

# 4.6

## Arithmetic Sequences

For use with Exploration 4.6

**Essential Question** How can you use an arithmetic sequence to describe a pattern?

An **arithmetic sequence** is an ordered list of numbers in which the difference between each pair of consecutive **terms**, or numbers in the list, is the same.

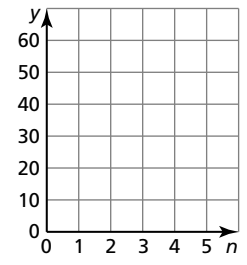
**1 EXPLORATION: Describing a Pattern**

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the  $y$ -values.

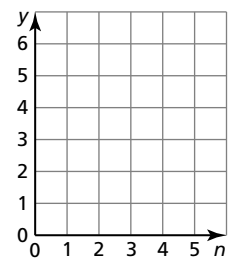
a.  $n = 1$        $n = 2$        $n = 3$        $n = 4$        $n = 5$

Number of stars, $n$	1	2	3	4	5
Number of sides, $y$					



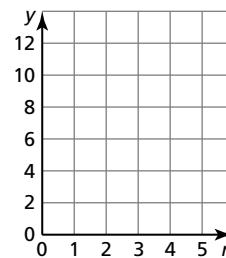
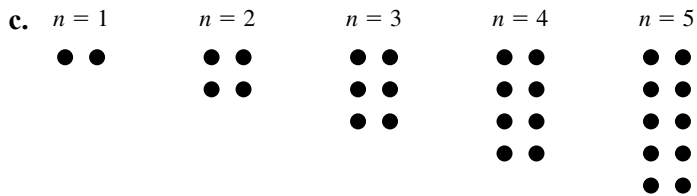
b.  $n = 1$        $n = 2$        $n = 3$        $n = 4$        $n = 5$

$n$	1	2	3	4	5
Number of circles, $y$					



**4.6 Arithmetic Sequences (continued)**

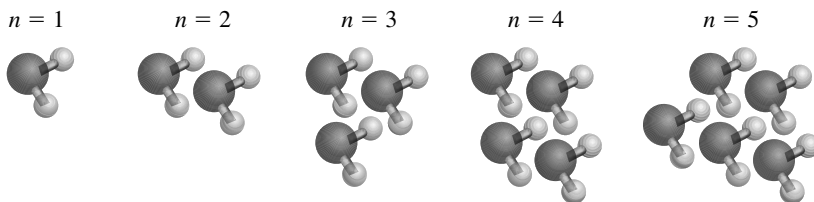
**1 EXPLORATION: Describing a Pattern (continued)**



Number of rows, $n$	1	2	3	4	5
Number of dots, $y$					

**Communicate Your Answer**

- How can you use an arithmetic sequence to describe a pattern? Give an example from real life.
- In chemistry, water is called  $H_2O$  because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.



# 4.6

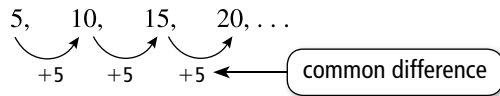
## Practice

For use after Lesson 4.6

### Core Concepts

#### Arithmetic Sequence

In an **arithmetic sequence**, the difference between each pair of consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.



Terms of an arithmetic sequence

#### Notes:

#### Equation for an Arithmetic Sequence

Let  $a_n$  be the  $n$ th term of an arithmetic sequence with first term  $a_1$  and common difference  $d$ . The  $n$ th term is given by

$$a_n = a_1 + (n - 1)d.$$

#### Notes:

### Worked-Out Examples

#### Example #1

Determine whether the sequence is arithmetic. If so, find the common difference.

48, 24, 12, 6, ...

<b>Position</b>	1	2	3	4
<b>Term</b>	48	24	12	6

+(-24) +(-12) +(-6)

Consecutive terms do *not* have a common difference.  
So, the sequence is *not* arithmetic.

**4.6 Practice (continued)**

**Example #2**

Write an equation for the  $n$ th term of the arithmetic sequence. Then find  $a_{10}$ .

10, 0, -10, -20, ...

$$d = 0 - 10 = -10$$

$$a_n = a_1 + (n - 1)d$$

$$a_n = 10 + (n - 1)(-10)$$

$$a_n = 10 + n(-10) - 1(-10)$$

$$a_n = 10 - 10n + 10$$

$$a_n = -10n + 20$$

An equation for the  $n$ th term of the arithmetic sequence is  $a_n = -10n + 20$ .

$$a_n = -10n + 20$$

$$a_{10} = -10(10) + 20$$

$$= -100 + 20$$

$$= -80$$

The 10th term of the arithmetic sequence is -80.

**Practice A**

In Exercises 1–6, write the next three terms of the arithmetic sequence.

1. 1, 8, 15, 22, ...

2. 20, 14, 8, 2, ...

3. 12, 21, 30, 39, ...

4. 5, 12, 19, 26, ...

5. 3, 7, 11, 15, ...

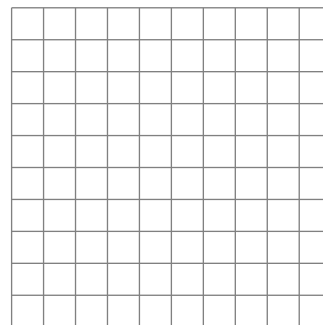
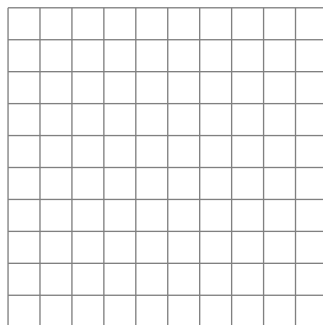
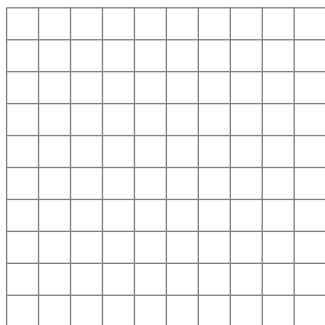
6. 2, 14, 26, 38, ...

In Exercises 7–12, graph the arithmetic sequence.

7. 1, 3, 5, 7, ...

8. 9, 6, 3, 0, ...

9.  $\frac{15}{2}, \frac{13}{2}, \frac{11}{2}, \frac{9}{2}, \dots$

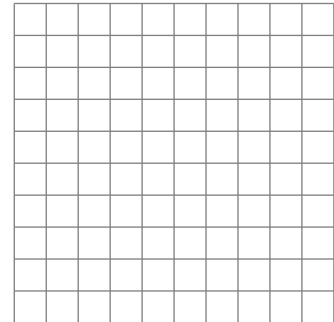
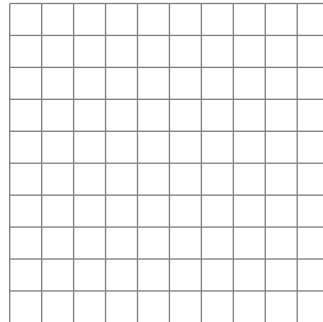
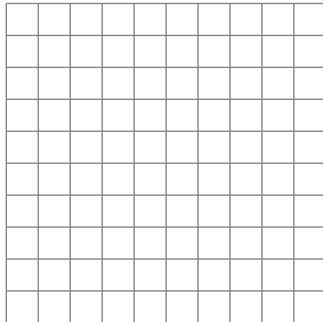


**4.6 Practice (continued)**

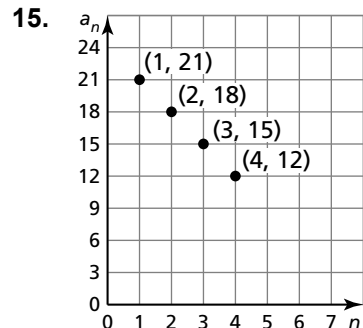
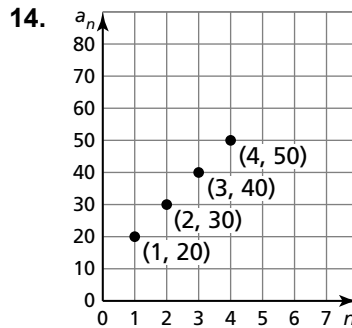
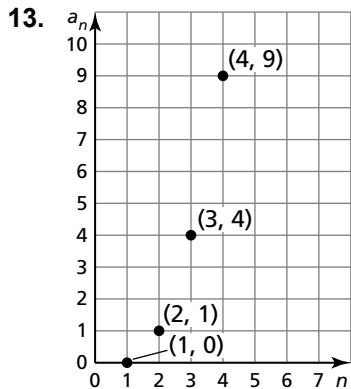
10. 1, 2.5, 4, 5.5, ...

11. 1, 4, 7, 10, ...

12.  $\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \dots$



In Exercises 13–15, determine whether the graph represents an arithmetic sequence. Explain.



In Exercises 16–21, write an equation for the *n*th term of the arithmetic sequence. Then find *a*<sub>10</sub>.

16. -5.4, -6.6, -7.8, -9.0, ...

17. 43, 38, 33, 28, ...

18. 6, 10, 14, 18, ...

19. -11, -9, -7, -5, ...

20. 34, 37, 40, 43, ...

21.  $\frac{9}{4}, \frac{7}{4}, \frac{5}{4}, \frac{3}{4}, \dots$

22. In an auditorium, the first row of seats has 30 seats. Each row behind the first row has 4 more seats than the row in front of it. How many seats are in the 25th row?

## Practice B

In Exercises 1 and 2, write the next three terms of the arithmetic sequence.

- |  |   |
|--|---|
| 1. First term: 8<br>Common difference: 5 | 2. First term: 40<br>Common difference: $-12$ |
|--|---|

In Exercises 3–6, find the common difference of the arithmetic sequence.

- |                                |  |
|--------------------------------|--|
| 3. $-4, -1, 2, 5, \dots$       | 4. $\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{8}{7}, \dots$ |
| 5. $8.6, 8.4, 8.2, 8.0, \dots$ | 6. $7\pi, 5\pi, 3\pi, \pi, \dots$                              |

In Exercises 7 and 8, graph the arithmetic sequence.

- |                           |                             |
|---------------------------|-----------------------------|
| 7. $4, 18, 32, 46, \dots$ | 8. $10, 7.5, 5, 2.5, \dots$ |
|---------------------------|-----------------------------|

In Exercises 9 and 10, determine whether the sequence is arithmetic. If so, find the common difference.

- |                            |                            |
|----------------------------|----------------------------|
| 9. $67, 52, 37, 22, \dots$ | 10. $128, 32, 8, 2, \dots$ |
|----------------------------|----------------------------|

In Exercises 11–14, write an equation for the  $n$ th term of the arithmetic sequence. Then find  $a_{10}$ .

- |                                     |   |
|-------------------------------------|---|
| 11. $-9, -1, 7, 15, \dots$          | 12. $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, \dots$    |
| 13. $-160, -180, -200, -220, \dots$ | 14. $-\frac{7}{3}, -\frac{5}{3}, -1, -\frac{1}{3}, \dots$ |
15. The first term of an arithmetic sequence is 3. The common difference of the sequence is 10 less than twice the first term. Write the next three terms of the sequence.
16. The volume (in cubic feet) of the water in a tank each hour after turning on a faucet can be estimated by the sequence in the table.

<b>Hours after turning on faucet</b>	1	2	3	4
<b>Volume (cubic feet)</b>	12	15	18	21

- a. Write a function that represents the arithmetic sequence.
- b. The tank is in the shape of a rectangular box. The length is 6 feet, the width is 3 feet, and the height is 2 feet. Find the  $n$ th term that represents a full tank. Explain.