Fluid Mechanics: Fundamentals and Applications, 2nd Edition
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Chapter 5
MASS, BERNOULLI AND ENERGY EQUATIONS

Lecture slides by

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Wind turbine “farms” are being constructed all over the world to extract kinetic energy from the wind and convert it to electrical energy. The mass, energy, momentum, and angular momentum balances are utilized in the design of a wind turbine. The Bernoulli equation is also useful in the preliminary design stage.

Objectives

- Apply the conservation of mass equation to balance the incoming and outgoing flow rates in a flow system.
- Recognize various forms of mechanical energy, and work with energy conversion efficiencies.
- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

5–1 ■ INTRODUCTION

You are already familiar with numerous **conservation laws** such as the laws of conservation of mass, conservation of energy, and conservation of momentum.

Historically, the conservation laws are first applied to a fixed quantity of matter called a *closed system* or just a *system*, and then extended to regions in space called *control volumes*.

The conservation relations are also called *balance equations* since any conserved quantity must balance during a process.

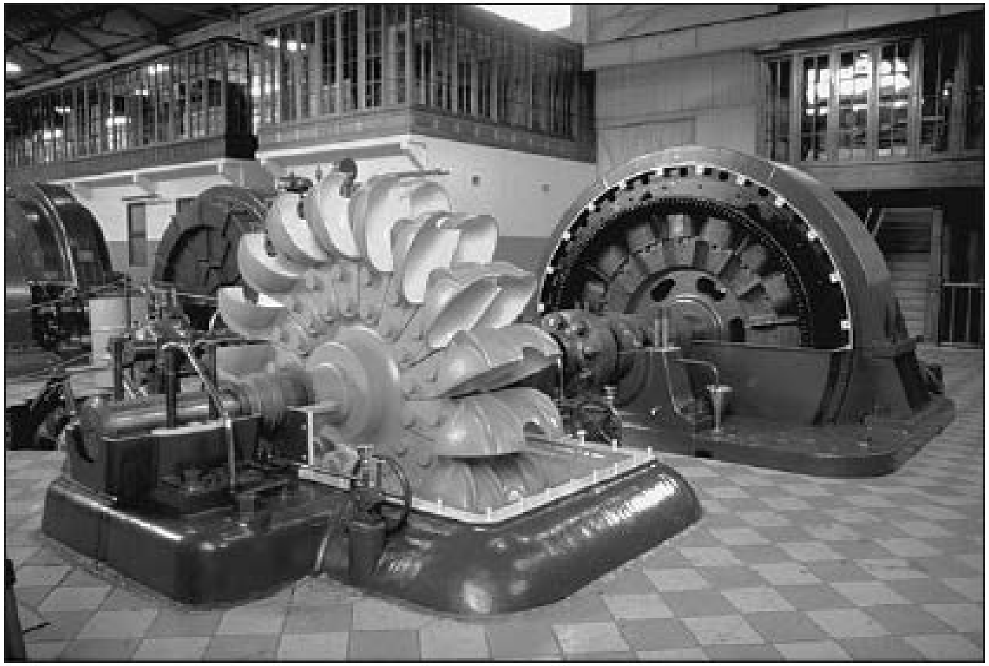


FIGURE 5–1
Many fluid flow devices such as this Pelton wheel hydraulic turbine are analyzed by applying the conservation of mass and energy principles, along with the linear momentum equation.

Conservation of Mass

The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = constant or $dm_{sys}/dt = 0$, which is the statement that the mass of the system remains constant during a process.

Mass balance for a control volume (CV) in rate form:

Conservation of mass: $$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}$$

$\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ the total rates of mass flow into and out of the control volume

dm_{CV}/dt the rate of change of mass within the control volume boundaries.

Continuity equation: In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the *continuity equation*.

The Linear Momentum Equation

Linear momentum: The product of the mass and the velocity of a body is called the *linear momentum* or just the *momentum* of the body.

The momentum of a rigid body of mass *m* moving with a velocity *V* is *mV.*

Newton's second law: The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.

Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved.

Linear momentum equation: In fluid mechanics, Newton's second law is usually referred to as the *linear momentum equation.*

Conservation of Energy

The conservation of energy principle (the energy balance): The net energy transfer to or from a system during a process be equal to the change in the energy content of the system.

Energy can be transferred to or from a closed system by heat or work.

Control volumes also involve energy transfer via mass flow.

Conservation of energy: $$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{CV}}{dt}$$

$\dot{E}_{in}$ and $\dot{E}_{out}$ the total rates of energy transfer into and out of the control volume

dE_{CV}/dt the rate of change of energy within the control volume boundaries

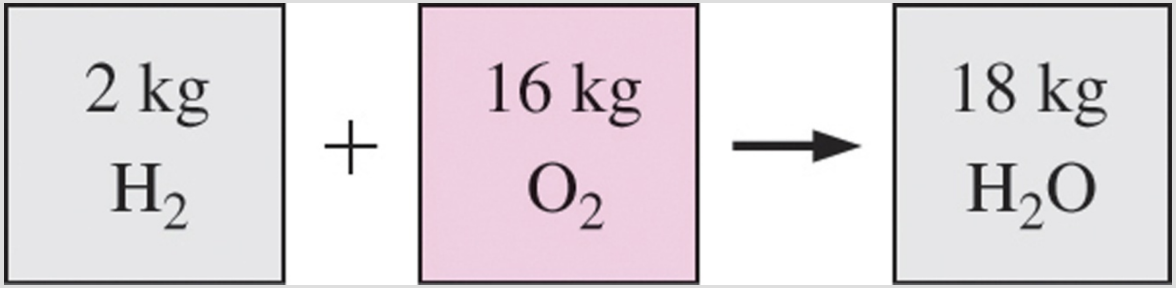
In fluid mechanics, we usually limit our consideration to mechanical forms of energy only.

5–2 ■ CONSERVATION OF MASS

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Mass is conserved even during chemical reactions.

Mass m and energy E can be converted to each other:

$$E = mc^2$$

c is the speed of light in a vacuum, $c = 2.9979 \times 10^8$ m/s

The mass change due to energy change is negligible.

Mass and Volume Flow Rates

Mass flow rate: The amount of mass flowing through a cross section per unit time.

The differential mass flow rate

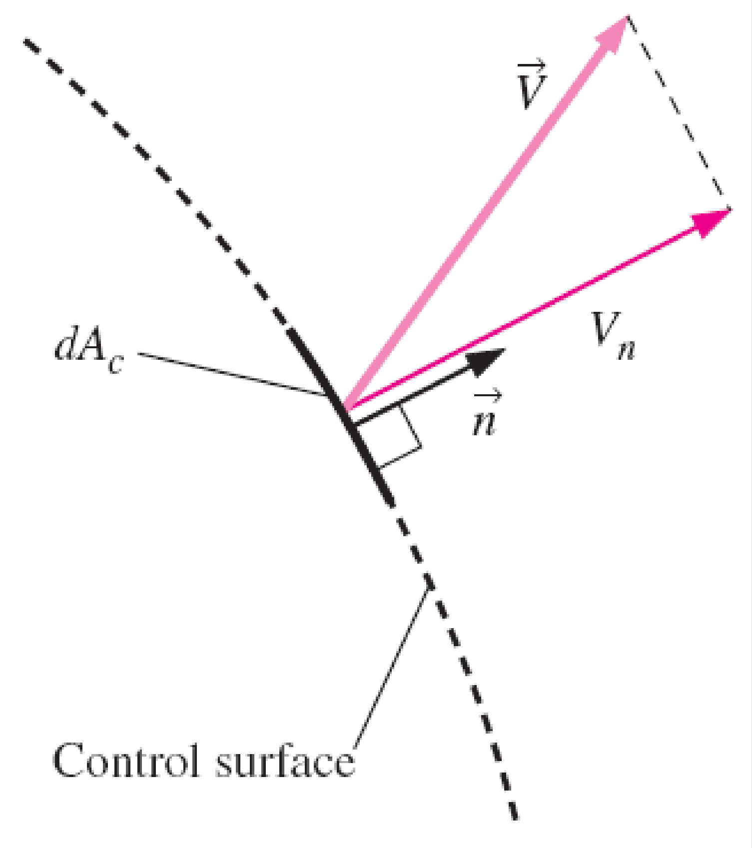
$$\delta \dot{m} = \rho V_n \, dA_c$$

Point functions have ***exact differentials***

$$\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^2 - r_1^2)$$

Path functions have ***inexact differentials***

$$\int_1^2 \delta \dot{m} = \dot{m}_{\text{total}} \qquad \text{not } \dot{m}_2 - \dot{m}_1$$



The normal velocity V_n for a surface is the component of velocity perpendicular to the surface.

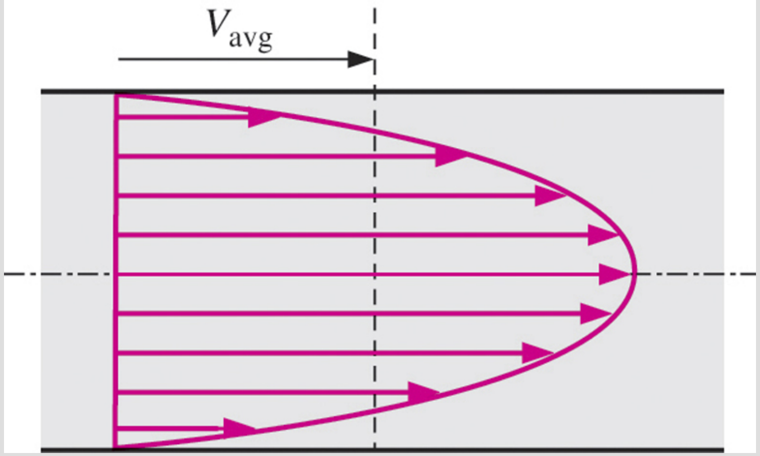
$$\delta\dot{m} = \rho V_n \, dA_c$$

$$\dot{m} = \int_{A_c} \delta\dot{m} = \int_{A_c} \rho V_n \, dA_c$$

Mass flow rate

$$\dot{m} = \rho V_{avg} A_c \qquad (\text{kg/s})$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$



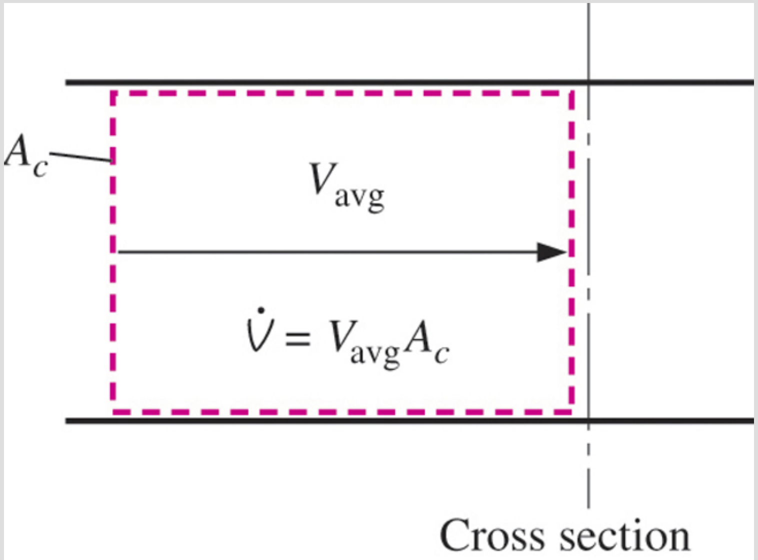
The average velocity V_{avg} is defined as the average speed through a cross section.

Average velocity

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$

Volume flow rate

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = V A_c \qquad (\text{m}^3/\text{s})$$



The volume flow rate is the volume of fluid flowing through a cross section per unit time.

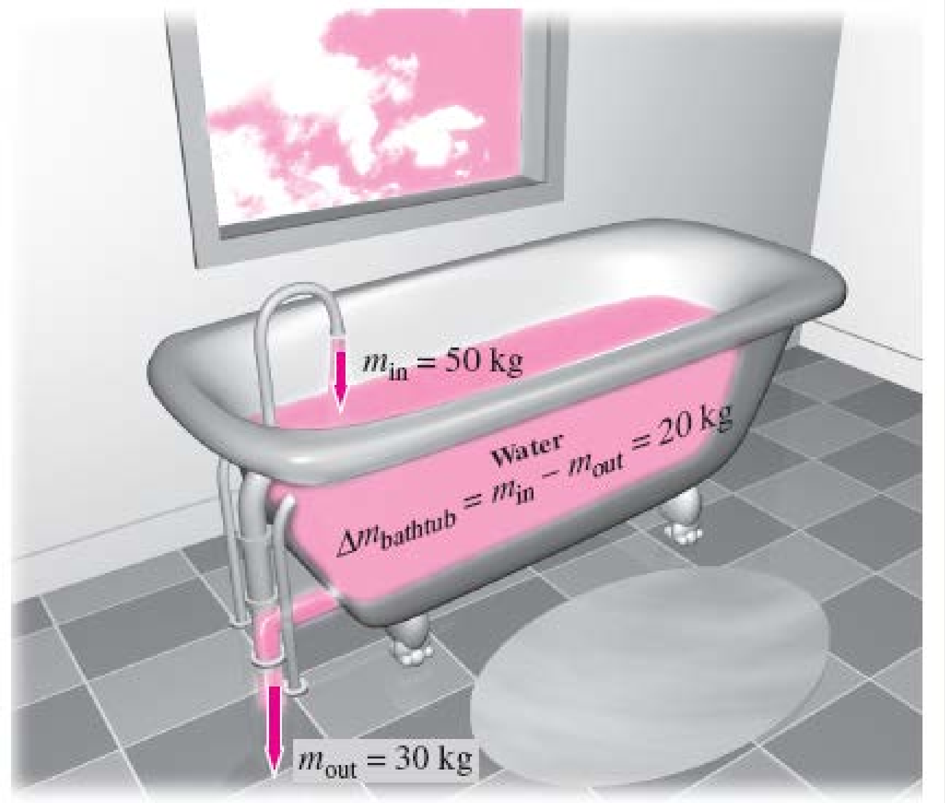
Conservation of Mass Principle

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\begin{pmatrix}\text{Total mass entering} \\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change of mass} \\ \text{within the CV during } \Delta t\end{pmatrix}$$

$$\boxed{m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}} \qquad (\text{kg})}$$

$$\boxed{\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt \qquad (\text{kg/s})}$$



$\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ the total rates of mass flow into and out of the control volume

dm_{CV}/dt the rate of change of mass within the control volume boundaries.

Mass balance is applicable to any control volume undergoing any kind of process.

Conservation of mass principle for an ordinary bathtub.

$dm = \rho \, dV$.

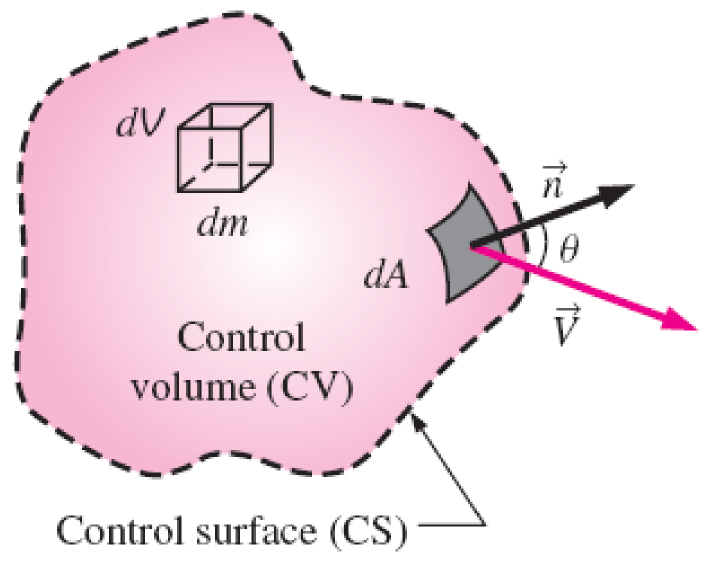
Total mass within the CV: $$m_{CV} = \int_{CV} \rho \, dV$$

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV$$

Normal component of velocity: $$V_n = V \cos\theta = \vec{V} \cdot \vec{n}$$

Differential mass flow rate: $$\delta\dot{m} = \rho V_n \, dA = \rho (V\cos\theta) \, dA = \rho(\vec{V} \cdot \vec{n}) \, dA$$

Net mass flow rate: $$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n \, dA = \int_{CS} \rho(\vec{V} \cdot \vec{n}) \, dA$$



The differential control volume *dV* and the differential control surface *dA* used in the derivation of the conservation of mass relation.

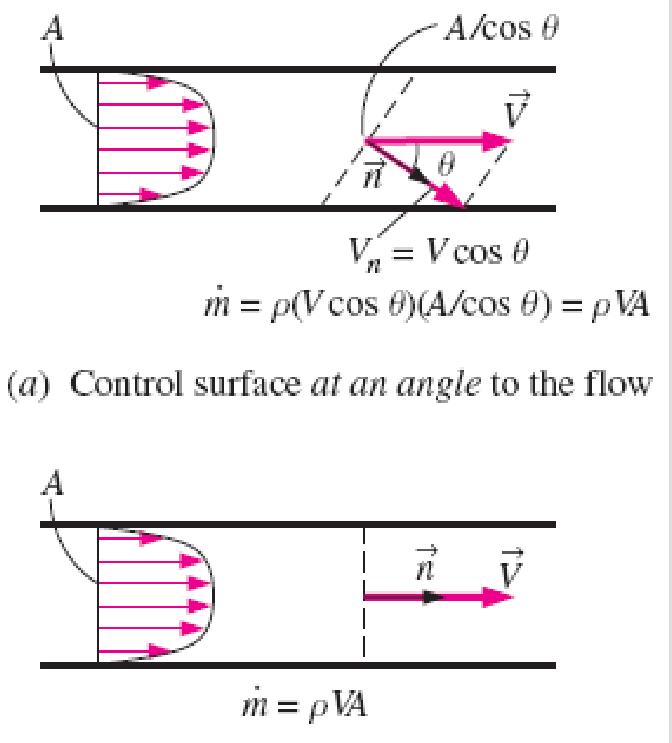
General conservation of mass: $$\frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt}\int_{CV} \rho \, dV + \sum_{\text{out}} \rho |V_n| A - \sum_{\text{in}} \rho |V_n| A = 0$$

$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{\text{in}} \dot{m} - \sum_{\text{out}} \dot{m}$$



(*a*) Control surface *at an angle* to the flow

(*b*) Control surface *normal* to the flow

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b(\vec{V}\cdot\vec{n})\, dA$$

$B = m$ $\qquad b = 1$ $\qquad b = 1$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

The conservation of mass equation is obtained by replacing *B* in the Reynolds transport theorem by mass *m*, and *b* by 1 (*m* per unit mass = *m*/*m* = 1).

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

Moving or Deforming Control Volumes

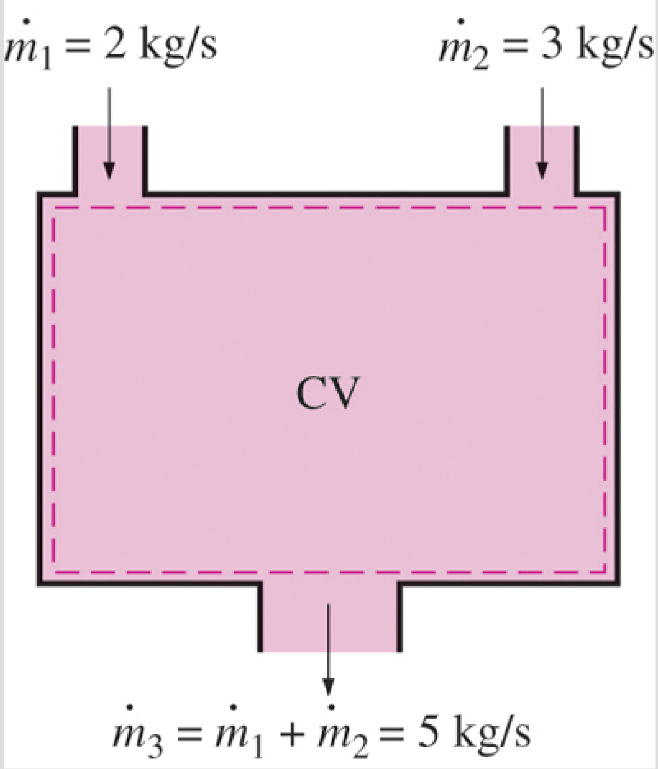
Equations 5–17 and 5–19 are also valid for moving control volumes provided that the *absolute velocity* $\vec{V}$ is replaced by the *relative velocity* $\vec{V}_r$, which is the fluid velocity relative to the control surface (Chap. 4). In the case of a moving but nondeforming control volume, relative velocity is the fluid velocity observed by a person moving with the control volume and is expressed as $\vec{V}_r = \vec{V} - \vec{V}_{CS}$, where $\vec{V}$ is the fluid velocity and $\vec{V}_{CS}$ is the velocity of the control surface, both relative to a fixed point outside. Note that this is a *vector* subtraction.

Some practical problems (such as the injection of medication through the needle of a syringe by the forced motion of the plunger) involve *deforming* control volumes. The conservation of mass relations developed can still be used for such deforming control volumes provided that the velocity of the fluid crossing a deforming part of the control surface is expressed relative to the control surface (that is, the fluid velocity should be expressed relative to a reference frame attached to the deforming part of the control surface). The relative velocity in this case at any point on the control surface is expressed again as $\vec{V}_r = \vec{V} - \vec{V}_{CS}$, where $\vec{V}_{CS}$ is the local velocity of the control surface at that point relative to a fixed point outside the control volume.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate*.

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

Multiple inlets and exits

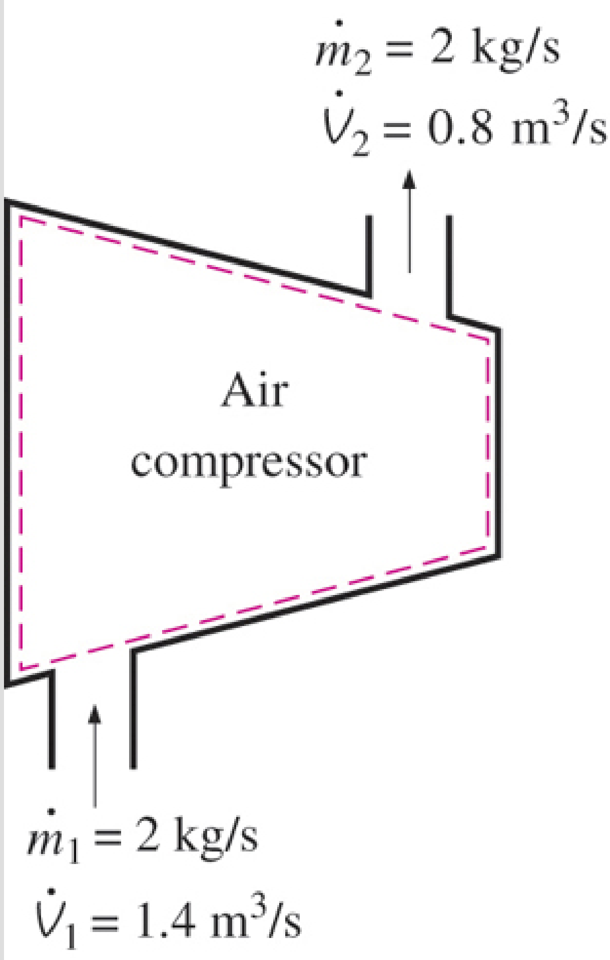
$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

However, for steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

EXAMPLE 5–1 Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (Fig. 5–12). If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the average velocity of water at the nozzle exit.

SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is a nearly incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

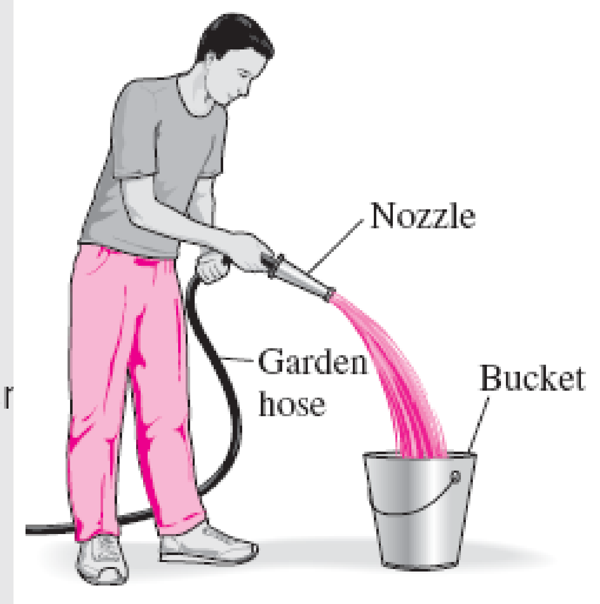
Properties We take the density of water to be 1000 kg/m^3 = 1 kg/L.

Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757 \text{ L/s}}$$

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757 \text{ kg/s}}$$

(*b*) The cross-sectional area of the nozzle exit is



The volume flow rate through the hose and the nozzle is constant. Ther average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1 \text{ m/s}}$$

Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

EXAMPLE 5–2 Discharge of Water from a Tank

A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out (Fig. 5–13). The average velocity of the jet is approximated as $V = \sqrt{2gh}$, where h is the height of water in the tank measured from the as center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop to 2 ft from the bottom.

Analysis We take the volume occupied by water as the control volume. The size of the control volume decreases in this case as the water level drops, and thus this is a variable control volume. (We could also treat this as a fixed control volume that consists of the interior volume of the tank by disregarding the air that replaces the space vacated by the water.) This is obviously an unsteady-flow problem since the properties (such as the amount of mass) within the control volume change with time.

The conservation of mass relation for a control volume undergoing any process is given in rate form as

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt} \tag{1}$$

During this process no mass enters the control volume ($\dot{m}_{\text{in}} = 0$), and the mass flow rate of discharged water is

$$\dot{m}_{\text{out}} = (\rho V A)_{\text{out}} = \rho\sqrt{2gh}\,A_{\text{jet}} \tag{2}$$

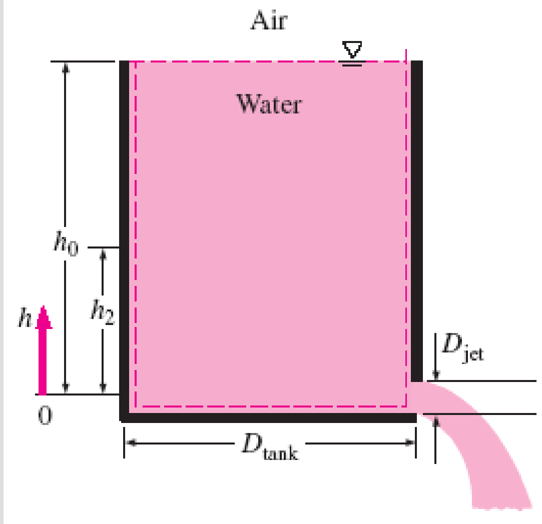


FIGURE 5–13
Schematic for Example 5–2.

where $A_{\text{jet}} = \pi D_{\text{jet}}^2/4$ is the cross-sectional area of the jet, which is constant. Noting that the density of water is constant, the mass of water in the tank at any time is

$$m_{CV} = \rho V = \rho A_{\text{tank}} h \tag{3}$$

where $A_{\text{tank}} = \pi D_{\text{tank}}^2/4$ is the base area of the cylindrical tank. Substituting Eqs. 2 and 3 into the mass balance relation (Eq. 1) gives

$$-\rho\sqrt{2gh}A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \rightarrow -\rho\sqrt{2gh}(\pi D_{\text{jet}}^2/4) = \frac{\rho(\pi D_{\text{tank}}^2/4)\,dh}{dt}$$

Canceling the densities and other common terms and separating the variables give

$$dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2}\frac{dh}{\sqrt{2gh}}$$

Integrating from $t = 0$ at which $h = h_0$ to $t = t$ at which $h = h_2$ gives

$$\int_0^t dt = -\frac{D_{\text{tank}}^2}{D_{\text{jet}}^2\sqrt{2g}}\int_{h_0}^{h_2}\frac{dh}{\sqrt{h}} \rightarrow t = \frac{\sqrt{h_0} - \sqrt{h_2}}{\sqrt{g/2}}\left(\frac{D_{\text{tank}}}{D_{\text{jet}}}\right)^2$$

Substituting, the time of discharge is determined to be

$$t = \frac{\sqrt{4\text{ ft}} - \sqrt{2\text{ ft}}}{\sqrt{32.2/2\text{ ft/s}^2}}\left(\frac{3 \times 12\text{ in}}{0.5\text{ in}}\right)^2 = 757\text{ s} = \mathbf{12.6\ min}$$

Therefore, it takes 12.6 min after the discharge hole is unplugged for half of the tank to be emptied.

Discussion Using the same relation with $h_2 = 0$ gives $t = 43.1$ min for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing h.

5–3 ■ MECHANICAL ENERGY AND EFFICIENCY

Mechanical energy: The form of energy that can be converted to mechanical work completely and directly by an ideal mechanical device such as an ideal turbine.

Mechanical energy of a flowing fluid per unit mass:

$$e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

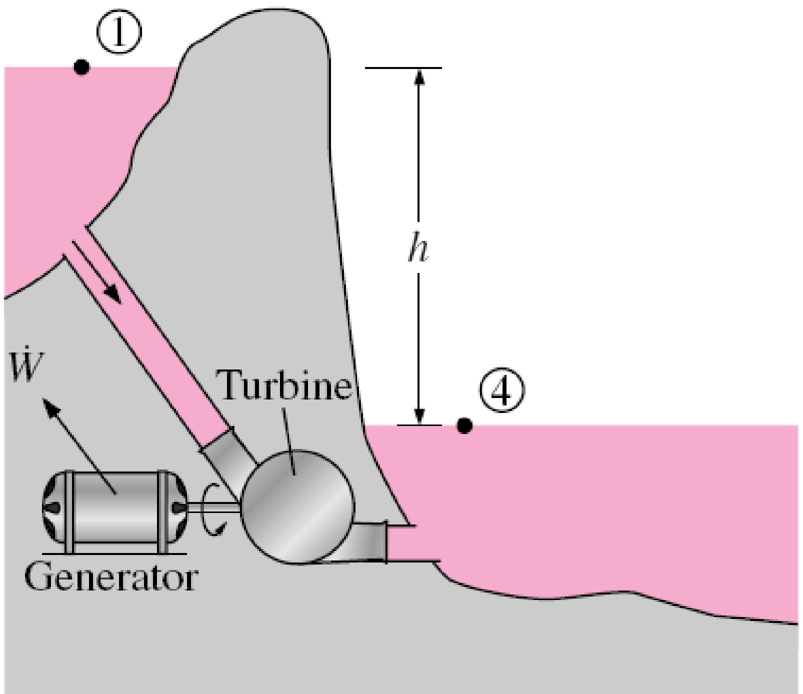
Flow energy + kinetic energy + potential energy

Mechanical energy change:

$$\Delta e_{\text{mech}} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \qquad \text{(kJ/kg)}$$

- The mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant.
- In the absence of any irreversible losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{\text{mech}} > 0$) or extracted from the fluid (if $\Delta e_{\text{mech}} < 0$).

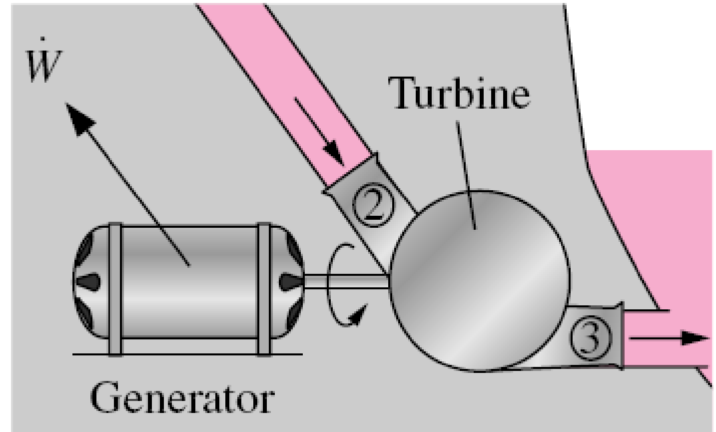
Mechanical energy is a useful concept for flows that do not involve significant heat transfer or energy conversion, such as the flow of gasoline from an underground tank into a car.



$$\dot{W}_{max} = \dot{m}\Delta e_{mech} = \dot{m}g(z_1 - z_4) = \dot{m}gh$$

$$\text{since } P_1 \approx P_4 = P_{atm} \text{ and } V_1 = V_4 \approx 0$$

(a)

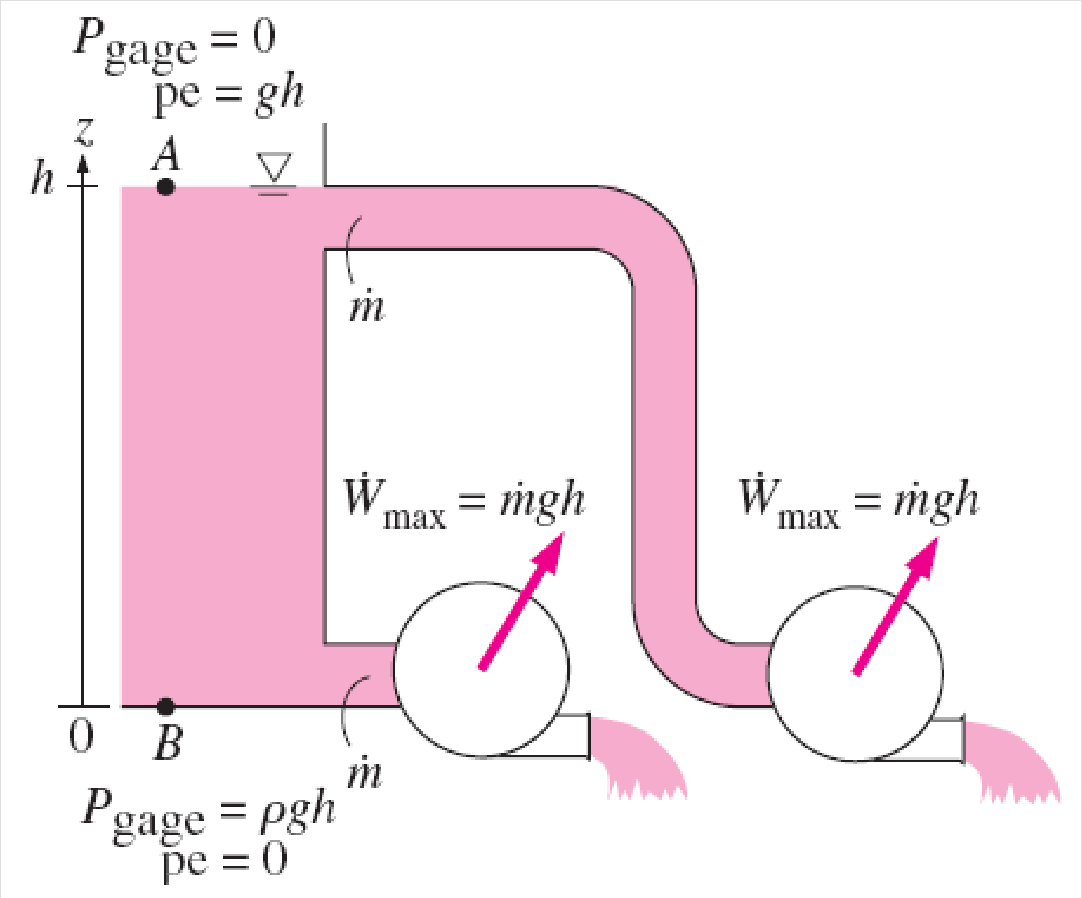


$$\dot{W}_{max} = \dot{m}\Delta e_{mech} = \dot{m}\frac{(P_2 - P_3)}{\rho} = \dot{m}\frac{\Delta P}{\rho}$$

$$\text{since } V_2 \approx V_3 \text{ and } z_2 \approx z_3$$

(b)

Mechanical energy is illustrated by an ideal hydraulic turbine coupled with an ideal generator. In the absence of irreversible losses, the maximum produced power is proportional to (*a*) the change in water surface elevation from the upstream to the downstream reservoir or (*b*) (close-up view) the drop in water pressure from just upstream to just downstream of the turbine.



The available mechanical energy of water at the bottom of a container is equal to the avaiable mechanical energy at any depth including the free surface of the container.

Shaft work: The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as *shaft work*.

A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses).

A turbine converts the mechanical energy of a fluid to shaft work.

$$\eta_{\text{mech}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{\text{mech, out}}}{E_{\text{mech, in}}} = 1 - \frac{E_{\text{mech, loss}}}{E_{\text{mech, in}}}$$

Mechanical efficiency of a device or process

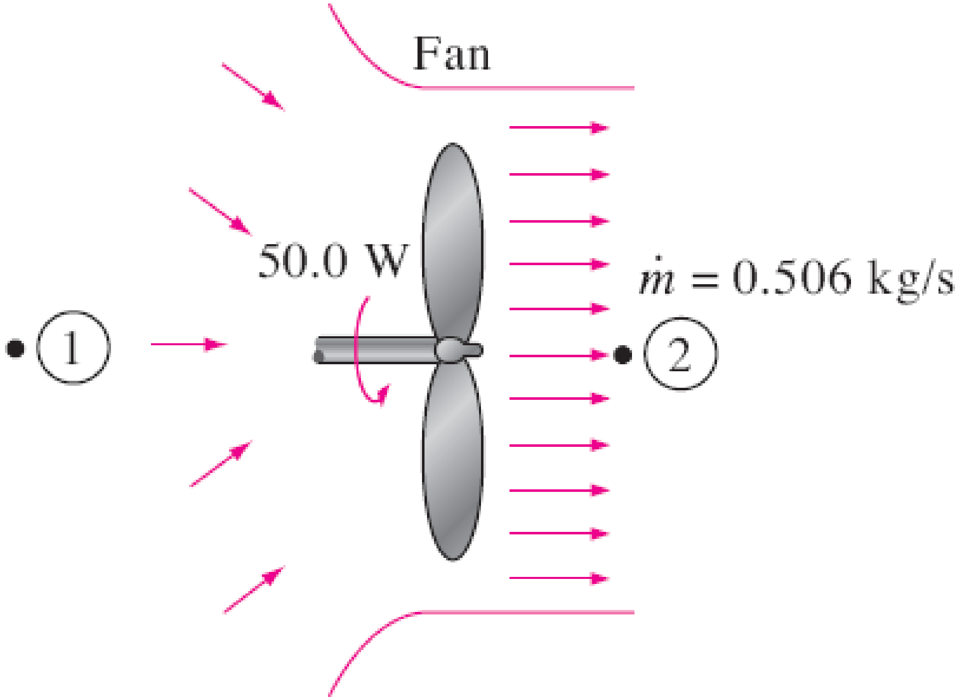
The effectiveness of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the **pump efficiency** and **turbine efficiency**,

$$\eta_{\text{pump}} = \frac{\text{Mechanical power increase of the fluid}}{\text{Mechanical power input}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump}}}$$

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}}$$

$$\eta_{\text{turbine}} = \frac{\text{Mechanical power output}}{\text{Mechanical power decrease of the fluid}} = \frac{\dot{W}_{\text{shaft, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine, }e}}$$

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{E}_{\text{mech, in}} - \dot{E}_{\text{mech, out}}$$



$V_1 \approx 0,\ \bar{V}_2 = 12.1$ m/s

$z_1 = z_2$

$P_1 \approx P_{atm}$ & $P_2 \approx P_{atm}$

$$\eta_{\text{mech, fan}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{m}V_2^2/2}{\dot{W}_{\text{shaft, in}}}$$

$$= \frac{(0.506 \text{ kg/s})(12.1 \text{ m/s})^2/2}{50.0 \text{ W}}$$

$$= 0.741$$

The mechanical efficiency of a fan is the ratio of the kinetic energy of air at the fan exit to the mechanical power input.

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electric power input}} = \frac{\dot{W}_{\text{shaft,out}}}{\dot{W}_{\text{elect,in}}}$$

Motor efficiency

$$\eta_{\text{generator}} = \frac{\text{Electric power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{shaft,in}}}$$

Generator efficiency

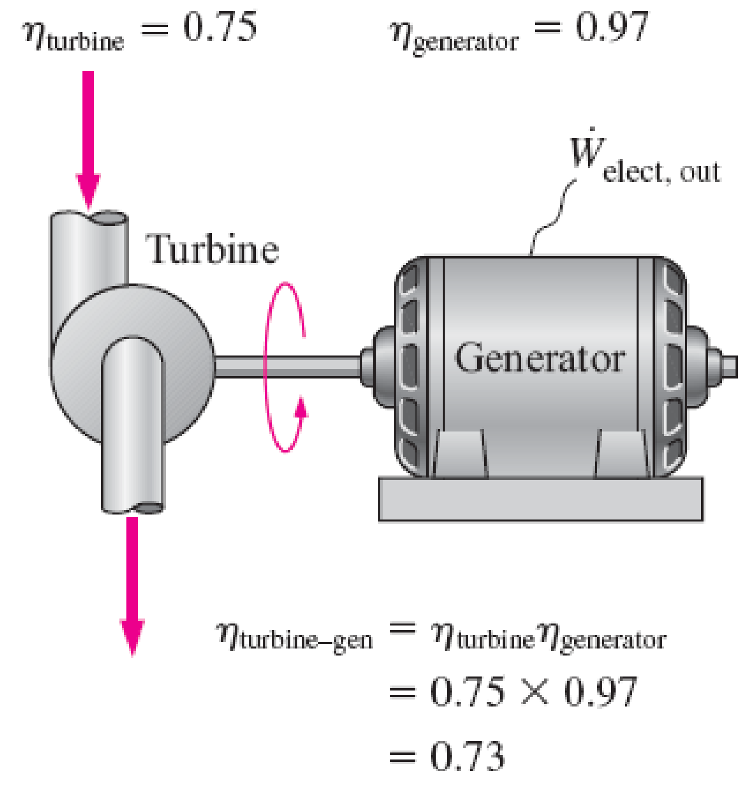
$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\eta_{\text{motor}} = \frac{\dot{W}_{\text{pump},u}}{\dot{W}_{\text{elect,in}}} = \frac{\Delta\dot{E}_{\text{mech,fluid}}}{\dot{W}_{\text{elect,in}}}$$

Pump-Motor overall efficiency

Turbine-Generator overall efficiency:

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect,out}}}{\dot{W}_{\text{turbine},e}} = \frac{\dot{W}_{\text{elect,out}}}{|\Delta\dot{E}_{\text{mech,fluid}}|}$$

The overall efficiency of a turbine–generator is the product of the efficiency of the turbine and the efficiency of the generator, and represents the fraction of the mechanical energy of the fluid converted to electric energy.



The efficiencies just defined range between 0 and 100%.

0% corresponds to the conversion of the entire mechanical or electric energy input to thermal energy, and the device in this case functions like a resistance heater.

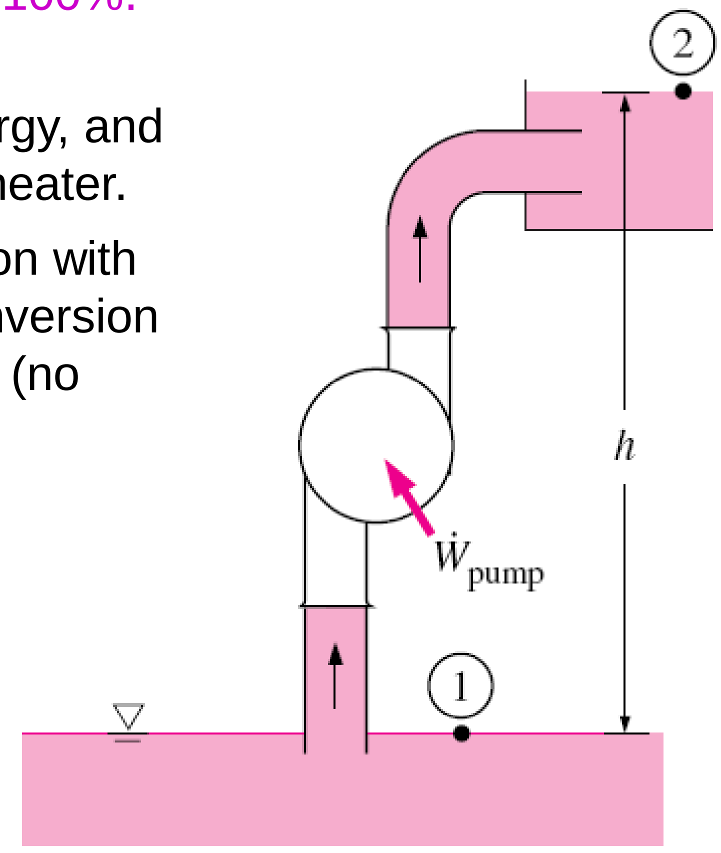
100% corresponds to the case of perfect conversion with no friction or other irreversibilities, and thus no conversion of mechanical or electric energy to thermal energy (no losses).

For systems that involve only *mechanical forms of energy* and its transfer as *shaft work*, the conservation of energy is

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}}$$

$E_{\text{mech, loss}}$: The conversion of mechanical energy to thermal energy due to irreversibilities such as friction.

Many fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a *mechanical energy* balance.



Steady flow

$$V_1 = V_2 \approx 0$$
$$z_2 = z_1 + h$$
$$P_1 = P_2 = P_{\text{atm}}$$

$$\dot{E}_{\text{mech, in}} = \dot{E}_{\text{mech, out}} + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} = \dot{m}gh + \dot{E}_{\text{mech, loss}}$$

EXAMPLE 5–3 **Performance of a Hydraulic Turbine–Generator**

The water in a large lake is to be used to generate electricity by the installation of a hydraulic turbine–generator. The elevation difference between the free surfaces upstream and downstream of the dam is 50 m (Fig. 5–19). Water is to be supplied at a rate of 5000 kg/s. If the electric power generated is measured to be 1862 kW and the generator efficiency is 95 percent, determine (*a*) the overall efficiency of the turbine–generator, (*b*) the mechanical efficiency of the turbine, and (*c*) the shaft power supplied by the turbine to the generator.

SOLUTION A hydraulic turbine–generator is to generate electricity from the water of a lake. The overall efficiency, the turbine efficiency, and the shaft power are to be determined.

Assumptions **1** The elevation of the lake and that of the discharge site remain constant. **2** Irreversible losses in the pipes are negligible.

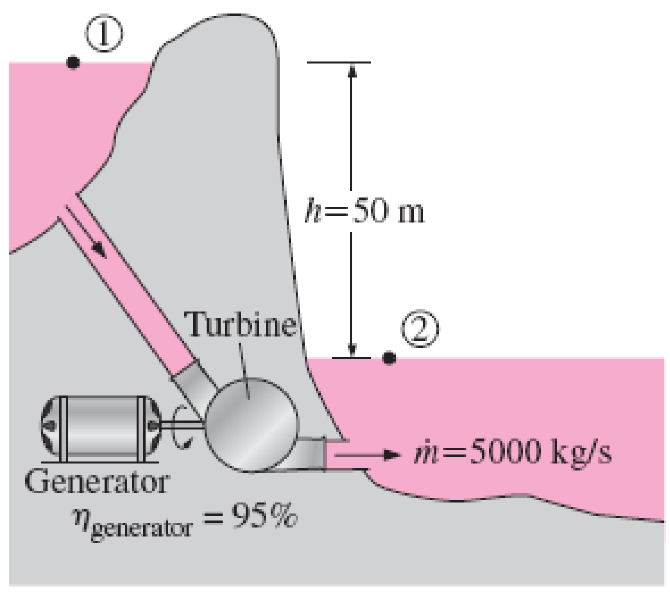
Properties The density of water is taken to be $\rho = 1000\ \text{kg/m}^3$.

Analysis (*a*) We perform our analysis from inlet (1) at the free surface of the lake to outlet (2) at the free surface of the downstream discharge site. At both free surfaces the pressure is atmospheric and the velocity is negligibly small. The change in the water's mechanical energy per unit mass is then

$$e_{\text{mech, in}} - e_{\text{mech, out}} = \underbrace{\cancel{\frac{P_{\text{in}} - P_{\text{out}}}{\rho}}}_{0} + \underbrace{\cancel{\frac{V_{\text{in}}^2 - V_{\text{out}}^2}{2}}}_{0} + g(z_{\text{in}} - z_{\text{out}})$$

$$= gh$$

$$= (9.81\ \text{m/s}^2)(50\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.491\frac{\text{kJ}}{\text{kg}}$$



Then the rate at which mechanical energy is supplied to the turbine by the fluid and the overall efficiency become

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{m}(e_{\text{mech, in}} - e_{\text{mech, out}}) = (5000 \text{ kg/s})(0.491 \text{ kJ/kg}) = 2455 \text{ kW}$$

$$\eta_{\text{overall}} = \eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{1862 \text{ kW}}{2455 \text{ kW}} = \mathbf{2.760}$$

(*b*) Knowing the overall and generator efficiencies, the mechanical efficiency of the turbine is determined from

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} \rightarrow \eta_{\text{turbine}} = \frac{\eta_{\text{turbine-gen}}}{\eta_{\text{generator}}} = \frac{0.76}{0.95} = \mathbf{0.800}$$

(*c*) The shaft power output is determined from the definition of mechanical efficiency,

$$\dot{W}_{\text{shaft, out}} = \eta_{\text{turbine}}|\Delta \dot{E}_{\text{mech, fluid}}| = (0.80)(2455 \text{ kW}) = 1964 \text{ kW} \approx \mathbf{1960 \text{ kW}}$$

Discussion Note that the lake supplies 2455 kW of mechanical power to the turbine, which converts 1964 kW of it to shaft power that drives the generator, which generates 1862 kW of electric power. There are irreversible losses through each component. Irreversible losses in the pipes are ignored here; you will learn how to account for these in Chap. 8.

EXAMPLE 5–4 Conservation of Energy for an Oscillating Steel Ball

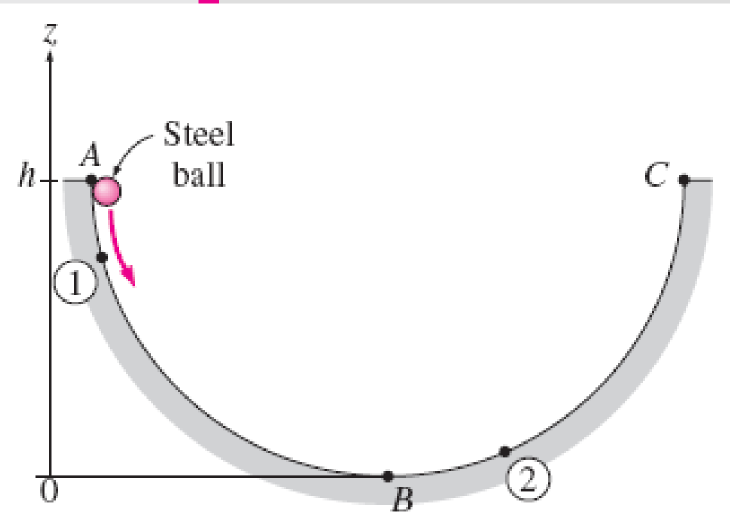
The motion of a steel ball in a hemispherical bowl of radius h shown in Fig. 5–20 is to be analyzed. The ball is initially held at the highest location at point A, and then it is released. Obtain relations for the conservation of energy of the ball for the cases of frictionless and actual motions.

SOLUTION A steel ball is released in a bowl. Relations for the energy balance are to be obtained.

Assumptions For the frictionless case, friction between the ball, the bowl, and the air is negligible.

Analysis When the ball is released, it accelerates under the influence of gravity, reaches a maximum velocity (and minimum elevation) at point B at the bottom of the bowl, and moves up toward point C on the opposite side. In the ideal case of frictionless motion, the ball will oscillate between points A and C. The actual motion involves the conversion of the kinetic and potential energies of the ball to each other, together with overcoming resistance to motion due to friction (doing frictional work). The general energy balance for any system undergoing any process is

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$



Then the energy balance (per unit mass) for the ball for a process from point 1 to point 2 becomes

$$-w_{\text{friction}} = (\text{ke}_2 + \text{pe}_2) - (\text{ke}_1 + \text{pe}_1)$$

or

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{\text{friction}}$$

since there is no energy transfer by heat or mass and no change in the internal energy of the ball (the heat generated by frictional heating is dissipated to the surrounding air). The frictional work term w_{friction} is often expressed as e_{loss} to represent the loss (conversion) of mechanical energy into thermal energy.

For the idealized case of frictionless motion, the last relation reduces to

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 \qquad \text{or} \qquad \frac{V^2}{2} + gz = C = \text{constant}$$

where the value of the constant is $C = gh$. That is, *when the frictional effects are negligible, the sum of the kinetic and potential energies of the ball remains constant.*

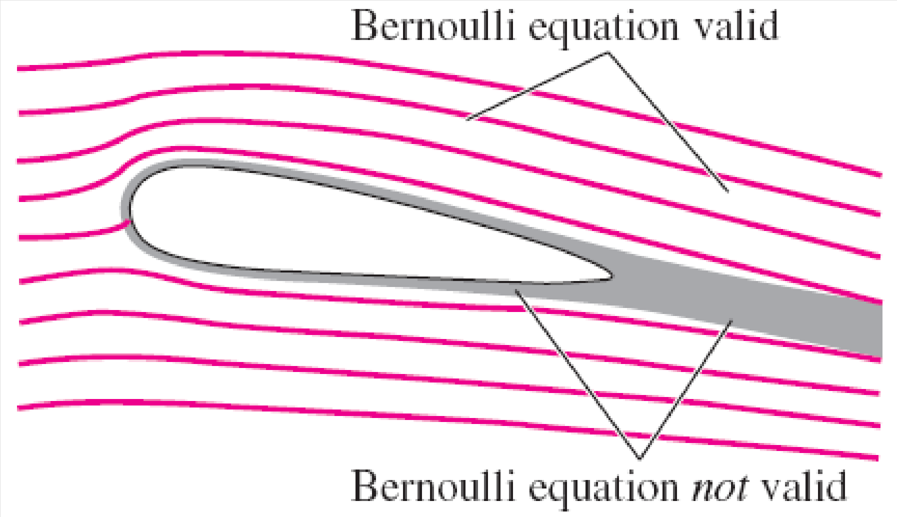
Discussion This is certainly a more intuitive and convenient form of the conservation of energy equation for this and other similar processes such as the swinging motion of a pendulum. The relation obtained is analogous to the Bernoulli equation derived in Section 5–4.

5–4 ■ THE BERNOULLI EQUATION

Bernoulli equation: An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

Acceleration of a Fluid Particle

In two-dimensional flow, the acceleration can be decomposed into two components:

streamwise acceleration $\boldsymbol{a_s}$ along the streamline and

normal acceleration $\boldsymbol{a_n}$ in the direction normal to the streamline, which is given as $a_n = V^2/R$.

Streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction.

For particles that move along a *straight path,* $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

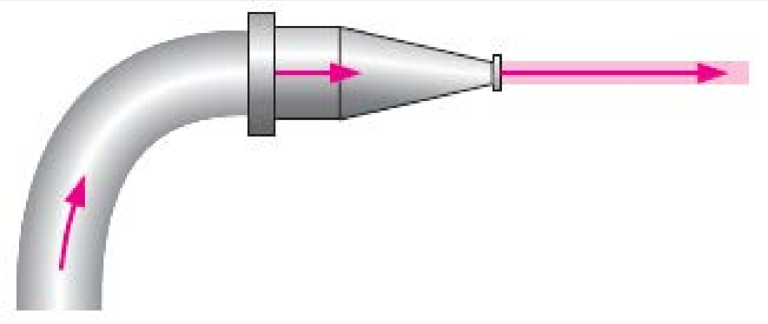
$$\partial V/\partial t = 0$$

$$V = V(s)$$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds}$$

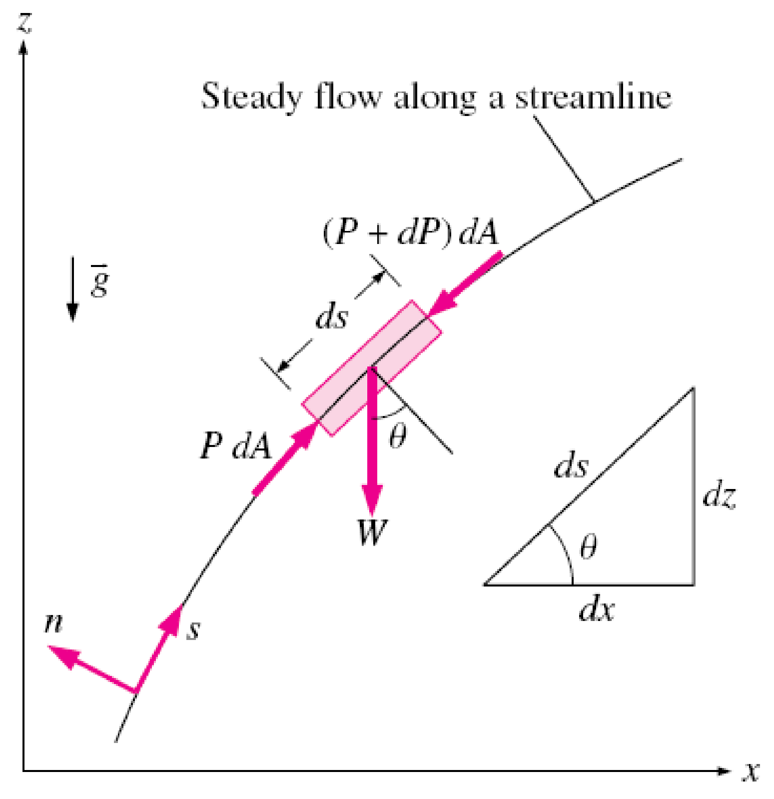
$$V = ds/dt$$

Acceleration in steady flow is due to the change of velocity with position.



During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

Derivation of the Bernoulli Equation



The forces acting on a fluid particle along a streamline.

$$\sum F_s = ma_s \qquad P\,dA - (P + dP)\,dA - W\sin\theta = mV\frac{dV}{ds}$$

$$m = \rho V = \rho\,dA\,ds \qquad W = mg = \rho g\,dA\,ds$$

$$\sin\theta = dz/ds \qquad -dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V\frac{dV}{ds}$$

$$-dP - \rho g\,dz = \rho V\,dV \qquad V\,dV = \tfrac{1}{2}\,d(V^2)$$

$$\frac{dP}{\rho} + \tfrac{1}{2}\,d(V^2) + g\,dz = 0$$

***Steady flow*:**

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

***Steady, incompressible flow*:**

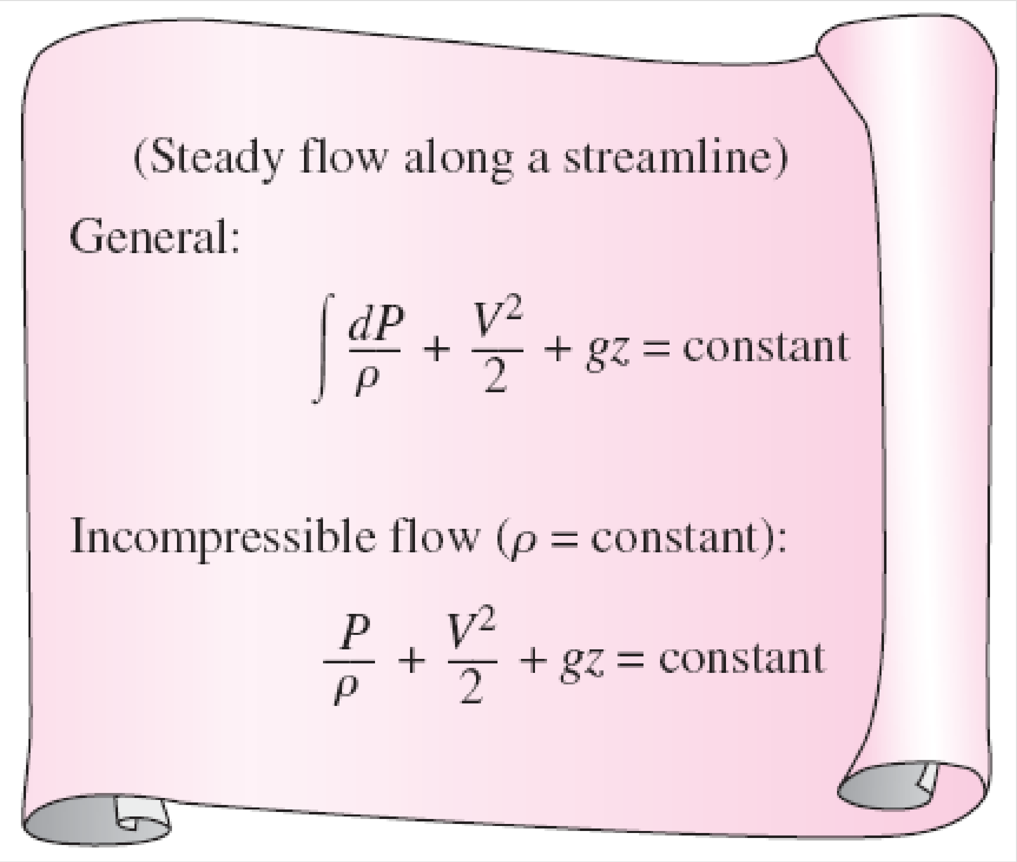
Bernoulli equation

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

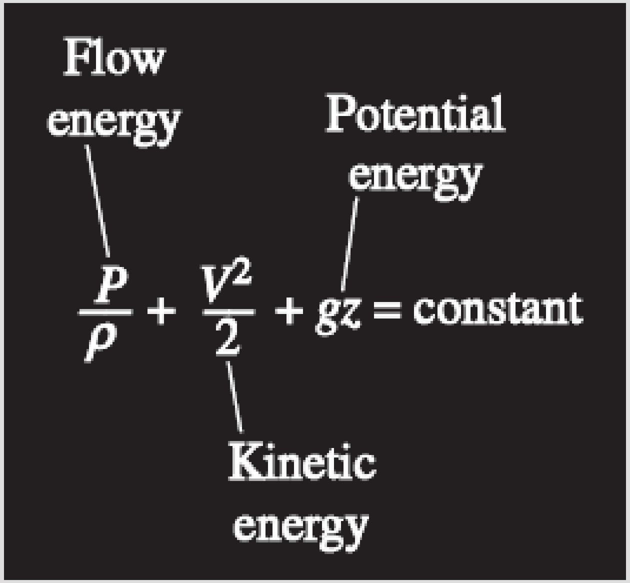
The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

The Bernoulli equation between any two points on the same streamline:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$



The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.



The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

- The Bernoulli equation can be viewed as the "*conservation of mechanical energy principle*."
- This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.
- The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.
- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.
- The Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it.

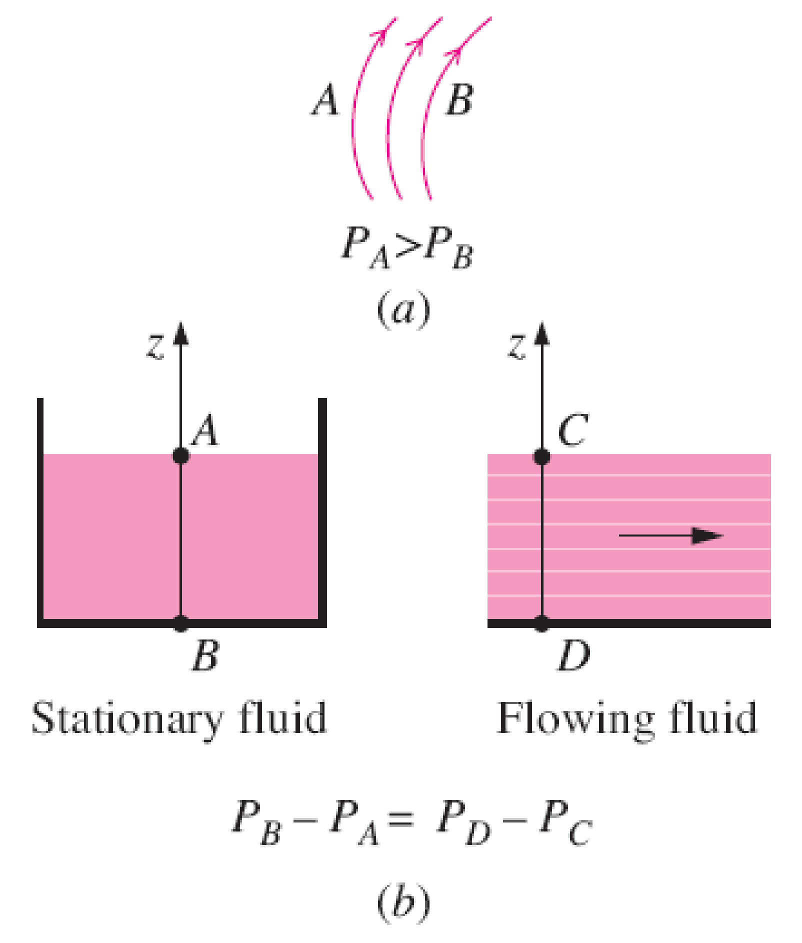
Force Balance across Streamlines

Force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R}\,dn + gz = \text{constant} \qquad \text{(across streamlines)}$$

For flow along a straight line, $R \rightarrow \infty$ and this equation reduces to $P/\rho + gz = \text{constant}$ or $P = -\rho gz + \text{constant}$, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body.

Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.



Unsteady, Compressible Flow

The Bernoulli equation for *unsteady, compressible flow:*

Unsteady, compressible flow: $$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{constant}$$

Static, Dynamic, and Stagnation Pressures

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

***P* is the static pressure:** It does not incorporate any dynamic effects; it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

$\rho V^2/2$ is the dynamic pressure: It represents the pressure rise when the fluid in motion is brought to a stop isentropically.

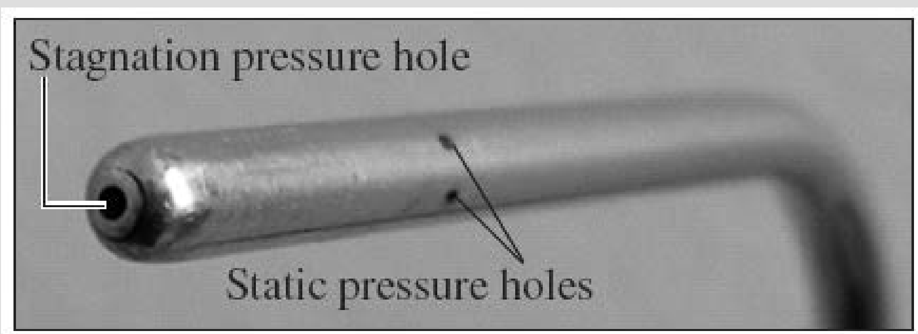
ρgz is the hydrostatic pressure: It is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure ρgh which *increases* with fluid depth h, the hydrostatic pressure term ρgz *decreases* with fluid depth.)

Total pressure: The sum of the static, dynamic, and hydrostatic pressures. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.

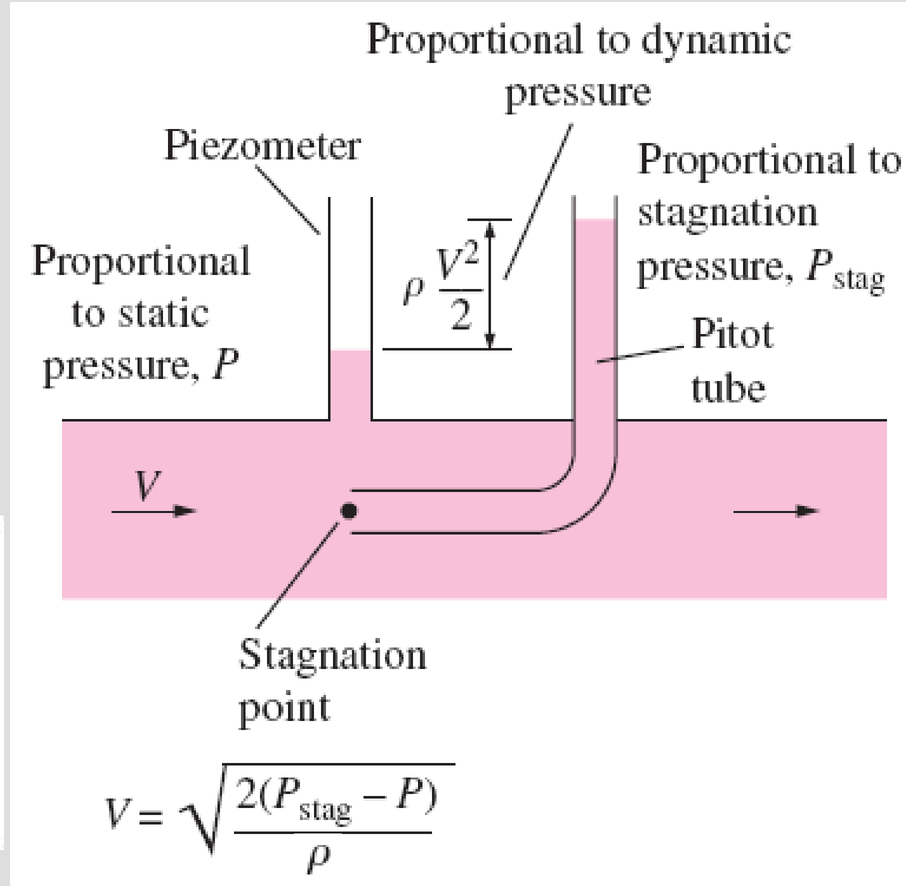
Stagnation pressure: The sum of the static and dynamic pressures. It represents the pressure at a point where the fluid is brought to a complete stop isentropically.

$$P_{stag} = P + \rho \frac{V^2}{2} \qquad \text{(kPa)}$$

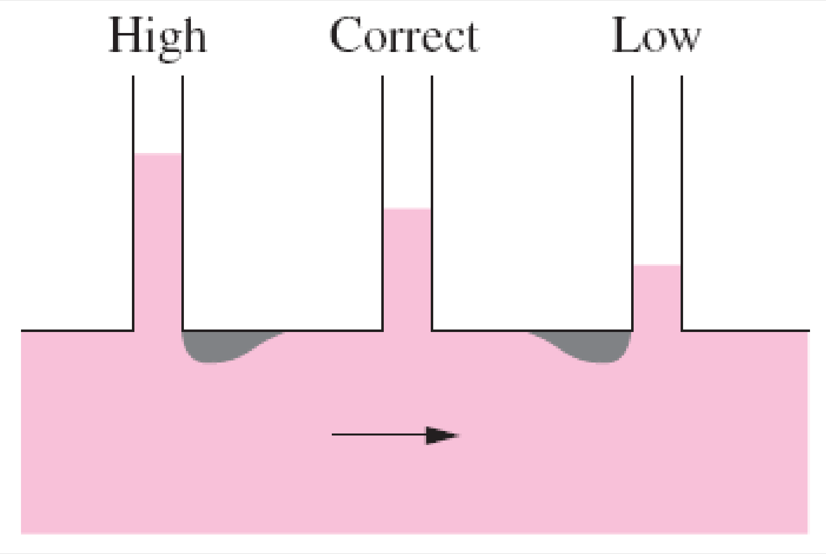
$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$



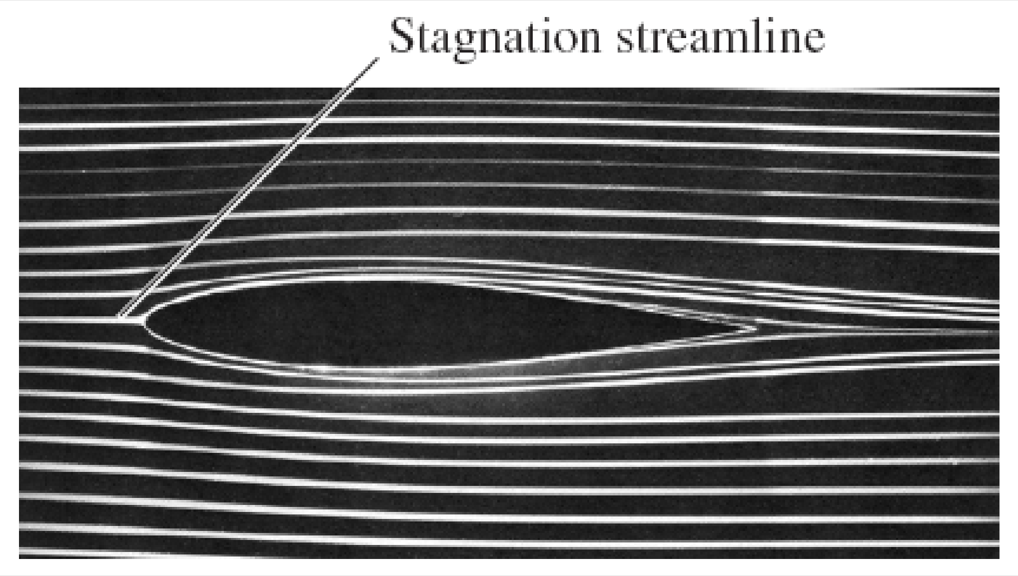
Close-up of a **Pitot-static probe**, showing the stagnation pressure hole and two of the five static circumferential pressure holes.



The static, dynamic, and stagnation pressures measured using **piezometer tubes**.



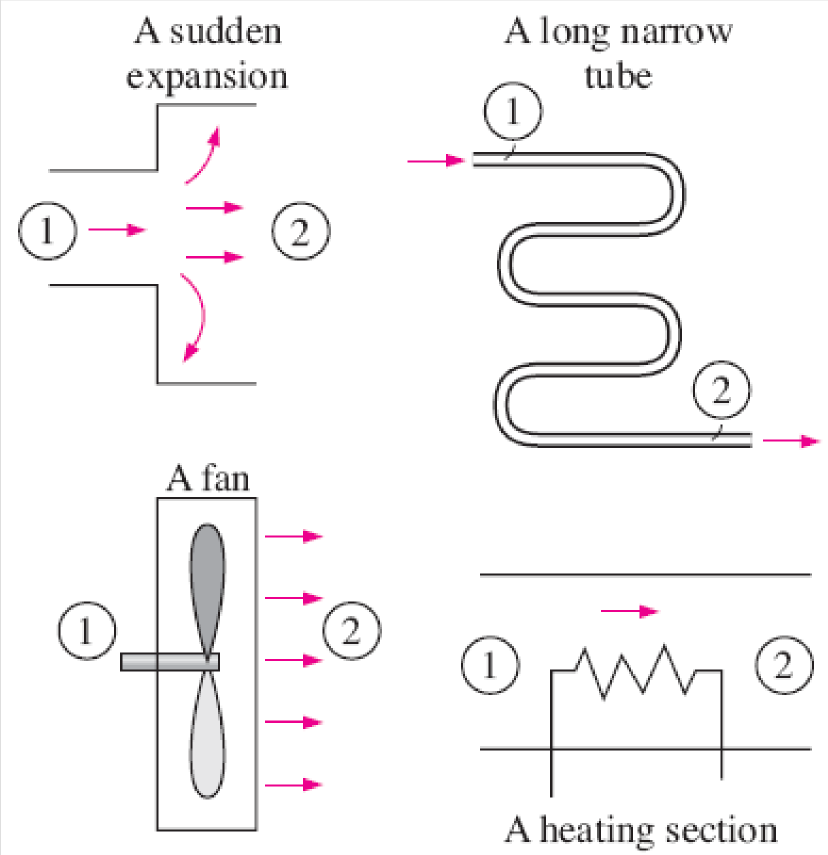
Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.

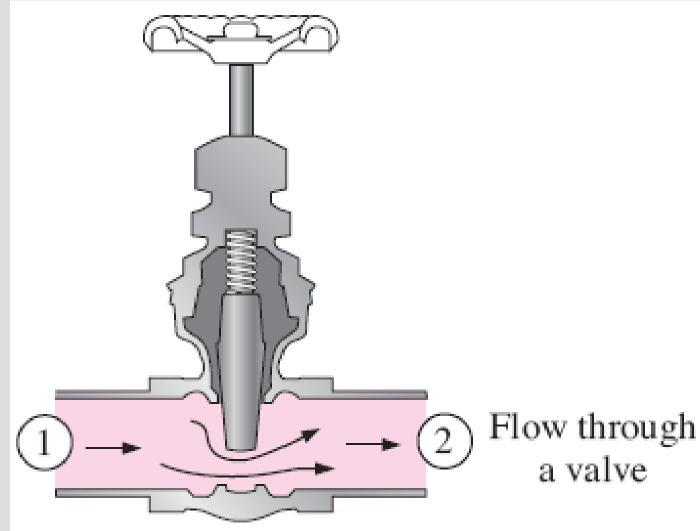


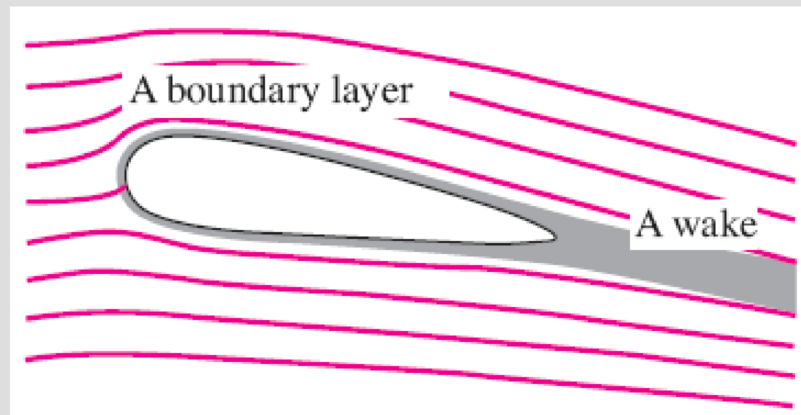
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The **stagnation streamline** is marked.

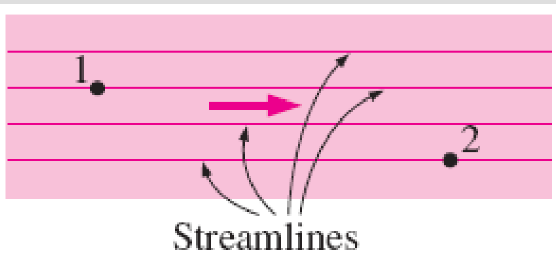
Limitations on the Use of the Bernoulli Equation

1. **Steady flow** The Bernoulli equation is applicable to *steady flow*.
2. **Frictionless flow** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.
3. **No shaft work** The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. When these devices exist, the energy equation should be used instead.
4. **Incompressible flow** Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.
5. **No heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
6. **Flow along a streamline** Strictly speaking, the Bernoulli equation is applicable along a streamline. However, when a region of the flow is *irrotational* and there is negligibly small *vorticity* in the flow field, the Bernoulli equation becomes applicable *across* streamlines as well.









$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

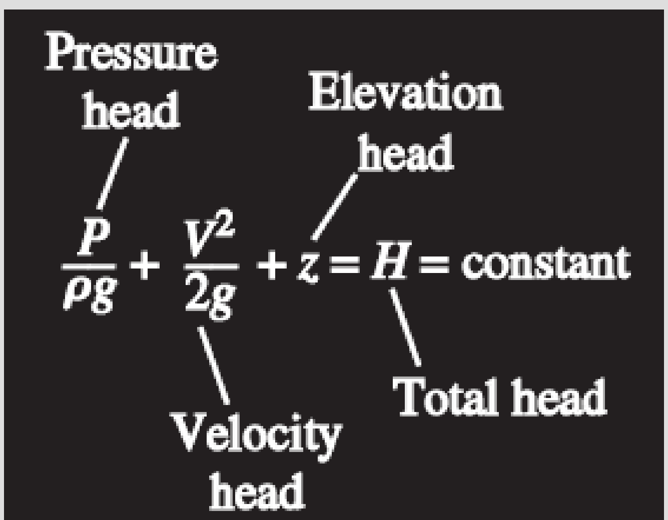
It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. Dividing each term of the Bernoulli equation by *g* gives

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)}$$

$P/\rho g$ is the pressure head; it represents the height of a fluid column that produces the static pressure *P*.

$V^2/2g$ is the velocity head; it represents the elevation needed for a fluid to reach the velocity *V* during frictionless free fall.

z is the elevation head; it represents the potential energy of the fluid.

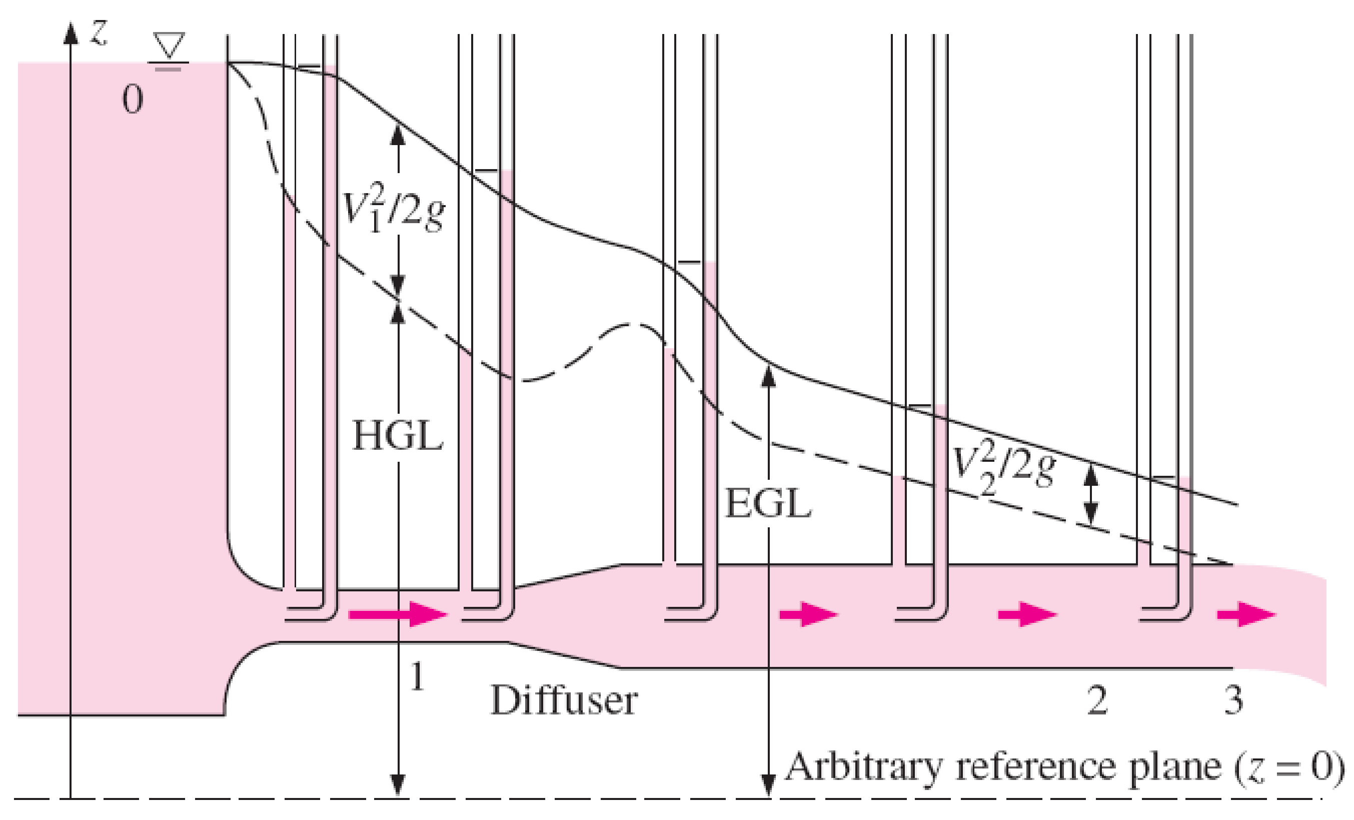


An alternative form of the Bernoulli equation is expressed in terms of heads as: ***The sum of the pressure, velocity, and elevation heads is constant along a streamline*.**

Hydraulic grade line (HGL), $P/\rho g + z$ The line that represents the sum of the static pressure and the elevation heads.

Energy grade line (EGL), $P/\rho g + V^2/2g + z$ The line that represents the total head of the fluid.

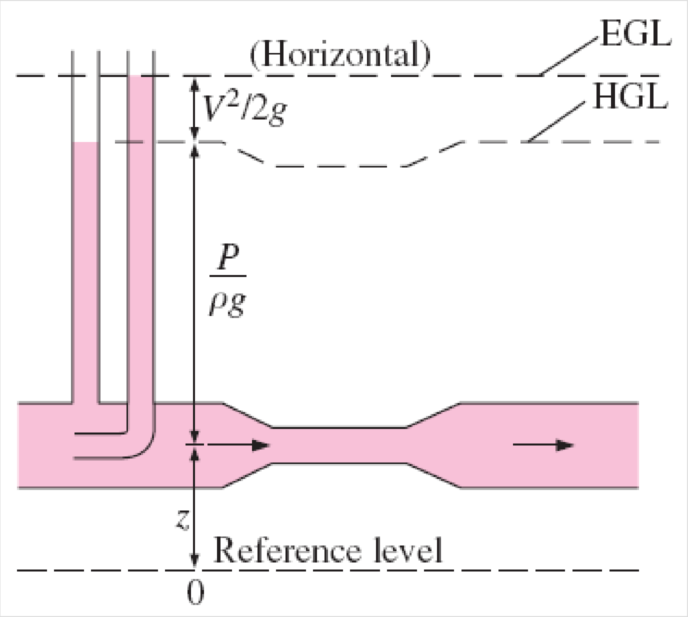
Dynamic head, $V^2/2g$ The difference between the heights of EGL and HGL.



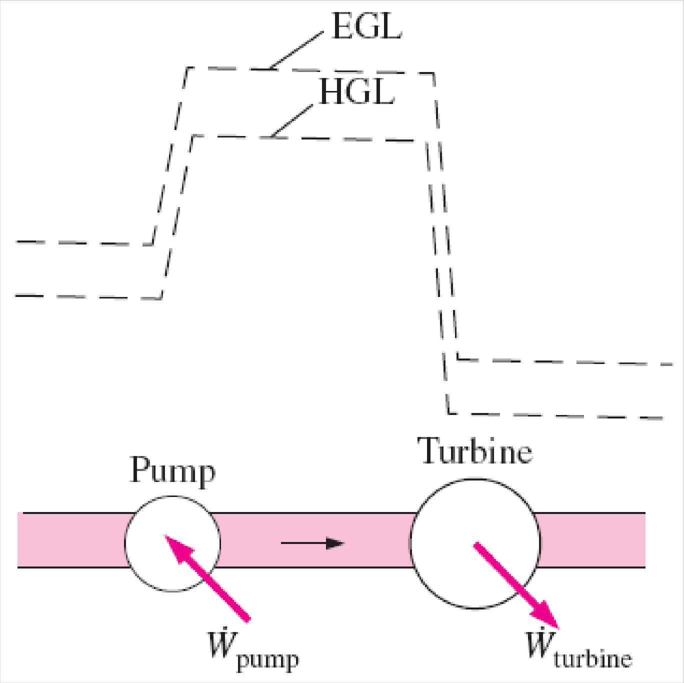
The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

Notes on HGL and EGL

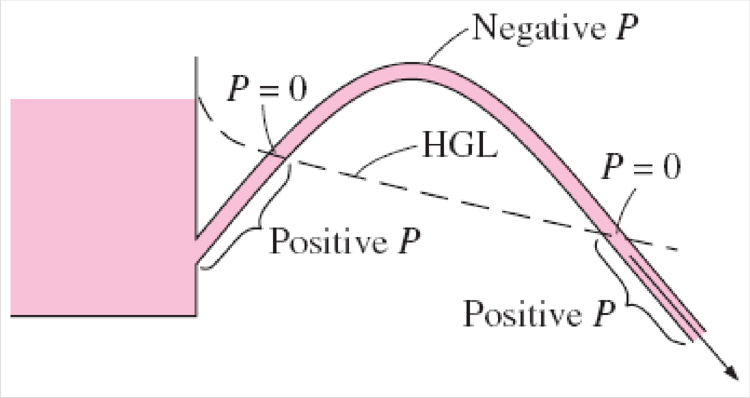
- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid.
- The EGL is always a distance $V^2/2g$ above the HGL. These two curves approach each other as the velocity decreases, and they diverge as the velocity increases.
- In an *idealized Bernoulli-type flow*, EGL is horizontal and its height remains constant.
- For *open-channel flow*, the HGL coincides with the free surface of the liquid, and the EGL is a distance $V^2/2g$ above the free surface.
- At a *pipe exit*, the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet.
- The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the head loss in the pipe. A component, such as a valve, that generates significant frictional effects causes a sudden drop in both EGL and HGL at that location.
- A *steep jump/drop* occurs in EGL and HGL whenever mechanical energy is added or removed to or from the fluid (pump, turbine).
- The (gage) pressure of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive.



In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.

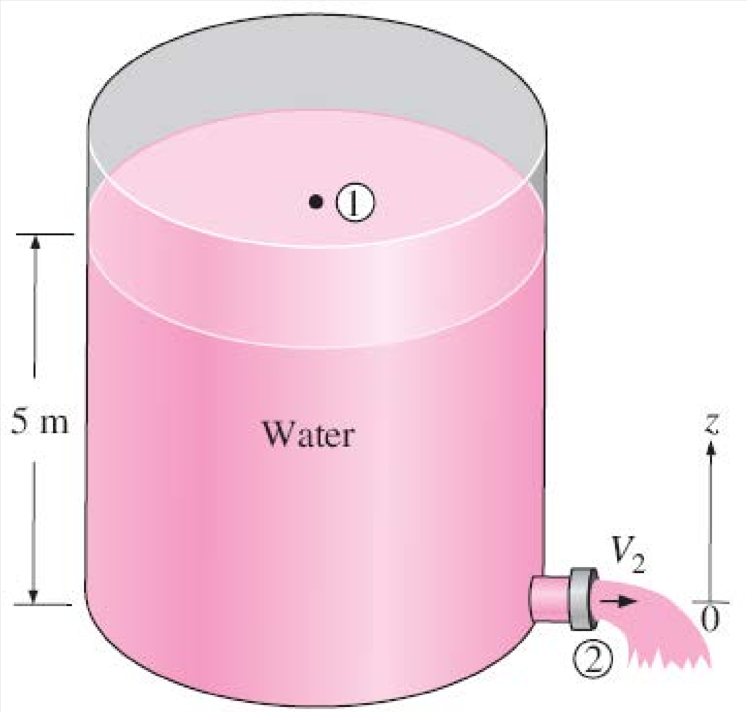


A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.



The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the pressure is negative (vacuum) in a flow section that lies above the HGL.

Example: Water Discharge from a Large Tank



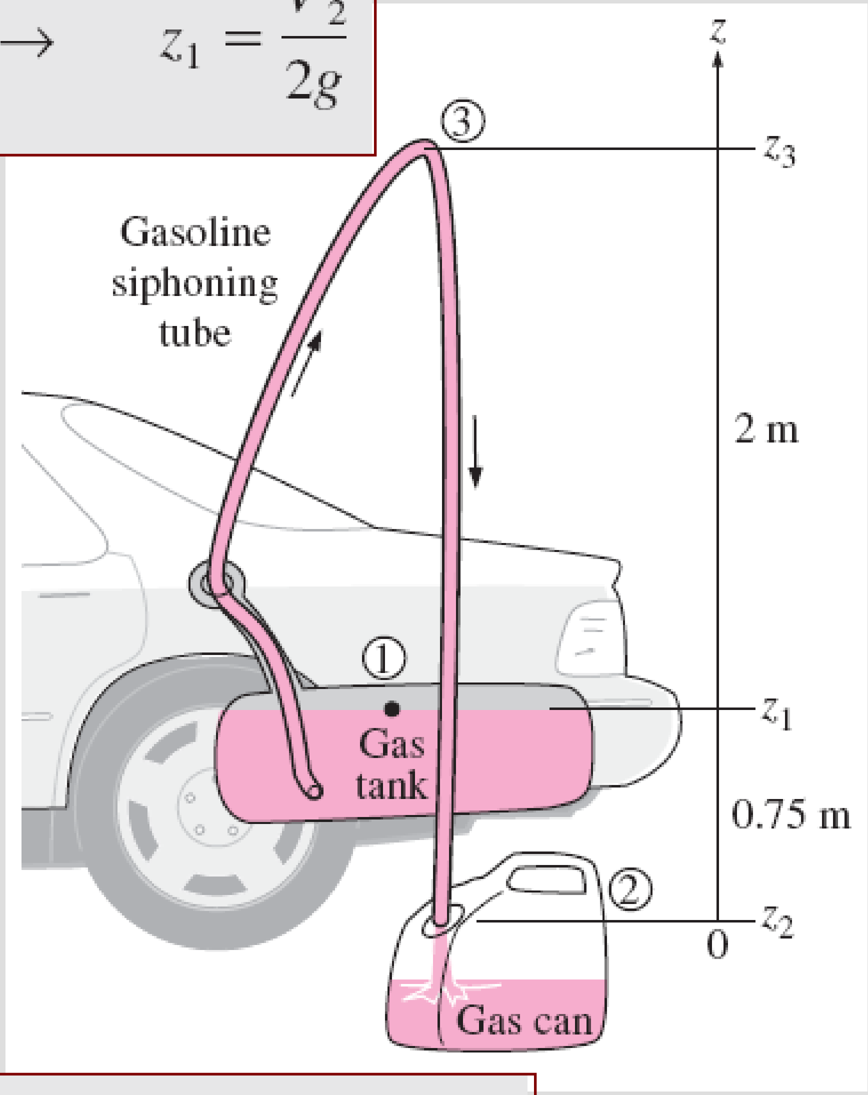
$$\frac{\cancel{P_1}}{\rho g} + \overset{\approx 0}{\frac{V_1^2}{2g}} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + \overset{0}{z_2} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Example: Spraying Water into the Air

$$\frac{P_1}{\rho g} + \overset{\approx 0}{\frac{V_1^2}{2g}} + \overset{0}{z_1} = \frac{P_2}{\rho g} + \overset{0}{\frac{V_2^2}{2g}} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

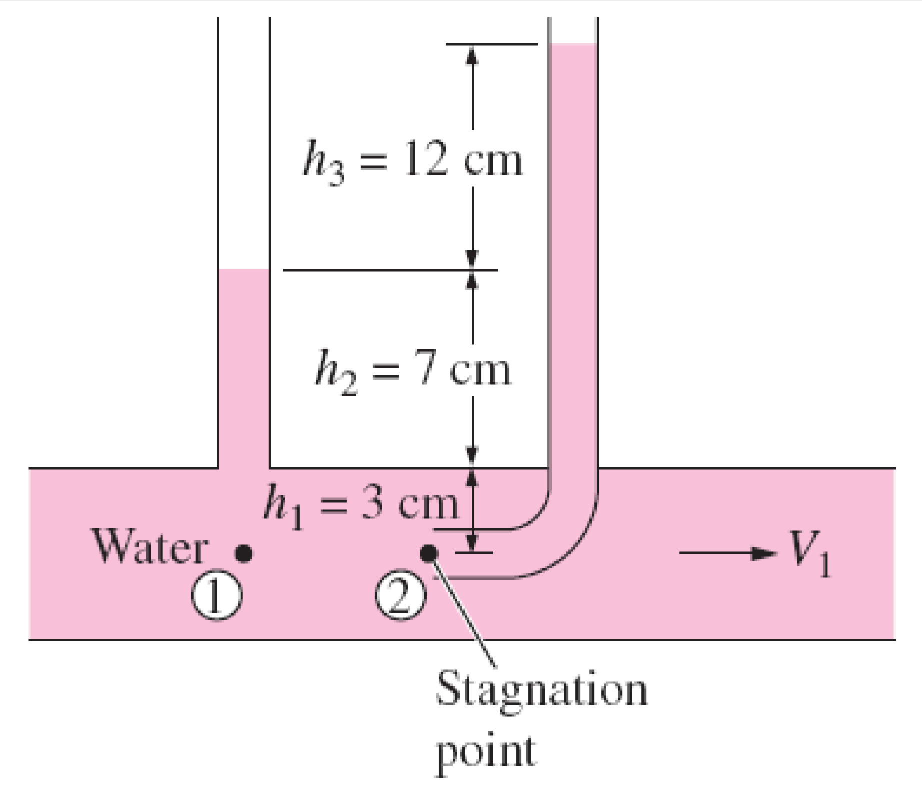
$$\frac{\cancel{P_1}}{\rho g} + \frac{V_1^2}{2g}^{\nearrow \approx 0} + z_1 = \frac{\cancel{P_2}}{\rho g} + \frac{V_2^2}{2g} + z_2^{\nearrow 0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Example: Siphoning Out Gasoline from a Fuel Tank



$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2^{\nearrow 0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

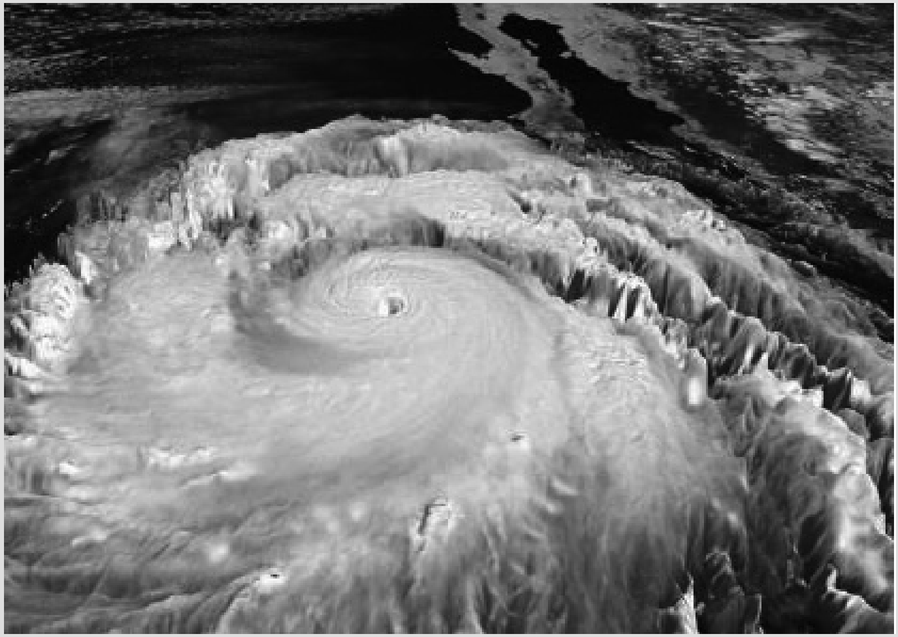
Example: Velocity Measurement by a Pitot Tube



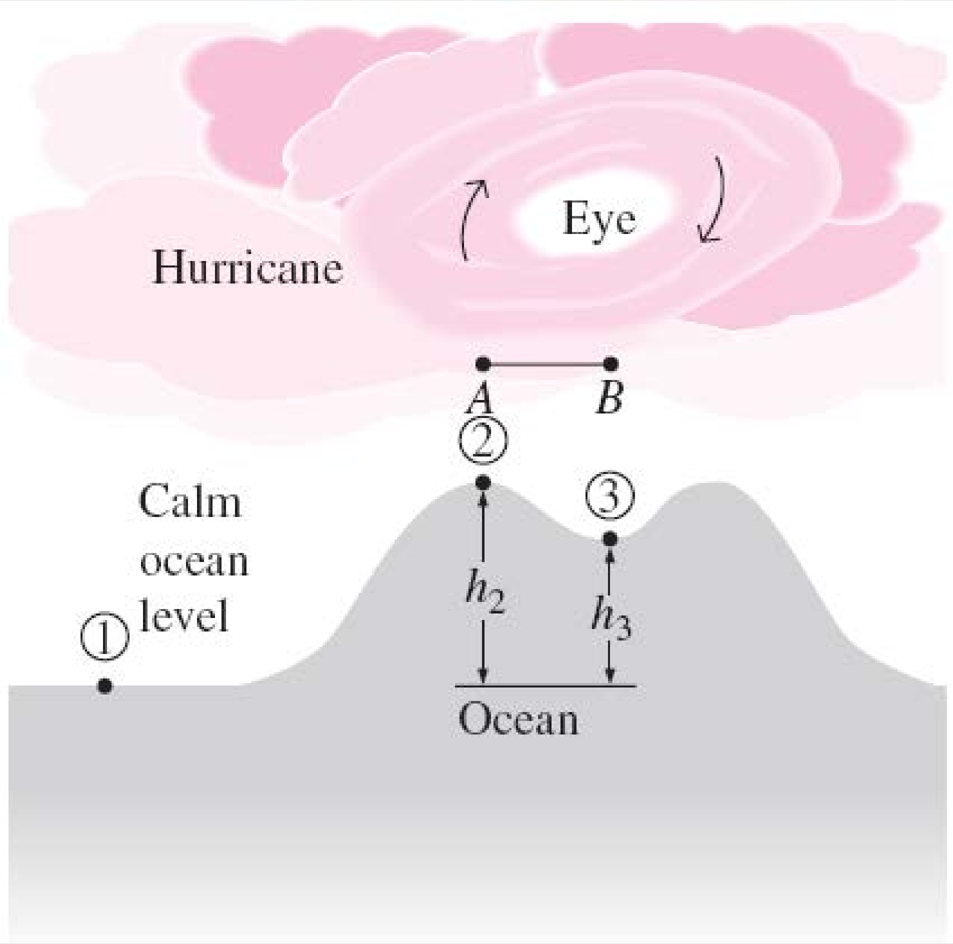
$$P_1 = \rho g(h_1 + h_2)$$
$$P_2 = \rho g(h_1 + h_2 + h_3)$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancelto{0}{\frac{V_2^2}{2g}} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Example: The Rise of the Ocean Due to a Hurricane



The eye of hurricane Linda (1997 in the Pacific Ocean near Baja California) is clearly visible in this satellite photo.



$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+\cancel{z_A}=\frac{P_B}{\rho g}+\cancelto{0}{\frac{V_B^2}{2g}}+\cancel{z_B} \quad\rightarrow\quad \frac{P_B-P_A}{\rho g}=\frac{V_A^2}{2g}$$

EXAMPLE 5–10 **Bernoulli Equation for Compressible Flow**

Derive the Bernoulli equation when the compressibility effects are not negligible for an ideal gas undergoing (*a*) an isothermal process and (*b*) an isentropic process.

SOLUTION The Bernoulli equation for compressible flow is to be obtained for an ideal gas for isothermal and isentropic processes.

Assumptions **1** The flow is steady and frictional effects are negligible. **2** The fluid is an ideal gas, so the relation $P = \rho RT$ is applicable. **3** The specific heats are constant so that P/ρ^k = constant during an isentropic process.

Analysis (*a*) When the compressibility effects are significant and the flow cannot be assumed to be incompressible, the Bernoulli equation is given by Eq. 5–40 as

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant} \qquad \text{(along a streamline)} \qquad (1)$$

The compressibility effects can be properly accounted for by performing the integration $\int dP/\rho$ in Eq. 1. But this requires a relation between P and ρ for the process. For the *isothermal* expansion or compression of an ideal gas, the integral in Eq. 1 is performed easily by noting that T = constant and substituting $\rho = P/RT$,

$$\int \frac{dP}{\rho} = \int \frac{dP}{P/RT} = RT \ln P$$

Substituting into Eq. 1 gives the desired relation,

Isothermal process: $$RT \ln P + \frac{V^2}{2} + gz = \text{constant} \qquad (2)$$

(b) A more practical case of compressible flow is the *isentropic flow of ideal gases* through equipment that involves high-speed fluid flow such as nozzles, diffusers, and the passages between turbine blades (Fig. 5–45). Isentropic (i.e., reversible and adiabatic) flow is closely approximated by these devices, and it is characterized by the relation $P/\rho^k = C =$ constant, where k is the specific heat ratio of the gas. Solving for ρ from $P/\rho^k = C$ gives $\rho = C^{-1/k}P^{1/k}$. Performing the integration,

$$\int \frac{dP}{\rho} = \int C^{1/k}P^{-1/k}\,dP = C^{1/k}\frac{P^{-1/k+1}}{-1/k+1} = \frac{P^{1/k}}{\rho}\frac{P^{-1/k+1}}{-1/k+1} = \left(\frac{k}{k-1}\right)\frac{P}{\rho} \qquad (3)$$

Substituting, the Bernoulli equation for steady, isentropic, compressible flow of an ideal gas becomes

Isentropic flow:

$$\left(\frac{k}{k-1}\right)\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant} \qquad (4a)$$

or

$$\left(\frac{k}{k-1}\right)\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \left(\frac{k}{k-1}\right)\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 \qquad (4b)$$

FIGURE 5–45
Compressible flow of a gas through turbine blades is often modeled as isentropic, and the compressible form of the Bernoulli equation is a reasonable approximation.

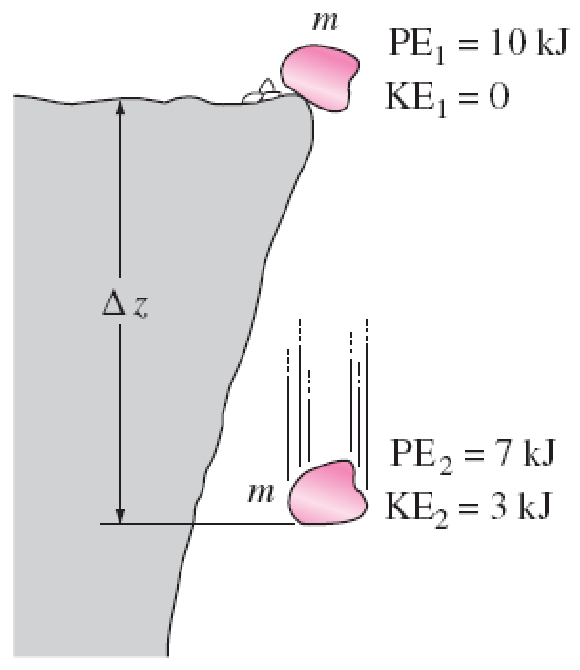
A common practical situation involves the acceleration of a gas from rest (stagnation conditions at state 1) with negligible change in elevation. In that case we have $z_1 = z_2$ and $V_1 = 0$. Noting that $\rho = P/RT$ for ideal gases, P/ρ^k = constant for isentropic flow, and the Mach number is defined as $\mathrm{Ma} = V/c$ where $c = \sqrt{kRT}$ is the local speed of sound for ideal gases, Eq. 4b simplifies to

$$\frac{P_1}{P_2} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{Ma}_2^2\right]^{k/(k-1)} \qquad \textbf{(4c)}$$

where state 1 is the stagnation state and state 2 is any state along the flow.

Discussion It can be shown that the results obtained using the compressible and incompressible equations deviate no more than 2 percent when the Mach number is less than 0.3. Therefore, the flow of an ideal gas can be considered to be incompressible when $\mathrm{Ma} \lesssim 0.3$. For atmospheric air at normal conditions, this corresponds to a flow speed of about 100 m/s or 360 km/h.

5–5 ■ GENERAL ENERGY EQUATION



The first law of thermodynamics (the conservation of energy principle): Energy cannot be created or destroyed during a process; it can only change forms.

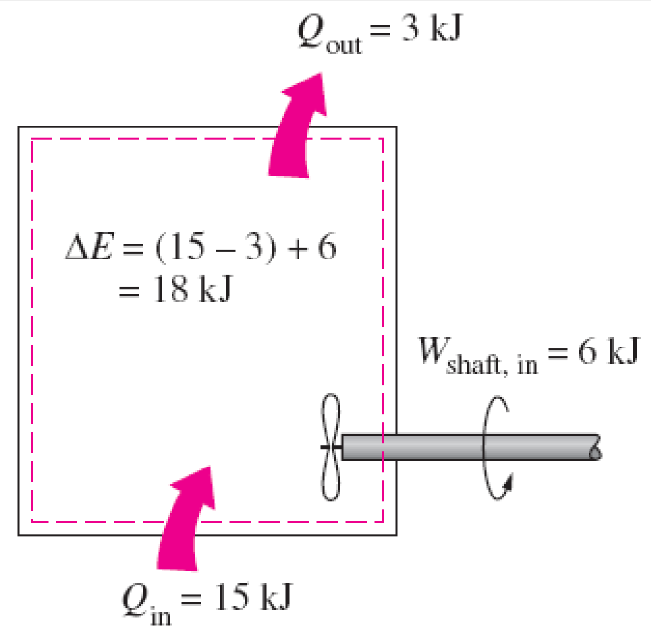
$$E_{in} - E_{out} = \Delta E$$

$$\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{dE_{sys}}{dt}$$

$$\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{d}{dt}\int_{sys} \rho e \, dV$$

$$\dot{Q}_{net\ in} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \dot{W}_{net\ in} = \dot{W}_{in} - \dot{W}_{out}$$

$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz$$



The energy change of a system during a process is equal to the *net* work and heat transfer between the system and its surroundings.

Energy Transfer by Heat, *Q*

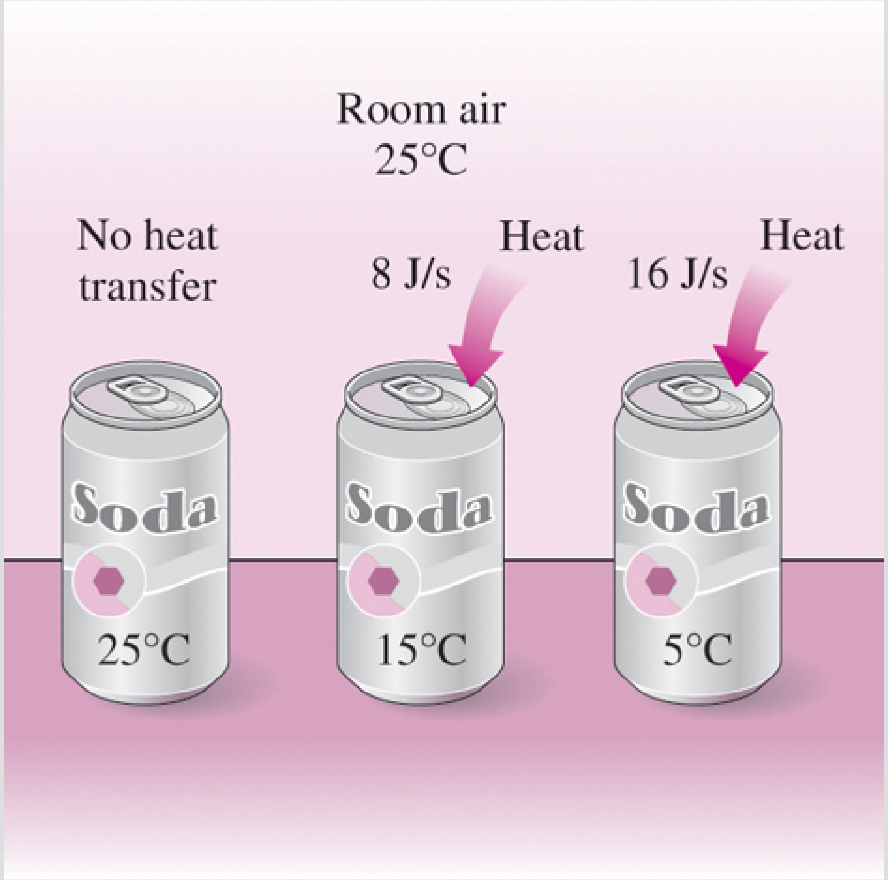
Thermal energy: The sensible and latent forms of internal energy.

Heat Transfer: The transfer of energy from one system to another as a result of a temperature difference.

The direction of heat transfer is always from the higher-temperature body to the lower-temperature one.

Adiabatic process: A process during which there is no heat transfer.

Heat transfer rate: The time rate of heat transfer.



Temperature difference is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.

Energy Transfer by Work, *W*

- **Work**: The energy transfer associated with a force acting through a distance.
- A rising piston, a rotating shaft, and an electric wire crossing the system boundaries are all associated with work interactions.
- **Power:** The time rate of doing work.
- Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, fans, and mixers consume work.

$$W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}}$$

W_{shaft} The work transmitted by a rotating shaft
$W_{pressure}$ The work done by the pressure forces on the control surface
$W_{viscous}$ The work done by the normal and shear components of viscous forces on the control surface
W_{other} The work done by other forces such as electric, magnetic, and surface tension

Shaft Work

A force F acting through a moment arm r generates a torque T

$$T = Fr \quad \rightarrow \quad F = \frac{T}{r}$$

This force acts through a distance s

$$s = (2\pi r)n$$

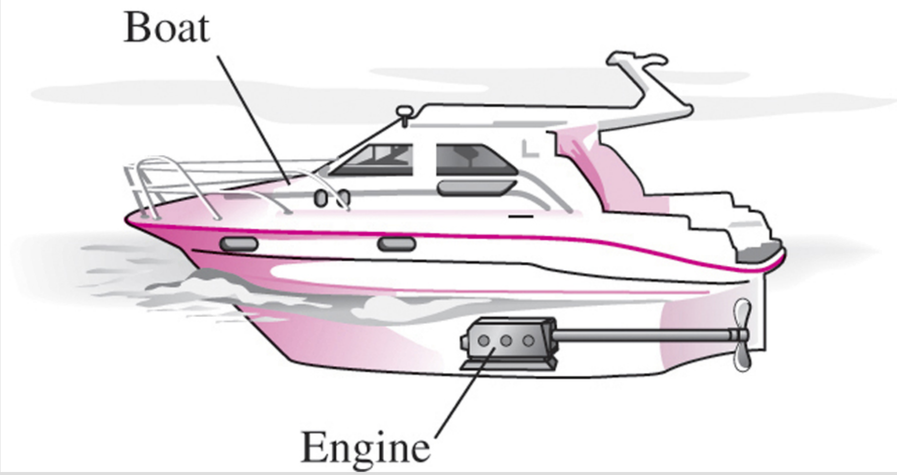
Shaft work

$$W_{sh} = Fs = \left(\frac{T}{r}\right)(2\pi rn) = 2\pi nT \qquad (kJ)$$

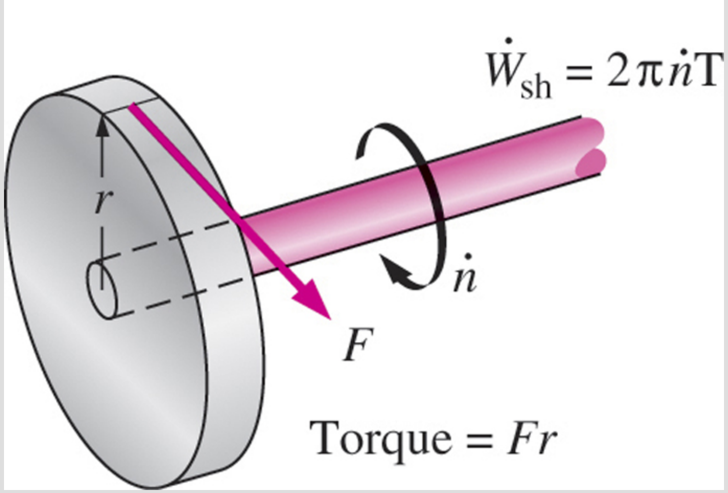
The power transmitted through the shaft is the shaft work done per unit time:

$$\dot{W}_{shaft} = \omega T_{shaft} = 2\pi\dot{n}T_{shaft}$$

$$\dot{W}_{sh} = 2\pi\dot{n}T \qquad (kW)$$



Energy transmission through rotating shafts is commonly encountered in practice.



Shaft work is proportional to the torque applied and the number of revolutions of the shaft.

Work Done by Pressure Forces

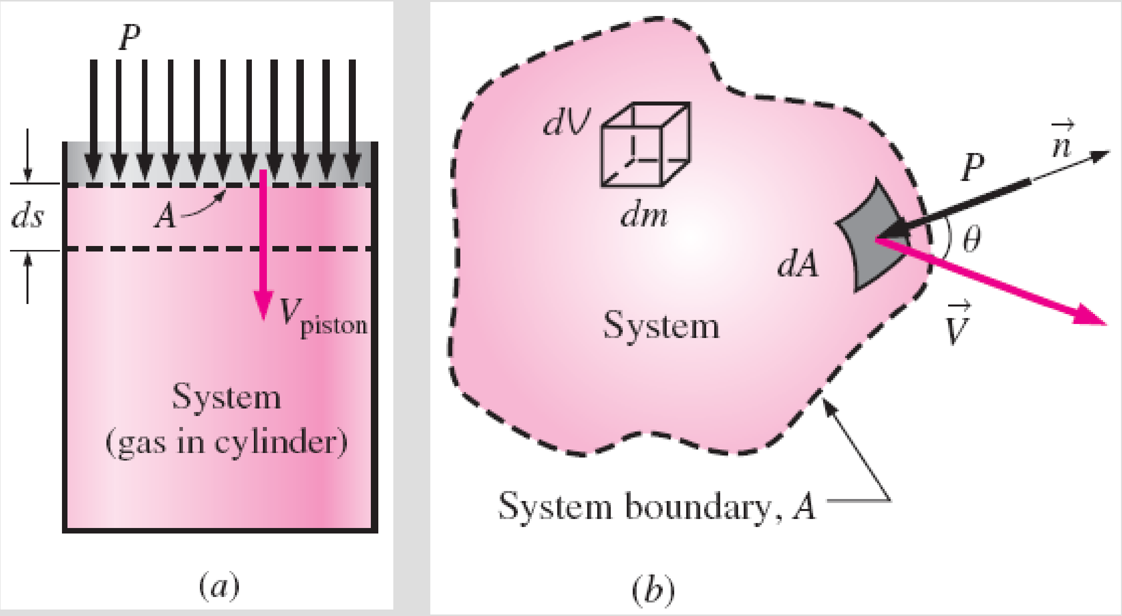
$$\delta W_{\text{boundary}} = PA\ ds$$

$$\delta \dot{W}_{\text{pressure}} = \delta \dot{W}_{\text{boundary}} = PAV_{\text{piston}} \qquad V_{\text{piston}} = ds/dt$$

$$\delta \dot{W}_{\text{pressure}} = -P\ dA\ V_n = -P\ dA(\vec{V}\cdot\vec{n})$$

$$\dot{W}_{\text{pressure, net in}} = -\int_A P(\vec{V}\cdot\vec{n})\ dA = -\int_A \frac{P}{\rho}\rho(\vec{V}\cdot\vec{n})\ dA$$

$$\dot{W}_{\text{net in}} = \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \dot{W}_{\text{shaft, net in}} - \int_A P(\vec{V}\cdot\vec{n})\ dA$$



The pressure force acting on (*a*) the moving boundary of a system in a piston-cylinder device, and (*b*) the differential surface area of a system of arbitrary shape.

The conservation of energy equation is obtained by replacing *B* in the Reynolds transport theorem by energy *E* and *b* by *e*.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho dV + \int_{CS} b\rho(V_r \cdot \vec{n})\, dA$$

$$B = E \qquad b = e \qquad b = e$$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{dE_{sys}}{dt}$$

$$e = u + \text{ke} + \text{pe} = u + V^2/2 + gz$$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})A$$

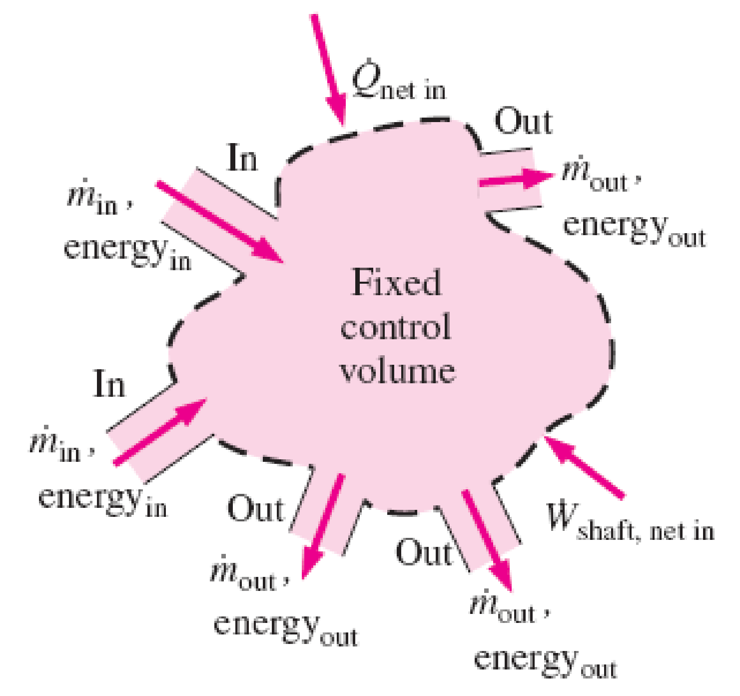
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} + \dot{W}_{\text{pressure, net in}} = \frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS} e\rho(\vec{V}_r \cdot \vec{n})\, dA$$

$$\begin{pmatrix}\text{The net rate of energy}\\ \text{transfer into a CV by}\\ \text{heat and work transfer}\end{pmatrix} = \begin{pmatrix}\text{The time rate of}\\ \text{change of the energy}\\ \text{content of the CV}\end{pmatrix} + \begin{pmatrix}\text{The net flow rate of}\\ \text{energy out of the control}\\ \text{surface by mass flow}\end{pmatrix}$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho(\vec{V}_r \cdot \vec{n})\, dA$$

Fixed CV: $$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(\frac{P}{\rho} + e\right)\rho(\vec{V} \cdot \vec{n})\, dA$$

In a typical engineering problem, the control volume may contain many inlets and outlets; energy flows in at each inlet, and energy flows out at each outlet. Energy also enters the control volume through net heat transfer and net shaft work.



$$\dot{m} = \int_{A_c} \rho(\vec{V} \cdot \vec{n})\, dA_c$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + e\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + e\right)$$

$$e = u + V^2/2 + gz$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, d \mp \sum_{\text{out}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(\frac{P}{\rho} + u + \frac{V^2}{2} + gz\right)$$

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$h = u + Pv = u + P/\rho$$

5–6 ■ ENERGY ANALYSIS OF STEADY FLOWS

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.

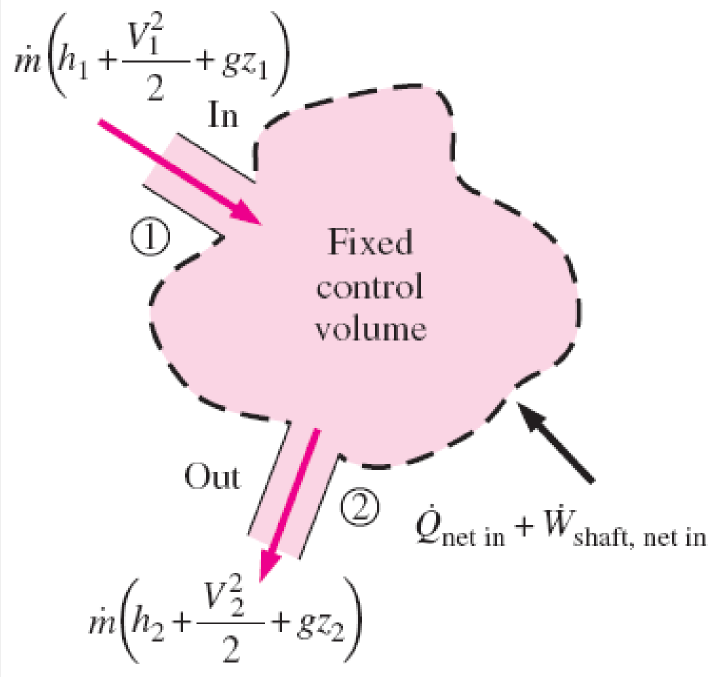
$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

single-stream devices

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h = u + Pv = u + P/\rho$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$



A control volume with only one inlet and one outlet and energy interactions.

Ideal flow (no mechanical energy loss):

$$q_{\text{net in}} = u_2 - u_1$$

Real flow (with mechanical energy loss):

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + e_{\text{mech, loss}}$$

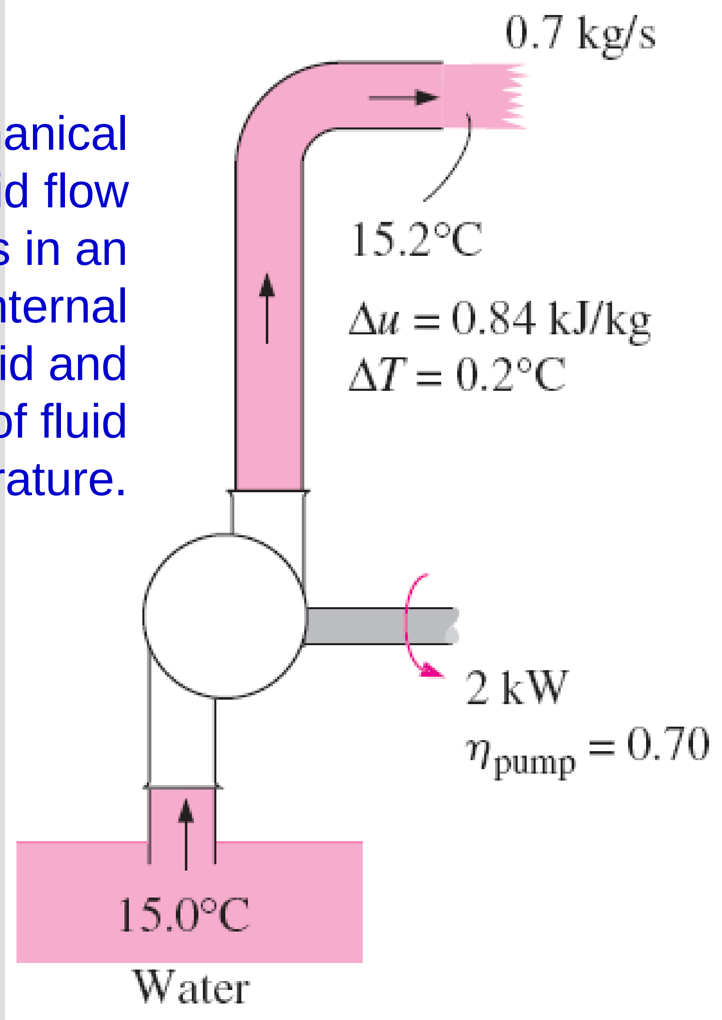
$$w_{\text{shaft, net in}} = w_{\text{pump}} - w_{\text{turbine}}$$

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\dot{E}_{\text{mech, loss}} = \dot{E}_{\text{mech loss, pump}} + \dot{E}_{\text{mech loss, turbine}} + \dot{E}_{\text{mech loss, piping}}$$

The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid and thus in a rise of fluid temperature.



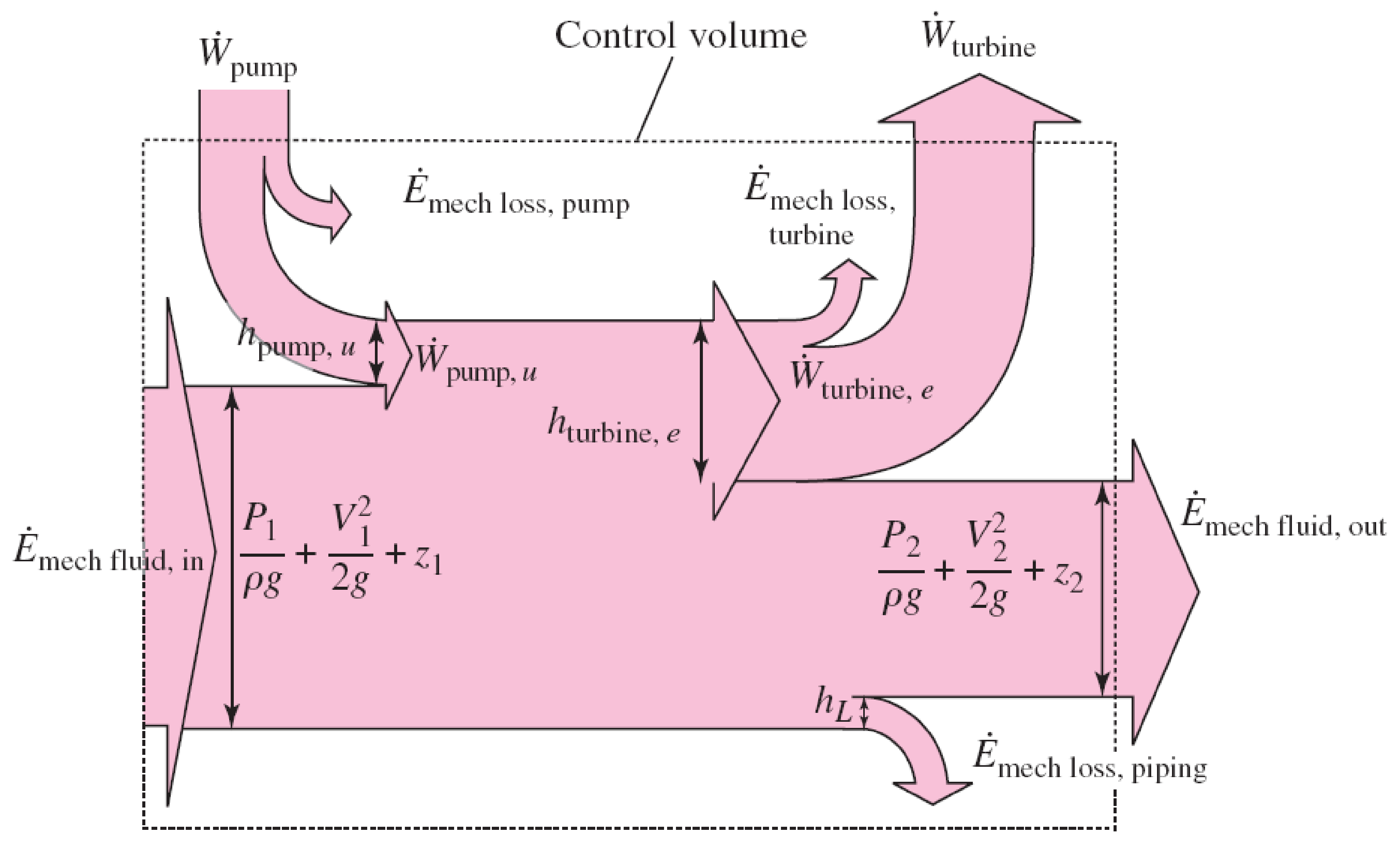
A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.

Energy equation in terms of *heads*

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

where

- $h_{\text{pump}, u} = \dfrac{w_{\text{pump}, u}}{g} = \dfrac{\dot{W}_{\text{pump}, u}}{\dot{m}g} = \dfrac{\eta_{\text{pump}}\dot{W}_{\text{pump}}}{\dot{m}g}$ is the *useful head delivered to the fluid by the pump*. Because of irreversible losses in the pump, $h_{\text{pump}, u}$ is *less* than $\dot{W}_{\text{pump}}/\dot{m}g$ by the factor η_{pump}.
- $h_{\text{turbine}, e} = \dfrac{w_{\text{turbine}, e}}{g} = \dfrac{\dot{W}_{\text{turbine}, e}}{\dot{m}g} = \dfrac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$ is the *extracted head removed from the fluid by the turbine*. Because of irreversible losses in the turbine, $h_{\text{turbine}, e}$ is greater than $\dot{W}_{\text{turbine}}/\dot{m}g$ by the factor η_{turbine}.
- $h_L = \dfrac{e_{\text{mech loss, piping}}}{g} = \dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the irreversible *head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine.



Mechanical energy flow chart for a fluid flow system that involves a pump and a turbine. Vertical dimensions show each energy term expressed as an equivalent column height of fluid, i.e., *head*.

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L \quad (5\text{-}74)$$

Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction

When piping losses are negligible, there is negligible dissipation of mechanical energy into thermal energy, and thus $h_L = e_{\text{mech loss, piping}}/g \cong 0$. Also, $h_{\text{pump, } u} = h_{\text{turbine, } e} = 0$ when there are no mechanical work devices such as fans, pumps, or turbines. Then Eq. 5–74 reduces to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \qquad \text{or} \qquad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

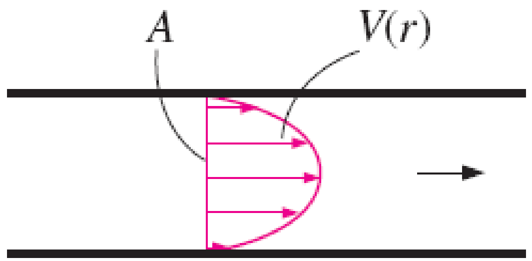
This is the **Bernoulli equation** derived earlier using Newton's second law of motion.

Thus, the Bernoulli equation can be thought of as a degenerate form of the energy equation.

Kinetic Energy Correction Factor, α

The kinetic energy of a fluid stream obtained from $V^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components.

This error can be corrected by replacing the kinetic energy terms $V^2/2$ in the energy equation by $\alpha V_{avg}^2/2$, where α is the **kinetic energy correction factor**.



$$\dot{m} = \rho V_{avg} A, \qquad \rho = \text{constant}$$

$$\dot{\mathrm{KE}}_{act} = \int \mathrm{ke}\,\delta\dot{m} = \int_A \frac{1}{2}[V(r)]^2[\rho V(r)\,dA]$$

$$= \frac{1}{2}\rho\int_A [V(r)]^3\,dA$$

$$\dot{\mathrm{KE}}_{avg} = \frac{1}{2}\dot{m}V_{avg}^2 = \frac{1}{2}\rho A V_{avg}^3$$

$$\alpha = \frac{\dot{\mathrm{KE}}_{act}}{\dot{\mathrm{KE}}_{avg}} = \frac{1}{A}\int_A\left(\frac{V(r)}{V_{avg}}\right)^3 dA$$

The correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a round pipe.

The determination of the *kinetic energy correction factor* using the actual velocity distribution $V(r)$ and the average velocity V_{avg} at a cross section.

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) + \dot{W}_{pump} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) + \dot{W}_{turbine} + \dot{E}_{mech,\,loss}$$

$$\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_{pump,\,u} = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + z_2 + h_{turbine,\,e} + h_L$$

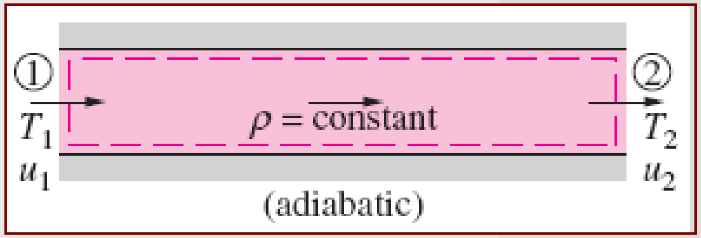
EXAMPLE 5–11 Effect of Friction on Fluid Temperature and Head Loss

Show that during steady and incompressible flow of a fluid in an adiabatic flow section (*a*) the temperature remains constant and there is no head loss when friction is ignored and (*b*) the temperature increases and some head loss occurs when frictional effects are considered. Discuss if it is possible for the fluid temperature to decrease during such flow (Fig. 5–57).

SOLUTION Steady and incompressible flow through an adiabatic section is considered. The effects of friction on the temperature and the heat loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow section is adiabatic and thus there is no heat transfer, $q_{\text{net in}} = 0$.

Analysis The density of a fluid remains constant during incompressible flow and the entropy change is



$$\Delta s = c_v \ln \frac{T_2}{T_1}$$

This relation represents the entropy change of the fluid per unit mass as it flows through the flow section from state 1 at the inlet to state 2 at the outlet. Entropy change is caused by two effects: (1) heat transfer and (2) irreversibilities. Therefore, in the absence of heat transfer, entropy change is due to irreversibilities only, whose effect is always to increase entropy.

(a) The entropy change of the fluid in an adiabatic flow section ($q_{\text{net in}} = 0$) is zero when the process does not involve any irreversibilities such as friction and swirling, and thus for *reversible flow* we have

Temperature change: $$\Delta s = c_v \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

Mechanical energy loss:

$$e_{\text{mech loss, piping}} = u_2 - u_1 - q_{\text{net in}} = c_v(T_2 - T_1) - q_{\text{net in}} = 0$$

Head loss: $$h_L = e_{\text{mech loss, piping}}/g = 0$$

Thus we conclude that when heat transfer and frictional effects are negligible, (1) the temperature of the fluid remains constant, (2) no mechanical energy is converted to thermal energy, and (3) there is no irreversible head loss.

(b) When irreversibilities such as friction are taken into account, the entropy change is positive and thus we have:

Temperature change: $$\Delta s = c_v \ln \frac{T_2}{T_1} > 0 \rightarrow T_2 > T_1$$

Mechanical energy loss: $$e_{\text{mech loss, piping}} = u_2 - u_1 - q_{\text{net in}} = c_v(T_2 - T_1) > 0$$

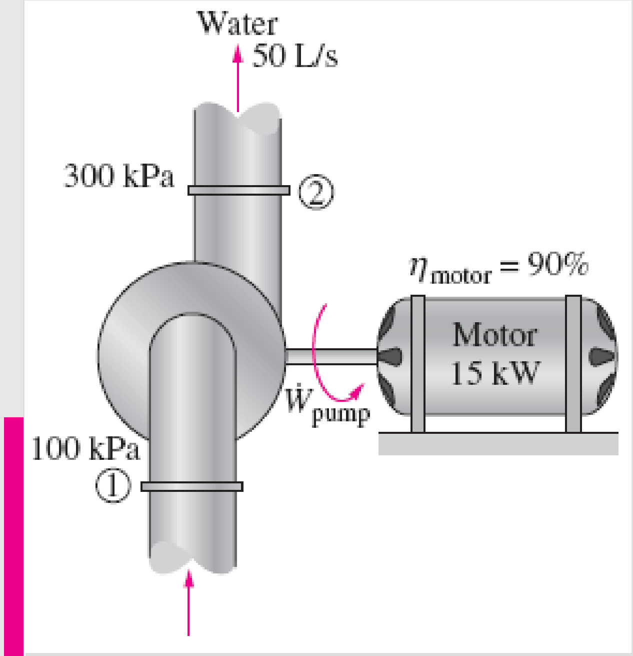
Head loss: $$h_L = e_{\text{mech loss, piping}}/g > 0$$

Thus we conclude that when the flow is adiabatic and irreversible, (1) the temperature of the fluid increases, (2) some mechanical energy is converted to thermal energy, and (3) some irreversible head loss occurs.

Discussion It is impossible for the fluid temperature to decrease during steady, incompressible, adiabatic flow since this would require the entropy of an adiabatic system to decrease, which would be a violation of the second law of thermodynamics.

EXAMPLE 5–12 **Pumping Power and Frictional Heating in a Pump**

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 5–58). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the absolute pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa, respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to mechanical inefficiencies.



Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg · °C.

Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho\dot{V} = (1\ \text{kg/L})(50\ \text{L/s}) = 50\ \text{kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}}\dot{W}_{\text{electric}} = (0.90)(15\ \text{kW}) = 13.5\ \text{kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta\dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50\ \text{kg/s})\left(\frac{(300-100)\ \text{kPa}}{1000\ \text{kg/m}^3}\right)\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}\right) = 10.0\ \text{kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10.0\ \text{kW}}{13.5\ \text{kW}} = \mathbf{0.741} \quad \text{or} \quad \mathbf{74.1\%}$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10.0 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10.0 = 3.5\ \text{kW}$$

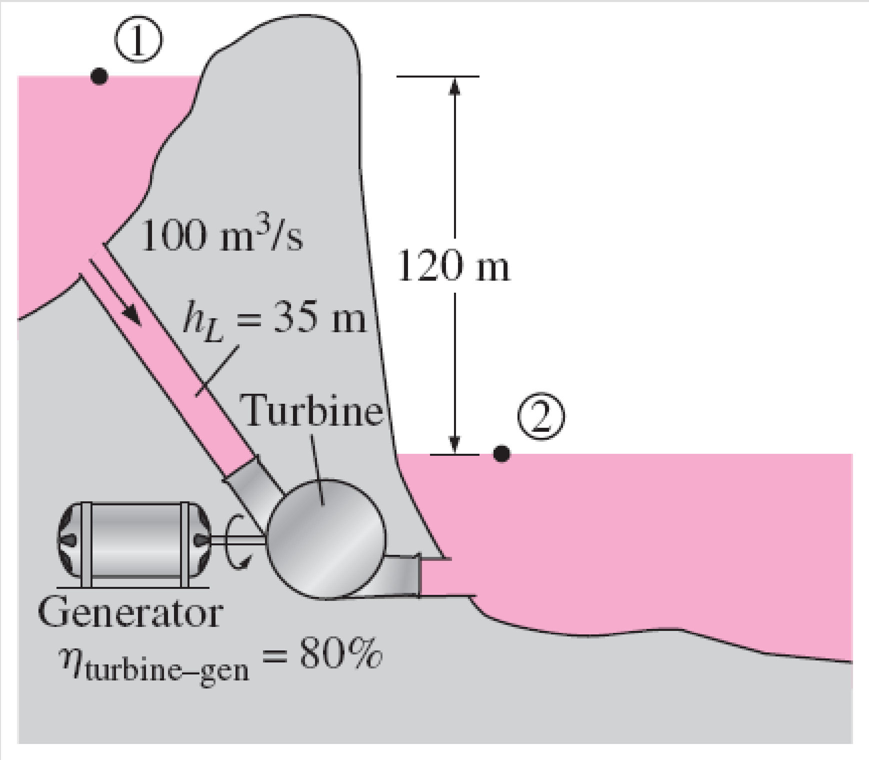
The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5\ \text{kW}}{(50\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{0.017^\circ C}$$

Therefore, the water experiences a temperature rise of 0.017°C which is very small, due to mechanical inefficiency, as it flows through the pump.

Discussion In an actual application, the temperature rise of water would probably be less since part of the heat generated would be transferred to the casing of the pump and from the casing to the surrounding air. If the entire pump and motor were submerged in water, then the 1.5 kW dissipated due to motor inefficiency would also be transferred to the surrounding water as heat.

Example: Hydroelectric Power Generation from a Dam

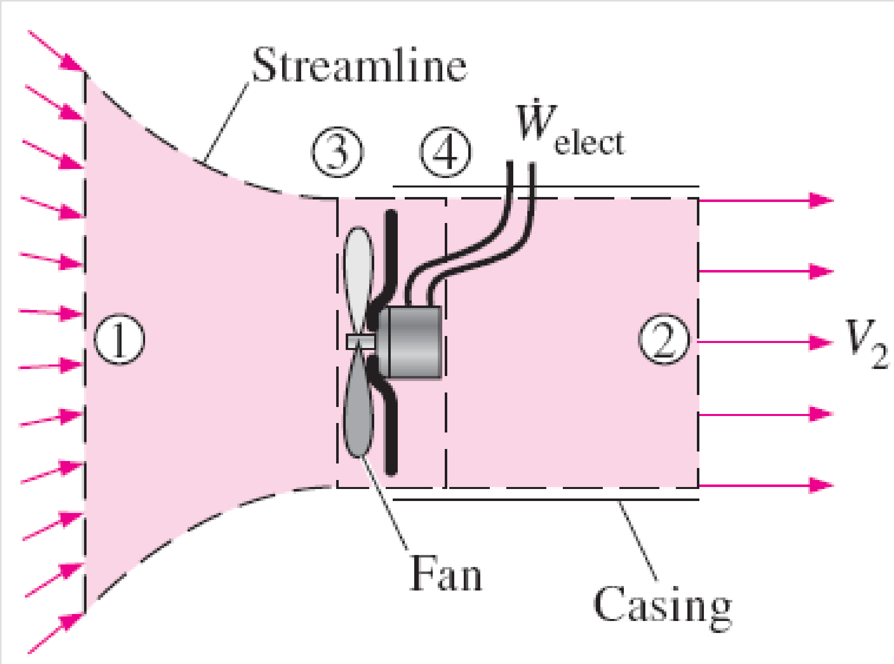


$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + \cancelto{0}{h_{\text{pump, }u}} = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + \cancelto{0}{z_2} + h_{\text{turbine, }e} + h_L$$

$$h_{\text{turbine, }e} = z_1 - h_L$$

$$\dot{W}_{\text{turbine, }e} = \dot{m} g h_{\text{turbine, }e} \qquad \dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}} \dot{W}_{\text{turbine, }e}$$

Example: Fan Selection for Air Cooling of a Computer



Energy equation between 3 and 4

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech loss, fan}}$$

$$\dot{W}_{\text{fan},\,u} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Energy equation between 1 and 2

$$\dot{m}\left(\cancel{\frac{P_1}{\rho}} + \cancelto{0}{\alpha_1\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{W}_{\text{fan}} = \dot{m}\left(\cancel{\frac{P_2}{\rho}} + \alpha_2\frac{V_2^2}{2} + \cancel{gz_2}\right) + \cancelto{0}{\dot{W}_{\text{turbine}}} + \dot{E}_{\text{mech loss, fan}}$$

$$\dot{W}_{\text{fan}} - \dot{E}_{\text{mech loss, fan}} = \dot{W}_{\text{fan},\,u}$$

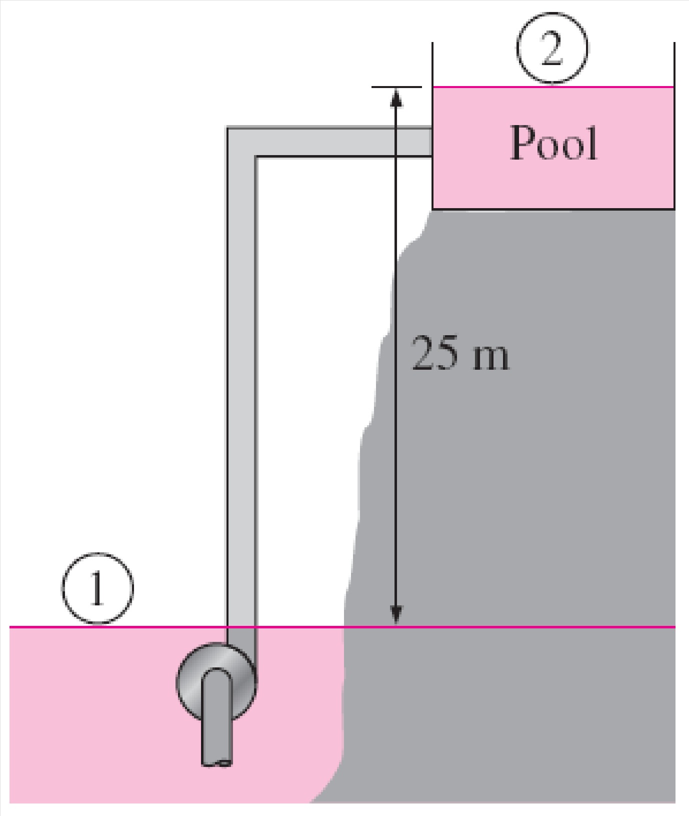
$$\dot{W}_{\text{fan},\,u} = \dot{m}\alpha_2\frac{V_2^2}{2} \qquad \dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{fan},\,u}}{\eta_{\text{fan-motor}}}$$

Example: Pumping Water from a Lake to a Reservoir

$$\dot{W}_{\text{pump } u} = \eta_{\text{pump}} \dot{W}_{\text{shaft}} = (0.72)(5\ \text{kW}) = 3.6\ \text{kW}$$

Energy equation between 1 and 2

$$\dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, } u} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine, } e} + \dot{E}_{\text{mech loss, piping}}$$



$$\dot{W}_{\text{pump, } u} = \dot{m} g z_2 + \dot{E}_{\text{mech loss, piping}}$$

$$\dot{E}_{\text{mech loss, piping}} = \dot{m} g h_L$$

For the pump

$$\Delta P = P_{\text{out}} - P_{\text{in}} = \frac{\dot{W}_{\text{pump, } u}}{\dot{V}}$$

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