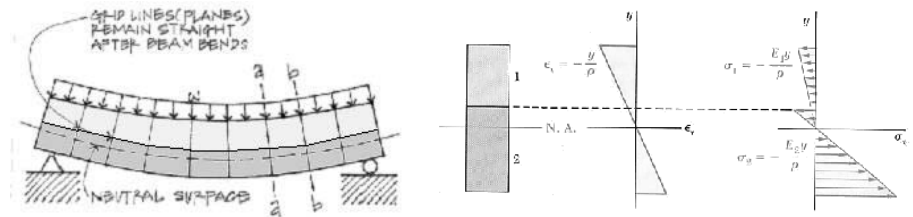
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## Behavior of Composite Members

- plane sections remain plane
- stress distribution changes



$$f_1 = E_1 \varepsilon = -\frac{E_1 y}{\rho} \qquad f_2 = E_2 \varepsilon = -\frac{E_2 y}{\rho}$$

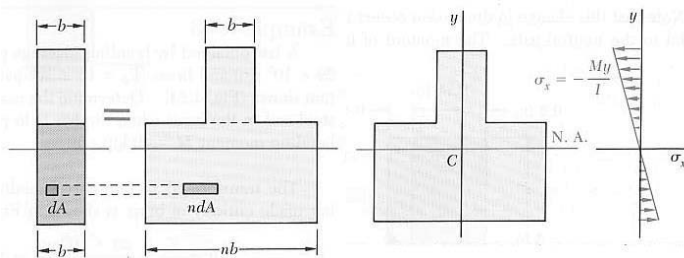
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## Transformation of Material

- $n$  is the ratio of  $E$ 's
 
$$n = \frac{E_2}{E_1}$$
- effectively widens a material to get same stress distribution



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## Stresses in Composite Section

- with a section transformed to one material, new  $I$ 
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by  $n$

$$n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$$

$$f_c = -\frac{My}{I_{transformed}}$$

$$f_s = -\frac{Myn}{I_{transformed}}$$

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# Reinforced Concrete - stress/strain

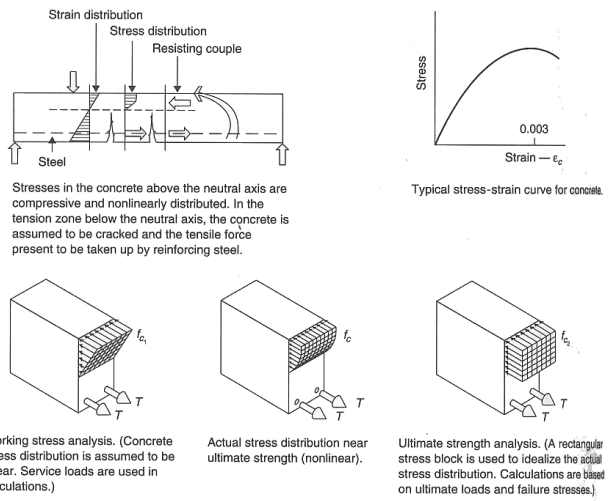
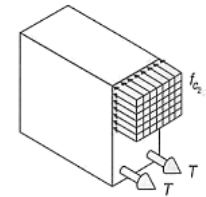


FIGURE 6-37 Reinforced concrete beams. ARCH 614

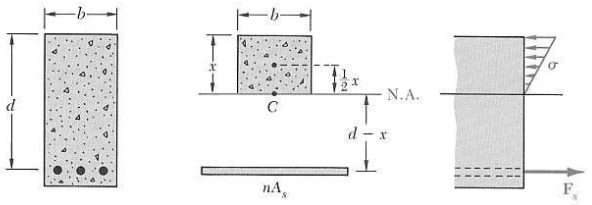
# Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure



## Location of n.a.

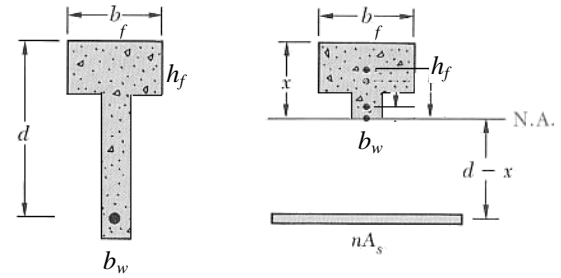
- ignore concrete below n.a.
- transform steel
- same area moments, solve for x



$$bx \cdot \frac{x}{2} - nA_s (d - x) = 0$$

## T sections

- n.a. equation is different if n.a. below flange



$$b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0$$

## ACI Load Combinations\*

- $1.4D$
- $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
- $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
- $1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
- $1.2D + 1.0E + 1.0L + 0.2S$
- $0.9D + 1.0W$
- $0.9D + 1.0E$

\*can also use old  
ACI factors

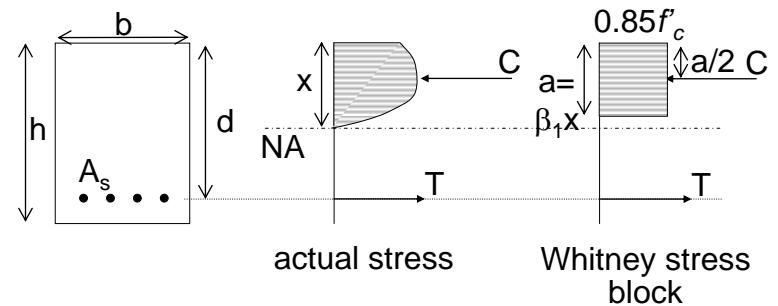
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## Reinforced Concrete Design

- stress distribution in bending



Wang & Salmon, Chapter 3

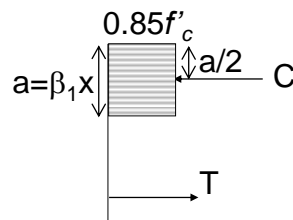
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## Force Equations

- $C = 0.85 f'_c b a$
- $T = A_s f_y$
- where
  - $f'_c$  = concrete compressive strength
  - $a$  = height of stress block
  - $\beta_1$  = factor based on  $f'_c$
  - $x$  = location to the n.a.
  - $b$  = width of stress block
  - $f_y$  = steel yield strength
  - $A_s$  = area of steel reinforcement



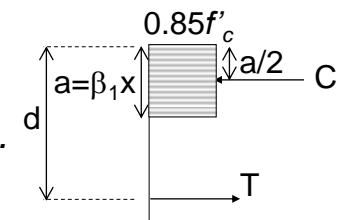
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## Equilibrium

- $T = C$
- $M_n = T(d-a/2)$ 
  - $d$  = depth to the steel n.a.
- with  $A_s$ 
  - $a = \frac{A_s f_y}{0.85 f'_c b}$
  - $M_u \leq \phi M_n$   $\phi = 0.9$  for flexure
  - $M_u = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$



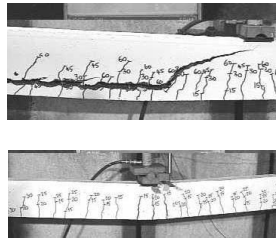
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## Over and Under-reinforcement

- over-reinforced
  - steel won't yield
- under-reinforced
  - steel will yield
- reinforcement ratio



[http://people.bath.ac.uk/abstj/concrete\\_video/virtual\\_lab.htm](http://people.bath.ac.uk/abstj/concrete_video/virtual_lab.htm)

- $\rho = \frac{A_s}{bd}$
- use as a design estimate to find  $A_s, b, d$
- max  $\rho$  is found with  $\epsilon_{steel} \geq 0.004$  (not  $\rho_{bal}$ )

## $A_s$ for a Given Section (cont)

- chart method
  - Wang & Salmon Fig. 3.8.1  $R_n$  vs.  $\rho$

1. calculate  $R_n = \frac{M_n}{bd^2}$

2. find curve for  $f'_c$  and  $f_y$  to get  $\rho$

3. calculate  $A_s$  and  $a$

- simplify by setting  $h = 1.1d$

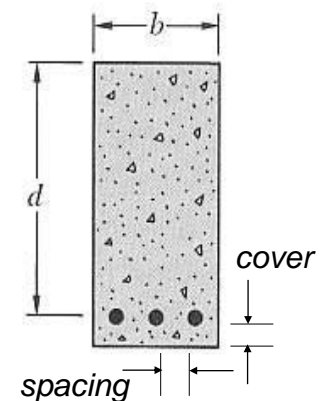
## $A_s$ for a Given Section

- several methods
  - guess  $a$  and iterate
    1. guess  $a$  (less than  $n.a.$ )
    2.  $A_s = \frac{0.85 f'_c b a}{f_y}$
    3. solve for  $a$  from  $M_u = \phi A_s f_y (d-a/2)$ 

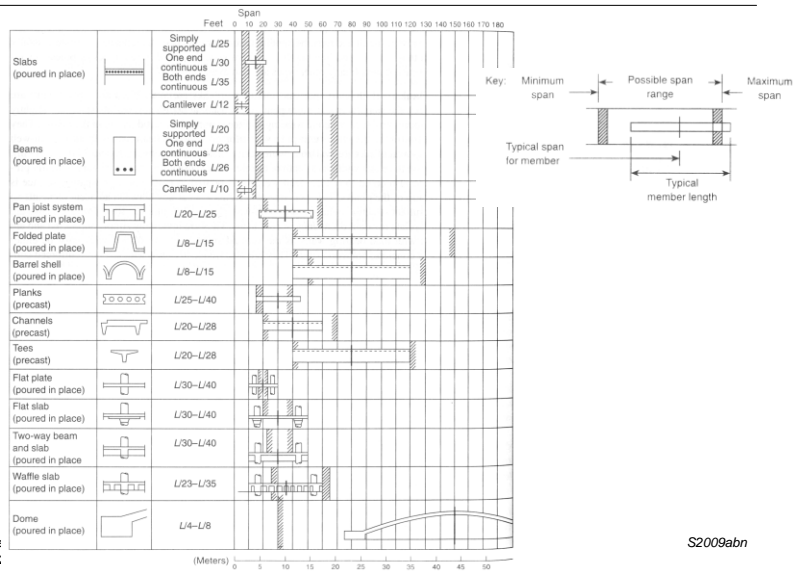
$$a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right)$$
    4. repeat from 2. until  $a$  from 3. matches  $a$  in 2.

## Reinforcement

- min for crack control
- required  $A_s = \frac{3\sqrt{f'_c}}{f_y} (bd)$
- not less than  $A_s = \frac{200}{f_y} (bd)$
- $A_{s-max} : a = \beta_1 (0.375d)$
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing



# Approximate Depths



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