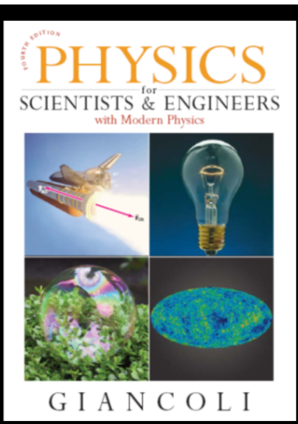


Lecture PowerPoints

Chapter 6

Physics for Scientists and Engineers, with Modern Physics, 4th edition

Giancoli

Chapter 6

Gravitation and Newton's Synthesis

Units of Chapter 6

- Newton's Law of Universal Gravitation
- Vector Form of Newton's Law of Universal Gravitation
- Gravity Near the Earth's Surface; Geophysical Applications
- Satellites and "Weightlessness"
- Kepler's Laws and Newton's Synthesis
- Gravitational Field

Opening Question

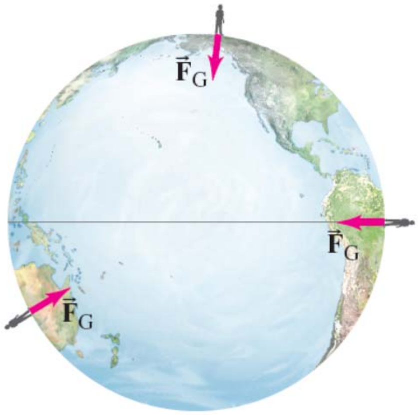
A space station revolves around the Earth as a satellite, 100 km above the Earth's surface. What is the net force on an astronaut at rest inside the space station?

(a) Equal to her weight on Earth.
(b) A little less than her weight on Earth.
(c) Less than half her weight on Earth.
(d) Zero (she is weightless).
(e) Somewhat larger than her weight on earth.

In fact only about 3% less

6-1 Newton’s Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the **origin** of that force?



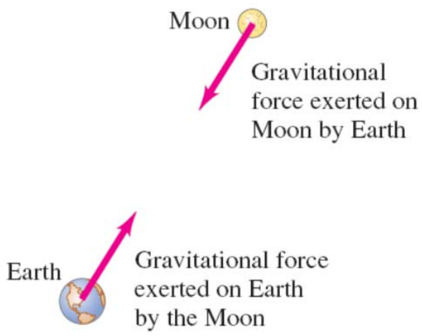
Newton’s realization was that the force must come from the **Earth.**

He further realized that this force must be what keeps the **Moon** in its orbit.

6-1 Newton’s Law of Universal Gravitation

The gravitational force on you is one-half of a third law pair: the **Earth** exerts a **downward** force on you, and you exert an **upward** force on the Earth.

When there is such a **disparity** in masses, the reaction force is undetectable, but for bodies more equal in mass it can be **significant**.



6-1 Newton's Law of Universal Gravitation

Therefore, the gravitational force must be proportional to **both** masses.

By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the **inverse** of the **square** of the distance between the masses.

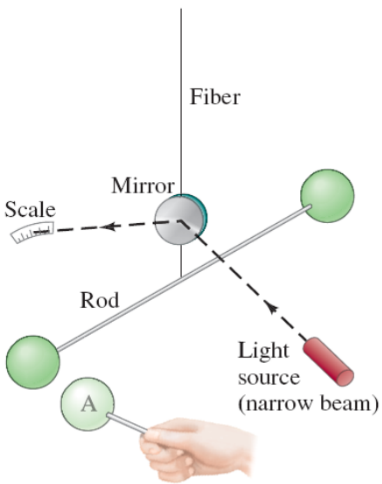
In its final form, the law of universal gravitation reads:

$$F = G\frac{m_1 m_2}{r^2},$$

where

$$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}.$$

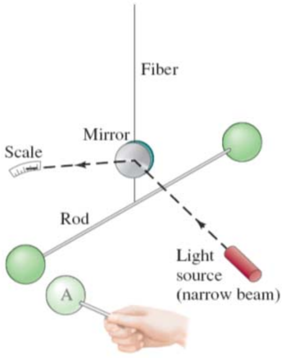
6-1 Newton's Law of Universal Gravitation



The **magnitude** of the gravitational constant *G* can be measured in the laboratory.

This is the **Cavendish** experiment.

6-1 Newton's Law of Universal Gravitation



Schematic diagram of Cavendish's apparatus. Two spheres are attached to a light horizontal rod, which is suspended at its center by a thin fiber. When a third sphere (A) is brought close to one of the suspended spheres, the gravitational force causes the latter to move, and this twists the fiber slightly. The tiny movement is magnified by the use of a narrow light beam directed at a mirror mounted on the fiber. The beam reflects onto a scale. Previous determination of how large a force will twist the fiber a given amount then allows the experimenter to determine the magnitude of the gravitational force between two objects

Do not need to know details of this.

6-1 Newton's Law of Universal Gravitation

Example 6-1: Can you attract another person gravitationally?

A 50 kg person and a 70 kg person are sitting on a bench about 0.5 m apart. Estimate the magnitude of the gravitational force each exerts on the other.

$$F = \frac{Gm_1 m_2}{r^2}$$

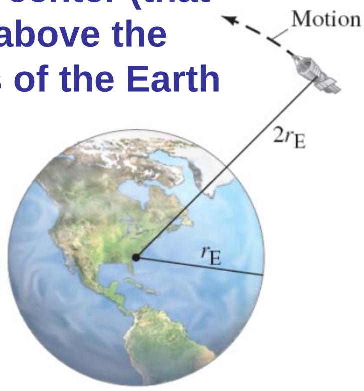
$$= \frac{6.67\times10^{-11}\times50\times70}{0.5^2} = 9.3\times10^{-7} \approx 10^{-6}\ \text{N}$$

This could not be detected without extremely sensitive instruments

6-1 Newton's Law of Universal Gravitation

Example 6-2: Spacecraft at 2r_E.

What is the force of gravity acting on a 2000 kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance r_E = 6380 km above the Earth's surface)? The mass of the Earth is m_E = 5.98 x 10^{24} kg.



Example 6-2: Spacecraft at 2r_E.

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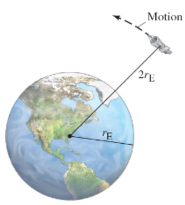
Hard Way:

$$F = \frac{G m_E m_S}{r_S^2} \qquad (r_S = 2r_E)$$

$$= \frac{6.67\times10^{-11}\times5.980\times10^{24}\times2000}{\left(2\times6.380\times10^{6}\right)^2} = 4900\text{ N}$$



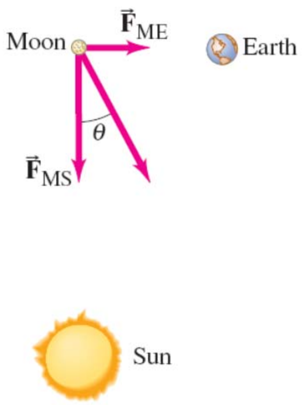
Easy Way:

$$\text{Since } F \propto \frac{1}{r^2}$$

$$\text{If } r \to 2r \quad \Rightarrow \quad F \to \frac{1}{2^2} = \frac{1}{4}$$

$$\therefore \quad F = \tfrac{1}{4} mg = \frac{2000\times9.8}{4} = 4900\text{ N}$$

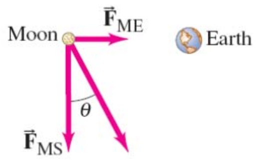
6-1 Newton's Law of Universal Gravitation



Example 6-3: Force on the Moon.

Find the net force on the Moon (m_M = 7.35 x 10^{22} kg) due to the gravitational attraction of both the Earth (m_E = 5.98 x 10^{24} kg) and the Sun (m_S = 1.99 x 10^{30} kg), assuming they are at right angles to each other.

6-1 Newton's Law of Universal Gravitation



Example 6-3: Force on the Moon.

m_M = 7.35 x 10^{22} kg ; r_{ME} = 3.84×10^8 m

m_E = 5.98 x 10^{24} kg

m_S = 1.99 x 10^{30} kg ; r_{MS} = 1.50×10^{11} m

$$F_{ME} = \frac{G m_M m_E}{r_{ME}^2} = \frac{6.67\times10^{-11}\times7.35\times10^{22}\times5.98\times10^{24}}{\left(3.84\times10^{8}\right)^2} = 1.99\times10^{20}\ \mathrm{N}$$

$$F_{MS} = \frac{G m_M m_S}{r_{MS}^2} = \frac{6.67\times10^{-11}\times7.35\times10^{22}\times1.99\times10^{30}}{\left(1.50\times10^{11}\right)^2} = 4.34\times10^{20}\ \mathrm{N}$$

$$F = \sqrt{F_{ME}^2 + F_{MS}^2} = \sqrt{1.99\times10^{20} + 4.34\times10^{20}} = 4.77\times10^{20}\ \mathrm{N}$$

$$\theta = \tan^{-1}\left(\frac{F_{ME}}{F_{MS}}\right) = \tan^{-1}\left(\frac{1.99\times10^{20}}{4.34\times10^{20}}\right) = 24.6°$$

6-1 Newton's Law of Universal Gravitation

Using calculus, you can show:

Particle **outside** a thin spherical shell: gravitational force is the **same** as if all mass were at center of shell

Particle **inside** a thin spherical shell: gravitational force is **zero**

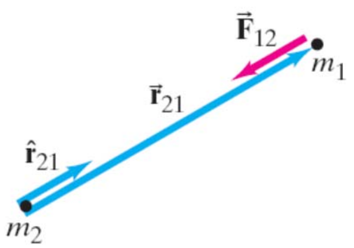
Can model a **sphere** as a series of thin shells; **outside** any spherically symmetric mass, gravitational force acts as though all mass is at center of sphere

6-2 Vector Form of Newton's Universal Gravitation

In vector form,

$$\vec{\mathbf{F}}_{12} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21}.$$

This figure gives the directions of the displacement and force vectors.



6-2 Vector Form of Newton's Universal Gravitation

If there are many particles, the total force is the vector sum of the individual forces:

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{14} + \cdots + \vec{\mathbf{F}}_{1n} = \sum_{i=2}^{n} \vec{\mathbf{F}}_{1i}.$$

6-3 Gravity Near the Earth's Surface; Geophysical Applications

Now we can relate the **gravitational constant** to the **local acceleration** of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{mm_{\mathrm{E}}}{r_{\mathrm{E}}^2}.$$

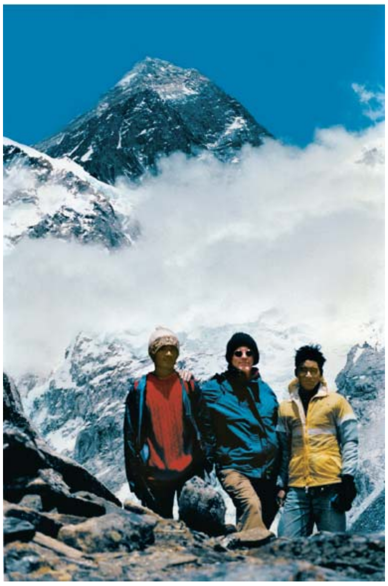
Solving for g gives:

$$g = G\frac{m_{\mathrm{E}}}{r_{\mathrm{E}}^2}.$$

Now, knowing g and the radius of the Earth, the mass of the Earth can be calculated:

$$m_{\mathrm{E}} = \frac{gr_{\mathrm{E}}^2}{G} = \frac{9.80\times\left(6.38\times10^6\right)^2}{6.67\times10^{-11}} = 5.98\times10^{24}\ \mathrm{kg}$$

6-3 Gravity Near the Earth's Surface; Geophysical Applications



Example 6-4: Gravity on Everest.

Estimate the effective value of g on the top of Mt. Everest, 8850 m above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

6-3 Gravity Near the Earth's Surface; Geophysical Applications

Example 6-4: Gravity on Everest.

Mt. Everest: h = 8850 m = 8.85 x 10^3 m above sea level.

r_E = 6380 km = 6.380 x 10^6 m

$r = r_E + h$ = 6.380 x 10^6 + 8.85 x 10^3 = 6.389 x 10^6 m

$$\text{From previous example:} \quad g = \frac{Gm_E}{r^2}$$

$$= \frac{6.67\times10^{-11}\times5.98\times10^{24}}{\left(6.389\times10^{6}\right)^2} = 9.77\ \text{m/s}^2$$

6-3 Gravity Near the Earth's Surface; Geophysical Applications

TABLE 6–1
Acceleration Due to Gravity at Various Locations on Earth

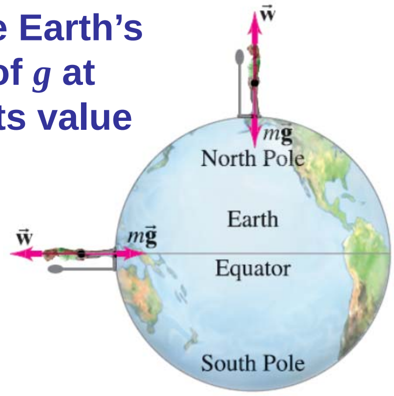
Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity **varies** over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

6-3 Gravity Near the Earth's Surface; Geophysical Applications

Example 6-5: Effect of Earth's rotation on g.

Assuming the Earth is a perfect sphere, determine how the Earth's rotation affects the value of g at the equator compared to its value at the poles.



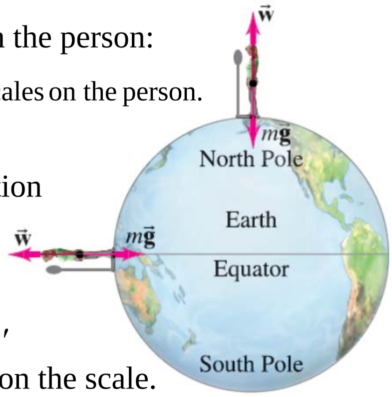
Example 6-5: Effect of Earth's rotation on *g*

At the pole there are two forces acting on the person:

$\vec{\mathbf{w}}$ and $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$ where $\vec{\mathbf{w}}$ is the reaction of the scales on the person.

As there is no rotation $w = mg$

At the equator there is a radial acceleration as the earth is rotating: $\left(a_R = \frac{mv^2}{r}\right)$



The magnitude of $F_G = mg$ is unchanged
The scale pushes upwards with a force w'
which is equal to the force of the person on the scale.
From Newton's 2nd law:

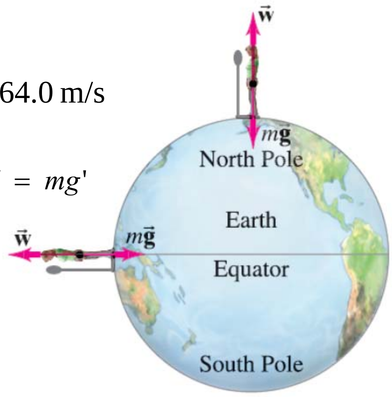
$$mg - w' = aR = \frac{mv^2}{r_E}$$

$$\text{solving for } w' \text{ gives: } w' = m\left(g - \frac{v^2}{r_E}\right)$$

Example 6-5: Effect of Earth's rotation on *g*

Next calculate velocity:

$$v = \frac{Circ}{time} = \frac{2\pi r_E}{1\,\text{day}} = \frac{2\pi \times 6.38\times10^6\ (\text{m})}{8.64\times10^4\ (\text{s})} = 464.0\ \text{m/s}$$



As $w' = m\left(g - \frac{v^2}{r_E}\right)$ The effective weight $w' = mg'$

where g' is the effective value of g

$$\rightarrow\ g' = \frac{w'}{m} = g - \frac{v^2}{r_E}$$

The change in g: $\Delta g = g - g' = \frac{v^2}{r_E}$

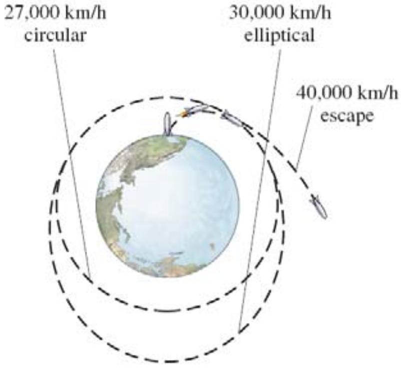
$$= \frac{464}{6.38\times10^6} = 0.0337\ \text{m/s}^2 \text{ or about } 0.3\%$$

Note that as Δg is positive $\rightarrow$ g' is negative

$\rightarrow$ g is slightly smaller at the equator compared to the poles.

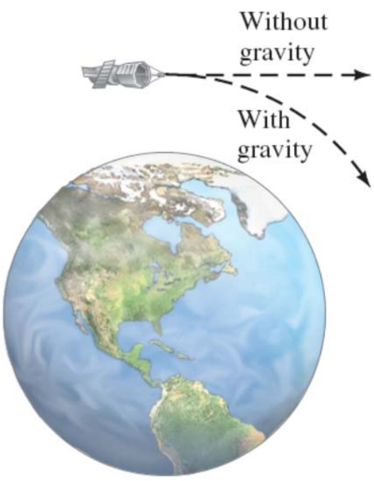
6-4 Satellites and "Weightlessness"

Satellites are routinely put into orbit around the Earth. The **tangential speed** must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.



6-4 Satellites and "Weightlessness"

The satellite is kept in **orbit** by its **speed**—it is continually falling, but the Earth curves from underneath it.



Example 6-6: Geosynchronous satellite.

(a) Relation between v and r $[r = r_E + h$; where $h =$ altitude$]$

$$\text{For circular orbit: } F_C = \frac{mv^2}{r} = \frac{Gmm_E}{r_2} \rightarrow \therefore \quad v^2 = \frac{Gm_E}{r}$$

For geosynchronous orbit:

$$v = \frac{Circ}{time} = \frac{2\pi r}{t} = \frac{2\pi r}{86400} \quad (1\text{ day} = 86400\text{ s})$$

Substitute into above equation and solve for r

$$r = \frac{Gm_E}{v^2} = \frac{6.67\times10^{-11}\times5.98\times10^{24}\times86400^2}{4\pi^2 r^2} = \frac{7.542\times10^{22}}{r^2}$$

$$\therefore \quad r = \sqrt[3]{7.542\times10^{22}} = 4.23\times10^7\text{ m} = 42300\text{ km}$$

$$\therefore \quad h = r - r_E = 42300 - 6380 = 35920\text{ km } (\approx 36000\text{ km})$$

Example 6-6: Geosynchronous satellite.

(b) the satellite's speed.

From previous slide:

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{6.67\times10^{-11}\times5.98\times10^{24}}{4.23\times10^7}} = 3070\text{ m/s}$$

(c) Compare to the speed of a satellite orbiting 200 km above Earth's surface.

From part (b) we see that $v \propto \sqrt{1/r}$

$$v' = v\sqrt{\frac{r}{r'}} = 3070\sqrt{\frac{42300}{6380+200}} = 7780\text{ m/s}$$

6-4 Satellites and "Weightlessness"

Conceptual Example 6-7: Catching a satellite.

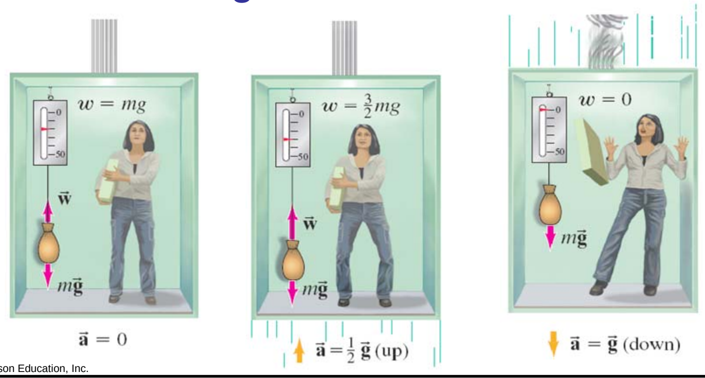
You are an astronaut in the space shuttle pursuing a satellite in need of repair. You find yourself in a circular orbit of the same radius as the satellite, but 30 km behind it. How will you catch up with it?

As $r = \dfrac{Gm_E}{v^2}$ If v increases – r must decrease
or if v decreases – r must increase

You have to drop into a lower orbit to speed up; when you get ahead of the satellite you need to slow down and get back into the higher orbit.

6-4 Satellites and "Weightlessness"

Objects in orbit are said to experience weightlessness. They do have a gravitational force acting on them, though!
The satellite and all its contents are in free fall, so there is no normal force. This is what leads to the experience of weightlessness.



6-4 Satellites and "Weightlessness"

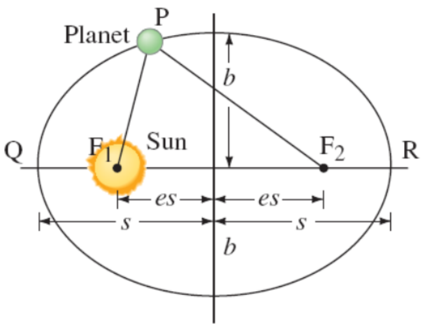
More properly, this effect is called **apparent weightlessness**, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:



6-5 Kepler's Laws and Newton's Synthesis

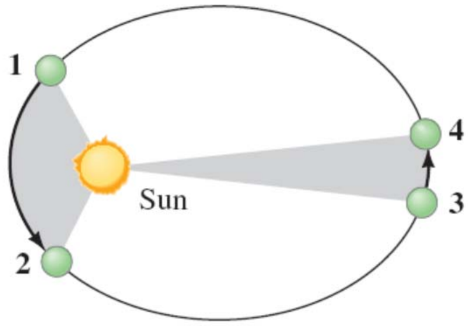
Kepler's laws describe **planetary motion.**

1. The orbit of each planet is an **ellipse**, with the Sun at one focus.



6-5 Kepler's Laws and Newton's Synthesis

2. An imaginary line drawn from each planet to the Sun sweeps out **equal areas** in **equal times**.



6-5 Kepler's Laws and Newton's Synthesis

3. The square of a planet's orbital **period** is proportional to the cube of its mean **distance** from the Sun.

TABLE 6–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth yr)	s^3/T^2 $\left(10^{24}\ \frac{\text{km}^3}{\text{yr}^2}\right)$
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

6-5 Kepler's Laws and Newton's Synthesis

Kepler's laws can be derived from **Newton's laws**. In particular, Kepler's third law follows directly from the law of universal gravitation—equating the gravitational force with the centripetal force shows that, for any two planets (assuming circular orbits, and that the only gravitational influence is the Sun):

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3.$$

6-5 Kepler's Laws and Newton's Synthesis

Example 6-8: Where is Mars?
Mars' period (its "year") was first noted by Kepler to be about 687 days (Earth-days),
which is (687 d/365 d) = 1.88 yr (Earth years). Determine the mean distance of Mars from the Sun using the Earth as a reference.
(r_{ES} = 1.50 x 10^{11} m)

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\therefore \quad \frac{r_{MS}}{r_{ES}} = \left(\frac{T_M}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1.88}{1}\right)^{\frac{2}{3}} = 1.52$$

$$\therefore \quad r_{MS} = 1.52 \times 1.50 \times 10^{11} = 2.28 \times 10^{11} \text{ m}$$

6-5 Kepler's Laws and Newton's Synthesis

Example 6-9: The Sun's mass determined.

Determine the mass of the Sun given the Earth's distance from the Sun as r_{ES} = 1.5 x 10^{11} m.

(Note: This an easier approach than Giancoli without having to prove Kepler's 3rd law.)

$$F_C = \frac{m_E v_E^2}{r_{ES}} = \frac{G m_E m_S}{(r_{ES})^2}$$

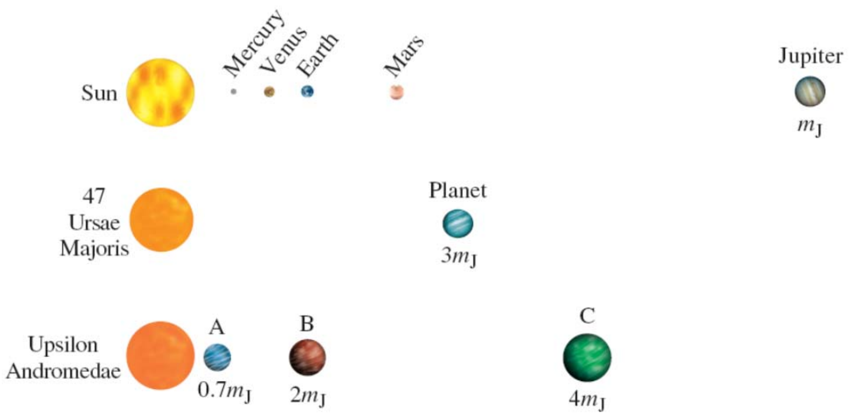
$$\text{and} \quad v_E = \frac{2\pi r_{ES}}{t} = \frac{2\pi \times 1.5\times 10^{11}}{365\times 24\times 60\times 60} = 3.00\times 10^4 \text{ m/s}$$

$$\therefore \quad m_S = \frac{r_{ES} v_E^2}{G} = \frac{1.5\times 10^{11}\times (3.00\times 10^4)^2}{6.67\times 10^{-11}}$$

$$= 2.0\times 10^{30} \text{ kg}$$

6-5 Kepler's Laws and Newton's Synthesis

Irregularities in planetary motion led to the discovery of Neptune, and irregularities in stellar motion have led to the discovery of many planets outside our solar system.



6-6 Gravitational Field

The gravitational **field** is the gravitational **force** per **unit mass**:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m}.$$

The gravitational field due to a **single mass** M is given by:

$$\vec{\mathbf{g}} = -\frac{GM}{r^2}\hat{\mathbf{r}}.$$

Summary of Chapter 6

- Newton's law of universal gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

- Total force is the vector sum of individual forces.
- Satellites are able to stay in Earth orbit because of their large tangential speed.
- Newton's laws provide a theoretical base for Kepler's laws.