GLENCOE MATHEMATICS

Geometry

Chapter 9 Resource Masters

McGraw Hill Glencoe

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Glencoe

The McGraw-Hill Companies

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-860186-X

Geometry
Chapter 9 Resource Masters

1 2 3 4 5 6 7 8 9 10 009 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 9 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 9 Resource Masters* includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 9-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 9-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 9 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 518–519. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ______________________________ DATE __________ PERIOD _____

9 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
component form		
dilation		
isometry		
line of reflection		
line of symmetry		
point of symmetry		
reflection		
regular tessellation		
resultant		
rotation		

(continued on the next page)

NAME ____ DATE ____________ PERIOD _____

9

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
rotational symmetry		
scalar		
semi-regular tessellation		
similarity transformation		
standard position		
tessellation		
transformation		
translation		
uniform tessellations		
vector		

9

Learning to Read Mathematics

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 9. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 9.1		
Theorem 9.2		
Postulate 9.1		

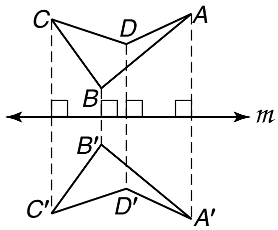
NAME ____ DATE ____________ PERIOD _____

9-1 Study Guide and Intervention

Reflections

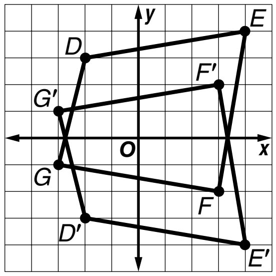
Draw Reflections The transformation called a **reflection** is a flip of a figure in a point, a line, or a plane. The new figure is the **image** and the original figure is the **preimage**. The preimage and image are congruent, so a reflection is a **congruence transformation** or **isometry**.

Example 1 **Construct the image of quadrilateral $ABCD$ under a reflection in line m.**



Draw a perpendicular from each vertex of the quadrilateral to m. Find vertices A', B', C', and D' that are the same distance from m on the other side of m. The image is $A'B'C'D'$.

Example 2 **Quadrilateral $DEFG$ has vertices $D(-2, 3)$, $E(4, 4)$, $F(3, -2)$, and $G(-3, -1)$. Find the image under reflection in the x-axis.**



To find an image for a reflection in the x-axis, use the same x-coordinate and multiply the y-coordinate by -1. In symbols, $(a, b) \rightarrow (a, -b)$. The new coordinates are $D'(-2, -3)$, $E'(4, -4)$, $F'(3, 2)$, and $G'(-3, 1)$. The image is $D'E'F'G'$.

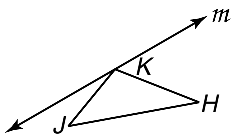
In Example 2, the notation $(a, b) \rightarrow (a, -b)$ represents a reflection in the x-axis. Here are three other common reflections in the coordinate plane.

- in the y-axis: $(a, b) \rightarrow (-a, b)$
- in the line $y = x$: $(a, b) \rightarrow (b, a)$
- in the origin: $(a, b) \rightarrow (-a, -b)$

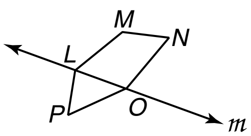
Exercises

Draw the image of each figure under a reflection in line m.

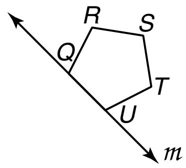
1.



2.

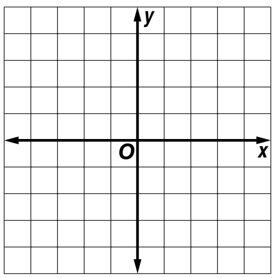


3.

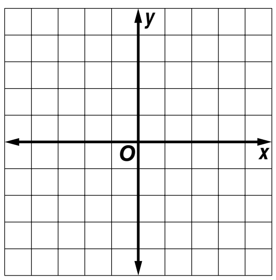


Graph each figure and its image under the given reflection.

4. $\triangle DEF$ with $D(-2, -1)$, $E(-1, 3)$, $F(3, -1)$ in the x-axis



5. $ABCD$ with $A(1, 4)$, $B(3, 2)$, $C(2, -2)$, $D(-3, 1)$ in the y-axis



Lesson 9-1

NAME ______________________________ DATE __________ PERIOD _____

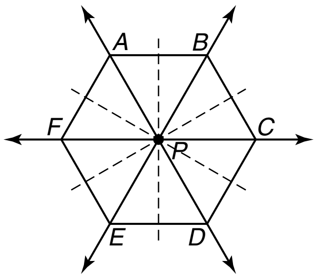
9-1 Study Guide and Intervention *(continued)*

Reflections

Lines and Points of Symmetry If a figure has a **line of symmetry**, then it can be folded along that line so that the two halves match. If a figure has a **point of symmetry**, it is the midpoint of all segments between the preimage and image points.

Example **Determine how many lines of symmetry a regular hexagon has. Then determine whether a regular hexagon has point symmetry.**

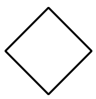
There are six lines of symmetry, three that are diagonals through opposite vertices and three that are perpendicular bisectors of opposite sides. The hexagon has point symmetry because any line through P identifies two points on the hexagon that can be considered images of each other.



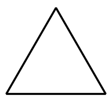
Exercises

Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

1.

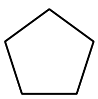


2.

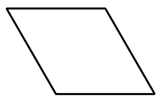


3.

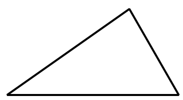
4.



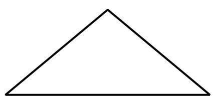
5.



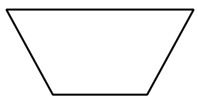
6.



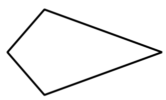
7.



8.



9.

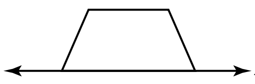


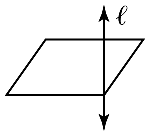
NAME ______________________________ DATE ____________ PERIOD _____

9-1 Skills Practice

Reflections

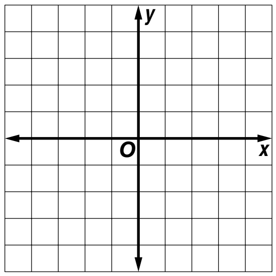
Draw the image of each figure under a reflection in line ℓ.

1. 

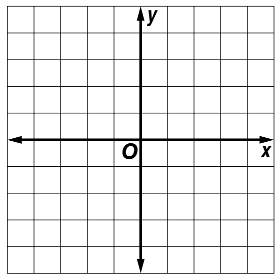
2. 

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

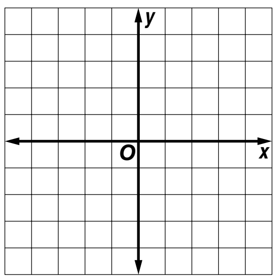
3. $\triangle ABC$ with vertices $A(-3, 2)$, $B(0, 1)$, and $C(-2, -3)$ in the origin



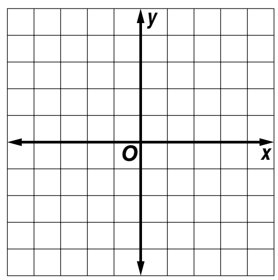
4. trapezoid $DEFG$ with vertices $D(0, -3)$, $E(1, 3)$, $F(3, 3)$, and $G(4, -3)$ in the y-axis



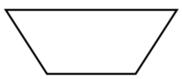
5. parallelogram $RSTU$ with vertices $R(-2, 3)$, $S(2, 4)$, $T(2, -3)$ and $U(-2, -4)$ in the line $y = x$



6. square $KLMN$ with vertices $K(-1, 0)$, $L(-2, 3)$, $M(1, 4)$, and $N(2, 1)$ in the x-axis



Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

7. 

8. 

9. **S**

Lesson 9-1

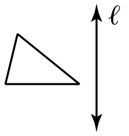
NAME ____________________________ DATE ____________ PERIOD _____

9-1 Practice

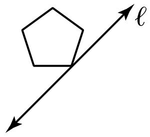
Reflections

Draw the image of each figure under a reflection in line ℓ.

1.

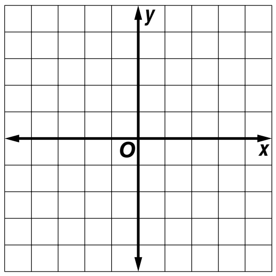


2.

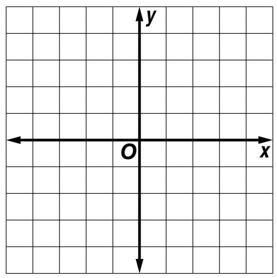


COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

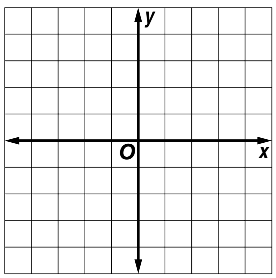
3. quadrilateral $ABCD$ with vertices $A(-3, 3)$, $B(1, 4)$, $C(4, 0)$, and $D(-3, -3)$ in the origin



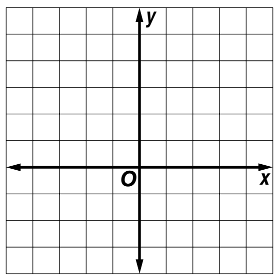
4. $\triangle FGH$ with vertices $F(-3, -1)$, $G(0, 4)$, and $H(3, -1)$ in the line $y = x$



5. rectangle $QRST$ with vertices $Q(-3, 2)$, $R(-1, 4)$, $S(2, 1)$, and $T(0, -1)$ in the x-axis

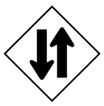


6. trapezoid $HIJK$ with vertices $H(-2, 5)$, $I(2, 5)$, $J(-4, -1)$, and $K(-4, 3)$ in the y-axis

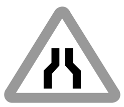


ROAD SIGNS Determine how many lines of symmetry each sign has. Then determine whether the sign has point symmetry.

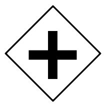
7.



8.



9.



9-1 Reading to Learn Mathematics

Reflections

Pre-Activity **Where are reflections found in nature?**

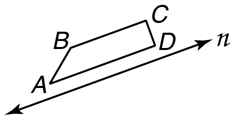
Read the introduction to Lesson 9-1 at the top of page 463 in your textbook.

Suppose you draw a line segment connecting a point at the peak of a mountain to its image in the lake. Where will the midpoint of this segment fall?

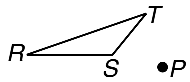
Reading the Lesson

1. Draw the reflected image for each reflection described below.

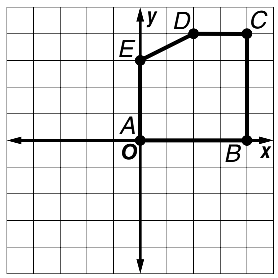
a. reflection of trapezoid $ABCD$ in the line n
Label the image of $ABCD$ as $A'B'C'D'$.



b. reflection of $\triangle RST$ in point P
Label the image of RST as $R'S'T'$.



c. reflection of pentagon $ABCDE$ in the origin
Label the image of $ABCDE$ as $A'B'C'D'E'$.



2. Determine the image of the given point under the indicated reflection.

a. (4, 6); reflection in the y-axis

b. (−3, 5); reflection in the x-axis

c. (−8, −2); reflection in the line $y = x$

d. (9, −3); reflection in the origin

3. Determine the number of lines of symmetry for each figure described below. Then determine whether the figure has point symmetry and indicate this by writing *yes* or *no*.

a. a square

b. an isosceles triangle (not equilateral)

c. a regular hexagon

d. an isosceles trapezoid

e. a rectangle (not a square)

f. the letter E

Helping You Remember

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *isometry* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part.

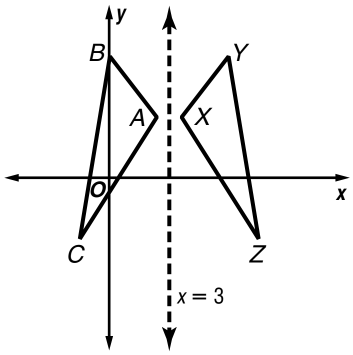
NAME ______________________ DATE ____________ PERIOD _____

9-1 Enrichment

Reflections in the Coordinate Plane

Study the diagram at the right. It shows how the triangle *ABC* is mapped onto triangle *XYZ* by the transformation $(x, y) \rightarrow (-x + 6, y)$. Notice that $\triangle XYZ$ is the reflection image with respect to the vertical line with equation $x = 3$.

$(x, y) \rightarrow (-x + 6, y)$

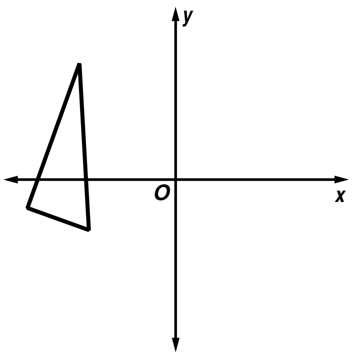


1. Prove that the vertical line with equation $x = 3$ is the perpendicular bisector of the segment with endpoints (x, y) and $(-x + 6, y)$. (Hint: Use the midpoint formula.)

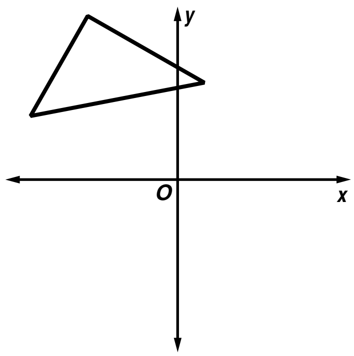
2. Every transformation of the form $(x, y) \rightarrow (-x + 2h, y)$ is a reflection with respect to the vertical line with equation $x = h$. What kind of transformation is $(x, y) \rightarrow (x, -y + 2k)$?

Draw the transformation image for each figure and the given transformation. Is it a reflection transformation? If so, with respect to what line?

3. $(x, y) \rightarrow (-x - 4, y)$



4. $(x, y) \rightarrow (x, -y + 8)$

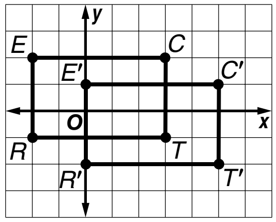


9-2 Study Guide and Intervention

Translations

Translations Using Coordinates A transformation called a **translation** slides a figure in a given direction. In the coordinate plane, a translation moves every preimage point $P(x, y)$ to an image point $P(x + a, y + b)$ for fixed values a and b. In words, a translation shifts a figure a units horizontally and b units vertically; in symbols, $(x, y) \rightarrow (x + a, y + b)$.

Example **Rectangle $RECT$ has vertices $R(-2, -1)$, $E(-2, 2)$, $C(3, 2)$, and $T(3, -1)$. Graph $RECT$ and its image for the translation $(x, y) \rightarrow (x + 2, y - 1)$.**



The translation moves every point of the preimage right 2 units and down 1 unit.

$(x, y) \rightarrow (x + 2, y - 1)$

$R(-2, -1) \rightarrow R'(-2 + 2, -1 - 1)$ or $R'(0, -2)$

$E(-2, 2) \rightarrow E'(-2 + 2, 2 - 1)$ or $E'(0, 1)$

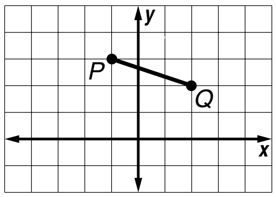
$C(3, 2) \rightarrow C'(3 + 2, 2 - 1)$ or $C'(5, 1)$

$T(3, -1) \rightarrow T'(3 + 2, -1 - 1)$ or $T'(5, -2)$

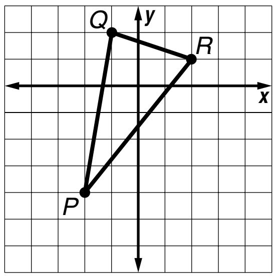
Exercises

Graph each figure and its image under the given translation.

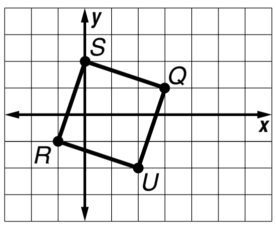
1. $\overline{PQ}$ with endpoints $P(-1, 3)$ and $Q(2, 2)$ under the translation left 2 units and up 1 unit



2. $\triangle PQR$ with vertices $P(-2, -4)$, $Q(-1, 2)$, and $R(2, 1)$ under the translation right 2 units and down 2 units



3. square $SQUR$ with vertices $S(0, 2)$, $Q(3, 1)$, $U(2, -2)$, and $R(-1, -1)$ under the translation right 3 units and up 1 unit



Lesson 9-2

NAME ____ DATE ____________ PERIOD _____

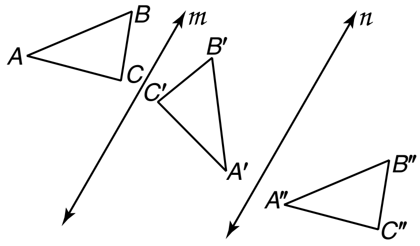
9-2 Study Guide and Intervention *(continued)*

Translations

Translations by Repeated Reflections Another way to find the image of a translation is to reflect the figure twice in parallel lines. This kind of translation is called a **composite of reflections**.

Example **In the figure, $m \parallel n$. Find the translation image of $\triangle ABC$.**

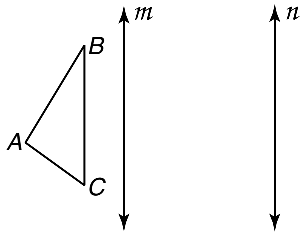
$\triangle A'B'C'$ is the image of $\triangle ABC$ reflected in line m.
$\triangle A''B''C''$ is the image of $\triangle A'B'C'$ reflected in line n.
The final image, $\triangle A''B''C''$, is a translation of $\triangle ABC$.



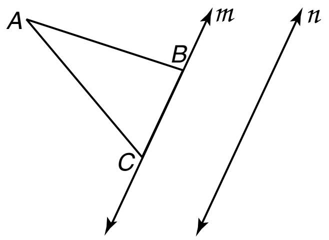
Exercises

In each figure, $m \parallel n$. Find the translation image of each figure by reflecting it in line m and then in line n.

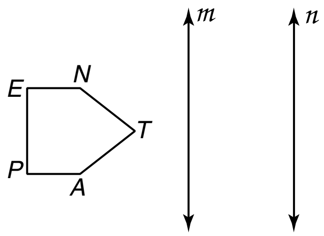
1.



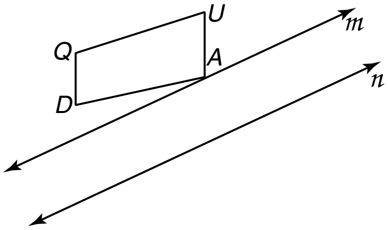
2.



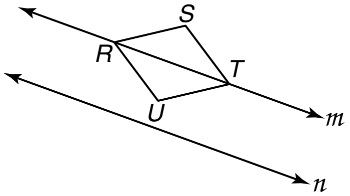
3.



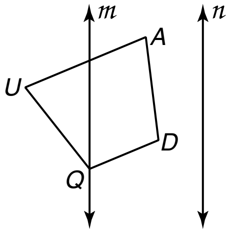
4.



5.



6.

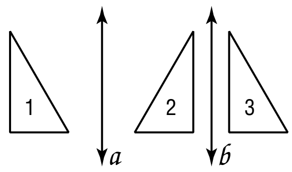


9-2 Skills Practice

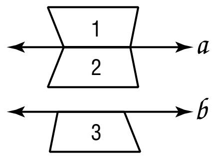
Translations

In each figure, $a \parallel b$. Determine whether figure 3 is a translation image of figure 1. Write *yes* or *no*. Explain your answer.

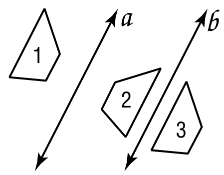
1.



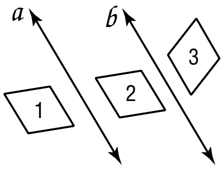
2.



3.

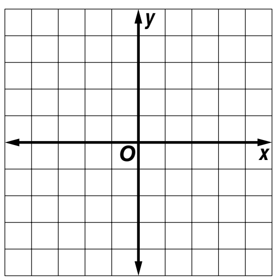


4.

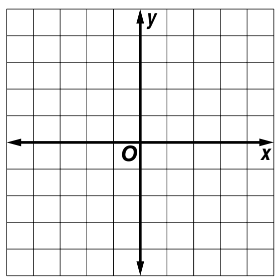


COORDINATE GEOMETRY Graph each figure and its image under the given translation.

5. $\triangle JKL$ with vertices $J(-4, -4)$, $K(-2, -1)$, and $L(2, -4)$ under the translation $(x, y) \rightarrow (x + 2, y + 5)$



6. quadrilateral $LMNP$ with vertices $L(4, 2)$, $M(4, -1)$, $N(0, -1)$, and $P(1, 4)$ under the translation $(x, y) \rightarrow (x - 4, y - 3)$



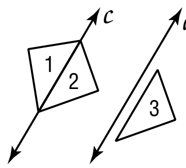
NAME ____ DATE ____________ PERIOD _____

9-2 Practice

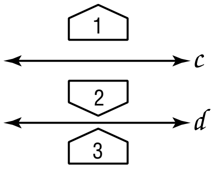
Translations

In each figure, $c \parallel d$. Determine whether figure 3 is a translation image of figure 1. Write *yes* or *no*. Explain your answer.

1.

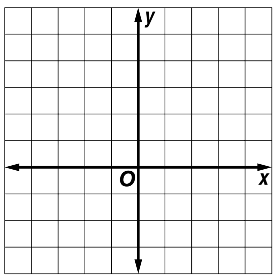


2.

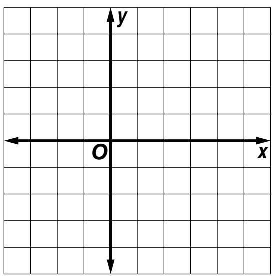


COORDINATE GEOMETRY Graph each figure and its image under the given translation.

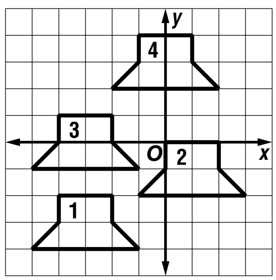
3. quadrilateral $TUWX$ with vertices $T(-1, 1)$, $U(4, 2)$, $W(1, 5)$, and $X(-1, 3)$ under the translation $(x, y) \rightarrow (x - 2, y - 4)$



4. pentagon $DEFGH$ with vertices $D(-1, -2)$, $E(2, -1)$, $F(5, -2)$, $G(4, -4)$, $H(1, -4)$ under the translation $(x, y) \rightarrow (x - 1, y + 5)$



ANIMATION Find the translation that moves the figure on the coordinate plane.



5. figure 1 → figure 2

6. figure 2 → figure 3

7. figure 3 → figure 4

NAME ______________________________ DATE __________ PERIOD _____

9-2 Reading to Learn Mathematics

Translations

Pre-Activity **How are translations used in a marching band show?**

Read the introduction to Lesson 9-2 at the top of page 470 in your textbook.

How do band directors get the marching band to maintain the shape of the figure they originally formed?

Reading the Lesson

1. Underline the correct word or phrase to form a true statement.

a. All reflections and translations are (opposites/isometries/equivalent).

b. The preimage and image of a figure under a reflection in a line have (the same orientation/opposite orientations).

c. The preimage and image of a figure under a translation have (the same orientation/opposite orientations).

d. The result of successive reflections over two parallel lines is a (reflection/rotation/translation).

e. Collinearity (is/is not) preserved by translations.

f. The translation $(x, y) \rightarrow (x + a, y + b)$ shifts every point a units (horizontally/vertically) and y units (horizontally/vertically).

2. Find the image of each preimage under the indicated translation.

a. (x, y); 5 units right and 3 units up

b. (x, y); 2 units left and 4 units down

c. (x, y); 1 unit left and 6 units up

d. (x, y); 7 units right

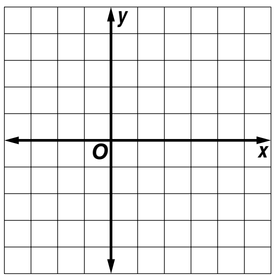
e. $(4, -3)$; 3 units up

f. $(-5, 6)$; 3 units right and 2 units down

g. $(-7, 5)$; 7 units right and 5 units down

h. $(-9, -2)$; 12 units right and 6 units down

3. $\triangle RST$ has vertices $R(-3, 3)$, $S(0, -2)$, and $T(2, 1)$. Graph $\triangle RST$ and its image $\triangle R'S'T'$ under the translation $(x, y) \rightarrow (x + 3, y - 2)$. List the coordinates of the vertices of the image.



Helping You Remember

4. A good way to remember a new mathematical term is to relate it to an everyday meaning of the same word. How is the meaning of *translation* in geometry related to the idea of *translation* from one language to another?

Lesson 9-2

9-2 Enrichment

Translations in The Coordinate Plane

You can use algebraic descriptions of reflections to show that the composite of two reflections with respect to parallel lines is a translation (that is, a slide).

1. Suppose a and b are two different real numbers. Let S and T be the following reflections.

$S: (x, y) \rightarrow (-x + 2a, y)$
$T: (x, y) \rightarrow (-x + 2b, y)$

S is reflection with respect to the line with equation $x = a$, and T is reflection with respect to the line with equation $x = b$.

a. Find an algebraic description (similar to those above for S and T) to describe the composite transformation "S followed by T."

b. Find an algebraic description for the composite transformation "T followed by S."

2. Think about the results you obtained in Exercise 1. What do they tell you about how the distance between two parallel lines is related to the distance between a preimage and image point for a composite of reflections with respect to these lines?

3. Illustrate your answers to Exercises 1 and 2 with sketches. Use a separate sheet if necessary.

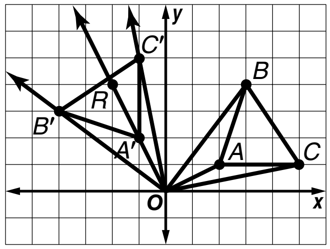
NAME ____ DATE ____________ PERIOD _____

9-3 Study Guide and Intervention

Rotations

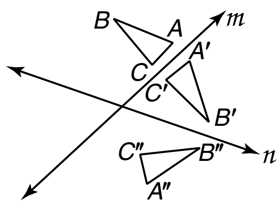
Draw Rotations A transformation called a **rotation** turns a figure through a specified angle about a fixed point called the **center of rotation**. To find the image of a rotation, one way is to use a protractor. Another way is to reflect a figure twice, in two intersecting lines.

Example 1 **$\triangle ABC$ has vertices $A(2, 1)$, $B(3, 4)$, and $C(5, 1)$. Draw the image of $\triangle ABC$ under a rotation of 90° counterclockwise about the origin.**



- First draw $\triangle ABC$. Then draw a segment from O, the origin, to point A.
- Use a protractor to measure 90° counterclockwise with $\overline{OA}$ as one side.
- Draw $\overrightarrow{OR}$.
- Use a compass to copy $\overline{OA}$ onto $\overrightarrow{OR}$. Name the segment $\overline{OA'}$.
- Repeat with segments from the origin to points B and C.

Example 2 **Find the image of $\triangle ABC$ under reflection in lines m and n.**



First reflect $\triangle ABC$ in line m. Label the image $\triangle A'B'C'$.

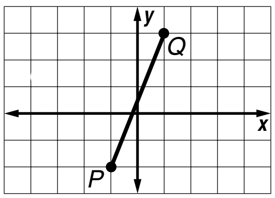
Reflect $\triangle A'B'C'$ in line n. Label the image $\triangle A''B''C''$.

$\triangle A''B''C''$ is a rotation of $\triangle ABC$. The center of rotation is the intersection of lines m and n. The angle of rotation is twice the measure of the acute angle formed by m and n.

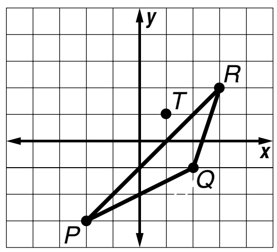
Exercises

Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.

1. $\overline{PQ}$ with endpoints $P(-1, -2)$ and $Q(1, 3)$ counterclockwise about the origin

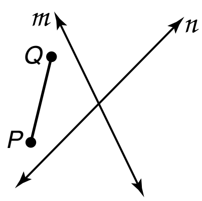


2. $\triangle PQR$ with vertices $P(-2, -3)$, $Q(2, -1)$, and $R(3, 2)$ clockwise about the point $T(1, 1)$

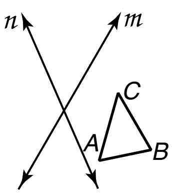


Find the rotation image of each figure by reflecting it in line m and then in line n.

3.



4.

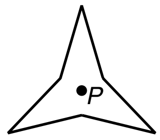


NAME ____ DATE ____________ PERIOD _____

9-3 Study Guide and Intervention *(continued)*

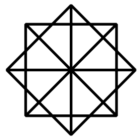
Rotations

Rotational Symmetry When the figure at the right is rotated about point P by 120° or 240°, the image looks like the preimage. The figure has **rotational symmetry**, which means it can be rotated less than 360° about a point and the preimage and image appear to be the same.



The figure has rotational symmetry of **order** 3 because there are 3 rotations less than 360° (0°, 120°, 240°) that produce an image that is the same as the original. The **magnitude** of the rotational symmetry for a figure is 360 degrees divided by the order. For the figure above, the rotational symmetry has magnitude 120 degrees.

Example **Identify the order and magnitude of the rotational symmetry of the design at the right.**



The design has rotational symmetry about the center point for rotations of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°.

There are eight rotations less than 360 degrees, so the order of its rotational symmetry is 8. The quotient 360 ÷ 8 is 45, so the magnitude of its rotational symmetry is 45 degrees.

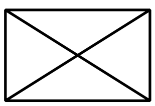
Exercises

Identify the order and magnitude of the rotational symmetry of each figure.

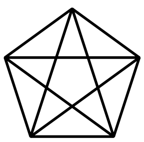
1. a square

2. a regular 40-gon

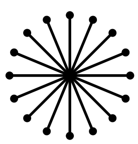
3.



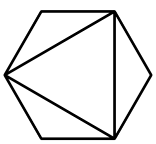
4.



5.



6.



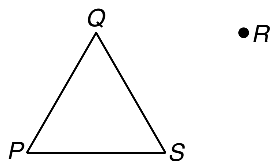
NAME ______________________________ DATE ____________ PERIOD _____

9-3 Skills Practice

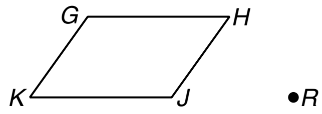
Rotations

Rotate each figure about point *R* under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. 90° counterclockwise

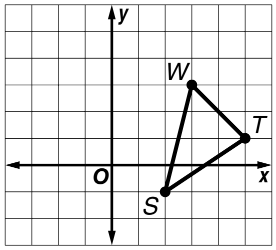


2. 90° clockwise

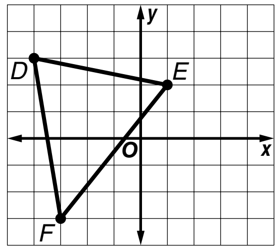


COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the origin and label the coordinates.

3. $\triangle STW$ with vertices $S(2, -1)$, $T(5, 1)$, and $W(3, 3)$ counterclockwise

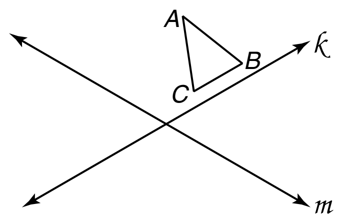


4. $\triangle DEF$ with vertices $D(-4, 3)$, $E(1, 2)$, and $F(-3, -3)$ clockwise

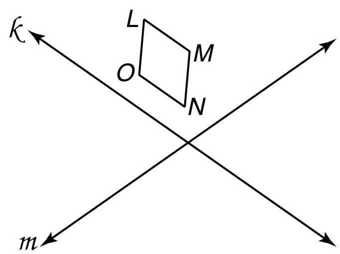


Use a composition of reflections to find the rotation image with respect to lines k and m. Then find the angle of rotation for each image.

5.



6.



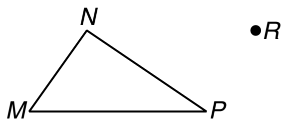
NAME ______________________________ DATE ____________ PERIOD _____

9-3 Practice

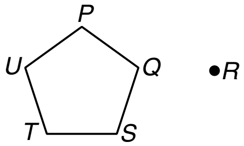
Rotations

Rotate each figure about point *R* under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. 80° counterclockwise

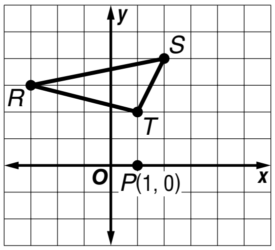


2. 100° clockwise

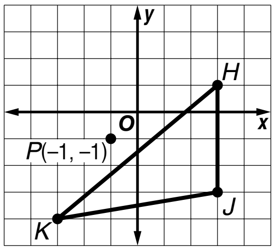


COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.

3. $\triangle RST$ with vertices $R(-3, 3)$, $S(2, 4)$, and $T(1, 2)$ clockwise about the point $P(1, 0)$

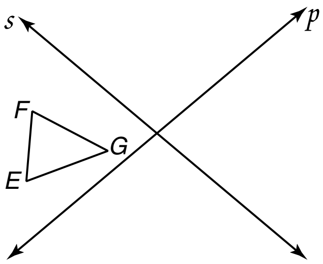


4. $\triangle HJK$ with vertices $H(3, 1)$, $J(3, -3)$, and $K(-3, -4)$ counterclockwise about the point $P(-1, -1)$

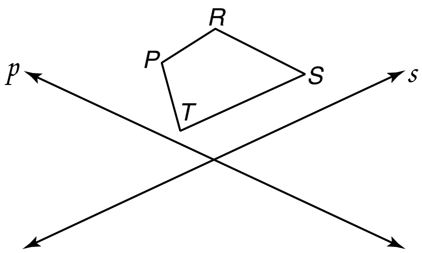


Use a composition of reflections to find the rotation image with respect to lines *p* and *s*. Then find the angle of rotation for each image.

5.



6.



7. STEAMBOATS A paddle wheel on a steamboat is driven by a steam engine and moves from one paddle to the next to propel the boat through the water. If a paddle wheel consists of 18 evenly spaced paddles, identify the order and magnitude of its rotational symmetry.

NAME ____ DATE ____________ PERIOD _____

9-3 Reading to Learn Mathematics

Rotations

Pre-Activity **How do some amusement rides illustrate rotations?**

Read the introduction to Lesson 9-3 at the top of page 476 in your textbook.

What are two ways that each car rotates?

Reading the Lesson

1. List all of the following types of transformations that satisfy each description: *reflection*, *translation*, *rotation*.

a. The transformation is an isometry.

b. The transformation preserves the orientation of a figure.

c. The transformation is the composite of successive reflections over two intersecting lines.

d. The transformation is the composite of successive reflections over two parallel lines.

e. A specific transformation is defined by a fixed point and a specified angle.

f. A specific transformation is defined by a fixed point, a fixed line, or a fixed plane.

g. A specific transformation is defined by $(x, y) \rightarrow (x + a, x + b)$, for fixed values of a and b.

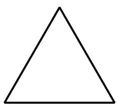
h. The transformation is also called a slide.

i. The transformation is also called a flip.

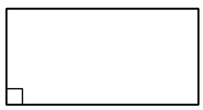
j. The transformation is also called a turn.

2. Determine the order and magnitude of the rotational symmetry for each figure.

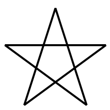
a.



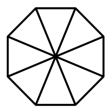
b.



c.



d.



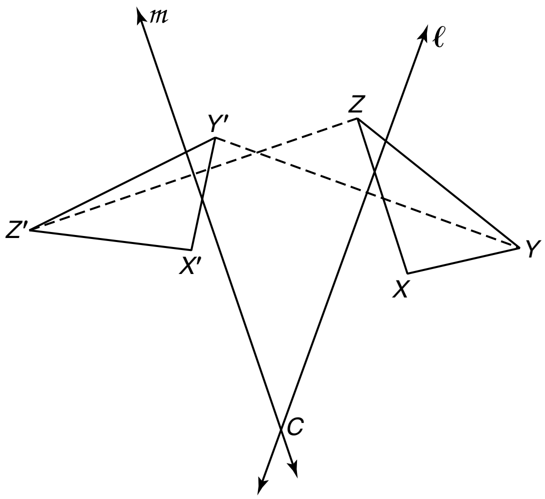
Helping You Remember

3. What is an easy way to remember the order and magnitude of the rotational symmetry of a regular polygon?

9-3 Enrichment

Finding the Center of Rotation

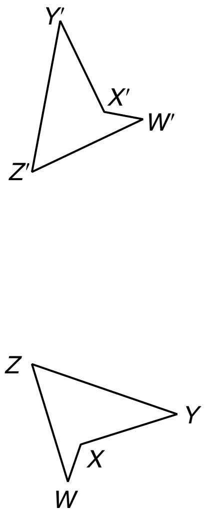
Suppose you are told that $\triangle X'Y'Z'$ is the rotation image of $\triangle XYZ$, but you are not told where the center of rotation is nor the measure of the angle of rotation. Can you find them? Yes, you can. Connect two pairs of corresponding vertices with segments. In the figure, the segments YY' and ZZ' are used. Draw the perpendicular bisectors, ℓ and m, of these segments. The point C where ℓ and m intersect is the center of rotation.



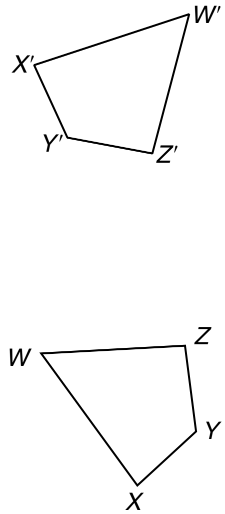
1. How can you find the measure of the angle of rotation in the figure above?

Locate the center of rotation for the rotation that maps *WXYZ* onto *W′X′Y′Z′*. Then find the measure of the angle of rotation.

2.



3.

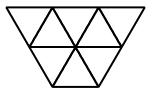


NAME ______________________________ DATE ____________ PERIOD _____

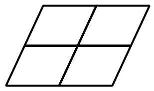
9-4 Study Guide and Intervention

Tessellations

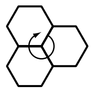
Regular Tessellations A pattern that covers a plane with repeating copies of one or more figures so that there are no overlapping or empty spaces is a **tessellation**. A **regular tessellation** uses only one type of regular polygon. In a tessellation, the sum of the measures of the angles of the polygons surrounding a vertex is 360. If a regular polygon has an interior angle that is a factor of 360, then the polygon will tessellate.



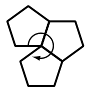
regular tessellation



tessellation



Copies of a regular hexagon can form a tessellation.



Copies of a regular pentagon cannot form a tessellation.

Example **Determine whether a regular 16-gon tessellates the plane. Explain.**

If $m\angle 1$ is the measure of one interior angle of a regular polygon, then a formula for $m\angle 1$ is $m\angle 1 = \frac{180(n-2)}{n}$. Use the formula with $n = 16$.

$$m\angle 1 = \frac{180(n-2)}{n}$$
$$= \frac{180(16-2)}{16}$$
$$= 157.5$$

The value 157.5 is not a factor of 360, so the 16-gon will not tessellate.

Exercises

Determine whether each polygon tessellates the plane. If so, draw a sample figure.

1. scalene right triangle

2. isosceles trapezoid

Determine whether each regular polygon tessellates the plane. Explain.

3. square

4. 20-gon

5. septagon

6. 15-gon

7. octagon

8. pentagon

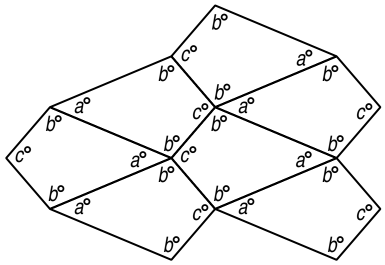
9-4 Study Guide and Intervention *(continued)*

Tessellations

Tessellations with Specific Attributes A tessellation pattern can contain any type of polygon. If the arrangement of shapes and angles at each vertex in the tessellation is the same, the tessellation is **uniform**. A **semi-regular tessellation** is a uniform tessellation that contains two or more regular polygons.

Example **Determine whether a kite will tessellate the plane. If so, describe the tessellation as *uniform*, *regular*, *semi-regular*, or *not uniform*.**

A kite will tessellate the plane. At each vertex the sum of the measures is $a + b + b + c$, which is 360. The tessellation is uniform.



Exercises

Determine whether a semi-regular tessellation can be created from each set of figures. If so, sketch the tessellation. Assume that each figure has a side length of 1 unit.

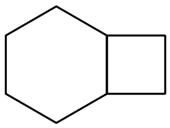
1. rhombus, equilateral triangle, and octagon

2. square and equilateral triangle

Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

3. rectangle

4. hexagon and square



NAME ____ DATE ____________ PERIOD _____

9-4 Skills Practice

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 15-gon

2. 18-gon

3. square

4. 20-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

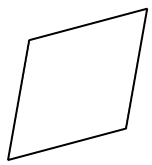
5. regular pentagons and equilateral triangles

6. regular dodecagons and equilateral triangles

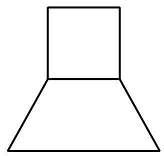
7. regular octagons and equilateral triangles

Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

8. rhombus



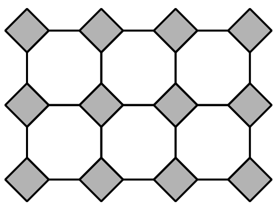
9. isosceles trapezoid and square



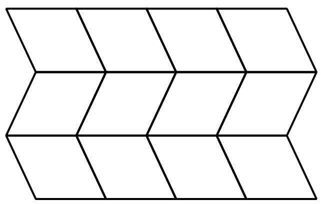
Lesson 9-4

Determine whether each pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.

10.



11.



9-4 Practice

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 22-gon

2. 40-gon

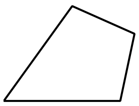
Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

3. regular pentagons and regular decagons

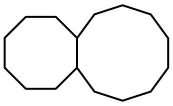
4. regular dodecagons, regular hexagons, and squares

Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

5. kite

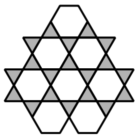


6. octagon and decagon

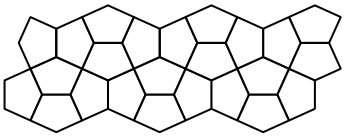


Determine whether each pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.

7.

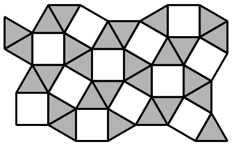


8.



FLOOR TILES For Exercises 9 and 10, use the following information.

Mr. Martinez chose the pattern of tile shown to retile his kitchen floor.



9. Determine whether the pattern is a tessellation. Explain

10. Is the pattern uniform, regular, or semi-regular?

9-4 Reading to Learn Mathematics

Tesselations

Pre-Activity **How are tessellations used in art?**

Read the introduction to Lesson 9-4 at the top of page 483 in your textbook.

- In the pattern shown in the picture in your textbook, how many small equilateral triangles make up one regular hexagon?
- In this pattern, how many fish make up one equilateral triangle?

Reading the Lesson

1. Underline the correct word, phrase, or number to form a true statement.

a. A tessellation is a pattern that covers a plane with the same figure or set of figures so that there are no (congruent angles/overlapping or empty spaces/right angles).

b. A tessellation that uses only one type of regular polygon is called a (uniform/regular/semi-regular) tessellation.

c. The sum of the measures of the angles at any vertex in any tessellation is (90/180/360).

d. A tessellation that contains the same arrangement of shapes and angles at every vertex is called a (uniform/regular/semi-regular) tessellation.

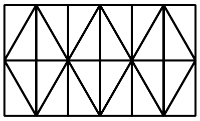
e. In a regular tessellation made up of hexagons, there are (3/4/6) hexagons meeting at each vertex, and the measure of each of the angles at any vertex is (60/90/120).

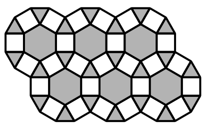
f. A uniform tessellation formed using two or more regular polygons is called a (rotational/regular/semi-regular) tessellation.

g. In a regular tessellation made up of triangles, there are (3/4/6) triangles meeting at each vertex, and the measure of each of the angles at any vertex is (30/60/120).

h. If a regular tessellation is made up of quadrilaterals, all of the quadrilaterals must be congruent (rectangles/parallelograms/squares/trapezoids).

2. Write all of the following words that describe each tessellation: *uniform*, *non-uniform*, *regular*, *semi-regular*.

a. 

b. 

Helping You Remember

3. Often the everyday meanings of a word can help you to remember its mathematical meaning. Look up *uniform* in your dictionary. How can its everyday meanings help you to remember the meaning of a *uniform* tessellation?

Lesson 9-4

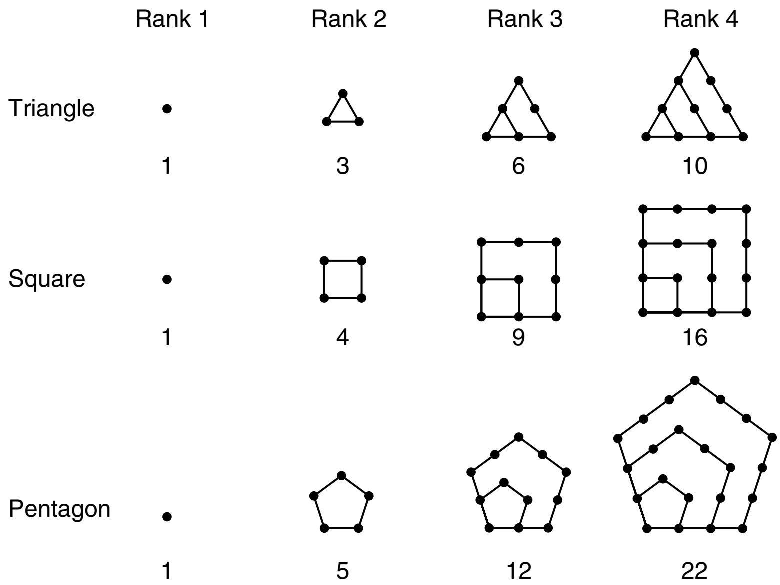
NAME ______________________________ DATE __________ PERIOD _____

9-4 Enrichment

Polygonal Numbers

Certain numbers related to regular polygons are called **polygonal numbers**. The chart shows several triangular, square, and pentagonal numbers. The **rank** of a polygon number is the number of dots on each "side" of the outer polygon. For example, the pentagonal number 22 has a rank of 4.

Polygonal numbers can be described with formulas. For example, a triangular number T of rank r can be described by $T = \frac{r(r+1)}{2}$.



Answer each question.

1. Draw a diagram to find the triangular number of rank 5.

2. Draw a diagram to find the pentagonal number of rank 5.

3. Write a formula for a square number S of rank r.

4. Write a formula for a pentagonal number P of rank r.

5. What is the rank of the pentagonal number 70?

6. List the hexagonal numbers for ranks 1 to 5. (Hint: Draw a diagram.)

NAME ____ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention

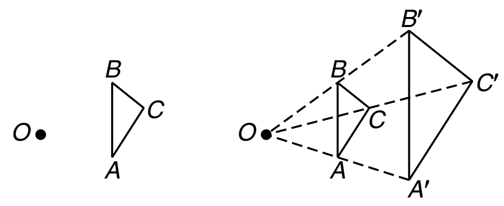
Dilations

Classify Dilations A **dilation** is a transformation in which the image may be a different size than the preimage. A dilation requires a center point and a scale factor, r.

Let r represent the scale factor of a dilation.
If $\|r\| > 1$, then the dilation is an enlargement.
If $\|r\| = 1$, then the dilation is a congruence transformation.
If $0 < \|r\| < 1$, then the dilation is a reduction.

Example **Draw the dilation image of $\triangle ABC$ with center O and $r = 2$.**

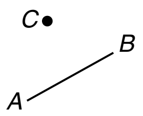
Draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$. Label points A', B', and C' so that $OA' = 2(OA)$, $OB' = 2(OB)$, and $OC' = 2(OC)$. $\triangle A'B'C'$ is a dilation of $\triangle ABC$.



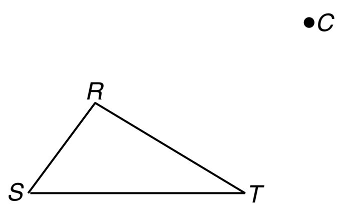
Exercises

Draw the dilation image of each figure with center C and the given scale factor. Describe each transformation as an *enlargement*, *congruence*, or *reduction*.

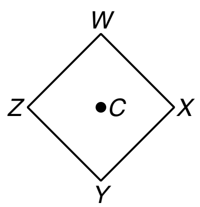
1. $r = 2$



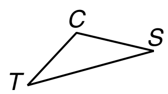
2. $r = \frac{1}{2}$



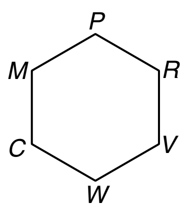
3. $r = 1$



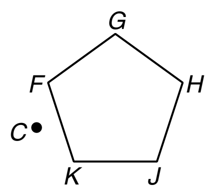
4. $r = 3$



5. $r = \frac{2}{3}$



6. $r = 1$



Lesson 9-5

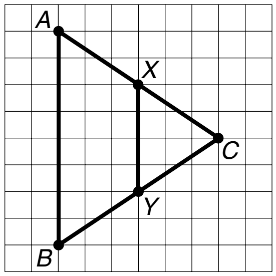
NAME ______________________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention *(continued)*

Dilations

Identify the Scale Factor If you know corresponding measurements for a preimage and its dilation image, you can find the scale factor.

Example **Determine the scale factor for the dilation of $\overline{XY}$ to $\overline{AB}$. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.**



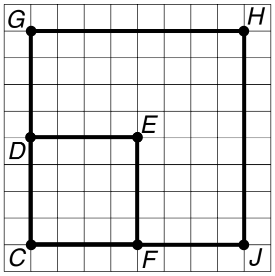
$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}} = \frac{8 \text{ units}}{4 \text{ units}} = 2$$

The scale factor is greater than 1, so the dilation is an enlargement.

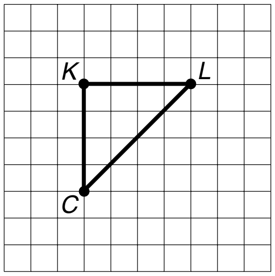
Exercises

Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

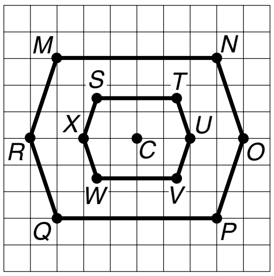
1. *CGHJ* is a dilation image of *CDEF*.



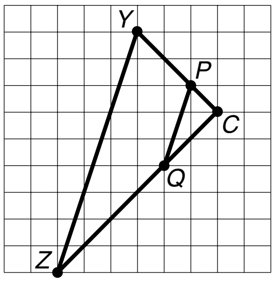
2. △*CKL* is a dilation image of △*CKL*.



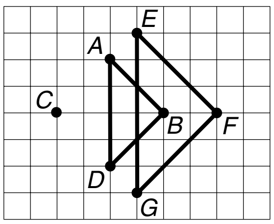
3. *STUVWX* is a dilation image of *MNOPQR*.



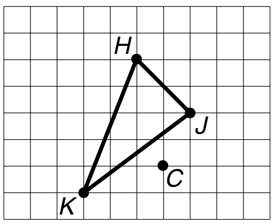
4. △*CPQ* is a dilation image of △*CYZ*.



5. △*EFG* is a dilation image of △*ABC*.



6. △*HJK* is a dilation image of △*HJK*.



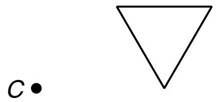
NAME ______________________________ DATE __________ PERIOD _____

9-5 Skills Practice

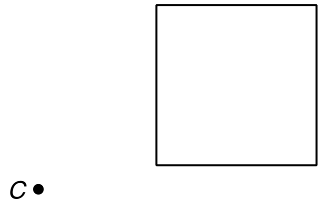
Dilations

Draw the dilation image of each figure with center C and the given scale factor.

1. $r = 2$



2. $r = \frac{1}{4}$



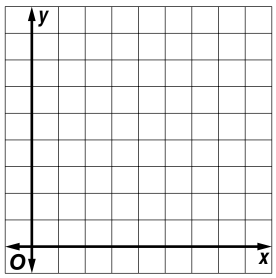
Find the measure of the dilation image $\overline{M'N'}$ or of the preimage $\overline{MN}$ using the given scale factor.

3. $MN = 3, r = 3$

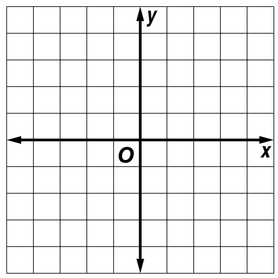
4. $M'N' = 7, r = 21$

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.**

5. $J(2, 4), K(4, 4), P(3, 2)$

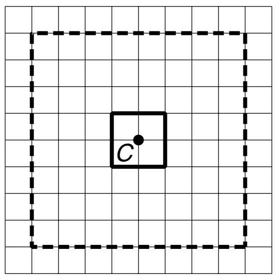


6. $D(-2, 0), G(0, 2), F(2, -2)$

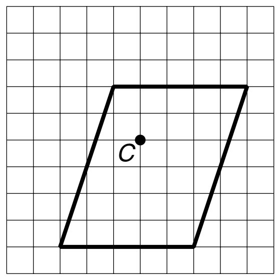


Determine the scale factor for each dilation with center C. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dashed figure is the dilation image.

7.



8.



Lesson 9-5

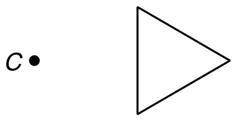
NAME ______________________________ DATE ____________ PERIOD _____

9-5 Practice

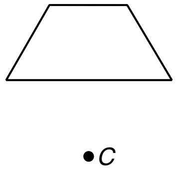
Dilations

Draw the dilation image of each figure with center *C* and the given scale factor.

1. $r = \frac{3}{2}$



2. $r = \frac{2}{3}$



Find the measure of the dilation image $\overline{A'T'}$ or of the preimage $\overline{AT}$ using the given scale factor.

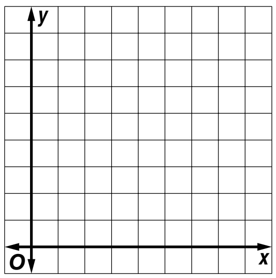
3. $AT = 15, r = \frac{3}{5}$

4. $AT = 30, r = -\frac{1}{6}$

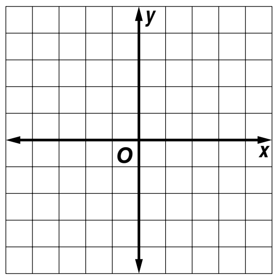
5. $A'T' = 12, r = \frac{4}{3}$

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.**

6. $A(1, 1), C(2, 3), D(4, 2), E(3, 1)$

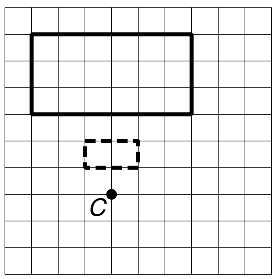


7. $Q(-1, -1), R(0, 2), S(2, 1)$

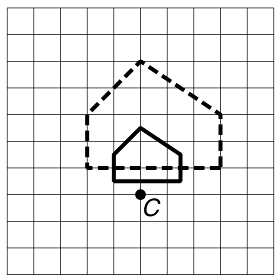


Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dotted figure is the dilation image.

8.



9.



10. PHOTOGRAPHY Estebe enlarged a 4-inch by 6-inch photograph by a factor of $\frac{5}{2}$. What are the new dimensions of the photograph?

9-5 Reading to Learn Mathematics

Dilations

Pre-Activity **How do you use dilations when you use a computer?**

Read the introduction to Lesson 9-5 at the top of page 490 in your textbook.

In addition to the example given in your textbook, give two everyday examples of scaling an object, one that makes the object larger and another that makes it smaller.

Reading the Lesson

1. Each of the values of r given below represents the scale factor for a dilation. In each case, determine whether the dilation is an *enlargement*, a *reduction*, or a *congruence transformation*.

a. $r = 3$

b. $r = 0.5$

c. $r = -0.75$

d. $r = -1$

e. $r = \frac{2}{3}$

f. $r = -\frac{3}{2}$

g. $r = -1.01$

h. $r = 0.999$

2. Determine whether each sentence is *always*, *sometimes*, or *never* true. If the sentence is not always true, explain why.

a. A dilation requires a center point and a scale factor.

b. A dilation changes the size of a figure.

c. A dilation changes the shape of a figure.

d. The image of a figure under a dilation lies on the opposite side of the center from the preimage.

e. A similarity transformation is a congruence transformation.

f. The center of a dilation is its own image.

g. A dilation is an isometry.

h. The scale factor for a dilation is a positive number.

i. Dilations produce similar figures.

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that your classmate Lydia is having trouble understanding the relationship between *similarity transformations* and *congruence transformations*. How can you explain this to her?

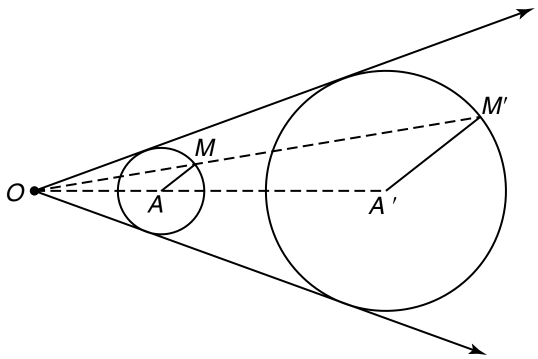
Lesson 9-5

9-5 Enrichment

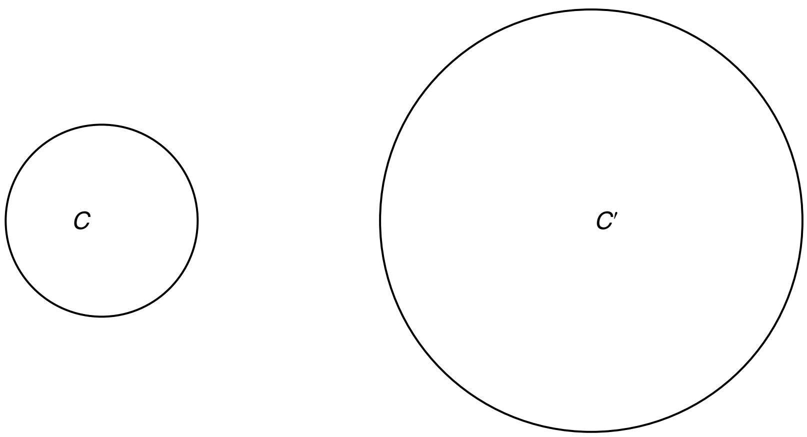
Similar Circles

You may be surprised to learn that two noncongruent circles that lie in the same plane and have no common interior points can be mapped one onto the other by more than one dilation.

1. Here is diagram that suggests one way to map a smaller circle onto a larger one using a dilation. The circles are given. The lines suggest how to find the center for the dilation. Describe how the center is found. Use segments in the diagram to name the scale factor.



2. Here is another pair of noncongruent circles with no common interior point. From Exercise 1, you know you can locate a point off to the left of the smaller circle that is the center for a dilation mapping $\odot C$ onto $\odot C'$. Find another center for another dilation that maps $\odot C$ onto $\odot C'$. Mark and label segments to name the scale factor.



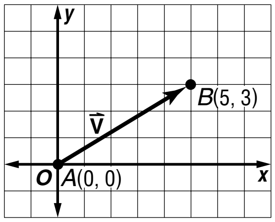
NAME ______________________ DATE ____________ PERIOD _____

9-6 Study Guide and Intervention

Vectors

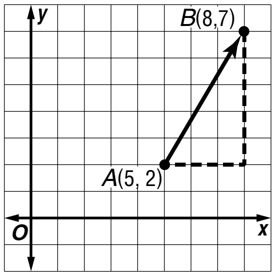
Magnitude and Direction A vector is a directed segment representing a quantity that has both **magnitude**, or length, and **direction**. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as $\overrightarrow{AB}$, where A is the initial point and B is the endpoint, or as $\vec{\mathbf{v}}$.

A vector in **standard position** has its initial point at (0, 0) and can be represented by the ordered pair for point B. The vector at the right can be expressed as $\vec{\mathbf{v}} = \langle 5, 3 \rangle$.



You can use the Distance Formula to find the magnitude $|\overrightarrow{AB}|$ of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive x-axis or with any other horizontal line.

Example **Find the magnitude and direction of $\overrightarrow{AB}$ for $A(5, 2)$ and $B(8, 7)$.**



Find the magnitude.

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 5)^2 + (7 - 2)^2}$$
$$= \sqrt{34} \text{ or about 5.8 units}$$

To find the direction, use the tangent ratio.

$\tan A = \frac{5}{3}$ The tangent ratio is opposite over adjacent.

$m\angle A \approx 59.0$ Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is 59°.

Exercises

Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates. Round to the nearest tenth.

1. $A(3, 1), B(-2, 3)$

2. $A(0, 0), B(-2, 1)$

3. $A(0, 1), B(3, 5)$

4. $A(-2, 2), B(3, 1)$

5. $A(3, 4), B(0, 0)$

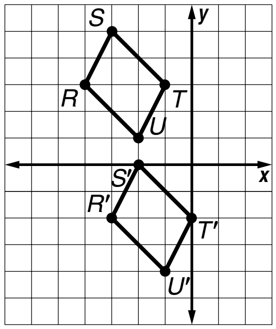
6. $A(4, 2), B(0, 3)$

9-6 Study Guide and Intervention *(continued)*

Vectors

Translations with Vectors Recall that the transformation $(a, b) \rightarrow (a + 2, b - 3)$ represents a translation right 2 units and down 3 units. The vector $\langle 2, -3 \rangle$ is another way to describe that translation. Also, two vectors can be added: $\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$. The sum of two vectors is called the **resultant**.

Example **Graph the image of parallelogram *RSTU* under the translation by the vectors $\overrightarrow{m} = \langle 3, -1 \rangle$ and $\overrightarrow{n} = \langle -2, -4 \rangle$.**



Find the sum of the vectors.

$\overrightarrow{m} + \overrightarrow{n} = \langle 3, -1 \rangle + \langle -2, -4 \rangle$

$= \langle 3 - 2, -1 - 4 \rangle$

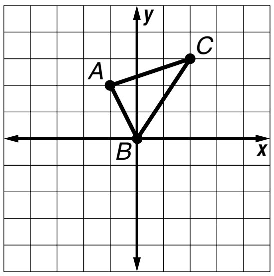
$= \langle 1, -5 \rangle$

Translate each vertex of parallelogram *RSTU* right 1 unit and down 5 units.

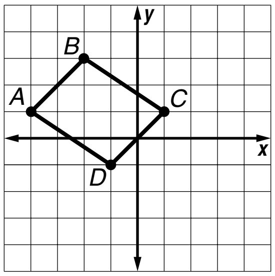
Exercises

Graph the image of each figure under a translation by the given vector(s).

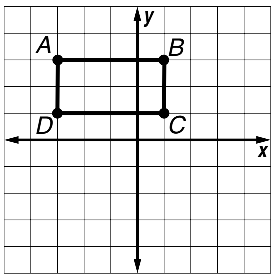
1. $\triangle ABC$ with vertices $A(-1, 2)$, $B(0, 0)$, and $C(2, 3)$; $\overrightarrow{m} = \langle 2, -3 \rangle$



2. *ABCD* with vertices $A(-4, 1)$, $B(-2, 3)$, $C(1, 1)$, and $D(-1, -1)$; $\overrightarrow{n} = \langle 3 -3 \rangle$



3. *ABCD* with vertices $A(-3, 3)$, $B(1, 3)$, $C(1, 1)$, and $D(-3, 1)$; the sum of $\overrightarrow{p} = \langle -2, 1 \rangle$ and $\overrightarrow{q} = \langle 5, -4 \rangle$



Given $\overrightarrow{m} = \langle 1, -2 \rangle$ and $\overrightarrow{n} = \langle -3, -4 \rangle$, represent each of the following as a single vector.

4. $\overrightarrow{m} + \overrightarrow{n}$

5. $\overrightarrow{n} - \overrightarrow{m}$

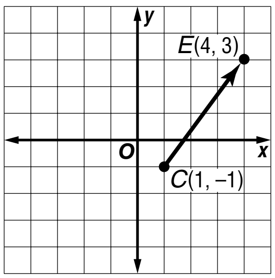
NAME ____ DATE ____________ PERIOD _____

9-6 Skills Practice

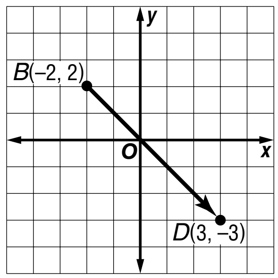
Vectors

Write the component form of each vector.

1.



2.



Find the magnitude and direction of $\overrightarrow{RS}$ for the given coordinates. Round to the nearest tenth.

3. $R(2, -3)$, $S(4, 9)$

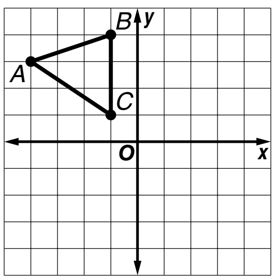
4. $R(0, 2)$, $S(3, 12)$

5. $R(5, 4)$, $S(-3, 1)$

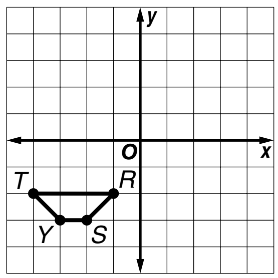
6. $R(1, 5)$, $S(-4, -6)$

Graph the image of each figure under a translation by the given vector(s).

7. $\triangle ABC$ with vertices $A(-4, 3)$, $B(-1, 4)$, $C(-1, 1)$; $\vec{\mathbf{t}} = \langle 4, -3 \rangle$



8. trapezoid with vertices $T(-4, -2)$, $R(-1, -2)$, $S(-2, -3)$, $Y(-3, -3)$; $\vec{\mathbf{a}} = \langle 3, 1 \rangle$ and $\vec{\mathbf{b}} = \langle 2, 4 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

9. $\vec{\mathbf{y}} = \langle 7, 0 \rangle$, $\vec{\mathbf{z}} = \langle 0, 6 \rangle$

10. $\vec{\mathbf{b}} = \langle 3, 2 \rangle$, $\vec{\mathbf{c}} = \langle -2, 3 \rangle$

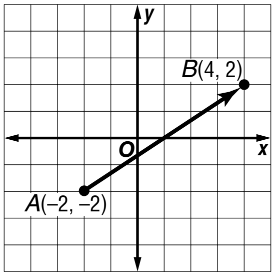
NAME ______________________________ DATE ____________ PERIOD _____

9-6 Practice

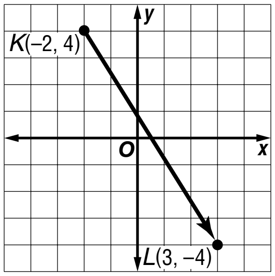
Vectors

Write the component form of each vector.

1.



2.



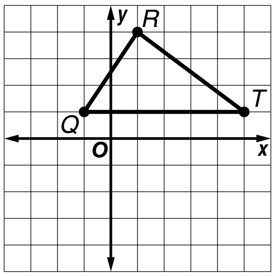
Find the magnitude and direction of $\overrightarrow{FG}$ for the given coordinates. Round to the nearest tenth.

3. $F(-8, -5)$, $G(-2, 7)$

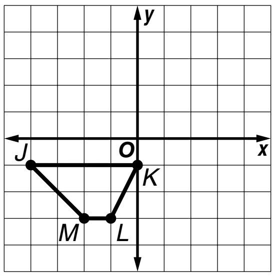
4. $F(-4, 1)$, $G(5, -6)$

Graph the image of each figure under a translation by the given vector(s).

5. $\triangle QRT$ with vertices $Q(-1, 1)$, $R(1, 4)$, $T(5, 1)$; $\vec{\mathbf{s}} = \langle -2, -5 \rangle$



6. trapezoid with vertices $J(-4, -1)$, $K(0, -1)$, $L(-1, -3)$, $M(-2, -3)$; $\vec{\mathbf{c}} = \langle 5, 4 \rangle$ and $\vec{\mathbf{d}} = \langle -2, 1 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

7. $\vec{\mathbf{a}} = \langle -6, 4 \rangle$, $\vec{\mathbf{b}} = \langle 4, 6 \rangle$

8. $\vec{\mathbf{e}} = \langle -4, -5 \rangle$, $\vec{\mathbf{f}} = \langle -1, 3 \rangle$

AVIATION For Exercises 9 and 10, use the following information.

A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

9. Find the resultant velocity of the plane.

10. Find the resultant direction of the plane.

NAME ____ DATE ____________ PERIOD _____

9-6 Reading to Learn Mathematics

Vectors

Pre-Activity **How do vectors help a pilot plan a flight?**

Read the introduction to Lesson 9-6 at the top of page 498 in your textbook.

Why do pilots often head their planes in a slightly different direction from their destination?

Reading the Lesson

1. Supply the missing words or phrases to complete the following sentences.

a. A _________________ is a directed segment representing a quantity that has both magnitude and direction.

b. The length of a vector is called its _________________.

c. Two vectors are parallel if and only if they have the same or _________________ direction.

d. A vector is in _____________________ if it is drawn with initial point at the origin.

e. Two vectors are equal if and only if they have the same _________________ and the same_________________.

f. The sum of two vectors is called the _________________.

g. A vector is written in _____________________ if it is expressed as an ordered pair.

h. The process of multiplying a vector by a constant is called _______________________.

2. Write each vector described below in component form.

a. a vector in standard position with endpoint (a, b)

b. a vector with initial point (a, b) and endpoint (c, d)

c. a vector in standard position with endpoint $(-3, 5)$

d. a vector with initial point $(2, -3)$ and endpoint $(6, -8)$

e. $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ if $\vec{\mathbf{a}} = \langle -3, 5 \rangle$ and $\vec{\mathbf{b}} = \langle 6, -4 \rangle$

f. $5\vec{\mathbf{u}}$ if $\vec{\mathbf{u}} = \langle 8, -6 \rangle$

g. $-\frac{1}{3}\vec{\mathbf{v}}$ if $\vec{\mathbf{v}} = \langle -15, 24 \rangle$

h. $0.5\vec{\mathbf{u}} + 1.5\vec{\mathbf{v}}$ if $\vec{\mathbf{u}} = \langle 10, -10 \rangle$ and $\vec{\mathbf{v}} = \langle -8, 6 \rangle$

Helping You Remember

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about *scale factors* when you studied similarity and dilations. How is the idea of a *scalar* related to *scale factors*?

9-6 Enrichment

Reading Mathematics

Many quantities in nature can be thought of as vectors. The science of physics involves many vector quantities. In reading about applications of mathematics, ask yourself whether the quantities involve only magnitude or both magnitude and direction. The first kind of quantity is called **scalar**. The second kind is a **vector**.

Classify each of the following. Write *scalar* or *vector*.

1. the mass of a book

2. a car traveling north at 55 mph

3. a balloon rising 24 feet per minute

4. the size of a shoe

5. a room temperature of 22 degrees Celsius

6. a west wind of 15 mph

7. the batting average of a baseball player

8. a car traveling 60 mph

9. a rock falling at 10 mph

10. your age

11. the force of Earth's gravity acting on a moving satellite

12. the area of a record rotating on a turntable

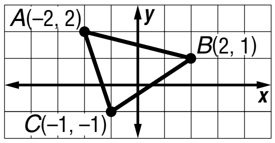
13. the length of a vector in the coordinate plane

9-7 Study Guide and Intervention

Transformations with Matrices

Translations and Dilations A **vector** can be represented by the ordered pair $\langle x, y \rangle$ or by the **column matrix** $\begin{bmatrix} x \\ y \end{bmatrix}$. When the ordered pairs for all the vertices of a polygon are placed together, the resulting matrix is called the **vertex matrix** for the polygon.

For $\triangle ABC$ with $A(-2, 2)$, $B(2, 1)$, and $C(-1, -1)$, the vertex matrix for the triangle is $\begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.



Example 1 **For $\triangle ABC$ above, use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ under the translation $(x, y) \rightarrow (x + 3, y - 1)$.**

To translate the figure 3 units to the right, add 3 to each x-coordinate. To translate the figure 1 unit down, add -1 to each y-coordinate.

Vertex Matrix of $\triangle ABC$		Translation Matrix		Vertex Matrix of $\triangle A'B'C'$
$\begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$	$+$	$\begin{bmatrix} 3 & 3 & 3 \\ -1 & -1 & -1 \end{bmatrix}$	$=$	$\begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & -2 \end{bmatrix}$

The coordinates are $A'(1, 1)$, $B'(5, 0)$, and $C'(2, -2)$.

Example 2 **For $\triangle ABC$ above, use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ for a dilation centered at the origin with scale factor 3.**

Scale Factor		Vertex Matrix of $\triangle ABC$		Vertex Matrix of $\triangle A'B'C'$
3	$\cdot$	$\begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$	$=$	$\begin{bmatrix} -6 & 6 & -3 \\ 6 & 3 & -3 \end{bmatrix}$

The coordinates are $A'(-6, 6)$, $B'(6, 3)$, and $C'(-3, -3)$.

Exercises

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations or dilations.

1. $\triangle ABC$ with $A(3, 1)$, $B(-2, 4)$, $C(-2, -1)$; $(x, y) \rightarrow (x - 1, y + 2)$

2. parallelogram $RSTU$ with $R(-4, -2)$, $S(-3, 1)$, $T(3, 4)$, $U(2, 1)$; $(x, y) \rightarrow (x - 4, y - 3)$

3. rectangle $PQRS$ with $P(4, 0)$, $Q(3, -3)$, $R(-3, -1)$, $S(-2, 2)$; $(x, y) \rightarrow (x - 2, y + 1)$

4. $\triangle ABC$ with $A(-2, -1)$, $B(-2, -3)$, $C(2, -1)$; dilation centered at the origin with scale factor 2

5. parallelogram $RSTU$ with $R(4, -2)$, $S(-4, -1)$, $T(-2, 3)$, $U(6, 2)$; dilation centered at the origin with scale factor 1.5

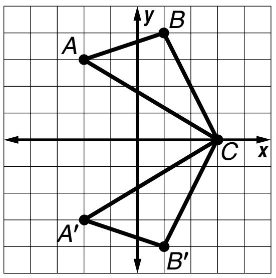
9-7 Study Guide and Intervention *(continued)*

Transformations with Matrices

Reflections and Rotations When you reflect an image, one way to find the coordinates of the reflected vertices is to multiply the vertex matrix of the object by a **reflection matrix**. To perform more than one reflection, multiply by one reflection matrix to find the first image. Then multiply by the second matrix to find the final image. The matrices for reflections in the axes, the origin, and the line $y = x$ are shown below.

For a reflection in the:	*x*-axis	*y*-axis	origin	line $y = x$
Multiply the vertex matrix by:	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Example **$\triangle ABC$ has vertices $A(-2, 3)$, $B(1, 4)$, and $C(3, 0)$. Use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the *x*-axis.**



To reflect in the *x*-axis, multiply the vertex matrix of $\triangle ABC$ by the reflection matrix for the *x*-axis.

Reflection Matrix for *x*-axis		Vertex Matrix of $\triangle ABC$		Vertex Matrix of $\triangle A'B'C'$
$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	$\cdot$	$\begin{bmatrix}-2 & 1 & 3\\3 & 4 & 0\end{bmatrix}$	$=$	$\begin{bmatrix}-2 & 1 & 3\\-3 & -4 & 0\end{bmatrix}$

Exercises

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

1. $\triangle ABC$ with $A(-3, 2)$, $B(-1, 3)$, $C(1, 0)$; reflection in the *x*-axis

2. $\triangle XYZ$ with $X(2, -1)$, $Y(4, -3)$, $Z(-2, 1)$; reflection in the *y*-axis

3. $\triangle ABC$ with $A(3, 4)$, $B(-1, 0)$, $C(-2, 4)$; reflection in the origin

4. parallelogram *RSTU* with $R(-3, 2)$, $S(3, 2)$, $T(5, -1)$, $U(-1, -1)$; reflection in the line $y = x$

5. $\triangle ABC$ with $A(2, 3)$, $B(-1, 2)$, $C(1, -1)$; reflection in the origin, then reflection in the line $y = x$

6. parallelogram *RSTU* with $R(0, 2)$, $S(4, 2)$, $T(3, -2)$, $U(-1, -2)$; reflection in the *x*-axis, then reflection in the *y*-axis

NAME ______________________________ DATE __________ PERIOD _____

9-7 Skills Practice

Transformations with Matrices

Lesson 9-7

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations.

1. $\triangle STU$ with $S(6, 4)$, $T(9, 7)$, and $U(14, 2)$; $(x, y) \rightarrow (x - 4, y + 3)$

2. $\triangle GHI$ with $G(-5, 0)$, $H(-3, 6)$, and $I(-2, 1)$; $(x, y) \rightarrow (x + 2, y + 6)$

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.

3. $\triangle DEF$ with $D(2, 1)$, $E(5, 4)$, and $F(7, 2)$; $r = 4$

4. quadrilateral $WXYZ$ with $W(-9, 6)$, $X(-6, 3)$, $Y(3, 12)$, and $Z(-6, 15)$; $r = \frac{1}{3}$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

5. $\triangle MNO$ with $M(-5, 1)$, $N(-2, 3)$, and $O(2, 0)$; y-axis

6. quadrilateral $ABCD$ with $A(3, 1)$, $B(6, -2)$, $C(5, -5)$, and $D(1, -6)$; x-axis

Use a matrix to find the coordinates of the vertices of the image of each figure under the given rotation.

7. $\triangle RST$ with $R(-2, -2)$, $S(-3, 3)$, and $T(2, 2)$; 90°counterclockwise

8. $\square LMNP$ with $L(3, 4)$, $M(7, 4)$, $N(9, -3)$, and $P(5, -3)$; 180° counterclockwise

NAME ____ DATE ____________ PERIOD _____

9-7 Practice

Transformations with Matrices

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations.

1. $\triangle KLM$ with $K(-7, -3)$, $L(4, 9)$, and $M(9, -6)$; $(x, y) \rightarrow (x - 7, y + 2)$

2. $\square ABCD$ with $A(-4, 3)$, $B(-2, 8)$, $C(3, 10)$, and $D(1, 5)$; $(x, y) \rightarrow (x + 3, y - 9)$

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.

3. quadrilateral $HIJK$ with $H(-2, 3)$, $I(2, 6)$, $J(8, 3)$, and $K(3, -4)$; $r = -\frac{1}{3}$

4. pentagon $DEFGH$ with $D(-8, -4)$, $E(-8, 2)$, $F(2, 6)$, $G(8, 0)$, and $H(4, -6)$; $r = \frac{5}{4}$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

5. $\triangle QRS$ with $Q(-5, -4)$, $R(-1, -1)$, and $S(2, -6)$; x-axis

6. quadrilateral $VXYZ$ with $V(-4, -2)$, $X(-3, 4)$, $Y(2, 1)$, and $Z(4, -3)$; $y = x$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given rotation.

7. $\square EFGH$ with $E(-5\ -4)$, $F(-3, -1)$, $G(5, -1)$, and $H(3, -4)$; 90° counterclockwise

8. quadrilateral $PSTU$ with $P(-3, 5)$, $S(2, 6)$, $T(8, 1)$, and $U(-6, -4)$; 270° counterclockwise

9. FORESTRY A research botanist mapped a section of forested land on a coordinate grid to keep track of endangered plants in the region. The vertices of the map are $A(-2, 6)$, $B(9, 8)$, $C(14, 4)$, and $D(1, -1)$. After a month, the botanist has decided to decrease the research area to $\frac{3}{4}$ of its original size. If the center for the reduction is $O(0, 0)$,what are the coordinates of the new research area?

NAME ______________________________ DATE __________ PERIOD _____

9-7 Reading to Learn Mathematics

Transformations with Matrices

Pre-Activity **How can matrices be used to make movies?**

Read the introduction to Lesson 9-7 at the top of page 506 in your textbook.

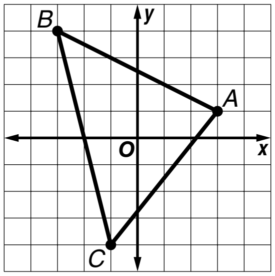
- What kind of transformation should be used to move a polygon?

- What kind of transformation should be used to resize a polygon?

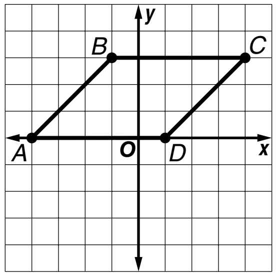
Reading the Lesson

1. Write a vertex matrix for each figure.

a. $\triangle ABC$



b. parallelogram *ABCD*



2. Match each transformation from the first column with the corresponding matrix from the second or third column. In each case, the vertex matrix for the preimage of a figure is multiplied on the left by one of the matrices below to obtain the image of the figure. All rotations listed are counterclockwise through the origin. (Some matrices may be used more than once or not at all.)

a. reflection over the y-axis
b. 90° rotation
c. reflection over the line $y = x$
d. 270° rotation
e. reflection over the origin
f. 180° rotation
g. reflection over the x-axis
h. 360° rotation

i. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **v.** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

ii. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **vi.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **vii.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

iv. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **viii.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Helping You Remember

3. How can you remember or quickly figure out the matrices for the transformations in Exercise 2?

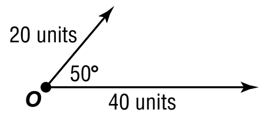
NAME ______________________ DATE __________ PERIOD _____

9-7 Enrichment

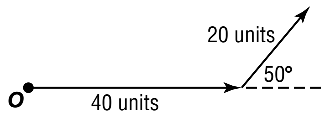
Vector Addition

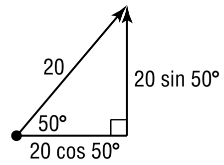
Vectors are physical quantities with magnitude and direction. Force and velocity are two examples. We will investigate adding vector quantities. The sum of two vectors is called a **resultant vector** or just the resultant.

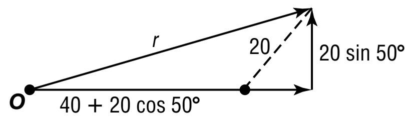
Example **Two separate forces, one measuring 20 units and the other measuring 40 units, act on an object. If the angle between the forces is 50°, find the magnitude and direction of the resultant force.**



First, the vectors must be rearranged by placing the tail of the 20-unit vector at the head of the 40-unit vector. Since these vectors are not perpendicular, the horizontal and vertical components of one of the vectors must be found. Using trigonometry, the horizontal component must be (20 cos 50°) units and the vertical component must be (20 sin 50°) units. Replacing the 20-unit vector with these components, we can now form two vectors perpendicular and use the Pythagorean Theorem to find the resultant.







$$r^2 = (40 + 20 \cos 50°)^2 + (20 \sin 50°)^2$$
$$r^2 \approx (52.9)^2 + (15.3)^2$$
$$r^2 \approx 3032.5$$
$$r^2 \approx 55.1$$

$$\tan O = \frac{20 \sin 50°}{40 + 20 \cos 50°}$$
$$\approx 0.2898$$
$$m\angle O \approx 16$$

Therefore, the resultant force is 55.1 units directed 16° from the 40-unit force.

Solve. Round all angle measures to the nearest degree. Round all other measures to the nearest tenth.

1. A plane flies due west at 250 kilometers per hour while the wind blows south at 70 kilometers per hour. Find the plane's resultant velocity.

2. A plane flies east for 200 km, then 60° south of east for 80 km. Find the plane's distance and direction from its starting point.

3. One force of 100 units acts on an object. Another force of 80 units acts on the object at a 40° angle from the first force. Find the resultant force on the object.

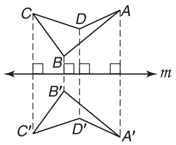
NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention

Reflections

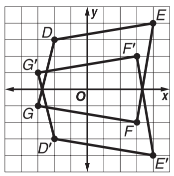
Draw Reflections The transformation called a **reflection** is a flip of a figure in a point, a line, or a plane. The new figure is the **image** and the original figure is the **preimage**. The preimage and image are congruent, so a reflection is a **congruence transformation** or **isometry**.

Example 1 **Construct the image of quadrilateral *ABCD* under a reflection in line *m*.**



Draw a perpendicular from each vertex of the quadrilateral to *m*. Find vertices *A'*, *B'*, *C'*, and *D'* that are the same distance from *m* on the other side of *m*. The image is *A'B'C'D'*.

Example 2 **Quadrilateral *DEFG* has vertices $D(-2, 3)$, $E(4, 4)$, $F(3, -2)$, and $G(-3, -1)$. Find the image under reflection in the *x*-axis.**



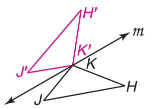
To find an image for a reflection in the *x*-axis, use the same *x*-coordinate and multiply the *y*-coordinate by -1. In symbols, $(a, b) \to (a, -b)$. The new coordinates are $D'(-2, -3)$, $E'(4, -4)$, $F'(3, 2)$, and $G'(-3, 1)$. The image is $D'E'F'G'$.

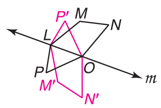
In Example 2, the notation $(a, b) \to (a, -b)$ represents a reflection in the *x*-axis. Here are three other common reflections in the coordinate plane.

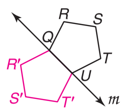
- in the *y*-axis: $(a, b) \to (-a, b)$
- in the line $y = x$: $(a, b) \to (b, a)$
- in the origin: $(a, b) \to (-a, -b)$

Exercises

Draw the image of each figure under a reflection in line *m*.

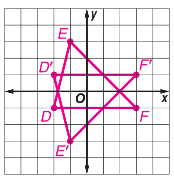
1. 

2. 

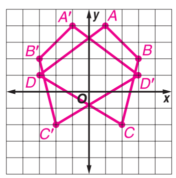
3. 

Graph each figure and its image under the given reflection.

4. $\triangle DEF$ with $D(-2, -1)$, $E(-1, 3)$, $F(3, -1)$ in the *x*-axis



5. *ABCD* with $A(1, 4)$, $B(3, 2)$, $C(2, -2)$, $D(-3, 1)$ in the *y*-axis



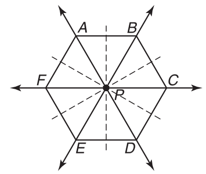
NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention *(continued)*

Reflections

Lines and Points of Symmetry If a figure has a **line of symmetry**, then it can be folded along that line so that the two halves match. If a figure has a **point of symmetry**, it is the midpoint of all segments between the preimage and image points.

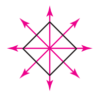
Example **Determine how many lines of symmetry a regular hexagon has. Then determine whether a regular hexagon has point symmetry.**

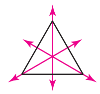


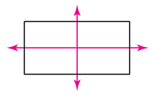
There are six lines of symmetry, three that are diagonals through opposite vertices and three that are perpendicular bisectors of opposite sides. The hexagon has point symmetry because any line through *P* identifies two points on the hexagon that can be considered images of each other.

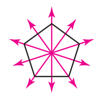
Exercises

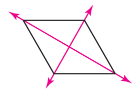
Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

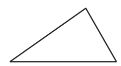
1. 
4; yes

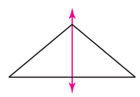
2. 
3; no

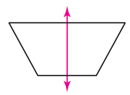
3. 
2; yes

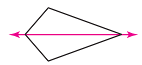
4. 
5; no

5. 
2; yes

6. 
0; no

7. 
1; no

8. 
1; no

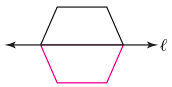
9. 
1; no

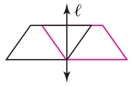
NAME ______________________ DATE ____________ PERIOD _____

9-1 Skills Practice

Reflections

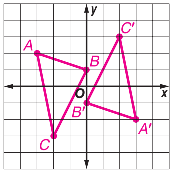
Draw the image of each figure under a reflection in line ℓ.

1. 

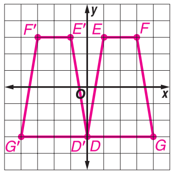
2. 

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

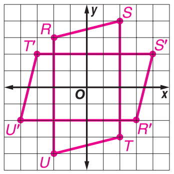
3. $\triangle ABC$ with vertices $A(-3, 2)$, $B(0, 1)$, and $C(-2, -3)$ in the origin



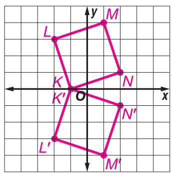
4. trapezoid $DEFG$ with vertices $D(0, -3)$, $E(1, 3)$, $F(3, 3)$, and $G(4, -3)$ in the y-axis



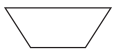
5. parallelogram $RSTU$ with vertices $R(-2, 3)$, $S(2, 4)$, $T(2, -3)$ and $U(-2, -4)$ in the line $y = x$



6. square $KLMN$ with vertices $K(-1, 0)$, $L(-2, 3)$, $M(1, 4)$, and $N(2, 1)$ in the x-axis



Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

7. 

1; no

8. 

4; yes

9. S

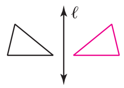
0; yes

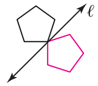
NAME ______________________ DATE ____________ PERIOD _____

9-1 Practice (Average)

Reflections

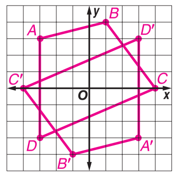
Draw the image of each figure under a reflection in line ℓ.

1. 

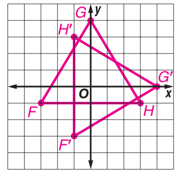
2. 

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

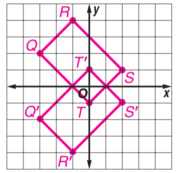
3. quadrilateral $ABCD$ with vertices $A(-3, 3)$, $B(1, 4)$, $C(4, 0)$, and $D(-3, -3)$ in the origin



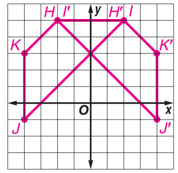
4. $\triangle FGH$ with vertices $F(-3, -1)$, $G(0, 4)$, and $H(3, -1)$ in the line $y = x$



5. rectangle $QRST$ with vertices $Q(-3, 2)$, $R(-1, 4)$, $S(2, 1)$, and $T(0, -1)$ in the x-axis



6. trapezoid $HIJK$ with vertices $H(-2, 5)$, $I(2, 5)$, $J(-4, -1)$, and $K(-4, 3)$ in the y-axis



ROAD SIGNS Determine how many lines of symmetry each sign has. Then determine whether the sign has point symmetry.

7. 

0; yes

8. 

1; no

9. 

4; yes

NAME ______________________ DATE ____________ PERIOD _____

9-1 Reading to Learn Mathematics

Reflections

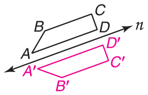
Pre-Activity **Where are reflections found in nature?**

Read the introduction to Lesson 9-1 at the top of page 463 in your textbook.

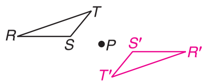
Suppose you draw a line segment connecting a point at the peak of a mountain to its image in the lake. Where will the midpoint of this segment fall? **on the boundary line between the shore and the surface of the lake**

Reading the Lesson

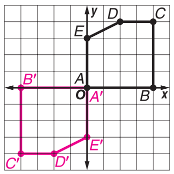
1. Draw the reflected image for each reflection described below.
   - **a.** reflection of trapezoid $ABCD$ in the line n. Label the image of $ABCD$ as $A'B'C'D'$.



   - **b.** reflection of $\triangle RST$ in point P. Label the image of RST as $R'S'T'$.



   - **c.** reflection of pentagon $ABCDE$ in the origin. Label the image of $ABCDE$ as $A'B'C'D'E'$.



2. Determine the image of the given point under the indicated reflection.
   - **a.** (4, 6); reflection in the y-axis **(−4, 6)**
   - **b.** (−3, 5); reflection in the x-axis **(−3, −5)**
   - **c.** (−8, −2); reflection in the line $y = x$ **(−2, −8)**
   - **d.** (9, −3); reflection in the origin **(−9, 3)**

3. Determine the number of lines of symmetry for each figure described below. Then determine whether the figure has point symmetry and indicate this by writing *yes* or *no*.
   - **a.** a square **4; yes**
   - **b.** an isosceles triangle (not equilateral) **1; no**
   - **c.** a regular hexagon **6; yes**
   - **d.** an isosceles trapezoid **1; no**
   - **e.** a rectangle (not a square) **2; yes**
   - **f.** the letter E **1; no**

Helping You Remember

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *isometry* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part. **Sample answer: The first part comes from *isos*, which means *equal*, as in *isosceles*. The second part comes from *metron*, which means *measure*, as in *geometry*.**

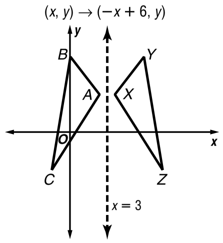
Lesson 9-1

NAME ______________________ DATE ____________ PERIOD _____

9-1 Enrichment

Reflections in the Coordinate Plane

Study the diagram at the right. It shows how the triangle ABC is mapped onto triangle XYZ by the transformation $(x, y) \to (-x + 6, y)$. Notice that $\triangle XYZ$ is the reflection image with respect to the vertical line with equation $x = 3$.



1. Prove that the vertical line with equation $x = 3$ is the perpendicular bisector of the segment with endpoints (x, y) and $(-x + 6, y)$. (Hint: Use the midpoint formula.)

Midpoint $= \left(\frac{x + (-x + 6)}{2}, \frac{y + y}{2}\right)$ **or (3, y)**

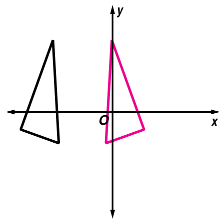
The segment joining (x, y) and ($-x + 6$, y) is horizontal and hence is perpendicular to the vertical line.

2. Every transformation of the form $(x, y) \to (-x + 2h, y)$ is a reflection with respect to the vertical line with equation $x = h$. What kind of transformation is $(x, y) \to (x, -y + 2k)$?

reflection with respect to the horizontal line with equation $y = k$

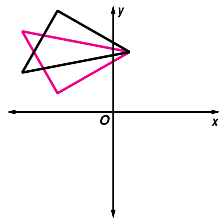
Draw the transformation image for each figure and the given transformation. Is it a reflection transformation? If so, with respect to what line?

3. $(x, y) \to (-x - 4, y)$



yes; $x = -2$

4. $(x, y) \to (x, -y + 8)$



yes; $y = 4$

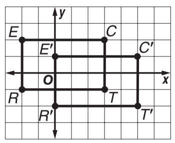
NAME ______________________ DATE __________ PERIOD _____

9-2 Study Guide and Intervention

Translations

Translations Using Coordinates A transformation called a **translation** slides a figure in a given direction. In the coordinate plane, a translation moves every preimage point $P(x, y)$ to an image point $P(x + a, y + b)$ for fixed values a and b. In words, a translation shifts a figure a units horizontally and b units vertically; in symbols, $(x, y) \rightarrow (x + a, y + b)$.

Example **Rectangle *RECT* has vertices $R(-2, -1)$, $E(-2, 2)$, $C(3, 2)$, and $T(3, -1)$. Graph *RECT* and its image for the translation $(x, y) \rightarrow (x + 2, y - 1)$.**



The translation moves every point of the preimage right 2 units and down 1 unit.

$(x, y) \rightarrow (x + 2, y - 1)$

$R(-2, -1) \rightarrow R'(-2 + 2, -1 - 1)$ or $R'(0, -2)$

$E(-2, 2) \rightarrow E'(-2 + 2, 2 - 1)$ or $E'(0, 1)$

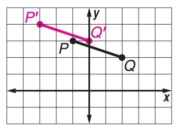
$C(3, 2) \rightarrow C'(3 + 2, 2 - 1)$ or $C'(5, 1)$

$T(3, -1) \rightarrow T'(3 + 2, -1 - 1)$ or $T'(5, -2)$

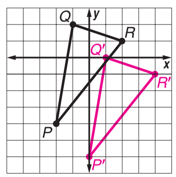
Exercises

Graph each figure and its image under the given translation.

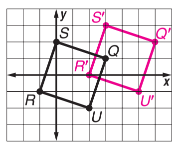
1. $\overline{PQ}$ with endpoints $P(-1, 3)$ and $Q(2, 2)$ under the translation left 2 units and up 1 unit



2. $\triangle PQR$ with vertices $P(-2, -4)$, $Q(-1, 2)$, and $R(2, 1)$ under the translation right 2 units and down 2 units



3. square $SQUR$ with vertices $S(0, 2)$, $Q(3, 1)$, $U(2, -2)$, and $R(-1, -1)$ under the translation right 3 units and up 1 unit



Lesson 9-2

NAME ______________________ DATE __________ PERIOD _____

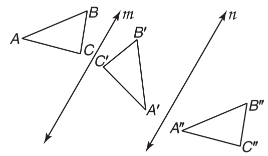
9-2 Study Guide and Intervention *(continued)*

Translations

Translations by Repeated Reflections Another way to find the image of a translation is to reflect the figure twice in parallel lines. This kind of translation is called a **composite of reflections**.

Example **In the figure, $m \parallel n$. Find the translation image of $\triangle ABC$.**

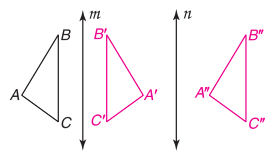
$\triangle A'B'C'$ is the image of $\triangle ABC$ reflected in line m.
$\triangle A''B''C''$ is the image of $\triangle A'B'C'$ reflected in line n.
The final image, $\triangle A''B''C''$, is a translation of $\triangle ABC$.



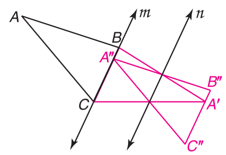
Exercises

In each figure, $m \parallel n$. Find the translation image of each figure by reflecting it in line m and then in line n.

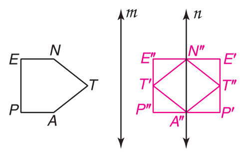
1.



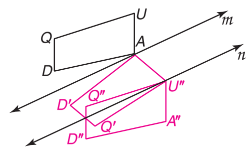
2.



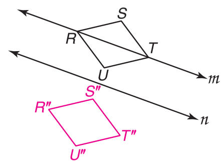
3.



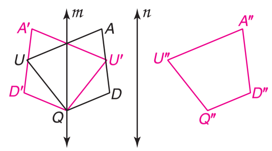
4.



5.



6.



Answers

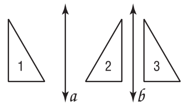
NAME ______________________ DATE ____________ PERIOD _____

9-2 Skills Practice

Translations

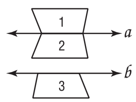
In each figure, $a \parallel b$. Determine whether figure 3 is a translation image of figure 1. Write *yes* or *no*. Explain your answer.

1.



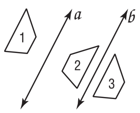
Yes; reflect figure 1 in line *a* to get figure 2 and figure 2 in line *b* to get figure 3.

2.



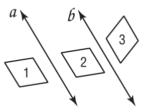
No; it is oriented differently than figure 1.

3.



Yes; reflect figure 1 in line *a* to get figure 2 and figure 2 in line *b* to get figure 3.

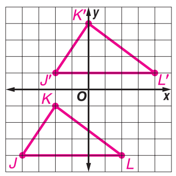
4.



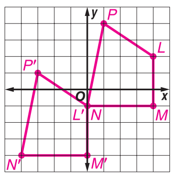
No; it is a reflection of a translation and is oriented differently than figure 1.

COORDINATE GEOMETRY **Graph each figure and its image under the given translation.**

5. $\triangle JKL$ with vertices $J(-4, -4)$, $K(-2, -1)$, and $L(2, -4)$ under the translation $(x, y) \to (x + 2, y + 5)$



6. quadrilateral $LMNP$ with vertices $L(4, 2)$, $M(4, -1)$, $N(0, -1)$, and $P(1, 4)$ under the translation $(x, y) \to (x - 4, y - 3)$



Lesson 9-2

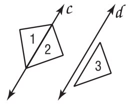
NAME ______________________ DATE ____________ PERIOD _____

9-2 Practice (Average)

Translations

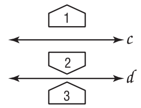
In each figure, $c \parallel d$. Determine whether figure 3 is a translation image of figure 1. Write *yes* or *no*. Explain your answer.

1.



No; it is a translation of a reflection and is oriented differently than figure 1.

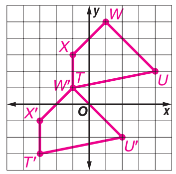
2.



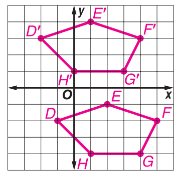
Yes; it is a reflection of a reflection with respect to the parallel lines.

COORDINATE GEOMETRY **Graph each figure and its image under the given translation.**

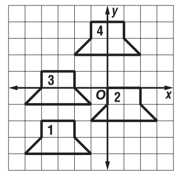
3. quadrilateral $TUWX$ with vertices $T(-1, 1)$, $U(4, 2)$, $W(1, 5)$, and $X(-1, 3)$ under the translation $(x, y) \to (x - 2, y - 4)$



4. pentagon $DEFGH$ with vertices $D(-1, -2)$, $E(2, -1)$, $F(5, -2)$, $G(4, -4)$, $H(1, -4)$ under the translation $(x, y) \to (x - 1, y + 5)$



ANIMATION **Find the translation that moves the figure on the coordinate plane.**



5. figure 1 → figure 2
(x + 4, y + 2)

6. figure 2 → figure 3
(x − 4, y + 1)

7. figure 3 → figure 4
(x + 3, y + 3)

NAME ______________________ DATE __________ PERIOD _____

9-2 Reading to Learn Mathematics

Translations

Pre-Activity **How are translations used in a marching band show?**

Read the introduction to Lesson 9-2 at the top of page 470 in your textbook.

How do band directors get the marching band to maintain the shape of the figure they originally formed? **Sample answer: The band practices marching in unison, with everyone making identical moves at the same time.**

Reading the Lesson

1. Underline the correct word or phrase to form a true statement.

a. All reflections and translations are (opposites/isometries/equivalent).

b. The preimage and image of a figure under a reflection in a line have (the same orientation/opposite orientations).

c. The preimage and image of a figure under a translation have (the same orientation/opposite orientations).

d. The result of successive reflections over two parallel lines is a (reflection/rotation/translation).

e. Collinearity (is/is not) preserved by translations.

f. The translation $(x, y) \rightarrow (x + a, y + b)$ shifts every point a units (horizontally/vertically) and y units (horizontally/vertically).

2. Find the image of each preimage under the indicated translation.

a. (x, y); 5 units right and 3 units up **$(x + 5, y + 3)$**

b. (x, y); 2 units left and 4 units down **$(x - 2, y - 4)$**

c. (x, y); 1 unit left and 6 units up **$(x - 1, y + 6)$**

d. (x, y); 7 units right **$(x + 7, y)$**

e. $(4, -3)$; 3 units up **$(4, 0)$**

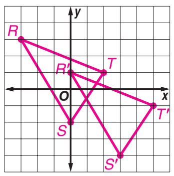
f. $(-5, 6)$; 3 units right and 2 units down **$(-2, 4)$**

g. $(-7, 5)$; 7 units right and 5 units down **$(0, 0)$**

h. $(-9, -2)$; 12 units right and 6 units down **$(3, -8)$**

3. $\triangle RST$ has vertices $R(-3, 3)$, $S(0, -2)$, and $T(2, 1)$. Graph $\triangle RST$ and its image $\triangle R'S'T'$ under the translation $(x, y) \rightarrow (x + 3, y - 2)$. List the coordinates of the vertices of the image.

$R'(0, 1)$, $S'(3, -4)$, $T'(5, -1)$



Helping You Remember

4. A good way to remember a new mathematical term is to relate it to an everyday meaning of the same word. How is the meaning of *translation* in geometry related to the idea of *translation* from one language to another? **Sample answer: When you translate from one language to another, you *carry over* the meaning from one language to another. When you translate a geometric figure, you *carry over* the figure from one position to another without changing its basic**

NAME ______________________ DATE __________ PERIOD _____

9-2 Enrichment

Translations in The Coordinate Plane

You can use algebraic descriptions of reflections to show that the composite of two reflections with respect to parallel lines is a translation (that is, a slide).

1. Suppose a and b are two different real numbers. Let S and T be the following reflections.

$S: (x, y) \rightarrow (-x + 2a, y)$
$T: (x, y) \rightarrow (-x + 2b, y)$

S is reflection with respect to the line with equation $x = a$, and T is reflection with respect to the line with equation $x = b$.

a. Find an algebraic description (similar to those above for S and T) to describe the composite transformation "S followed by T."

$(x, y) \rightarrow (x + 2(b - a), y)$

b. Find an algebraic description for the composite transformation "T followed by S."

$(x, y) \rightarrow (x + 2(a - b), y)$

2. Think about the results you obtained in Exercise 1. What do they tell you about how the distance between two parallel lines is related to the distance between a preimage and image point for a composite of reflections with respect to these lines?

The distance between a preimage and its image point is twice the distance between the parallel lines.

3. Illustrate your answers to Exercises 1 and 2 with sketches. Use a separate sheet if necessary.

See students' work.

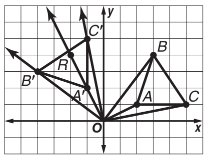
Answers

9-3 Study Guide and Intervention

Rotations

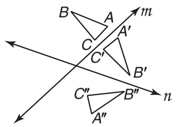
Draw Rotations A transformation called a **rotation** turns a figure through a specified angle about a fixed point called the **center of rotation**. To find the image of a rotation, one way is to use a protractor. Another way is to reflect a figure twice, in two intersecting lines.

Example 1 **$\triangle ABC$ has vertices $A(2, 1)$, $B(3, 4)$, and $C(5, 1)$. Draw the image of $\triangle ABC$ under a rotation of 90° counterclockwise about the origin.**



- First draw $\triangle ABC$. Then draw a segment from O, the origin, to point A.
- Use a protractor to measure 90° counterclockwise with $\overline{OA}$ as one side.
- Draw $\overrightarrow{OR}$.
- Use a compass to copy $\overline{OA}$ onto $\overrightarrow{OR}$. Name the segment $\overline{OA'}$.
- Repeat with segments from the origin to points B and C.

Example 2 **Find the image of $\triangle ABC$ under reflection in lines m and n.**

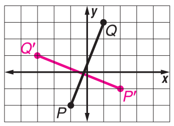


First reflect $\triangle ABC$ in line m. Label the image $\triangle A'B'C'$.
Reflect $\triangle A'B'C'$ in line n. Label the image $\triangle A''B''C''$.
$\triangle A''B''C''$ is a rotation of $\triangle ABC$. The center of rotation is the intersection of lines m and n. The angle of rotation is twice the measure of the acute angle formed by m and n.

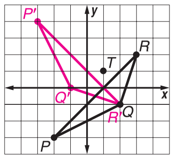
Exercises

Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.

1. $\overline{PQ}$ with endpoints $P(-1, -2)$ and $Q(1, 3)$ counterclockwise about the origin

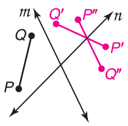


2. $\triangle PQR$ with vertices $P(-2, -3)$, $Q(2, -1)$, and $R(3, 2)$ clockwise about the point $T(1, 1)$

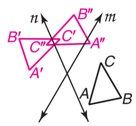


Find the rotation image of each figure by reflecting it in line m and then in line n.

3.



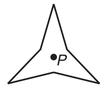
4.



9-3 Study Guide and Intervention *(continued)*

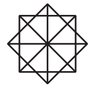
Rotations

Rotational Symmetry When the figure at the right is rotated about point P by 120° or 240°, the image looks like the preimage. The figure has **rotational symmetry**, which means it can be rotated less than 360° about a point and the preimage and image appear to be the same.



The figure has rotational symmetry of **order** 3 because there are 3 rotations less than 360° (0°, 120°, 240°) that produce an image that is the same as the original. The **magnitude** of the rotational symmetry for a figure is 360 degrees divided by the order. For the figure above, the rotational symmetry has magnitude 120 degrees.

Example **Identify the order and magnitude of the rotational symmetry of the design at the right.**



The design has rotational symmetry about the center point for rotations of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°.

There are eight rotations less than 360 degrees, so the order of its rotational symmetry is 8. The quotient 360 ÷ 8 is 45, so the magnitude of its rotational symmetry is 45 degrees.

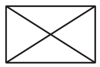
Exercises

Identify the order and magnitude of the rotational symmetry of each figure.

1. a square
order: 4; magnitude: 90°

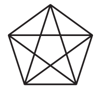
2. a regular 40-gon
order: 40; magnitude: 9°

3.



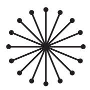
order: 2; magnitude: 180°

4.



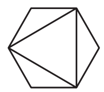
order: 5; magnitude: 72°

5.



order: 16; magnitude: 22.5°

6.



order: 3; magnitude: 120°

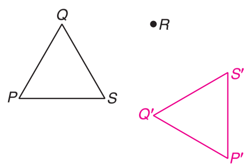
NAME ______________________ DATE ____________ PERIOD _____

9-3 Skills Practice

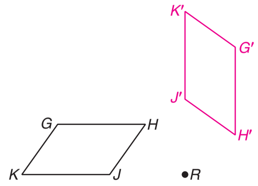
Rotations

Rotate each figure about point *R* under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. 90° counterclockwise

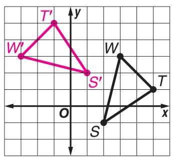


2. 90° clockwise

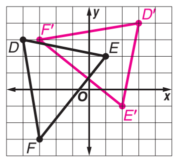


COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the origin and label the coordinates.

3. $\triangle STW$ with vertices $S(2, -1)$, $T(5, 1)$, and $W(3, 3)$ counterclockwise

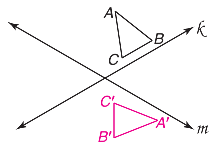


4. $\triangle DEF$ with vertices $D(-4, 3)$, $E(1, 2)$, and $F(-3, -3)$ clockwise



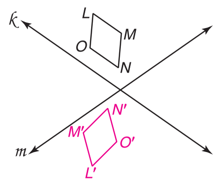
Use a composition of reflections to find the rotation image with respect to lines *k* and *m*. Then find the angle of rotation for each image.

5.



120° clockwise

6.



140° counterclockwise

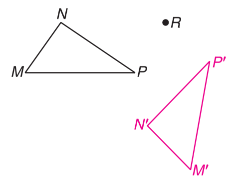
NAME ______________________ DATE ____________ PERIOD _____

9-3 Practice (Average)

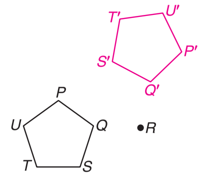
Rotations

Rotate each figure about point *R* under the given angle of rotation and the given direction. Label the vertices of the rotation image.

1. 80° counterclockwise

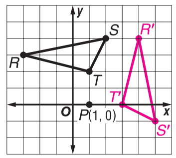


2. 100° clockwise

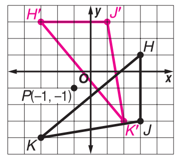


COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.

3. $\triangle RST$ with vertices $R(-3, 3)$, $S(2, 4)$, and $T(1, 2)$ clockwise about the point $P(1, 0)$

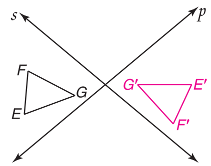


4. $\triangle HJK$ with vertices $H(3, 1)$, $J(3, -3)$, and $K(-3, -4)$ counterclockwise about the point $P(-1, -1)$



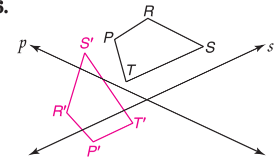
Use a composition of reflections to find the rotation image with respect to lines *p* and *s*. Then find the angle of rotation for each image.

5.



160° counterclockwise

6.



100° counterclockwise

7. STEAMBOATS A paddle wheel on a steamboat is driven by a steam engine and moves from one paddle to the next to propel the boat through the water. If a paddle wheel consists of 18 evenly spaced paddles, identify the order and magnitude of its rotational symmetry. order 18 and magnitude 20°

NAME ______________________ DATE ____________ PERIOD _____

9-3 Reading to Learn Mathematics

Rotations

Pre-Activity **How do some amusement rides illustrate rotations?**

Read the introduction to Lesson 9-3 at the top of page 476 in your textbook.

What are two ways that each car rotates?

Each car spins around its own center and each car rotates around a point in the center of the circular track.

Reading the Lesson

1. List all of the following types of transformations that satisfy each description: *reflection*, *translation*, *rotation*.

a. The transformation is an isometry. **reflection, translation, rotation**

b. The transformation preserves the orientation of a figure. **translation, rotation**

c. The transformation is the composite of successive reflections over two intersecting lines. **rotation**

d. The transformation is the composite of successive reflections over two parallel lines. **translation**

e. A specific transformation is defined by a fixed point and a specified angle. **rotation**

f. A specific transformation is defined by a fixed point, a fixed line, or a fixed plane. **reflection**

g. A specific transformation is defined by $(x, y) \rightarrow (x + a, x + b)$, for fixed values of a and b. **translation**

h. The transformation is also called a slide. **translation**

i. The transformation is also called a flip. **reflection**

j. The transformation is also called a turn. **rotation**

2. Determine the order and magnitude of the rotational symmetry for each figure.

a. 

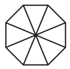
order 3; magnitude 120°

b. 

order 2; magnitude 180°

c. 

order 5; magnitude 72°

d. 

order 8; magnitude 45°

Helping You Remember

3. What is an easy way to remember the order and magnitude of the rotational symmetry of a regular polygon?

Sample answer: The order is the same as the number of sides. To find the magnitude, divide 360 by the number of sides.

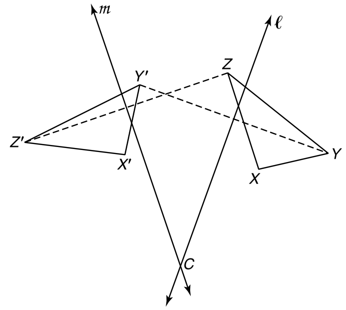
Lesson 9-3

NAME ______________________ DATE ____________ PERIOD _____

9-3 Enrichment

Finding the Center of Rotation

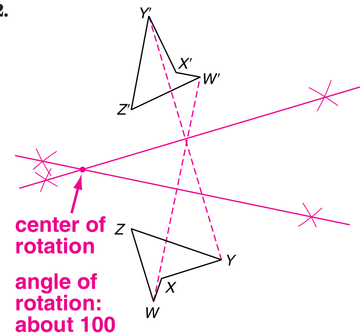
Suppose you are told that $\triangle X'Y'Z'$ is the rotation image of $\triangle XYZ$, but you are not told where the center of rotation is nor the measure of the angle of rotation. Can you find them? Yes, you can. Connect two pairs of corresponding vertices with segments. In the figure, the segments YY' and ZZ' are used. Draw the perpendicular bisectors, ℓ and m, of these segments. The point C where ℓ and m intersect is the center of rotation.



1. How can you find the measure of the angle of rotation in the figure above?

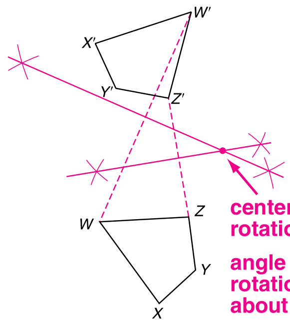
Draw $\angle XCX'$ ($\angle YCY'$ or $\angle ZCZ'$ would also do) and measure it with a protractor.

Locate the center of rotation for the rotation that maps $WXYZ$ onto $W'X'Y'Z'$. Then find the measure of the angle of rotation.

2. 

center of rotation

angle of rotation: about 100

3. 

center of rotation

angle of rotation: about 110

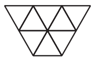
The segments students choose to bisect may vary. See students' work.

NAME ________________________________ DATE __________ PERIOD _____

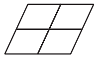
9-4 Study Guide and Intervention

Tessellations

Regular Tessellations A pattern that covers a plane with repeating copies of one or more figures so that there are no overlapping or empty spaces is a **tessellation**. A **regular tessellation** uses only one type of regular polygon. In a tessellation, the sum of the measures of the angles of the polygons surrounding a vertex is 360. If a regular polygon has an interior angle that is a factor of 360, then the polygon will tessellate.

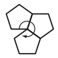


regular tessellation



tessellation

Copies of a regular hexagon can form a tessellation.



Copies of a regular pentagon cannot form a tessellation.

Example **Determine whether a regular 16-gon tessellates the plane. Explain.**

If $m\angle 1$ is the measure of one interior angle of a regular polygon, then a formula for $m\angle 1$ is $m\angle 1 = \frac{180(n-2)}{n}$. Use the formula with $n = 16$.

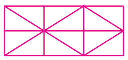
$$m\angle 1 = \frac{180(n-2)}{n}$$
$$= \frac{180(16-2)}{16}$$
$$= 157.5$$

The value 157.5 is not a factor of 360, so the 16-gon will not tessellate.

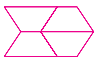
Exercises

Determine whether each polygon tessellates the plane. If so, draw a sample figure.

1. scalene right triangle **yes**



2. isosceles trapezoid **yes**



Determine whether each regular polygon tessellates the plane. Explain.

3. square
Yes; the measure of each interior angle is 90, and 90 is a factor of 360.

4. 20-gon
No; the measure of each interior angle is 162, and 162 is not a factor of 360.

5. septagon
No; the measure of each interior angle is 128.6, and 128.6 is not a factor of 360.

6. 15-gon
No; the measure of each interior angle is 156, and 156 is not a factor of 360.

7. octagon
No; the measure of each interior angle is 135, and 135 is not a factor of 360.

8. pentagon
No; the measure of each interior angle is 108, and 108 is not a factor of 360.

NAME ________________________________ DATE __________ PERIOD _____

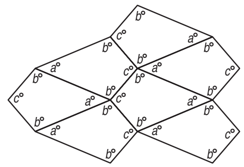
9-4 Study Guide and Intervention *(continued)*

Tessellations

Tessellations with Specific Attributes A tessellation pattern can contain any type of polygon. If the arrangement of shapes and angles at each vertex in the tessellation is the same, the tessellation is **uniform**. A **semi-regular tessellation** is a uniform tessellation that contains two or more regular polygons.

Example **Determine whether a kite will tessellate the plane. If so, describe the tessellation as *uniform*, *regular*, *semi-regular*, or *not uniform*.**

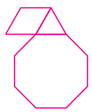
A kite will tessellate the plane. At each vertex the sum of the measures is $a + b + b + c$, which is 360. The tessellation is uniform.



Exercises

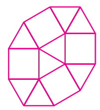
Determine whether a semi-regular tessellation can be created from each set of figures. If so, sketch the tessellation. Assume that each figure has a side length of 1 unit.

1. rhombus, equilateral triangle, and octagon



no

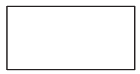
2. square and equilateral triangle



yes

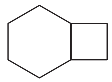
Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

3. rectangle



yes, uniform

4. hexagon and square



no

NAME ________________________ DATE __________ PERIOD _____

9-4 Skills Practice

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 15-gon
no, interior angle = 156

2. 18-gon
no, interior angle = 160

3. square
yes, interior angle = 90

4. 20-gon
no, interior angle = 162

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

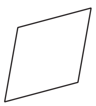
5. regular pentagons and equilateral triangles
no

6. regular dodecagons and equilateral triangles
yes

7. regular octagons and equilateral triangles
no

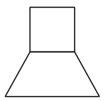
Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

8. rhombus



yes; uniform

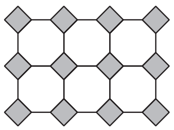
9. isosceles trapezoid and square



no

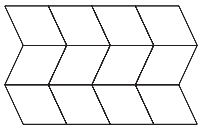
Determine whether each pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.

10.



yes; uniform, semi-regular

11.



yes; uniform

NAME ________________________ DATE __________ PERIOD _____

9-4 Practice (Average)

Tessellations

Determine whether each regular polygon tessellates the plane. Explain.

1. 22-gon
no, interior angle ≈ 163.6

2. 40-gon
no, interior angle = 171

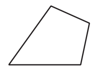
Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

3. regular pentagons and regular decagons
no (will not tessellate the plane)

4. regular dodecagons, regular hexagons, and squares
yes

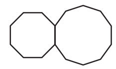
Determine whether each polygon tessellates the plane. If so, describe the tessellation as *uniform*, *not uniform*, *regular*, or *semi-regular*.

5. kite



yes; uniform

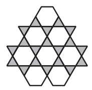
6. octagon and decagon



no

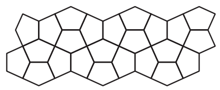
Determine whether each pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.

7.



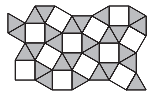
yes; uniform, semi-regular

8.



yes, not uniform

FLOOR TILES For Exercises 9 and 10, use the following information.
Mr. Martinez chose the pattern of tile shown to retile his kitchen floor.



9. Determine whether the pattern is a tessellation. Explain
Yes; since there are 2 squares and 3 triangles at each vertex, the sum of the angles at the vertices is 360°.

10. Is the pattern uniform, regular, or semi-regular?
uniform and semi-regular

NAME ______________________ DATE ____________ PERIOD _____

9-4 Reading to Learn Mathematics

Tesselations

Pre-Activity How are tessellations used in art?

Read the introduction to Lesson 9-4 at the top of page 483 in your textbook.

- In the pattern shown in the picture in your textbook, how many small equilateral triangles make up one regular hexagon? **6**
- In this pattern, how many fish make up one equilateral triangle? **$1\frac{1}{2}$**

Reading the Lesson

1. Underline the correct word, phrase, or number to form a true statement.

a. A tessellation is a pattern that covers a plane with the same figure or set of figures so that there are no (congruent angles/overlapping or empty spaces/right angles).

b. A tessellation that uses only one type of regular polygon is called a (uniform/regular/semi-regular) tessellation.

c. The sum of the measures of the angles at any vertex in any tessellation is (90/180/360).

d. A tessellation that contains the same arrangement of shapes and angles at every vertex is called a (uniform/regular/semi-regular) tessellation.

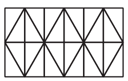
e. In a regular tessellation made up of hexagons, there are (3/4/6) hexagons meeting at each vertex, and the measure of each of the angles at any vertex is (60/90/120).

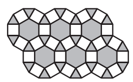
f. A uniform tessellation formed using two or more regular polygons is called a (rotational/regular/semi-regular) tessellation.

g. In a regular tessellation made up of triangles, there are (3/4/6) triangles meeting at each vertex, and the measure of each of the angles at any vertex is (30/60/120).

h. If a regular tessellation is made up of quadrilaterals, all of the quadrilaterals must be congruent (rectangles/parallelograms/squares/trapezoids).

2. Write all of the following words that describe each tessellation: *uniform*, *non-uniform*, *regular*, *semi-regular*.

a.  **non-uniform**

b.  **uniform, semi-regular**

Helping You Remember

3. Often the everyday meanings of a word can help you to remember its mathematical meaning. Look up *uniform* in your dictionary. How can its everyday meanings help you to remember the meaning of a *uniform* tessellation? **Sample answer: Three similar meanings of *uniform* are *unvarying, consistent, and identical*. In a uniform tessellation, the arrangement of shapes and angles at each vertex is the same. Each of the three everyday meanings describes this situation.**

Lesson 9-4

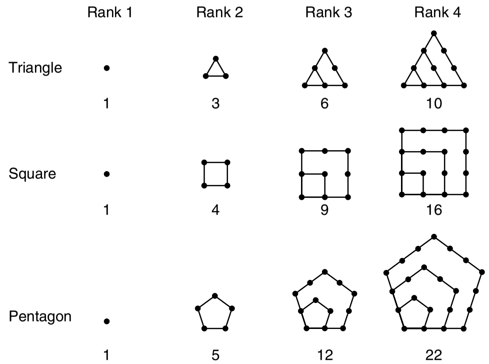
NAME ______________________ DATE ____________ PERIOD _____

9-4 Enrichment

Polygonal Numbers

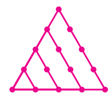
Certain numbers related to regular polygons are called **polygonal numbers**. The chart shows several triangular, square, and pentagonal numbers. The **rank** of a polygon number is the number of dots on each "side" of the outer polygon. For example, the pentagonal number 22 has a rank of 4.

Polygonal numbers can be described with formulas. For example, a triangular number T of rank r can be described by $T = \frac{r(r+1)}{2}$.

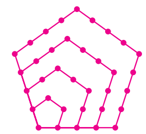


Answer each question.

1. Draw a diagram to find the triangular number of rank 5. **15**



2. Draw a diagram to find the pentagonal number of rank 5. **35**



3. Write a formula for a square number S of rank r.
$S = r^2$

4. Write a formula for a pentagonal number P of rank r.
$P = \frac{r(3r-1)}{2}$

5. What is the rank of the pentagonal number 70? **7**

6. List the hexagonal numbers for ranks 1 to 5. (Hint: Draw a diagram.)
1, 6, 15, 28, 45

Answers

NAME ______________________ DATE ________ PERIOD ____

9-5 Study Guide and Intervention

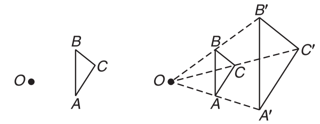
Dilations

Classify Dilations A **dilation** is a transformation in which the image may be a different size than the preimage. A dilation requires a center point and a scale factor, r.

> Let r represent the scale factor of a dilation.
> If $|r| > 1$, then the dilation is an enlargement.
> If $|r| = 1$, then the dilation is a congruence transformation.
> If $0 < |r| < 1$, then the dilation is a reduction.

Example **Draw the dilation image of $\triangle ABC$ with center O and $r = 2$.**

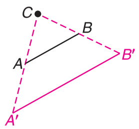
Draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$. Label points A', B', and C' so that $OA' = 2(OA)$, $OB' = 2(OB)$, and $OC' = 2(OC)$. $\triangle A'B'C'$ is a dilation of $\triangle ABC$.



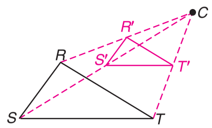
Exercises

Draw the dilation image of each figure with center C and the given scale factor. Describe each transformation as an *enlargement*, *congruence*, or *reduction*.

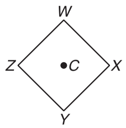
1. $r = 2$ enlargement



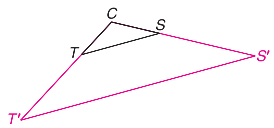
2. $r = \frac{1}{2}$ reduction



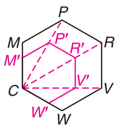
3. $r = 1$ congruence



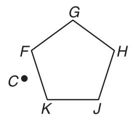
4. $r = 3$ enlargement



5. $r = \frac{2}{3}$ reduction



6. $r = 1$ congruence



Lesson 9-5

NAME ______________________ DATE ________ PERIOD ____

9-5 Study Guide and Intervention *(continued)*

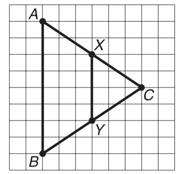
Dilations

Identify the Scale Factor If you know corresponding measurements for a preimage and its dilation image, you can find the scale factor.

Example **Determine the scale factor for the dilation of $\overline{XY}$ to $\overline{AB}$. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.**

$$\text{scale factor} = \frac{\text{image length}}{\text{preimage length}} = \frac{8 \text{ units}}{4 \text{ units}} = 2$$

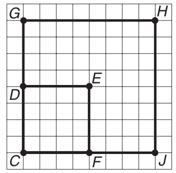
The scale factor is greater than 1, so the dilation is an enlargement.



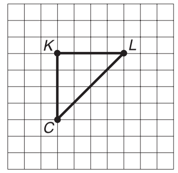
Exercises

Determine the scale factor for each dilation with center C. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

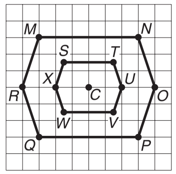
1. $CGHJ$ is a dilation image of $CDEF$. 2; enlargement



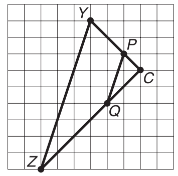
2. $\triangle CKL$ is a dilation image of $\triangle CKL$. 1; congruence



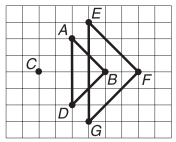
3. $STUVWX$ is a dilation image of $MNOPQR$. $\frac{1}{2}$; reduction



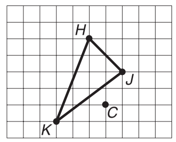
4. $\triangle CPQ$ is a dilation image of $\triangle CYZ$. $\frac{1}{3}$; reduction



5. $\triangle EFG$ is a dilation image of $\triangle ABC$. 1.5; enlargement



6. $\triangle HJK$ is a dilation image of $\triangle HJK$. 1; congruence

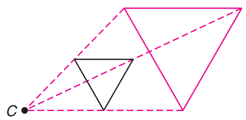


NAME ______________________ DATE ____________ PERIOD _____

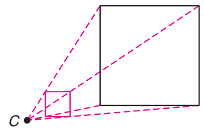
9-5 Skills Practice
Dilations

Draw the dilation image of each figure with center *C* and the given scale factor.

1. $r = 2$



2. $r = \frac{1}{4}$



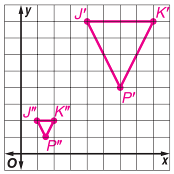
Find the measure of the dilation image $\overline{M'N'}$ or of the preimage $\overline{MN}$ using the given scale factor.

3. $MN = 3, r = 3$
M'N' = 9

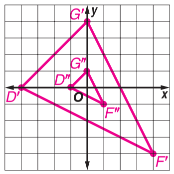
4. $M'N' = 7, r = 21$
***MN* = $\frac{1}{3}$**

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.**

5. $J(2, 4), K(4, 4), P(3, 2)$

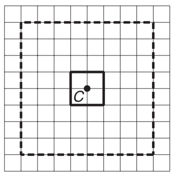


6. $D(-2, 0), G(0, 2), F(2, -2)$



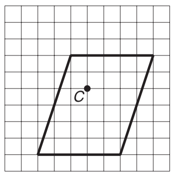
Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dashed figure is the dilation image.

7.



4; enlargement

8.



1; congruence transformation

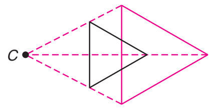
Lesson 9-5

NAME ______________________ DATE ____________ PERIOD _____

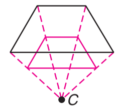
9-5 Practice (Average)
Dilations

Draw the dilation image of each figure with center *C* and the given scale factor.

1. $r = \frac{3}{2}$



2. $r = \frac{2}{3}$



Find the measure of the dilation image $\overline{A'T'}$ or of the preimage $\overline{AT}$ using the given scale factor.

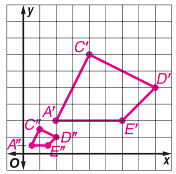
3. $AT = 15, r = \frac{3}{5}$
A'T' = 9

4. $AT = 30, r = -\frac{1}{6}$
A'T' = 5

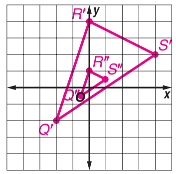
5. $A'T' = 12, r = \frac{4}{3}$
AT = 9

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.**

6. $A(1, 1), C(2, 3), D(4, 2), E(3, 1)$

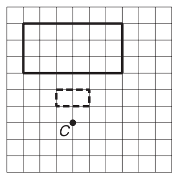


7. $Q(-1, -1), R(0, 2), S(2, 1)$



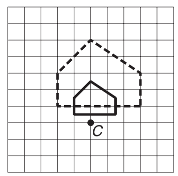
Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*. The dotted figure is the dilation image.

8.



$\frac{1}{3}$; reduction

9.



2; enlargement

10. PHOTOGRAPHY Estebe enlarged a 4-inch by 6-inch photograph by a factor of $\frac{5}{2}$. What are the new dimensions of the photograph? **10 in. by 15 in.**

Answers

NAME ______ DATE ______ PERIOD ______

9-5 Reading to Learn Mathematics

Dilations

Pre-Activity **How do you use dilations when you use a computer?**

Read the introduction to Lesson 9-5 at the top of page 490 in your textbook.

In addition to the example given in your textbook, give two everyday examples of scaling an object, one that makes the object larger and another that makes it smaller. **Sample answer: larger: enlarging a photograph; smaller: making a scale model of a building**

Reading the Lesson

1. Each of the values of r given below represents the scale factor for a dilation. In each case, determine whether the dilation is an *enlargement*, a *reduction*, or a *congruence transformation*.

a. $r = 3$ **enlargement**
b. $r = 0.5$ **reduction**
c. $r = -0.75$ **reduction**
d. $r = -1$ **congruence transformation**
e. $r = \frac{2}{3}$ **reduction**
f. $r = -\frac{3}{2}$ **enlargement**
g. $r = -1.01$ **enlargement**
h. $r = 0.999$ **reduction**

2. Determine whether each sentence is *always*, *sometimes*, or *never* true. If the sentence is not always true, explain why. **For explanations, sample answers are given.**

a. A dilation requires a center point and a scale factor. **always**

b. A dilation changes the size of a figure. **Sometimes; if the dilation is a congruence transformation, the size of the figure is unchanged.**

c. A dilation changes the shape of a figure. **Never; all dilations produce similar figures, and similar figures have the same shape.**

d. The image of a figure under a dilation lies on the opposite side of the center from the preimage. **Sometimes; this is only true when the scale factor is negative.**

e. A similarity transformation is a congruence transformation. **Sometimes; this is true only when the scale factor is 1 or −1.**

f. The center of a dilation is its own image. **always**

g. A dilation is an isometry. **Sometimes; this is true only when the dilation is a congruence transformation.**

h. The scale factor for a dilation is a positive number. **Sometimes; the scale factor can be any positive or negative number.**

i. Dilations produce similar figures. **always**

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that your classmate Lydia is having trouble understanding the relationship between *similarity transformations* and *congruence transformations*. How can you explain this to her? **Sample answer: A congruence transformation preserves the size and shape of a figure, that is, the image is congruent to the preimage. A similarity transformation preserves the shape of a figure, that is, the image is similar to the preimage. A similarity transformation is a congruence transformation if the scale factor is 1 or −1.**

Lesson 9-5

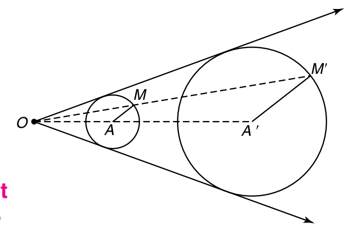
NAME ______ DATE ______ PERIOD ______

9-5 Enrichment

Similar Circles

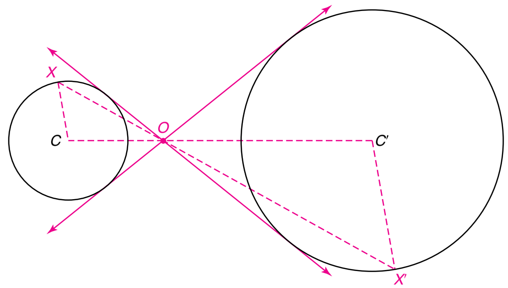
You may be surprised to learn that two noncongruent circles that lie in the same plane and have no common interior points can be mapped one onto the other by more than one dilation.

1. Here is diagram that suggests one way to map a smaller circle onto a larger one using a dilation. The circles are given. The lines suggest how to find the center for the dilation. Describe how the center is found. Use segments in the diagram to name the scale factor.



Draw the two common external tangents. The point where they intersect is the center of a dilation that maps ⊙A onto ⊙A′; scale factor = $\frac{A'M'}{AM}$.

2. Here is another pair of noncongruent circles with no common interior point. From Exercise 1, you know you can locate a point off to the left of the smaller circle that is the center for a dilation mapping ⊙C onto ⊙C'. Find another center for another dilation that maps ⊙C onto ⊙C'. Mark and label segments to name the scale factor.



Find the intersection of the two common internal tangents; scale factor: $-\frac{C'X'}{CX}$.

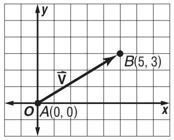
NAME ______________________ DATE ________ PERIOD ____

9-6 Study Guide and Intervention

Vectors

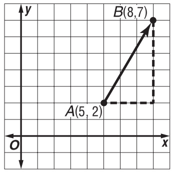
Magnitude and Direction A vector is a directed segment representing a quantity that has both **magnitude**, or length, and **direction**. For example, the speed and direction of an airplane can be represented by a vector. In symbols, a vector is written as $\overrightarrow{AB}$, where A is the initial point and B is the endpoint, or as $\vec{v}$.

A vector in **standard position** has its initial point at (0, 0) and can be represented by the ordered pair for point B. The vector at the right can be expressed as $\vec{v} = \langle 5, 3 \rangle$.



You can use the Distance Formula to find the magnitude $|\overrightarrow{AB}|$ of a vector. You can describe the direction of a vector by measuring the angle that the vector forms with the positive x-axis or with any other horizontal line.

Example **Find the magnitude and direction of $\overrightarrow{AB}$ for $A(5, 2)$ and $B(8, 7)$.**



Find the magnitude.

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 5)^2 + (7 - 2)^2}$

$= \sqrt{34}$ or about 5.8 units

To find the direction, use the tangent ratio.

$\tan A = \frac{5}{3}$ The tangent ratio is opposite over adjacent.

$m\angle A \approx 59.0$ Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is 59°.

Exercises

Find the magnitude and direction of $\overrightarrow{AB}$ for the given coordinates. Round to the nearest tenth.

1. $A(3, 1), B(-2, 3)$ **5.4; 158.2°**
2. $A(0, 0), B(-2, 1)$ **2.2; 153.4°**
3. $A(0, 1), B(3, 5)$ **5; 53.1°**
4. $A(-2, 2), B(3, 1)$ **5.1; 348.7°**
5. $A(3, 4), B(0, 0)$ **5; 233.1°**
6. $A(4, 2), B(0, 3)$ **4.1; 166.0°**

Lesson 9-6

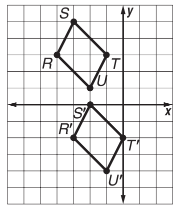
NAME ______________________ DATE ________ PERIOD ____

9-6 Study Guide and Intervention (continued)

Vectors

Translations with Vectors Recall that the transformation $(a, b) \rightarrow (a + 2, b - 3)$ represents a translation right 2 units and down 3 units. The vector $\langle 2, -3 \rangle$ is another way to describe that translation. Also, two vectors can be added: $\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$. The sum of two vectors is called the **resultant**.

Example **Graph the image of parallelogram $RSTU$ under the translation by the vectors $\vec{m} = \langle 3, -1 \rangle$ and $\vec{n} = \langle -2, -4 \rangle$.**



Find the sum of the vectors.

$\vec{m} + \vec{n} = \langle 3, -1 \rangle + \langle -2, -4 \rangle$

$= \langle 3 - 2, -1 - 4 \rangle$

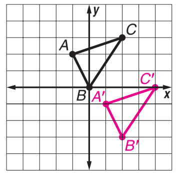
$= \langle 1, -5 \rangle$

Translate each vertex of parallelogram $RSTU$ right 1 unit and down 5 units.

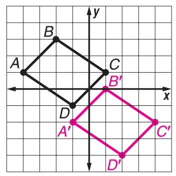
Exercises

Graph the image of each figure under a translation by the given vector(s).

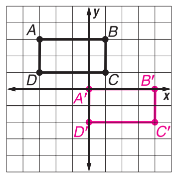
1. $\triangle ABC$ with vertices $A(-1, 2), B(0, 0)$, and $C(2, 3)$; $\vec{m} = \langle 2, -3 \rangle$



2. $ABCD$ with vertices $A(-4, 1), B(-2, 3)$, $C(1, 1)$, and $D(-1, -1)$; $\vec{n} = \langle 3\ -3 \rangle$



3. $ABCD$ with vertices $A(-3, 3), B(1, 3), C(1, 1)$, and $D(-3, 1)$; the sum of $\vec{p} = \langle -2, 1 \rangle$ and $\vec{q} = \langle 5, -4 \rangle$



Given $\vec{m} = \langle 1, -2 \rangle$ and $\vec{n} = \langle -3, -4 \rangle$, represent each of the following as a single vector.

4. $\vec{m} + \vec{n}$ **⟨−2, −6⟩**
5. $\vec{n} - \vec{m}$ **⟨−4, −2⟩**

Answers

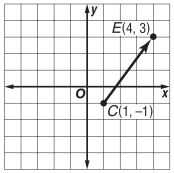
NAME ______________________ DATE __________ PERIOD _____

9-6 Skills Practice

Vectors

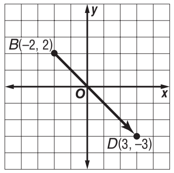
Write the component form of each vector.

1.



⟨3, 4⟩

2.



⟨5, –5⟩

Find the magnitude and direction of $\overrightarrow{RS}$ for the given coordinates. Round to the nearest tenth.

3. $R(2, -3)$, $S(4, 9)$
$2\sqrt{37} \approx 12.2, 80.5°$

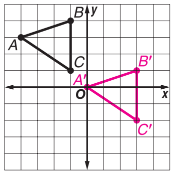
4. $R(0, 2)$, $S(3, 12)$
$\sqrt{109} \approx 10.4, 73.3°$

5. $R(5, 4)$, $S(-3, 1)$
$\sqrt{73} \approx 8.5, 200.6°$

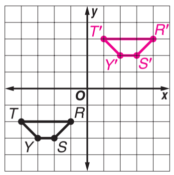
6. $R(1, 5)$, $S(-4, -6)$
$\sqrt{146} \approx 12.1, 245.6°$

Graph the image of each figure under a translation by the given vector(s).

7. $\triangle ABC$ with vertices $A(-4, 3)$, $B(-1, 4)$, $C(-1, 1)$; $\vec{t} = \langle 4, -3 \rangle$



8. trapezoid with vertices $T(-4, -2)$, $R(-1, -2)$, $S(-2, -3)$, $Y(-3, -3)$; $\vec{a} = \langle 3, 1 \rangle$ and $\vec{b} = \langle 2, 4 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

9. $\vec{y} = \langle 7, 0 \rangle$, $\vec{z} = \langle 0, 6 \rangle$
$\sqrt{85} \approx 9.2, 40.6°$

10. $\vec{b} = \langle 3, 2 \rangle$, $\vec{c} = \langle -2, 3 \rangle$
$\sqrt{26} \approx 5.1, 78.7°$

Lesson 9-6

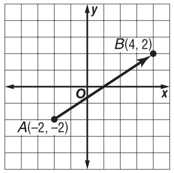
NAME ______________________ DATE __________ PERIOD _____

9-6 Practice (Average)

Vectors

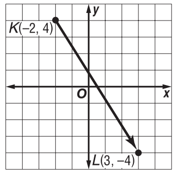
Write the component form of each vector.

1.



⟨6, 4⟩

2.



⟨5, –8⟩

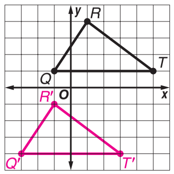
Find the magnitude and direction of $\overrightarrow{FG}$ for the given coordinates. Round to the nearest tenth.

3. $F(-8, -5)$, $G(-2, 7)$
$6\sqrt{5} \approx 13.4, 63.4°$

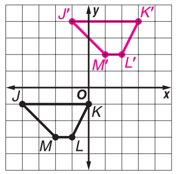
4. $F(-4, 1)$, $G(5, -6)$
$\sqrt{130} \approx 11.4, 322.1°$

Graph the image of each figure under a translation by the given vector(s).

5. $\triangle QRT$ with vertices $Q(-1, 1)$, $R(1, 4)$, $T(5, 1)$; $\vec{s} = \langle -2, -5 \rangle$



6. trapezoid with vertices $J(-4, -1)$, $K(0, -1)$, $L(-1, -3)$, $M(-2, -3)$; $\vec{c} = \langle 5, 4 \rangle$ and $\vec{d} = \langle -2, 1 \rangle$



Find the magnitude and direction of each resultant for the given vectors.

7. $\vec{a} = \langle -6, 4 \rangle$, $\vec{b} = \langle 4, 6 \rangle$
$2\sqrt{26} \approx 10.2, 101.3°$

8. $\vec{e} = \langle -4, -5 \rangle$, $\vec{f} = \langle -1, 3 \rangle$
$\sqrt{29} \approx 5.4, 201.8°$

AVIATION For Exercises 9 and 10, use the following information.

A jet begins a flight along a path due north at 300 miles per hour. A wind is blowing due west at 30 miles per hour.

9. Find the resultant velocity of the plane. **about 301.5 mph**

10. Find the resultant direction of the plane. **about 5.7° west of due north**

NAME ______________ DATE ________ PERIOD ____

9-6 Reading to Learn Mathematics

Vectors

Pre-Activity **How do vectors help a pilot plan a flight?**

Read the introduction to Lesson 9-6 at the top of page 498 in your textbook.

Why do pilots often head their planes in a slightly different direction from their destination? **Sample answer: to compensate for the effect of the wind so that the result will be that the plane will actually fly in the correct direction**

Reading the Lesson

1. Supply the missing words or phrases to complete the following sentences.

a. A **vector** is a directed segment representing a quantity that has both magnitude and direction.

b. The length of a vector is called its **magnitude**.

c. Two vectors are parallel if and only if they have the same or **opposite** direction.

d. A vector is in **standard position** if it is drawn with initial point at the origin.

e. Two vectors are equal if and only if they have the same **magnitude** and the same **direction**.

f. The sum of two vectors is called the **resultant**.

g. A vector is written in **component form** if it is expressed as an ordered pair.

h. The process of multiplying a vector by a constant is called **scalar multiplication**.

2. Write each vector described below in component form.

a. a vector in standard position with endpoint (a, b) **$\langle a, b\rangle$**

b. a vector with initial point (a, b) and endpoint (c, d) **$\langle c - a, d - b\rangle$**

c. a vector in standard position with endpoint $(-3, 5)$ **$\langle -3, 5\rangle$**

d. a vector with initial point $(2, -3)$ and endpoint $(6, -8)$ **$\langle 4, -5\rangle$**

e. $\vec{a} + \vec{b}$ if $\vec{a} = \langle -3, 5\rangle$ and $\vec{b} = \langle 6, -4\rangle$ **$\langle 3, 1\rangle$**

f. $5\vec{u}$ if $\vec{u} = \langle 8, -6\rangle$ **$\langle 40, -30\rangle$**

g. $-\frac{1}{3}\vec{v}$ if $\vec{v} = \langle -15, 24\rangle$ **$\langle 5, -8\rangle$**

h. $0.5\vec{u} + 1.5\vec{v}$ if $\vec{u} = \langle 10, -10\rangle$ and $\vec{v} = \langle -8, 6\rangle$ **$\langle -7, 4\rangle$**

Helping You Remember

3. A good way to remember a new mathematical term is to relate it to a term you already know. You learned about *scale factors* when you studied similarity and dilations. How is the idea of a *scalar* related to *scale factors*? **Sample answer: A *scalar* is the term used for a constant (a specific real number) when working with vectors. A vector has both magnitude and direction, while a scalar is just a magnitude. Multiplying a vector by a positive scalar changes the magnitude of the vector, but not the direction, so it represents a change in scale.**

NAME ______________ DATE ________ PERIOD ____

9-6 Enrichment

Reading Mathematics

Many quantities in nature can be thought of as vectors. The science of physics involves many vector quantities. In reading about applications of mathematics, ask yourself whether the quantities involve only magnitude or both magnitude and direction. The first kind of quantity is called **scalar**. The second kind is a **vector**.

Classify each of the following. Write *scalar* or *vector*.

1. the mass of a book **scalar**

2. a car traveling north at 55 mph **vector**

3. a balloon rising 24 feet per minute **vector**

4. the size of a shoe **scalar**

5. a room temperature of 22 degrees Celsius **scalar**

6. a west wind of 15 mph **vector**

7. the batting average of a baseball player **scalar**

8. a car traveling 60 mph **vector**

9. a rock falling at 10 mph **vector**

10. your age **scalar**

11. the force of Earth's gravity acting on a moving satellite **vector**

12. the area of a record rotating on a turntable **scalar**

13. the length of a vector in the coordinate plane **scalar**

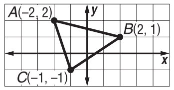
NAME ______________ DATE ________ PERIOD _____

9-7 Study Guide and Intervention

Transformations with Matrices

Translations and Dilations A **vector** can be represented by the ordered pair $\langle x, y\rangle$ or by the **column matrix** $\begin{bmatrix} x \\ y \end{bmatrix}$. When the ordered pairs for all the vertices of a polygon are placed together, the resulting matrix is called the **vertex matrix** for the polygon.

For $\triangle ABC$ with $A(-2, 2)$, $B(2, 1)$, and $C(-1, -1)$, the vertex matrix for the triangle is $\begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.



Example 1 **For $\triangle ABC$ above, use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ under the translation $(x, y) \rightarrow (x + 3, y - 1)$.**

To translate the figure 3 units to the right, add 3 to each x-coordinate. To translate the figure 1 unit down, add -1 to each y-coordinate.

Vertex Matrix of $\triangle ABC$ + Translation Matrix = Vertex Matrix of $\triangle A'B'C'$

$$\begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

The coordinates are $A'(1, 1)$, $B'(5, 0)$, and $C'(2, -2)$.

Example 2 **For $\triangle ABC$ above, use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ for a dilation centered at the origin with scale factor 3.**

Scale Factor · Vertex Matrix of $\triangle ABC$ = Vertex Matrix of $\triangle A'B'C'$

$$3 \cdot \begin{bmatrix} -2 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 6 & -3 \\ 6 & 3 & -3 \end{bmatrix}$$

The coordinates are $A'(-6, 6)$, $B'(6, 3)$, and $C'(-3, -3)$.

Exercises

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations or dilations.

1. $\triangle ABC$ with $A(3, 1)$, $B(-2, 4)$, $C(-2, -1)$; $(x, y) \rightarrow (x - 1, y + 2)$
   $A'(2, 3)$, $B'(-3, 6)$, $C'(-3, 1)$
2. parallelogram $RSTU$ with $R(-4, -2)$, $S(-3, 1)$, $T(3, 4)$, $U(2, 1)$; $(x, y) \rightarrow (x - 4, y - 3)$
   $R'(-8, -5)$, $S'(-7, -2)$, $T'(-1, 1)$, $U'(-2, -2)$
3. rectangle $PQRS$ with $P(4, 0)$, $Q(3, -3)$, $R(-3, -1)$, $S(-2, 2)$; $(x, y) \rightarrow (x - 2, y + 1)$
   $P'(2, 1)$, $Q'(1, -2)$, $R'(-5, 0)$, $S'(-4, 3)$
4. $\triangle ABC$ with $A(-2, -1)$, $B(-2, -3)$, $C(2, -1)$; dilation centered at the origin with scale factor 2
   $A'(-4, -2)$, $B'(-4, -6)$, $C'(4, -2)$
5. parallelogram $RSTU$ with $R(4, -2)$, $S(-4, -1)$, $T(-2, 3)$, $U(6, 2)$; dilation centered at the origin with scale factor 1.5
   $R'(6, -3)$, $S'(-6, -1.5)$, $T'(-3, 4.5)$, $U'(9, 3)$

Lesson 9-7

NAME ______________ DATE ________ PERIOD _____

9-7 Study Guide and Intervention *(continued)*

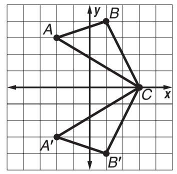
Transformations with Matrices

Reflections and Rotations When you reflect an image, one way to find the coordinates of the reflected vertices is to multiply the vertex matrix of the object by a **reflection matrix**. To perform more than one reflection, multiply by one reflection matrix to find the first image. Then multiply by the second matrix to find the final image. The matrices for reflections in the axes, the origin, and the line $y = x$ are shown below.

For a reflection in the:	x-axis	y-axis	origin	line $y = x$
Multiply the vertex matrix by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Example **$\triangle ABC$ has vertices $A(-2, 3)$, $B(1, 4)$, and $C(3, 0)$. Use a matrix to find the coordinates of the vertices of the image of $\triangle ABC$ after a reflection in the x-axis.**

To reflect in the x-axis, multiply the vertex matrix of $\triangle ABC$ by the reflection matrix for the x-axis.



Reflection Matrix for x-axis · Vertex Matrix of $\triangle ABC$ = Vertex Matrix of $\triangle A'B'C'$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 3 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 3 \\ -3 & -4 & 0 \end{bmatrix}$$

Exercises

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

1. $\triangle ABC$ with $A(-3, 2)$, $B(-1, 3)$, $C(1, 0)$; reflection in the x-axis
   $A'(-3, -2)$, $B'(-1, -3)$, $C'(1, 0)$
2. $\triangle XYZ$ with $X(2, -1)$, $Y(4, -3)$, $Z(-2, 1)$; reflection in the y-axis
   $X'(-2, -1)$, $Y'(-4, -3)$, $Z'(2, 1)$
3. $\triangle ABC$ with $A(3, 4)$, $B(-1, 0)$, $C(-2, 4)$; reflection in the origin
   $A'(-3, -4)$, $B'(1, 0)$, $C'(2, -4)$
4. parallelogram $RSTU$ with $R(-3, 2)$, $S(3, 2)$, $T(5, -1)$, $U(-1, -1)$; reflection in the line $y = x$
   $R'(2, -3)$, $S'(2, 3)$, $T'(-1, 5)$, $U'(-1, -1)$
5. $\triangle ABC$ with $A(2, 3)$, $B(-1, 2)$, $C(1, -1)$; reflection in the origin, then reflection in the line $y = x$
   $A'(-3, -2)$, $B'(-2, 1)$, $C'(1, -1)$
6. parallelogram $RSTU$ with $R(0, 2)$, $S(4, 2)$, $T(3, -2)$, $U(-1, -2)$; reflection in the x-axis, then reflection in the y-axis
   $R'(0, -2)$, $S'(-4, -2)$, $T'(-3, 2)$, $U'(1, 2)$

NAME ______________________ DATE ________ PERIOD _____

9-7 Skills Practice

Transformations with Matrices

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations.

1. $\triangle STU$ with $S(6, 4)$, $T(9, 7)$, and $U(14, 2)$; $(x, y) \to (x - 4, y + 3)$
S′(2, 7), T′(5, 10), U′(10, 5)

2. $\triangle GHI$ with $G(-5, 0)$, $H(-3, 6)$, and $I(-2, 1)$; $(x, y) \to (x + 2, y + 6)$
G′(−3, 6), H′(−1, 12), I′(0, 7)

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.

3. $\triangle DEF$ with $D(2, 1)$, $E(5, 4)$, and $F(7, 2)$; $r = 4$
D′(8, 4), E′(20, 16), F′(28, 8)

4. quadrilateral $WXYZ$ with $W(-9, 6)$, $X(-6, 3)$, $Y(3, 12)$, and $Z(-6, 15)$; $r = \frac{1}{3}$
W′(−3, 2), X′(−2, 1), Y′(1, 4), Z′(−2, 5)

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

5. $\triangle MNO$ with $M(-5, 1)$, $N(-2, 3)$, and $O(2, 0)$; y-axis
M′(5, 1), N′(2, 3), O′(−2, 0)

6. quadrilateral $ABCD$ with $A(3, 1)$, $B(6, -2)$, $C(5, -5)$, and $D(1, -6)$; x-axis
A′(3, −1), B′(6, 2), C′(5, 5), D′(1, 6)

Use a matrix to find the coordinates of the vertices of the image of each figure under the given rotation.

7. $\triangle RST$ with $R(-2, -2)$, $S(-3, 3)$, and $T(2, 2)$; 90°counterclockwise
R′(2, −2), S′(−3, −3), T′(−2, 2)

8. ▱$LMNP$ with $L(3, 4)$, $M(7, 4)$, $N(9, -3)$, and $P(5, -3)$; 180° counterclockwise
L′(−3, −4), M′(−7, −4), N′(−9, 3), P′(−5, 3)

Lesson 9-7

NAME ______________________ DATE ________ PERIOD _____

9-7 Practice (Average)

Transformations with Matrices

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translations.

1. $\triangle KLM$ with $K(-7, -3)$, $L(4, 9)$, and $M(9, -6)$; $(x, y) \to (x - 7, y + 2)$
K′(−14, −1), L′(−3, 11), M′(2, −4)

2. ▱$ABCD$ with $A(-4, 3)$, $B(-2, 8)$, $C(3, 10)$, and $D(1, 5)$; $(x, y) \to (x + 3, y - 9)$
A′(−1, −6), B′(1, −1), C′(6, 1), D′(4, −4)

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.

3. quadrilateral $HIJK$ with $H(-2, 3)$, $I(2, 6)$, $J(8, 3)$, and $K(3, -4)$; $r = -\frac{1}{3}$
$H'\left(\frac{2}{3}, -1\right)$, $I'\left(-\frac{2}{3}, -2\right)$, $J'\left(-\frac{8}{3}, -1\right)$, $K'\left(-1, \frac{4}{3}\right)$

4. pentagon $DEFGH$ with $D(-8, -4)$, $E(-8, 2)$, $F(2, 6)$, $G(8, 0)$, and $H(4, -6)$; $r = \frac{5}{4}$
$D'(-10, -5)$, $E'\left(-10, \frac{5}{2}\right)$, $F'\left(\frac{5}{2}, \frac{15}{2}\right)$, $G'(10, 0)$, $H'\left(5, -\frac{15}{2}\right)$

Use a matrix to find the coordinates of the vertices of the image of each figure under the given reflection.

5. $\triangle QRS$ with $Q(-5, -4)$, $R(-1, -1)$, and $S(2, -6)$; x-axis
Q′(−5, 4), R′(−1, 1), S′(2, 6)

6. quadrilateral $VXYZ$ with $V(-4, -2)$, $X(-3, 4)$, $Y(2, 1)$, and $Z(4, -3)$; $y = x$
V′(−2, −4), X′(4, −3), Y′(1, 2), Z′(−3, 4)

Use a matrix to find the coordinates of the vertices of the image of each figure under the given rotation.

7. ▱$EFGH$ with $E(-5\ -4)$, $F(-3, -1)$, $G(5, -1)$, and $H(3, -4)$; 90° counterclockwise
E′(4, −5), F′(1, −3), G′(1, 5), H′(4, 3)

8. quadrilateral $PSTU$ with $P(-3, 5)$, $S(2, 6)$, $T(8, 1)$, and $U(-6, -4)$; 270° counterclockwise
P′(5, 3), S′(6, −2), T′(1, −8), U′(−4, 6)

9. FORESTRY A research botanist mapped a section of forested land on a coordinate grid to keep track of endangered plants in the region. The vertices of the map are $A(-2, 6)$, $B(9, 8)$, $C(14, 4)$, and $D(1, -1)$. After a month, the botanist has decided to decrease the research area to $\frac{3}{4}$ of its original size. If the center for the reduction is $O(0, 0)$,what are the coordinates of the new research area?
$A\left(-\frac{3}{2}, \frac{9}{2}\right)$, $B\left(\frac{27}{4}, 6\right)$, $C\left(\frac{21}{2}, 3\right)$, $D\left(\frac{3}{4}, -\frac{3}{4}\right)$

Answers

NAME ______________ DATE ________ PERIOD ____

9-7 Reading to Learn Mathematics

Transformations with Matrices

Pre-Activity **How can matrices be used to make movies?**

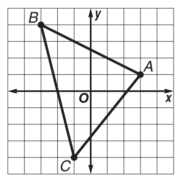
Read the introduction to Lesson 9-7 at the top of page 506 in your textbook.

- What kind of transformation should be used to move a polygon? **reflection, translation, or rotation**
- What kind of transformation should be used to resize a polygon? **dilation**

Reading the Lesson

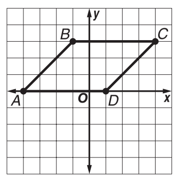
1. Write a vertex matrix for each figure.

a. $\triangle ABC$



$\begin{bmatrix} 3 & -3 & -1 \\ 1 & 4 & -4 \end{bmatrix}$

b. parallelogram *ABCD*



$\begin{bmatrix} -4 & -1 & 4 & 1 \\ 0 & 3 & 3 & 0 \end{bmatrix}$

2. Match each transformation from the first column with the corresponding matrix from the second or third column. In each case, the vertex matrix for the preimage of a figure is multiplied on the left by one of the matrices below to obtain the image of the figure. All rotations listed are counterclockwise through the origin. (Some matrices may be used more than once or not at all.)

a. reflection over the *y*-axis **vi**
b. 90° rotation **viii**
c. reflection over the line $y = x$ **iii**
d. 270° rotation **v**
e. reflection over the origin **ii**
f. 180° rotation **ii**
g. reflection over the *x*-axis **vii**
h. 360° rotation **i**

i. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **v.** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

ii. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **vi.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **vii.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

iv. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **viii.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Helping You Remember

3. How can you remember or quickly figure out the matrices for the transformations in Exercise 2? **Visualize or sketch how the "unit points" (1, 0) on the *x*-axis and (0,1) on the *y*-axis are moved by the transformation. Write the ordered pair for the image points in a 2 × 2 matrix, with the coordinates of the image of the *x*-axis unit point in the first column and the coordinates of the image of the *y*-axis unit point in the second column.**

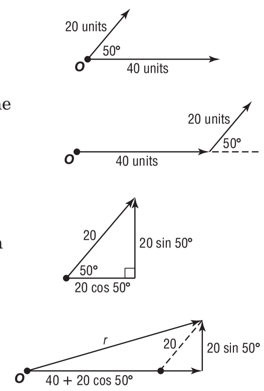
NAME ______________ DATE ________ PERIOD ____

9-7 Enrichment

Vector Addition

Vectors are physical quantities with magnitude and direction. Force and velocity are two examples. We will investigate adding vector quantities. The sum of two vectors is called a **resultant vector** or just the resultant.

Example **Two separate forces, one measuring 20 units and the other measuring 40 units, act on an object. If the angle between the forces is 50°, find the magnitude and direction of the resultant force.**



First, the vectors must be rearranged by placing the tail of the 20-unit vector at the head of the 40-unit vector. Since these vectors are not perpendicular, the horizontal and vertical components of one of the vectors must be found. Using trigonometry, the horizontal component must be (20 cos 50°) units and the vertical component must be (20 sin 50°) units. Replacing the 20-unit vector with these components, we can now form two vectors perpendicular and use the Pythagorean Theorem to find the resultant.

$r^2 = (40 + 20 \cos 50°)^2 + (20 \sin 50°)^2$
$r^2 \approx (52.9)^2 + (15.3)^2$
$r^2 \approx 3032.5$
$r^2 \approx 55.1$

$\tan O = \dfrac{20 \sin 50°}{40 + 20 \cos 50°}$
≈ 0.2898
$m\angle O \approx 16$

Therefore, the resultant force is 55.1 units directed 16° from the 40-unit force.

Solve. Round all angle measures to the nearest degree. Round all other measures to the nearest tenth.

1. A plane flies due west at 250 kilometers per hour while the wind blows south at 70 kilometers per hour. Find the plane's resultant velocity.
259.6 km/h, 16° south of west

2. A plane flies east for 200 km, then 60° south of east for 80 km. Find the plane's distance and direction from its starting point.
249.8 km, 16° south of east

3. One force of 100 units acts on an object. Another force of 80 units acts on the object at a 40° angle from the first force. Find the resultant force on the object.
169.3 units, 18° from the 100-unit force