

**MODELING
METHODOLOGY****A Theory of Monitoring Credit Risk****Author**

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Abstract

On any given day, credit analysts monitor multiple names. Some names have been reviewed recently, but not all. Some names represent large exposures, while others are small. Some are known high credit risks, while others are low credit risks. The risk profile of some exposures may have changed recently, while others remain unchanged. Exposures to some names can be meaningfully reduced, while some names are more difficult to hedge. Which names should the analyst review first? Next? Take action on? Ultimately, how does the institution value a credit review?

Information's value lies in its potential to change actions. This paper focuses on measuring the economic value of a review and deriving an optimal review policy. We develop a framework for constructing a review strategy as well as its associated costs and benefits. For specific assumptions, our methodology derives an optimal monitoring strategy. The actual value of the information depends upon the risk return profile of the loan, the cost of obtaining the information, and the ability of the lender to reduce their exposure to the borrower if the review is negative. Timely information regarding credit risk is especially valuable when credit risk is elevated (the return may or may not justify the risk), and the bank can meaningfully reduce their exposure to the borrower.

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1. Introduction

Text Information's value lies in its potential to change actions, and credit managers act based upon updated information. Actions may include originating new loans, extending additional credit to existing borrowers, not refinancing a maturing term loan, cutting lines of credit, buying insurance against a borrower defaulting, selling the exposure to another financial institution, working with the borrower to establish an alternative source of financing, and finally, securitizing the exposure off the balance sheet via a collateralized loan obligation. In consumer credit, managers may make loan decisions using only quantitative risk metrics (i.e., a credit score). In contrast, financial institutions use a combination of quantitative risk metrics and qualitative information to manage risk. These analyses require resources that have a direct cost.¹

Banks lend in illiquid markets, with access to information not publicly available. In the *middle market*, a borrower typically uses one bank.² There is scope for a bank to reduce its exposure to a borrower should the borrower receive a negative review. The borrower often utilizes a mixture of revolving lines of credit and term loans, and standard bank practice is to review the relationship annually. If a bank decides to reduce exposure to a specific borrower, they can demand more collateral, cut lines of credit, increase reporting requirements, and/or refuse to renew the relationship. When a bank pursues such a strategy, the borrower typically shops for another bank with either a different *risk appetite* or a different *view* of its credit risk.³ Should the borrower successfully find another bank, the lender may suffer little to no loss on the outstanding principle.

Risk management departments review credits periodically, according to specific requirements set by regulators and financial institutions. The OCC prescribes reviewing high risk credits more frequently. It allows for smaller, performing credits to be reviewed less often (page 9, Rating Credit Risk, OCC, 2001).

This paper focuses on measuring the economic value of a review and deriving an optimal review policy within a theoretical setting. In the core framework, prior to the review, the credit analyst has a partial information PD with which to assess the risk-return trade-off of the exposure. If the credit analyst chooses to conduct the review, the credit analyst has a full information PD with better discriminatory power. The outcome of the review is both a new and more informative PD, as well as a decision regarding whether or not to take action on the exposure; the lender's action is to sell or hold the exposure. If the lender sells the exposure, he loses the spread income while preserving the principal.

Ex post, the review adds value when it leads a manager to a different action — to either hold a name they would otherwise sell or to sell a name they would otherwise hold. The *ex ante* value of the review per unit of exposure is the difference of the expected return on the loan under full information with optimal selling versus partial information with optimal selling, given partial information. The value of the review is greatest for the names the portfolio manager is indifferent between holding and selling under partial information.

We extend the framework to include a time dimension. The information content of a review erodes over time. We derive an optimal review policy, in which some relationships are reviewed frequently and others less often. We also consider the impact that costs associated with terminating the relationship have on the optimal review strategy.

We show how the optimal review strategy changes for different types of exposures. We first look at how the strategy changes when holding the spread on the loan constant. We then characterize how the review strategy changes when the loan spread reflects the "partial information PD" but not the "full information PD." Such a setting can be thought of as a competitive equilibrium, in which prices reflect "public information" but not "private information." The safest loans are reviewed less often than the riskier loans.

We contrast the value of the relationship under an optimal review policy with the value of the relationship under an annual, fixed review cycle. The incremental value of the review is greatest when the optimal review policy is to review the loan more than once each year. This multiyear review occurs for loans with elevated risk, whose private information places them close to the termination threshold.

Relation to the Literature

Much research deals with incomplete information regarding credit risk (cf., Duffie and Lando, 2001; or Giesecke and Goldberg, 2004). These papers specify an exogenous arrival process for information. In this paper, information is costly, and we model the

¹ Dwyer and Russell (2010) present one approach to combining qualitative factors with a quantitative risk metric.

² There are many definitions of a "middle market" borrower, but most definitions consider a firm with sales \$2mm to \$100mm as a middle market borrower. A middle market borrower is generally large enough so that the finances of the firm are independent of the finances of the owner. A middle market borrower is generally not large enough to have either listed equity or an agency rating.

³ Stephanie Simon, reporting in the *Wall Street Journal*, provides one such example in "Town's Friendly Bank Left a Nasty Mess" (June 16, 2009).

decision on how often to update one's information. To our knowledge, treating the acquisition of information as endogenous is new to the credit risk literature.

This paper draws upon two different bodies of literature: power curves and the economics of inaction. Researchers use power curves to quantify the predictive power of a medical test or a credit risk model. We draw from the economics of inaction literature to derive the optimal dynamic policy for reviewing a credit.

In medical testing, the outcome of a test can be a continuous variable, for which higher levels are associated with higher incidences of a disease (e.g., higher blood pressure is associated with a higher likelihood of heart disease). One may want to determine a threshold for the test outcome. If the test outcome is above this threshold, the disease is treated (e.g., medication is given to prevent heart disease) and left untreated, otherwise. In determining this threshold, one considers the rates of true positives (treating someone who does have the disease), false positives, true negatives, and false negatives implied by the threshold, as well as their associated costs and benefits, then optimizes accordingly. Power curves are analytic tools used in the biostatistics literature for this purpose.⁴ Roger Stein (2005) drew upon this literature to determine optimal lending policies in the context of managing credit risk. He also showed how to compute the benefit to a financial institution of using one model relative to another in a variety of settings. This paper uses the same tools, but takes a step back. It views the "test" as having a cost to administer, and it considers whether or not the value of knowing the outcome justifies the cost.

Economists have developed theories in which it is optimal for agents to do nothing for periods of time in a dynamic setting (c.f., Stokey, 2009). When an action has a fixed cost, one can typically derive an optimal range of inaction. Economists use such theories to explain stickiness in prices and wages, investment spikes, and the slow adoption of new technologies. In this line of research, one paper explicitly models the fixed cost as the cost of acquiring and analyzing updated information (Reis, 2006). Reis models the fixed cost of changing a price as the cost of updating one's information, and the new information is used to reset the price to an optimal level. He focuses on explaining the dynamics of inflation. While the contexts differ, the structures of the dynamic programming problems are similar: we both model the optimal wait time to update one's information in an infinite horizon setting.

The remainder of the paper is organized as follows:

- Section 2 presents the basic framework of the model, in which there is both a partial information PD and a full information PD. We show how to derive the CAP plots as well as the value of information in a static setting.
- Section 3 incorporates a time dimension and shows how to solve for the optimal review policy in a dynamic setting.
- Section 4 provides partial equilibrium comparative analysis that shows how the optimal review policy changes as the scope for risk mitigation changes.
- Section 5 analyzes an equilibrium setting (prices reflect public information)
- Section 6 looks at the question: how much value does pursuing an optimal review strategy create?
- Section 7 provides concluding remarks.
- Appendix A outlines symbols used in the paper.
- Appendix B derives the distribution of PDs and the Cumulative Accuracy Profiles implied by this framework.
- Appendix C describes the algorithm used to calibrate the model parameters for the benchmark case in the dynamic setting.

2. The Model

In this section, we model how to determine the value of information for specific names as a one period problem. We presume the resources required to execute a review are costly, and the outcome may be a decision to sell. The credit analyst should choose to sell the exposure if the credit risk is very elevated relative to the sum of the spread and the selling cost. The credit analyst can make this decision using either partial information or full information. The marginal cost of the partial information is assumed to be zero, whereas, obtaining full information for a specific credit has a cost. We compute the value of information per unit of exposure as the difference between the expected return under full information, where the credit analyst acts optimally, and the

⁴ Power curves include Receiver Operator Characteristic curves (ROC curves) and Cumulative Accuracy Profiles (CAP curves).

return based upon partial information, where the credit analyst acts optimally using the available information. In this setting, the information is valuable when it leads an analyst to act differently.

2.1 The Basic Framework

Suppose you have an exposure with a full information PD defined by:

$$N(\beta_0 + \mathbf{X}\beta)$$

In this equation, \mathbf{X} is a row vector, the elements of the row vector are standard normal variables independent of each other, β is a column vector of coefficients, and β_0 is a scalar, i.e., the intercept.⁵ The function, N , is the standard cumulative normal distribution. Without loss of generality, we assume that all the elements of β are positive, so that large values of \mathbf{X} correspond to elevated credit risk. We assume that the intercept, β_0 , is negative, which implies that the median credit has a probability of default of less than 50%. We can give a latent variable interpretation to the full information PD as follows. Let ε be a latent variable that has a standard normal distribution. Default occurs if:

$$\varepsilon < \beta_0 + \mathbf{X}\beta$$

The probability of ε being less than $\beta_0 + \mathbf{X}\beta$ is given by $N(\beta_0 + \mathbf{X}\beta)$, i.e., default occurs when there is a sufficiently bad realization of the latent variable ε . Partition \mathbf{x} into two matrices Y and Z , so that $\mathbf{X}\beta = Y\gamma + Z\lambda$, where Y is known and Z is unknown.⁶ Using this notation, we define the full information probability of default as:

$$PD_{PI}(Y; \beta_0, \gamma, \lambda) = N\left(\frac{\beta_0 + Y\gamma}{\sqrt{1 + \lambda'\lambda}}\right) \quad (1)$$

We now derive the partial information PD. Default occurs when:

$$\varepsilon < \beta_0 + Y\gamma + Z\lambda$$

or

$$\frac{\varepsilon - Z\lambda}{\sqrt{1 + \lambda'\lambda}} < \frac{\beta_0 + Y\gamma}{\sqrt{1 + \lambda'\lambda}}$$

Note, the left-hand side of this last equation has a standard normal distribution. Therefore, the partial information PD is given by:

$$PD_{PI}(Y; \beta_0, \gamma, \lambda) = N\left(\frac{\beta_0 + Y\gamma}{\sqrt{1 + \lambda'\lambda}}\right) \quad (2)$$

By the same argument, if both Y and Z are unknown, then the PD is given by $N\left(\frac{\beta_0}{\sqrt{1 + \beta'\beta}}\right)$. For convenience, we refer to this quantity as the *CDT*, the Central Default Tendency. The *CDT* is the average default rate of the population.

⁵ For purposes of this paper, I treat β_0 and the vector β as known. In practice, they are estimated, and there are a number of challenging issues remaining in how to estimate these parameters. Incorporating this type of uncertainty into the framework is outside the scope of this paper, but a possible topic for future research.

⁶ For the examples in this paper, \mathbf{X} is a row vector of two elements, so that Y and Z are scalars (single element vectors). In other contexts, \mathbf{X} may have more than two elements, and Y is the subset of \mathbf{X} that is known, and Z is the subset of \mathbf{X} that is unknown.

It will be convenient to define $\hat{\gamma} = \gamma(1 + \lambda'\lambda)^{-1/2}$; $\hat{\beta}_0 = \beta_0(1 + \lambda'\lambda)^{-1/2}$ so the partial information PD can be written as $N(\hat{\beta}_0 + Y\hat{\gamma})$.

2.2 Distribution of PDs and Cumulative Accuracy Profiles

In Appendix B, we construct the distribution of the partial information PDs and the full information PDs. We also derive an expression for the Cumulative Accuracy Profile (CAP curves) and the corresponding Accuracy Ratio (AR). These constructs allow us to calibrate the parameters using empirical reference points. We show that the quantile functions of the full information and partial information PDs are given by:

$$F_{FI}(q) = N\left(N^{-1}(q)\sqrt{\beta'\beta} + \beta_0\right) \text{ and } F_{PI}(q) = N\left(N^{-1}(q)\sqrt{\hat{\gamma}'\hat{\gamma}} + \hat{\beta}_0\right)$$

These functions take a percentile as an input and produce the PD associated with that percentile. For example, if $q=0.975$, then 97.5% of the population has a full information PD of less than $F_{FI}(0.975)$. The CAP curve for the full information and partial information PDs are given by:

$$CAP_{FI}(x) = \int_0^x N\left(-N^{-1}(q)\sqrt{\beta'\beta} + \beta_0\right) dq / CDT$$

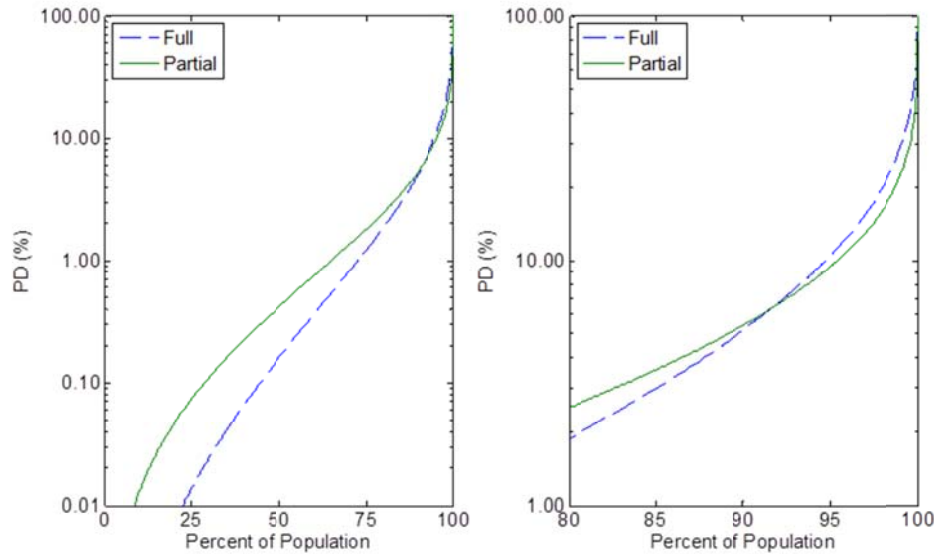
and

$$CAP_{PI}(x) = \int_0^x N\left(-N^{-1}(q)\sqrt{\hat{\gamma}'\hat{\gamma}} + \hat{\beta}_0\right) dq / CDT$$

Figure 1 presents the quantile function for full information and partial information PDs based on a CDT of 2% and γ of 0.9 and λ of 0.5. Under this parameterization, the AR of the partial information PD is 77.8%, and for the full information PD, the AR is 85.5%. An AR of 85.5% is roughly comparable with the accuracy ratio of the Moody's KMV public firm model on a large sample at a one-year horizon and exceeds the accuracy ratios typically found in private firm models. An AR of 77.8% is roughly comparable to the AR that can be achieved using a financial statement-based model on a comparable sample. A population default rate of 2% is a reasonable value for the long-run average default rate for a Commercial and Industrial portfolio.⁷ The figure compares the distribution of the full information PD with that of the partial information PD. We find that the distribution of the full information PD is more dispersed than the distribution of partial information PDs. Figure 1's right hand panel focuses on upper percentiles of the distribution. Note, the partial information PD is larger than the full information PD for 91% of the sample, but for the riskiest names, the full information PD is larger than the partial information PD. Also note that the median partial information PD (about 0.40%) is three times larger than the median full information PD (about 0.13%).

⁷ See Sections 2.4 and 4.8 of Dwyer and Zhao (2009). In this paper, we find that a model that uses financial statement information alone achieves an AR of 79.1% (RiskCalc Large Firm Model). Whereas, the Moody's KMV Public Firm model achieves an AR of 84.2% on the same sample. Accuracy ratios vary tremendously from sample to sample and from model to model. The reason for using this pair of AR ratios is the following: the RiskCalc Large Firm Model utilizes information drawn from financial statements. It is intended to be used on firms large enough to have traded common stock but do not. The Moody's KMV public firm model uses equity price information to predict default, and it has a higher AR than the RC large firm model on a common sample. One can think about the RC EDF (the PD generated by the RiskCalc Large Firm model) as being a "partial information PD" as it incorporates only financial statement information. The KMV EDF (the EDF generated by the Moody's KMV Public Firm Model) utilizes equity price information has higher discriminatory power than the RC EDF. Therefore, the KMV EDF credit measure is closer to a "full information PD" than the RiskCalc Large Firm EDF.

Figure 1 Quantile Function for Full and Partial Information PD



2.3 Determining the Value of Information

Consider an exposure that pays back the principal plus a coupon in one time period (e.g., one year) from now. We refer to the principal as the exposure or size of the loan. The coupon is equal to $s + r$ per unit exposure, where r is the risk free rate, and s is the spread over the risk free rate. The transaction cost associated with selling the loan is h . For convenience, we assume that the coupon is paid whether or not default occurs, and, if default occurs, the LGD is 100% of the principal. We assume that the risk manager is risk-neutral, and that the risk manager currently holds the exposure. The risk manager sells the exposure if the probability of default exceeds the spread plus the transaction cost. Under full information, the risk manager sells the exposure if the risk of holding the exposure (the full information PD) exceeds the return on holding the exposure, plus the transaction cost ($s+h$):

$$N(\beta_0 + Y\gamma + Z\lambda) > s + h$$

If the complete information PD is not available, the risk manager sells the exposure if:

$$N(\hat{\beta}_0 + Y\hat{\gamma}) > s + h.$$

Before deriving the value of information, in general, going through a specific example is instructive.

Suppose that the transaction cost is 0%, the spread earned on the loan equals 2%, and the partial information PD is 2.01%. If full information is not available, then the risk manager sells the exposure, in which case he loses 0% versus an expected loss of 0.01% associated with holding the position.

Let the vectors Y and Z each have one element, and let $\gamma=0.9$ and $\lambda=0.5$. We can solve for β_0 so that the default rate of the population is 2%:

$$\beta_0 = N^{-1}(.02)\sqrt{1 + \gamma_1^2 + \lambda_1^2} = -2.948$$

$$\hat{\beta}_0 = \frac{\beta_0}{\sqrt{1 + \lambda_1^2}} = -2.636$$

$$\hat{\gamma}_1 = \frac{\gamma_1}{\sqrt{1 + \lambda_1^2}} = 0.805$$

The partial information PD is given by:

$$N(-2.636 + 0.805 \times Y)$$

One can verify that if Y equals 0.726, the partial information PD is 2.01%. As Y has a standard normal distribution, this implies that 76.6% ($N(0.726)$) of the population has a partial information PD of less than or equal to 2.01%. Suppose that the risk manager conducts a loan review and obtains Z , and that the realized value of Z turns out to be 0. Then the full information PD is given by $N(-2.948 + 0.9 \times 0.726 + 0.5 \times 0)$ or 1.090%. Therefore, a *neutral realization* for Z lowers the PD of the exposure and leads a risk manager to hold the exposure. In this case, the expected return on the exposure is now 0.91%. The additional information saves the risk manager 0.91% (the expected return on the loan). For this example, whether or not the *fully-informed risk manager* chooses to hedge the loan depends on the realization of Z . The *fully-informed risk manager* sells the exposure if the full information PD exceeds 2%:

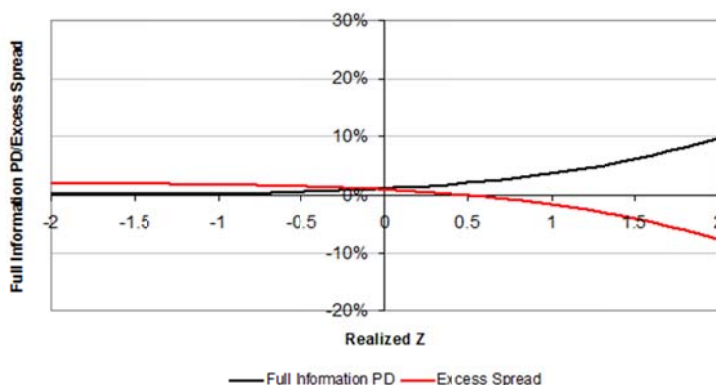
$$N(-2.948 + 0.9 \times 0.726 + 0.5 \times Z) > 2\%$$

Taking the normal inverse of both sides and solving for Z reveals that the fully informed risk manager hedges whenever Z is greater than 0.480, approximately 31.6% of the time. In this example, full information leads the risk manager to do what he would have done under partial information approximately 31.6% of the time and to do something different 68.4% of the time, i.e., he chooses not to hedge the loan. If he chooses to conduct a credit review, the value of the credit review equals the probability that he acts differently as a result of the review (he does not hedge the loan), $N(0.480)$, multiplied by the expected gain from not hedging the loan, given that he does not hedge the loan. The value of information is given by:

$$N(0.480) \left(0.02 - \frac{\int_{-\infty}^{0.480} N(-2.948 + 0.9 \times 0.726 + 0.5 \times Z) n(Z) dZ}{N(0.480)} \right)$$

where n is the density function of a standard normal distribution. Using numerical techniques to evaluate the integral, we find that the value of information is 0.860% for this example. Therefore, the risk manager would pay up to 0.860% of the amount of the exposure for knowledge of Z . Figure 2 depicts this example plotting the full information PD and the excess spread, $h+s-PD_{FI}$, as a function of Z . Note, it is a skewed distribution, in that the downside associated with large realizations of Z is larger than the upside associated with positive realizations of Z . Under partial information, the value of the loan is close to zero. Under full information, the value of the loan equals the probability of the excess spread being positive multiplied by the expectation of the distribution of the excess spread truncated to the positive region.

Figure 2 Full Information PD and Excess Spread, as a function of Z



The general solution to this example for the case that the partial information PD leads the risk manager to sell the exposure (partial information PD > $h + s$) is:

$$N(cv) \left((h + s) - \frac{\int_{-\infty}^{cv} N(\beta_0 + Y\gamma + Z\lambda) n(Z) dZ}{N(cv)} \right)$$

where

$$cv = \frac{N^{-1}(h+s) - (\beta_0 + Y\gamma)}{\lambda}$$

Here, cv is the realized value of Z that leads the risk manager to be indifferent between hedging and not hedging. $N(cv)$ is the probability that the risk manager does not hedge (i.e., the probability of doing something different), $h + s$ is the spread on the loan

plus the transaction cost, and $\frac{\int_{-\infty}^{cv} N(\beta_0 + Y\gamma + Z\lambda)n(Z)dZ}{N(cv)}$ is the expected loss on the loan, conditional on a good outcome for the review (i.e., $Z < cv$).

For the case in which the partial information PD leads the risk manager to hold the loan (partial information $PD < h + s$) the value of information is

$$(1 - N(cv)) \left(\frac{\int_{cv}^{\infty} N(\beta_0 + Y\gamma + Z\lambda)n(Z)dZ}{1 - N(cv)} - s - h \right)$$

In this expression, each term has an analogous interpretation as in the prior expression. $(1 - N(cv))$ is the probability that the risk manager sells the loan (i.e., the probability of doing something different), $\frac{\int_{cv}^{\infty} N(\beta_0 + Y\gamma + Z\lambda)n(Z)dZ}{1 - N(cv)}$ is the expected loss on the loan conditional on a bad outcome for the review (i.e., $Z > cv$), this loss is saved as the loan is sold, and the spread and the transaction cost are subtracted as this money when the loan is sold.

In the general case, the value of information can be expressed as:

$$VI(Y, h, s; \beta_0, \gamma, \lambda) = (PD_{PI} > h + s) N(cv) \left(h + s - \frac{\int_{-\infty}^{cv} N(\beta_0 + Y\gamma + Z\lambda)n(Z)dZ}{N(cv)} \right) + (PD_{PI} < h + s) (1 - N(cv)) \left(\frac{\int_{cv}^{\infty} N(\beta_0 + Y\gamma + Z\lambda)n(Z)dZ}{N(cv)} - h - s \right) \tag{3}$$

where cv and PD_{PI} are defined as above, and (x) takes on a value of 1, if the statement x is true and 0 otherwise. The value of information is expressed as a proportion of the exposure.

Table 1 VALUE OF THE REVIEW (BPS)

		Partial Information PD								
		0.10%	0.20%	0.50%	1.00%	2.00%	5.00%	10.00%	20.00%	50.00%
h+s (bps)	10	6	4	2	1	0	0	0	0	0
	20	4	11	6	3	1	0	0	0	0
	50	2	6	25	16	7	1	0	0	0
	100	1	3	16	47	27	7	1	0	0
	200	0	1	7	27	86	33	8	1	0
	500	0	0	1	7	33	186	75	14	0

1000	0	0	0	1	8	75	320	94	1
2000	0	0	0	0	1	14	94	515	22
5000	0	0	0	0	0	0	1	22	738

Table 1 presents the value of information as a function of $(h+s)$ and the partial information PD for a CDT of 2% and γ of 0.9 and λ of 0.5. Note, holding the partial information PD constant (moving down a column), the value of information is greatest where the firm is indifferent between holding and selling the exposure under partial information. Further, note that the maximum value of information for each partial information PD increases with the PD. The value of information has the potential to be the largest for the loans with the largest expected losses. The value of information is fairly sizable. For example, suppose a financial institution has a \$10mm exposure to a firm with a partial information PD of 2% and a spread of 100bps. Under this framework, the financial institution should review the loan provided the cost of the review is less than \$27,000.

Such a table could be used to determine the exposures to review first – to triage. Nevertheless, up to this point, the framework abstracts away from the fact that credit analysts manage a portfolio over time. We now turn to incorporating a time dimension into the framework.

3. Incorporating a Time Dimension

Banks manage credit risk through time. In this section, we show how to incorporate a time dimension into the problem by assuming that both the terms of the loan and the public information are fixed over time, but the private information varies over time. We provide a general setup of the problem using transition matrices.⁸ In subsequent sections, we also characterize the solution to the framework within the context of the price being fixed, as well as the context in which prices reflect public information. In Section 7, we characterize the value of information as the value of the relationship with the borrower under the optimal strategy versus the value of the relationship under an alternative review strategy.

The Dynamic Programming Problem

Suppose a bank has an option to lend to a specific borrower at a fixed spread until the lender defaults. Suppose that the partial information for the borrower is time-invariant, U , while the additional piece of information required to determine the full information PD, S_t , changes over time.

The timing of events within the period is as follows. Going into the period, there exists a value of S_t . At the start of the period, the lender first chooses whether or not to review the loan. After reviewing the loan (or not), the lender can choose to terminate the relationship for a specified cost. Then, the borrower defaults (or not), with the probability of default being determined by S_t . If the lender chooses to review the loan in period t , they know the exact value of S_t , and carry this knowledge into the next period. Finally, the new value of S_{t+1} is realized at the end of the period. If the lender chooses to terminate the relationship, the lender forever loses the option to lend to the borrower. There is a cost of termination that may depend on S_t . Payments made j periods from now are discounted at β^j .

Let there be N possible states, and let the i -th element of the vector \mathbf{S} imply the value of S for state i with S increasing in i . Let "state N " be the absorbing state that implies default. Let $\mathbf{D}(j)$ be an N element row vector, where the i th element represents the probability that the borrower is in state i in period j . This vector will sum to one. Let \mathbf{A} be the $N \times N$ transition matrix, where the element in the i th row and k th column denotes the probability of transitioning from state i to state k . The rows of \mathbf{A} will sum to 1. As the last state is the absorbing state, the last column of this matrix is the probability of default from each state. For convenience, let \mathbf{PD} denote the last column of \mathbf{A} . The last row of \mathbf{A} represents transitions from the absorbing state and is therefore a vector of zeros except for the last element which is 1. Given the initial distribution, $\mathbf{D}(0)$, the distribution across the states in period j is given by:

$$\mathbf{D}(j) = \mathbf{D}(0)\mathbf{A}^j$$

⁸ We could have set up the problem in continuous time and a continuous state space. There are pros and cons to both approaches. For a continuous time and a continuous state space, analytic solutions to the dynamic programming problem are very useful when they are attainable, but when they are not, one typically needs to employ analytic methods to solve the problem, in general, and the numerical methods involve discrete approximations. We will show how continuous time processes can be approximated within this framework. In fact, the limit of these discrete time approximations define the continuous time and space analogues. Whether or not one can obtain an analytic solution to the dynamic programming problems considered in this paper is an open question.

The expected cash flows for each credit state is given by the $N \times 1$ vector \mathbf{II} . Each element of \mathbf{II} could be the spread on the loan less the expected loss associated with each state ($PD_i \times LGD_i$). The last element of \mathbf{II} represents the cash flows from a borrower who has already defaulted, which will be zero in the examples we consider.

Let CoR be the cost of a review, which is constant over time. Let CoT be an $N \times 1$ vector that represents the cost of terminating the relationship, which can depend upon the credit state.

Before setting up the Bellman equation, we first describe our approach to solving the dynamic programming problem. One way to think about dynamic programming is to divide an infinite horizon problem that has many decisions at many points in time into a two-step problem, in which, there is one decision. The value function at the start of the current period is written as the sum of the payoffs during the current period plus the expected discounted value of the value function at the start of the next period. A strategy specifies what actions the agent will take in every possible state of the world. One can show that a "candidate-optimal strategy" is an optimal strategy, if one can show that in every state of the world the next decision according to the strategy is the optimal decision, given that one follows the "candidate-optimal strategy" for all the other future decisions.

In the current context, the next decision becomes when to take the next action, which could be either to review the loan or to terminate the relationship. Our procedure for finding the optimal strategy is the following. Upon completing a review, for each state of the world (for each value of S), there is one dynamic decision, which is how long to wait to conduct the next action (review or terminate). A "strategy" specifies how long to wait to take the next action and what the action will be. Let \mathbf{R}_0 be an $N \times 2$ matrix denoting the initial strategy. The first column specifies how long to wait to perform an action. The second column specifies which action to take: review the loan or terminate the relationship. If the strategy for credit state i is to wait five periods to do the next review, then $\mathbf{R}(i,1) = 5$, and $\mathbf{R}(i,2) = 1$. If the strategy for credit state i is terminate the relationship immediately, then $\mathbf{R}(i,1) = 0$ and $\mathbf{R}(i,2) = 2$. For each state of the world and a given review strategy (\mathbf{R}_0), one can compute the optimal waiting times assuming that one will follow \mathbf{R}_0 for every action thereafter. Let this strategy be denoted as \mathbf{R}_1 . If \mathbf{R}_1 does not equal \mathbf{R}_0 that indicates that \mathbf{R}_0 is not an optimal strategy, but \mathbf{R}_1 could be. We can test whether or not \mathbf{R}_1 is an optimal strategy by computing \mathbf{R}_2 using the same procedure. We can iterate and if we find an \mathbf{R}_k that is equal to \mathbf{R}_{k+1} then we have found the optimal strategy.

Let \mathbf{R}_k be a matrix representing a given review strategy. Let \mathbf{V}_k be the value of having a relationship with a borrower known to be in state i given review strategy \mathbf{R}_k . Define the i -th row of the matrix \mathbf{R}_{k+1} as:

$$\mathbf{R}_{k+1}(i,1) = \arg \max_{r>0} \left(\sum_{j=0}^{r-1} \beta^j \mathbf{D}(j) \Pi + \beta^r \max(\mathbf{D}(r) \mathbf{V}_k - CoR, -\mathbf{D}(r) CoT) \right) \quad (4)$$

and

$$\mathbf{R}_{k+1}(i,2) = \begin{cases} 1 & \text{if } \mathbf{D}(r) \mathbf{V}_k - CoR > -\mathbf{D}(r) CoT \\ 2 & \text{otherwise} \end{cases}$$

Define the i -th element of the vector \mathbf{V}_{k+1} as:

$$\mathbf{V}_{k+1}(i; \mathbf{R}_{k+1}) = \begin{cases} \sum_{j=0}^{\mathbf{R}_{k+1}(i,1)-1} \beta^j \mathbf{D}(j) \Pi + \beta^{\mathbf{R}_{k+1}(i,1)} (\mathbf{D}(\mathbf{R}_{k+1}(i,1)) \mathbf{V}_{k+1} - CoR) & \text{if } \mathbf{R}_{k+1}(i,2) = 1 \\ \sum_{j=0}^{\mathbf{R}_{k+1}(i,1)-1} \beta^j \mathbf{D}(j) \Pi - \beta^{\mathbf{R}_{k+1}(i,1)} \mathbf{D}(\mathbf{R}_{k+1}(i,1)) CoT & \text{if } \mathbf{R}_{k+1}(i,2) = 2 \end{cases} \quad (5)$$

In equation (4), the expression can be extended to include the strategy of setting $\mathbf{R}(i,1) = 0$ (taking immediate action) by setting

the summation term $\left(\sum_{j=0}^{\mathbf{R}(i,1)-1} \beta^j () \right)$ to zero when $r = 0$, and likewise for equation (5).

As equations (4) and (5) form the *crux* of the paper, I will review each component in detail.

The expression:

$$\mathbf{D}(j) \Pi$$

represents the expected discounted cash flows from the relationship in period j . The discounted sum of these terms,

$$\left(\sum_{j=0}^{r-1} \beta^j (\quad) \right),$$

is the expected value of the cash flows until the next action is taken.

The term $\beta^r (\max(\mathbf{D}(r)\mathbf{V}_k - \text{CoR}, -\mathbf{D}(r)\text{CoT})$ reflects the discounted expected value of the relationship after waiting r periods to take an action given the strategy \mathbf{R}_k . The value is the better of the two possible actions: review the loan or terminate the relationship prior to conducting the review. If the lender chooses to review the loan after r periods, the expected discounted value of the relationship in period r after the review is given by $\beta^r (\mathbf{D}(r)\mathbf{V}_k - \text{CoR})$. If one chooses to terminate the relationship after r periods without a review, the expected value of the relationship in period r is given by $-\beta^r \mathbf{D}(r)\text{CoT}$.

Equation (4) produces an optimal strategy, \mathbf{R}_{k+1} , for timing the next action given that strategy \mathbf{R}_k is followed for every action thereafter. Further, equation (4) takes \mathbf{V}_k as given. Equation (5) produces the discounted expected value, \mathbf{V}_{k+1} , associated with following the strategy \mathbf{R}_{k+1} for every action. Computationally, solving equation (4) is done by computing the value of each possible review strategy for each possible credit state i and then choosing the review strategy that produces the highest value. As \mathbf{V}_{k+1} is on both side of equation (5) we solve for \mathbf{V}_{k+1} iteratively.⁹ Note, the expression $\mathbf{D}(r)\mathbf{V}_k$ in equation (4) becomes

$\mathbf{D}(\mathbf{R}_{k+1}(i,1))\mathbf{V}_{k+1}$ in equation (5). In equation (4), we take \mathbf{V}_k as given and solve for time until the next review, r , which maximizes equation (4). In equation (5), we take the time until the next action, $\mathbf{D}(\mathbf{R}_{k+1}(i,1))$, as given and find the value of \mathbf{V}_{k+1} that solves the equation.

If $\mathbf{R}_{k+1} = \mathbf{R}_k$ then the strategy is optimal, and the vector \mathbf{V}_{k+1} is the value of having a relationship with this borrower for each possible initial state.¹⁰ One numerical approach to solving this value function is the following. First, compute the value function, \mathbf{V}_0 , using an \mathbf{R}_0 , in which every element is four (i.e., a review every four periods). Then solve for $\mathbf{R}_1, \mathbf{V}_1, \mathbf{R}_2$, and \mathbf{V}_2 . Test to determine if $\mathbf{V}_1 = \mathbf{V}_2$ and if not continue until convergence.

We write the dynamic programming problem so that the borrower chooses to wait to take an action and then chooses which action to take (review the loan or terminate the relationship without doing a review). It is possible that the outcome of the review could be so bad that the lender would choose to terminate the relationship immediately. If this were the case for credit states $k > i$, the optimal strategy would be $\mathbf{R}(k,1)=0$ and $\mathbf{R}(k,2)=2$ for all $k > i$, i.e., take immediate action and the action is to terminate the

relationship. Equations (4) and (5) still work in this case, provided we define $\left(\sum_{j=0}^{R(i,2)-1} \beta^j (\quad) \right)$ to be zero when $\mathbf{R}(i,2)=0$.

Remarks

This problem's setup for different "accounting." Depending upon how $\mathbf{\Pi}$ and the cost of termination are specified, \mathbf{V} can be interpreted differently. For the examples considered in this paper, $\mathbf{\Pi}$ represents cash flow per unit principal in excess of the risk free rate (i.e., the spread) less the expected loss per unit principal should the borrower default, and the Cost of Termination is the loss per unit principal from terminating the relationship. The termination cost will be non-negative and bounded above by 100%. Consequently, if \mathbf{V} is 2%, the lender would be indifferent to selling their relationship with the borrower for 102% of the principal.

This setup allows for the case that a lender is required to review a loan at least every x periods, but is free to review more often. One constrains the strategy space such that $\mathbf{R}(i,1)$ is less than or equal to x for all i .

3.1 Determining the Transition Matrix

Given the partial information on the borrower assumed to be fixed, \mathbf{U} , the value of the relationship depends upon the transition matrix that governs the migration of the time-varying information (\mathbf{S}_t) regarding credit risk. We use a discrete approximation for a

⁹ The iterative approach we use to compute \mathbf{V}_{k+1} is to use \mathbf{V}_k for the initial value for iteration and then compute the \mathbf{V}_{k+1} as the expected value until the next action, plus the discounted value of the loan at the time of the next action computed using \mathbf{V}_k (the value vector from the prior iteration). We iterate until convergence. For \mathbf{V}_0 , we start with the full information costless hedging value of the loan, which has an analytic solution:

$$\mathbf{V} = \text{Max}(\mathbf{\Pi}, 0) + \beta \mathbf{\Lambda} \mathbf{V} \Rightarrow \mathbf{V} = \text{Max}(\mathbf{\Pi}, 0) (\mathbf{I} - \beta \mathbf{\Lambda})^{-1}$$

where \mathbf{I} is an Identity matrix.

¹⁰ This result is an application of Theorem 9.2 of Stokey and Lucas (1989). Invoking this theorem requires that the transversality condition holds, which holds if all the elements of $\mathbf{\Pi}$ are finite and the discount factor is less than 1.

simple stochastic process to derive this transition matrix for the time-varying information (S_t). Let \mathbf{G} be the transition matrix Λ constructed using this setup described below.

Suppose the full information PD is given by:

$$PD_{FI,t}(U, S_t; \beta_0, \alpha, \psi) = N(\beta_0 + \alpha U + \psi S_t) \quad (6)$$

where U partially characterizes the credit risk of the borrower and is fixed for each borrower. We allow S_t to vary with time, and S_t is what the credit analyst learns when conducting the review at time t . The time varying information, S_t , follows an Ornstein-Uhlenbeck process that has a long run mean of 0 and long run variance of $(2\rho)^{-1}$:

$$dS_t = -\rho S_{t-1} + dW_t$$

where dW is a standard Wiener process.¹¹

This process is the continuous time analogue to an AR1 process, sometimes termed an “elastic random walk,” in that, S_t is always being pulled back to the long term mean of 0 (c.f., Vasicek, 1977). Given S_{t-x} , the expectation and variance of S_t are given by:

$$E(S_t | S_{t-x}) = S_{t-x} \exp(-\rho x) \text{ and } Var(S_t | S_{t-x}) = \frac{1 - \exp(-2\rho x)}{2\rho}$$

One can represent a discrete approximation to an Ornstein-Uhlenbeck process as follows. Subdivide a time interval T into n equal partitions. Let the time increment, Δ_n , be defined as T/n . Define a step size as $h_n = \sqrt{\Delta_n}$, and a grid

$\mathbf{X} = \{\dots, -2h_n, -h_n, 0, h_n, 2h_n, \dots\}$. If the probability of moving from x_i to x_{i+1} is equal to $0.5 + \frac{\rho\Delta}{2h} x_i$ and the probability of

moving from x_i to x_{i-1} is given by $0.5 - \frac{\rho\Delta}{2h} x_i$ then this process converges to an Ornstein-Uhlenbeck process with a mean of 0

and an unconditional variance of $1/(2\rho)$ as n goes to infinity (c.f., Stokoy, 2009, pages 27-28). In order to specify this process as a transition matrix, we define a state space that has a finite number of elements by choosing the smallest S_j and the largest S_M , such that, the state space contains plus or minus five standard deviations of the unconditional variance of the process. Then the

transition matrix, \mathbf{T} can be defined as $\mathbf{T}_{ii-1} = 0.5 + \frac{\rho\Delta}{2h} S_i$ for $i \in \{2, \dots, M\}$, $\mathbf{T}_{ii+1} = 0.5 - \frac{\rho\Delta}{2h} S_i$ for $i \in \{1, \dots, M-1\}$,

$\mathbf{T}_{11} = 0.5 + \frac{\rho\Delta}{2h} S_1$, $\mathbf{T}_{MM} = 0.5 - \frac{\rho\Delta}{2h} S_M$, and zero elsewhere. We use this transition matrix to build the transition matrix \mathbf{G} .

The timing of events is important. We assume that $T=1$, and we interpret this time interval to represent a quarter of a year. We assume that the probability of default within the quarter is determined by the state at the beginning of the quarter. Further, the probability of transition from one state to another state conditional upon not defaulting is represented by \mathbf{T}^n . We now construct the matrix \mathbf{G} . Define the column vector $\mathbf{PD} = [PD_{FI}(S_1), PD_{FI}(S_2), \dots, PD_{FI}(S_M)]'$. Let $\mathbf{1}$ be a column vector of ones with M elements and $\mathbf{0}$ be row vectors of zeros with M elements. The transition matrix \mathbf{G} can then be constructed as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{T}^n \cdot * (\mathbf{1} - \mathbf{PD}) & \mathbf{PD} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

The last row represents the probability of transitioning from default to another state, and the probability of transitioning from a default state to a default state is 1. Note, we use $A \cdot * B$ to represent element by element multiplication of the matrix A by the column vector B .¹² In order to ensure that the other rows sum to 1, we multiple each element of the matrix \mathbf{T}^n by one minus the probability of default.

¹¹ We choose the Ornstein-Uhlenbeck process as it is one of the simplest continuous time mean reverting processes.

¹² This notation is used in Matlab.

This transition matrix, \mathbf{G} , can be specified using $\{\beta_0, \alpha U, \psi, \rho\}$ and a value of n . For a small n , the state space is “discrete,” and, as T and n become large, the state space converges to the continuous time analogue. Loosely speaking, the parameter β_0 governs the default rate of the population, αU is the fixed component of the credit risk of the borrower, ψ determines that importance of the time varying information for determining default risk, and ρ determines the persistence of the time varying information. In Appendix C, we assume that U has a discrete approximation to a standard normal distribution in the population, and we argue that certain parameter values for $\{\beta_0, \alpha, \psi, \rho\}$ and a value of n (we use an n of 20) are consistent with specific Central Default Tendencies and Partial and Full Information Accuracy Ratios to provide some empirical reference points. In order to do so, we need to make additional assumptions on what happens to borrowers that default and the distribution of S for new borrowers.

4. Comparative Analyses

In the problem outlined in Section 3, the lender takes as given the $\mathbf{\Pi}$ and \mathbf{CoT} . These parameters are more realistically viewed as being determined by prices, which are, in turn, determined by market forces (e.g., a market equilibrium). In future sections, we consider examples where prices reflect public information. Before doing so, we provide an example of a solution to the dynamic programming problem and show how the solution changes as different parameters change. In this section, the lender takes the $\mathbf{\Pi}$ and \mathbf{CoT} as given and prices need not reflect public information.

Table 2 TWO SCENARIOS AND ONE COMPARATIVE ANALYSIS

Comparative Analyses	Scenario A	Scenario B
Case #1 The relationship can be terminated without cost (Scenario B) versus a cost of termination equal to 10% of the principal (Scenario C).	Scenario A $\mathbf{\Pi} = \text{spread} - \mathbf{EL}$, $\mathbf{\Pi}_H = \mathbf{\Pi}$ $\mathbf{CoT} = 0$	Scenario B $\mathbf{\Pi} = \text{spread} - \mathbf{EL}$, $\mathbf{\Pi}_H = \mathbf{\Pi}$ $\mathbf{CoT} = 0.1$
Baseline Parameters: U=1.5 $PD_t = N(-3.51+0.83U+0.31S_t)$; LGD=0.67; EL=PD*LGD; = $\exp(\log(.5)/4)$; =0.98; coR=0.005; spread=0.0142;		

Baseline Parameters:

U=1.5
 $PD_t = N(-3.51+0.83U+0.31S_t)$; LGD=0.67; EL=PD*LGD;
= $\exp(\log(.5)/4)$; =0.98; coR=0.005; spread=0.0142;

We solve for the optimal monitoring policy for two different scenarios and compare the solutions side-by-side. We choose the baseline parameters as reasonable starting points in which a time interval is one quarter of a year. In Appendix C, we argue that the baseline parameters are consistent with a full information accuracy ratio of 85.5, a partial information AR of 77.8, and a CDT of 2.0%. In all scenarios, the lender reviews the loans a minimum of every 100 quarters.¹³ In these comparative analyses, we look at the implications of different risk mitigation mechanisms on the optimal monitoring policy.

In Scenario A, the lender can permanently terminate the relationship at any time period prior to default at no cost. This scenario makes the most sense when the lender can refuse to refinance the borrower, and, there are other lenders with different risk tolerances willing to take on the borrower. In Scenario B, there is an explicit termination cost of 10% of the principal amount of the loan, provided the lender terminates the relationship before the actual default event. This scenario makes sense for secured lending, when collateral is worth 90% of the principal, provided the lender terminates the relationship prior to the borrower actually defaulting.

¹³ This restriction is a computational convenience, which could be relaxed. It becomes a binding constraint when either the cost of review is high or the risk mitigation options available to the lender are limited.

Figure 3 Comparative Analysis: No Termination Costs versus Termination Costs

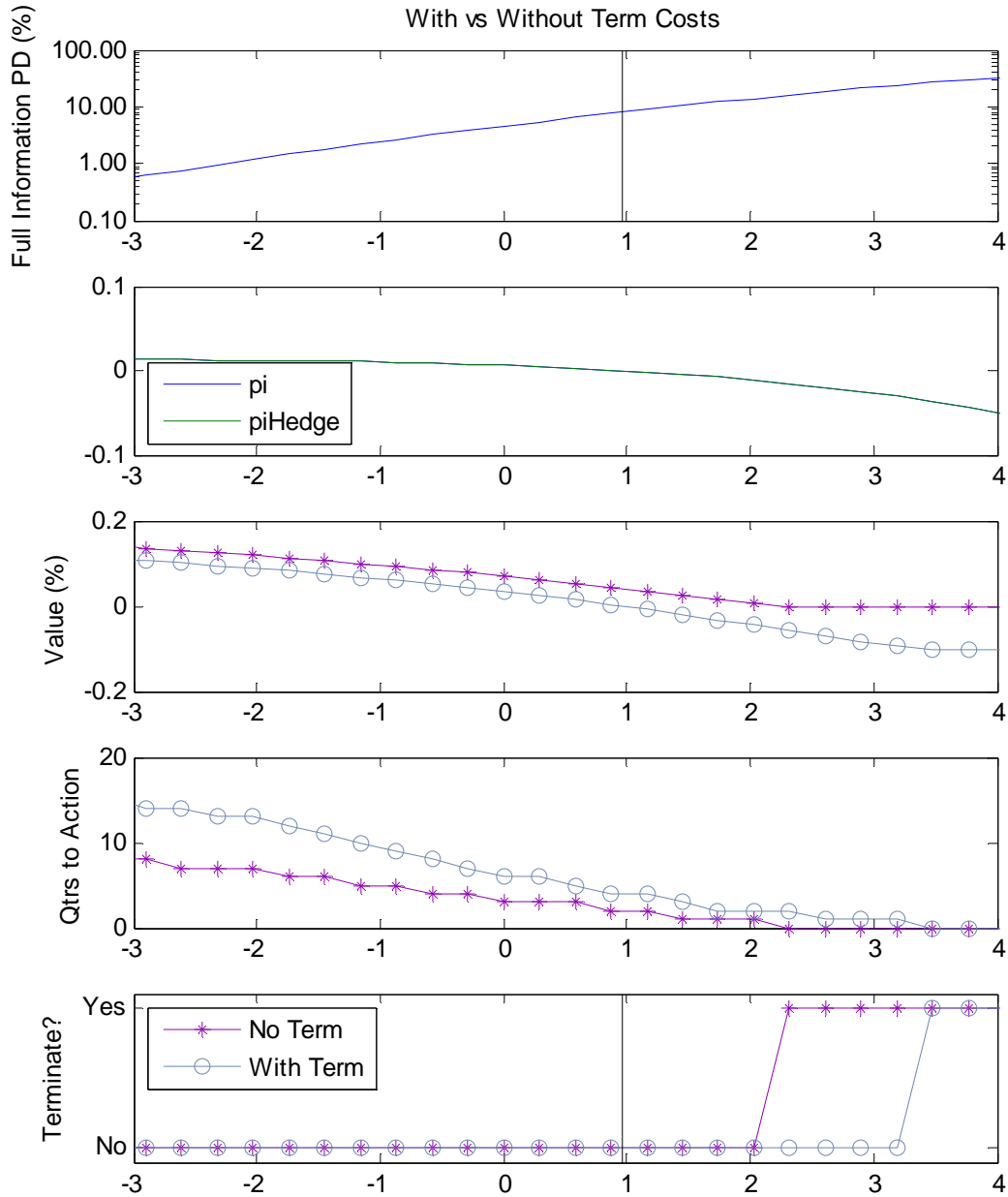


Figure 3 presents the solution to the lender's problem for Scenario A and Scenario B. In Figure 3, the '*' represents Scenario A no termination costs and the 'o' represents the Scenario B. In all panels, the horizontal axis represents the value of $S_t \sqrt{2\rho}$ observed at the time of the last review.¹⁴ The first panel presents the probability of default, the same for both scenarios. The second panel

¹⁴ Recall that the long run variance of S_t is $(1/2\rho)$, so we have normalized S_t scaled so that it has the interpretation of a standard deviation relative to the population.

presents the expected cash flow function for Scenarios A and B (II). Note that the profit function is concave, as would typically be the case for lenders, as the downside risk is more severe than the upside benefit. The third panel presents the value of the loan. Without the termination costs, the value of the loan "smoothly pastes" zero, as credit risk increases and the likelihood of the lender terminating the relationship becomes very high. With the termination costs the value of the asymptotes -10% as the value of the loan is bounded below by the termination costs. The fourth panel presents the time until action for the two scenarios. In both cases, the time until action decreases as credit risk increases, but the termination costs increase the time until action. The larger termination costs the less incentive the lender has to conduct a review. The bottom panel presents the action (terminate or not).

In both cases, the lender does not immediately terminate a relationship when the expected cash flows become negative. The decision to terminate is irreversible. By terminating the relationship today, the lender loses the option to terminate at some point in the future. The option to terminate in the future has a positive value. The termination costs cause the lender to make the termination decision at a higher level of credit risk in Scenario B relative to Scenario A.

In this section, we demonstrated some properties of the solution to the lenders problem (partial equilibrium setting). The lender takes the spread on the loan and the cost of terminating the lending relationship as given and optimizes accordingly. We see that the more scope the lender has for risk mitigation actions, the more valuable the loan, and the more frequent the review process. We also show that, typically, risky loans are reviewed more often, provided they are not so risky the lender chooses to terminate. In the next section, we show how the solution changes when the value of the loan changes to reflect publicly available information within an equilibrium context.

5. Equilibrium Examples

In most general equilibrium models, the arrival of information is an exogenous process. In this framework, the lender's information is costly to obtain. General equilibrium models with this feature are relatively rare in the economics literature (cf., Grossman and Stiglitz, 1980 and Muendler, 2007). Up to this point, the framework shows how the lender can make an optimal decision, taking both the credit risk of the borrower and the spread on a loan as given. In a competitive general equilibrium setting with full information, the spread on the loan typically depends upon the borrower's probability of default.¹⁵ For the remainder of the paper, we view the partial information as public information that is reflected in prices, and the information that is as costly to acquire as private information, which is not reflected in prices. Given the characteristics of the loan, we solve for the spread that makes the expected value under public information equal to the costs of administering the loan.

In this section, we characterize the solution to the dynamic programming problem for three specific scenarios. These examples allow us to demonstrate how the equilibrium spreads vary with credit risk in different scenarios. They also allow us to show how both credit risk and risk mitigation activities influence the review strategy.

This framework's calibration requires values for the parameters: $\{\beta_0, \alpha, \psi, \rho\}$. Recall that, roughly, the parameter β_0 determines the average default rate for the quarter, the parameter α is the sensitivity of the probability of default to public information, and the parameter ψ is the sensitivity of the probability of default to private information. The parameter ρ determines the information decay rate. In Appendix B, we argue that the parameter values for the base case in Table 3 are consistent with a *CDT* of 2.0% and a public information accuracy ratio of 77.8%, a full information accuracy ratio of 85.5%, and the assumption that private information decays over time with a half-life of four quarters.¹⁶

Table 3 presents two other scenarios. In the second scenario, private information is more informative, in that the full information AR is 93%, but the public information is the same. In the third scenario, public information is weak; the public information AR is 35%, while the full information AR is comparable to the base scenario.

Table 3 CALIBRATION SCENARIOS

Parameters	Base Case	Strong	Weak
		Private Information	Public Information

¹⁵ For example, Black and Scholes (pages 649-652,1973) argue that the Black Scholes formula can be used to determine the discount on a bond due to default risk.

¹⁶ By a half-life of approximately four quarters, we mean that $E(S_{t+4} | S_t) \approx 0.5S_t$.

β_0	-3.51	-5.77	-4.84
α	0.83	1.62	0.54
ψ	0.31	0.92	0.95
ρ	0.16	0.16	0.16
Public Information AR	77.8%	77.8%	35.0%
Full Information AR	85.5%	93.0%	85.5%
Central Default Tendency	2%	2%	2%

We solve for the equilibrium spread for different scenarios under the assumption that the initial value of the relationship is 2% of the principal, the cost of review is 0.5% of the principal, the quarterly discount factor is 0.98, and LGD is 67%. We assume that the relationship can be terminated prior to default without cost (Scenario A from the prior section). We also look at the case where the cost of review is relatively small (0.10%) and relatively large (2.0%). We present five different values (-2,-1,0,1,2) of U in the tables and more granular values (-3,-2.5,...3) for U in the figures. For each value of U , we solve for the spread, such that, the expected value of the loan is 2% after the review has been conducted, assuming that the borrower is drawn at random from the steady state population of potential borrowers with that U that have not yet defaulted.

Table 4 and 0 show the equilibrium spread for the three different scenarios. Columns 2, 4, and 6 are the annualized probability of default for the different values of U . In all cases, the equilibrium spread increases in risk. Further, the spread is positive for the safest names. In order for the value of the relationship to be 2%, a positive spread is required.¹⁷ For the base case, with elevated credit risk, the expected loss based upon public information exceeds the spread by a significant amount (from the last row of Table 4, 19.14% x 0.67 is 1,282bps versus an equilibrium spread of 724 bps). For the case of strong private information, the expected loss based on public information exceeds the probability of default by a considerable amount (2150bps versus 358bps), as well as for the case of weak public information (683 bps versus 183bps). The reason for this difference: when the relative importance of private information is large, the lender chooses to terminate the relationship if the outcome of the review is poor and, therefore, is not exposed to the full downside risk that investing solely on the basis of public information would imply.¹⁸

The relationship between PD and the spread is the "steepest" for the base case scenario. Under the base case scenario, the incremental predictive power of the private information is the least; therefore, the equilibrium coupon better reflects the public information. As the relative importance of private information increases, the sensitivity of the spread to the PD based upon public information becomes flatter.

Table 4 EQUILIBRIUM SPREAD (BPS PER ANNUM) FOR A 0.50% COR

U	Base Case		Strong Private Information		Weak Public Information	
	Public Information PD	Spread (bps)	Public Information PD	Spread (bps)	Public Information PD	Spread (bps)
-2	0.001	21	0	21	0.472	45
-1	0.028	22	0.018	22	1.143	74
0	0.412	49	0.487	46	2.56	103
1	3.66	219	5.77	145	5.31	138
2	19.14	724	32.076	358	10.187	183

¹⁷ We compute equilibrium spreads as low as 21bps per annum or 7bps per quarter. The value of 7bps per quarter in perpetuity discounted at 2% is 2.62%, somewhat above the minimum required value of the relationship, plus the cost of the initial review.

¹⁸ In this context, the public information Probability of Default is the probability that a borrower would default were they granted credit without regard to whether or not they actually get credit. In this equilibrium, some potential borrowers would be denied credit, based upon the outcome of the reviews.

Figure 4 Spread and Public Information PD for Different Scenarios with a 0.5% Review Cost

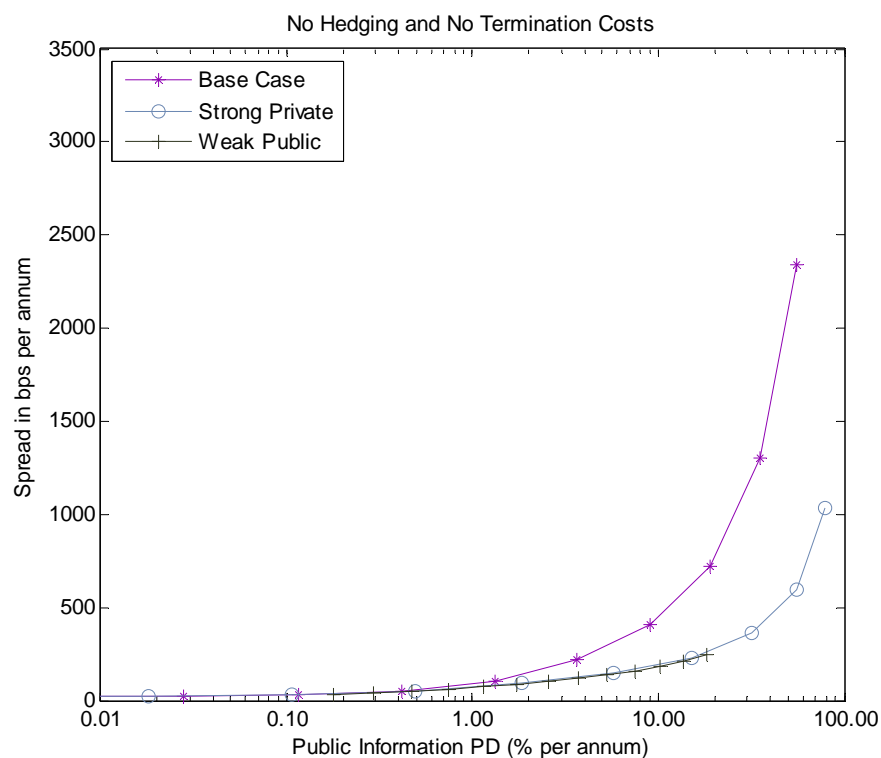


Table 5 and Figure 5 present the equilibrium spread for the base case scenario under different review costs. When the cost of review is low, the spreads are relatively flat, and the largest spread falls significantly below the full information PD. In this case, the lender is likely to exercise their option to review and choose not to extend credit, unless the review comes back favorably. As a result, spreads are low even for borrowers whom appear very risky, but only the safe borrowers are extended credit. When the cost of review is large, the spread profile becomes steeper, as reviews will be conducted less often, making it necessary for the spread to better reflect the public information PD. The spread on the riskiest loan remains less than the full information PD, which reflects that, for the high risk loans, the lender will periodically exercise their option to review and terminate the relationship when the review's outcome is poor.

Table 5 EQUILIBRIUM COUPON AND PUBLIC INFORMATION PD FOR DIFFERENT REVIEW COSTS

U	Public Information PD(%)	0.10%	0.50%	2%
-2	0.001	17	21	35
-1	0.028	19	22	36
0	0.412	45	49	63
1	3.66	172	219	266
2	19.14	605	724	1016

Figure 5 Spread and Public Information PD for Different Review

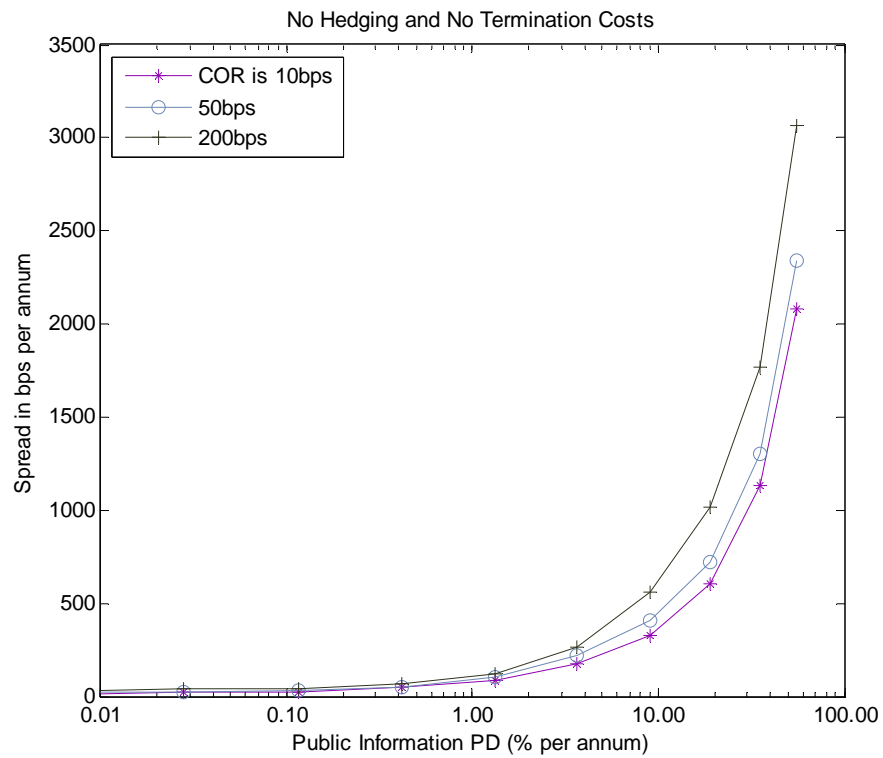


Figure 6 Value of the Relationship and Time to Next Review for the Base Case with a 0.50% Cost of Review

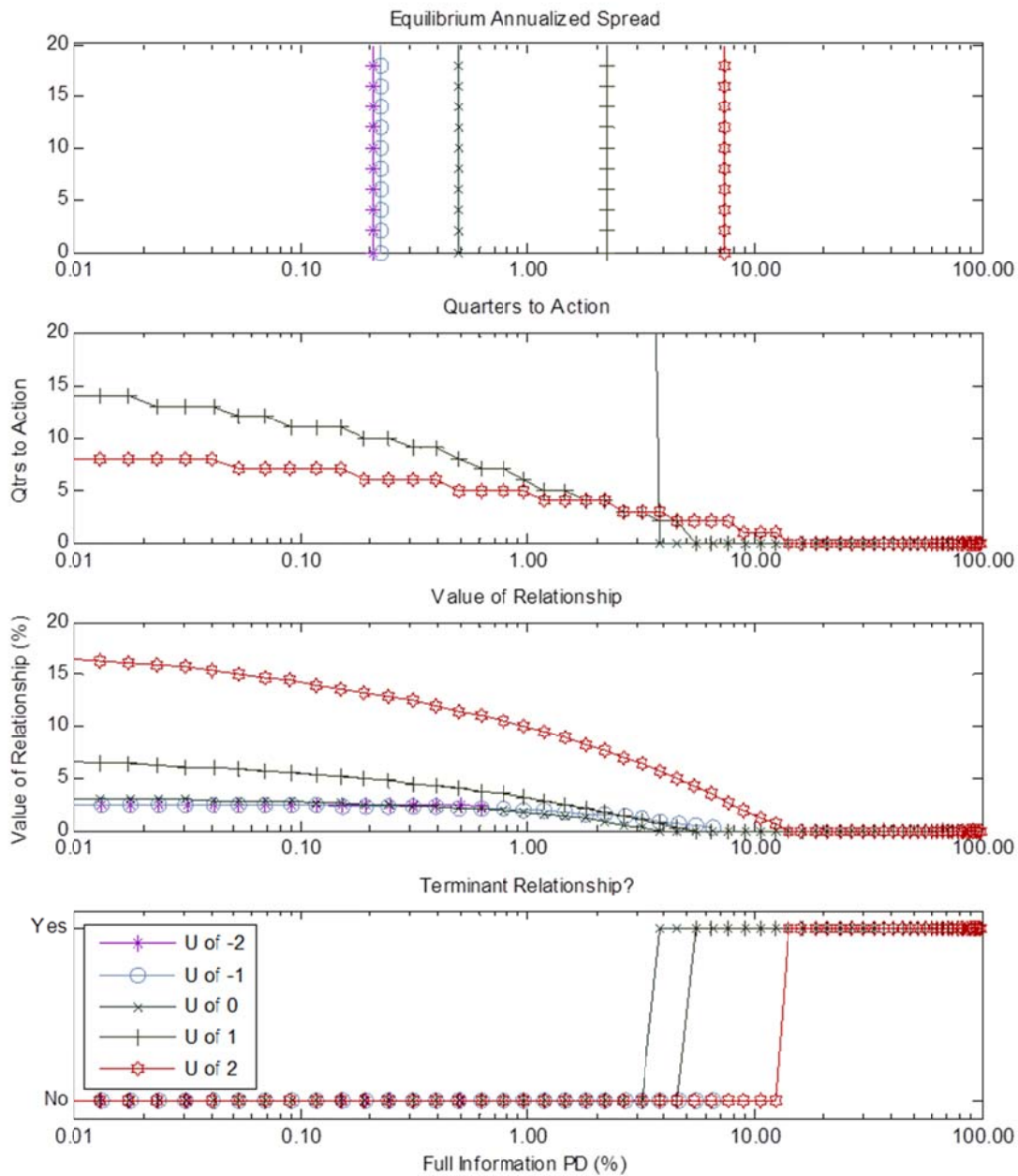


Figure 6 illustrates the optimal monitoring strategy for different values of U in equilibrium using four panels. In each panel, the horizontal axis is the annualized full information PD. The top panel of Figure 6 presents the equilibrium spreads for different values of U (public information) graphed as vertical lines on the "full information PD axis." The second panel presents time to action against the most recently observed full information PD for different values of U . The third panel represents the loan value expressed as a percentage of the principal. The bottom panel presents whether or not the optimal action is to terminate the loan rather than go to the expense of conducting a review. The value of the relationship obeys a "smooth pasting" condition. As the full information PD becomes large, the value becomes close to zero, and the derivative of the value of the loan with respect to the full information PD approaches zero.

For the safest loans (a negative U), the optimal strategy is to wait as long as possible to review the loan (we impose the constraint that every loan must be reviewed at least every 100 quarters) and to never terminate the loan. The reason for this result: these loans are very safe. The public information PDs are 0.00% and 0.03%, and the spreads are 21 and 22bps, respectively. The 21–22bps of spread pays the administrative costs of originating the loan. In the case of $U = -1$, the full information annualized PD can be as high as 6%. Nevertheless, it is not optimal to terminate this loan. The probability of default within the quarter is approximately 1.5%, which translates to an expected loss of approximately 1%. It is optimal to absorb this loss, as there is a high likelihood that the loan mean reverts back to a much safer level of credit risk, in which case, the loan becomes worth 2.5%. It is never optimal to pay 50bps to review the loan as regardless of the outcome of the review, one would not do anything differently.

For the borrower with moderate credit risk (U of 0, Public Information PD of 0.41% and spread of 49bps) the optimal strategy is to wait the full 100 periods to review the loan and to terminate the loan immediately if the full information PD exceeds 3.8%. If the full information PD exceeds 3.8%, the expected loss that quarter is 65bps, which is partially offset by 12bps of spread income. If the full information PD is less than this threshold, the optimal strategy is to hold the loan. Below this threshold, the lender should absorb some expected losses in the short-term, on the expectation that the loan mean reverts to more profitable levels of credit risk. Above this threshold, the short-term expected losses do not justify the long-term gain.

For the loans with high levels of credit risk (U of 1 and 2, public information PDs of 3.6% and 19.1%, and spreads of 219bps and 724bps, respectively) the loans are potentially very valuable provided the full information PD is low. For the U of 2, the value of the loan can be in excess of 15% of the principal. For these loans, it pays to review them, and to review them more often if the full information PD is higher. When the full information PD is in excess of 2%, it pays to review at least every year. As the full information PD becomes close to the termination threshold, for a U of 1, it is optimal to review it twice a year and for a U of 2 it is optimal to review it every quarter. In both of these cases, at the termination threshold the expected losses exceed the spread on the loan as the lender suffers short-term expected losses, offset by the potential of the borrower to migrate to lower levels of credit risk. For a U of 2 (1), the lender suffers a 32bps (23bps) of losses in expected cash flows at the last value of S before the termination threshold.

In this section, we showed how equilibrium spreads vary with public information, and how the nature of the equilibrium spreads change as we change the framework's calibration. Increasing the cost of review increases the sensitivity of spreads to the public information PD. Conversely, as the relative importance of private information increases the sensitivity of spreads to the public information PD declines. The optimal monitoring strategy changes with the level of credit risk. With low credit risk (based on public information), it is optimal to review the borrower only rarely and, as credit risk increases, it becomes optimal to review them more often. It also can be optimal to suffer short term negative expected cash flows in the hope of being rewarded with positive cash flows in the future, if the borrower migrates to lower levels of credit risk. Optimal monitoring is most frequent for the risky loans whose full information PD is close to the termination threshold.

6. Value of a Review in a Dynamic Setting

An important question remains: how much economic value does pursuing an optimal review strategy create? In the dynamic setting, one can express the value of pursuing an optimal review strategy as the difference between the value of the relationship under the optimal review strategy and the value of the relationship if the lender pursues some other policy. In this section, we use reviewing each borrower every four quarters as the alternative strategy. One reason for choosing this strategy as a reference point: often regulatory guidelines require an annual review. Also, the work flow of the review process can be tied to the receipt of the new, full-year financial statements.

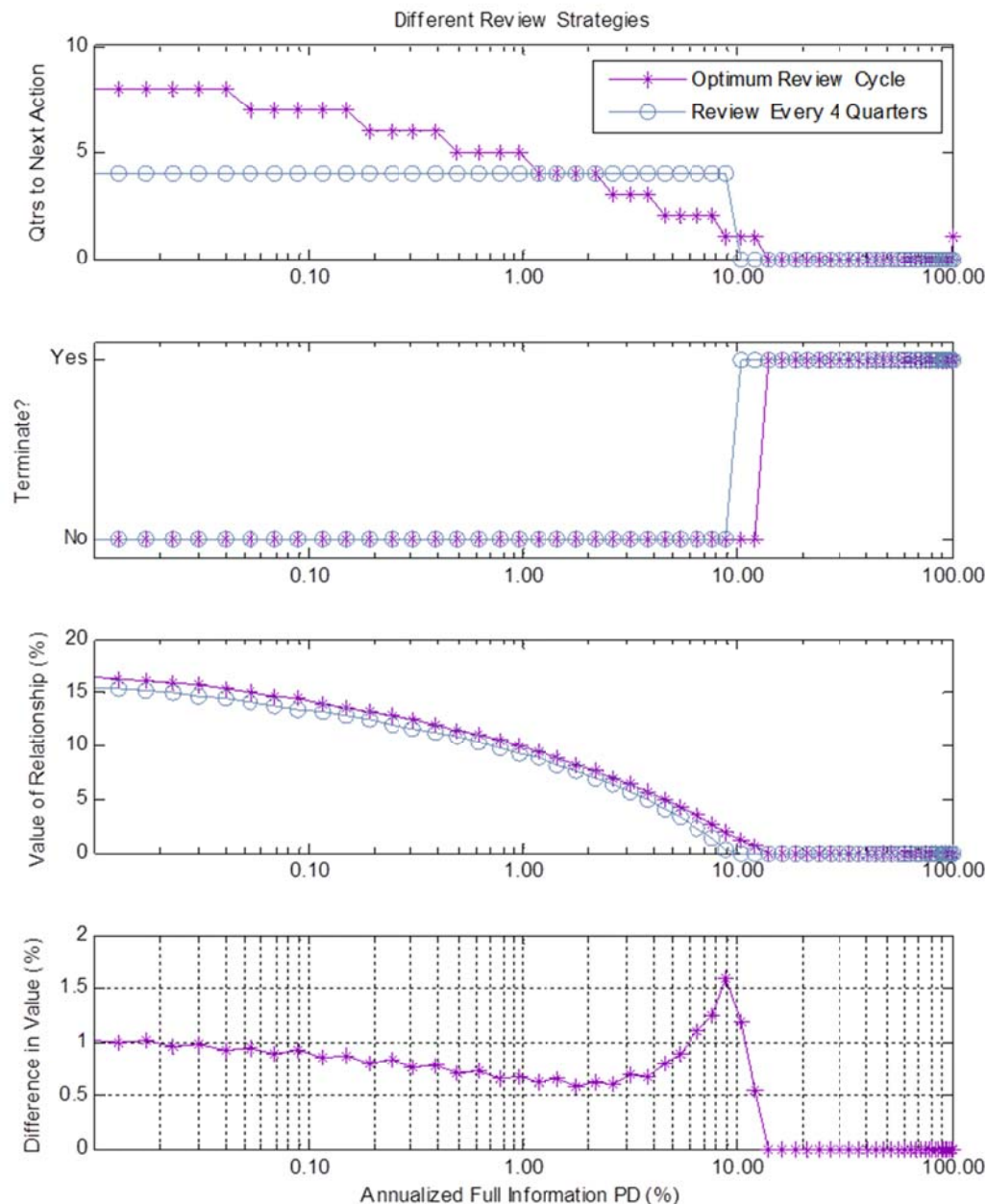
We solve for the value of the relationship under a fixed annual review policy by setting every element of \mathbf{R} to 4 in equations (4) and (5). We then solve for the optimal termination decision, given the constrained review policy. The assumption here: if the regulatory requirement for the review process becomes too expensive, the lender has the option to terminate the relationship.

We focus on a firm with a U of 2, which corresponds to an annualized PD based on public information of 19.14% and a spread of 724bps (Table 4). We focus on a U of 2 rather than a lower level of credit risk, because for this level of risk, it is often optimal to review the loan more often than once a year (Figure 6). In this case, the incremental value of the review is likely elevated.

Figure 7 presents four panels in which the horizontal axis is the annualized full information PD. The first panel compares the optimal review policy for different full information PDs, with the every four quarters constraint. For low levels of credit risk, it is optimal to wait eight quarters to conduct a review, whereas, for the highest levels of credit risk, one will want to review the loan every quarter. This optimal policy is contrasted with the fixed four quarters review cycle that is maintained unless it is optimal to terminate the loan. The second panel presents the termination decision. Under the fixed review policy it is optimal to terminate the loan at a lower level of credit risk, as one does not monitor the loan as closely. The third panel presents the value of the loan

under the two review policies. The bottom panel presents the difference between the values of the loan under the two policies. This difference reaches a peak of 1.6%. This peak occurs where it is optimal to monitor the loan very closely, and if lender is constrained to an annual review cycle, the lender chooses to terminate the loan. The review process generates the most value on risky loans being monitored closely, rather than terminated prematurely.

Figure 7 Value of a Review in a Dynamic Setting



In this section, we demonstrated the value of the review by contrasting the value of the relationship under an optimal review cycle versus a fixed review cycle. The value of the review is largest when credit risk is the highest, but it is still optimal not to terminate the relationship.

7. Conclusion

This paper presents a basic framework for computing the value of a credit review. Our framework is consistent with some basic characteristics of PDs. For example, the distribution of PDs in a portfolio typically skews to the right as default probabilities are bounded between 0 and 1, and the largest PDs in a population are typically many times larger than the average PD of the population.

Our core assumption is that the outcome of the credit review is a *full information PD* with better discriminatory power than a *partial information PD*, and the *full information PD* leads to a decision on how to manage the credit risk of the exposure. In this framework, the relationship between the value of a review and the characteristics of the exposure is multidimensional: it depends upon the *partial information PD*, the spread, the cost of selling the exposure, the central default tendency, and the accuracy ratios of the full and partial information PDs. Simple policy rules, such as review each loan each year or review each loan whose PD moves outside a certain threshold range, applied without regard to the spread on the loan, are not optimal within this framework.

In general, the value of a review is lower when the decision to hedge (or not) is a foregone conclusion. The highest values of a review occur near the point where the lender is indifferent between hedging and not hedging, given partial information. Holding the excess spread constant, the value of a review increases as the partial information probability of default increases.

Our framework provides a theoretical interpretation for practices found in the review process. For certain credit classes, a financial institution may take action without a formal review, based upon the outcome of a quantitative scoring model (e.g., a pre-approved credit card), whereas, for other asset classes, an action is never taken solely on the basis of a quantitative scoring model (e.g., the renewal of a line of credit to a large corporation). As the cost of review per unit of exposure is likely higher in consumer credit than corporate credit, these different practices could be justified within this framework. Another common practice is to *watch list* the riskiest loans and review them more often. In this framework, one should review loan A more often than loan B if it is riskier and both are in the portfolio (spread exceeds expected loss under partial information) and pay the same spread. Therefore "*watch-listing* the riskiest credits" can be justified under this framework.

We show how the framework can be extended to incorporate a time dimension. In this setting, we compute the "partial information PD" utilizing "public information" and the "full information PD" utilizing both "public information" and "private information." The relevance of the information acquired during a review process decays with time. The optimal time until the next review depends upon the spread on the loan, the cost of the review, the cost of terminating the relationship with the borrower, the public information on the borrower, and the outcome of the most recent review. It also depends upon the characteristics of the asset class: how informative is the partial information PD? The full information PD? How quickly does information acquired during the review process change? For reasonable parameter values, it may be optimal to review the loan during the next quarter and, in other cases, it may be optimal to review the loan only rarely.

We assess the value of an optimal review policy by comparing it to a fixed review cycle of once every four quarters. We show that the incremental value of the review is largest for the loans that should be reviewed more often than every four quarters.

At present, banks do have policies for collecting information on borrowers, processing this information, and conducting a review process. As the IT infrastructure of banks is changing rapidly, these policies are changing as well. Establishing the value of the review process requires multiple parameters. Some of these parameters are known (e.g., the terms of the loan). Some of these can be estimated empirically (e.g., PD, LGD). Some of these parameters could be estimated using business logic (e.g., the cost of a review). Some of these parameters are more difficult to empirically estimate (e.g., the cost of terminating a relationship with a borrower). One could use the framework to determine an internally consistent review policy by using reasonable values for the "knowable" parameters and "guessing" at the "unknown" parameters, and then revisiting the "unknown" parameters to obtain a review policy that makes intuitive sense. Such a process could lead to a more effective review strategy than attempting to collect and process effectively all available information on all borrowers.

The "realized value of a review" for a good credit (one for which the excess spread is positive) is asymmetric in this setting. In most cases, the *ex post* value of a review equals zero. Most reviews determine that the full information PD is less than the partial information PD, and no action is taken on the loan. In some cases, however, the full information PD turns out to be poorer than the partial information PD, enough so that the institution acts on the loan. In these relatively rare cases, the lender's savings are substantial. As a result, one can under-appreciate a review's value if looking at the *ex post* realized values on a relatively small sample. This particular scenario speaks to the difficulty of properly assessing the value that analysts bring to managing a credit portfolio.

Appendix A Table of Symbols

Symbol	Meaning
PD	Probability of Default
\mathbf{X}	Row Vector of both partial and full information default drivers
β_0 and β	The intercept and a column vector of coefficients for the full information PD
Y and Z	The partial information and full information default drivers
γ and λ	The coefficients for the partial and full information default drivers
CDT	The average default rate of the population
$\hat{\gamma}$ and $\hat{\beta}_0$	The coefficient for the partial information driver and the intercept under partial information
CAP and AR	Cumulative Accuracy Profile and Accuracy Ratio
s and h	The spread on the loan and the transaction cost of selling the credit
<i>Dynamic Version of the Framework</i>	
S_t	The time-varying private information
U	The public information that is fixed for each borrower
β	The discount factor
M and N	The number of possible values for S without the absorbing state (M) and with the absorbing state (N)
$D(j)$	A row vector representing the distribution of a borrower across different states from the perspective of the lender
Π	Column vector representing the expected cash flows for each credit state
A and G	For a given borrower, A represents the transition matrix governing the evolution of S (the private information). G is the calibrated version of this matrix.
CoR , s and CoT	The cost of review and spread on the loan. CoT is a column vector that represents the cost of terminating the relationship.
R and V	R is a $2 \times N$ matrix that represents the number of periods until the next review and what the action will be. V is an N element column vector that represents the value of the option to lend for each possible value of S
α and ψ	The parameters governing the sensitivity of the PD to the public information, U and private information, S_t
ρ	The parameter governing the persistence of the private information

Appendix B

In this Appendix, we show how to derive the distribution of the partial information PDs and the full information PDs. We also derive an expression for the Cumulative Accuracy Profile (CAP curves) and the corresponding Accuracy Ratio (AR). These constructs allow us to calibrate the parameters using empirical reference points. We show how the framework enables comparative statics that change the partial information AR, while holding the full information AR constant and vice versa.

In validating a PD model, key workhorses are CAP curves and their corresponding ARs (Engelmann et. al., 2003). These constructs assess a credit risk measure's ability to rank order firms in terms of separating out the bad borrowers from the good borrowers. CAP curves are typically defined with respect to a specific sample. In this section, we work with the *population* analogues to these measures. The cumulative accuracy profile (CAP plot) of a credit risk measure can be defined as:

$$CAP(q) = \int_0^q D(x)dx / \int_0^1 D(x)dx = \int_0^q D(x)dx / CDT$$

Where $D(q)$ is defined as follows. Consider an arbitrarily small interval, $(q-\delta, q+\delta)$, which is a subset of $[0,1]$. The portion of the exposures in the population riskier (according to some credit risk measure) than the exposures within the interval is $q-\delta$. The portion of the exposures in the population safer than the exposures within the interval is $1-q-\delta$. $D(q)$ is defined to provide the

rate at which exposures within the interval $(q-\delta, q+\delta)$ default. The Accuracy Ratio is defined as $AR = \frac{\int_0^1 CAP(q)dq - 0.5}{0.5(1 - CDT)}$.

The AR is the ratio of the area between the CAP Curve and the 45 degree line, divided by the area between the CAP Curve of a Perfect Model and the 45 degree line. A perfect model has an AR of 1, and a random model has an AR of 0.

In our context, for the full information PD, $N^{-1}(PD)$ has a normal distribution with a mean of β_0 and a variance of $\beta' \beta$. Therefore, the quantile function of the full information PD is given by:

$$F_{FI}(q) = N\left(N^{-1}(q)\sqrt{\beta' \beta} + \beta_0\right)$$

This function takes a percentile as an input and produces the PD associated with that percentile. For example, if $q=0.975$, then 97.5% of the population has a PD less than $F_{FI}(0.975)$. It is convenient to define $PD_{FI}(q) = 1 - F_{FI}(q)$, so that $PD_{FI}(0.025)$ has the interpretation that 2.5% of the population has PD of greater than $PD_{FI}(0.025)$. By symmetry of the normal distribution, we see that:

$$PD_{FI}(q) = 1 - N\left(N^{-1}(q)\sqrt{\beta' \beta} + \beta_0\right) = N\left(-N^{-1}(q)\sqrt{\beta' \beta} + \beta_0\right)$$

Note, in this context, $PD_{FI}(q)$ is the $D(q)$ used above in defining a CAP curve. Therefore, the CAP curve for the full information PD is given by:

$$CAP_{FI}(x) = \int_0^x N\left(-N^{-1}(q)\sqrt{\beta' \beta} + \beta_0\right)dq / CDT$$

We evaluate this function using numerical integration. Computing the corresponding AR involves evaluating a double numerical integral.

Appendix C Calibration

In Section 4, we state that specific values of $\{\alpha, \beta_0, \rho, \psi\}$ imply specific values for the CDT, partial information AR, and full information AR at the one-year horizon. In this section, we describe the calibration procedure used to make this assertion. We make three assumptions. First, we assume a specific distribution of the private information (S) of entrants, given their public information (U). Second, in order for a steady state population of firms to exist, it is necessary that all firms have a positive probability of default. Third, we compute the average CDT, partial information AR, and full information AR, with respect to the whole population, without regard to whether or not they are actually extended credit.

We describe a procedure that takes as inputs $\{\alpha, \beta_0, \rho, \psi\}$ and produces $\{CDT, AR_{PI}, AR_{FI}\}$ at the one-year horizon. In order to do so, we need to compute a steady state distribution of firms in terms of U and S . We then must compute the partial and full information probability of default at a one-year horizon. Finally, we must compute the average default rate, cumulative accuracy profiles, and the corresponding accuracy ratios. Once we define the function, we can compute the parameters $\{\alpha, \beta_0, \rho, \psi\}$ that are consistent with the desired $\{CDT, AR_{PI}, AR_{FI}\}$ by minimizing a loss function.

First, we use a discrete space approximation to a standard normal distribution via a binomial distribution: Let

$U = (X - 12) / \sqrt{6}$ where X has a binomial distribution with the probability of success being 0.5 and the number of trials being 24.¹⁹

For each value of U , we solve for the steady state distribution of S as follows. First, we assume that there are entrants in each period. The new entrants are distributed according to the unconditional distribution of an Ornstein-Uhlenbeck process using the discrete approximation. We subdivide a time period into 20 equal increments ($n=20$) and compute the transition matrix T , defined in Section 5. We then compute the unconditional distribution of the Ornstein-Uhlenbeck process by taking any row of T^{500} . Let \mathbf{E} be this row vector with a 0 added to the end to represent the absorbing state (i.e., firms in the default state do not enter).

For each value of U , we utilize $\sum_{i=1}^{\infty} \mathbf{E} \mathbf{G}_U^i$ to compute the steady state distribution. Note, each term in the sum is a row vector

that represents the distribution in the current period of the cohort that entered i periods ago. Finally, we convert it into a distribution by setting the population in the absorbing set to 0 and then normalizing so that the row vector sums to one. The early cohorts have an approximately normal distribution, while the latter cohorts have a distribution that is shifted to the left due to the firms with low credit risk being more likely to survive. Let SS_U be the steady state distribution associated with a specific value of U .

We now have the state steady distributions for both S and U . Given $\{U, S\}$, the one-year full information probability of default is given by the last element of $\mathbf{D} \mathbf{G}_U^4$, where \mathbf{D} is a row vector of zeros, with a 1 indicating the current values of S and \mathbf{G}_U is the transition matrix associated with the specific value of U . For a given U , the public information probability of default is given by the last element $SS_U \mathbf{G}_U^4$. The average default rate of the population is computed as the weighted sum of the full information one-year default probabilities for each value pair of $\{U, S\}$. The weights are the product of the probability of U_i and the probability of S_i given U , where the latter is given by the steady state distribution of S given U . The computation of the partial information AR and the full information AR are then computed using the discrete analogues to the definitions provided in Appendix C. By treating ρ as fixed, we can solve for the values of $\{\alpha, \beta_0, \psi\}$ that produce specific values of $\{CDT, AR_{PI}, AR_{FI}\}$ by minimizing a loss function.²⁰

Of course, the results change somewhat if one changes the fineness of the grid used to approximate U and S that we are targeting. We have not explored this topic. Our results are valid for the specific distributions of U and S that we use.

¹⁹ As X has a mean of $(1/2)*24$ and a variance of $24*(1/2)*(1/2)$, U has a mean of 0 and a variance of 1.

²⁰ We minimize the sum of the squared difference between the values of $\{CDT, AR_{PI}, AR_{FI}\}$ implied by $\{\alpha, \beta_0, \psi\}$ and their target values. This loss function obtained a lower bound of very close to zero.

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