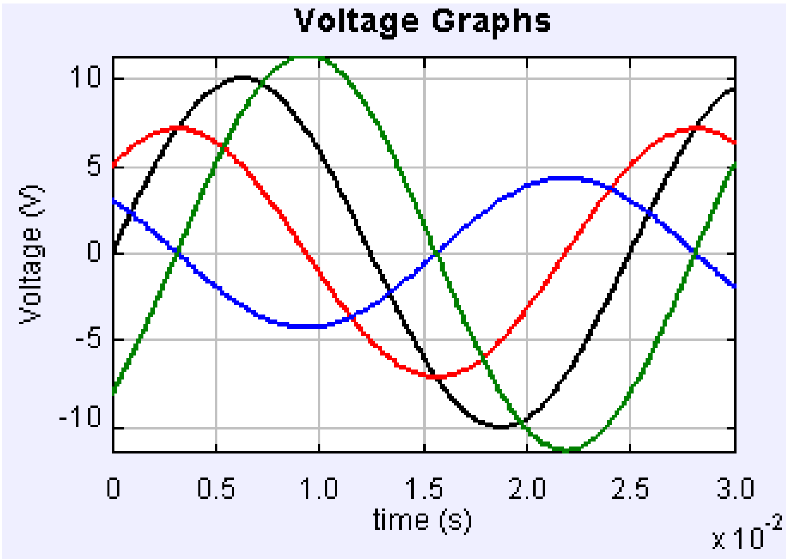


At first glance, this graph just shows a jumble of sine waves. They all have the same frequency, but their amplitudes and phases are different. One of our goals in this chapter is to make some sense of this jumble. The graph actually shows the voltage from an alternating current source (like a wall socket), as well as the voltage across a resistor, a capacitor, and an inductor (a coil), all of which are connected in a series circuit to the source. By the end of this chapter, you should be able to determine which graph is which, and to explain their phase relationships and the relationships between the amplitudes.

Chapter AC – Alternating Current Circuits

CHAPTER CONTENTS

When you switch on a light at home, you are turning on an AC circuit. AC stands for alternating current – in fact, both the current and the voltage oscillate sinusoidally. What this means for the light bulb filament is that the current reverses direction at regular intervals. In North America, in fact, where the frequency of the alternating voltage is 60 Hz, the current changes direction 120 times every second! In addition, at 120 instants during each second, the current is zero, and if you average over each cycle the average current is zero. The light bulb filament shines brightly despite this, because of all the energy transferred to it from the net motion of the electrons during the periods of non-zero current.

In this chapter, we will investigate some of the issues that come up when resistors, capacitors, and inductors (coils) are connected in AC circuits. To understand these circuits, we can apply many of the ideas that we applied when we learned about DC circuits. Ohm's law, for instance, applies, as does the loop rule. We will learn some new ideas, too. As usual, the key will be to see how all the new concepts fit into the framework we built when we looked at DC circuits. We should also keep in mind that AC circuits are all around us – anything that plugs into a wall socket is part of an AC circuit, and many practical devices, such as metal detectors and circuits in stereo systems, exploit various properties of AC circuits.

AC-1 Inductors and Inductance

In Essential Physics Chapter 20, Generating Electricity, we discussed Faraday's law and Lenz's law, and explored the tendency of a coil of wire to oppose changes in the magnetic flux passing through the coil. When we use a coil of wire as part of a circuit (as a circuit element, that is), we call the coil an **inductor**.

With an inductor in a circuit, the magnetic field in the coil comes from the current that passes through the coil. If the current is constant, the magnetic flux is constant, and there is no change in flux to oppose. If the current changes, however, there is a corresponding change in magnetic flux, and the coil then acts to oppose the change in flux. Through Faraday's law, there is an induced voltage across the coil – the coil actually acts like a battery while the flux is changing. If the current through the coil is decreasing, the induced voltage causes an induced current that opposes the change, adding some current back to try to counteract the loss of current. If the current through the coil is increasing, the induced voltage again causes an induced current that opposes the change, adding some current in the opposite direction to cancel out a fraction of the increased current. All of this is shown below in Figure AC.1.

When we're talking about inductors (coils in circuits), we use a form of Faraday's law that shows that any change in flux is the direct result of a changing current in the inductor.

The induced voltage, or induced emf, in an inductor is given by:

$$\Delta V_L = -L\frac{\Delta I}{\Delta t} \quad , \qquad \text{(Equation AC.1: \textbf{Induced voltage for an inductor})}$$

where L is called the self-inductance of the inductor, generally referred to as the **inductance**. The inductance is a measure of how much opposition an inductor provides to a change in the current in the coil. Similar to a capacitor, the inductance of an inductor depends on the geometry (such as the number of turns, and the length), as well as on whether there is a magnetic material, such as a piece of iron, inside the coil. The SI unit of inductance is the henry (H).

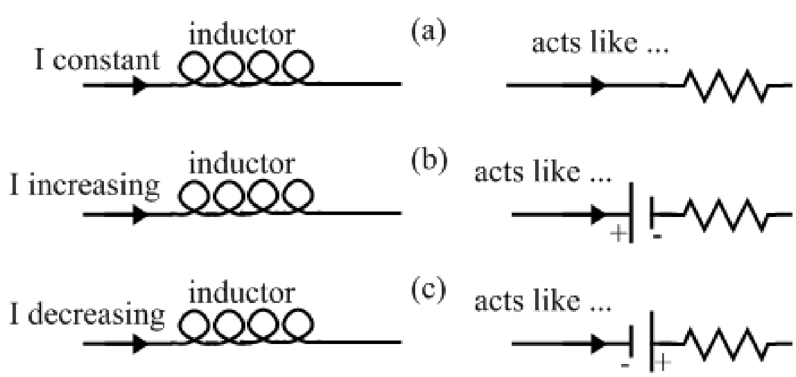


Figure AC.1: This figure shows a graphic representation of how an inductor acts under various conditions in a circuit. In (a), showing a constant current, the inductor acts as a resistor, because the length of wire from which the inductor is made has a resistance. In (b), the current in the circuit is directed to the right, and is increasing. This causes an increase in magnetic flux through the coils of the inductor (the magnetic field being proportional to the current through the inductor), and the inductor responds by acting as a battery, connected so as to cancel out some of the increase in current. It also acts as a resistor, too, as before. In (c), the current in the circuit is directed to the right, and is decreasing. This causes a decrease in magnetic flux through the coils of the inductor, and the inductor responds by acting as a battery, connected so as to cancel out some of the decrease in current. As before, it also acts as a resistor.

EXPLORATION AC.1 – Potential differences in a circuit

Figure AC.2 shows part of a circuit, in which an inductor, a resistor, and a capacitor are connected in series. At a particular instant in time, the current in the circuit is 2.0 A, and the current is decreasing at the rate of 0.2 A/s. At this instant, the potential difference across the capacitor is 5.0 V, with the positive plate on the right.

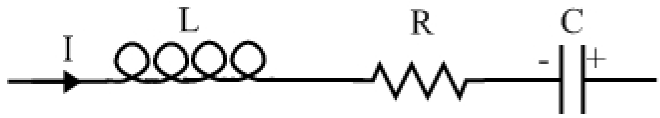


Figure AC.2: Part of a circuit, showing an inductor, a resistor, and a capacitor connected in series with one another, for Exploration AC.1.

Step 1 – ***The resistor in the circuit has a resistance of 2.0 ohms. At the instant in time described above, what is the potential difference across the resistor, and which end of the resistor is at a higher potential?***

The fact that the current is changing is not relevant here. All we need to know is the value of the current in the circuit at the instant we're interested in, and we know that to be 2.0 A. The potential difference across the resistor is then given by Ohm's law:

$$\Delta V_R = IR = (2.0 \text{ A}) \times (2.0 \ \Omega) = 4.0 \text{ V} .$$

Current passes through a resistor from the high potential side to the low potential side, so the left end of the resistor has the higher potential.

Step 2 – ***The inductor in the circuit has an inductance of 5.0 H. At the instant in time described above, what is the potential difference across the inductor, and which end of the inductor is at a higher potential? Assume that the resistance of the inductor is negligible.***

The magnitude of the potential difference across the inductor is given by:

$$|\Delta V_L| = L\left|\frac{\Delta I}{\Delta t}\right| = (5.0 \text{ H}) \times (0.2 \text{ A/s}) = 1.0 \text{ V} .$$

The current is directed to the right, but decreasing, so the induced voltage across the inductor acts to increase the current. This situation is exactly like that shown in part (c) of Figure AC.1, in fact. Thus, the right end of the inductor has a higher potential.

Step 3 – ***At the instant referred to above, what is the potential difference across the entire L-R-C (inductor-resistor-capacitor) series combination, and which end is at a higher potential?***

The potential difference across the inductor has the same polarity as the potential difference across the capacitor, so these combine to 6.0 volts. The potential difference across the resistor has the opposite polarity, so we subtract that 4.0 volts from the 6.0 volts to get a net potential difference of 2.0 V, with the right end having the higher potential.

Key ideas: The three different circuit components, resistors, capacitors, and inductors, behave differently from one another, but we can still apply basic rules, such as the loop rule and the junction rule, to understanding circuits that contain these components.
Related End-of-Chapter Exercises: ?, ?.

Essential Question AC.1: Return to the situation described in Exploration AC.1. If everything was the same, except that the current was increasing instead of decreasing, which answers would stay the same and which would be different? What would the new answers be?

Answer to Essential Question AC.1: The potential difference across the resistor would be the same. With the current increasing instead of decreasing, however, the potential difference across the inductor would have the same magnitude as before, but the left end of the inductor would have a higher potential. In that case, the inductor and resistor would combine to 5.0 V, cancelling the potential difference across the capacitor. The net potential difference would be zero.

AC-2 RL Circuits

RL circuits are circuits that contain both a resistor (the R) and an inductor (the L). In section AC-4, we will address what happens when an alternating current is applied to an RL circuit. For now, however, consider a series RL circuit consisting of a resistor, an inductor, a battery, and a switch. Such a circuit is similar in form to the RC circuit we investigated in Chapter 18. As we will see, the behavior of the current and voltage in this RL circuit is in many ways opposite to the behavior of current and voltage in the RC circuit, in the sense that the current in the RL circuit behaves like the voltage in the RC circuit, and vice versa.

EXPLORATION AC.2 – RL Circuits

In the RL circuit in Figure AC.3, the resistor and inductor are in series with one another. There is also a battery of emf ε , and a switch that is initially in the "no battery" position. Initially, there is no current in the circuit.

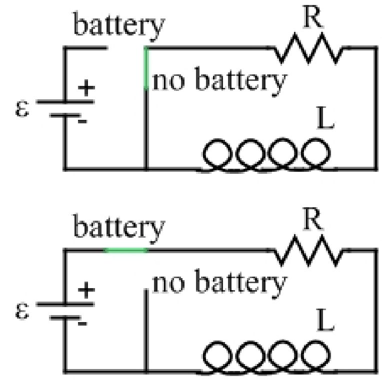


Figure AC.3: An RL circuit with a battery, resistor, inductor, and switch.

Step 1 – ***What are the general equations for the potential difference across a resistor, and the potential difference across an inductor?*** The potential difference across a resistor is given by Ohm's law, $\Delta V_R = IR$, while the potential difference across an inductor is given by $\Delta V_L = -L\frac{\Delta I}{\Delta t}$.

Step 2 – ***Use the loop rule to find the potential difference across the resistor, and across the inductor, immediately after the switch is moved to the "battery" position.*** If the inductor were not present, the current would change immediately from zero to $I = \varepsilon / R$. The inductor opposes changes in current, however, so immediately after the switch is closed, the current is still zero. This means the potential difference across the resistor is zero, and the potential difference across the inductor equals the emf of the battery.

Step 3 – ***What happens to the potential difference across the inductor, the potential difference across the resistor, and the current in the circuit as time goes by?*** The current in the circuit, and the potential difference across the resistor, which is proportional to the current, both increase as time goes by. By the loop rule, the potential difference across the inductor decreases as time goes by. The potential difference across the inductor is proportional to the slope of the current graph – thus, the slope of the current graph gradually decreases in magnitude as time goes by. This gives rise to the exponential relationships reflected in Figure AC.4, and characterized by the value of the inductance divided by the resistance, which has units of time.

$\tau = L / R$. (Equation AC.2: **Time constant for a series RL circuit**)

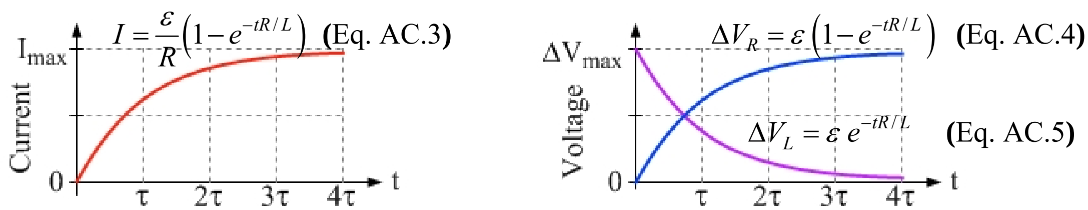


Figure AC.4: Plots of the current, resistor voltage, and inductor voltage, as a function of time after the switch is closed to bring the battery into the circuit.

Step 4 – *If we wanted the resistor voltage to increase more quickly, could we change the resistance? If so, how? Could we accomplish this by changing the inductance? If so, how?* To change the resistor voltage more quickly, we could change the resistance or the inductance. Increasing the resistance decreases the maximum current, so the inductor has a smaller change in current to oppose. Decreasing the inductance, on the other hand, means that the inductor is less effective at opposing change, so changes occur more quickly. This is consistent with the definition of the time constant, the value of L/R. Decreasing the time constant means that quantities change more quickly.

Step 5 – *When the switch has been in the "battery" position for a long time, the circuit approaches a steady state, in which the current and the resistor voltage both approach their maximum values, and the inductor voltage approaches zero. If the switch is now moved to the "no battery" position, what happens to the potential difference across the inductor, the potential difference across the resistor, and the current in the circuit as time goes by?* If the inductor was not present, switching the battery out of the circuit would instantly bring the current to zero. With the inductor, the inductor prevents this instantaneous change in current. Instead, the current decreases exponentially to zero, as does the voltage across the resistor. The inductor voltage is the negative of the resistor voltage, by the loop rule, so it also decreases in magnitude as time goes by. This gives rise to the relationships shown in Figure AC.5.

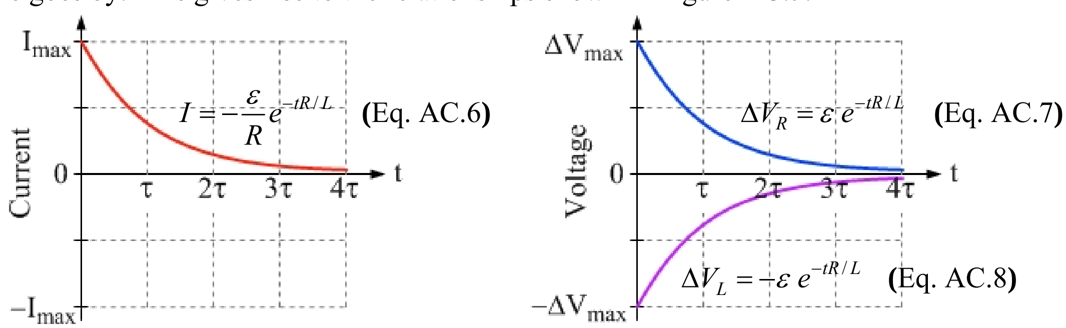


Figure AC.5: Plots of the current, resistor voltage, and inductor voltage, as a function of time after the battery is switched out of the circuit.

Key ideas for RL Circuits: Because of the response of the inductor to changes – it opposes any change in current – the current in an RL circuit changes as time goes by. The potential differences across the resistor and inductor also change, but the loop rule is satisfied at all times.
Related End-of-Chapter Exercises: 11, 12, 63, 64.

Essential Question AC.2: A particular RL circuit is connected like that in Figure AC.3. The battery emf is 12 V, and the resistor has a resistance of 20 Ω. When the switch is placed in the "battery" position, it takes 7.5 ms for the resistor voltage to increase from 0 V to 6.0 V. What is the inductance?

Answer to Essential Question AC.2: Here, we can substitute what we know into Equation AC.4.

$$6.0 \text{ V} = (12 \text{ V})\left(1 - e^{-(0.0075 \text{ s})R/L}\right).$$

Divide both sides by 12 V: $0.5 = 1 - e^{-(0.0075 \text{ s})R/L}$.

Re-arrange, then take the natural log of both sides: $\ln(0.5) = -\frac{(0.0075 \text{ s})R}{L}$.

Solving for the inductance gives: $L = -\frac{(0.0075 \text{ s}) \times (20\,\Omega)}{\ln(0.5)} = 0.22 \text{ H}$.

AC-3 AC Circuits with One Circuit Element, part I

In this section, we will investigate a circuit in which a single circuit element (either a resistor, a capacitor, or an inductor) is connected to a source of alternating voltage, such as a wall socket. The source voltage is a single frequency, so the voltage as a function of time is given by an equation of the form:

$$\Delta V = \varepsilon_{\max} \sin(\omega t) \,.$$ (Equation AC.9: **Source voltage in an AC circuit)**

In North America, the maximum emf provided by a standard wall socket is $\varepsilon_{\max}$ = 170 V, and the frequency of the oscillations is f = 60 Hz. Thus, $\omega = 2\pi f$ = 377 rad/s. Many people have heard that the voltage from a wall socket is 110 V or 120 V, which clearly differs from the 170 V stated here. This is because 110 to 120 V is an effective average voltage known as the rms (root-mean-square) voltage. For a sine wave, the relationship between the peak value and the rms average value is:

$$\varepsilon_{\text{rms}} = \frac{\varepsilon_{\max}}{\sqrt{2}} \,.$$ (Eq. AC.10: **Relationship between rms and peak values for a sine wave)**

AC circuit with just a resistor

When there is just a resistor, of resistance R, connected to the AC source, Ohm's law tells us that the current, as a function of time, in the circuit is given by:

$$I(t) = \frac{\varepsilon_{\max}}{R} \sin(\omega t) \,.$$ (Eq. AC.11: **Current in the case when there is just a resistor)**

The current peaks when the voltage peaks, so we say that the current is in phase with the voltage.

AC circuit with just a capacitor

In analyzing any AC circuit, the loop rule must be obeyed at all times. We investigated the loop rule in Chapter 18 – it tells us that the sum of the potential differences around a closed loop in a circuit is always zero. When we have just one circuit element connected to an AC source, the implication of the loop rule is that the potential difference across that circuit element must equal the source voltage. The source voltage oscillates sinusoidally, so the potential difference across the single circuit element must oscillate sinusoidally, as well.

Recall from Chapter 18 that the potential difference across a capacitor is given by:

$$\Delta V_C = \frac{Q}{C} \quad \text{(Eq. AC.12: \textbf{Potential difference across a capacitor})}$$

Here, Q is the charge stored on the capacitor, and C is the capacitance of the capacitor. For a particular capacitor, the capacitance is constant. Thus, when the voltage oscillates like a sine wave, the charge on the capacitor also follows a sine wave. If we look at the time rate of change of the capacitor voltage (the slope of the capacitor voltage vs. time graph), we get:

$$\frac{\Delta(\Delta V_C)}{\Delta t} = \frac{1}{C}\frac{\Delta Q}{\Delta t} = \frac{I}{C} \quad \text{(Eq. AC.13: \textbf{Current proportional to slope of the voltage graph})}$$

So, for a resistor, the current is proportional to the voltage. With a capacitor, the current is proportional to the slope of the voltage vs. time graph. With just a capacitor, the current is, in fact, given by a cosine, when the voltage is give by a sine. Let's make sense of this using the graphs shown in Figure AC.6. One cycle of the oscillations is split into four equal phases, each phase covering one quarter of the cycle.

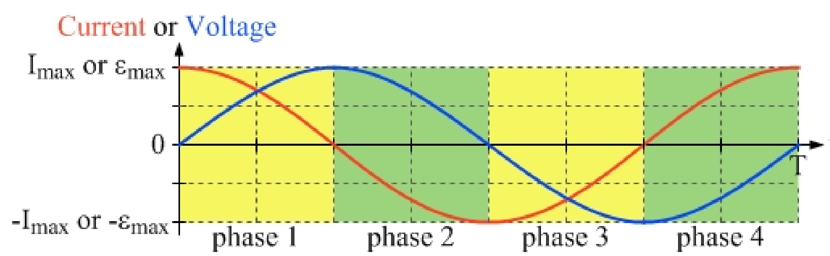


Figure AC.6: Plots of current as a function of time, and potential difference as a function of time, over one cycle for a capacitor connected to an AC source. The current peaks before the voltage does, by one-quarter cycle, so we say that the current leads the voltage by 90°.

Phase 1 represents the first quarter-cycle of the oscillations, during which time the voltage changes from 0 to the maximum positive voltage. At the beginning of phase 1, the capacitor is uncharged, but it needs to charge quickly at the beginning of phase 1, when the voltage increases quickly. As the slope of the voltage graph decreases, so does the current. At the very end of phase 1, the voltage peaks, so for an instant it is neither increasing nor decreasing. The current passes through zero at this instant, to match the zero rate of change of voltage – the capacitor is fully charged.

During phase 2, the capacitor discharges, first slowly, and then more quickly as the voltage decreases more rapidly. By the end of phase 2, the capacitor is fully discharged.

Phase 3 is similar to phase 1, except the current direction is opposite to what it was in phase 1, so the capacitor charges such that the plate that was positive in phase 1 becomes negative in phase 3, and vice verse. During phase 4, the capacitor discharges again. This whole sequence of events is illustrated in Figure AC.7.

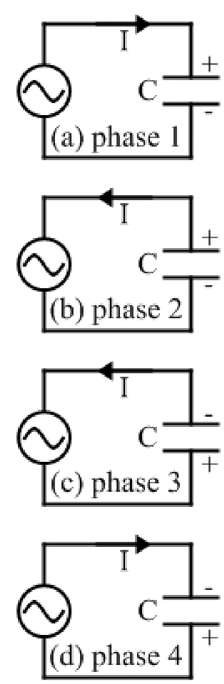


Figure AC.7: During one cycle of the voltage, the capacitor (a) charges one way, (b) discharges, (c) charges the other way, and (d) discharges again.

Essential Question AC.3: For the circuit with just a capacitor connected to an AC source, what happens to the maximum current if the capacitance is increased? If the frequency is increased?

Answer to Essential Question AC.3: From Equation AC.13, we can see that, if the capacitance is increased, the current must also increase. To achieve the same potential difference with a larger capacitance, we need more charge, which requires a larger current. If, instead, the frequency is increased, the voltage must increase by as much as it did before but in less time, so the current must increase in this case, too. We will address these ideas further in Section AC.4.

AC-4 AC Circuits with One Circuit Element, part II

Now, let's examine what happens when just an inductor is connected to an AC source.

AC circuit with just an inductor

In Section AC.1, we learned that the potential difference across an inductor is given by:

$$\Delta V_L = -L\frac{\Delta I}{\Delta t} \quad . \qquad \text{(Equation AC.1: \textbf{Induced voltage for an inductor})}$$

As with the previous cases, the loop rule must be obeyed at all times. Applying the loop rule here gives

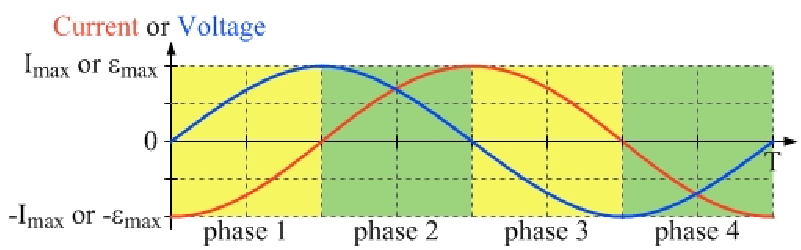
$$\varepsilon - \Delta V_L = 0;$$

$$\varepsilon_{\max}\sin(\omega t) - L\frac{\Delta I}{\Delta t} = 0;$$

$$\varepsilon_{\max}\sin(\omega t) = L\frac{\Delta I}{\Delta t}. \qquad \text{(Eq. AC.14: \textbf{Voltage proportional to the slope of the current})}$$

This is essentially the opposite of the behavior that we saw with the capacitor. With a capacitor, the current is proportional to the slope of the voltage vs. time graph. With an inductor, the inductor voltage is proportional to the slope of the current vs. time graph. Again, we can make sense of this using the graphs shown in Figure AC.8. One cycle of the oscillations is split into four equal phases, each phase covering one quarter of the cycle.

Figure AC.8: Plots of current as a function of time, and potential difference as a function of time, over one cycle for an inductor connected to an AC source.



Phase 1 represents the first quarter-cycle of the oscillations, during which time the voltage changes from 0 to the maximum positive voltage. To get zero voltage, Equation AC.14 tells us that the current vs. time graph must have zero slope at that instant. As the voltage increases, the slope of the current vs. time graph must also increase, and the slope of the current vs. time graph is maximum when the voltage peaks at the end of phase 1. This behavior is consistent with the current being given by a negative cosine function, when the voltage is given by a sine function.

During phases 2, 3, and 4, the slope of the current vs. time graph is proportional to the voltage at all times. For an inductor, we see that the voltage across the inductor is ahead of the current through the inductor by a quarter-cycle - we say that the voltage leads the current by 90°.

EXPLORATION AC.4 – Effective resistance in an AC circuit

In general, the resistance of a resistor depends only on its geometry (its length and cross-sectional area) as well as on the material it is made from (specifically, on the resistivity of that material). The frequency of the source of voltage that is connected to the resistor has no impact on the resistance of the resistor.

The same cannot be said for capacitors and inductors – their effective resistance is frequency dependent, which we will explore here.

Step 1 – ***Think about an AC circuit with just a capacitor connected to a source of alternating voltage. Based on what we learned in Section AC.3, do you expect the maximum current in this circuit to increase, decrease, or remain the same when we increase the frequency of the alternating voltage? Assume that the maximum voltage stays the same. Hint: think about what happens to the current during what we referred to as phase 1, when the capacitor goes from being uncharged to being charged with the maximum voltage across it.*** With an increased frequency, the capacitor experiences the same change in potential difference (and the same change in charge), in less time. The current is given by $I = \Delta Q / \Delta t$, so decreasing the time interval produces a larger current.

Step 2 – ***Given how the current changes when we increase the frequency, how does the effective resistance of the capacitor depend on frequency? Hint: use Ohm's law.*** In a situation like this, we can define the effective resistance as the maximum voltage divided by the maximum current, a definition consistent with Ohm's law. Keeping the maximum voltage the same, and observing that the current increases when frequency increases, we can conclude that the capacitor's effective resistance decreases as frequency increases – it is inversely proportional to frequency.

Step 3 – ***Now, think about an AC circuit with just an inductor connected to a source of alternating voltage. Based on what we know about inductors, how do you expect the effective resistance of an inductor to depend on frequency?*** We could start with equation AC.14, first thinking about what happens to the current, and then applying Ohm's law. Another way to answer this, however, is to recall that inductors tend to oppose a change in current. By increasing frequency, we are trying to change current more quickly, so the inductor will provide more resistance to change – its effective resistance is proportional to frequency.

Key ideas about effective resistance in AC circuits. We can be much more quantitative then we were in our analysis above. The effective resistance of a capacitor is known as the capacitive reactance, which has units of resistance, and is given by

$$X_C = \frac{1}{\omega C} \quad . \qquad \text{(Equation AC.15: \textbf{Capacitive reactance})}$$

The effective resistance of an inductor is known as the inductive reactance, which has units of resistance, and is given by

$$X_L = \omega L \quad . \qquad \text{(Equation AC.16: \textbf{Inductive reactance})}$$

Related End-of-Chapter Exercises: 11, 12, 63, 64.

Essential Question AC.4: A capacitor is connected to a source of alternating voltage. Find three different ways to double the maximum current in the circuit.

Answer to Essential Question AC.4: Ohm's law tells us that the maximum current in the circuit is given by $I_{max} = \varepsilon_{max} / X_C = \omega C \varepsilon_{max}$. In this form, coming up with three ways to double the maximum current is straightforward. We could double the frequency of the alternating voltage, we could double the capacitance of the capacitor, or, we could double the maximum voltage.

AC-5 AC Circuits with Two Circuit Elements

We can make some practical devices by combining two circuit elements together in a series circuit with a source of alternating voltage. We will investigate two such devices in this section.

To get us started, let's look at how to add resistances in an AC series circuit. Remember that in a series circuit with two resistors, the total resistance is simply the sum of the individual resistances. With a series circuit consisting of a resistor and an inductor, however, we find the net resistance by adding the individual resistances as vectors that are at 90° to one another. This is because, in the resistor, the voltage is in phase with the current, while in the inductor, the voltage is 90° ahead of the current. Being a series circuit, the current is the same through both devices, so the voltage across the inductor is 90° ahead of the voltage across the resistor – thus, we maintain this 90° angle when we find the equivalent resistance. Note that the equivalent resistance of an AC circuit is known as the **impedance**, *Z*, which has units of resistance.

As an example, let's say we have a circuit like that shown in Figure AC.10, where the resistance of the resistor is 40 ohms, and the resistance of the inductor (that is, the inductive reactance, X_L) is 30 ohms. Adding these as vectors gives an impedance of 50 ohms (the square root of the sum of the squares of *R* and X_L), as demonstrated in Figure AC.9.

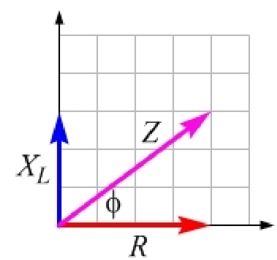


Figure AC.9: To find the equivalent resistance (the impedance) of an AC series circuit with an inductor and a resistor, we add, as vectors, the effective resistance of the inductor (the inductive reactance, X_L) and the resistance of the resistor.

EXPLORATION AC.5 – A dimmer switch

A variable inductor (an inductor with a variable inductance) can be used to cause a light bulb to dim or brighten in a circuit. We'll investigate how that works in this Exploration, using a series circuit consisting of a source of alternating voltage, an inductor, and a light bulb, which acts as a resistor in the circuit. This circuit is shown in Figure AC.10.

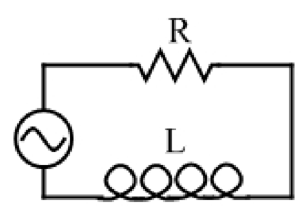


Figure AC.10: In this circuit, the resistor is a light bulb, and the value of the inductance can be changed by inserting an iron rod (or a rod made of any other ferromagnetic material) into the coil that is the inductor. Inserting the iron rod can dramatically increase the inductance.

Step 1 – ***If the peak voltage is 100 V, what is the peak current in this circuit when the resistor's resistance is 40 ohms, and the inductive reactance is 30 ohms?*** As we learned above, the impedance in this case is $Z = 50\ \Omega$. We can apply Ohm's law, to find that the peak current is given by $I_{max} = \varepsilon_{max} / Z = 2.0$ A .

Step 2 – ***A piece of iron is now inserted into the inductor, increasing the inductance by a factor of 10. By what factor does the inductive reactance increase? By what factor does the impedance increase?*** As we can see from Equation AC.16, the inductive reactance is proportional to the inductance. Thus, increasing the inductance by a factor of 10 will increase the inductive reactance by a factor of 10, also.

To determine the factor by which the impedance increases, we have to find the new impedance and compare it to the original value of 50 ohms. The new value is given by:

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{\left(300\,\Omega\right)^2 + \left(40\,\Omega\right)^2} = 303\,\Omega\,.$$

So, the impedance has increased by a factor of just over six.

Step 3 – ***What is the new value of the maximum current in the circuit? What effect does inserting the iron rod into the inductor have on the brightness of the light bulb?*** Increasing the impedance by a factor of six decreases the maximum current in the circuit by a factor of six, so the maximum current is about 0.33 A. As we learned from DC circuits, decreasing the current through a light bulb causes the light bulb to get dimmer. Thus, inserting the iron rod into the inductor causes the bulb to dim. The power dissipated is proportional to the square of the current, so decreasing the current by a factor of six decreases the power dissipated by a factor of 36 – it is the power dissipated that tells us about the brightness of the bulb.

Key ideas for a dimmer switch: Placing a variable inductor in series with a light bulb in an AC circuit is one way to make a dimmer switch. Using a variable resistor instead of a variable inductor would have the same effect on the bulb, but the variable resistor would dissipate energy, transforming the electrical energy into thermal energy. There is no such loss of energy with the inductor, because the inductor stores energy in the magnetic field, and then turns the stored energy back into electrical energy when the field goes to zero.
Related End-of-Chapter Exercises: 11, 12, 63, 64.

Power in an AC circuit

The instantaneous power dissipated in an AC circuit is the product of the emf and the current at that instant. In general, we are more interested in the average power dissipated in the circuit, so we average over one complete cycle. This gives:

$$P_{average} = \varepsilon_{rms} I_{rms} \cos\phi\,,$$ (Eq. AC.17: **Average power dissipated in an AC circuit**)

Where the rms (root-mean-square) values of the voltage and the current are smaller than the corresponding peak values by a factor of the square root of 2, and is the phase angle between the voltage and the current in the circuit. This is the same angle as that between *Z*, the impedance, and *R*, the resistance, in an impedance diagram like that in Figure AC.9. In the next section, we will explore this phase angle in a little more detail.

With a capacitor storing energy in the electric field between the capacitor plates, and an inductor storing energy in the magnetic field inside the coil, the average power dissipated in any AC circuit is always dissipated in the resistor. It can be shown that Equation AC.17 is completely equivalent to $P_{average} = \left(I_{rms}\right)^2 R$, which looks very much like the equation we used for DC circuits.

Essential Question AC.5: Could you make a dimmer switch using a variable capacitor, instead of a variable inductor, connected in series with alight bulb? If so, explain how it would work.

Answer to Essential Question AC.5: Yes, this is possible. To cause the bulb to dim, the capacitive reactance would need to increase, which requires decreasing the capacitance. That could be accomplished by removing a dielectric from the capacitor.

AC-6 RLC Circuits and the Impedance Triangle

In this section, we will look at some implications of connecting a resistor (R), an inductor (L), and a capacitor (C) together in what is called a series RLC circuit. The circuit diagram for such a circuit is shown in Figure AC.11.

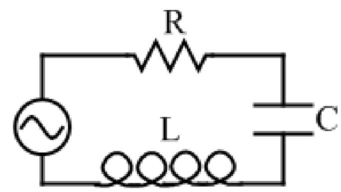


Figure AC.11: A series RLC circuit, in which a resistor, an inductor, and a capacitor are all connected in series with a source of alternating voltage.

As with a DC circuit, to find the current in the circuit we must first find the equivalent resistance, which we call the impedance in an AC circuit. The impedance is found by adding, as vectors, the resistance, the capacitive reactance, and the inductive reactance. Figure AC.12 illustrates the process. The voltage across the resistor is in phase with the current, so we place the resistance on the *x*-axis, at an angle of 0°. The voltage across the inductor is 90° ahead of the current, so we place the inductive reactance on the positive *y*-axis. The voltage across the capacitor is 90° behind the current, so we place the capacitive reactance on the negative *y*-axis.

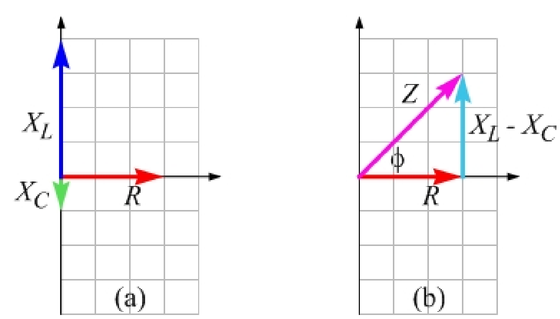


Figure AC.12: (a) In a particular case for an AC circuit, the resistance (R), inductive reactance (X_L), and capacitive reactance (X_C) are as shown in the diagram. (b) Combining the inductive reactance and capacitive reactance into one net vector allows us to find the impedance easily, because it is the hypotenuse of a right-angled triangle. We can call this the ***impedance triangle.***

In Figure AC.12, the phase angle is +45°. That is for that particular case – the phase angle can be anything between +90° and –90°, as we will explore in Exploration AC.6.

The equivalent resistance of a series RLC circuit is known as the ***impedance***. The impedance is the hypotenuse of an impedance triangle, such as that shown in Figure AC.12. Applying the Pythagorean theorem, we get:

$$Z = \sqrt{(X_L - X_C)^2 + R^2} \quad .$$ (Equation AC.18: **Impedance in a series RLC circuit**)

The impedance depends on the values of the resistance (R), inductance (L), capacitance (C), as well as on the frequency of the alternating voltage.

The ***phase angle***, ϕ, is the angle between the source voltage and the current in the circuit. From the geometry of right-angled triangles, we have:

$$\tan\phi = \frac{X_L - X_C}{R} \quad .$$ (Equation AC.19: **Phase angle in a series RLC circuit**)

In a circuit in which $X_L > X_C$ (the inductor dominates), the phase angle is positive, and the voltage leads the current. When $X_C > X_L$ (the capacitor dominates), the phase angle is negative, and the current leads the voltage. The phrase "ELI the ICE man" can help us to remember this. ELI tells us that for an inductor (L), the emf (E) leads the current (I). ICE tells us that for a capacitor (C), the current (I) leads the emf (E).

EXPLORATION AC.6 – The effect of changing frequency on the impedance

Resistance is independent of frequency, but the inductive reactance ($X_L = \omega L$) and the capacitive reactance ($X_C = 1/\omega C$) both depend on frequency. Thus, the impedance of a series RLC circuit depends on frequency. This dependence is rather interesting, in fact, because X_L increases with frequency and X_C decreases with frequency. We will explore this issue a little here.

Step 1 – ***In Figure AC.12, the inductive reactance is shown as four units, the capacitive reactance is shown as 1 unit, and the resistance is shown as 3 units. If the frequency of the alternating voltage is then decreased by a factor of 2, what happens to the values of X_L, X_C, and R?*** X_L is proportional to frequency, so decreasing the frequency by a factor of 2 also decreases the value of X_L by a factor of 2 – it drops from 4 units to 2 units. The opposite happens for X_C, which is inversely proportional to frequency. It doubles, from 1 unit to 2 units. R remains at 3 units, because changing frequency has no effect on the resistance.

Step 2 – ***For the situation of step 1, sketch a diagram like that of Figure AC.12, to see the new value of the impedance.*** This diagram is shown as Figure AC.13. Note that this is a special case, because the inductive reactance and capacitive reactance cancel one another, so the impedance is equal to the resistance. We will investigate this special case further in Section AC.7.

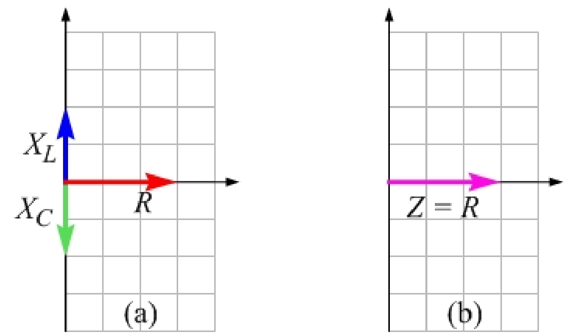


Figure AC.13: (a) When the frequency is decreased by a factor of 2 from that corresponding to Figure AC.12, the values of X_L and X_C also change by a factor of 2, with X_L decreasing and X_C increasing. (b) In this special case, the impedance is just the resistance**.**

Step 3 – ***Starting from the situation of steps 1 and 2 above, the frequency is decreased again by a factor of 2? What happens to the values of X_L, X_C, and R? Once again, sketch a diagram like that of Figure AC.12, to see the new value of the impedance.*** As before, X_L decreases by a factor of 2 – it drops from 2 units to 1 unit. X_C increases by a factor of 2 – it increases from 2 units to 4 units. R stays the same. As we can see in Figure AC.14, this brings the impedance back to its original value, although the voltage would now be behind the current by 45°.

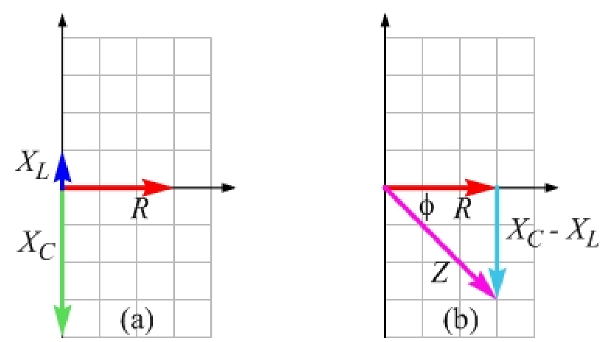


Figure AC.14: (a) After decreasing the frequency by an overall factor of 4 compared to Figure AC.12, X_L and X_C have traded values. (b) The impedance in this case has the same value as that in Figure AC.12, with a negative phase angle instead of a positive one**.**

Key ideas for the impedance triangle: The inductive reactance is proportional to frequency, and the capacitive reactance is inversely proportional to frequency, so the values of the impedance and the phase angle depend on frequency. We find these values with the aid of the impedance triangle, which uses vector addition. **Related End-of-Chapter Exercises: 11, 12, 63, 64.**

Essential Question AC.6: As we discussed above, the frequency at which $X_L = X_C$ is a special case. In terms of L and C, what is the value of the angular frequency, ω_0, when $X_L = X_C$?

Answer to Essential Question AC.6: If $X_L = X_C$, then we can say that:

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad \Rightarrow \qquad \omega_0 = \frac{1}{\sqrt{LC}} .$$

This is known as the ***resonance frequency***. We will investigate resonance in detail in Section AC-7.

AC-7 Resonance

The phenomenon of resonance is well known to you already. If you push someone on a swing, the most effective way to build up large-amplitude oscillations is to match the frequency of your pushes to the natural oscillation frequency (the resonance frequency) of the swing. Similarly, a series RLC circuit has a natural oscillation frequency that is given by the values of the inductance and the capacitance. As we learned from Essential Question AC.6, the angular frequency corresponding to the resonance frequency is:

$$\omega_0 = \frac{1}{\sqrt{LC}} .$$ (Equation AC.20: **Resonance angular frequency in a series RLC circuit**)

As we explored in Section AC-6, a circuit is in resonance when $X_L = X_C$. In that case, the inductive reactance cancels the capacitive reactance. This leads to a number of special cases.

Special cases at resonance

- $R = Z$. The impedance is equal to the resistance. This minimizes the value of the impedance.
- Because the impedance is minimized, the current in the circuit is maximized.
- $\phi = 0$. This means that the source voltage and the circuit current oscillate in phase with one another.
- $\cos\phi = 1$. $\cos\phi$ is known as the power factor, because of the fact that $\cos\phi$ appears in the equation for average power dissipated in the circuit (see Equation AC.17). The power dissipated in the circuit is a maximum.

Figure AC.15 shows a graph of the rms current in a particular RLC circuit, as a function of the frequency of the alternating voltage. The peak current occurs at the resonance frequency, and the current decreases the more the frequency is moved away from the resonance frequency.

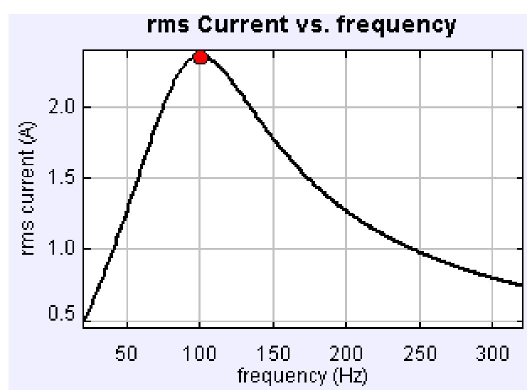


Figure AC.15: a graph of the rms current in a particular RLC circuit, as a function of the frequency of the alternating voltage.

Resonance can be exploited in many circuits, such as in the metal detectors you walk through at the airport or which trigger a traffic light to change, or as in the tuning circuits of older AM-FM radios. Let's explore how a simple metal detector might work. A better name for such a device is a ferromagnetic material detector, because ferromagnetic materials change the inductance of an inductor to a much greater extent than do other metals.

EXPLORATION AC.7 – A (ferromagnetic) metal detector

Let's explore a basic metal detector, such as one used at an airport security checkpoint. The basic design is a series RLC circuit, in which the inductor is the large rectangular loop that you walk through. An AC ammeter is the detector – it will sound an alarm if the rms current drops too far below the value at resonance. Let's say that the inductor has an inductance of 5.0 mH, the resistance is 4.0 Ω, and the source of alternating voltage is a transformer plugged into a North American wall socket, with a frequency of 60 Hz and an rms voltage of 6.0 V.

Step 1 – ***If we want to design the circuit so that the resonant frequency is 60 Hz (this is f, rather than ω), what is the value of the capacitance in the circuit?*** Using $\omega = 2\pi f$, we find that the angular frequency corresponding to resonance is ω_0 = 377 rad/s. We can then re-arrange Equation AC.20 to solve for the capacitance:

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(377 \text{ rad/s})^2 (0.005 \text{ H})} = 1.4 \text{ mF} .$$

Step 2 – ***What is the rms current in the circuit at resonance?*** Here, we can exploit the fact that, at resonance, the impedance is equal to the resistance, so Z = 4.0 Ω. Applying Ohm's law to the circuit as a whole, we find:

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{6.0 \text{ V}}{4.0 \ \Omega} = 1.5 \text{ A}$$

Step 3 – ***You now walk through the inductor. Usually, this would not cause much change in the inductance, but today you are wearing a steel belt buckle, which causes the inductance to increase by a factor of 5. Find the value of the circuit's impedance now.*** Let's first find the value of the capacitive reactance. Because the circuit was initially at resonance, the original value of the inductive reactance is equal to the capacitive reactance. The capacitive reactance is given by:

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{(377 \text{ rad/s})(1.4 \text{ mF})} = 1.885 \ \Omega .$$

This is also the value of the original inductive reactance, so the new value of the inductive reactance is five times larger than this (because the inductive reactance is proportional to the inductance), so is now X_L = 9.425 Ω. The new value of the impedance is given by:

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(9.425 \ \Omega - 1.885 \ \Omega)^2 + (4.0 \ \Omega)^2} = 8.535 \ \Omega$$

Step 4 –***Calculate the new value of the rms current in the circuit.*** Once again, we can apply Ohm's law to the circuit as a whole. This gives:

$$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{6.0 \text{ V}}{8.535 \ \Omega} = 0.70 \text{ A} .$$

Thus, the current has decreased from its value at resonance by more than a factor of 2. This drop in current can be used to trigger an alarm, to tell the checkpoint personnel that someone has brought something unusual through the checkpoint.

Key ideas: Resonant circuits are used in many practical devices. Generally, these circuits exploit the fact that the inductance increases whenever an object made from ferromagnetic material is brought near the inductor. **Related End-of-Chapter Exercises: 11, 12, 63, 64.**

Essential Question AC.7: In Figure AC.15, the resonance frequency is shown to be 100 Hz. If the inductance in the circuit is 5.0 mH, what is the capacitance?

Answer to Essential Question AC.7: Once again, we can use the resonance relationship, $X_L = X_C$. This gives: $\omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 100 \text{ Hz})^2 (0.005 \text{ H})} = 5.1\times 10^{-4} \text{ F}$.

AC-8 The Loop Rule in AC Circuits

In this section, we will explore how to apply the loop rule to an AC circuit. In particular, we'll look at a series RLC (resistor-inductor-capacitor) circuit. We'll also learn how not to apply the loop rule, too.

The loop rule is something we learned in DC circuits. There, we said that the sum of the potential differences around a complete loop in a circuit must be zero. For a circuit consisting of a battery and a few resistors in series, an equivalent statement is that the sum of the potential differences across all the resistors is equal to the battery voltage. We also observed that the potential difference across any one resistor was always less than or equal to the battery voltage.

It's easy to make an analogous statement for a series AC circuit. This is only slightly complicated by the fact that the source voltage in an AC circuit changes as time goes by, something we did not usually have to worry about for the DC circuit (an exception being a resistor-capacitor circuit, where the loop rule could be stated in the same way we're just about to give the loop rule for AC circuits). Here, we will just add that the loop rule applies at every instant in time.

The loop rule for AC circuits is that the sum of the potential differences around a closed loop, at any instant in time, is equal to zero. For a series RLC circuit, an equivalent statement for a series RLC circuit is that the sum of the potential differences across the resistor, the inductor, and the capacitor at any instant is equal to the source voltage at that instant.

EXPLORATION AC.8 – Applying the loop rule

Consider a particular RLC circuit, connected to an alternating voltage source that has a frequency of 70 Hz and a peak voltage of 10 V. The resistance is 2.0 Ω, the inductance is 15 mH, and the capacitance is 1.5 mF. There is nothing particularly special about these conditions. For instance, the circuit is not at resonance. Our goal will be to work out the peak values of the potential difference across the resistor, the inductor, and the capacitor, and then to compare the sum of those peak values to the maximum source voltage.

Step 1 – *Calculate the impedance of the circuit.* Using $\omega = 2\pi f$, we find that the angular frequency corresponding to 70 Hz is $\omega = 439.8$ rad/s. This helps us to calculate the inductive reactance and the capacitive reactance.

$$X_L = \omega L = (439.8 \text{ rad/s})(0.015 \text{ H}) = 6.60 \ \Omega .$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(439.8 \text{ rad/s})(0.0015 \text{ F})} = 1.52 \ \Omega .$$

Using these values in the impedance equation gives:

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(6.60 \ \Omega - 1.52 \ \Omega)^2 + (2.0 \ \Omega)^2} = 5.46 \ \Omega .$$

Step 2 – *Calculate the peak current in the circuit.* That is, calculate the peak current at this frequency. This comes directly from applying Ohm's law to the circuit as a whole:

$$I_{\max} = \frac{\varepsilon_{\max}}{Z} = \frac{10.0 \text{ V}}{5.46 \ \Omega} = 1.83 \text{ A}$$

Step 3 – ***Applying Ohm's law individually to the resistor, the inductor, and the capacitor, calculate the peak voltage across each of these components.*** Here, we're exploiting the fact that the current through each component is the same, because this is a series circuit.

Across the resistor: $\Delta V_{\max,R} = I_{\max} R = 1.83 \text{ A} \times 2.0 \ \Omega = 3.66 \text{ V}$.

Across the inductor: $\Delta V_{\max,L} = I_{\max} X_L = 1.83 \text{ A} \times 6.60 \ \Omega = 12.1 \text{ V}$.

Across the capacitor: $\Delta V_{\max,C} = I_{\max} X_C = 1.83 \text{ A} \times 1.52 \ \Omega = 2.78 \text{ V}$.

Note that this already seems a little strange – the peak inductor voltage all by itself is larger than the peak source voltage. We certainly never saw anything like that in DC circuits.

Step 4 – ***Add the three individual peak voltages, from step 3, and compare the sum to the peak voltage from the source. Comment on whether or not this result indicates that the loop rule is being violated in this situation.*** Adding the three peak voltages from step 3, we find that their sum is 15.8 V. This is more than 50% larger than the peak voltage from the source. However, this does not mean that the loop rule is being violated. In fact, the loop rule works just fine in this situation - adding the three peak voltages actually results in a meaningless number.

The key here is that, to check the loop rule, we have to add the resistor, inductor, and capacitor voltages at a particular instant in time. We can't just add the peak voltages, because the peak voltages occur at different times. The graphs in Figure AC.16 help to illustrate this point. If you choose a particular instant in time, you will always find that the sum of the resistor, inductor, and capacitor voltages at that instant add up to the source voltage at that instant. Check this at a few different times to make sure you believe it. There is never a time when you add the three peak voltages together. In fact, the inductor and capacitor voltages are always 180° out of phase with one another, so when one of these voltages is positive the other is always negative. That helps to explain how one (or even both) of these voltages can exceed the peak source voltage.

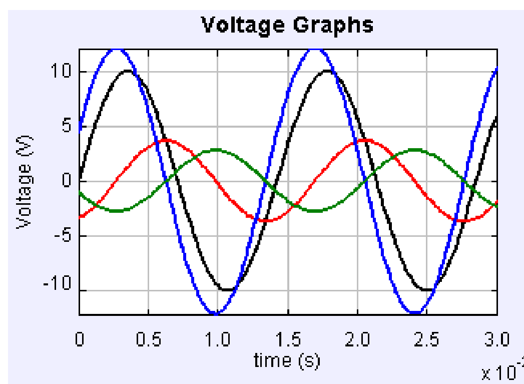


Figure AC.16: a graph of the voltages in our particular RLC circuit, as a function of time. The source voltage is shown in black – it has a peak voltage of 10 V. The inductor voltage is shown in blue – it has a peak voltage of about 12 V. The resistor voltage is shown in red. Its peak voltage is a little higher than the capacitor voltage, which is shown in green.

Key ideas for applying the loop rule: One thing to remember about AC circuits is that the current and the various voltages change with time. This is quite different from DC circuits with resistors, in which the currents and voltages were constant. Thus, to correctly apply the loop rule, we need to apply it at an instant in time – we can't just add the peak voltages together and expect to get something meaningful. **Related End-of-Chapter Exercises: 11, 12, 63, 64.**

Essential Question AC.8: In Figure AC.16, is the current leading the voltage, or is the voltage leading the current?

Answer to Essential Question AC.8: The first thing to realize is that we have quite a few different voltages, so which of these is "the" voltage? In this context, we want to know the phase relationship between the source voltage and the current. The second thing to realize is that the graph of current vs. time is not shown – all the graphs are voltage graphs. However, for a resistor, the resistor voltage is in phase with the current, so the phase relationship between the source voltage and the resistor voltage is the same as the phase relationship between the voltage and the current. In looking at the graph, we can see that the source voltage graph peaks at a little earlier time than the resistor voltage graph, so the voltage leads the current here. Note that the current and voltage are always within a quarter-period of one another, so we can always compare them when the peaks are close together on the graph.

Note that are answer here is consistent with the "ELI the ICE man" memory device we introduced in section AC-6. In the circuit of Exploration AC.8, the inductor dominates compared to the capacitor. Whenever the inductor dominates, the voltage (emf) leads the current (I). The opposite is true in circuits in which the capacitor dominates compared to the inductor.

Chapter Summary

Essential Idea: AC Circuits.

Just like in DC circuits, analyzing an AC circuit means finding the current through, and the potential difference across, each component. As with DC circuits, we first find the equivalent resistance in the circuit (this is the **impedance** in an AC circuit), and then we can apply basic rules (such as Ohm's law) to determine the potential difference across every component. Unlike DC circuits (generally, at least), however, in AC circuits the voltage and current continually change with time.

Inductors and Inductive Reactance

An inductor is simply a wire coil that is connected in a circuit. Through the process of electromagnetic induction, a voltage is induced in such a device whenever the current through it is changing, as it is at almost all times in an AC circuit. The effective resistance (known as the inductive reactance, X_L) of an inductor is proportional to the frequency of the alternating current.

$X_L = \omega L$.   (Equation AC.16: **Inductive reactance**)

Capacitive Reactance

Capacitors also have an effective resistance (the capacitive reactance), but it is inversely proportional to the frequency.

$X_C = \dfrac{1}{\omega C}$.   (Equation AC.15: **Capacitive reactance**)

The impedance (the equivalent resistance) of a series AC circuit

For an AC circuit with a resistor, capacitor, and/or inductor connected in series to one another and to an AC source, the impedance is given by:

$Z = \sqrt{(X_L - X_C)^2 + R^2}$.   (Equation AC.18: **Impedance in a series RLC circuit**)

The impedance triangle

The impedance triangle (see, for example Figure AC.12) is a method we use to add, as vectors, the various contributions to the impedance. The resistance is placed on the *x*-axis. The inductive reactance is placed on the positive y-axis, because the voltage across the inductor is 90° (one quarter-cycle) ahead of the current. The capacitive reactance is placed on the negative y-axis, because the voltage across the capacitor is 90° behind the current. The impedance is the vector sum of these three vectors, resulting in the impedance equation given above.

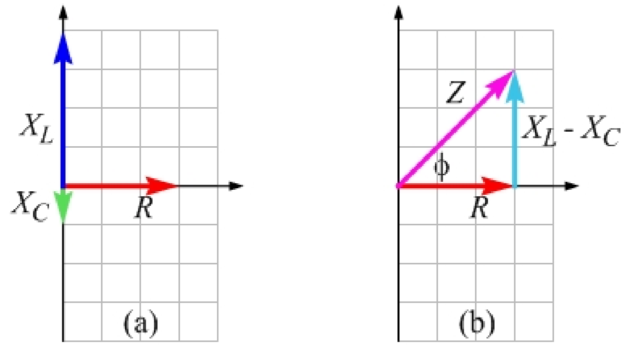


Figure AC.12: (a) In a particular case for an AC circuit, the resistance (R), inductive reactance (X_L), and capacitive reactance (X_C) are as shown in the diagram. (b) Combining the inductive reactance and capacitive reactance into one net vector allows us to find the impedance easily, because it is the hypotenuse of a right-angled triangle. We call this the ***impedance triangle.***

The phase angle

The phase angle, ϕ , in the impedance triangle is also the angle between the source voltage and the current in the circuit. From the geometry of right-angled triangles, we have:

$$\tan\phi = \frac{X_L - X_C}{R} \quad .$$

(Equation AC.19: **Phase angle in a series RLC circuit**)

In a circuit in which $X_L > X_C$ (the inductor dominates), the phase angle is positive, and the voltage leads the current. When $X_C > X_L$ (the capacitor dominates), the phase angle is negative, and the current leads the voltage. The phrase "ELI the ICE man" can help us to remember this. ELI tells us that for an inductor (L), the emf (E) leads the current (I). ICE tells us that for a capacitor (C), the current (I) leads the emf (E).

Resonance

A series RLC circuit has a natural oscillation frequency, just like a mass on a spring. In fact, each of these systems is an analog of the other. The condition for resonance is that the inductive reactance is equal to the capacitive reactance, which leads to the following expression for the resonance frequency of the circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}} \, .$$

(Equation AC.20: **Resonance angular frequency in a series RLC circuit**)

Special cases at resonance

- $R = Z$. The impedance is equal to the resistance. This minimizes the value of the impedance.
- Because the impedance is minimized, the current in the circuit is maximized.
- $\phi = 0$. This means that the source voltage and the circuit current oscillate in phase with one another.
- $\cos\phi = 1$. $\cos\phi$ is known as the power factor, because of the fact that $\cos\phi$ appears in the equation for average power dissipated in the circuit (see Equation AC.17). The power dissipated in the circuit is a maximum.

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter.

1. Figure AC.17 shows a circuit in which a resistor with a resistance of 3.0 Ω is connected in series with an inductor and a variable power supply. The inductor has an inductance of 4.0 H. At a particular instant, you are turning the knob on the power supply, and the potential difference from the power supply is changing. At this instant, the current is 2.0 A, directed clockwise, and the power supply voltage is 4.0 V. (a) At this instant, what is the magnitude of the potential difference across the inductor? (b) At this instant, what is the time rate of change of the current, and is the current increasing or decreasing in magnitude?

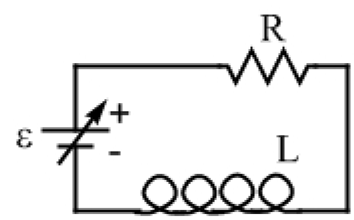


Figure AC.17: A circuit consisting of a resistor, an inductor, and a variable power supply (a voltage source with a variable voltage), all connected in series. For Exercise 1.

2. Figure AC.18 shows a series RLC AC circuit, in which the resistor is a light bulb of constant resistance. As you adjust the frequency of the alternating voltage source, you observe that the bulb is brightest at a frequency of 90 Hz, and gets dimmer if you increase or decrease the frequency from there. Explain these observations.

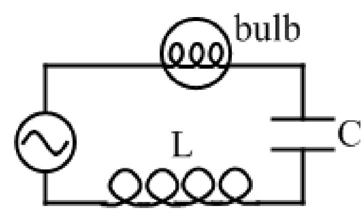


Figure AC.18: A series RLC AC circuit, in which the resistor is a light bulb, for Exercise 2.

3. In a stereo speaker, the tweeter emits the high-frequency sounds and the woofer emits the low-frequency sounds. The voltage signal representing the music being played by this speaker is placed across a resistor and a capacitor that are in series with one another. The voltage across the resistor is connected to the tweeter, and the voltage across the capacitor is connected to the woofer. Explain why the connections are made in this way.

4. The graphs in Figure AC.19 show current and voltage, as a function of time, for an AC circuit with only one circuit element. Which graph (a or b) shows the current and which shows the voltage if that one circuit element is (a) a capacitor, (b) an inductor, or (c) a resistor?

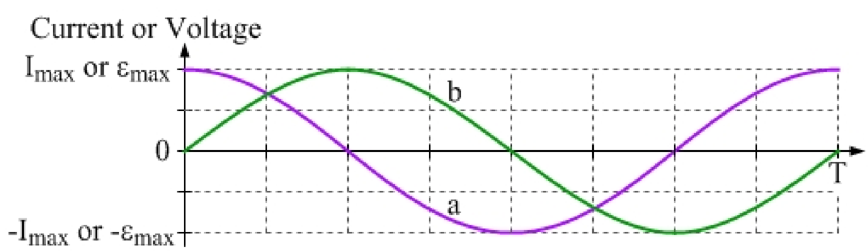
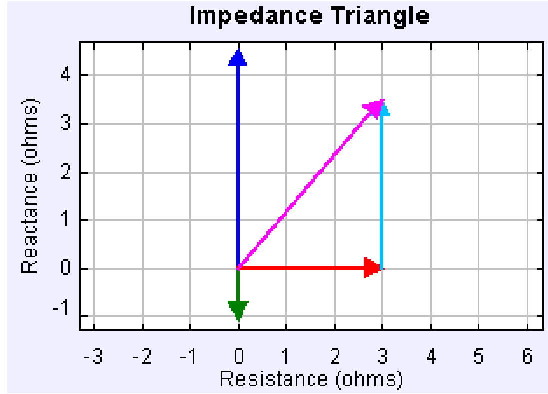


Figure AC.19: Graphs of current and voltage as a function of time, for an AC circuit with only one circuit element. For Exercise 4.

5. Figure AC.20 shows the impedance triangle for a particular RLC series AC circuit, at a particular frequency. For this situation, rank the peak voltages across the resistor, the inductor, and the capacitor, from largest to smallest. Your ranking should have the form $\Delta V_{\max,R} > \Delta V_{\max,L} = \Delta V_{\max,C}$.

Figure AC.20: The impedance triangle for a particular RLC series AC circuit, at a particular frequency. For Exercises 5 and 6.



6. Figure AC.20 shows the impedance triangle for a particular RLC series AC circuit, at a particular frequency. If the frequency is increased, state whether the following will increase in magnitude, decrease in magnitude, or stay the same. (a) The resistance. (b) The inductive reactance. (c) The capacitive reactance. (d) The impedance. (e) The rms current. (f) The phase angle. (g) The voltage across the resistor.

7. In a particular RLC series AC circuit, at a particular source frequency, the inductive reactance is larger than the capacitive reactance. (a) Does the source voltage lead the current at this frequency, or does the current lead the source voltage? (b) The frequency is then increased a little. Will this change in frequency cause the phase difference between the source voltage and the current to become larger in magnitude or smaller in magnitude?

8. In a particular RLC series AC circuit, at a particular source frequency, the inductive reactance is smaller than the capacitive reactance. (a) Does the source voltage lead the current at this frequency, or does the current lead the source voltage? (b) The frequency is then increased a little (the inductive reactance is still smaller than the capacitive reactance, however). Will this change in frequency cause the phase difference between the source voltage and the current to become larger in magnitude or smaller in magnitude?

9. In a particular RLC series AC circuit, at a particular source frequency, the inductive reactance is smaller than the capacitive reactance. The frequency is then increased a little. Will this change in frequency cause the rms current in the circuit to increase or decrease? Justify your answer.

10. In an RLC series AC circuit, the impedance is always minimized when the source frequency is set to the resonance frequency. If the frequency is increased from resonance by some factor k, the impedance increases. Explain why the impedance increases to exactly the same value whether the frequency is increased by a factor of k, or decreased by a factor of k. In other words, if the resonance frequency is f_0, explain why the impedance is the same at a frequency of $k \times f_0$ as it is at a frequency of f_0/k.

11. Figure AC.21 shows a graph of the voltages across the resistor, the inductor, and the capacitor in a series RLC circuit. Identify which curve goes with which component.

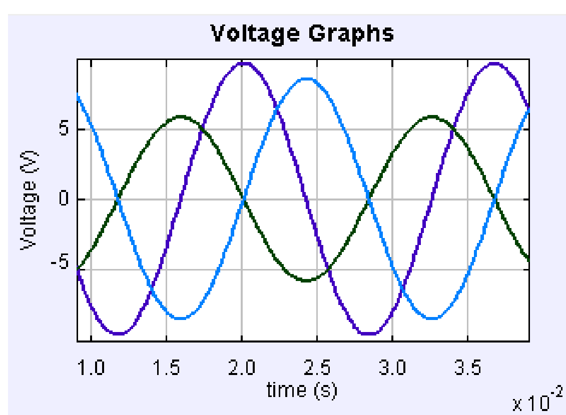


Figure AC.21: A graph of the voltages across the resistor, the inductor, and the capacitor in a series RLC circuit, for Exercises 11 and 12.

12. Figure AC.21 shows a graph of the voltages across the resistor, the inductor, and the capacitor in a series RLC circuit. How does the frequency of the voltage source compare to the resonance frequency? All you have to do is to specify whether the frequency is greater than, equal to, or less than the resonance frequency.

Exercises 13 – 16 involve RL circuits.

13. A series RL circuit consists of a 12-volt battery, a resistor, an inductor, and a switch, all connected in series. The switch is open and there is no current, initially, in the circuit. The switch is closed at $t = 0$ and the current increases from zero. At $t = 4.0$ s, the potential difference across the inductor is 9.0 V. (a) What is the potential difference across the inductor at $t = 8.0$ s? (b) At what time is the potential difference across the inductor equal to 6.0 V? (c) At what time is it equal to 3.0 V?

14. A particular series RL circuit consists of a 12-volt battery, a resistor with a resistance of 3.0 Ω, an inductor with an inductance of 6.0 H, and a switch, as shown in Figure AC.22. The switch has been in the "battery" position for a long time. (a) Under these conditions, what is the current in the circuit? The switch is then moved to the "no battery" position at $t = 0$. At $t = 1.0$ s, determine (b) the current in the circuit, (c) the potential difference across the resistor, and (d) the potential difference across the inductor.

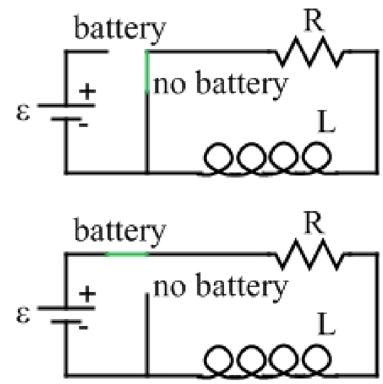


Figure AC.22: An RL series circuit, with a switch that can either connect or disconnect the battery, for Exercise 14.

15. Table AC.1 shows the voltages across the battery, resistor, and the inductor, for a circuit in which those three components are wired together in series. The times shown are 2.0 s and 5.0 s after the circuit is connected. Three of the voltage readings are not shown in the table – complete the table.

Table AC.1: A particular circuit consists of a battery, a resistor, and an inductor, connected in series. Initially, one of the wires in the circuit is disconnected, so there is no current. At $t = 0$, the wire is connected, and the current increases from zero. The voltages across the battery, the resistor, and the inductor, at two times after the wire has been connected. are shown. For Exercise 15.

Time	Battery voltage	Resistor voltage	Inductor voltage
2.0 s	6.0 V	2.0 V	_____ V
5.0 s	6.0 V	____ V	_____ V

16. A series circuit consists of a resistor, an inductor, and a 6.0-volt battery. 20 ms after the battery is first connected, the voltage across the resistor is 1.0 V. After a long time, the current in the circuit reaches a constant value of 1.5 A. (a) What is the current in the circuit when the resistor voltage is 1.0 V? (b) What is the value of the resistance? (c) What is the value of the inductance?

Exercises 17 – 20 involve AC circuits with two circuit elements.

17. A particular AC circuit consists of a resistor, an inductor, and a source of alternating voltage, all connected in series. When the source frequency is 100 Hz, the phase angle is

45°, with the source voltage leading the current. When the source frequency is changed to 200 Hz, what is the phase angle?

18. In a dimmer-switch circuit, a 120 V, 100 W light bulb is connected to a 120 V (rms) wall socket, in which the frequency is 60 Hz. Also in the circuit, in series with the light bulb, is an inductor. What is the value of the inductance when the light bulb is dissipating 60 watts? Assume that the resistance of the light bulb is constant.

19. Consider the impedance triangle shown in Figure AC.23, which is for an AC circuit consisting of a resistor and a capacitor connected to a source of alternating voltage. The source frequency is 60 Hz, the resistance is 4.0 Ω, and the capacitive reactance is 3.0 Ω. Determine the values of (a) the impedance, (b) the phase angle, and (c) the capacitance.

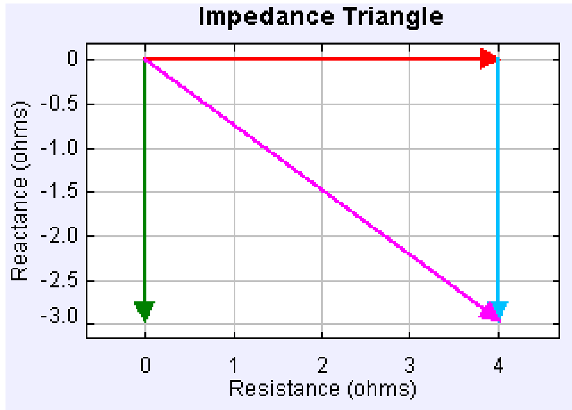


Figure AC.23: An impedance triangle for an AC circuit consisting of a resistor and a capacitor connected to a source of alternating voltage, for Exercises 19 and 20.

20. Consider the impedance triangle shown in Figure AC.23, which is for an AC circuit consisting of a resistor and a capacitor connected to a source of alternating voltage. The source frequency is 60 Hz, the resistance is 4.0 Ω, and the capacitive reactance is 3.0 Ω. At what frequency would the impedance be doubled?

Exercises 21 – 24 involve the impedance triangle.

21. In a particular AC circuit, a resistor and a capacitor are connected in series with a source of alternating voltage. The frequency is 60 Hz, the resistance is 3.0 Ω, and the capacitance is 2.5 mF. (a) Sketch the impedance triangle corresponding to this situation. (b) Calculate the impedance. (c) Calculate the phase angle.

22. In a particular AC circuit, a resistor, a capacitor, and an inductor are connected in series with a source of alternating voltage. The frequency is 50 Hz, the resistance is 2.0 Ω, the capacitance is 3.0 mF, and the inductance is 6.0 mH. (a) Sketch the impedance triangle corresponding to this situation. (b) Calculate the impedance. (c) Calculate the phase angle.

23. Figure AC.24 shows an impedance triangle for a particular RLC series AC circuit, at a particular frequency. (a) Determine the impedance in this situation. (b) If the frequency is increased by just the right factor, the impedance will be the same as it is at the frequency shown. What factor do we need to multiply the frequency by for the impedance to be the same?

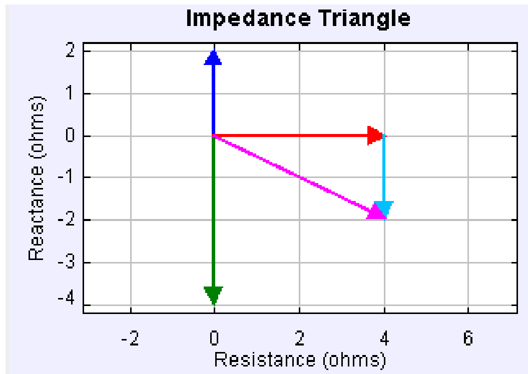


Figure AC.24: An impedance triangle for a particular RLC series AC circuit, at a particular frequency. For Exercises 23, 24, and 25.

24. Figure AC.24 shows an impedance triangle for a particular RLC series AC circuit, at a particular frequency. The inductance is 8.0 mH. (a) What is the frequency of the alternating voltage source? (b) What is the capacitance?

Exercises 25 – 28 involve resonance.

25. Figure AC.24 shows an impedance triangle for a particular RLC series AC circuit, at a particular frequency. (a) By what factor would the inductance need to change by for the circuit to be in resonance? (b) If, instead, you brought the circuit to resonance by changing the capacitance, by what factor would the capacitance need to change by for the circuit to be in resonance? (c) If, instead, you brought the circuit to resonance by changing the frequency, by what factor would the frequency need to change by for the circuit to be in resonance? (d) What is the impedance of the circuit at resonance?

26. You are designing a series RLC AC circuit in which the resonance frequency should be 120 Hz. (a) If the inductance in the circuit is 1.0 mH, what is the capacitance? (b) Does the value of the resistance have any impact on the value of the resonance frequency? Justify your answer.

27. In a particular RLC AC circuit, the peak voltage is 10.0 V, and the resonance frequency is 75 Hz. The resistance in the circuit is 2.0 Ω, and the inductance is 5.0 mH. (a) Find the value of the peak current in the circuit. (b) If the frequency is doubled to 150 Hz, find the new value of the peak current (at 150 Hz) in the circuit. (c) Find the ratio of the peak current at 75 Hz to the peak current at 150 Hz.

28. Repeat Exercise 27, but now use a resistance of 10 Ω. (d) Comment on how the ratio of the peak currents changes when the resistance is increased.

Exercises 29 – 34 involve practical applications of AC circuits.

29. A series circuit consisting of a source of alternating voltage connected to a resistor and an inductor can be used as a ***low-pass filter*** (passing low-frequency signals, but blocking high-frequency signals) and/or a ***high-pass filter*** (passing high-frequency signals, but blocking low-frequency signals). Let's explore how this works. The maximum source voltage is 10.0 V, the frequency is 200 Hz, the resistance is 8.00 Ω, and the inductance is 5.00 mH. Determine the maximum voltage across (a) the inductor, and (b) the resistor. The frequency is then changed to 1000 Hz. Once again, determine the maximum voltage across (c) the inductor, and (d) the resistor. In general, one of these voltages steadily increases as the frequency increases, while the other decreases. (e) Identify which voltage (the voltage across the inductor, or the voltage across the resistor) increases as frequency increases. Note that the other increases as frequency decreases. (f) Which of these signals serves as the low-pass filter, and which as the high-pass filter?

30. Continuing from Exercise 29, note that the series combination of a resistor and a capacitor can be used in a stereo speaker, with the source voltage being the voltage that represents the music being played through the speaker. The tweeter in the speaker emits the high-frequency sounds and the woofer emits the low-frequency sounds. (a) Identify

which voltage (the voltage across the inductor, or the voltage across the resistor) is connected to the inductor, and which is connected to the resistor. (b) If the resistance is 4.00 Ω, what should the value of the inductance be if the maximum voltage across the resistor and the maximum voltage across the inductor are equal to one another at a frequency of 500 Hz?

31. In a transistor radio, the antenna may be picking up signals from dozens of radio stations simultaneously. How does it know which signal to play through the speaker? The radio makes use of a series RLC circuit, with the resonance frequency matching the broadcast frequency of the radio station. Let's say you are listening to an FM radio station broadcasting at a frequency of 101.5 MHz. The inductor in the circuit has an inductance of 100 pH (pico is 10^{-12}). (a) What is the capacitance of the capacitor? (b) To tune the radio to a different station, you move a dial – this dial changes the value of the inductance, by moving a ferromagnetic material either closer to, or farther from, the inductor. To tune to a station at 90.9 MHz, what does the new value of the inductance have to be?

32. You may have noticed that, at particular traffic lights, when you stop at a red light the light will soon change, as if the light sensed your presence. This, in fact, is the case. At such lights, you may also notice large rectangular loops cut into the pavement – these are from conducting wires that make up the inductor in a series RLC circuit. The frequency is 60 Hz, and the resistance is 30 Ω. With no cars present, the circuit is in resonance. When your car stops over the induction loop, however, the inductance increases by a large factor – let's say that the value of $X_L - X_C$ changes from 0 to 40 Ω. By what factor does this cause the current to decrease by in the circuit? This change is what triggers the light to change.

33. In a dimmer-switch circuit, a 120 V, 100 W light bulb is connected to a 120 V (rms) wall socket, in which the frequency is 60 Hz. Also in the circuit, in series with the light bulb, is an inductor. The inductance of the inductor can be changed, to change the current in the circuit and therefore to change the brightness of the bulb. The same result (a dimming of the light bulb) could be obtained by using a variable resistor instead of a variable inductor. What is the advantage of using the inductor?

34. In a factory that operates a lot of motors, the inductive reactance can be quite large. In the AC circuit that represents the factory, this large inductive reactance produces a relatively large phase angle, and a correspondingly low power factor (the cosine of the phase angle). It is more advantageous for the factory to have a high power factor. If the inductance is 0.80 H and the frequency is 60 Hz, what value of capacitance can be added in series to maximize the power factor?

General problems and conceptual questions

35. Consider a circuit consisting of a coil of wire connected to a battery by means of a switch. You have to be careful when you open the switch in such a circuit, because there can be a sizable spark across the gap in the switch when the switch is opened. Based on what you have learned about inductors, explain why the spark happens.

36. Figure AC.25 shows a section of a circuit, in which an inductor, a resistor, and a capacitor are connected in series. At a particular instant in time, the current in this section of the circuit is 3.0 A, directed left, and the magnitude of the current is increasing at the

rate of 0.2 A/s. At this instant, the potential difference across the capacitor is 2.0 V, with the positive plate on the right. The resistor has a resistance of 3.0 Ω. (a) The inductor has an inductance of 2.0 H – what is the potential difference across the inductor at this instant, and which end of the inductor is at a higher potential? (b) What is the net potential difference across the entire inductor – resistor – capacitor combination?

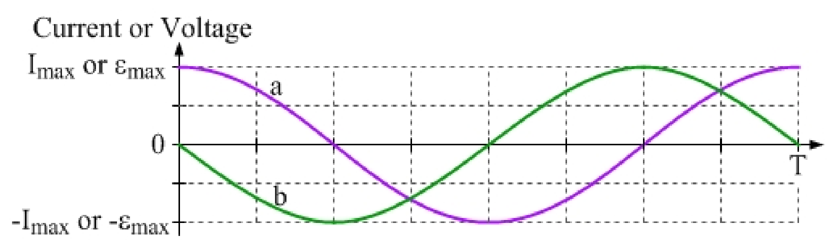
Figure AC.25: A section of a circuit, showing an inductor, a resistor, and a capacitor connected in series with one another. For Exercises 36 and 37.

37. Figure AC.25 shows a section of a circuit, in which an inductor, a resistor, and a capacitor are connected in series. At a particular instant in time, the current in this section of the circuit is 3.0 A, directed left, and the magnitude of the current is decreasing at the rate of 0.2 A/s. At this instant, the potential difference across the capacitor is 2.0 V, with the positive plate on the right. The resistor has a resistance of 3.0 Ω. (a) The inductor has an inductance of 2.0 H – what is the potential difference across the inductor at this instant, and which end of the inductor is at a higher potential? (b) What is the net potential difference across the entire inductor – resistor – capacitor combination?

38. A capacitor is the only circuit element connected to a source of alternating voltage, and the maximum current in the circuit is 1.0 A. What is the value of the maximum current (a) if the frequency of the source voltage is doubled? (b) if, instead, the distance between the capacitor plates is decreased by a factor of 2? (c) if, instead, a dielectric material with a dielectric constant of 4.0 fills the space between the capacitor plates (assume that space was originally empty)?

39. The graphs in Figure AC.26 show current and voltage, as a function of time, for an AC circuit with only one circuit element, a capacitor. The maximum voltage in this case is 12.0 V, the maximum current is 2.00 A, and *T*, the period of a complete cycle, is 20.0 milliseconds. (a) What is the capacitance? (b) Which graph shows the current and which graph shows the voltage?

Figure AC.22: Graphs of current and voltage as a function of time, for an AC circuit with only one circuit element. For Exercises 39 – 40.



40. The graphs in Figure AC.26 show current and voltage, as a function of time, for an AC circuit with only one circuit element, an inductor. The maximum voltage in this case is 12.0 V, the maximum current is 3.00 A, and *T*, the period of a complete cycle, is 25.0 milliseconds. (a) What is the inductance? (b) Which graph shows the current and which graph shows the voltage?

41. An inductor is the only circuit element connected to a source of alternating voltage, and the maximum current in the circuit is 2.0 A. What is the value of the maximum current (a) if the frequency of the source voltage is doubled? (b) if, instead, a piece of ferromagnetic material is inserted into the inductor, increasing its inductance by a factor of 40?

42. In the circuit shown in Figure AC.27, a source of alternating voltage is connected across a parallel circuit. The two bulbs are identical. (a) At a particular frequency, the inductive reactance of the inductor is equal to the capacitive reactance of the capacitor. At this frequency, which bulb is brighter? When the frequency is then increased, what happens to the brightness of (b) bulb A, and (c) bulb B?

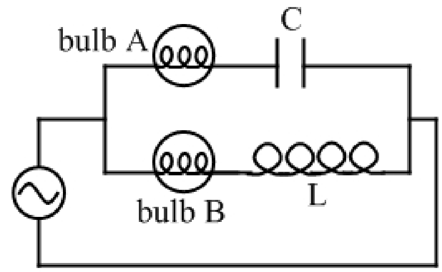


Figure AC.27: In this circuit, a source of alternating voltage is connected across a parallel circuit. One of the parallel branches has a light bulb and a capacitor, and the other branch has an identical light bulb and an inductor. For Exercises 42 – 43.

43. In the circuit shown in Figure AC.27, a source of alternating voltage is connected across a parallel circuit. The two bulbs are identical. At a frequency of 120 Hz, the bulbs are equally bright. (a) If the inductance of the inductor is 8.0 mH, what is the capacitance of the capacitor? (b) If the rms current through bulb A is 1.0 amp when the frequency is 120 Hz, is the rms current through the voltage source equal to 2.0 amps, more than 2.0 amps, or less than 2.0 amps? Briefly justify your answer.

44. In the circuit shown in Figure AC.28, a source of alternating voltage is connected across a parallel circuit. Bulb a is in one branch, and an identical bulb (bulb b) is in the other branch, along with a capacitor and a resistor. (a) In general, which bulb will be brighter? (b) Connected like this, can the two bulbs ever be equally bright? If so, under what condition(s)?

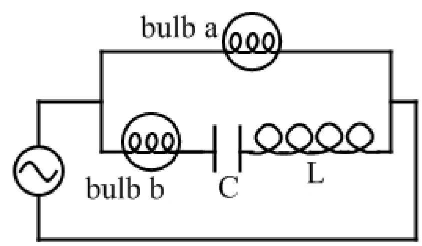


Figure AC.28: In this circuit, a source of alternating voltage is connected across a parallel circuit. One of the parallel branches has a light bulb, and the other branch has an identical light bulb along with a capacitor and an inductor. For Exercise 44.

45. Figure AC.29 shows graphs of the voltages across the resistor, the inductor, and the capacitor in a series RLC AC circuit. The maximum source voltage in this circuit is 10.0 V. (a) Determine which graph goes with which component. Then, using the information shown on the graph, determine the values of (b) the frequency, (b) the peak current (at this frequency), (c) the phase angle.

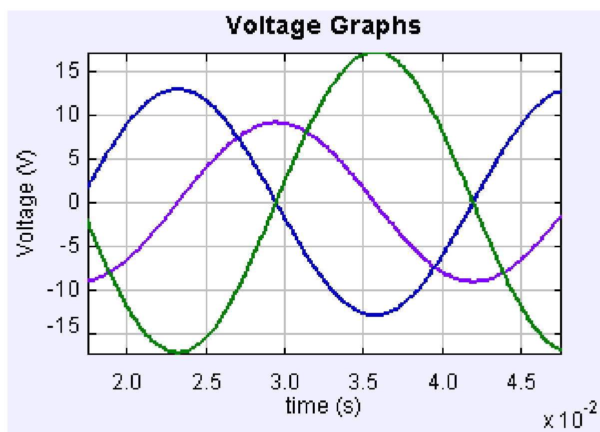


Figure AC.29: A graph of the voltages across the resistor, the inductor, and the capacitor in a series RLC AC circuit, for Exercises 45 and 46. Note that the scale on the time axis is in units of 0.01 s.

46. Figure AC.29 shows graphs of the voltages across the resistor, the inductor, and the capacitor in a series RLC AC circuit. (a) Is the frequency in this situation above, below,

or equal to the resonance frequency? (b) Using the information given in the graph, estimate the value of the resonance frequency.

47. In a particular RLC series AC circuit, at a particular frequency, the impedance is 8.0 Ω and the phase angle is +30°. What is the resistance in this circuit?

48. A particular AC circuit consists of a resistor (R = 5.0 Ω) and a capacitor (C = 6.0 mF) connected in series to a source of alternating voltage. The source frequency is 40 Hz, and the maximum voltage is 9.0 V. Determine (a) the impedance, (b) the phase angle, (c) the rms value of the current, and (d) the power dissipated in the circuit.

49. A particular AC circuit consists of a resistor (R = 8.0 Ω), a capacitor (C = 3.0 mF), and an inductor (L = 5.0 mH) connected in series to a source of alternating voltage. The source frequency is 80 Hz, and the maximum voltage is 9.0 V. Determine (a) the impedance, (b) the phase angle, (c) the rms value of the current, and (d) the power dissipated in the circuit.

50. A particular AC circuit consists of a resistor, a capacitor (C = 4.0 mF), and an inductor (L = 6.0 mH) connected in series to a source of alternating voltage. At a frequency of 75 Hz, the maximum current in the circuit is 2.0 A. The maximum voltage is 12.0 V. What is the resistance in the circuit?

51. A particular AC circuit consists of a resistor (R = 2.0 Ω), a capacitor (C = 5.0 mF), and an inductor connected in series to a source of alternating voltage. The maximum voltage is 12.0 V. At a frequency of 60 Hz, the maximum current in the circuit is equal to 4.0 A. What is the value of the inductance? State all possible answers.

52. A particular AC circuit consists of a resistor (R = 3.0 Ω), a capacitor (C = 2.0 mF), and an inductor (L = 8.0 mH) connected in series to a source of alternating voltage. The maximum voltage is 12.0 V. At what frequency is the maximum current in the circuit equal to 3.0 A? State all possible answers.

53. In a particular RLC series AC circuit, the resistance is 5.0 Ω. At a particular frequency, the capacitive reactance is 12 Ω and the inductive reactance is 2.0 Ω. (a) Calculate the impedance. (b) The frequency is then doubled. Calculate the new value of the impedance.

54. In a particular RLC series AC circuit, the resistance is 6.0 Ω. At a particular frequency, the capacitive reactance is 15 Ω and the inductive reactance is 3.0 Ω. (a) Calculate the phase angle. (b) The frequency is then doubled. Calculate the new phase angle. (c) Starting with the original frequency, determine by what factor the frequency must be changed by for the circuit to be in resonance.

55. In a particular RLC series AC circuit, at the resonance frequency, the inductive reactance is 6.0 Ω? When the frequency is three times the resonance frequency, the phase angle between the voltage and current is 45°. Calculate the resistance in this circuit.

56. Table AC.2 shows peak voltages across the resistor, the inductor, and the capacitor, at various frequencies, for a particular RLC series AC circuit. (a) What is the peak voltage of the source? (b) Complete the table, filling in the six missing values.

Table AC.2: A table of peak voltages across the resistor, the inductor, and the capacitor, at various frequencies, for a particular RLC series AC circuit. Six values are missing. For Exercise 47.

Frequency (Hz)	$\Delta V_{\max,R}$	$\Delta V_{\max,L}$	$\Delta V_{\max,C}$
60 Hz	_____ V	_____ V	_____ V
120 Hz	6.0 V	8.0 V	8.0 V
200 Hz	_____ V	_____ V	_____ V

57. Figure AC.30 shows an impedance triangle for a particular series RLC AC circuit, at a particular frequency. The capacitance in this case is 1.0 mF, and the resistance is 6.0 Ω. The inductive reactance is 10.0 Ω and the capacitive reactance is 2.0 Ω. Determine the values of (a) the phase angle, (b) the frequency of the AC source, and (c) the inductance.

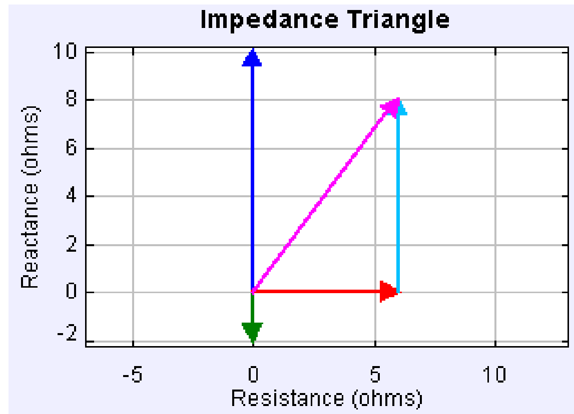
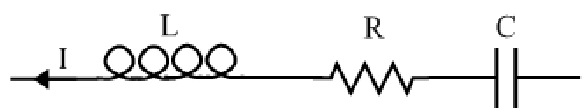


Figure AC.30: An impedance triangle for a particular RLC series AC circuit, at a particular frequency. For Exercises 57 and 58.

58. Figure AC.30 shows an impedance triangle for a particular series RLC AC circuit, at a particular frequency. The resistance is 6.0 Ω, the inductive reactance is 10.0 Ω, and the capacitive reactance is 2.0 Ω. Compare the following to the peak source voltage – be as quantitative as possible. (a) The peak voltage across the inductor, (b) the peak voltage across the resistor, and (c) the peak voltage across the capacitor.

59. For a particular series RLC AC circuit, at a particular frequency, the peak voltage across the inductor is 12.0 V, the peak voltage across the resistor is 5.0 V, and the peak voltage across the capacitor is 3.0 V. (a) Sketch an impedance triangle for this situation. Note that you cannot find values in ohms, but you can draw a diagram to scale. (b) Determine the phase angle. (c) Determine the peak source voltage.

Figure AC.31: A section of a circuit, showing an inductor, a resistor, and a capacitor connected in series with one another. The current, at the particular instant shown, is directed to the left. For Exercise 60.



60. Two students are considering a particular question, relating to the picture shown above in Figure AC.31. Comment on the part of their conversation that is reported below.

Jack: The current is directed to the left. The inductor always opposes the current, so that means the voltage across the inductor must be set up to create a current going to the right.

Casey: So, you mean that the right end of the inductor is the positive end? If we had a battery with a positive end on the right, that would create a current to the right.

Jack: That sounds good.

Casey: So, does it matter what the current is doing? Does it matter whether it's increasing or decreasing? What if it's constant?

Jack: No, it makes no difference. The inductor just opposes the current, no matter what.