Name____ Date__________

Chapter 9 Maintaining Mathematical Proficiency

Simplify the expression.

1. $\sqrt{500}$

2. $\sqrt{189}$

3. $\sqrt{252}$

4. $\dfrac{4}{\sqrt{3}}$

5. $\dfrac{11}{\sqrt{5}}$

6. $\dfrac{8}{\sqrt{2}}$

Solve the proportion.

7. $\dfrac{x}{21} = \dfrac{2}{7}$

8. $\dfrac{x}{5} = \dfrac{9}{4}$

9. $\dfrac{3}{x} = \dfrac{14}{42}$

10. $\dfrac{20}{27} = \dfrac{6}{x}$

11. $\dfrac{x - 4}{5} = \dfrac{10}{9}$

12. $\dfrac{15}{5x + 25} = \dfrac{3}{9}$

13. The Pythagorean Theorem states that $a^2 + b^2 = c^2$, where a and b are legs of a right triangle and c is the hypotenuse. Are you able to simplify the Pythagorean Theorem further to say that $a + b = c$? Explain.

Name ____ Date __________

9.1 The Pythagorean Theorem
For use with Exploration 9.1

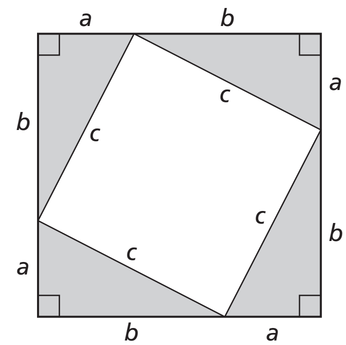
Essential Question
How can you prove the Pythagorean Theorem?

1 EXPLORATION: Proving the Pythagorean Theorem without Words

Work with a partner.

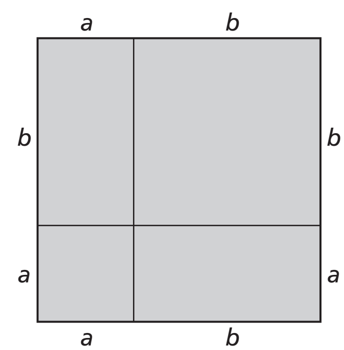
a. Draw and cut out a right triangle with legs a and b, and hypotenuse c.

b. Make three copies of your right triangle. Arrange all four triangles to form a large square as shown.



c. Find the area of the large square in terms of a, b, and c by summing the areas of the triangles and the small square.

d. Copy the large square. Divide it into two smaller squares and two equally-sized rectangles, as shown.



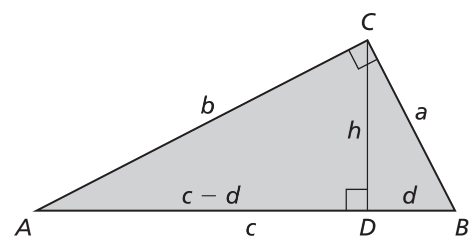
e. Find the area of the large square in terms of a and b by summing the areas of the rectangles and the smaller squares.

f. Compare your answers to parts (c) and (e). Explain how this proves the Pythagorean Theorem.

2 EXPLORATION: Proving the Pythagorean Theorem

Work with a partner.

a. Consider the triangle shown.



b. Explain why $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are similar.

c. Write a two-column proof using the similar triangles in part (b) to prove that $a^2 + b^2 = c^2$.

Communicate Your Answer

3. How can you prove the Pythagorean Theorem?

4. Use the Internet or some other resource to find a way to prove the Pythagorean Theorem that is different from Explorations 1 and 2.

Name ____ Date __________

9.1 Notetaking with Vocabulary

For use after Lesson 9.1

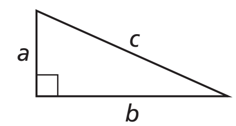
In your own words, write the meaning of each vocabulary term.

Pythagorean triple

Theorems

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$$c^2 = a^2 + b^2$$

Notes:

Core Concepts

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

Notes:

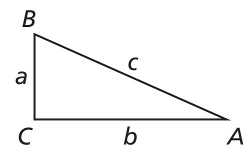
Name____ Date__________

9.1 **Notetaking with Vocabulary (continued)**

Theorems

Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

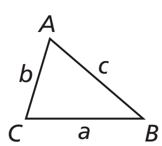
Notes:

Theorem 9.3 Pythagorean Inequalities Theorem

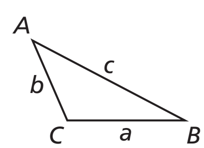
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 < a^2 + b^2$



$c^2 > a^2 + b^2$

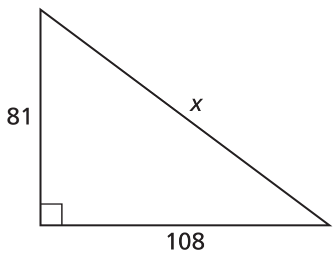
Notes:

Name ____ Date __________

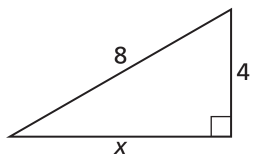
Extra Practice

In Exercises 1–6, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.

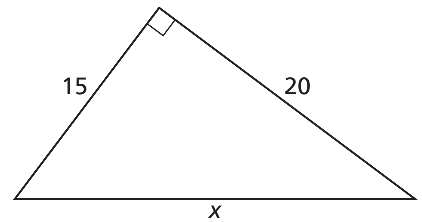
1.



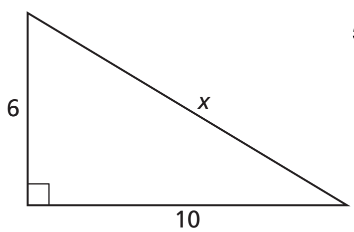
2.



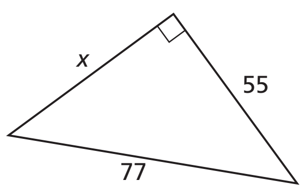
3.



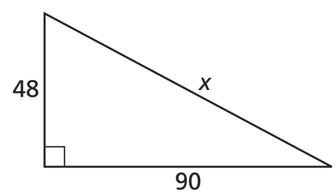
4.



5.



6.



7. From school, you biked 1.2 miles due south and then 0.5 mile due east to your house. If you had biked home on the street that runs directly diagonal from your school to your house, how many fewer miles would you have biked?

In Exercises 8 and 9, verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

8. 90, 216, and 234

9. $1, 1,$ and $\sqrt{3}$

Name____ Date__________

9.2 Special Right Triangles

For use with Exploration 9.2

Essential Question What is the relationship among the side lengths of 45°-45°-90° triangles? 30°-60°-90° triangles?

1 EXPLORATION: Side Ratios of an Isosceles Right Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to construct an isosceles right triangle with a leg length of 4 units.

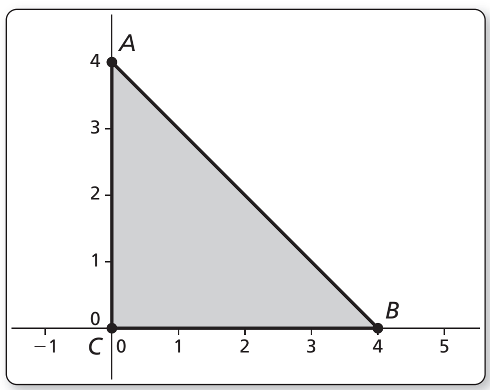
b. Find the acute angle measures. Explain why this triangle is called a 45°-45°-90° triangle.

c. Find the exact ratios of the side lengths (using square roots).

$\dfrac{AB}{AC} =$ _______

$\dfrac{AB}{BC} =$ _______

$\dfrac{AC}{BC} =$ _______



Sample

Points
$A(0, 4)$
$B(4, 0)$
$C(0, 0)$

Segments
$AB = 5.66$
$BC = 4$
$AC = 4$

Angles
$m\angle A = 45°$
$m\angle B = 45°$

d. Repeat parts (a) and (c) for several other isosceles right triangles. Use your results to write a conjecture about the ratios of the side lengths of an isosceles right triangle.

2 EXPLORATION: Side Ratios of a $30°$-$60°$-$90°$ Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

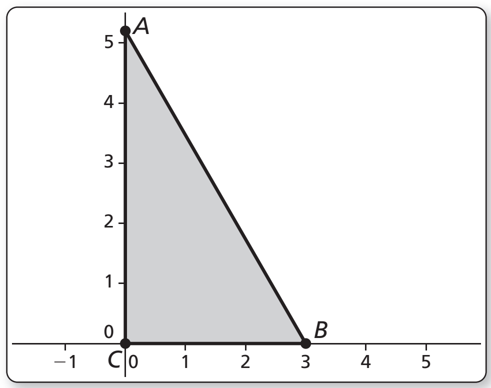
a. Use dynamic geometry software to construct a right triangle with acute angle measures of 30° and 60° (a $30°$-$60°$-$90°$ triangle), where the shorter leg length is 3 units.

b. Find the exact ratios of the side lengths (using square roots).

$\dfrac{AB}{AC} =$ _______

$\dfrac{AB}{BC} =$ _______

$\dfrac{AC}{BC} =$ _______



Sample

Points

$A(0, 5.20)$

$B(3, 0)$

$C(0, 0)$

Segments

$AB = 6$

$BC = 3$

$AC = 5.20$

Angles

$m\angle A = 30°$

$m\angle B = 60°$

c. Repeat parts (a) and (b) for several other $30°$-$60°$-$90°$ triangles. Use your results to write a conjecture about the ratios of the side lengths of a $30°$-$60°$-$90°$ triangle.

Communicate Your Answer

3. What is the relationship among the side lengths of $45°$-$45°$-$90°$ triangles? $30°$-$60°$-$90°$ triangles?

Name____ Date__________

9.2 Notetaking with Vocabulary
For use after Lesson 9.2

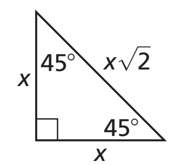
In your own words, write the meaning of each vocabulary term.

isosceles triangle

Theorems

Theorem 9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

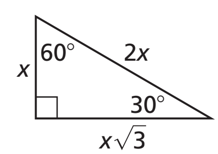


hypotenuse = leg • $\sqrt{2}$

Notes:

Theorem 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



hypotenuse = shorter leg • 2

longer leg = shorter leg • $\sqrt{3}$

Notes:

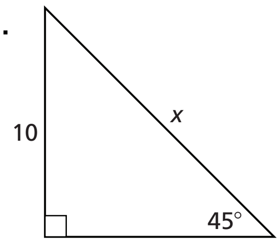
Name ____ Date __________

9.2 Notetaking with Vocabulary (continued)

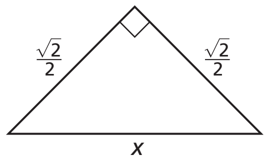
Extra Practice

In Exercises 1–4, find the value of *x*. Write your answer in simplest form.

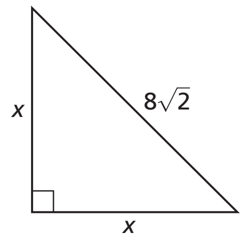
1.



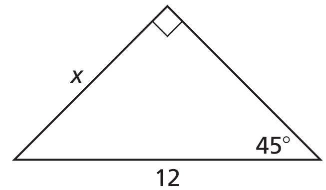
2.



3.

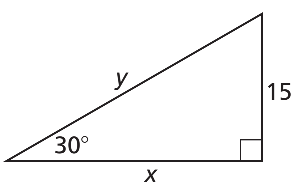


4.

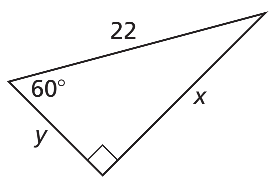


In Exercises 5–7, find the values of *x* and *y*. Write your answers in simplest form.

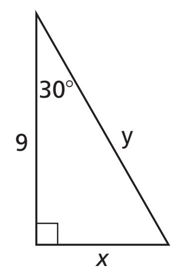
5.



6.



7.



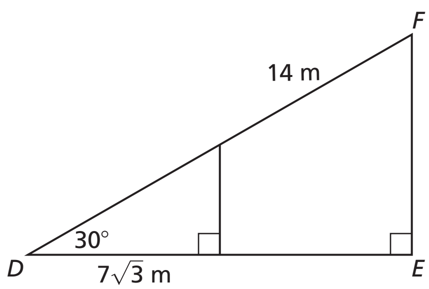
9.2 Notetaking with Vocabulary (continued)

In Exercises 8 and 9, sketch the figure that is described. Find the indicated length. Round decimal answers to the nearest tenth.

8. The length of a diagonal in a square is 32 inches. Find the perimeter of the square.

9. An isosceles triangle with $30°$ base angles has an altitude of $\sqrt{3}$ meters. Find the length of the base of the isosceles triangle.

10. Find the area of $\triangle DEF$. Round decimal answers to the nearest tenth.



Name ____ Date _________

9.3 Similar Right Triangles

For use with Exploration 9.3

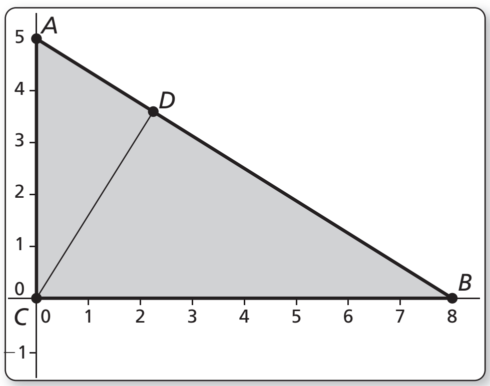
Essential Question How are altitudes and geometric means of right triangles related?

1 EXPLORATION: Writing a Conjecture

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to construct right $\triangle ABC$, as shown. Draw $\overline{CD}$ so that it is an altitude from the right angle to the hypotenuse of $\triangle ABC$.



Points
$A(0, 5)$
$B(8, 0)$
$C(0, 0)$
$D(2.25, 3.6)$

Segments
$AB = 9.43$
$BC = 8$
$AC = 5$

b. The **geometric mean** of two positive numbers a and b is the positive number x that satisfies

$$\frac{a}{\boldsymbol{x}} = \frac{\boldsymbol{x}}{b}.$$

x is the geometric mean of a and b.

Write a proportion involving the side lengths of $\triangle CBD$ and $\triangle ACD$ so that CD is the geometric mean of two of the other side lengths. Use similar triangles to justify your steps.

1 EXPLORATION: Writing a Conjecture (continued)

c. Use the proportion you wrote in part (b) to find *CD*.

d. Generalize the proportion you wrote in part (b). Then write a conjecture about how the geometric mean is related to the altitude from the right angle to the hypotenuse of a right triangle.

2 EXPLORATION: Comparing Geometric and Arithmetic Means

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use a spreadsheet to find the arithmetic mean and the geometric mean of several pairs of positive numbers. Compare the two means. What do you notice?

	A	B	C	D
1	a	b	Arithmetic Mean	Geometric Mean
2	3	4	3.5	3.464
3	4	5		
4	6	7		
5	0.5	0.5		
6	0.4	0.8		
7	2	5		
8	1	4		
9	9	16		
10	10	100		
11				

Communicate Your Answer

3. How are altitudes and geometric means of right triangles related?

Name ______________________________ Date __________

9.3 Notetaking with Vocabulary
For use after Lesson 9.3

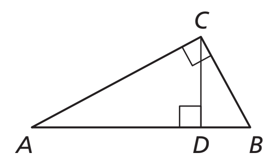
In your own words, write the meaning of each vocabulary term.

geometric mean

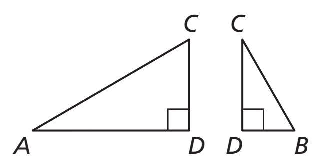
Theorems

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



Notes:

Core Concepts

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number $\boldsymbol{x}$ that satisfies $\frac{a}{\boldsymbol{x}} = \frac{\boldsymbol{x}}{b}$. So, $\boldsymbol{x}^2 = ab$ and $\boldsymbol{x} = \sqrt{ab}$.

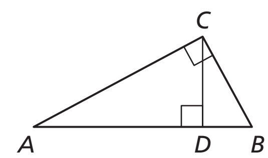
Notes:

Theorems

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



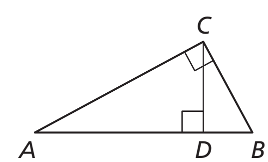
$CD^2 = AD \bullet BD$

Notes:

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



$CB^2 = DB \bullet AB$

$AC^2 = AD \bullet AB$

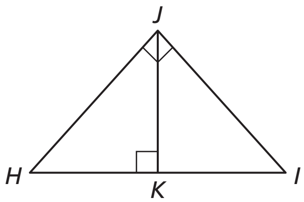
Notes:

Name ______________________________ Date __________

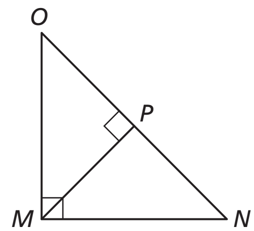
Extra Practice

In Exercises 1 and 2, identify the similar triangles.

1.



2.



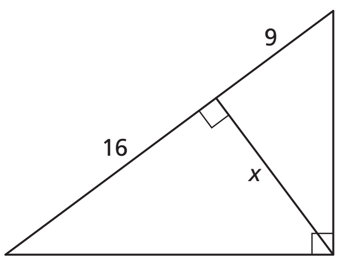
In Exercises 3 and 4, find the geometric mean of the two numbers.

3. 2 and 6

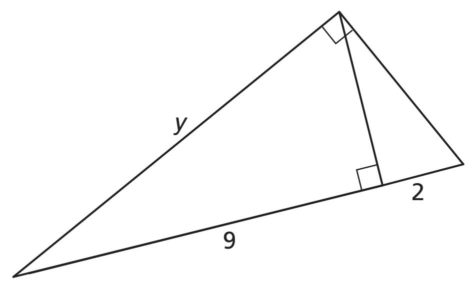
4. 5 and 45

In Exercises 5–8, find the value of the variable.

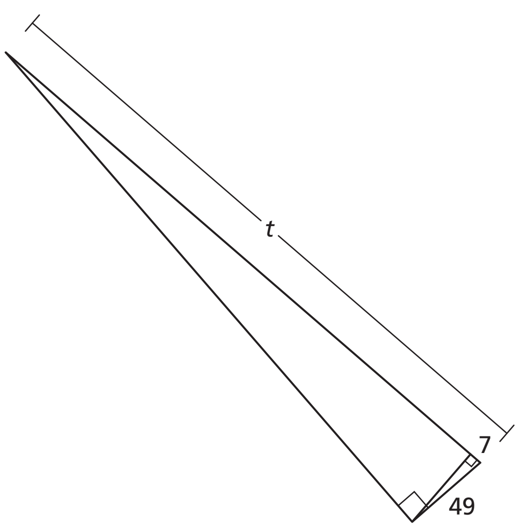
5.



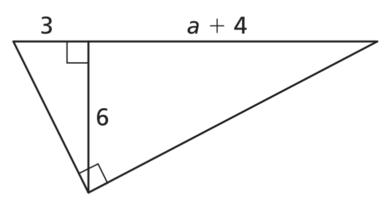
6.



7.



8.



Name______________________________ Date__________

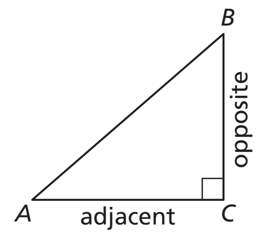
9.4 The Tangent Ratio

For use with Exploration 9.4

Essential Question How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *tangent* of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

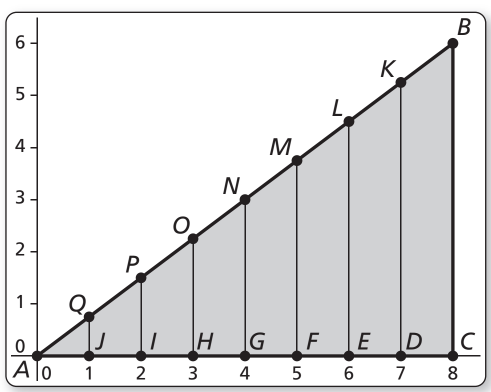


1 EXPLORATION: Calculating a Tangent Ratio

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample
Points
$A(0, 0)$
$B(8, 6)$
$C(8, 0)$
Angle
$m\angle BAC = 36.87°$

b. Calculate each given ratio to complete the table for the decimal value of $\tan A$ for each right triangle. What can you conclude?

Ratio	$\frac{BC}{AC}$	$\frac{KD}{AD}$	$\frac{LE}{AE}$	$\frac{MF}{AF}$	$\frac{NG}{AG}$	$\frac{OH}{AH}$	$\frac{PI}{AI}$	$\frac{QJ}{AJ}$
tan A								

2 EXPLORATION: Using a Calculator

Work with a partner. Use a calculator that has a tangent key to calculate the tangent of $36.87°$. Do you get the same result as in Exploration 1? Explain.

Communicate Your Answer

3. Repeat Exploration 1 for $\triangle ABC$ with vertices $A(0, 0)$, $B(8, 5)$, and $C(8, 0)$.

Construct the seven perpendicular segments so that not all of them intersect $\overline{AC}$ at integer values of x. Discuss your results.

4. How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Name____ Date__________

9.4 Notetaking with Vocabulary

For use after Lesson 9.4

In your own words, write the meaning of each vocabulary term.

trigonometric ratio

tangent

angle of elevation

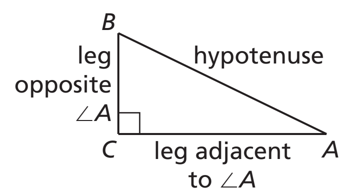
Core Concepts

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as tan A) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



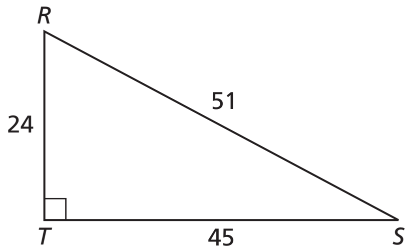
Notes:

Name ____ Date __________

Extra Practice

In Exercises 1–3, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.

1.



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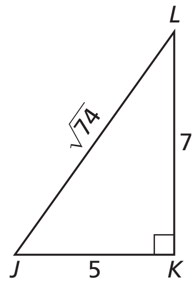

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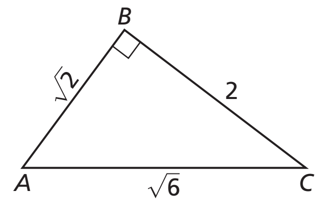
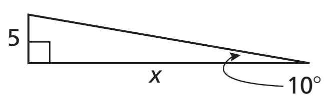


In Exercises 4–6, find the value of *x*. Round your answer to the nearest tenth.

4.



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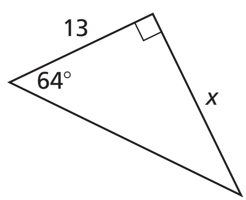
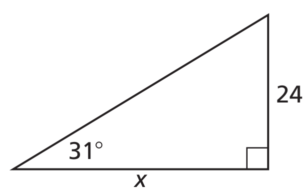


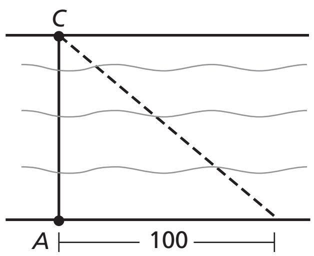
6.



7. In $\triangle CDE$, $\angle E = 90°$ and $\tan C = \frac{4}{3}$. Find tan D? Write your answer as a fraction.

8. An environmentalist wants to measure the width of a river to monitor its erosion. From point *A*, she walks downstream 100 feet and measures the angle from this point to point *C* to be $40°$.

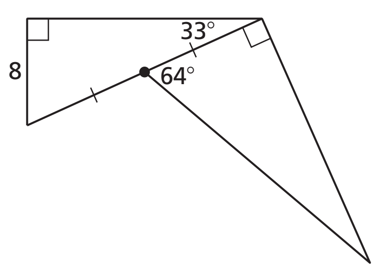
a. How wide is the river? Round to the nearest tenth.



b. One year later, the environmentalist returns to measure the same river. From point *A*, she again walks downstream 100 feet and measures the angle from this point to point *C* to be now $51°$. By how many feet has the width of the river increased?

9. A boy flies a kite at an angle of elevation of $18°$. The kite reaches its maximum height 300 feet away from the boy. What is the maximum height of the kite? Round to the nearest tenth.

10. Find the perimeter of the figure.



Name ____ Date __________

9.5 The Sine and Cosine Ratios

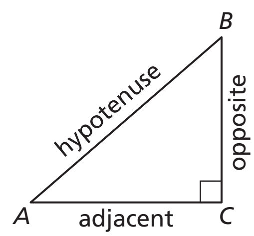
For use with Exploration 9.5

Essential Question How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *sine* of $\angle A$ and *cosine* of $\angle A$ (written as $\sin A$ and $\cos A$, respectively) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

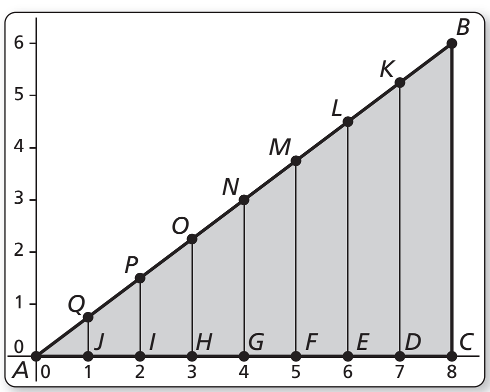


1 EXPLORATION: Calculating Sine and Cosine Ratios

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample

Points

$A(0, 0)$

$B(8, 6)$

$C(8, 0)$

Angle

$m\angle BAC = 36.87°$

1 EXPLORATION: Calculating Sine and Cosine Ratios (continued)

b. Calculate each given ratio to complete the table for the decimal values of sin A and cos A for each right triangle. What can you conclude?

Sine ratio	$\frac{BC}{AB}$	$\frac{KD}{AK}$	$\frac{LE}{AL}$	$\frac{MF}{AM}$	$\frac{NG}{AN}$	$\frac{OH}{AO}$	$\frac{PI}{AP}$	$\frac{QJ}{AQ}$
sin *A*								
Cosine ratio	$\frac{AC}{AB}$	$\frac{AD}{AK}$	$\frac{AE}{AL}$	$\frac{AF}{AM}$	$\frac{AG}{AN}$	$\frac{AH}{AO}$	$\frac{AI}{AP}$	$\frac{AJ}{AQ}$
cos *A*								

Communicate Your Answer

2. How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

3. In Exploration 1, what is the relationship between $\angle A$ and $\angle B$ in terms of their measures? Find sin B and cos B. How are these two values related to sin A and cos A? Explain why these relationships exist.

Name ____ Date __________

9.5 Notetaking with Vocabulary

For use after Lesson 9.5

In your own words, write the meaning of each vocabulary term.

sine

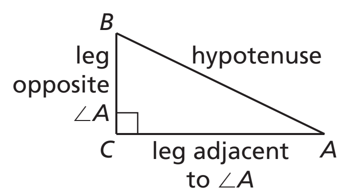
cosine

angle of depression

Core Concepts

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as $\sin A$ and $\cos A$) are defined as follows.



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

Notes:

9.5 Notetaking with Vocabulary (continued)

Sine and Cosine of Complementary Angles

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

Let A and B be complementary angles. Then the following statements are true.

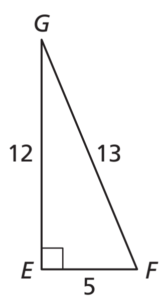
$$\sin A = \cos(90° - A) = \cos B \qquad \sin B = \cos(90° - B) = \cos A$$
$$\cos A = \sin(90° - A) = \sin B \qquad \cos B = \sin(90° - B) = \sin A$$

Notes:

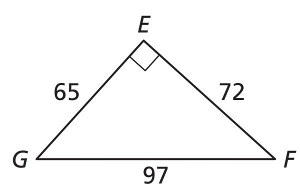
Extra Practice

In Exercises 1–3, find sin *F*, sin *G*, cos *F*, and cos *G*. Write each answer as a fraction and as a decimal rounded to four places.

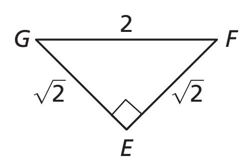
1.



2.



3.



In Exercises 4–6, write the expression in terms of cosine.

4. $\sin 9°$

5. $\sin 30°$

6. $\sin 77°$

Name ____ Date __________

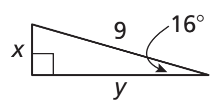
9.5 Notetaking with Vocabulary (continued)

In Exercises 7–9, write the expression in terms of sine.

7. $\cos 15°$ **8.** $\cos 83°$ **9.** $\cos 45°$

In Exercises 10–13, find the value of each variable using sine and cosine. Round your answers to the nearest tenth.

10.



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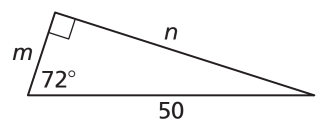

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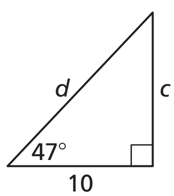
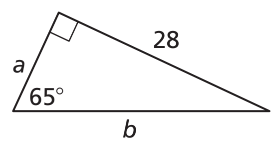


13.



14. A camera attached to a kite is filming the damage caused by a brush fire in a closed-off area. The camera is directly above the center of the closed-off area.

a. A person is standing 100 feet away from the center of the closed-off area. The angle of depression from the camera to the person flying the kite is $25°$. How long is the string on the kite?

b. If the string on the kite is 200 feet long, how far away must the person flying the kite stand from the center of the closed-off area, assuming the same angle of depression of $25°$, to film the damage?

Name____ Date__________

9.6 Solving Right Triangles

For use with Exploration 9.6

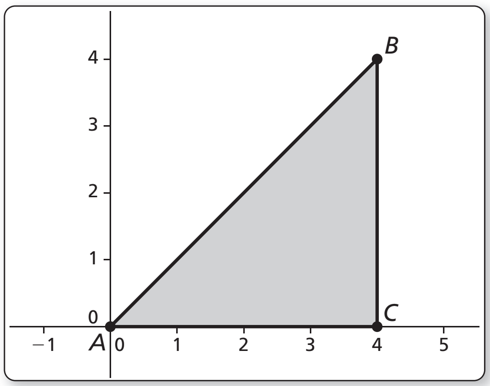
Essential Question When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

1 EXPLORATION: Solving Special Right Triangles

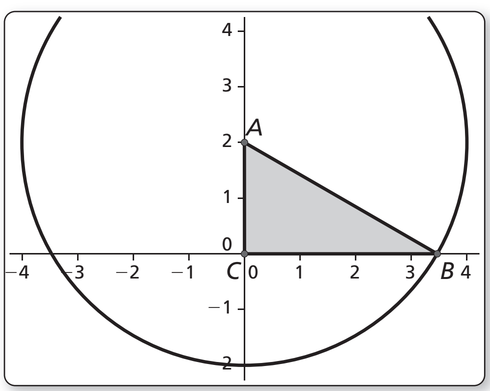
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the figures to find the values of the sine and cosine of $\angle A$ and $\angle B$. Use these values to find the measures of $\angle A$ and $\angle B$. Use dynamic geometry software to verify your answers.

a.



b.

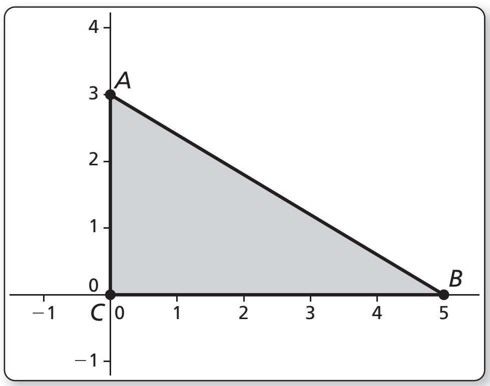


2 EXPLORATION: Solving Right Triangles

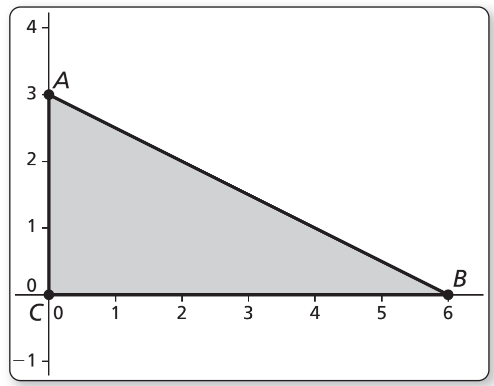
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. You can use a calculator to find the measure of an angle when you know the value of the sine, cosine, or tangent of the angle. Use the inverse sine, inverse cosine, or inverse tangent feature of your calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree. Then use dynamic geometry software to verify your answers.

a.



b.



Communicate Your Answer

3. When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

4. A ladder leaning against a building forms a right triangle with the building and the ground. The legs of the right triangle (in meters) form a 5-12-13 Pythagorean triple. Find the measures of the two acute angles to the nearest tenth of a degree.

Name____ Date__________

9.6 Notetaking with Vocabulary
For use after Lesson 9.6

In your own words, write the meaning of each vocabulary term.

inverse tangent

inverse sine

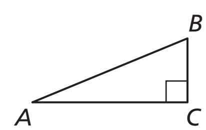
inverse cosine

solve a right triangle

Core Concepts

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \boldsymbol{\tan^{-1}\frac{BC}{AC} = m\angle A}$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \boldsymbol{\sin^{-1}\frac{BC}{AB} = m\angle A}$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \boldsymbol{\cos^{-1}\frac{AC}{AB} = m\angle A}$

Notes:

9.6 Notetaking with Vocabulary (continued)

Solving a Right Triangle

To **solve a right triangle** means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following.

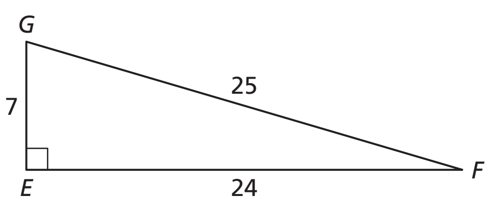
- two side lengths
- one side length and the measure of one acute angle

Notes:

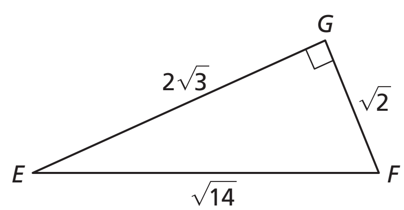
Extra Practice

In Exercises 1 and 2, determine which of the two acute angles has the given trigonometric ratio.

1. The cosine of the angle is $\frac{24}{25}$.



2. The sine of the angle is about 0.38.



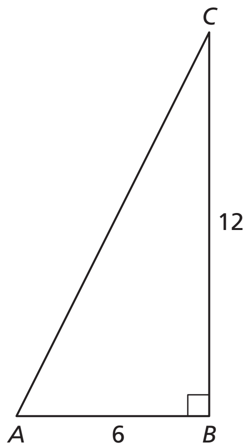
In Exercises 3–6, let $\angle H$ be an acute angle. Use a calculator to approximate the measure of $\angle H$ to the nearest tenth of a degree.

3. $\sin H = 0.2$ **4.** $\tan H = 1$ **5.** $\cos H = 0.33$ **6.** $\sin H = 0.89$

9.6 Notetaking with Vocabulary (continued)

In Exercises 7–10, solve the right triangle. Round decimal answers to the nearest tenth.

7.



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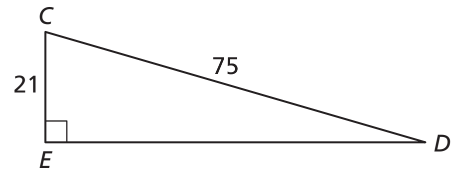

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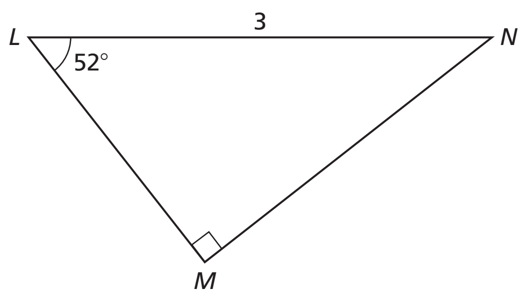

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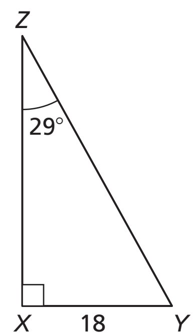
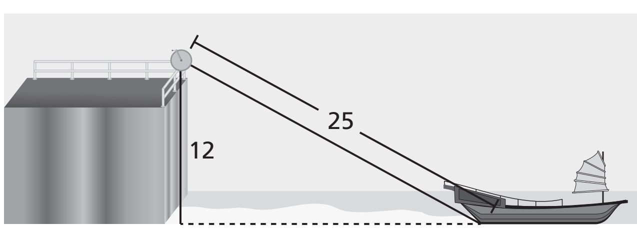


11. A boat is pulled in by a winch on a dock 12 feet above the deck of the boat. When the winch is fully extended to 25 feet, what is the angle of elevation from the boat to the winch?



Name ______________________________ Date __________

9.7 Law of Sines and Law of Cosines

For use with Exploration 9.7

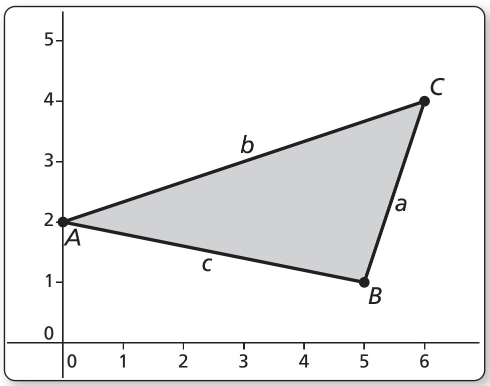
Essential Question What are the Law of Sines and the Law of Cosines?

1 EXPLORATION: Discovering the Law of Sines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for the triangle shown. What can you conclude?



Sample
Segments
$a = 3.16$
$b = 6.32$
$c = 5.10$
Angles
$m\angle A = 29.74°$
$m\angle B = 97.13°$
$m\angle C = 53.13°$

$m\angle A$	a	$\frac{\sin A}{a}$	$m\angle B$	b	$\frac{\sin B}{b}$	$m\angle C$	c	$\frac{\sin C}{c}$

b. Use dynamic geometry software to draw two other triangles. Complete a table for each triangle. Use your results to write a conjecture about the relationship between the sines of the angles and the lengths of the sides of a triangle.

$m\angle A$	a	$\frac{\sin A}{a}$	$m\angle B$	b	$\frac{\sin B}{b}$	$m\angle C$	c	$\frac{\sin C}{c}$

$m\angle A$	a	$\frac{\sin A}{a}$	$m\angle B$	b	$\frac{\sin B}{b}$	$m\angle C$	c	$\frac{\sin C}{c}$

2 EXPLORATION: Discovering the Law of Cosines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for the triangle in Exploration 1(a). What can you conclude?

c	c^2	a	a^2	b	b^2	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

b. Use dynamic geometry software to draw two other triangles. Complete a table for each triangle. Use your results to write a conjecture about what you observe in the completed tables.

c	c^2	a	a^2	b	b^2	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

c	c^2	a	a^2	b	b^2	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

Communicate Your Answer

3. What are the Law of Sines and the Law of Cosines?

4. When would you use the Law of Sines to solve a triangle? When would you use the Law of Cosines to solve a triangle?

Name ____ Date __________

9.7 Notetaking with Vocabulary

For use after Lesson 9.7

In your own words, write the meaning of each vocabulary term.

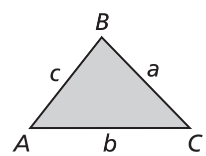
Law of Sines

Law of Cosines

Core Concepts

Area of a Triangle

The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For $\triangle ABC$ shown, there are three ways to calculate the area.

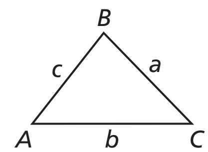


$$\text{Area} = \frac{1}{2}bc \sin A \qquad \text{Area} = \frac{1}{2}ac \sin B \qquad \text{Area} = \frac{1}{2}ab \sin C$$

Notes:

Theorems

Theorem 9.9 Law of Sines

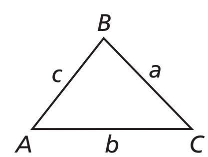


The Law of Sines can be written in either of the following forms for $\triangle ABC$ with sides of length a, b, and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Notes:

Theorem 9.10 Law of Cosines



If $\triangle ABC$ has sides of length a, b, and c, as shown, then the following are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Notes:

Name ____ Date _________

9.7 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–3, use a calculator to find the trigonometric ratio. Round your answer to four decimal places.

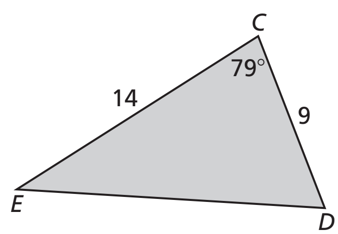
1. $\sin 225°$

2. $\cos 111°$

3. $\tan 96°$

In Exercises 4 and 5, find the area of the triangle. Round your answer to the nearest tenth.

4.



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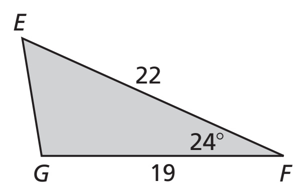
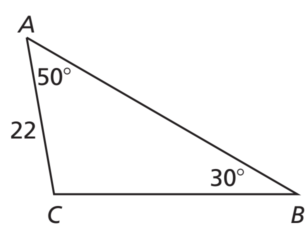


In Exercises 6-8, solve the triangle. Round decimal answers to the nearest tenth.

6.



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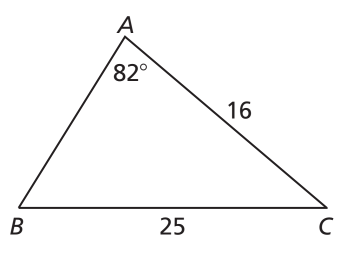

8.