CHAPTER 5
Analytic Trigonometry

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Section 5.1 Using Fundamental Identities

- You should know the fundamental trigonometric identities.
  (a) Reciprocal Identities

  $\sin u = \dfrac{1}{\csc u}$ $\qquad$ $\csc u = \dfrac{1}{\sin u}$

  $\cos u = \dfrac{1}{\sec u}$ $\qquad$ $\sec u = \dfrac{1}{\cos u}$

  $\tan u = \dfrac{1}{\cot u} = \dfrac{\sin u}{\cos u}$ $\qquad$ $\cot u = \dfrac{1}{\tan u} = \dfrac{\cos u}{\sin u}$

  (b) Pythagorean Identities

  $\sin^2 u + \cos^2 u = 1$

  $1 + \tan^2 u = \sec^2 u$

  $1 + \cot^2 u = \csc^2 u$

  (c) Cofunction Identities

  $\sin\left(\dfrac{\pi}{2} - u\right) = \cos u$ $\qquad$ $\cos\left(\dfrac{\pi}{2} - u\right) = \sin u$

  $\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$ $\qquad$ $\cot\left(\dfrac{\pi}{2} - u\right) = \tan u$

  $\sec\left(\dfrac{\pi}{2} - u\right) = \csc u$ $\qquad$ $\csc\left(\dfrac{\pi}{2} - u\right) = \sec u$

  (d) Even/Odd Identities

  $\sin(-x) = -\sin x$ $\qquad$ $\csc(-x) = -\csc x$

  $\cos(-x) = \cos x$ $\qquad$ $\sec(-x) = \sec x$

  $\tan(-x) = -\tan x$ $\qquad$ $\cot(-x) = -\cot x$
- You should be able to use these fundamental identities to find function values.
- You should be able to convert trigonometric expressions to equivalent forms by using the fundamental identities.

Vocabulary Check

1. $\tan u$ **2.** $\cos u$ **3.** $\cot u$

4. $\csc u$ **5.** $\cot^2 u$ **6.** $\sec^2 u$

7. $\cos u$ **8.** $\csc u$ **9.** $\cos u$

10. $-\tan u$

1. $\sin x = \frac{\sqrt{3}}{2},\ \cos x = -\frac{1}{2} \implies x$ is in Quadrant II.

$$\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{-1/2} = -2$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

2. $\tan x = \frac{\sqrt{3}}{3}, \cos x = -\frac{\sqrt{3}}{2}$

x is in Quadrant III.

$$\sin x = -\sqrt{1 - \left(-\frac{\sqrt{3}}{2}\right)^2} = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$$

$$\csc x = \frac{1}{\sin x} = -2$$

$$\sec x = \frac{1}{\cos x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

3. $\sec \theta = \sqrt{2},\ \sin \theta = -\frac{\sqrt{2}}{2} \implies \theta$ is in Quadrant IV.

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

$$\cot \theta = \frac{1}{\tan \theta} = -1$$

$$\csc \theta = \frac{1}{\sin \theta} = -\sqrt{2}$$

4. $\csc \theta = \frac{5}{3}, \tan \theta = \frac{3}{4}$

θ is in Quadrant I.

$$\sin \theta = \frac{1}{\csc \theta} = \frac{3}{5}$$

$$\cos \theta = \frac{\sin \theta}{\tan \theta} = \frac{3}{5} \cdot \frac{4}{3} = \frac{4}{5}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{5}{4}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{4}{3}$$

5. $\tan x = \frac{5}{12},\ \sec x = -\frac{13}{12} \implies x$ is in Quadrant III.

$$\cos x = \frac{1}{\sec x} = -\frac{12}{13}$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$$

$$\cot x = \frac{1}{\tan x} = \frac{12}{5}$$

$$\csc x = \frac{1}{\sin x} = -\frac{13}{5}$$

6. $\cot \phi = -3, \sin \phi = \frac{\sqrt{10}}{10}$

ϕ is in Quadrant II.

$$\cos \phi = \cot \phi \sin \phi = -\frac{3\sqrt{10}}{10}$$

$$\tan \phi = \frac{1}{\cot \phi} = -\frac{1}{3}$$

$$\csc \phi = \frac{1}{\sin \phi} = \sqrt{10}$$

$$\sec \phi = \frac{1}{\cos \phi} = -\frac{10}{3\sqrt{10}} = -\frac{\sqrt{10}}{3}$$

7. $\sec \phi = \frac{3}{2},\ \csc \phi = -\frac{3\sqrt{5}}{5} \implies \phi$ is in Quadrant IV.

$$\sin \phi = \frac{1}{\csc \phi} = \frac{1}{-3\sqrt{5}/5} = -\frac{\sqrt{5}}{3}$$

$$\cos \phi = \frac{1}{\sec \phi} = \frac{1}{3/2} = \frac{2}{3}$$

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{-\sqrt{5}/3}{2/3} = -\frac{\sqrt{5}}{2}$$

$$\cot \phi = \frac{1}{\tan \phi} = \frac{1}{-\sqrt{5}/2} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

8. $\cos\left(\frac{\pi}{2} - x\right) = \frac{3}{5}, \cos x = \frac{4}{5}$, x is in Quadrant I.

$$\sin x = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{4}$$

$$\cot x = \frac{1}{\tan x} = \frac{4}{3}$$

9. $\sin(-x) = -\frac{1}{3} \implies \sin x = \frac{1}{3},\ \tan x = -\frac{\sqrt{2}}{4} \implies x$ is in Quadrant II.

$\cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-\frac{1}{9}} = -\frac{2\sqrt{2}}{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{-\sqrt{2}/4} = -2\sqrt{2}$

$\sec x = \frac{1}{\cos x} = \frac{1}{-2\sqrt{2}/3} = -\frac{3\sqrt{2}}{4}$

$\csc x = \frac{1}{\sin x} = \frac{1}{1/3} = 3$

10. $\sec x = 4, \sin x > 0$

x is in Quadrant I.

$\cos x = \frac{1}{\sec x} = \frac{1}{4}$

$\sin x = \sqrt{1-\left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{15}}{4} \cdot \frac{4}{1} = \sqrt{15}$

$\csc x = \frac{1}{\sin x} = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$

11. $\tan\theta = 2,\ \sin\theta < 0 \implies \theta$ is in Quadrant III.

$\sec\theta = -\sqrt{\tan^2\theta + 1} = -\sqrt{4+1} = -\sqrt{5}$

$\cos\theta = \frac{1}{\sec\theta} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$\sin\theta = -\sqrt{1-\cos^2\theta}$

$= -\sqrt{1-\frac{1}{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{5}}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2}$

12. $\csc\theta = -5, \cos\theta < 0$

θ is in Quadrant III.

$\sin\theta = \frac{1}{\csc\theta} = -\frac{1}{5}$

$\cos\theta = -\sqrt{1-\left(-\frac{1}{5}\right)^2} = \frac{-2\sqrt{6}}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{1}{5} \cdot -\frac{5}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{2\sqrt{6}} = -\frac{5\sqrt{6}}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{\sqrt{6}} = 2\sqrt{6}$

13. $\sin\theta = -1,\ \cot\theta = 0 \implies \theta = \frac{3\pi}{2}$

$\cos\theta = \sqrt{1-\sin^2\theta} = 0$

$\sec\theta$ is undefined.

$\tan\theta$ is undefined.

$\csc\theta = -1$

14. $\tan\theta$ is undefined, $\sin\theta > 0$.

$\theta = \frac{\pi}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ is undefined $\implies \cos\theta = 0$

$\sin\theta = \sqrt{1-0^2} = 1$

$\csc\theta = \frac{1}{\sin\theta} = 1$

$\sec\theta = \frac{1}{\cos\theta}$ is undefined.

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0}{1} = 0$

15. $\sec x \cos x = \sec x \cdot \frac{1}{\sec x} = 1$

The expression is matched with (d).

16. $\tan x \csc x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \sec x$

Matches (a).

17. $\cot^2 x - \csc^2 x = \cot^2 x - (1 + \cot^2 x) = -1$

The expression is matched with (b).

18. $(1-\cos^2 x)(\csc x) = (\sin^2 x)\frac{1}{\sin x} = \sin x$

Matches (f).

19. $\dfrac{\sin(-x)}{\cos(-x)} = \dfrac{-\sin x}{\cos x} = -\tan x$

The expression is matched with (e).

20. $\dfrac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]} = \dfrac{\cos x}{\sin x} = \cot x$

Matches (c).

21. $\sin x \sec x = \sin x \cdot \dfrac{1}{\cos x} = \tan x$

The expression is matched with (b).

22. $\cos^2 x(\sec^2 x - 1) = \cos^2 x(\tan^2 x)$

$$= \cos^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right)$$

$$= \sin^2 x$$

Matches (c).

23. $\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$$= (\sec^2 x + \tan^2 x)(1) = \sec^2 x + \tan^2 x$$

The expression is matched with (f).

24. $\cot x \sec x = \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x} = \dfrac{1}{\sin x} = \csc x$

Matches (a).

25. $\dfrac{\sec^2 x - 1}{\sin^2 x} = \dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{1}{\sin^2 x} = \sec^2 x$

The expression is matched with (e).

26. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x} = \dfrac{\sin^2 x}{\cos x} = \dfrac{\sin x}{\cos x}\sin x = \tan x \sin x$

Matches (d).

27. $\cot \theta \sec \theta = \dfrac{\cos \theta}{\sin \theta} \cdot \dfrac{1}{\cos \theta} = \dfrac{1}{\sin \theta} = \csc \theta$

28. $\cos \beta \tan \beta = \cos \beta \dfrac{\sin \beta}{\cos \beta} = \sin \beta$

29. $\sin \phi(\csc \phi - \sin \phi) = (\sin \phi)\dfrac{1}{\sin \phi} - \sin^2 \phi$

$$= 1 - \sin^2 \phi = \cos^2 \phi$$

30. $\sec^2 x(1 - \sin^2 x) = \sec^2 x - \sec^2 x \sin^2 x$

$$= \sec^2 x - \frac{1}{\cos^2 x} \cdot \sin^2 x$$

$$= \sec^2 x - \frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - \tan^2 x$$

$$= 1$$

31. $\dfrac{\cot x}{\csc x} = \dfrac{\cos x/\sin x}{1/\sin x}$

$$= \frac{\cos x}{\sin x} \cdot \frac{\sin x}{1}$$

$$= \cos x$$

32. $\dfrac{\csc \theta}{\sec \theta} = \dfrac{1/(\sin \theta)}{1/(\cos \theta)} = \dfrac{\cos \theta}{\sin \theta} = \cot \theta$

33. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1} = \dfrac{\cos^2 x}{\cot^2 x} = \cos^2 x \tan^2 x = (\cos^2 x)\dfrac{\sin^2 x}{\cos^2 x}$

$$= \sin^2 x$$

34. $\dfrac{1}{\tan^2 x + 1} = \dfrac{1}{\sec^2 x} = \dfrac{1}{1/(\cos^2 x)} = \cos^2 x$

35. $\sec \alpha \dfrac{\sin \alpha}{\tan \alpha} = \dfrac{1}{\cos \alpha}(\sin \alpha) \cot \alpha$

$$= \frac{1}{\cos \alpha}(\sin \alpha)\left(\frac{\cos \alpha}{\sin \alpha}\right) = 1$$

36. $\dfrac{\tan^2 \theta}{\sec^2 \theta} = \dfrac{\sin^2 \theta}{\cos^2 \theta} \cdot \dfrac{1}{\sec^2 \theta}$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{1/(\cos^2 \theta)} = \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = \sin^2 \theta$$

37. $\cos\left(\dfrac{\pi}{2} - x\right) \sec x = (\sin x)(\sec x)$

$$= (\sin x)\left(\frac{1}{\cos x}\right) = \frac{\sin x}{\cos x} = \tan x$$

38. $\cot\left(\frac{\pi}{2} - x\right)\cos x = \tan x \cos x = \frac{\sin x}{\cos x} \cdot \cos x = \sin x$

39. $\frac{\cos^2 y}{1 - \sin y} = \frac{1 - \sin^2 y}{1 - \sin y}$

$= \frac{(1 + \sin y)(1 - \sin y)}{1 - \sin y} = 1 + \sin y$

40. $(\cos t)(1 + \tan^2 t) = (\cos t)(\sec^2 t) = \frac{\cos t}{\cos^2 t} = \frac{1}{\cos t} = \sec t$

41. $\sin \beta \tan \beta + \cos \beta = (\sin \beta)\frac{\sin \beta}{\cos \beta} + \cos \beta$

$= \frac{\sin^2 \beta}{\cos \beta} + \frac{\cos^2 \beta}{\cos \beta}$

$= \frac{\sin^2 \beta + \cos^2 \beta}{\cos \beta}$

$= \frac{1}{\cos \beta}$

$= \sec \beta$

42. $\csc \phi \tan \phi + \sec \phi = \frac{1}{\sin \phi} \cdot \frac{\sin \phi}{\cos \phi} + \sec \phi$

$= \frac{1}{\cos \phi} + \sec \phi$

$= 2 \sec \phi$

43. $\cot u \sin u + \tan u \cos u = \frac{\cos u}{\sin u}(\sin u) + \frac{\sin u}{\cos u}(\cos u)$

$= \cos u + \sin u$

44. $\sin \theta \sec \theta + \cos \theta \csc \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$

$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$

$= \frac{1}{\cos \theta \sin \theta}$

$= \sec \theta \csc \theta$

45. $\tan^2 x - \tan^2 x \sin^2 x = \tan^2 x(1 - \sin^2 x)$

$= \tan^2 x \cos^2 x$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$

$= \sin^2 x$

46. $\sin^2 x \csc^2 x - \sin^2 x = \sin^2 x(\csc^2 x - 1)$

$= \sin^2 x \cot^2 x$

$= \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$

$= \cos^2 x$

47. $\sin^2 x \sec^2 x - \sin^2 x = \sin^2 x(\sec^2 x - 1)$

$= \sin^2 x \tan^2 x$

48. $\cos^2 x + \cos^2 x \tan^2 x = \cos^2 x(1 + \tan^2 x)$

$= \cos^2 x(\sec^2 x)$

$= \cos^2 x\left(\frac{1}{\cos^2 x}\right)$

$= 1$

49. $\frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x + 1)(\sec x - 1)}{\sec x - 1}$

$= \sec x + 1$

50. $\frac{\cos^2 x - 4}{\cos x - 2} = \frac{(\cos x + 2)(\cos x - 2)}{\cos x - 2}$

$= \cos x + 2$

51. $\tan^4 x + 2\tan^2 x + 1 = (\tan^2 x + 1)^2$

$= (\sec^2 x)^2$

$= \sec^4 x$

52. $1 - 2\cos^2 x + \cos^4 x = (1 - \cos^2 x)^2$

$= (\sin^2 x)^2$

$= \sin^4 x$

53. $\sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$
$= (1)(\sin^2 x - \cos^2 x)$
$= \sin^2 x - \cos^2 x$

54. $\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$
$= (\sec^2 x + \tan^2 x)(1)$
$= \sec^2 x + \tan^2 x$

55. $\csc^3 x - \csc^2 x - \csc x + 1 = \csc^2 x(\csc x - 1) - 1(\csc x - 1)$
$= (\csc^2 x - 1)(\csc x - 1)$
$= \cot^2 x(\csc x - 1)$

56. $\sec^3 x - \sec^2 x - \sec x + 1 = \sec^2 x(\sec x - 1) - (\sec x - 1)$
$= (\sec^2 x - 1)(\sec x - 1)$
$= \tan^2 x(\sec x - 1)$

57. $(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$
$= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$
$= 1 + 2 \sin x \cos x$

58. $(\cot x + \csc x)(\cot x - \csc x) = \cot^2 x - \csc^2 x$
$= -1$

59. $(2 \csc x + 2)(2 \csc x - 2) = 4 \csc^2 x - 4$
$= 4(\csc^2 x - 1)$
$= 4 \cot^2 x$

60. $(3 - 3 \sin x)(3 + 3 \sin x) = 9 - 9 \sin^2 x$
$= 9(1 - \sin^2 x)$
$= 9 \cos^2 x$

61. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = \dfrac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$
$= \dfrac{2}{1 - \cos^2 x}$
$= \dfrac{2}{\sin^2 x}$
$= 2 \csc^2 x$

62. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1} = \dfrac{\sec x - 1 - (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$
$= \dfrac{\sec x - 1 - \sec x - 1}{\sec^2 x - 1}$
$= \dfrac{-2}{\tan^2 x}$
$= -2\left(\dfrac{1}{\tan^2 x}\right)$
$= -2 \cot^2 x$

63. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x} = \dfrac{\cos^2 x + (1 + \sin x)^2}{\cos x(1 + \sin x)} = \dfrac{\cos^2 x + 1 + 2 \sin x + \sin^2 x}{\cos x(1 + \sin x)}$
$= \dfrac{2 + 2 \sin x}{\cos x(1 + \sin x)}$
$= \dfrac{2(1 + \sin x)}{\cos x(1 + \sin x)}$
$= \dfrac{2}{\cos x}$
$= 2 \sec x$

64. $\tan x - \dfrac{\sec^2 x}{\tan x} = \dfrac{\tan^2 x - \sec^2 x}{\tan x}$
$= \dfrac{-1}{\tan x} = -\cot x$

65. $\dfrac{\sin^2 y}{1 - \cos y} = \dfrac{1 - \cos^2 y}{1 - \cos y}$
$= \dfrac{(1 + \cos y)(1 - \cos y)}{1 - \cos y} = 1 + \cos y$

66. $\dfrac{5}{\tan x + \sec x} \cdot \dfrac{\tan x - \sec x}{\tan x - \sec x} = \dfrac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$

$= \dfrac{5(\tan x - \sec x)}{-1}$

$= 5(\sec x - \tan x)$

67. $\dfrac{3}{\sec x - \tan x} \cdot \dfrac{\sec x + \tan x}{\sec x + \tan x} = \dfrac{3(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$

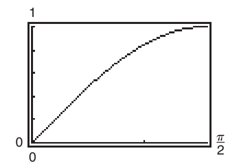
$= \dfrac{3(\sec x + \tan x)}{1}$

$= 3(\sec x + \tan x)$

68. $\dfrac{\tan^2 x}{\csc x + 1} \cdot \dfrac{\csc x - 1}{\csc x - 1} = \dfrac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1} = \dfrac{\tan^2 x(\csc x - 1)}{\cot^2 x} = \tan^2 x(\csc x - 1)\tan^2 x = \tan^4 x(\csc x - 1)$

69. $y_1 = \cos\left(\dfrac{\pi}{2} - x\right),\ y_2 = \sin x$

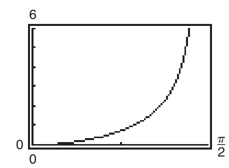
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854
y_2	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854



Conclusion: $y_1 = y_2$

70. $y_1 = \sec x - \cos x, y_2 = \sin x \tan x$

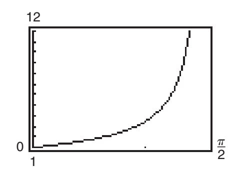
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135
y_2	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135



It appears that $y_1 = y_2$.

71. $y_1 = \dfrac{\cos x}{1 - \sin x},\ y_2 = \dfrac{1 + \sin x}{\cos x}$

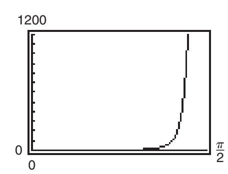
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814
y_2	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814



Conclusion: $y_1 = y_2$

72. $y_1 = \sec^4 x - \sec^2 x, y_2 = \tan^2 x + \tan^4 x$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143
y_2	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143



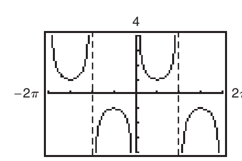
It appears that $y_1 = y_2$.

73. $y_1 = \cos x \cot x + \sin x = \csc x$

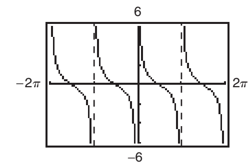
$\cos x \cot x + \sin x = \cos x\left(\dfrac{\cos x}{\sin x}\right) + \sin x$

$= \dfrac{\cos^2 x}{\sin x} + \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin x} = \dfrac{1}{\sin x} = \csc x$

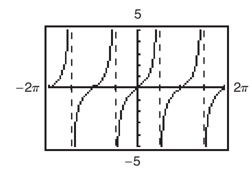


74. $y_1 = \sec x \csc x - \tan x = \cot x$



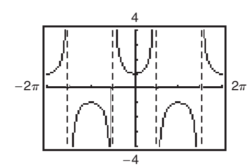
$$\sec x \csc x - \tan x = \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x}$$
$$= \frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x}$$
$$= \frac{1 - \sin^2 x}{\cos x \sin x}$$
$$= \frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x} = \cot x$$

75. $y_1 = \frac{1}{\sin x}\left(\frac{1}{\cos x} - \cos x\right) = \tan x$



$$\frac{1}{\sin x}\left(\frac{1}{\cos x} - \cos x\right) = \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x}$$
$$= \frac{1 - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$$

76. $y_1 = \frac{1}{2}\left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}\right)$



$$\frac{1}{2}\left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}\right) = \frac{1}{2}\left(\frac{(1 + \sin \theta)(1 + \sin \theta)}{(\cos \theta)(1 + \sin \theta)} + \frac{(\cos \theta)(\cos \theta)}{(\cos \theta)(1 + \sin \theta)}\right)$$
$$= \frac{1}{2}\left(\frac{1 + 2 \sin \theta + \sin^2 \theta + \cos^2 \theta}{(\cos \theta)(1 + \sin \theta)}\right)$$
$$= \frac{1}{2}\left(\frac{1 + 2 \sin \theta + 1}{(\cos \theta)(1 + \sin \theta)}\right)$$
$$= \frac{1}{2}\left(\frac{2 + 2 \sin \theta}{(\cos \theta)(1 + \sin \theta)}\right)$$
$$= \frac{1 + \sin \theta}{(\cos \theta)(1 + \sin \theta)} = \frac{1}{\cos \theta} = \sec \theta$$

77. Let $x = 3 \cos \theta$, then

$$\sqrt{9 - x^2} = \sqrt{9 - (3 \cos \theta)^2} = \sqrt{9 - 9 \cos^2 \theta} = \sqrt{9(1 - \cos^2 \theta)}$$
$$= \sqrt{9 \sin^2 \theta} = 3 \sin \theta.$$

78. Let $x = 2 \cos \theta$.

$$\sqrt{64 - 16x^2} = \sqrt{64 - 16(2 \cos \theta)^2}$$
$$= \sqrt{64(1 - \cos^2 \theta)}$$
$$= \sqrt{64 \sin^2 \theta}$$
$$= 8 \sin \theta$$

79. Let $x = 3 \sec \theta$, then

$$\sqrt{x^2 - 9} = \sqrt{(3 \sec \theta)^2 - 9}$$
$$= \sqrt{9 \sec^2 \theta - 9}$$
$$= \sqrt{9(\sec^2 \theta - 1)}$$
$$= \sqrt{9 \tan^2 \theta}$$
$$= 3 \tan \theta.$$

80. Let $x = 2 \sec \theta$.

$$\sqrt{x^2 - 4} = \sqrt{(2 \sec \theta)^2 - 4}$$
$$= \sqrt{4(\sec^2 \theta - 1)}$$
$$= \sqrt{4 \tan^2 \theta}$$
$$= 2 \tan \theta$$

81. Let $x = 5 \tan \theta$, then

$$\sqrt{x^2 + 25} = \sqrt{(5 \tan \theta)^2 + 25}$$
$$= \sqrt{25 \tan^2 \theta + 25}$$
$$= \sqrt{25(\tan^2 \theta + 1)}$$
$$= \sqrt{25 \sec^2 \theta}$$
$$= 5 \sec \theta.$$

82. Let $x = 10 \tan \theta$.

$$\sqrt{x^2 + 100} = \sqrt{(10 \tan \theta)^2 + 100}$$
$$= \sqrt{100(\tan^2 \theta + 1)}$$
$$= \sqrt{100 \sec^2 \theta}$$
$$= 10 \sec \theta$$

83. Let $x = 3 \sin \theta$, then $\sqrt{9 - x^2} = 3$ becomes

$$\sqrt{9 - (3 \sin \theta)^2} = 3$$
$$\sqrt{9 - 9 \sin^2 \theta} = 3$$
$$\sqrt{9(1 - \sin^2 \theta)} = 3$$
$$\sqrt{9 \cos^2 \theta} = 3$$
$$3 \cos \theta = 3$$
$$\cos \theta = 1$$
$$\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - (1)^2} = 0.$$

84. $x = 6 \sin \theta$

$$3 = \sqrt{36 - x^2}$$
$$= \sqrt{36 - (6 \sin \theta)^2}$$
$$= \sqrt{36(1 - \sin^2 \theta)}$$
$$= \sqrt{36 \cos^2 \theta}$$
$$= 6 \cos \theta$$
$$\cos \theta = \frac{3}{6} = \frac{1}{2}$$
$$\sin \theta = \pm\sqrt{1 - \cos^2 \theta}$$
$$= \pm\sqrt{1 - \left(\frac{1}{2}\right)^2}$$
$$= \pm\sqrt{\frac{3}{4}}$$
$$= \pm\frac{\sqrt{3}}{2}$$

85. Let $x = 2 \cos \theta$, then $\sqrt{16 - 4x^2} = 2\sqrt{2}$ becomes

$$\sqrt{16 - 4(2 \cos \theta)^2} = 2\sqrt{2}$$
$$\sqrt{16 - 16 \cos^2 \theta} = 2\sqrt{2}$$
$$\sqrt{16(1 - \cos^2 \theta)} = 2\sqrt{2}$$
$$\sqrt{16 \sin^2 \theta} = 2\sqrt{2}$$
$$4 \sin \theta = 2\sqrt{2}$$
$$\sin \theta = \frac{\sqrt{2}}{2}$$
$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$
$$= \sqrt{1 - \frac{1}{2}}$$
$$= \sqrt{\frac{1}{2}}$$
$$= \frac{\sqrt{2}}{2}.$$

86. $x = 10 \cos \theta$

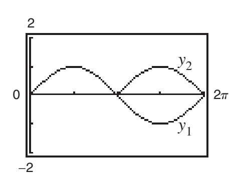
$$-5\sqrt{3} = \sqrt{100 - x^2}$$
$$= \sqrt{100 - (10 \cos \theta)^2}$$
$$= \sqrt{100(1 - \cos^2 \theta)}$$
$$= \sqrt{100 \sin^2 \theta}$$
$$= 10 \sin \theta$$
$$\sin \theta = -\frac{5\sqrt{3}}{10} = -\frac{\sqrt{3}}{2}$$
$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(-\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}$$

87. $\sin \theta = \sqrt{1 - \cos^2 \theta}$

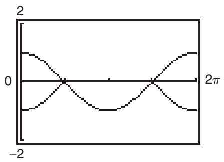
Let $y_1 = \sin x$ and $y_2 = \sqrt{1 - \cos^2 x}$, $0 \le x \le 2\pi$.

$y_1 = y_2$ for $0 \le x \le \pi$, so we have

$\sin \theta = \sqrt{1 - \cos^2 \theta}$ for $0 \le \theta \le \pi$.



88. $\cos\theta = -\sqrt{1 - \sin^2\theta}$



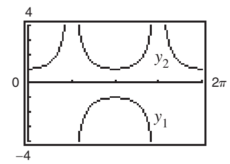
$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

89. $\sec\theta = \sqrt{1 + \tan^2\theta}$

Let $y_1 = \frac{1}{\cos x}$ and $y_2 = \sqrt{1 + \tan^2 x}$, $0 \le x \le 2\pi$.

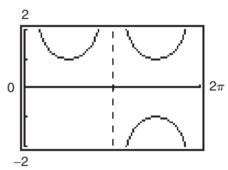
$y_1 = y_2$ for $0 \le x < \frac{\pi}{2}$ and $\frac{3\pi}{2} < x \le 2\pi$, so we have

$\sec\theta = \sqrt{1 + \tan^2\theta}$ for $0 \le \theta < \frac{\pi}{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$.



90. $\csc\theta = \sqrt{1 + \cot^2\theta}$

$0 < \theta < \pi$



91. $\ln|\cos x| - \ln|\sin x| = \ln\frac{|\cos x|}{|\sin x|} = \ln|\cot x|$

92. $\ln|\sec x| + \ln|\sin x| = \ln|\sec x \sin x|$

$$= \ln\left|\frac{1}{\cos x} \cdot \sin x\right|$$

$$= \ln|\tan x|$$

93. $\ln|\cot t| + \ln(1 + \tan^2 t) = \ln\left[|\cot t|(1 + \tan^2 t)\right]$

$$= \ln|\cot t \sec^2 t|$$

$$= \ln\left|\frac{\cos t}{\sin t} \cdot \frac{1}{\cos^2 t}\right|$$

$$= \ln\left|\frac{1}{\sin t \cos t}\right| = \ln|\csc t \sec t|$$

94. $\ln(\cos^2 t) + \ln(1 + \tan^2 t) = \ln[\cos^2 t(1 + \tan^2 t)]$

$$= \ln[\cos^2 t \sec^2 t]$$

$$= \ln\left(\cos^2 t \cdot \frac{1}{\cos^2 t}\right)$$

$$= \ln(1) = 0$$

95. (a) $\csc^2 132° - \cot^2 132° \approx 1.8107 - 0.8107 = 1$

(b) $\csc^2\frac{2\pi}{7} - \cot^2\frac{2\pi}{7} \approx 1.6360 - 0.6360 = 1$

96. $\tan^2\theta + 1 = \sec^2\theta$

(a) $\theta = 346°$

$(\tan 346°)^2 + 1 \approx 1.0622$

$(\sec 346°)^2 = \left(\frac{1}{\cos 346°}\right)^2 \approx 1.0622$

(b) $\theta = 3.1$

$(\tan 3.1)^2 + 1 \approx 1.00173$

$(\sec 3.1)^2 = \left(\frac{1}{\cos 3.1}\right)^2 \approx 1.00173$

97. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

(a) $\theta = 80°$

$\cos(90° - 80°) = \sin 80°$

$0.9848 = 0.9848$

(b) $\theta = 0.8$

$\cos\left(\frac{\pi}{2} - 0.8\right) = \sin 0.8$

$0.7174 = 0.7174$

98. $\sin(-\theta) = -\sin\theta$

(a) $\theta = 250°$

$\sin(-250°) \approx 0.9397$

$-(\sin 250°) \approx 0.9397$

(b) $\theta = \frac{1}{2}$

$\sin\left(-\frac{1}{2}\right) \approx -0.4794$

$-\left(\sin\frac{1}{2}\right) \approx -0.4794$

99. $\mu W\cos\theta = W\sin\theta$

$$\mu = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$$

100.
$$\begin{aligned}\csc x\cot x - \cos x &= \frac{1}{\sin x}\left(\frac{\cos x}{\sin x}\right) - \cos x\\ &= \frac{\cos x}{\sin^2 x} - \cos x\\ &= \frac{\cos x - \sin^2 x\ \cos x}{\sin^2 x}\\ &= \frac{\cos x(1 - \sin^2 x)}{\sin^2 x}\\ &= \frac{\cos x\cos^2 x}{\sin^2 x} = \cos x\cot^2 x\end{aligned}$$

101. True. For example, $\sin(-x) = -\sin x$ means that the graph of $\sin x$ is symmetric about the origin.

102. False. A cofunction identity can be used to transform a tangent function so that it can be represented by a cotangent function.

103. As $x \longrightarrow \frac{\pi}{2}^-$, $\sin x \longrightarrow 1$ and $\csc x \longrightarrow 1$.

104. As $x \longrightarrow 0^+$, $\cos x \longrightarrow 1$ and $\sec x = \frac{1}{\cos x} \longrightarrow 1$.

105. As $x \longrightarrow \frac{\pi}{2}^-$, $\tan x \longrightarrow \infty$ and $\cot x \longrightarrow 0$.

106. As $x \longrightarrow \pi^+$, $\sin x \longrightarrow 0$ and $\csc x = \frac{1}{\sin x} \longrightarrow -\infty$.

107. $\cos\theta = \sqrt{1 - \sin^2\theta}$ *is not* an identity.

$\cos^2\theta + \sin^2\theta = 1 \implies \cos\theta = \pm\sqrt{1 - \sin^2\theta}$

108. The equation *is not* an identity.

$\cot\theta = \pm\sqrt{\csc^2\theta - 1}$

109. $\frac{\sin k\theta}{\cos k\theta} = \tan\theta$ *is not* an identity.

$\frac{\sin k\theta}{\cos k\theta} = \tan k\theta$

110. The equation *is not* an identity.

$\frac{1}{5\cos\theta} = \frac{1}{5}\left(\frac{1}{\cos\theta}\right) = \frac{1}{5}\sec\theta \neq 5\sec\theta$

111. $\sin\theta\csc\theta = 1$ *is* an identity.

$\sin\theta \cdot \frac{1}{\sin\theta} = 1$, provided $\sin\theta \neq 0$.

112. The equation *is not* an identity. The angles are not the same.

$\sin\theta\csc\phi = \sin\theta \cdot \frac{1}{\sin\phi} = \frac{\sin\theta}{\sin\phi} \neq 1$, in general

113. Let (x, y) be any point on the terminal side of θ.

Then, $r = \sqrt{x^2 + y^2}$ and

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2\\ &= \frac{y^2 + x^2}{r^2}\\ &= \frac{r^2}{r^2}\\ &= 1.\end{aligned}$$

114. Divide both sides of $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$:

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Divide both sides of $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$:

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Discussion for remembering identities will vary, but one key is first to learn the identities that concern the sine and cosine functions thoroughly, and then to use these as a basis to establish the other identities when necessary.

115. $(\sqrt{x} + 5)(\sqrt{x} - 5) = (\sqrt{x})^2 - (5)^2 = x - 25$

116. $$\begin{aligned}(2\sqrt{z} + 3)^2 &= (2\sqrt{z})^2 + 2(2\sqrt{z})(3) + (3)^2\\ &= 4z + 12\sqrt{z} + 9\end{aligned}$$

117. $$\begin{aligned}\frac{1}{x+5} + \frac{x}{x-8} &= \frac{(x-8) + x(x+5)}{(x+5)(x-8)}\\ &= \frac{x^2 + 6x - 8}{(x+5)(x-8)}\end{aligned}$$

118. $$\begin{aligned}\frac{6x}{x-4} - \frac{3}{4-x} &= \frac{6x}{x-4} + \frac{3}{x-4}\\ &= \frac{6x+3}{x-4}\\ &= \frac{3(2x+1)}{x-4}\end{aligned}$$

119. $$\begin{aligned}\frac{2x}{x^2-4} - \frac{7}{x+4} &= \frac{2x(x+4) - 7(x^2-4)}{(x^2-4)(x+4)}\\ &= \frac{2x^2 + 8x - 7x^2 + 28}{(x^2-4)(x+4)}\\ &= \frac{-5x^2 + 8x + 28}{(x^2-4)(x+4)}\end{aligned}$$

120. $$\begin{aligned}\frac{x}{x^2-25} + \frac{x^2}{x-5} &= \frac{x}{(x-5)(x+5)} + \frac{x^2(x+5)}{(x-5)(x+5)}\\ &= \frac{x + x^3 + 5x^2}{(x-5)(x+5)}\\ &= \frac{x(1 + x^2 + 5x)}{(x-5)(x+5)}\\ &= \frac{x(x^2 + 5x + 1)}{x^2 - 25}\end{aligned}$$

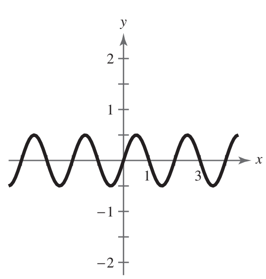
121. $f(x) = \frac{1}{2}\sin(\pi x)$

Amplitude: $\frac{1}{2}$

Period: $\frac{2\pi}{\pi} = 2$

Key points:

$(0, 0), \left(\frac{1}{2}, \frac{1}{2}\right), (1, 0), \left(\frac{3}{2}, -\frac{1}{2}\right), (2, 0)$



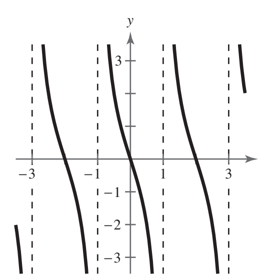
122. $f(x) = -2\tan\left(\frac{\pi x}{2}\right)$

Amplitude: 2

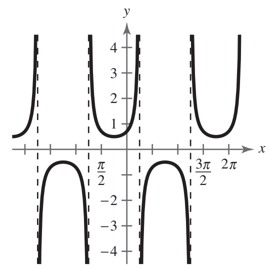
Period: $\frac{\pi}{\pi/2} = 2$

Two consecutive vertical asymptotes: $x = -1, x = 1$

Key points: $\left(-\frac{1}{2}, 2\right), (0, 0), \left(\frac{1}{2}, 2\right)$



123. $f(x) = \frac{1}{2}\sec\left(x + \frac{\pi}{4}\right)$



Sketch the graph of $y = \frac{1}{2}\cos\left(x + \frac{\pi}{4}\right)$ first.

Amplitude: $\frac{1}{2}$

Period: 2π

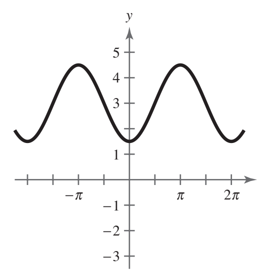
One cycle: $x + \frac{\pi}{4} = 0 \implies x = -\frac{\pi}{4}$

$x + \frac{\pi}{4} = 2\pi \implies x = \frac{7\pi}{4}$

The x-intercepts of $y = \frac{1}{2}\cos\left(x + \frac{\pi}{4}\right)$ correspond to the vertical asymptotes of $f(x) = \frac{1}{2}\sec\left(x + \frac{\pi}{4}\right)$.

$x = \frac{\pi}{4}, x = \frac{5\pi}{4}, \ldots$

124. $f(x) = \frac{3}{2}\cos(x - \pi) + 3$



Using $y = a\cos bx$, $a = \frac{3}{2}$ so the amplitude is $\frac{3}{2}$.

$b = 1$ so the period is $\frac{2\pi}{1} = 2\pi$.

$(x - \pi)$ shifts the graph right by π and $+3$ shifts the graph upward by 3.

Section 5.2 Verifying Trigonometric Identities

- You should know the difference between an expression, a conditional equation, and an identity.
- You should be able to solve trigonometric identities, using the following techniques.
  - (a) Work with *one* side at a time. Do not "cross" the equal sign.
  - (b) Use algebraic techniques such as combining fractions, factoring expressions, rationalizing denominators, and squaring binomials.
  - (c) Use the fundamental identities.
  - (d) Convert all the terms into sines and cosines.

Vocabulary Check

1. identity

2. conditional equation

3. $\tan u$

4. $\cot u$

5. $\cos^2 u$

6. $\sin u$

7. $-\csc u$

8. $\sec u$

1. $\sin t \csc t = \sin t\left(\frac{1}{\sin t}\right) = 1$

2. $\sec y \cos y = \left(\frac{1}{\cos y}\right)\cos y = 1$

3. $(1 + \sin\alpha)(1 - \sin\alpha) = 1 - \sin^2\alpha = \cos^2\alpha$

4. $\cot^2 y(\sec^2 y - 1) = \cot^2 y \tan^2 y = 1$

5. $\cos^2 \beta - \sin^2 \beta = (1 - \sin^2 \beta) - \sin^2 \beta$
$= 1 - 2\sin^2 \beta$

6. $\cos^2 \beta - \sin^2 \beta = \cos^2 \beta - (1 - \cos^2 \beta)$
$= 2\cos^2 \beta - 1$

7. $\sin^2 \alpha - \sin^4 \alpha = \sin^2 \alpha(1 - \sin^2 \alpha)$
$= (1 - \cos^2 \alpha)(\cos^2 \alpha)$
$= \cos^2 \alpha - \cos^4 \alpha$

8. $\cos x + \sin x \tan x = \cos x + \sin x\left(\dfrac{\sin x}{\cos x}\right)$
$= \dfrac{\cos^2 x + \sin^2 x}{\cos x}$
$= \dfrac{1}{\cos x}$
$= \sec x$

9. $\dfrac{\csc^2 \theta}{\cot \theta} = \csc^2 \theta\left(\dfrac{1}{\cot \theta}\right)$
$= \csc^2 \theta \tan \theta$
$= \left(\dfrac{1}{\sin^2 \theta}\right)\left(\dfrac{\sin \theta}{\cos \theta}\right)$
$= \left(\dfrac{1}{\sin \theta}\right)\left(\dfrac{1}{\cos \theta}\right)$
$= \csc \theta \sec \theta$

10. $\dfrac{\cot^3 t}{\csc t} = \dfrac{\cot t \cot^2 t}{\csc t}$
$= \dfrac{\cot t(\csc^2 t - 1)}{\csc t}$
$= \dfrac{\dfrac{\cos t}{\sin t}(\csc^2 t - 1)}{\dfrac{1}{\sin t}}$
$= \dfrac{\cos t \sin t}{\sin t}(\csc^2 t - 1)$
$= \cos t(\csc^2 t - 1)$

11. $\dfrac{\cot^2 t}{\csc t} = \dfrac{\cos^2 t}{\sin^2 t} \cdot \sin t$
$= \dfrac{\cos^2 t}{\sin t}$
$= \dfrac{1 - \sin^2 t}{\sin t} = \dfrac{1}{\sin t} - \dfrac{\sin^2 t}{\sin t}$
$= \csc t - \sin t$

12. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{1 + \tan^2 \beta}{\tan \beta}$
$= \dfrac{\sec^2 \beta}{\tan \beta}$

13. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \sin^{1/2} x \cos x(1 - \sin^2 x) = \sin^{1/2} x \cos x \cdot \cos^2 x = \cos^3 x\sqrt{\sin x}$

14. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^4 x(\sec x \tan x)(\sec^2 x - 1)$
$= \sec^4 x(\sec x \tan x)\tan^2 x$
$= \sec^5 x \tan^3 x$

15. $\dfrac{1}{\sec x \tan x} = \cos x \cot x = \cos x \cdot \dfrac{\cos x}{\sin x}$
$= \dfrac{\cos^2 x}{\sin x}$
$= \dfrac{1 - \sin^2 x}{\sin x}$
$= \dfrac{1}{\sin x} - \sin x$
$= \csc x - \sin x$

16. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \dfrac{\sec \theta - 1}{1 - (1/\sec \theta)} \cdot \dfrac{\sec \theta}{\sec \theta}$
$= \dfrac{\sec \theta(\sec \theta - 1)}{\sec \theta - 1}$
$= \sec \theta$

17. $\csc x - \sin x = \dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{1 - \sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x}$

$= \dfrac{\cos x}{1} \cdot \dfrac{\cos x}{\sin x}$

$= \cos x \cot x$

18. $\sec x - \cos x = \dfrac{1}{\cos x} - \cos x$

$= \dfrac{1 - \cos^2 x}{\cos x}$

$= \dfrac{\sin^2 x}{\cos x}$

$= \sin x \cdot \dfrac{\sin x}{\cos x}$

$= \sin x \tan x$

19. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \dfrac{\cot x + \tan x}{\tan x \cot x}$

$= \dfrac{\cot x + \tan x}{1}$

$= \tan x + \cot x$

20. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \dfrac{\csc x - \sin x}{\sin x \csc x}$

$= \dfrac{\csc x - \sin x}{1}$

$= \csc x - \sin x$

21. $\dfrac{\cos\theta \cot\theta}{1 - \sin\theta} - 1 = \dfrac{\cos\theta\cot\theta - (1 - \sin\theta)}{1 - \sin\theta}$

$= \dfrac{\cos\theta\left(\dfrac{\cos\theta}{\sin\theta}\right) - 1 + \sin\theta}{1 - \sin\theta} \cdot \dfrac{\sin\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta - \sin\theta + \sin^2\theta}{\sin\theta(1 - \sin\theta)}$

$= \dfrac{1 - \sin\theta}{\sin\theta(1 - \sin\theta)}$

$= \dfrac{1}{\sin\theta}$

$= \csc\theta$

22. $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{(1 + \sin\theta)^2 + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{2 + 2\sin\theta}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{2}{\cos\theta}$

$= 2\sec\theta$

23. $\dfrac{1}{\sin x + 1} + \dfrac{1}{\csc x + 1} = \dfrac{\csc x + 1 + \sin x + 1}{(\sin x + 1)(\csc x + 1)}$

$= \dfrac{\sin x + \csc x + 2}{\sin x \csc x + \sin x + \csc x + 1}$

$= \dfrac{\sin x + \csc x + 2}{1 + \sin x + \csc x + 1}$

$= \dfrac{\sin x + \csc x + 2}{\sin x + \csc x + 2}$

$= 1$

24. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\cos x(1 - \tan x) - \cos x}{1 - \tan x}$

$= \dfrac{-\cos x \tan x}{1 - \tan x}$

$= \dfrac{-\cos x(\sin x/\cos x)}{1 - (\sin x/\cos x)} \cdot \dfrac{\cos x}{\cos x}$

$= \dfrac{-\sin x \cos x}{\cos x - \sin x}$

$= \dfrac{\sin x \cos x}{\sin x - \cos x}$

25. $\tan\left(\dfrac{\pi}{2} - \theta\right)\tan\theta = \cot\theta\tan\theta$

$= \left(\dfrac{1}{\tan\theta}\right)\tan\theta$

$= 1$

26. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \dfrac{\sin x}{\cos x} = \tan x$

27. $\dfrac{\csc(-x)}{\sec(-x)} = \dfrac{1/\sin(-x)}{1/\cos(-x)}$

$= \dfrac{\cos(-x)}{\sin(-x)}$

$= \dfrac{\cos x}{-\sin x}$

$= -\cot x$

28. $(1 + \sin y)[1 + \sin(-y)] = (1 + \sin y)(1 - \sin y)$

$= 1 - \sin^2 y$

$= \cos^2 y$

29. $\dfrac{\tan x \cot x}{\cos x} = \dfrac{1}{\cos x} = \sec x$

30. $$\frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{1}{\cot x} + \frac{1}{\cot y}}{1 - \frac{1}{\cot x} \cdot \frac{1}{\cot y}} \cdot \frac{\cot x \cot y}{\cot x \cot y}$$

$$= \frac{\cot y + \cot x}{\cot x \cot y - 1}$$

31. $$\frac{\tan x + \cot y}{\tan x \cot y} = \frac{\frac{1}{\cot x} + \frac{1}{\tan y}}{\frac{1}{\cot x} \cdot \frac{1}{\tan y}} \cdot \frac{\cot x \tan y}{\cot x \tan y}$$

$$= \tan y + \cot x$$

32. $$\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = \frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= \frac{(\cos^2 x + \sin^2 x) - (\cos^2 y + \sin^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= 0$$

33. $$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} = \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta} \cdot \frac{1 + \sin\theta}{1 + \sin\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}$$

$$= \sqrt{\frac{(1 + \sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1 + \sin\theta}{|\cos\theta|}$$

34. $$\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta} \cdot \frac{1 - \cos\theta}{1 - \cos\theta}}$$

$$= \sqrt{\frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}}$$

$$= \sqrt{\frac{(1 - \cos\theta)^2}{\sin^2\theta}}$$

$$= \frac{1 - \cos\theta}{|\sin\theta|}$$

35. $\cos^2\beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = \cos^2\beta + \sin^2\beta = 1$

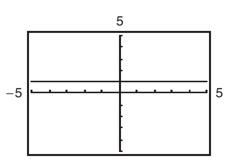
36. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = \sec^2 y - \tan^2 y = 1$

37. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \sin t \sec t = \sin t\left(\dfrac{1}{\cos t}\right)$

$= \dfrac{\sin t}{\cos t} = \tan t$

38. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \csc^2 x - 1 = \cot^2 x$

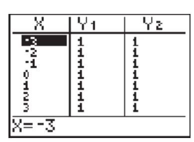
39. (a)



Let $y_1 = \dfrac{2}{(\cos x)^2} - \dfrac{2(\sin x)^2}{(\cos x)^2} - (\sin x)^2 - (\cos x)^2$ and $y_2 = 1$.

Identity

(b)



Identity

—CONTINUED—

39. —CONTINUED—

(c) $2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 2\sec^2 x(1 - \sin^2 x) - (\sin^2 x + \cos^2 x)$

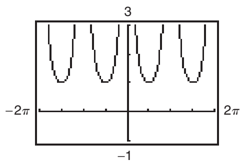
$= 2\sec^2 x(\cos^2 x) - 1$

$= 2 \cdot \dfrac{1}{\cos^2 x} \cdot \cos^2 x - 1$

$= 2 - 1$

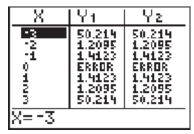
$= 1$

40. (a)



Identity

(b)



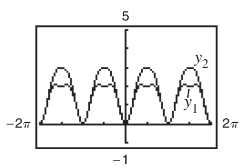
Identity

(c) $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x - \csc x \sin x + 1 - \dfrac{\cos x}{\sin x} + \cot x$

$= \csc^2 x - 1 + 1 - \cot x + \cot x$

$= \csc^2 x$

41. (a)

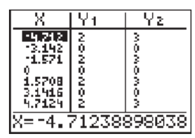


Let $y_1 = 2 + (\cos x)^2 - 3(\cos x)^4$ and

$y_2 = (\sin x)^2(3 + 2(\cos x)^2)$.

Not an identity

(b)



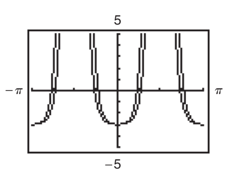
Not an identity

(c) $2 + \cos^2 x - 3\cos^4 x = (1 - \cos^2 x)(2 + 3\cos^2 x)$

$= \sin^2 x(2 + 3\cos^2 x)$

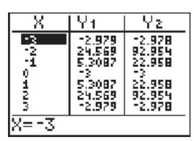
$\neq \sin^2 x(3 + 2\cos^2 x)$

42. (a)



Not an identity

(b)



Not an identity

(c) $\tan^4 x + \tan^2 x - 3 = \dfrac{\sin^4 x}{\cos^4 x} + \dfrac{\sin^2 x}{\cos^2 x} - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^4 x}{\cos^2 x} + \sin^2 x\right) - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x}\right) - 3$

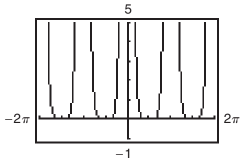
$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^2 x}{\cos^2 x}\right)(\sin^2 x + \cos^2 x) - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^2 x}{\cos^2 x} \cdot 1\right) - 3$

$= \sec^2 x \tan^2 x - 3$

$\neq \sec^2 x(4\tan^2 x - 3)$

43. (a)



Let $y_1 = \dfrac{1}{(\sin x)^4} - \dfrac{2}{(\sin x)^2} + 1$ and $y_2 = \dfrac{1}{(\tan x)^4}$.

Identity

(b)

X	Y1	Y2
-3	2422	2422
-2	.04387	.04387
-1	.16998	.16998
0	ERROR	ERROR
1	.16998	.16998
2	.04387	.04387
3	2422	2422

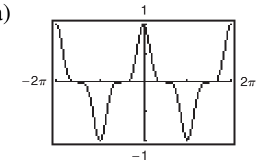
X=-3

Identity

(c) $\csc^4 x - 2\csc^2 x + 1 = (\csc^2 x - 1)^2$

$= (\cot^2 x)^2 = \cot^4 x$

44. (a)



Identity

(b)

X	Y1	Y2
-4.712	0	0
-3.142	-1	-1
-1.571	0	0
0	1	1
1.5708	0	0
3.1416	-1	-1
4.7124	0	0

X=-4.71238898038

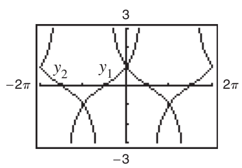
Identity

(c) $(\sin^4 \beta - 2\sin^2 \beta + 1)\cos \beta = (\sin^2 \beta - 1)^2 \cos \beta$

$= (-\cos^2 \beta)^2 \cos \beta$

$= \cos^5 \beta$

45. (a)



Let $y_1 = \dfrac{\cos x}{(1 - \sin x)}$ and $y_2 = \dfrac{(1 - \sin x)}{\cos x}$.

Not an identity

(b)

X	Y1	Y2
-3	-.8676	-1.153
-2	-.218	-4.588
-1	.29341	3.4082
0	1	1
1	3.4082	.29341
2	-4.588	-.218
3	-1.153	-.8676

X=-3

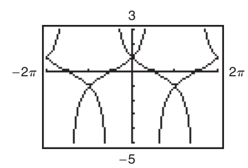
Not an identity

(c) $\dfrac{\cos x}{1 - \sin x} = \dfrac{\cos x}{1 - \sin x} \cdot \dfrac{1 + \sin x}{1 + \sin x}$

$= \dfrac{\cos x(1 + \sin x)}{1 - \sin^2 x}$

$= \dfrac{\cos x(1 + \sin x)}{\cos^2 x} = \dfrac{1 + \sin x}{\cos x}$

46. (a)



Not an identity

(b)

X	Y1	Y2
-3	-1.153	-.8676
-2	-4.588	-.218
-1	3.4082	.29341
0	ERROR	ERROR
1	.29341	3.4082
2	-.218	-4.588
3	-.8676	-1.153

X=-3

Not an identity

(c) $\dfrac{\cot \alpha}{\csc \alpha + 1}$ is the reciprocal of $\dfrac{\csc \alpha + 1}{\cot \alpha}$.

They will only be equivalent at isolated points in their respective domains. Hence, not an identity.

47. $\tan^3 x \sec^2 x - \tan^3 x = \tan^3 x(\sec^2 x - 1)$

$= \tan^3 x \tan^2 x$

$= \tan^5 x$

48. $(\tan^2 x + \tan^4 x)\sec^2 x = \left(\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\sin^4 x}{\cos^4 x}\right)\dfrac{1}{\cos^2 x}$

$= \dfrac{1}{\cos^4 x}\left(\sin^2 x + \dfrac{\sin^4 x}{\cos^2 x}\right)$

$= \dfrac{1}{\cos^4 x}\left(\dfrac{\sin^2 x \cos^2 x + \sin^4 x}{\cos^2 x}\right)$

$= \dfrac{1}{\cos^4 x}\left(\dfrac{\sin^2 x(\cos^2 x + \sin^2 x)}{\cos^2 x}\right)$

$= \dfrac{1}{\cos^4 x}\left(\dfrac{\sin^2 x}{\cos^2 x} \cdot 1\right) = \sec^4 x \cdot \tan^2 x$

49. $(\sin^2 x - \sin^4 x)\cos x = \sin^2 x(1 - \sin^2 x)\cos x$

$= \sin^2 x \cos^2 x \cos x$

$= \sin^2 x \cos^3 x$

50. $\sin^4 x + \cos^4 x = \sin^2 x \sin^2 x + \cos^4 x$

$= (1 - \cos^2 x)(1 - \cos^2 x) + \cos^4 x$

$= 1 - 2\cos^2 x + \cos^4 x + \cos^4 x$

$= 1 - 2\cos^2 x + 2\cos^4 x$

51. $\sin^2 25° + \sin^2 65° = \sin^2 25° + \cos^2(90° - 65°)$

$= \sin^2 25° + \cos^2 25°$

$= 1$

52. $\cos^2 55° + \cos^2 35° = \cos^2 55° + \sin^2(90° - 35°)$

$= \cos^2 55° + \sin^2 55°$

$= 1$

53. $\cos^2 20° + \cos^2 52° + \cos^2 38° + \cos^2 70° = \cos^2 20° + \cos^2 52° + \sin^2(90° - 38°) + \sin^2(90° - 70°)$

$= \cos^2 20° + \cos^2 52° + \sin^2 52° + \sin^2 20°$

$= (\cos^2 20° + \sin^2 20°) + (\cos^2 52° + \sin^2 52°)$

$= 1 + 1$

$= 2$

54. $\sin^2 12° + \sin^2 40° + \sin^2 50° + \sin^2 78° = \sin^2 12° + \sin^2 78° + \sin^2 40° + \sin^2 50°$

$= \cos^2(90° - 12°) + \sin^2 78° + \cos^2(90° - 40°) + \sin^2 50°$

$= \cos^2 78° + \sin^2 78° + \cos^2 50° + \sin^2 50°$

$= 1 + 1 = 2$

55. $\cos x - \csc x \cot x = \cos x - \dfrac{1}{\sin x}\dfrac{\cos x}{\sin x}$

$= \cos x\left(1 - \dfrac{1}{\sin^2 x}\right)$

$= \cos x(1 - \csc^2 x)$

$= -\cos x(\csc^2 x - 1)$

$= -\cos x \cot^2 x$

56. (a) $\dfrac{h\sin(90° - \theta)}{\sin\theta} = \dfrac{h\cos\theta}{\sin\theta} = h\cot\theta$

(b)

θ	10°	20°	30°	40°	50°	60°	70°	80°	90°
s	28.36	13.74	8.66	5.96	4.20	2.89	1.82	0.88	0

(c) Greatest: 10°, Least: 90°

(d) Noon

57. False. For the equation to be an identity, it must be true for all values of θ in the domain.

58. True. An identity is an equation that is true for all real values in the domain of the variable.

59. Since $\sin^2\theta = 1 - \cos^2\theta$, then $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$; $\sin\theta \neq \sqrt{1 - \cos^2\theta}$ if θ lies in Quadrant III or IV. One such angle is $\theta = \dfrac{7\pi}{4}$.

60. $\tan\theta = \sqrt{\sec^2\theta - 1}$

True identity: $\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$ is not true for $\pi/2 < \theta < \pi$ or $3\pi/2 < \theta < 2\pi$. Thus, the equation is not true for $\theta = 3\pi/4$.

61. $(2 + 3i) - \sqrt{-26} = 2 + 3i - \sqrt{26}i$
$= 2 + \left(3 - \sqrt{26}\right)i$

62. $(2 - 5i)^2 = (2 - 5i)(2 - 5i)$
$= 4 - 20i + 25i^2$
$= 4 - 20i - 25$
$= -21 - 20i$

63. $\sqrt{-16}\left(1 + \sqrt{-4}\right) = 4i(1 + 2i)$
$= 4i + 8i^2$
$= 4i - 8$
$= -8 + 4i$

64. $(3 + 2i)^3 = (3 + 2i)(3 + 2i)(3 + 2i)$
$= (9 + 12i + 4i^2)(3 + 2i)$
$= (5 + 12i)(3 + 2i)$
$= 15 + 10i + 36i + 24i^2$
$= -9 + 46i$

65. $x^2 + 6x - 12 = 0$

$a = 1, b = 6, c = -12$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 + 48}}{2}$$
$$= \frac{-6 \pm \sqrt{84}}{2}$$
$$= \frac{-6 \pm 2\sqrt{21}}{2}$$
$$= -3 \pm \sqrt{21}$$

66. $x^2 + 5x - 7 = 0$

$a = 1, b = 5, c = -7$

$$x = \frac{-5 \pm \sqrt{5^2 + (4)(1)(7)}}{2(1)}$$
$$x = \frac{-5 \pm \sqrt{53}}{2}$$

67. $3x^2 - 6x - 12 = 0$

$3(x^2 - 2x - 4) = 0$

$x^2 - 2x - 4 = 0$

$a = 1, b = -2, c = -4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4 + 16}}{2}$$
$$= \frac{2 \pm \sqrt{20}}{2}$$
$$= \frac{2 \pm 2\sqrt{5}}{2}$$
$$= 1 \pm \sqrt{5}$$

68. $8x^2 - 4x - 3 = 0$

$a = 8, b = -4, c = -3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 + 4(8)(3)}}{2(8)}$$
$$= \frac{4 \pm \sqrt{112}}{16}$$
$$x = \frac{4 \pm 4\sqrt{7}}{16}$$
$$x = \frac{1}{4}\left(1 \pm \sqrt{7}\right)$$

Section 5.3 Solving Trigonometric Equations

- You should be able to identify and solve trigonometric equations.
- A trigonometric equation is a conditional equation. It is true for a specific set of values.
- To solve trigonometric equations, use algebraic techniques such as collecting like terms, extracting square roots, factoring, squaring, converting to quadratic type, using formulas, and using inverse functions. Study the examples in this section.

Vocabulary Check

1. general **2.** quadratic **3.** extraneous

1. $2\cos x - 1 = 0$

(a) $2\cos\frac{\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

(b) $2\cos\frac{5\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

2. $\sec x - 2 = 0$

(a) $x = \frac{\pi}{3}$

$$\sec\frac{\pi}{3} - 2 = \frac{1}{\cos(\pi/3)} - 2$$
$$= \frac{1}{1/2} - 2 = 2 - 2 = 0$$

(b) $x = \frac{5\pi}{3}$

$$\sec\frac{5\pi}{3} - 2 = \frac{1}{\cos(5\pi/3)} - 2$$
$$= \frac{1}{1/2} - 2 = 2 - 2 = 0$$

3. $3\tan^2 2x - 1 = 0$

(a) $$3\left[\tan 2\left(\frac{\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{\pi}{6} - 1$$
$$= 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1$$
$$= 0$$

(b) $$3\left[\tan 2\left(\frac{5\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{5\pi}{6} - 1$$
$$= 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1$$
$$= 0$$

4. $2\cos^2 4x - 1 = 0$

(a) $x = \frac{\pi}{16}$

$$2\cos^2\left[4\left(\frac{\pi}{16}\right)\right] - 1 = 2\cos^2\frac{\pi}{4} - 1$$
$$= 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1$$
$$= 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$

(b) $x = \frac{3\pi}{16}$

$$2\cos^2\left[4\left(\frac{3\pi}{16}\right)\right] - 1 = 2\cos^2\frac{3\pi}{4} - 1$$
$$= 2\left(-\frac{\sqrt{2}}{2}\right)^2 - 1$$
$$= 2\left(\frac{1}{2}\right) - 1 = 0$$

5. $2\sin^2 x - \sin x - 1 = 0$

(a) $2\sin^2\frac{\pi}{2} - \sin\frac{\pi}{2} - 1 = 2(1)^2 - 1 - 1$

$= 0$

(b) $2\sin^2\frac{7\pi}{6} - \sin\frac{7\pi}{6} - 1 = 2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1$

$= \frac{1}{2} + \frac{1}{2} - 1$

$= 0$

6. $\csc^4 x - 4\csc^2 x = 0$

(a) $x = \frac{\pi}{6}$

$$\csc^4\frac{\pi}{6} - 4\csc^2\frac{\pi}{6} = \frac{1}{\sin^4(\pi/6)} - \frac{4}{\sin^2(\pi/6)}$$

$$= \frac{1}{(1/2)^4} - \frac{4}{(1/2)^2}$$

$$= 16 - 16 = 0$$

(b) $x = \frac{5\pi}{6}$

$$\csc^4\frac{5\pi}{6} - 4\csc\frac{5\pi}{6} = \frac{1}{\sin^4(5\pi/6)} - \frac{4}{\sin^2(5\pi/6)}$$

$$= \frac{1}{(1/2)^4} - \frac{4}{(1/2)^2}$$

$$= 16 - 16 = 0$$

7. $2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2n\pi$

or $x = \frac{4\pi}{3} + 2n\pi$

8. $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6} + 2n\pi$

or $x = \frac{11\pi}{6} + 2n\pi$

9. $\sqrt{3}\csc x - 2 = 0$

$\sqrt{3}\csc x = 2$

$\csc x = \frac{2}{\sqrt{3}}$

$x = \frac{\pi}{3} + 2n\pi$

or $x = \frac{2\pi}{3} + 2n\pi$

10. $\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3} + n\pi$

11. $3\sec^2 x - 4 = 0$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm\frac{2}{\sqrt{3}}$

$x = \frac{\pi}{6} + n\pi$

or $x = \frac{5\pi}{6} + n\pi$

12. $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{3} + n\pi$

or $x = \frac{2\pi}{3} + n\pi$

13. $\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -1$

$x = n\pi$ $\quad$ $x = \frac{3\pi}{2} + 2n\pi$

14. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$ or $\tan^2 x - 3 = 0$

$\tan x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6} + n\pi$

or $x = \frac{5\pi}{6} + n\pi$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3} + n\pi$

or $x = \frac{2\pi}{3} + n\pi$

15. $4\cos^2 x - 1 = 0$

$$\cos^2 x = \frac{1}{4}$$

$$\cos^2 x = \pm\frac{1}{2}$$

$$x = \frac{\pi}{3} + n\pi \quad \text{or} \quad x = \frac{2\pi}{3} + n\pi$$

16. $\sin^2 x = 3\cos^2 x$

$$\sin^2 x - 3(1 - \sin^2 x) = 0$$

$$4\sin^2 x = 3$$

$$\sin x = \pm\frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + n\pi \quad \text{or} \quad x = \frac{2\pi}{3} + n\pi$$

17. $2\sin^2 2x = 1$

$$\sin 2x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4} + 2n\pi,\ 2x = \frac{3\pi}{4} + 2n\pi,$$

$$2x = \frac{5\pi}{4} + 2n\pi, 2x = \frac{7\pi}{4} + 2n\pi$$

Thus, $x = \frac{\pi}{8} + n\pi,\ \frac{3\pi}{8} + n\pi,\ \frac{5\pi}{8} + n\pi,\ \frac{7\pi}{8} + n\pi.$

We can combine these as follows:

$$x = \frac{\pi}{8} + \frac{n\pi}{2}, x = \frac{3\pi}{8} + \frac{n\pi}{2}$$

18. $\tan^2 3x = 3$

$$\tan 3x = \pm\sqrt{3}$$

$$3x = \frac{\pi}{3} + n\pi \implies x = \frac{\pi}{9} + \frac{n\pi}{3}$$

or

$$3x = \frac{2\pi}{3} + n\pi \implies x = \frac{2\pi}{9} + \frac{n\pi}{3}$$

19. $\tan 3x(\tan x - 1) = 0$

$\tan 3x = 0$ or $\tan x - 1 = 0$

$3x = n\pi$ $\tan x = 1$

$x = \frac{n\pi}{3}$ $x = \frac{\pi}{4} + n\pi$

20. $\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$ or $2\cos x + 1 = 0$

$2x = \frac{\pi}{2} + n\pi$ $\cos x = -\frac{1}{2}$

$x = \frac{\pi}{4} + \frac{n\pi}{2}$ $x = \frac{2\pi}{3} + 2n\pi$

or $x = \frac{4\pi}{3} + 2n\pi$

21. $\cos^3 x = \cos x$

$$\cos^3 x - \cos x = 0$$

$$\cos x(\cos^2 x - 1) = 0$$

$\cos x = 0$ or $\cos^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\cos x = \pm 1$

$x = 0, \pi$

22. $\sec^2 x - 1 = 0$

$$\sec^2 x = 1$$

$$\sec x = \pm 1$$

$$x = 0 \text{ or } x = \pi$$

23. $3\tan^3 x - \tan x = 0$

$$\tan x(3\tan^2 x - 1) = 0$$

$\tan x = 0$ or $3\tan^2 x - 1 = 0$

$x = 0, \pi$ $\tan x = \pm\frac{\sqrt{3}}{3}$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

24. $2\sin^2 x = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$2\cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ or $2\cos x + 1 = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad 2\cos x = -1$

$\cos x = -\dfrac{1}{2}$

$x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

25. $\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$ or $\sec x + 1 = 0$

$\sec x = 2$ $\qquad \sec x = -1$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\qquad x = \pi$

26. $\sec x \csc x = 2\csc x$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ or $\sec x - 2 = 0$

No solution $\qquad \sec x = 2$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

27. $2\sin x + \csc x = 0$

$2\sin x + \dfrac{1}{\sin x} = 0$

$2\sin^2 x + 1 = 0$

$\sin^2 x = -\dfrac{1}{2} \implies$ No solution

28. $\sec x + \tan x = 1$

$\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x} = 1$

$1 + \sin x = \cos x$

$(1 + \sin x)^2 = \cos^2 x$

$1 + 2\sin x + \sin^2 x = \cos^2 x$

$1 + 2\sin x + \sin^2 x = 1 - \sin^2 x$

$2\sin^2 x + 2\sin x = 0$

$2\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x + 1 = 0$

$x = 0, \pi$ $\qquad \sin x = -1$

(π is extraneous.) $\qquad x = \dfrac{3\pi}{2}$

$\left(\dfrac{3\pi}{2} \text{ is extraneous.}\right)$

$x = 0$ is the only solution.

29. $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \dfrac{1}{2}$ $\qquad \cos x = -1$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\qquad x = \pi$

30. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$\sin x = -\dfrac{1}{2}$ $\qquad \sin x = -1$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ $\qquad x = \dfrac{3\pi}{2}$

31. $2\sec^2 x + \tan^2 x - 3 = 0$

$2(\tan^2 x + 1) + \tan^2 x - 3 = 0$

$3\tan^2 x - 1 = 0$

$\tan x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

32.
$$\cos x + \sin x \tan x = 2$$
$$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right) = 2$$
$$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$$
$$\frac{1}{\cos x} = 2$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

33.
$$\csc x + \cot x = 1$$
$$(\csc x + \cot x)^2 = 1^2$$
$$\csc^2 x + 2\csc x \cot x + \cot^2 x = 1$$
$$\cot^2 x + 1 + 2\csc x \cot x + \cot^2 x = 1$$
$$2\cot^2 x + 2\csc x \cot x = 0$$
$$2\cot x(\cot x + \csc x) = 0$$
$$2\cot x = 0 \quad \text{or} \quad \cot x + \csc x = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \frac{\cos x}{\sin x} = -\frac{1}{\sin x}$$
$$\cos x = -1$$
$$x = \pi$$

By checking in the original equation, we find that $x = \pi$ and $x = 3\pi/2$ are extraneous. The only solution to the equation in the interval $[0, 2\pi)$ is $x = \pi/2$.

34. $\sin x - 2 = \cos x - 2$
$$\sin x = \cos x$$
$$\frac{\sin x}{\cos x} = 1$$
$$\tan x = 1$$
$$x = \tan^{-1} 1$$
$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

35. $\cos 2x = \frac{1}{2}$
$$2x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{3} + 2n\pi$$
$$x = \frac{\pi}{6} + n\pi \qquad x = \frac{5\pi}{6} + n\pi$$

36. $\sin 2x = -\frac{\sqrt{3}}{2}$
$$2x = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{3} + 2n\pi$$
$$x = \frac{2\pi}{3} + n\pi \qquad x = \frac{5\pi}{6} + n\pi$$

37. $\tan 3x = 1$
$$3x = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad 3x = \frac{5\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{12} + \frac{2n\pi}{3} \qquad x = \frac{5\pi}{12} + \frac{2n\pi}{3}$$

These can be combined as $x = \frac{\pi}{12} + \frac{n\pi}{3}$.

38. $\sec 4x = 2$
$$4x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 4x = \frac{5\pi}{3} + 2n\pi$$
$$x = \frac{\pi}{12} + \frac{n\pi}{2} \qquad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

39. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}$
$$\frac{x}{2} = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad \frac{x}{2} = \frac{7\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{2} + 4n\pi \qquad x = \frac{7\pi}{2} + 4n\pi$$

40. $\sin\frac{x}{2} = -\frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{4\pi}{3} + 2n\pi$ or $\frac{x}{2} = \frac{5\pi}{3} + 2n\pi$

$x = \frac{8\pi}{3} + 4n\pi$ $\quad x = \frac{10\pi}{3} + 4n\pi$

41. $y = \sin\frac{\pi x}{2} + 1$

From the graph in the textbook we see that the curve has x-intercepts at $x = -1$ and at $x = 3$.

In general, we have: $\sin\left(\frac{\pi x}{2}\right) = -1$

$$\frac{\pi x}{2} = \frac{3\pi}{2} + 2n\pi$$

$$x = 3 + 4n$$

42. $y = \sin \pi x + \cos \pi x$

$$\sin \pi x + \cos \pi x = 0$$

$$\sin \pi x = -\cos \pi x$$

$$\pi x = -\frac{\pi}{4} + n\pi$$

$$x = -\frac{1}{4} + n$$

For $-1 < x < 3$ the intercepts are $-\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{11}{4}$.

43. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$

From the graph in the textbook we see that the curve has x-intercepts at $x = \pm 2$.

In general, we have: $\tan^2\left(\frac{\pi x}{6}\right) = 3$

$$\tan\left(\frac{\pi x}{6}\right) = \pm\sqrt{3}$$

$$\frac{\pi x}{6} = \pm\frac{\pi}{3} + n\pi$$

$$x = \pm 2 + 6n$$

44. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$

$$\sec^4\left(\frac{\pi x}{8}\right) - 4 = 0$$

$$\sec^4\left(\frac{\pi x}{8}\right) = 4$$

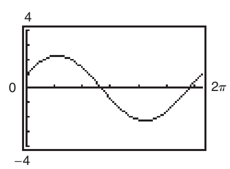
$$\sec\left(\frac{\pi x}{8}\right) = \pm\sqrt{2}$$

$$\frac{\pi x}{8} = \frac{\pi}{4} + \frac{\pi}{2}n$$

$$x = 2 + 4n$$

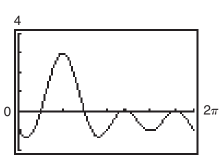
For $-3 < x < 3$ the intercepts are -2 and 2.

45. Graph $y_1 = 2\sin x + \cos x$.



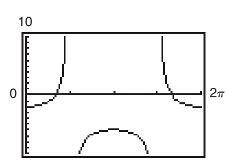
The x-intercepts occur at $x \approx 2.678$ and $x \approx 5.820$.

46. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$



$x \approx 0.785, 2.356, 3.665, 3.927, 5.498, 5.760$

47. Graph $y_1 = \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} - 4$.

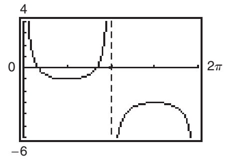


The x-intercepts occur at $x = \frac{\pi}{3} \approx 1.047$ and $x = \frac{5\pi}{3} \approx 5.236$.

48. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

$$y_1 = \left(\frac{\dfrac{\cos x}{\tan x}}{1 - \sin x}\right) - 3$$

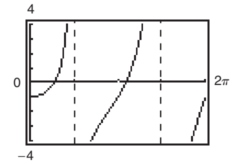
$x \approx 0.524, 2.618$



49. $x \tan x - 1 = 0$

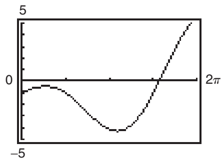
Graph $y_1 = x \tan x - 1$.

The x-intercepts occur at $x \approx 0.860$ and $x \approx 3.426$.



50. $x \cos x - 1 = 0$

$x \approx 4.917$



51. $\sec^2 x + 0.5 \tan x - 1 = 0$

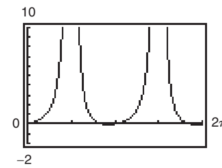
Graph $y_1 = \dfrac{1}{(\cos x)^2} + 0.5 \tan x - 1$.

The x-intercepts occur at

$x = 0$, $x \approx 2.678$,

$x = \pi \approx 3.142$, and

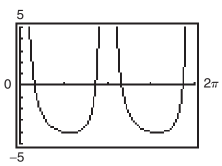
$x \approx 5.820$.



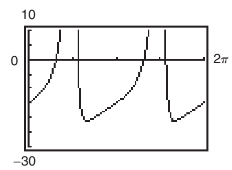
52. $\csc^2 x + 0.5 \cot x - 5 = 0$

$$y_1 = \left(\frac{1}{\sin x}\right)^2 + \frac{1}{2 \tan x} - 5$$

$x \approx 0.515, 2.726, 3.657, 5.868$



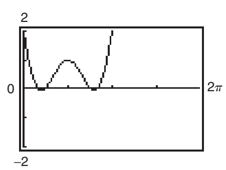
53. Graph $y_1 = 2 \tan^2 x + 7 \tan x - 15$.



The x-intercepts occur at $x \approx 0.983$, $x \approx 1.768$, $x \approx 4.124$ and $x \approx 4.910$.

54. $6 \sin^2 x - 7 \sin x + 2 = 0$

$x \approx 0.524, 0.730, 2.412, 2.618$



55. $12 \sin^2 x - 13 \sin x + 3 = 0$

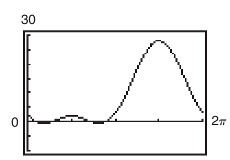
$$\sin x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(12)(3)}}{2(12)}$$

$$= \frac{13 \pm 5}{24}$$

$\sin x = \dfrac{1}{3}$ or $\sin x = \dfrac{3}{4}$

$x \approx 0.3398,\ 2.8018$ $\qquad$ $x \approx 0.8481,\ 2.2935$

Graph $y_1 = 12 \sin^2 x - 13 \sin x + 3$.



The x-intercepts occur at $x \approx 0.3398$, $x \approx 0.8481$, $x \approx 2.2935$, and $x \approx 2.8018$.

56. $3\tan^2 x + 4\tan x - 4 = 0$

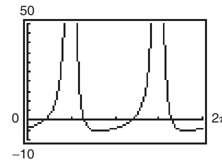
$$\tan x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-4)}}{2(3)} = \frac{-4 \pm \sqrt{64}}{6} = -2, \frac{2}{3}$$

$\tan x = -2$ $\qquad$ $\tan x = \frac{2}{3}$

$x = \arctan(-2) + n\pi$ $\qquad$ $x = \arctan\left(\frac{2}{3}\right) + n\pi$

$\approx -1.1071 + n\pi$ $\qquad$ $\approx 0.5880 + n\pi$

The values of x in $[0, 2\pi)$ are 0.5880, 3.7296, 2.0344, 5.1760.



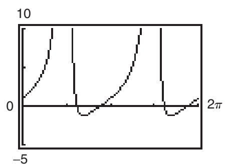
57. $\tan^2 x + 3\tan x + 1 = 0$

$$\tan x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

$\tan x = \frac{-3 - \sqrt{5}}{2}$ or $\tan x = \frac{-3 + \sqrt{5}}{2}$

$x \approx 1.9357,\ 5.0773$ $\qquad$ $x \approx 2.7767,\ 5.9183$

Graph $y_1 = \tan^2 x + 3\tan x + 1$.



The x-intercepts occur at $x \approx 1.9357$, $x \approx 2.7767$, $x \approx 5.0773$, and $x \approx 5.9183$.

58. $4\cos^2 x - 4\cos x - 1 = 0$

$$\cos x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2(4)}$$

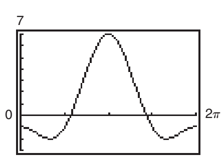
$$= \frac{4 \pm \sqrt{32}}{8} = \frac{1 \pm \sqrt{2}}{2}$$

$\cos x = \frac{1 - \sqrt{2}}{2}$ $\qquad$ $\cos x = \frac{1 + \sqrt{2}}{2}$

$x = \arccos\left(\frac{1 - \sqrt{2}}{2}\right)$ $\qquad$ No solution

≈ 1.7794 $\qquad$ $\left(\frac{1 + \sqrt{2}}{2} > 1\right)$

Solutions in $[0, 2\pi)$ are $\arccos\left(\frac{1 - \sqrt{2}}{2}\right)$ and $2\pi - \arccos\left(\frac{1 - \sqrt{2}}{2}\right)$: 1.7794, 4.5038.



59. $\tan^2 x - 6\tan x + 5 = 0$

$(\tan x - 1)(\tan x - 5) = 0$

$\tan x - 1 = 0$ or $\tan x - 5 = 0$

$\tan x = 1$ $\qquad$ $\tan x = 5$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ $\qquad$ $x = \arctan 5, \arctan 5 + \pi$

60. $\sec^2 x + \tan x - 3 = 0$

$1 + \tan^2 x + \tan x - 3 = 0$

$\tan^2 x + \tan x - 2 = 0$

$(\tan x + 2)(\tan x - 1) = 0$

$\tan x + 2 = 0$ $\qquad$ $\tan x - 1 = 0$

$\tan x = -2$ $\qquad$ $\tan x = 1$

$x = \arctan(-2) + n\pi$ $\qquad$ $x = \arctan(1) + n\pi$

$\approx -1.1071 + n\pi$ $\qquad$ $= \dfrac{\pi}{4} + n\pi$

Solutions in $[0, 2\pi)$ are $\arctan(-2) + \pi$, $\arctan(-2) + 2\pi$, $\dfrac{\pi}{4}, \dfrac{5\pi}{4}$.

61. $2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$2\cos x - 1 = 0$ or $\cos x - 2 = 0$

$\cos x = \dfrac{1}{2}$ $\qquad$ $\cos x = 2$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\qquad$ No solution

62. $2\sin^2 x - 7\sin x + 3 = 0$

$(\sin x - 3)(2\sin x - 1) = 0$

$\sin x - 3 = 0$ $\qquad$ $2\sin x - 1 = 0$

No solution $\qquad$ $\sin x = \dfrac{1}{2}$

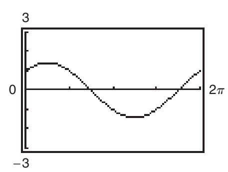
$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Solutions in $[0, 2\pi)$ are $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$.

63. (a) $f(x) = \sin x + \cos x$

Maximum: $\left(\dfrac{\pi}{4}, \sqrt{2}\right)$

Minimum: $\left(\dfrac{5\pi}{4}, -\sqrt{2}\right)$



(b) $\cos x - \sin x = 0$

$\cos x = \sin x$

$1 = \dfrac{\sin x}{\cos x}$

$\tan x = 1$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

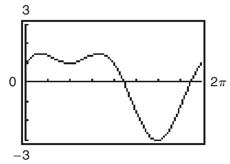
$$f\left(\frac{5\pi}{4}\right) = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\sin\frac{\pi}{4} + \left(-\cos\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

Therefore, the maximum point in the interval $[0, 2\pi)$ is $(\pi/4, \sqrt{2})$ and the minimum point is $(5\pi/4, -\sqrt{2})$.

64. (a) $f(x) = 2\sin x + \cos 2x$

max: $(0.5240, 1.5)$ min: $(1.5708, 1.0)$

max: $(2.6180, 1.5)$ min: $(4.7124, -3.0)$



(b) $2\cos x - 4\sin x\cos x = 0$

$2\cos x(1 - 2\sin x) = 0$

$2\cos x = 0$ $\quad\quad$ $1 - 2\sin x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad\quad$ $\sin x = \frac{1}{2}$

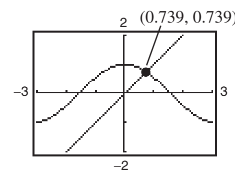
$\approx 1.5708, 4.7124$ $\quad\quad$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\approx 0.5240, 2.6180$

65. $f(x) = \tan\frac{\pi x}{4}$

Since $\tan \pi/4 = 1$, $x = 1$ is the smallest nonnegative fixed point.

66. Graph $y = \cos x$ and $y = x$ on the same set of axes. Their point of intersection gives the value of c such that $f(c) = c \implies \cos c = c$.



$c \approx 0.739$

67. $f(x) = \cos\frac{1}{x}$

(a) The domain of $f(x)$ is all real numbers x except $x = 0$.

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 1$.

(c) As $x \longrightarrow 0$, $f(x)$ oscillates between -1 and 1.

(d) There are infinitely many solutions in the interval $[-1, 1]$. They occur at $x = \frac{2}{(2n + 1)\pi}$ where n is any integer.

(e) The greatest solution appears to occur at $x \approx 0.6366$.

68. $f(x) = \frac{\sin x}{x}$

(a) Domain: all real numbers except $x = 0$.

(b) The graph has y-axis symmetry.

(c) As $x \longrightarrow 0, f(x) \longrightarrow 1$.

(d) $\frac{\sin x}{x} = 0$ has four solutions in the interval $[-8, 8]$.

$\sin x\left(\frac{1}{x}\right) = 0$

$\sin x = 0$

$x = -2\pi, -\pi, \pi, 2\pi$

69. $y = \frac{1}{12}(\cos 8t - 3\sin 8t)$

$\frac{1}{12}(\cos 8t - 3\sin 8t) = 0$

$\cos 8t = 3\sin 8t$

$\frac{1}{3} = \tan 8t$

$8t \approx 0.32175 + n\pi$

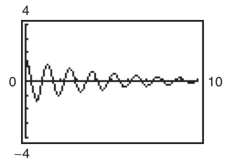
$t \approx 0.04 + \frac{n\pi}{8}$

In the interval $0 \le t \le 1$, $t \approx 0.04, 0.43$, and 0.83.

70. $y_1 = 1.56e^{-0.22t} \cos 4.9t$

Right-most point of intersection: $(1.96, -1)$

The displacement does not exceed one foot from equilibrium after $t = 1.96$ seconds.



71. $S = 74.50 + 43.75 \sin \dfrac{\pi t}{6}$

t	1	2	3	4	5	6	7	8	9	10	11	12
S	96.4	112.4	118.3	112.4	96.4	74.5	52.6	36.6	30.8	36.6	52.6	74.5

Sales exceed 100,000 units during February, March, and April.

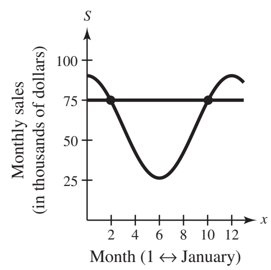
72. Graph $y_1 = 58.3 + 32 \cos\left(\dfrac{\pi t}{6}\right)$

$y_2 = 75.$

Left point of intersection: $(1.95, 75)$

Right point of intersection: $(10.05, 75)$

So, sales exceed 7500 in January, November, and December.



73.

$$\text{Range} = 300 \text{ feet}$$
$$v_0 = 100 \text{ feet per second}$$
$$r = \tfrac{1}{32}v_0^2 \sin 2\theta$$
$$\tfrac{1}{32}(100)^2 \sin 2\theta = 300$$
$$\sin 2\theta = 0.96$$
$$2\theta = \arcsin(0.96) \approx 73.74°$$
$$\theta \approx 36.9°$$

or

$$2\theta = 180° - \arcsin(0.96) \approx 106.26°$$
$$\theta \approx 53.1°$$

74.

$$\text{Range} = 1000 \text{ yards} = 3000 \text{ feet}$$
$$v_0 = 1200 \text{ feet per second}$$
$$f = \tfrac{1}{32} v_0^2 \sin 2\theta$$
$$3000 = \tfrac{1}{32}(1200)^2 \sin 2\theta$$
$$\sin 2\theta \approx 0.066667$$
$$2\theta \approx 3.8°$$
$$\theta \approx 1.9°$$

75. $h(t) = 53 + 50 \sin\left(\dfrac{\pi}{16}t - \dfrac{\pi}{2}\right)$

(a) $h(t) = 53$ when $50 \sin\left(\dfrac{\pi}{16}t - \dfrac{\pi}{2}\right) = 0.$

$$\frac{\pi}{16}t - \frac{\pi}{2} = 0 \qquad \text{or} \qquad \frac{\pi}{16}t - \frac{\pi}{2} = \pi$$
$$\frac{\pi}{16}t = \frac{\pi}{2} \qquad\qquad \frac{\pi}{16}t = \frac{3\pi}{2}$$
$$t = 8 \qquad\qquad t = 24$$

The Ferris wheel will be 53 feet above ground at 8 seconds and at 24 seconds.

—CONTINUED—

75. —CONTINUED—

(b) The person will be at the top of the Ferris wheel when

$$\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right) = 1$$

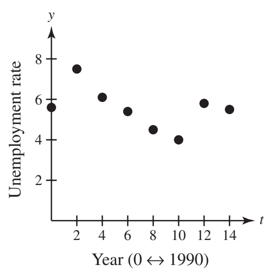
$$\frac{\pi}{16}t - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\frac{\pi}{16}t = \pi$$

$$t = 16.$$

The first time this occurs is after 16 seconds. The period of this function is $\frac{2\pi}{\pi/16} = 32$. During 160 seconds, 5 cycles will take place and the person will be at the top of the ride 5 times, spaced 32 seconds apart. The times are: 16 seconds, 48 seconds, 80 seconds, 112 seconds, and 144 seconds.

76. (a)



(b) By graphing the curves, we see that
(1) $r = 1.24\sin(0.47t + 0.40) + 5.45$ best fits the data.

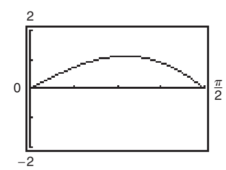
(c) The constant term gives the average unemployment rate of 5.45%.

(d) Period: $\frac{2\pi}{0.47} = 13.4$ years

(e) $r = 5$ when $t = 20$ which corresponds to the year 2010.

77. $A = 2x\cos x,\ 0 < x < \frac{\pi}{2}$

(a)



The maximum area of $A \approx 1.12$ occurs when $x \approx 0.86$.

(b) $A \ge 1$ for $0.6 < x < 1.1$

78. $f(x) = 3\sin(0.6x - 2)$

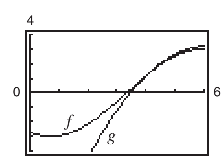
(a) Zero: $\sin(0.6x - 2) = 0$

$$0.6x - 2 = 0$$

$$0.6x = 2$$

$$x = \frac{2}{0.6} = \frac{10}{3}$$

(b) $g(x) = -0.45x^2 + 5.52x - 13.70$



For $3.5 \le x \le 6$ the approximation appears to be good.

(c) $-0.45x^2 + 5.52x - 13.70 = 0$

$$x = \frac{-5.52 \pm \sqrt{(5.52)^2 - 4(-0.45)(-13.70)}}{2(-0.45)}$$

$$x \approx 3.46, 8.81$$

The zero of g on $[0, 6]$ is 3.46. The zero is close to the zero $\frac{10}{3} \approx 3.33$ of f.

79. True. The period of $2\sin 4t - 1$ is $\frac{\pi}{2}$ and the period of $2\sin t - 1$ is 2π.

In the interval $[0, 2\pi)$ the first equation has four cycles whereas the second equation has only one cycle, thus the first equation has four times the x-intercepts (solutions) as the second equation.

80. False.
$\sin x = 3.4$ has no solution since 3.4 is outside the range of sin. Also, 3.4 is outside the domain of arcsin, so $x = \arcsin(3.4)$ is an invalid equation.

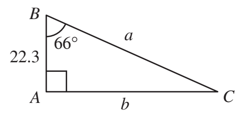
81. $y_1 = 2 \sin x$

$y_2 = 3x + 1$

From the graph we see that there is only one point of intersection.

82. By inspecting the graphs of y_1 and y_2, it appears they intersect at three points.

83. $C = 90° - 66° = 24°$



$$\cos 66° = \frac{22.3}{a}$$

$$a \cos 66° = 22.3$$

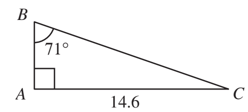
$$a = \frac{22.3}{\cos 66°} \approx 54.8$$

$$\tan 66° = \frac{b}{22.3}$$

$$b = 22.3 \tan 66° \approx 50.1$$

84. Given: $A = 90°, B = 71°, b = 14.6$

$$C = 90° - 71° = 19°$$



$$\sin 71° = \frac{14.6}{a}$$

$$a = \frac{14.6}{\sin 71} \approx 15.4$$

$$\tan 71° = \frac{14.6}{c}$$

$$c = \frac{14.6}{\tan 71°} \approx 5.0$$

85. $\theta = 390°,\ \theta' = 390° - 360° = 30°$, θ is in Quadrant I.

$$\sin 390° = \sin 30° = \frac{1}{2}$$

$$\cos 390° = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 390° = \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

86. $600°$

$600° - 360° = 240°$, Quadrant III

Reference angle: $60°$

$$\sin 600° = -\sin 60° = -\frac{\sqrt{3}}{2}$$

$$\cos 600° = -\cos 60° = -\frac{1}{2}$$

$$\tan 600° = \tan 60° = \sqrt{3}$$

87. $\theta = -1845°,\ \theta' = 45°$, θ is in Quadrant IV.

$$\sin(-1845°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$

$$\cos(-1845°) = \cos 45° = \frac{\sqrt{2}}{2}$$

$$\tan(-1845°) = -\tan 45° = -1$$

88. $-1410°$

$-1410° + 4(360°) = 30°$, Quadrant I

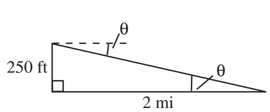
$$\sin(-1410°) = \sin 30° = \frac{1}{2}$$

$$\cos(-1410°) = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan(-1410°) = \tan 30° = \frac{\sqrt{3}}{3}$$

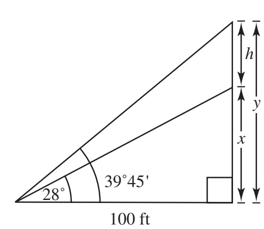
89. $\tan \theta = \dfrac{250 \text{ feet}}{2 \text{ miles}} \times \dfrac{1 \text{ mile}}{5280 \text{ feet}} \approx 0.02367$

$$\theta \approx 1.36°$$



Not drawn to scale

90. $h = y - x$



$$\tan 39.75° = \frac{y}{100}$$

$$100 \tan 39.75° = y$$

$$\tan 28° = \frac{x}{100}$$

$$100 \tan 28° = x$$

$$h = 100 \tan 39.75° - 100 \tan 28°$$

$$h \approx 30 \text{ feet}$$

91. Answers will vary.

Section 5.4 Sum and Difference Formulas

- You should know the sum and difference formulas.

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- You should be able to use these formulas to find the values of the trigonometric functions of angles whose sums or differences are special angles.
- You should be able to use these formulas to solve trigonometric equations.

Vocabulary Check

1. $\sin u \cos v - \cos u \sin v$

2. $\cos u \cos v - \sin u \sin v$

3. $\dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

4. $\sin u \cos v + \cos u \sin v$

5. $\cos u \cos v + \sin u \sin v$

6. $\dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

1. (a) $\cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

$$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{-\sqrt{2} - \sqrt{6}}{4}$$

(b) $\cos 120° + \cos 45° = -\dfrac{1}{2} + \dfrac{\sqrt{2}}{2} = \dfrac{-1 + \sqrt{2}}{2}$

2. (a) $\sin(135° - 30°) = \sin 135° \cos 30° - \cos 135° \sin 30°$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

(b) $\sin 135° - \cos 30° = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{2} - \sqrt{3}}{2}$

3. (a) $\cos\left(\dfrac{\pi}{4} + \dfrac{\pi}{3}\right) = \cos\dfrac{\pi}{4}\cos\dfrac{\pi}{3} - \sin\dfrac{\pi}{4}\sin\dfrac{\pi}{3}$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

(b) $\cos\dfrac{\pi}{4} + \cos\dfrac{\pi}{3} = \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{2} + 1}{2}$

4. (a) $\sin\left(\dfrac{3\pi}{4} + \dfrac{5\pi}{6}\right) = \sin\dfrac{3\pi}{4}\cos\dfrac{5\pi}{6} + \cos\dfrac{3\pi}{4}\sin\dfrac{5\pi}{6}$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$= -\frac{\sqrt{6} + \sqrt{2}}{4}$$

(b) $\sin\dfrac{3\pi}{4} + \sin\dfrac{5\pi}{6} = \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{2} + 1}{2}$

5. (a) $\sin\left(\dfrac{7\pi}{6} - \dfrac{\pi}{3}\right) = \sin\dfrac{5\pi}{6} = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$

(b) $\sin\dfrac{7\pi}{6} - \sin\dfrac{\pi}{3} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{-1 - \sqrt{3}}{2}$

6. (a) $\sin(315° - 60°) = \sin 315° \cos 60° - \cos 315° \sin 60°$

$$= \frac{-\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

(b) $\sin 315° - \sin 60° = \dfrac{-\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{-\sqrt{2} - \sqrt{3}}{2}$

7. $\sin 105° = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\tan 105° = \tan(60° + 45°)$

$= \frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$

8. $165° = 135° + 30°$

$\sin 165° = \sin(135° + 30°)$

$= \sin 135° \cos 30° + \sin 30° \cos 135°$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$

$\cos 165° = \cos(135° + 30°)$

$= \cos 135° \cos 30° - \sin 135° \sin 30°$

$= -\cos 45° \cos 30° - \sin 45° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\tan 165° = \tan(135° + 30°)$

$= \frac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°}$

$= \frac{-\tan 45° + \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$

$= -2 + \sqrt{3}$

9. $\sin 195° = \sin(225° - 30°)$

$= \sin 225° \cos 30° - \cos 225° \sin 30°$

$= -\sin 45° \cos 30° + \cos 45° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$

$\cos 195° = \cos(225° - 30°)$

$= \cos 225° \cos 30° + \sin 225° \sin 30°$

$= -\cos 45° \cos 30° - \sin 45° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$

$\tan 195° = \tan(225° - 30°)$

$= \frac{\tan 225° - \tan 30°}{1 + \tan 225° \tan 30°}$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - \left(\frac{\sqrt{3}}{3}\right)}{1 + \left(\frac{\sqrt{3}}{3}\right)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$

10. $255° = 300° - 45°$

$$\sin 255° = \sin(300° - 45°)$$
$$= \sin 300° \cos 45° - \sin 45° \cos 300°$$
$$= -\sin 60° \cos 45° - \sin 45° \cos 60°$$
$$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos 255° = \cos(300° - 45°)$$
$$= \cos 300° \cos 45° + \sin 300° \sin 45°$$
$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan 255° = \tan(300° - 45°)$$
$$= \frac{\tan 300° - \tan 45°}{1 + \tan 300° \tan 45°}$$
$$= \frac{-\tan 60° - \tan 45°}{1 - \tan 60° \tan 45°}$$
$$= \frac{-\sqrt{3} - 1}{1 - \sqrt{3}} = 2 + \sqrt{3}$$

11. $\sin \frac{11\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$$= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6}$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\frac{1}{2}$$
$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$$\cos \frac{11\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$
$$= \cos \frac{3\pi}{4} \cos \frac{\pi}{6} - \sin \frac{3\pi}{4} \sin \frac{\pi}{6}$$
$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\tan \frac{11\pi}{4} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$
$$= \frac{\tan \frac{3\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{3\pi}{4} \tan \frac{\pi}{6}}$$
$$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 - (-1)\frac{\sqrt{3}}{3}}$$
$$= \frac{-3 + \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$
$$= \frac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$$

12. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$$\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{\pi}{3}$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4}$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan \frac{7\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$$
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$
$$= -2 - \sqrt{3}$$

13. $\sin\frac{17\pi}{12} = \sin\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right)$

$= \sin\frac{9\pi}{4}\cos\frac{5\pi}{6} - \cos\frac{9\pi}{4}\sin\frac{5\pi}{6}$

$= \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos\frac{17\pi}{12} = \cos\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right)$

$= \cos\frac{9\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{9\pi}{4}\sin\frac{5\pi}{6}$

$= \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\tan\frac{17\pi}{12} = \tan\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right)$

$= \frac{\tan(9\pi/4) - \tan(5\pi/6)}{1 + \tan(9\pi/4)\tan(5\pi/6)}$

$= \frac{1 - \left(-\sqrt{3}/3\right)}{1 + \left(-\sqrt{3}/3\right)}$

$= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$

$= \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}$

14. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$

$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$= \frac{\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3}}$

$= -2 + \sqrt{3}$

15. $285 = 225 + 60$

$\sin 285° = \sin(225° + 60°)$

$= \sin 225°\cos 60° + \cos 225°\sin 60°$

$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos 285° = \cos(225° + 60°)$

$= \cos 225°\cos 60° - \sin 225°\sin 60°$

$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\tan 285° = \tan(225° + 60°)$

$= \frac{\tan 225° + \tan 60°}{1 - \tan 225°\tan 60°} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3} = -\left(2 + \sqrt{3}\right)$

16. $-105 = 30° - 135°$

$$\begin{aligned}\sin(30° - 135°) &= \sin 30° \cos 135° - \cos 30° \sin 135°\\ &= \sin 30°(-\cos 45°) - \cos 30° \sin 45°\\ &= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)\end{aligned}$$

$$\begin{aligned}\cos(30° - 135°) &= \cos 30° \cos 135° + \sin 30° \sin 135°\\ &= \cos 30°(-\cos 45°) + \sin 30° \sin 45°\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\\ &= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)\end{aligned}$$

$$\begin{aligned}\tan(30° - 135°) &= \frac{\tan 30° - \tan 135°}{1 + \tan 30° \tan 135°}\\ &= \frac{\tan 30° - (-\tan 45°)}{1 + \tan 30°(-\tan 45°)}\\ &= \frac{\frac{\sqrt{3}}{3} - (-1)}{1 + \left(\frac{\sqrt{3}}{3}\right)(-1)} = 2 + \sqrt{3}\end{aligned}$$

17. $-165° = -(120° + 45°)$

$$\begin{aligned}\sin(-165°) &= \sin[-(120° + 45°)]\\ &= -\sin(120° + 45°)\\ &= -[\sin 120° \cos 45° + \cos 120° \sin 45°]\\ &= -\left[\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right]\\ &= -\frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)\end{aligned}$$

$$\begin{aligned}\cos(-165°) &= \cos[-(120° + 45°)]\\ &= \cos(120° + 45°)\\ &= \cos 120° \cos 45° - \sin 120° \sin 45°\\ &= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\\ &= -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)\end{aligned}$$

$$\begin{aligned}\tan(-165°) &= \tan[-(120° + 45°)]\\ &= -\tan(120° + \tan 45°)\\ &= -\frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}\\ &= -\frac{-\sqrt{3} + 1}{1 - \left(-\sqrt{3}\right)(1)}\\ &= -\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}\\ &= -\frac{4 - 2\sqrt{3}}{-2}\\ &= 2 - \sqrt{3}\end{aligned}$$

18. $15° = 45° - 30°$

$$\begin{aligned}\sin 15° = \sin(45° - 30°) &= \sin 45° \cos 30° - \cos 45° \sin 30°\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3} - 1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)\end{aligned}$$

$$\begin{aligned}\cos 15° = \cos(45° - 30°) &= \cos 45° \cos 30° + \sin 45° \sin 30°\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3} + 1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)\end{aligned}$$

$$\begin{aligned}\tan 15° = \tan(45° - 30°) &= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}\\ &= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}\end{aligned}$$

19. $\frac{13\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$

$$\sin\frac{13\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

$$\tan\frac{13\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= \frac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$$

$$= \frac{-1 + \sqrt{3}}{1 - (-1)\left(\sqrt{3}\right)}$$

$$= -\frac{1 - \sqrt{3}}{1 + \sqrt{3}}\cdot\frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$= -\frac{4 - 2\sqrt{3}}{-2}$$

$$= 2 - \sqrt{3}$$

20. $-\frac{7\pi}{12} = -\frac{\pi}{3} - \frac{\pi}{4}$

$$\sin\left(-\frac{7\pi}{12}\right) = \sin\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\left(-\frac{7\pi}{12}\right) = \cos\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan\left(-\frac{7\pi}{12}\right) = \tan\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\left(-\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(-\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{-\sqrt{3} - 1}{1 + \left(-\sqrt{3}\right)(1)} = 2 + \sqrt{3}$$

21. $-\frac{13\pi}{12} = -\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$$\sin\left[-\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)\right] = -\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= -\left[\sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}\right]$$

$$= -\left[\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= -\frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$$\cos\left[-\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)\right] = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\tan\left[-\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)\right] = -\tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= -\frac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{3\pi}{4}\tan\frac{\pi}{3}} = -\frac{-1 + \sqrt{3}}{1 - \left(-\sqrt{3}\right)}$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{4 - 2\sqrt{3}}{-2} = -2 + \sqrt{3}$$

22. $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

$$\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$$\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)} = \sqrt{3} + 2$$

23. $\cos 25^\circ \cos 15^\circ - \sin 25^\circ \sin 15^\circ = \cos(25^\circ + 15^\circ) = \cos 40^\circ$

24. $\sin 140^\circ \cos 50^\circ + \cos 140^\circ \sin 50^\circ = \sin(140^\circ + 50^\circ) = \sin 190^\circ$

25. $\dfrac{\tan 325° - \tan 86°}{1 + \tan 325° \tan 86°} = \tan(325° - 86°) = \tan 239°$

26. $\dfrac{\tan 140° - \tan 60°}{1 + \tan 140° \tan 60°} = \tan(140° - 60°) = \tan 80°$

27. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2 = \sin(3 - 1.2) = \sin 1.8$

28. $\cos \dfrac{\pi}{7} \cos \dfrac{\pi}{5} - \sin \dfrac{\pi}{7} \sin \dfrac{\pi}{5} = \cos\left(\dfrac{\pi}{7} + \dfrac{\pi}{5}\right)$

$= \cos \dfrac{12\pi}{35}$

29. $\dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x} = \tan(2x + x) = \tan 3x$

30. $\cos 3x \cos 2y + \sin 3x \sin 2y = \cos(3x - 2y)$

31. $\sin 330° \cos 30° - \cos 330° \sin 30° = \sin(330° - 30°)$

$= \sin 300°$

$= -\dfrac{\sqrt{3}}{2}$

32. $\cos 15° \cos 60° + \sin 15° \sin 60° = \cos(15° - 60°)$

$= \cos(-45°) = \dfrac{\sqrt{2}}{2}$

33. $\sin \dfrac{\pi}{12} \cos \dfrac{\pi}{4} + \cos \dfrac{\pi}{12} \sin \dfrac{\pi}{4} = \sin\left(\dfrac{\pi}{12} + \dfrac{\pi}{4}\right)$

$= \sin \dfrac{\pi}{3}$

$= \dfrac{\sqrt{3}}{2}$

34. $\cos \dfrac{\pi}{16} \cos \dfrac{3\pi}{16} - \sin \dfrac{\pi}{16} \sin \dfrac{3\pi}{16} = \cos\left(\dfrac{\pi}{16} + \dfrac{3\pi}{16}\right)$

$= \cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

35. $\dfrac{\tan 25° + \tan 110°}{1 - \tan 25° \tan 110°} = \tan(25° + 110°)$

$= \tan 135°$

$= -1$

36. $\dfrac{\tan\left(\dfrac{5\pi}{4}\right) - \tan\left(\dfrac{\pi}{12}\right)}{1 + \tan\left(\dfrac{5\pi}{4}\right)\tan\left(\dfrac{\pi}{12}\right)} = \tan\left(\dfrac{5\pi}{4} - \dfrac{\pi}{12}\right)$

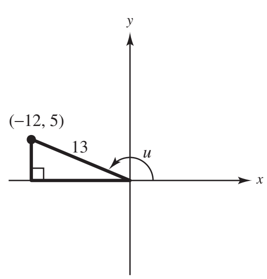
$= \tan\left(\dfrac{7\pi}{6}\right)$

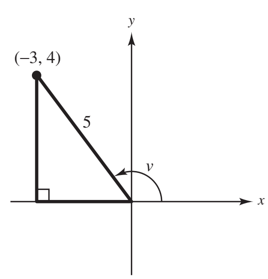
$= \tan\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{3}$

For Exercises 37–44, we have:

$\sin u = \frac{5}{13}$, u in Quadrant II $\Rightarrow$ $\cos u = -\frac{12}{13}$, $\tan u = -\frac{5}{12}$

$\cos v = -\frac{3}{5}$, v in Quadrant II $\Rightarrow$ $\sin v = \frac{4}{5}$, $\tan v = -\frac{4}{3}$,





Figures for Exercises 37–44

37. $\sin(u + v) = \sin u \cos v + \cos u \sin v$

$$= \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(-\frac{12}{13}\right)\left(\frac{4}{5}\right)$$

$$= -\frac{63}{65}$$

38. $\cos(u - v) = \cos u \cos v + \sin u \sin v$

$$= \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

$$= \frac{36}{65} + \frac{20}{65} = \frac{56}{65}$$

39. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$$= \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$$

$$= \frac{16}{65}$$

40. $\sin(v - u) = \sin v \cos u - \cos v \sin u$

$$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= -\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$$

41. $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{-\frac{5}{12} + \left(-\frac{4}{3}\right)}{1 - \left(-\frac{5}{12}\right)\left(-\frac{4}{3}\right)} = \dfrac{-\frac{21}{12}}{1 - \frac{5}{9}}$

$$= \left(-\frac{7}{4}\right)\left(\frac{9}{4}\right) = -\frac{63}{16}$$

42. $\csc(u - v) = \dfrac{1}{\sin(u - v)} = \dfrac{1}{-\sin(v - u)}$

$$= \frac{1}{-\left(-\frac{33}{65}\right)} = \frac{65}{33}$$

43. $\sec(v - u) = \dfrac{1}{\cos(v - u)} = \dfrac{1}{\cos v \cos u + \sin v \sin u}$

$$= \frac{1}{\left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)} = \frac{1}{\left(\frac{36}{65}\right) + \left(\frac{20}{65}\right)} = \frac{1}{\frac{56}{65}}$$

$$= \frac{65}{56}$$

44. $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\left(-\frac{5}{12}\right) + \left(-\frac{4}{3}\right)}{1 - \left(-\frac{5}{12}\right)\left(-\frac{4}{3}\right)}$

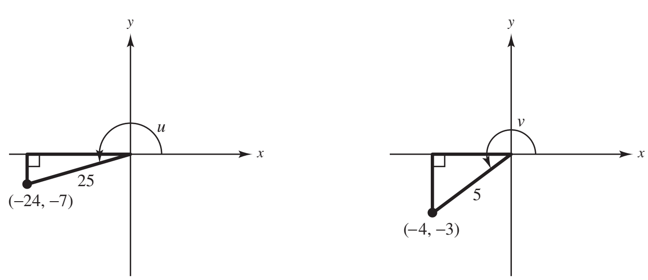
$$= \frac{-\frac{7}{4}}{\frac{4}{9}} = -\frac{63}{16}$$

$$\cot(u + v) = \frac{1}{\tan(u + v)} = \frac{1}{-\frac{63}{16}} = -\frac{16}{63}$$

For Exercises 45–50, we have:

$\sin u = -\frac{7}{25}$, u in Quadrant III $\Longrightarrow \cos u = -\frac{24}{25}$, $\tan u = \frac{7}{24}$

$\cos v = -\frac{4}{5}$, v in Quadrant III $\Longrightarrow \sin v = -\frac{3}{5}$, $\tan v = \frac{3}{4}$



Figures for Exercises 45–50

45. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$$= \left(-\tfrac{24}{25}\right)\left(-\tfrac{4}{5}\right) - \left(-\tfrac{7}{25}\right)\left(-\tfrac{3}{5}\right)$$

$$= \tfrac{3}{5}$$

46. $\sin(u + v) = \sin u \cos v + \cos u \sin v$

$$= \left(-\tfrac{7}{25}\right)\left(-\tfrac{4}{5}\right) + \left(-\tfrac{24}{25}\right)\left(-\tfrac{3}{5}\right)$$

$$= \tfrac{28}{125} + \tfrac{72}{125} = \tfrac{100}{125} = \tfrac{4}{5}$$

47. $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$= \dfrac{\frac{7}{24} - \frac{3}{4}}{1 + \left(\frac{7}{24}\right)\left(\frac{3}{4}\right)} = \dfrac{-\frac{11}{24}}{\frac{39}{32}} = -\dfrac{44}{117}$

48. $\tan(v - u) = \dfrac{\tan v - \tan u}{1 + \tan v \tan u} = \dfrac{\left(\frac{3}{4}\right) - \left(\frac{7}{24}\right)}{1 + \left(\frac{3}{4}\right)\left(\frac{7}{24}\right)}$

$= \dfrac{\frac{11}{24}}{\frac{39}{32}} = \dfrac{44}{117}$

$\cot(v - u) = \dfrac{1}{\tan(v - u)} = \dfrac{1}{\frac{44}{117}} = \dfrac{117}{44}$

49. $\sec(u + v) = \dfrac{1}{\cos(u + v)} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

Use Exercise 45 for $\cos(u + v)$.

50. $\cos(u - v) = \cos u \cos v + \sin u \sin v = \left(-\dfrac{24}{25}\right)\left(-\dfrac{4}{5}\right) + \left(-\dfrac{7}{25}\right)\left(-\dfrac{3}{5}\right)$

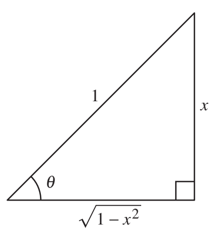
$= \dfrac{96}{125} + \dfrac{21}{125} = \dfrac{117}{125}$

51. $\sin(\arcsin x + \arccos x) = \sin(\arcsin x)\cos(\arccos x) + \sin(\arccos x)\cos(\arcsin x)$

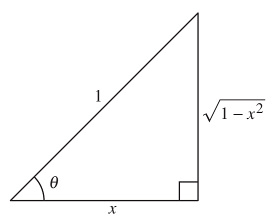
$= x \cdot x + \sqrt{1 - x^2} \cdot \sqrt{1 - x^2}$

$= x^2 + 1 - x^2$

$= 1$



$\theta = \arcsin x$

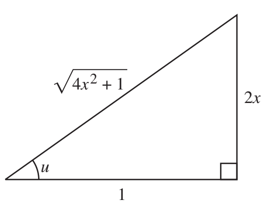


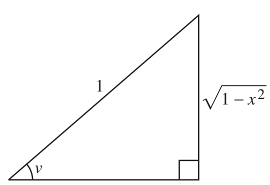
$\theta = \arccos x$

52. Let

$u = \arctan 2x$ and $v = \arccos x$

$\tan u = 2x$ $\qquad \cos v = x.$





$\sin(\arctan 2x - \arccos x) = \sin(u - v)$

$= \sin u \cos v - \cos u \sin v$

$= \dfrac{2x}{\sqrt{4x^2 + 1}}(x) - \dfrac{1}{\sqrt{4x^2 + 1}}\left(\sqrt{1 - x^2}\right)$

$= \dfrac{2x^2 - \sqrt{1 - x^2}}{\sqrt{4x^2 + 1}}$

53. $\cos(\arccos x + \arcsin x) = \cos(\arccos x)\cos(\arcsin x) - \sin(\arccos x)\sin(\arcsin x)$

$$= x \cdot \sqrt{1 - x^2} - \sqrt{1 - x^2} \cdot x$$

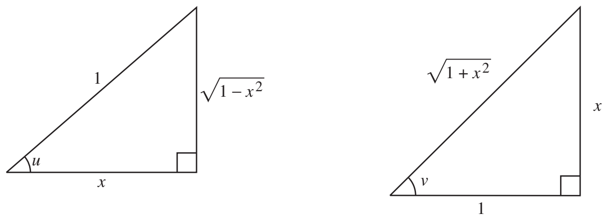
$$= 0$$

(Use the triangles in Exercise 51.)

54. Let

$u = \arccos x$ and $v = \arctan x$

$\cos u = x$ $\tan v = x.$



$$\cos(\arccos x - \arctan x) = \cos(u - v)$$

$$= \cos u \cos v + \sin u \sin v$$

$$= (x)\left(\frac{1}{\sqrt{1 + x^2}}\right) + \left(\sqrt{1 - x^2}\right)\left(\frac{x}{\sqrt{1 + x^2}}\right)$$

$$= \frac{x + x\sqrt{1 - x^2}}{\sqrt{1 + x^2}}$$

55. $\sin(3\pi - x) = \sin 3\pi \cos x - \sin x \cos 3\pi$

$$= (0)(\cos x) - (-1)(\sin x)$$

$$= \sin x$$

56. $\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \sin x \cos\frac{\pi}{2}$

$$= (1)(\cos x) + (\sin x)(0)$$

$$= \cos x$$

57. $\sin\left(\frac{\pi}{6} + x\right) = \sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x$

$$= \frac{1}{2}\left(\cos x + \sqrt{3}\sin x\right)$$

58. $\cos\left(\frac{5\pi}{4} - x\right) = \cos\frac{5\pi}{4}\cos x + \sin\frac{5\pi}{4}\sin x$

$$= -\frac{\sqrt{2}}{2}(\cos x + \sin x)$$

59. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = \cos\pi\cos\theta + \sin\pi\sin\theta + \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$$= (-1)(\cos\theta) + (0)(\sin\theta) + (1)(\cos\theta) + (\sin\theta)(0)$$

$$= -\cos\theta + \cos\theta$$

$$= 0$$

60. $\tan\left(\frac{\pi}{4} - \theta\right) = \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

61. $\cos(x + y)\cos(x - y) = (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x(1 - \sin^2 y) - \sin^2 x \sin^2 y$$

$$= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 x \sin^2 y$$

$$= \cos^2 x - \sin^2 y(\cos^2 x + \sin^2 x)$$

$$= \cos^2 x - \sin^2 y$$

62. $\sin(x + y)\sin(x - y) = (\sin x \cos y + \sin y \cos x)(\sin x \cos y - \sin y \cos x)$

$$= \sin^2 x \cos^2 y - \sin^2 y \cos^2 x$$

$$= \sin^2 x(1 - \sin^2 y) - \sin^2 y \cos^2 x$$

$$= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y \cos^2 x$$

$$= \sin^2 x - \sin^2 y(\sin^2 x + \cos^2 x)$$

$$= \sin^2 x - \sin^2 y$$

63. $\sin(x + y) + \sin(x - y) = \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y$

$$= 2 \sin x \cos y$$

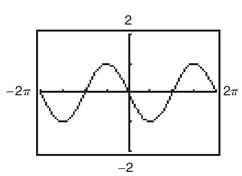
64. $\cos(x + y) + \cos(x - y) = \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$

$$= 2 \cos x \cos y$$

65. $\cos\left(\dfrac{3\pi}{2} - x\right) = \cos\dfrac{3\pi}{2}\cos x + \sin\dfrac{3\pi}{2}\sin x$

$$= (0)(\cos x) + (-1)(\sin x)$$

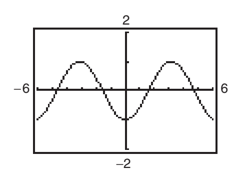
$$= -\sin x$$



66. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x$

$$= (-1)\cos x - (0)\sin x$$

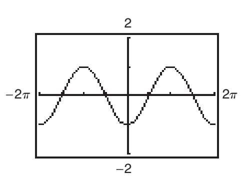
$$= -\cos x$$



67. $\sin\left(\dfrac{3\pi}{2} + \theta\right) = \sin\dfrac{3\pi}{2}\cos\theta + \cos\dfrac{3\pi}{2}\sin\theta$

$$= (-1)(\cos\theta) + (0)(\sin\theta)$$

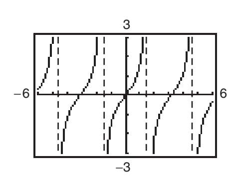
$$= -\cos\theta$$



68. $\tan(\pi + \theta) = \dfrac{\tan \pi + \tan\theta}{1 - \tan \pi \tan\theta}$

$$= \frac{0 + \tan\theta}{1 - (0)\tan\theta}$$

$$= \tan\theta$$



69.

$$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$$

$$\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = 1$$

$$2 \sin x(0.5) = 1$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

70.

$$\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} - \left(\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6}\right) = \frac{1}{2}$$

$$2 \cos x(0.5) = \frac{1}{2}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

71.

$$\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} - \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = 1$$

$$-2 \sin x\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$-\sqrt{2} \sin x = 1$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

72. $$\tan(x + \pi) + 2\sin(x + \pi) = 0$$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2(\sin x \cos \pi + \cos x \sin \pi) = 0$$

$$\frac{\tan x + 0}{1 - \tan x(0)} + 2[\sin x(-1) + \cos x(0)] = 0$$

$$\frac{\tan x}{1} - 2\sin x = 0$$

$$\frac{\sin x}{\cos x} = 2\sin x$$

$$\sin x = 2\sin x \cos x$$

$$\sin x(1 - 2\cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

73. Analytically: $$\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} + \cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} = 1$$

$$2\cos x\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$\sqrt{2}\cos x = 1$$

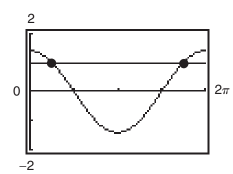
$$\cos x = \frac{1}{\sqrt{2}}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

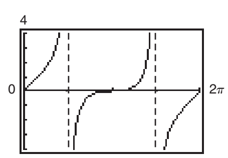
Graphically: Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)$ and $y_2 = 1$.

The points of intersection occur at $x = \frac{\pi}{4}$ and $x = \frac{7\pi}{4}$.



74. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$

Answers: $(0, 0), (3.14, 0) \implies x = 0, \pi$



75. $y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$

(a) $a = \frac{1}{3},\ b = \frac{1}{4},\ B = 2$

$C = \arctan\frac{b}{a} = \arctan\frac{3}{4} \approx 0.6435$

$y \approx \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}\sin(2t + 0.6435)$

$= \frac{5}{12}\sin(2t + 0.6435)$

(b) Amplitude: $\frac{5}{12}$ feet

(c) Frequency: $\frac{1}{\text{period}} = \frac{B}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$ cycle per second

76.

$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$y_1 + y_2 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$y_1 + y_2 = A\left[\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda} + \sin 2\pi\frac{t}{T}\sin 2\pi\frac{x}{\lambda}\right] + A\left[\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda} - \sin 2\pi\frac{t}{T}\sin 2\pi\frac{x}{\lambda}\right]$$

$$= 2A\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda}$$

77. False.

$\sin(u \pm v) = \sin u\cos v \pm \cos u\sin v$

78. False.

$\cos(u \pm v) = \cos u\cos v \mp \sin u\sin v$

79. False. $\cos\left(x - \frac{\pi}{2}\right) = \cos x\cos\frac{\pi}{2} + \sin x\sin\frac{\pi}{2}$

$= (\cos x)(0) + (\sin x)(1)$

$= \sin x$

80. True.

$\sin\left(x - \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2} - x\right) = -\cos x$

81. $\cos(n\pi + \theta) = \cos n\pi\cos\theta - \sin n\pi\sin\theta$

$= (-1)^n(\cos\theta) - (0)(\sin\theta)$

$= (-1)^n(\cos\theta)$, where n is an integer.

82. $\sin(n\pi + \theta) = \sin n\pi\cos\theta + \sin\theta\cos n\pi$

$= (0)(\cos\theta) + (\sin\theta)(-1)^n$

$= (-1)^n(\sin\theta)$, where n is an integer.

83. $C = \arctan\frac{b}{a} \Rightarrow \sin C = \frac{b}{\sqrt{a^2 + b^2}},\ \cos C = \frac{a}{\sqrt{a^2 + b^2}}$

$$\sqrt{a^2 + b^2}\sin(B\theta + C) = \sqrt{a^2 + b^2}\left(\sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} \cdot \cos B\theta\right) = a\sin B\theta + b\cos B\theta$$

84. $C = \arctan\frac{a}{b} \Rightarrow \sin C = \frac{a}{\sqrt{a^2 + b^2}},\ \cos C = \frac{b}{\sqrt{a^2 + b^2}}$

$$\sqrt{a^2 + b^2}\cos(B\theta - C) = \sqrt{a^2 + b^2}\left(\cos B\theta \cdot \frac{b}{\sqrt{a^2 + b^2}} + \sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}}\right)$$

$$= b\cos B\theta + a\sin B\theta$$

$$= a\sin B\theta + b\cos B\theta$$

85. $\sin\theta + \cos\theta$

$a = 1,\ b = 1,\ B = 1$

(a) $C = \arctan\dfrac{b}{a} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$= \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$= \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

86. $3\sin 2\theta + 4\cos 2\theta$

$a = 3, b = 4, B = 2$

(a) $C = \arctan\dfrac{b}{a} = \arctan\dfrac{4}{3} \approx 0.9273$

$$3\sin 2\theta + 4\cos 2\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$\approx 5\sin(2\theta + 0.9273)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan\dfrac{3}{4} \approx 0.6435$

$$3\sin 2\theta + 4\cos 2\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$\approx 5\cos(2\theta - 0.6435)$$

87. $12\sin 3\theta + 5\cos 3\theta$

$a = 12,\ b = 5,\ B = 3$

(a) $C = \arctan\dfrac{b}{a} = \arctan\dfrac{5}{12} \approx 0.3948$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$\approx 13\sin(3\theta + 0.3948)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan\dfrac{12}{5} \approx 1.1760$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$\approx 13\cos(3\theta - 1.1760)$$

88. $\sin 2\theta - \cos 2\theta$

$a = 1, b = -1, B = 2$

(a) $C = \arctan\dfrac{b}{a} = \arctan(-1) = -\dfrac{\pi}{4}$

$$\sin 2\theta - \cos 2\theta = \sqrt{a^2 + b^2}\sin(B\theta + C)$$
$$= \sqrt{2}\sin\left(2\theta - \frac{\pi}{4}\right)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan(-1) = -\dfrac{\pi}{4}$

$$\sin 2\theta - \cos 2\theta = \sqrt{a^2 + b^2}\cos(B\theta - C)$$
$$= \sqrt{2}\cos\left(2\theta + \frac{\pi}{4}\right)$$

89. $C = \arctan\dfrac{b}{a} = \dfrac{\pi}{2} \implies a = 0$

$\sqrt{a^2 + b^2} = 2 \implies b = 2$

$B = 1$

$$2\sin\left(\theta + \frac{\pi}{2}\right) = (0)(\sin\theta) + (2)(\cos\theta) = 2\cos\theta$$

90. $C = \arctan\dfrac{a}{b} = -\dfrac{3\pi}{4} \implies a = b, a < 0, b < 0$

$\sqrt{a^2 + b^2} = 5 \implies a = b = \dfrac{-5\sqrt{2}}{2}$

$B = 1$

$$5\cos\left(\theta + \frac{3\pi}{4}\right) = -\frac{5\sqrt{2}}{2}\sin\theta - \frac{5\sqrt{2}}{2}\cos\theta$$

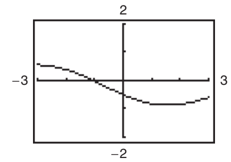
91.
$$\frac{\cos(x + h) - \cos x}{h} = \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$
$$= \frac{\cos x\cos h - \cos x - \sin x\sin h}{h}$$
$$= \frac{\cos x(\cos h - 1) - \sin x\sin h}{h}$$
$$= \frac{\cos x(\cos h - 1)}{h} - \frac{\sin x\sin h}{h}$$

92. (a) The domains of f and g are the sets of real numbers, $h \neq 0$.

(b)

h	0.01	0.02	0.05	0.1	0.2	0.5
$f(h)$	-0.504	-0.509	-0.521	-0.542	-0.583	-0.691
$g(h)$	-0.504	-0.509	-0.521	-0.542	-0.583	-0.691

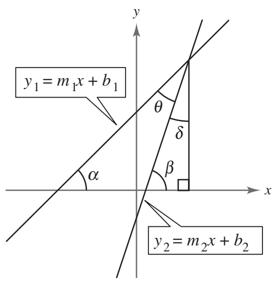
(c) The graphs are the same.



(d) As $h \to 0, f(h)$ approaches -0.5.

As $h \to 0$, $g(h)$ approaches -0.5.

93.



$m_1 = \tan \alpha$ and $m_2 = \tan \beta$

$\beta + \delta = 90° \implies \delta = 90° - \beta$

$\alpha + \theta + \delta = 90° \implies \alpha + \theta + (90° - \beta) = 90° \implies \theta = \beta - \alpha$

Therefore, $\theta = \arctan m_2 - \arctan m_1$.

For $y = x$ and $y = \sqrt{3}x$ we have $m_1 = 1$ and $m_2 = \sqrt{3}$.

$$\theta = \arctan \sqrt{3} - \arctan 1$$
$$= 60° - 45°$$
$$= 15°$$

94. For $m_2 > m_1 > 0$, the angle θ between the lines is:

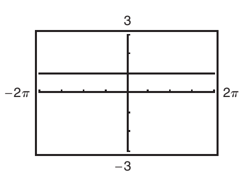
$$\theta = \arctan\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

$$m_2 = 1$$

$$m_1 = \frac{1}{\sqrt{3}}$$

$$\theta = \arctan\left(\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}\right) = \arctan\left(2 - \sqrt{3}\right) = 15°$$

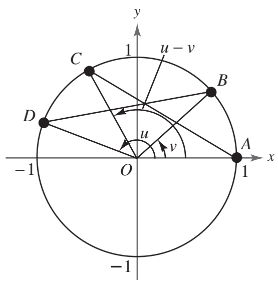
95.



Conjecture: $\sin^2\left(\theta + \frac{\pi}{4}\right) + \sin^2\left(\theta - \frac{\pi}{4}\right) = 1$

$$\sin^2\left(\theta + \frac{\pi}{4}\right) + \sin^2\left(\theta - \frac{\pi}{4}\right) = \left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right]^2 + \left[\sin\theta\cos\frac{\pi}{4} - \cos\theta\sin\frac{\pi}{4}\right]^2$$
$$= \left[\frac{\sin\theta}{\sqrt{2}} + \frac{\cos\theta}{\sqrt{2}}\right]^2 + \left[\frac{\sin\theta}{\sqrt{2}} - \frac{\cos\theta}{\sqrt{2}}\right]^2$$
$$= \frac{\sin^2\theta}{2} + \sin\theta\cos\theta + \frac{\cos^2\theta}{2} + \frac{\sin^2\theta}{2} - \sin\theta\cos\theta + \frac{\cos^2\theta}{2}$$
$$= \sin^2\theta + \cos^2\theta$$
$$= 1$$

96. (a) To prove the identity for $\sin(u + v)$ we first need to prove the identityfor $\cos(u - v)$. Assume $0 < v < u < 2\pi$ and locate u, v, and $u - v$ on the unit circle.



The coordinates of the points on the circle are:

$A = (1, 0)$, $B = (\cos v, \sin v)$, $C = (\cos(u - v), \sin(u - v))$, and $D = (\cos u, \sin u)$.

Since $\angle DOB = \angle COA$, chords AC and BD are equal. By the distance formula we have:

$$\sqrt{[\cos(u - v) - 1]^2 + [\sin(u - v) - 0]^2} = \sqrt{(\cos u - \cos v)^2 + (\sin u - \sin v)^2}$$

$$\cos^2(u - v) - 2\cos(u - v) + 1 + \sin^2(u - v) = \cos^2 u - 2\cos u \cos v + \cos^2 v + \sin^2 u - 2\sin u \sin v + \sin^2 v$$

$$[\cos^2(u - v) + \sin^2(u - v)] + 1 - 2\cos(u - v) = (\cos^2 u + \sin^2 u) + (\cos^2 v + \sin^2 v) - 2\cos u \cos v - 2\sin u \sin v$$

$$2 - 2\cos(u - v) = 2 - 2\cos u \cos v - 2\sin u \sin v$$

$$-2\cos(u - v) = -2(\cos u \cos v + \sin u \sin v)$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

Now, to prove the identity for $\sin(u + v)$, use cofunction identities.

$$\sin(u + v) = \cos\left[\frac{\pi}{2} - (u + v)\right] = \cos\left[\left(\frac{\pi}{2} - u\right) - v\right]$$

$$= \cos\left(\frac{\pi}{2} - u\right)\cos v + \sin\left(\frac{\pi}{2} - u\right)\sin v$$

$$= \sin u \cos v + \cos u \sin v$$

(b) First, prove $\cos(u - v) = \cos u \cos v + \sin u \sin v$ using the figure containing points

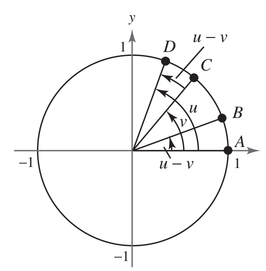
$A(1, 0)$

$B(\cos(u - v), \sin(u - v))$

$C(\cos v, \sin v)$

$D(\cos u, \sin u)$

on the unit circle.



Since chords AB and CD are each subtended by angle $u - v$, their lengths are equal. Equating $[d(A, B)]^2 = [d(C, D)]^2$ we have $(\cos(u - v) - 1)^2 + \sin^2(u - v) = (\cos u - \cos v)^2 + (\sin u - \sin v)^2$. Simplifying and solving for $\cos(u - v)$, we have $\cos(u - v) = \cos u \cos v + \sin u \sin v$.

Using $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ we have

$$\sin(u - v) = \cos\left[\frac{\pi}{2} - (u - v)\right] = \cos\left[\left(\frac{\pi}{2} - u\right) - (-v)\right]$$

$$= \cos\left(\frac{\pi}{2} - u\right)\cos(-v) + \sin\left(\frac{\pi}{2} - u\right)\sin(-v)$$

$$= \sin u \cos v - \cos u \sin v.$$

97.

$$f(x) = 5(x - 3)$$
$$y = 5(x - 3)$$
$$\frac{y}{5} = x - 3$$
$$\frac{y}{5} + 3 = x$$
$$\frac{x}{5} + 3 = y$$
$$f^{-1}(x) = \frac{x + 15}{5}$$
$$f(f^{-1}(x)) = f\left(\frac{x + 15}{5}\right) = 5\left[\frac{x + 15}{5} - 3\right]$$
$$= 5\left(\frac{x + 15}{5}\right) - 5(3)$$
$$= x + 15 - 15$$
$$= x$$
$$f^{-1}(f(x)) = f^{-1}(5(x - 3)) = \frac{5(x - 3) + 15}{5}$$
$$= \frac{5x - 15 + 15}{5}$$
$$= \frac{5x}{5}$$
$$= x$$

98.

$$f(x) = \frac{7 - x}{8}$$
$$y = \frac{7 - x}{8}$$
$$8y = 7 - x$$
$$x = 7 - 8y \implies f^{-1}(x) = -8x + 7$$
$$f(f^{-1}(x)) = \frac{7 - f^{-1}(x)}{8}$$
$$= \frac{7 - (-8x + 7)}{8}$$
$$= x$$
$$f^{-1}(f(x)) = -8\left(\frac{7 - x}{8}\right) + 7$$
$$= x$$

99. $f(x) = x^2 - 8$

f is not one-to-one so f^{-1} does not exist.

100.

$$f(x) = \sqrt{x - 16},\ x \ge 16$$
$$y = \sqrt{x - 16}$$
$$y^2 = x - 16$$
$$x = y^2 + 16 \implies f^{-1}(x) = x^2 + 16,\ x \ge 0$$
$$f(f^{-1}(x)) = \sqrt{(x^2 + 16) - 16} = x$$
$$f^{-1}(f(x)) = \left(\sqrt{x - 16}\right)^2 + 16 = x$$

101. $\log_3 3^{4x-3} = 4x - 3$

102. $\log_8 8^{3x^2} = 3x^2$

103. $e^{\ln(6x-3)} = 6x - 3$

104.

$$12x + e^{\ln x(x-2)} = 12x + x(x - 2)$$
$$= 12x + x^2 - 2x$$
$$= x^2 + 10x$$

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

- You should know the following double-angle formulas.

  (a) $\sin 2u = 2 \sin u \cos u$

  (b) $\cos 2u = \cos^2 u - \sin^2 u$

  $= 2\cos^2 u - 1$

  $= 1 - 2\sin^2 u$

  (c) $\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

- You should be able to reduce the power of a trigonometric function.

  (a) $\sin^2 u = \dfrac{1 - \cos 2u}{2}$

  (b) $\cos^2 u = \dfrac{1 + \cos 2u}{2}$

  (c) $\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

- You should be able to use the half-angle formulas. The signs of $\sin \dfrac{u}{2}$ and $\cos \dfrac{u}{2}$ depend on the quadrant in which $\dfrac{u}{2}$ lies.

  (a) $\sin \dfrac{u}{2} = \pm\sqrt{\dfrac{1 - \cos u}{2}}$

  (b) $\cos \dfrac{u}{2} = \pm\sqrt{\dfrac{1 + \cos u}{2}}$

  (c) $\tan \dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u} = \dfrac{\sin \text{u}}{1 + \cos u}$

- You should be able to use the product-sum formulas.

  (a) $\sin u \sin v = \dfrac{1}{2}[\cos(u - v) - \cos(u + v)]$

  (b) $\cos u \cos v = \dfrac{1}{2}[\cos(u - v) + \cos(u + v)]$

  (c) $\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$

  (d) $\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$

- You should be able to use the sum-product formulas.

  (a) $\sin x + \sin y = 2 \sin\left(\dfrac{x + y}{2}\right) \cos\left(\dfrac{x - y}{2}\right)$

  (b) $\sin x - \sin y = 2 \cos\left(\dfrac{x + y}{2}\right) \sin\left(\dfrac{x - y}{2}\right)$

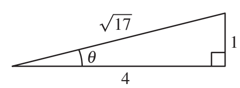
  (c) $\cos x + \cos y = 2 \cos\left(\dfrac{x + y}{2}\right) \cos\left(\dfrac{x - y}{2}\right)$

  (d) $\cos x - \cos y = -2 \sin\left(\dfrac{x + y}{2}\right) \sin\left(\dfrac{x - y}{2}\right)$

Vocabulary Check

1. $2 \sin u \cos u$
2. $\cos^2 u$
3. $\cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$
4. $\tan^2 u$
5. $\pm\sqrt{\dfrac{1-\cos u}{2}}$
6. $\dfrac{1-\cos u}{\sin u} = \dfrac{\sin u}{1+\cos u}$
7. $\dfrac{1}{2}[\cos(u-v) + \cos(u+v)]$
8. $\dfrac{1}{2}[\sin(u+v) + \sin(u-v)]$
9. $2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$
10. $-2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

Figure for Exercises 1–8



$\sin\theta = \dfrac{\sqrt{17}}{17}$

$\cos\theta = \dfrac{4\sqrt{17}}{17}$

$\tan\theta = \dfrac{1}{4}$

1. $\sin\theta = \dfrac{\sqrt{17}}{17}$

2. $\tan\theta = \dfrac{1}{4}$

3. $\cos 2\theta = 2\cos^2\theta - 1$
$$= 2\left(\frac{4\sqrt{17}}{17}\right)^2 - 1 = \frac{32}{17} - 1 = \frac{15}{17}$$

4. $\sin 2\theta = 2\sin\theta\cos\theta$
$$= 2\left(\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) = \frac{8}{17}$$

5. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
$$= \frac{2\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)^2} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{1}{2}\cdot\frac{16}{15} = \frac{8}{15}$$

6. $\sec 2\theta = \dfrac{1}{\cos 2\theta}$
$$= \frac{1}{\cos^2\theta - \sin^2\theta} = \frac{1}{\left(\frac{4}{\sqrt{17}}\right)^2 - \left(\frac{1}{\sqrt{17}}\right)^2} = \frac{1}{\frac{16}{17} - \frac{1}{17}} = \frac{17}{15}$$

7. $\csc 2\theta = \dfrac{1}{\sin 2\theta} = \dfrac{1}{2\sin\theta\cos\theta} = \dfrac{1}{2\left(\frac{\sqrt{17}}{17}\right)\left(\frac{4\sqrt{17}}{17}\right)}$
$$= \frac{17}{8}$$

8. $\cot 2\theta = \dfrac{1}{\tan 2\theta} = \dfrac{1-\tan^2\theta}{2\tan\theta} = \dfrac{1-\left(\frac{1}{4}\right)^2}{2\left(\frac{1}{4}\right)}$
$$= \frac{15}{8}$$

9.

$$\begin{aligned} \sin 2x - \sin x &= 0 \\ 2\sin x\cos x - \sin x &= 0 \\ \sin x(2\cos x - 1) &= 0 \end{aligned}$$

$\sin x = 0$ or $2\cos x - 1 = 0$

$x = 0, \pi$ $\qquad \cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$x = 0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

10.

$$\begin{aligned} \sin 2x + \cos x &= 0 \\ 2\sin x\cos x + \cos x &= 0 \\ \cos x(2\sin x + 1) &= 0 \end{aligned}$$

$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad \sin x = -\dfrac{1}{2}$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

11. $4\sin x\cos x = 1$

$$\begin{aligned} 2\sin 2x &= 1 \\ \sin 2x &= \frac{1}{2} \end{aligned}$$

$2x = \dfrac{\pi}{6} + 2n\pi$ or $2x = \dfrac{5\pi}{6} + 2n\pi$

$x = \dfrac{\pi}{12} + n\pi$ $\qquad x = \dfrac{5\pi}{12} + n\pi$

$x = \dfrac{\pi}{12}, \dfrac{13\pi}{12}$ $\qquad x = \dfrac{5\pi}{12}, \dfrac{17\pi}{12}$

12.

$$\begin{aligned} \sin 2x\sin x &= \cos x \\ 2\sin x\cos x\sin x - \cos x &= 0 \\ \cos x(2\sin^2 x - 1) &= 0 \end{aligned}$$

$\cos x = 0$ or $2\sin^2 x - 1 = 0$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad \sin^2 x = \dfrac{1}{2}$

$\sin x = \pm\dfrac{\sqrt{2}}{2}$

$x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

13.

$$\begin{aligned} \cos 2x - \cos x &= 0 \\ \cos 2x &= \cos x \\ \cos^2 x - \sin^2 x &= \cos x \\ \cos^2 x - (1 - \cos^2 x) - \cos x &= 0 \\ 2\cos^2 x - \cos x - 1 &= 0 \\ (2\cos x + 1)(\cos x - 1) &= 0 \end{aligned}$$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$\cos x = -\dfrac{1}{2}$ $\qquad \cos x = 1$

$x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ $\qquad x = 0$

14.

$$\begin{aligned} \cos 2x + \sin x &= 0 \\ 1 - 2\sin^2 x + \sin x &= 0 \\ 2\sin^2 x - \sin x - 1 &= 0 \\ (2\sin x + 1)(\sin x - 1) &= 0 \end{aligned}$$

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$\sin x = -\dfrac{1}{2}$ $\qquad \sin x = 1$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ $\qquad x = \dfrac{\pi}{2}$

15.

$$\tan 2x - \cot x = 0$$
$$\frac{2\tan x}{1-\tan^2 x} = \cot x$$
$$2\tan x = \cot x(1-\tan^2 x)$$
$$2\tan x = \cot x - \cot x\tan^2 x$$
$$2\tan x = \cot x - \tan x$$
$$3\tan x = \cot x$$
$$3\tan x - \cot x = 0$$
$$3\tan x - \frac{1}{\tan x} = 0$$
$$\frac{3\tan^2 x - 1}{\tan x} = 0$$
$$\frac{1}{\tan x}(3\tan^2 x - 1) = 0$$
$$\cot x(3\tan^2 x - 1) = 0$$

$\cot x = 0$ or $3\tan^2 x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

16.

$$\tan 2x - 2\cos x = 0$$
$$\frac{2\tan x}{1-\tan^2 x} = 2\cos x$$
$$2\tan x = 2\cos x(1-\tan^2 x)$$
$$2\tan x = 2\cos x - 2\cos x\tan^2 x$$
$$2\tan x = 2\cos x - 2\cos x\frac{\sin^2 x}{\cos^2 x}$$
$$2\tan x = 2\cos x - 2\frac{\sin^2 x}{\cos x}$$
$$\tan x = \cos x - \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin x}{\cos x} = \cos x - \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin x}{\cos x} + \frac{\sin^2 x}{\cos x} - \cos x = 0$$
$$\frac{\sin x + \sin^2 x - \cos^2 x}{\cos x} = 0$$
$$\frac{1}{\cos x}[\sin x + \sin^2 x - (1-\sin^2 x)] = 0$$
$$\sec x[2\sin^2 x + \sin x - 1] = 0$$
$$\sec x(2\sin x - 1)(\sin x + 1) = 0$$

$\sec x = 0$ or $2\sin x - 1 = 0$ or $\sin x + 1 = 0$

No solution $\quad$ $\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{3\pi}{2}$

Also, values for which $\cos x = 0$ need to be checked.

$\frac{\pi}{2}, \frac{3\pi}{2}$ are solutions.

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

17.

$$\sin 4x = -2\sin 2x$$
$$\sin 4x + 2\sin 2x = 0$$
$$2\sin 2x\cos 2x + 2\sin 2x = 0$$
$$2\sin 2x(\cos 2x + 1) = 0$$

$2\sin 2x = 0$ or $\cos 2x + 1 = 0$

$\sin 2x = 0$ $\quad$ $\cos 2x = -1$

$2x = n\pi$ $\quad$ $2x = \pi + 2n\pi$

$x = \frac{n}{2}\pi$ $\quad$ $x = \frac{\pi}{2} + n\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ $\quad$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

18.
$$\begin{aligned}(\sin 2x + \cos 2x)^2 &= 1\\ \sin^2 2x + 2\sin 2x\cos 2x + \cos^2 2x &= 1\\ 2\sin 2x\cos 2x &= 0\\ \sin 4x &= 0\\ 4x &= n\pi\\ x &= \frac{n\pi}{4}\\ x &= 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\end{aligned}$$

19. $6\sin x\cos x = 3(2\sin x\cos x)$
$= 3\sin 2x$

20. $6\cos^2 x - 3 = 3(2\cos^2 x - 1)$
$= 3\cos 2x$

21. $4 - 8\sin^2 x = 4(1 - 2\sin^2 x)$
$= 4\cos 2x$

22. $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x$
$= \cos 2x$

23. $\sin u = -\frac{4}{5}, \pi < u < \frac{3\pi}{2} \Rightarrow \cos u = -\frac{3}{5}$

$\sin 2u = 2\sin u\cos u = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$

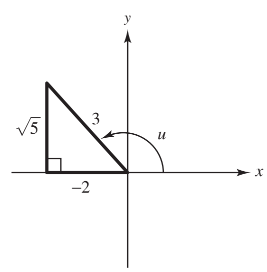
$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(\frac{4}{3}\right)}{1 - \frac{16}{9}} = \frac{8}{3}\left(-\frac{9}{7}\right) = -\frac{24}{7}$

24. $\cos u = -\frac{2}{3}, \frac{\pi}{2} < u < \pi$

$\sin 2u = 2\sin u\cos u = 2\cdot\frac{\sqrt{5}}{3}\left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{\sqrt{5}}{2}\right)}{1 - \frac{5}{4}} = 4\sqrt{5}$



$\csc u = 3, \frac{\pi}{2} < u < \pi$

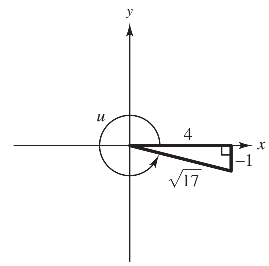
25. $\tan u = \frac{3}{4}, 0 < u < \frac{\pi}{2} \Rightarrow \sin u = \frac{3}{5}$ and $\cos u = \frac{4}{5}$

$\sin 2u = 2\sin u\cos u = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}} = \frac{3}{2}\left(\frac{16}{7}\right) = \frac{24}{7}$

26. $\cot u = -4, \dfrac{3\pi}{2} < u < 2\pi$



$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) = -\frac{8}{17}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= \left(\frac{4}{\sqrt{17}}\right)^2 - \left(-\frac{1}{\sqrt{17}}\right)^2 = \frac{15}{17}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{1}{4}\right)}{1-\left(-\frac{1}{4}\right)^2} = -\frac{8}{15}$$

27. $\sec u = -\dfrac{5}{2}, \dfrac{\pi}{2} < u < \pi \implies \sin u = \dfrac{\sqrt{21}}{5}$ and $\cos u = -\dfrac{2}{5}$

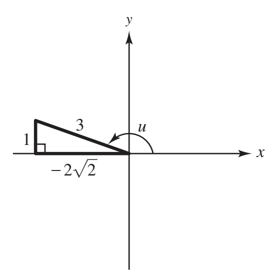
$$\sin 2u = 2\sin u\cos u = 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right) = -\frac{4\sqrt{21}}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2 = -\frac{17}{25}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{\sqrt{21}}{2}\right)}{1-\left(-\frac{\sqrt{21}}{2}\right)^2}$$

$$= \frac{-\sqrt{21}}{1-\frac{21}{4}} = \frac{4\sqrt{21}}{17}$$

28. $\sin 2u = 2\sin u\cos u = 2\cdot\dfrac{1}{3}\left(-\dfrac{2\sqrt{2}}{3}\right) = -\dfrac{4\sqrt{2}}{9}$



$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{7}{9}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(-\frac{\sqrt{2}}{4}\right)}{1-\left(-\frac{\sqrt{2}}{4}\right)^2} = -\frac{4\sqrt{2}}{7}$$

29. $\cos^4 x = (\cos^2 x)(\cos^2 x) = \left(\dfrac{1+\cos 2x}{2}\right)\left(\dfrac{1+\cos 2x}{2}\right) = \dfrac{1+2\cos 2x+\cos^2 2x}{4}$

$$= \frac{1+2\cos 2x+\frac{1+\cos 4x}{2}}{4}$$

$$= \frac{2+4\cos 2x+1+\cos 4x}{8}$$

$$= \frac{3+4\cos 2x+\cos 4x}{8}$$

$$= \frac{1}{8}(3+4\cos 2x+\cos 4x)$$

30. $\sin^8 x = \sin^4 x \sin^4 x = (\sin^2 x \sin^2 x)(\sin^2 x \sin^2 x)$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin^4 x = \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)$$

$$= \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$$

$$= \frac{1}{4}\left(1 - 2\cos 2x + \left(\frac{1 + \cos 4x}{2}\right)\right)$$

$$= \frac{1}{8}(3 - 4\cos 2x + \cos 4x)$$

$$\sin^8 x = \sin^4 x \sin^4 x$$

$$= \frac{1}{64}(3 - 4\cos 2x + \cos 4x)(3 - 4\cos 2x + \cos 4x)$$

$$= \frac{1}{64}(9 - 24\cos 2x + 16\cos^2 2x + 6\cos 4x - 8\cos 2x\cos 4x + \cos^2 4x)$$

$$= \frac{1}{64}\left[9 - 24\cos 2x + 16\left(\frac{1 + \cos 4x}{2}\right) + 6\cos 4x - (8)\left(\frac{1}{2}\right)(\cos 6x + \cos 2x) + \left(\frac{1 + \cos 8x}{2}\right)\right]$$

$$= \frac{1}{64}\left[\frac{35}{2} - 28\cos 2x + 14\cos 4x - 4\cos 6x + \frac{1}{2}\cos 8x\right]$$

$$= \frac{1}{128}[35 - 56\cos 2x + 28\cos 4x - 8\cos 6x + \cos 8x]$$

In the above, we used $\cos 2x \cos 4x = \frac{1}{2}(\cos 6x + \cos 2x)$.

31. $(\sin^2 x)(\cos^2 x) = \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$

$$= \frac{1 - \cos^2 2x}{4}$$

$$= \frac{1}{4}\left(1 - \frac{1 + \cos 4x}{2}\right)$$

$$= \frac{1}{8}(2 - 1 - \cos 4x)$$

$$= \frac{1}{8}(1 - \cos 4x)$$

32. $\sin^4 x \cos^4 x = \sin^2 x \sin^2 x \cos^2 x \cos^2 x$

$$= (\sin^2 x \cos^2 x)(\sin^2 x \cos^2 x)$$

$$= \left(\frac{1}{4}\sin^2 2x\right)\left(\frac{1}{4}\sin^2 2x\right)$$

$$= \left[\frac{1}{4}\left(\frac{1 - \cos 4x}{2}\right)\right]\left[\frac{1}{4}\left(\frac{1 - \cos 4x}{2}\right)\right]$$

$$= \frac{1}{64}[1 - 2\cos 4x + \cos^2 4x]$$

$$= \frac{1}{64}\left[1 - 2\cos 4x + \left(\frac{1 + \cos 8x}{2}\right)\right]$$

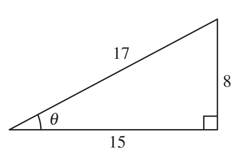
$$= \frac{1}{64}\left[\frac{3}{2} - 2\cos 4x + \frac{1}{2}\cos 8x\right]$$

$$= \frac{1}{128}(3 - 4\cos 4x + \cos 8x)$$

33. $$\sin^2 x \cos^4 x = \sin^2 x \cos^2 x \cos^2 x = \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$
$$= \frac{1}{8}(1 - \cos 2x)(1 + \cos 2x)(1 + \cos 2x)$$
$$= \frac{1}{8}(1 - \cos^2 2x)(1 + \cos 2x)$$
$$= \frac{1}{8}(1 + \cos 2x - \cos^2 2x - \cos^3 2x)$$
$$= \frac{1}{8}\left[1 + \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) - \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$
$$= \frac{1}{16}[2 + 2\cos 2x - 1 - \cos 4x - \cos 2x - \cos 2x \cos 4x]$$
$$= \frac{1}{16}(1 + \cos 2x - \cos 4x - \cos 2x \cos 4x)$$

34. $$\sin^4 x \cos^2 x = \sin^2 x \sin^2 x \cos^2 x$$
$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$
$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x)$$
$$= \frac{1}{8}(1 - \cos 2x - \cos^2 2x + \cos^3 2x)$$
$$= \frac{1}{8}\left[1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) + \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$
$$= \frac{1}{16}[2 - 2\cos 2x - 1 - \cos 4x + \cos 2x + \cos 2x \cos 4x]$$
$$= \frac{1}{16}\left[1 - \cos 2x - \cos 4x + \frac{1}{2}\cos 2x + \frac{1}{2}\cos 6x\right]$$
$$= \frac{1}{32}[2 - 2\cos 2x - 2\cos 4x + \cos 2x + \cos 6x]$$
$$= \frac{1}{32}[2 - \cos 2x - 2\cos 4x + \cos 6x]$$

Figure for Exercises 35–40



$\sin\theta = \frac{8}{17}$

$\cos\theta = \frac{15}{17}$

35. $$\cos\frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}} = \sqrt{\frac{1 + \frac{15}{17}}{2}} = \sqrt{\frac{32}{34}} = \sqrt{\frac{16}{17}} = \frac{4\sqrt{17}}{17}$$

36. $$\sin\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 - \frac{15}{17}}{2}} = \sqrt{\frac{\frac{2}{17}}{2}} = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

37. $\tan\dfrac{\theta}{2} = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{8/17}{1+(15/17)} = \dfrac{8}{17}\cdot\dfrac{17}{32} = \dfrac{1}{4}$

38. $\sec\dfrac{\theta}{2} = \dfrac{1}{\cos(\theta/2)} = \dfrac{1}{\sqrt{(1+\cos\theta)/2}}$

$= \dfrac{1}{\sqrt{[1+(15/17)]/2}} = \dfrac{1}{\sqrt{16/17}}$

$= \dfrac{\sqrt{17}}{4}$

39. $\csc\dfrac{\theta}{2} = \dfrac{1}{\sin(\theta/2)} = \dfrac{1}{\sqrt{(1-\cos\theta)/2}}$

$= \dfrac{1}{\sqrt{[1-(15/17)]/2}} = \dfrac{1}{\sqrt{1/17}} = \sqrt{17}$

40. $\cot\dfrac{\theta}{2} = \dfrac{1}{\tan(\theta/2)} = \dfrac{\sin\theta}{1-\cos\theta} = \dfrac{8/17}{1-(15/17)}$

$= \dfrac{8/17}{2/17} = 4$

41. $\sin 75° = \sin\left(\dfrac{1}{2}\cdot 150°\right) = \sqrt{\dfrac{1-\cos 150°}{2}} = \sqrt{\dfrac{1+\left(\sqrt{3}/2\right)}{2}}$

$= \dfrac{1}{2}\sqrt{2+\sqrt{3}}$

$\cos 75° = \cos\left(\dfrac{1}{2}\cdot 150°\right) = \sqrt{\dfrac{1+\cos 150°}{2}} = \sqrt{\dfrac{1-\left(\sqrt{3}/2\right)}{2}}$

$= \dfrac{1}{2}\sqrt{2-\sqrt{3}}$

$\tan 75° = \tan\left(\dfrac{1}{2}\cdot 150°\right) = \dfrac{\sin 150°}{1+\cos 150°} = \dfrac{1/2}{1-\left(\sqrt{3}/2\right)}$

$= \dfrac{1}{2-\sqrt{3}}\cdot\dfrac{2+\sqrt{3}}{2+\sqrt{3}} = \dfrac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

42. $\sin 165 = \sin\left(\dfrac{1}{2}\cdot 330\right) = \sqrt{\dfrac{1-\cos 330}{2}} = \sqrt{\dfrac{1-\left(\sqrt{3}/2\right)}{2}} = \dfrac{1}{2}\sqrt{2-\sqrt{3}}$

$\cos 165 = \cos\left(\dfrac{1}{2}\cdot 330\right) = -\sqrt{\dfrac{1+\cos 330}{2}} = -\sqrt{\dfrac{1+\left(\sqrt{3}/2\right)}{2}} = -\dfrac{1}{2}\sqrt{2+\sqrt{3}}$

$\tan 165 = \tan\left(\dfrac{1}{2}\cdot 330\right) = \dfrac{\sin 330}{1+\cos 330} = \dfrac{-1/2}{1+\left(\sqrt{3}/2\right)} = \dfrac{-1}{2+\sqrt{3}} = \sqrt{3}-2$

43. $\sin 112°\,30' = \sin\left(\dfrac{1}{2}\cdot 225°\right) = \sqrt{\dfrac{1-\cos 225°}{2}} = \sqrt{\dfrac{1+\left(\sqrt{2}/2\right)}{2}} = \dfrac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos 112°\,30' = \cos\left(\dfrac{1}{2}\cdot 225°\right) = -\sqrt{\dfrac{1+\cos 225°}{2}} = -\sqrt{\dfrac{1-\left(\sqrt{2}/2\right)}{2}} = -\dfrac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan 112°\,30' = \tan\left(\dfrac{1}{2}\cdot 225°\right) = \dfrac{\sin 225°}{1+\cos 225°} = \dfrac{-\sqrt{2}/2}{1-\left(\sqrt{2}/2\right)} = -1-\sqrt{2}$

44. $\sin 67°\,30' = \sin\left(\dfrac{1}{2}\cdot 135°\right) = \sqrt{\dfrac{1-\cos 135°}{2}} = \sqrt{\dfrac{1+\left(\sqrt{2}/2\right)}{2}} = \dfrac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos 67°\,30' = \cos\left(\dfrac{1}{2}\cdot 135°\right) = \sqrt{\dfrac{1+\cos 135°}{2}} = \sqrt{\dfrac{1-\left(\sqrt{2}/2\right)}{2}} = \dfrac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan 67°\,30' = \tan\left(\dfrac{1}{2}\cdot 135°\right) = \dfrac{\sin 135°}{1+\cos 135°} = \dfrac{\sqrt{2}/2}{1-\left(\sqrt{2}/2\right)} = 1+\sqrt{2}$

45. $\sin\frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\cos\frac{\pi}{8} = \cos\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\tan\frac{\pi}{8} = \tan\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \sqrt{2}-1$

46. $\sin\frac{\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \frac{1}{2}\sqrt{2-\sqrt{3}}$

$\cos\frac{\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{3}}$

$\tan\frac{\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right]\frac{\sin\frac{\pi}{6}}{1+\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = 2-\sqrt{3}$

47. $\sin\frac{3\pi}{8} = \sin\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$

$\cos\frac{3\pi}{8} = \cos\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

$\tan\frac{3\pi}{8} = \tan\left(\frac{1}{2}\cdot\frac{3\pi}{4}\right) = \frac{\sin\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{(2-\sqrt{2})}{2}} = \frac{\sqrt{2}}{2-\sqrt{2}} = \sqrt{2}+1$

48. $\sin\frac{7\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = \sqrt{\frac{1-\cos(7\pi(6))}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{1}{2}\sqrt{2+\sqrt{3}}$

$\cos\frac{7\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = -\sqrt{\frac{1+\cos\frac{7\pi}{6}}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\frac{1}{2}\sqrt{2-\sqrt{3}}$

$\tan\frac{7\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = \frac{\sin\frac{7\pi}{6}}{1+\cos\frac{7\pi}{6}} = \frac{-\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} = -2-\sqrt{3}$

49. $\sin u = \frac{5}{13}, \frac{\pi}{2} < u < \pi \Longrightarrow \cos u = -\frac{12}{13}$

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+\frac{12}{13}}{2}} = \frac{5\sqrt{26}}{26}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}} = \frac{\sqrt{26}}{26}$

$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u} = \frac{\frac{5}{13}}{1-\frac{12}{13}} = 5$

50. $\cos u = \frac{3}{5}, 0 < u < \frac{\pi}{2}$

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \frac{\sqrt{5}}{5}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \frac{2\sqrt{5}}{5}$

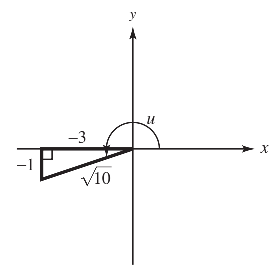
51. $\tan u = -\frac{5}{8}, \frac{3\pi}{2} < u < 2\pi \implies \sin u = -\frac{5}{\sqrt{89}}$ and $\cos u = \frac{8}{\sqrt{89}}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\frac{8}{\sqrt{89}}}{2}}\sqrt{\frac{\sqrt{89}-8}{2\sqrt{89}}} = \sqrt{\frac{89-8\sqrt{89}}{178}}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{8}{\sqrt{89}}}{2}} = -\sqrt{\frac{\sqrt{89}+8}{2\sqrt{89}}} = -\sqrt{\frac{89+8\sqrt{89}}{178}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1-\frac{8}{\sqrt{89}}}{-\frac{5}{\sqrt{89}}} = \frac{8-\sqrt{89}}{5}$$

52. $\cot u = 3, \pi < u < \frac{3\pi}{2}$



$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+\frac{3}{\sqrt{10}}}{2}} = \sqrt{\frac{10+3\sqrt{10}}{20}} = \frac{1}{2}\sqrt{\frac{10+3\sqrt{10}}{5}}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1-\frac{3}{\sqrt{10}}}{2}} = -\sqrt{\frac{10-3\sqrt{10}}{20}} = -\frac{1}{2}\sqrt{\frac{10-3\sqrt{10}}{5}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1+\frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}} = -\sqrt{10}-3$$

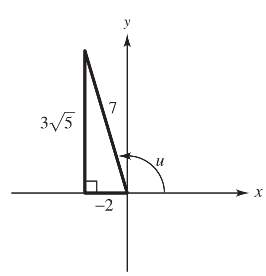
53. $\csc u = -\frac{5}{3}, \pi < u < \frac{3\pi}{2} \implies \sin u = -\frac{3}{5}$ and $\cos u = -\frac{4}{5}$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \frac{3\sqrt{10}}{10}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1-\frac{4}{5}}{2}} = -\frac{\sqrt{10}}{10}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1+\frac{4}{5}}{-\frac{3}{5}} = -3$$

54. $\sec u = -\frac{7}{2}, \frac{\pi}{2} < u < \pi$



$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1+\frac{2}{7}}{2}} = \frac{3\sqrt{14}}{14}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1-\frac{2}{7}}{2}} = \frac{\sqrt{70}}{14}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u} = \frac{1+\frac{2}{7}}{3\frac{\sqrt{5}}{7}} = \frac{3\sqrt{5}}{5}$$

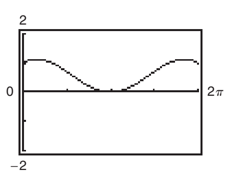
55. $\sqrt{\dfrac{1 - \cos 6x}{2}} = |\sin 3x|$

56. $\sqrt{\dfrac{1 + \cos 4x}{2}} = \left|\cos \dfrac{4x}{2}\right| = |\cos 2x|$

57. $$-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}} = -\frac{\sqrt{\dfrac{1 - \cos 8x}{2}}}{\sqrt{\dfrac{1 + \cos 8x}{2}}} = -\left|\frac{\sin 4x}{\cos 4x}\right| = -|\tan 4x|$$

58. $-\sqrt{\dfrac{1 - \cos(x - 1)}{2}} = -\left|\sin\left(\dfrac{x - 1}{2}\right)\right|$

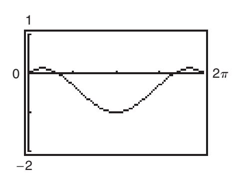
59.
$$\sin \frac{x}{2} + \cos x = 0$$
$$\pm\sqrt{\frac{1 - \cos x}{2}} = -\cos x$$
$$\frac{1 - \cos x}{2} = \cos^2 x$$
$$0 = 2\cos^2 x + \cos x - 1$$
$$= (2\cos x - 1)(\cos x + 1)$$
$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -1$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$$



By checking these values in the original equation, we see that $x = \pi/3$ and $x = 5\pi/3$ are extraneous, and $x = \pi$ is the only solution.

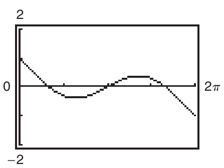
60.
$$h(x) = \sin \frac{x}{2} + \cos x - 1$$
$$\sin \frac{x}{2} + \cos x - 1 = 0$$
$$\pm\sqrt{\frac{1 - \cos x}{2}} = 1 - \cos x$$
$$\frac{1 - \cos x}{2} = 1 - 2\cos x + \cos^2 x$$
$$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$$
$$2\cos^2 x - 3\cos x + 1 = 0$$
$$(2\cos x - 1)(\cos x - 1) = 0$$
$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$
$$\cos x = \frac{1}{2} \qquad \cos x = 1$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = 0$$

$0, \dfrac{\pi}{3}$, and $\dfrac{5\pi}{3}$ are all solutions to the equation.



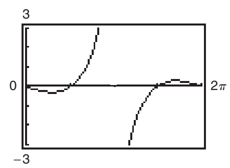
61.
$$\cos\frac{x}{2} - \sin x = 0$$
$$\pm\sqrt{\frac{1+\cos x}{2}} = \sin x$$
$$\frac{1+\cos x}{2} = \sin^2 x$$
$$1 + \cos x = 2\sin^2 x$$
$$1 + \cos x = 2 - 2\cos^2 x$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$
$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x + 1 = 0$$
$$\cos x = \frac{1}{2} \qquad \cos x = -1$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad x = \pi$$
$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$\pi/3$, π, and $5\pi/3$ are all solutions to the equation.



62.
$$g(x) = \tan\frac{x}{2} - \sin x$$
$$\tan\frac{x}{2} - \sin x = 0$$
$$\frac{1-\cos x}{\sin x} = \sin x$$
$$1 - \cos x = \sin^2 x$$
$$1 - \cos x = 1 - \cos^2 x$$
$$\cos^2 x - \cos x = 0$$
$$\cos x(\cos x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \cos x - 1 = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \cos x = 1$$
$$x = 0$$

$0, \frac{\pi}{2}$, and $\frac{3\pi}{2}$ are all solutions to the equation.



63. $6\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 6\cdot\frac{1}{2}\left[\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}-\frac{\pi}{4}\right)\right] = 3\left(\sin\frac{\pi}{2} + \sin 0\right)$

64. $4\cos\frac{\pi}{3}\sin\frac{5\pi}{6} = 4\cdot\frac{1}{2}\left[\sin\left(\frac{\pi}{3}+\frac{5\pi}{6}\right) - \sin\left(\frac{\pi}{3}-\frac{5\pi}{6}\right)\right] = 2\left[\sin\left(\frac{7\pi}{6}\right) - \sin\left(-\frac{\pi}{2}\right)\right] = 2\left[\sin\left(\frac{7\pi}{6}\right) + \sin\left(\frac{\pi}{2}\right)\right]$

65. $10\cos 75^\circ\cos 15^\circ = 10\left(\frac{1}{2}\right)[\cos(75^\circ - 15^\circ) + \cos(75^\circ + 15^\circ)] = 5[\cos 60^\circ + \cos 90^\circ]$

66. $6\sin 45^\circ\cos 15^\circ = 6\left(\frac{1}{2}\right)(\sin 60^\circ + \sin 30^\circ) = 3(\sin 60^\circ + \sin 30^\circ)$

67. $\cos 4\theta\sin 6\theta = \frac{1}{2}[\sin(4\theta + 6\theta) - \sin(4\theta - 6\theta)] = \frac{1}{2}[\sin 10\theta - \sin(-2\theta)] = \frac{1}{2}(\sin 10\theta + \sin 2\theta)$

68. $3\sin 2\alpha\sin 3\alpha = 3\cdot\frac{1}{2}[\cos(2\alpha - 3\alpha) - \cos(2\alpha + 3\alpha)] = \frac{3}{2}[\cos(-\alpha) - \cos 5\alpha] = \frac{3}{2}(\cos\alpha - \cos 5\alpha)$

69. $5\cos(-5\beta)\cos 3\beta = 5\cdot\frac{1}{2}[\cos(-5\beta - 3\beta) + \cos(-5\beta + 3\beta)] = \frac{5}{2}[\cos(-8\beta) + \cos(-2\beta)]$
$= \frac{5}{2}(\cos 8\beta + \cos 2\beta)$

70. $\cos 2\theta\cos 4\theta = \frac{1}{2}[\cos(2\theta - 4\theta) + \cos(2\theta + 4\theta)] = \frac{1}{2}[\cos(-2\theta) + \cos 6\theta] = \frac{1}{2}(\cos 2\theta + \cos 6\theta)$

71. $\sin(x + y)\sin(x - y) = \frac{1}{2}(\cos 2y - \cos 2x)$

72. $\sin(x + y)\cos(x - y) = \frac{1}{2}(\sin 2x + \sin 2y)$

73. $\cos(\theta - \pi)\sin(\theta + \pi) = \frac{1}{2}[\sin 2\theta - \sin(-2\pi)]$

$= \frac{1}{2}(\sin 2\theta + \sin 2\pi)$

74. $\sin(\theta + \pi)\sin(\theta - \pi) = \frac{1}{2}(\cos 2\pi - \cos 2\theta)$

75. $\sin 5\theta - \sin 3\theta = 2\cos\left(\frac{5\theta + 3\theta}{2}\right)\sin\left(\frac{5\theta - 3\theta}{2}\right)$

$= 2\cos 4\theta \sin \theta$

76. $\sin 3\theta + \sin \theta = 2\sin\left(\frac{3\theta + \theta}{2}\right)\cos\left(\frac{3\theta - \theta}{2}\right)$

$= 2\sin 2\theta \cos \theta$

77. $\cos 6x + \cos 2x = 2\cos\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right) = 2\cos 4x \cos 2x$

78. $\sin x + \sin 5x = 2\sin\left(\frac{x + 5x}{2}\right)\cos\left(\frac{x - 5x}{2}\right) = 2\sin 3x\cos(-2x) = 2\sin 3x \cos 2x$

79. $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\left(\frac{\alpha + \beta + \alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta - \alpha + \beta}{2}\right) = 2\cos\alpha\sin\beta$

80. $\cos(\phi + 2\pi) + \cos\phi = 2\cos\left(\frac{\phi + 2\pi + \phi}{2}\right)\cos\left(\frac{\phi + 2\pi - \phi}{2}\right) = 2\cos(\phi + \pi)\cos(\pi)$

81. $\cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right) = -2\sin\left[\frac{\left(\theta + \frac{\pi}{2}\right) + \left(\theta - \frac{\pi}{2}\right)}{2}\right]\sin\left[\frac{\left(\theta + \frac{\pi}{2}\right) - \left(\theta - \frac{\pi}{2}\right)}{2}\right] = -2\sin\theta\sin\frac{\pi}{2}$

82. $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right) = 2\sin\left(\frac{x + \frac{\pi}{2} + x - \frac{\pi}{2}}{2}\right)\cos\left(\frac{x + \frac{\pi}{2} - \left(x - \frac{\pi}{2}\right)}{2}\right) = 2\sin x\cos\frac{\pi}{2}$

83. $\sin 60° + \sin 30° = 2\sin\left(\frac{60° + 30°}{2}\right)\cos\left(\frac{60° - 30°}{2}\right) = 2\sin 45°\cos 15°$

$\sin 60° + \sin 30° = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$

84. $\cos 120° + \cos 30° = 2\cos\left(\frac{120° + 30°}{2}\right)\cos\left(\frac{120° - 30°}{2}\right) = 2\cos 75°\cos 45°$

$\cos 120° + \cos 30° = -\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} - 1}{2}$

85. $\cos\frac{3\pi}{4} - \cos\frac{\pi}{4} = -2\sin\left(\frac{\frac{3\pi}{4} + \frac{\pi}{4}}{2}\right)\sin\left(\frac{\frac{3\pi}{4} - \frac{\pi}{4}}{2}\right) = -2\sin\frac{\pi}{2}\sin\frac{\pi}{4}$

$\cos\frac{3\pi}{4} - \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

86. $\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4} = 2\cos\left(\frac{\frac{5\pi}{4} + \frac{3\pi}{4}}{2}\right)\sin\left(\frac{\frac{5\pi}{4} - \frac{3\pi}{4}}{2}\right) = 2\cos\pi\sin\frac{\pi}{4}$

$\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

87. $$\sin 6x + \sin 2x = 0$$

$$2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right) = 0$$

$$2(\sin 4x)\cos 2x = 0$$

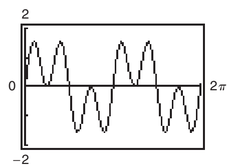
$\sin 4x = 0$ or $\cos 2x = 0$

$4x = n\pi$ $\qquad$ $2x = \dfrac{\pi}{2} + n\pi$

$x = \dfrac{n\pi}{4}$ $\qquad$ $x = \dfrac{\pi}{4} + \dfrac{n\pi}{2}$

In the interval $[0, 2\pi)$ we have

$x = 0, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}, \dfrac{3\pi}{2}, \dfrac{7\pi}{4}.$



88. $$h(x) = \cos 2x - \cos 6x$$

$$\cos 2x - \cos 6x = 0$$

$$-2\sin 4x \sin(-2x) = 0$$

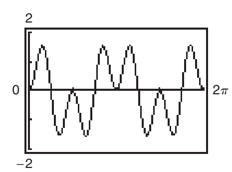
$$2\sin 4x \sin 2x = 0$$

$\sin 4x = 0$ or $\sin 2x = 0$

$4x = n\pi$ $\qquad$ $2x = n\pi$

$x = \dfrac{n\pi}{4}$ $\qquad$ $x = \dfrac{n\pi}{2}$

$x = 0, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}, \dfrac{3\pi}{2}, \dfrac{7\pi}{4}$ $\qquad$ $x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$



89. $$\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$$

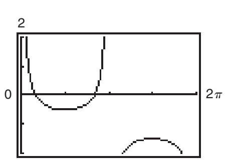
$$\frac{\cos 2x}{\sin 3x - \sin x} = 1$$

$$\frac{\cos 2x}{2\cos 2x \sin x} = 1$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$



90. $$f(x) = \sin^2 3x - \sin^2 x$$

$$\sin^2 3x - \sin^2 x = 0$$

$$(\sin 3x + \sin x)(\sin 3x - \sin x) = 0$$

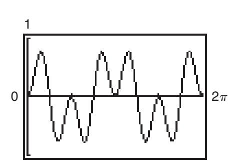
$$(2\sin 2x \cos x)(2\cos 2x \sin x) = 0$$

$\sin 2x = 0 \implies x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$ or

$\cos x = 0 \implies x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ or

$\cos 2x = 0 \implies x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ or

$\sin x = 0 \implies x = 0, \pi$



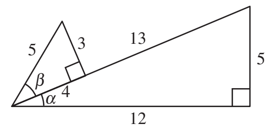


Figure for Exercises 91–94

91. $\sin^2 \alpha = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \left(\frac{12}{13}\right)^2$$
$$= 1 - \frac{144}{169} = \frac{25}{169}$$

92. $\cos^2 \alpha = (\cos \alpha)^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$

$$\cos^2 \alpha = 1 - \sin^2 \alpha$$
$$= 1 - \left(\frac{5}{13}\right)^2$$
$$= 1 - \frac{25}{169} = \frac{144}{169}$$

93. $\sin \alpha \cos \beta = \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$

$$\sin \alpha \cos \beta = \cos\left(\frac{\pi}{2} - \alpha\right) \sin\left(\frac{\pi}{2} - \beta\right)$$
$$= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$$

94. $\cos \alpha \sin \beta = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$

$$\cos \alpha \sin \beta = \sin\left(\frac{\pi}{2} - \alpha\right) \cos\left(\frac{\pi}{2} - \beta\right)$$
$$= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$$

95. $\csc 2\theta = \frac{1}{\sin 2\theta}$

$$= \frac{1}{2 \sin \theta \cos \theta}$$
$$= \frac{1}{\sin \theta} \cdot \frac{1}{2 \cos \theta}$$
$$= \frac{\csc \theta}{2 \cos \theta}$$

96. $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta}$

$$= \frac{\frac{1}{\cos^2 \theta}}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}$$
$$= \frac{\sec^2 \theta}{1 - \tan^2 \theta}$$
$$= \frac{\sec^2 \theta}{1 - (\sec^2 \theta - 1)}$$
$$= \frac{\sec^2 \theta}{2 - \sec^2 \theta}$$

97. $\cos^2 2\alpha - \sin^2 2\alpha = \cos[2(2\alpha)]$

$$= \cos 4\alpha$$

98. $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$$= (\cos 2x)(1)$$
$$= \cos 2x$$

99. $(\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$

$$= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$$
$$= 1 + \sin 2x$$

100. $\sin\left(\frac{\alpha}{3}\right) \cos\left(\frac{\alpha}{3}\right) = \frac{1}{2}\left[2\left(\sin\left(\frac{\alpha}{3}\right) \cos\left(\frac{\alpha}{3}\right)\right)\right]$

$$= \frac{1}{2} \sin \frac{2\alpha}{3}$$

101. $1 + \cos 10y = 1 + \cos^2 5y - \sin^2 5y$

$$= 1 + \cos^2 5y - (1 - \cos^2 5y)$$
$$= 2 \cos^2 5y$$

102. $\dfrac{\cos 3\beta}{\cos \beta} = \dfrac{\cos(2\beta + \beta)}{\cos \beta}$

$= \dfrac{\cos 2\beta \cos \beta - \sin 2\beta \sin \beta}{\cos \beta}$

$= \dfrac{(1 - 2\sin^2 \beta)\cos \beta - (2\cos \beta \sin \beta)\sin \beta}{\cos \beta}$

$= 1 - 2\sin^2 \beta - 2\sin^2 \beta$

$= 1 - 4\sin^2 \beta$

103. $\sec \dfrac{u}{2} = \dfrac{1}{\cos \dfrac{u}{2}}$

$= \pm\sqrt{\dfrac{2}{1 + \cos u}}$

$= \pm\sqrt{\dfrac{2\sin u}{\sin u(1 + \cos u)}}$

$= \pm\sqrt{\dfrac{2\sin u}{\sin u + \sin u \cos u}}$

$= \pm\sqrt{\dfrac{\dfrac{2\sin u}{\cos u}}{\dfrac{\sin u}{\cos u} + \dfrac{\sin u \cos u}{\cos u}}}$

$= \pm\sqrt{\dfrac{2\tan u}{\tan u + \sin u}}$

104. $\tan \dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u}$

$= \dfrac{1}{\sin u} - \dfrac{\cos u}{\sin u}$

$= \csc u - \cot u$

105. $\dfrac{\sin x \pm \sin y}{\cos x + \cos y} = \dfrac{2\sin\left(\dfrac{x \pm y}{2}\right)\cos\left(\dfrac{x \mp y}{2}\right)}{2\cos\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)}$

$= \tan\left(\dfrac{x \pm y}{2}\right)$

106. $\dfrac{\sin x + \sin y}{\cos x - \cos y} = \dfrac{2\sin\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)}{-2\sin\left(\dfrac{x + y}{2}\right)\sin\left(\dfrac{x - y}{2}\right)}$

$= -\cot\left(\dfrac{x - y}{2}\right)$

107. $\dfrac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} = \dfrac{2\cos\left(\dfrac{4x + 2x}{2}\right)\cos\left(\dfrac{4x - 2x}{2}\right)}{2\sin\left(\dfrac{4x + 2x}{2}\right)\cos\left(\dfrac{4x - 2x}{2}\right)}$

$= \dfrac{2\cos 3x \cos x}{2\sin 3x \cos x} = \cot 3x$

108. $\dfrac{\cos t + \cos 3t}{\sin 3t - \sin t} = \dfrac{2\cos\left(\dfrac{4t}{2}\right)\cos\left(-\dfrac{2t}{2}\right)}{2\cos\left(\dfrac{4t}{2}\right)\sin\left(\dfrac{2t}{2}\right)}$

$= \dfrac{\cos(-t)}{\sin(t)}$

$= \dfrac{\cos(t)}{\sin(t)} = \cot t$

109. $\sin\left(\dfrac{\pi}{6} + x\right) + \sin\left(\dfrac{\pi}{6} - x\right) = 2\sin \dfrac{\pi}{6}\cos x$

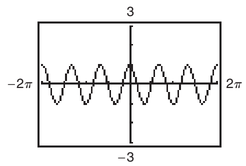
$= 2 \cdot \dfrac{1}{2}\cos x$

$= \cos x$

110. $\cos\left(\dfrac{\pi}{3} + x\right) + \cos\left(\dfrac{\pi}{3} - x\right) = 2\cos\left(\dfrac{\dfrac{\pi}{3} + x + \dfrac{\pi}{3} - x}{2}\right)\cos\left(\dfrac{\dfrac{\pi}{3} + x - \left(\dfrac{\pi}{3} - x\right)}{2}\right)$

$= 2\cos\left(\dfrac{\pi}{3}\right)\cos(x)$

$= 2\left(\dfrac{1}{2}\right)\cos x = \cos x$

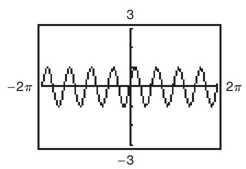
111. 

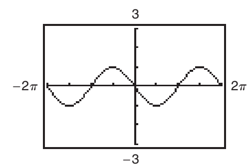
Let $y_1 = \cos(3x)$ and

$y_2 = (\cos x)^3 - 3(\sin x)^2 \cos x$.

$$\begin{aligned}\cos 3\beta &= \cos(2\beta + \beta)\\ &= \cos 2\beta \cos\beta - \sin 2\beta \sin \beta\\ &= (\cos^2 \beta - \sin^2 \beta) \cos \beta - 2 \sin \beta \cos \beta \sin \beta\\ &= \cos^3 \beta - \sin^2 \beta \cos \beta - 2 \sin^2 \beta \cos \beta\\ &= \cos^3 \beta - 3 \sin^2 \beta \cos \beta\end{aligned}$$

112. $$\begin{aligned}\sin 4\beta &= 2 \sin 2\beta \cos 2\beta\\ &= 2(2 \sin \beta \cos \beta)(1 - 2 \sin^2 \beta)\\ &= 4 \sin \beta \cos \beta(1 - 2 \sin^2 \beta)\end{aligned}$$



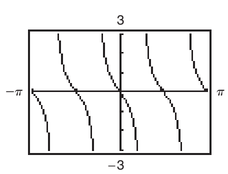
113. 

Let $y_1 = \dfrac{(\cos 4x - \cos 2x)}{(2 \sin 3x)}$

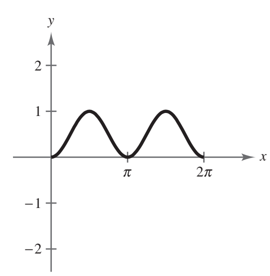
and $y_2 = -\sin x$.

$$\begin{aligned}\frac{\cos 4x - \cos 2x}{2 \sin 3x} &= \frac{-2 \sin\left(\frac{4x + 2x}{2}\right) \sin\left(\frac{4x - 2x}{2}\right)}{2 \sin 3x}\\ &= \frac{-2 \sin 3x \sin x}{2 \sin 3x} = -\sin x\end{aligned}$$

114. $$\begin{aligned}\frac{\cos 3x - \cos x}{\sin 3x - \sin x} &= \frac{-2 \sin\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)}{2 \cos\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)}\\ &= \frac{-2 \sin 2x \sin x}{2 \cos 2x \sin x}\\ &= -\tan 2x\end{aligned}$$



115. $\sin^2 x = \dfrac{1 - \cos 2x}{2} = \dfrac{1}{2} - \dfrac{\cos 2x}{2}$

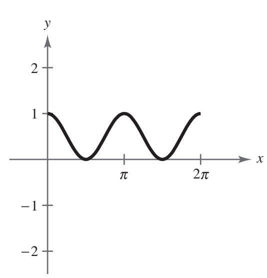


116. $f(x) = \cos^2 x = \dfrac{1 + \cos 2x}{2} = \dfrac{1}{2} + \dfrac{\cos 2x}{2}$

Shifted upward by $\frac{1}{2}$ unit.

Amplitude: $|a| = \dfrac{1}{2}$

Period: $\dfrac{2\pi}{2} = \pi$



117. $$\begin{aligned}\sin(2 \arcsin x) &= 2 \sin(\arcsin x) \cos(\arcsin x)\\ &= 2x\sqrt{1 - x^2}\end{aligned}$$

118. $\cos(2 \arccos x) = \cos^2(\arccos x) - \sin^2(\arccos x)$

$= x^2 - (1 - x^2) = 2x^2 - 1$

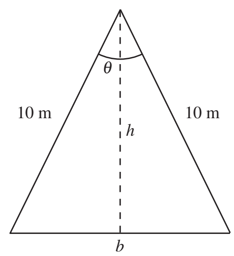
119. $\frac{1}{32}(75)^2 \sin 2\theta = 130$

$\sin 2\theta = \frac{130(32)}{75^2}$

$\theta = \frac{1}{2} \sin^{-1}\left(\frac{130(32)}{75^2}\right)$

$\theta \approx 23.85°$

120. (a) $A = \frac{1}{2}bh$



$\cos \frac{\theta}{2} = \frac{h}{10} \Rightarrow h = 10 \cos \frac{\theta}{2}$

$\sin \frac{\theta}{2} = \frac{(1/2)b}{10} \Rightarrow \frac{1}{2}b = 10 \sin \frac{\theta}{2}$

$A = 10 \sin \frac{\theta}{2} 10 \cos \frac{\theta}{2} \Rightarrow A = 100 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

(b) $A = 100 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

$A = 50\left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}\right)$

$A = 50 \sin \theta$

When $\theta = \pi/2$, $\sin \theta = 1 \Rightarrow$ the area is a maximum.

$A = 50 \sin \frac{\pi}{2} = 50(1) = 50$ square feet

121. $\sin \frac{\theta}{2} = \frac{1}{M}$

(a) $\sin \frac{\theta}{2} = 1$

$\frac{\theta}{2} = \arcsin 1$

$\frac{\theta}{2} = \frac{\pi}{2}$

$\theta = \pi$

(b) $\sin \frac{\theta}{2} = \frac{1}{4.5}$

$\frac{\theta}{2} = \arcsin\left(\frac{1}{4.5}\right)$

$\theta = 2 \arcsin\left(\frac{1}{4.5}\right)$

$\theta \approx 0.4482$

(c) $\frac{S}{760} = 1$

$S = 760$ miles per hour

$\frac{S}{760} = 4.5$

$S = 3420$ miles per hour

(d) $\sin \frac{\theta}{2} = \frac{1}{M}$

$\frac{\theta}{2} = \arcsin\left(\frac{1}{M}\right)$

$\theta = 2 \arcsin\left(\frac{1}{M}\right)$

122. $\frac{x}{2} = 2r \sin^2 \frac{\theta}{2} = 2r\left(\frac{1 - \cos \theta}{2}\right)$

$= r(1 - \cos \theta)$

So, $x = 2r(1 - \cos \theta)$.

123. False. For $u < 0$,

$\sin 2u = -\sin(-2u)$

$= -2 \sin(-u) \cos(-u)$

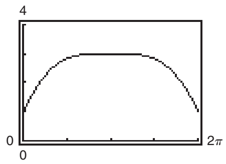
$= -2(-\sin u) \cos u$

$= 2 \sin u \cos u$.

124. False. If $90° < u < 180°$, $\frac{u}{2}$ is in the first quadrant and

$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}}$.

125. (a) $y = 4\sin\dfrac{x}{2} + \cos x$

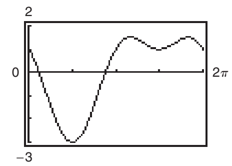


Maximum: $(\pi, 3)$

(b)
$$2\cos\frac{x}{2} - \sin x = 0$$
$$2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right) = \sin x$$
$$4\left(\frac{1+\cos x}{2}\right) = \sin^2 x$$
$$2(1+\cos x) = 1 - \cos^2 x$$
$$\cos^2 x + 2\cos x + 1 = 0$$
$$(\cos x + 1)^2 = 0$$
$$\cos x = -1$$
$$x = \pi$$

126. $f(x) = \cos 2x - 2\sin x$

(a)



Maximum points: $(3.6652, 1.5), (5.7596, 1.5)$

Minimum points: $(1.5708, -3), (4.7124, 1)$

(b) $-2\cos x(2\sin x + 1) = 0$

$-2\cos x = 0$ or $2\sin x + 1 = 0$

$\cos x = 0$ $\qquad$ $\sin x = -\dfrac{1}{2}$

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\qquad$ $x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\dfrac{\pi}{2} \approx 1.5708$ $\qquad$ $\dfrac{7\pi}{6} \approx 3.6652$

$\dfrac{3\pi}{2} \approx 4.7124$ $\qquad$ $\dfrac{11\pi}{2} \approx 5.7596$

127. $f(x) = \sin^4 x + \cos^4 x$

(a)
$$\begin{aligned}
\sin^4 x + \cos^4 x &= (\sin^2 x)^2 + (\cos^2 x)^2 \\
&= \left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2 \\
&= \frac{1}{4}[(1-\cos 2x)^2 + (1+\cos 2x)^2] \\
&= \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x) \\
&= \frac{1}{4}(2 + 2\cos^2 2x) \\
&= \frac{1}{4}\left[2 + 2\left(\frac{1+\cos 2(2x)}{2}\right)\right] \\
&= \frac{1}{4}(3 + \cos 4x)
\end{aligned}$$

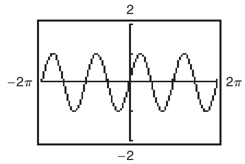
(b)
$$\begin{aligned}
\sin^4 x + \cos^4 x &= (\sin^2 x)^2 + \cos^4 x \\
&= (1 - \cos^2 x)^2 + \cos^4 x \\
&= 1 - 2\cos^2 x + \cos^4 x + \cos^4 x \\
&= 2\cos^4 x - 2\cos^2 x + 1
\end{aligned}$$

(c)
$$\begin{aligned}
\sin^4 x + \cos^4 x &= \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x - 2\sin^2 x\cos^2 x \\
&= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x \\
&= 1 - 2\sin^2 x\cos^2 x
\end{aligned}$$

(d)
$$\begin{aligned}
1 - 2\sin^2 x\cos^2 x &= 1 - (2\sin x\cos x)(\sin x\cos x) \\
&= 1 - (\sin 2x)\left(\frac{1}{2}\sin 2x\right) \\
&= 1 - \frac{1}{2}\sin^2 2x
\end{aligned}$$

(e) No, it does not mean that one of you is wrong. There is often more than one way to rewrite a trigonometric expression.

128. (a)

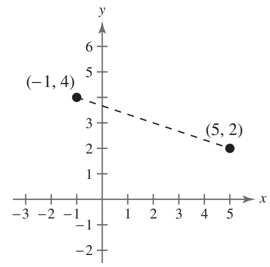


(b) The graph appears to be that of $\sin 2x$.

(c) $2 \sin x\left[2 \cos^2\left(\frac{x}{2}\right) - 1\right] = 2 \sin x \cos x$

$= \sin 2x$

129. (a)

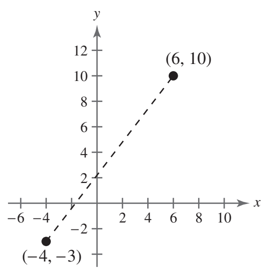


(b) $d = \sqrt{(-1 - 5)^2 + (4 - 2)^2} = \sqrt{(-6)^2 + (2)^2}$

$= \sqrt{40} = 2\sqrt{10}$

(c) Midpoint: $\left(\frac{5 + (-1)}{2}, \frac{2 + 4}{2}\right) = (2, 3)$

130. (a)



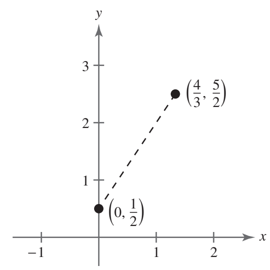
(b) $d = \sqrt{(-4 - 6)^2 + (-3 - 10)^2}$

$= \sqrt{(-10)^2 + (-13)^2}$

$= \sqrt{100 + 169} = \sqrt{269}$

(c) Midpoint: $\left(\frac{-4 + 6}{2}, \frac{-3 + 10}{2}\right) = \left(1, \frac{7}{2}\right)$

131. (a)

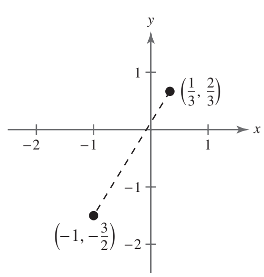


(b) $d = \sqrt{\left(\frac{4}{3} - 0\right)^2 + \left(\frac{5}{2} - \frac{1}{2}\right)^2} = \sqrt{\frac{16}{9} + 4}$

$= \sqrt{\frac{52}{9}} = \frac{2\sqrt{13}}{3}$

(c) Midpoint: $\left(\frac{0 + \frac{4}{3}}{2}, \frac{\frac{1}{2} + \frac{5}{2}}{2}\right) = \left(\frac{2}{3}, \frac{3}{2}\right)$

132. (a)



(b) $d = \sqrt{\left(\frac{1}{3} + 1\right)^2 + \left(\frac{2}{3} + \frac{3}{2}\right)^2} = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{13}{6}\right)^2}$

$= \sqrt{\frac{16}{9} + \frac{169}{36}} = \sqrt{\frac{233}{36}} = \frac{1}{6}\sqrt{233}$

(c) Midpoint:

$\left(\frac{\frac{1}{3} + (-1)}{2}, \frac{\frac{2}{3} + \left(-\frac{3}{2}\right)}{2}\right) = \left(\frac{-\frac{2}{3}}{2}, \frac{-\frac{5}{6}}{2}\right) = \left(-\frac{1}{3}, \frac{-5}{12}\right)$

133. (a) Complement: $90° - 55° = 35°$

Supplement: $180° - 55° = 125°$

(b) Complement: Not possible. $162° > 90°$

Supplement: $180° - 162° = 18°$

134. (a) The supplement is $180° - 109° = 71°$.

There is no complement.

(b) The supplement is $180° - 78° = 102°$.

The complement is $90° - 78° = 12°$.

135. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{18} = \frac{4\pi}{9}$

Supplement: $\pi - \frac{\pi}{18} = \frac{17\pi}{18}$

(b) Complement: $\frac{\pi}{2} - \frac{9\pi}{20} = \frac{\pi}{20}$

Supplement: $\pi - \frac{9\pi}{20} = \frac{11\pi}{20}$

136. (a) The supplement is $\pi - 0.95 = 2.19$.

The complement is $\frac{\pi}{2} - 0.95 = 0.62$.

(b) The supplement is $\pi - 2.76 = 0.38$.

There is no complement.

137. Let x = profit for September,
then $x + 0.16x$ = profit for October.

$$x + (x + 0.16x) = 507{,}600$$
$$2.16x = 507{,}600$$
$$x = 235{,}000$$
$$x + 0.16x = 272{,}600$$

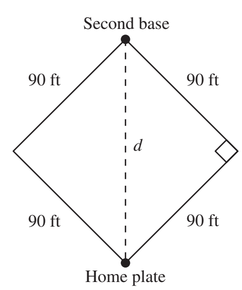
Profit for September: \$235,000

Profit for October: \$272,600

138. Let x = number of gallons of 100% concentrate.

$$0.30(55 - x) + 1.00x = 0.50(55)$$
$$16.50 - 0.30x + x = 27.50$$
$$0.70x = 11$$
$$x \approx 15.7 \text{ gallons}$$

139.



$$d^2 = 90^2 + 90^2$$
$$= 16{,}200$$
$$d = \sqrt{16{,}200}$$
$$= 90\sqrt{2}$$
$$\approx 127 \text{ feet}$$

Review Exercises for Chapter 5

1. $\frac{1}{\cos x} = \sec x$

2. $\frac{1}{\sin x} = \csc x$

3. $\frac{1}{\sec x} = \cos x$

4. $\frac{1}{\tan x} = \cot x$

5. $\frac{\cos x}{\sin x} = \cot x$

6. $\sqrt{1 + \tan^2 x} = \sqrt{\sec^2 x} = |\sec x|$

7. $\sin x = \frac{3}{5}, \ \cos x = \frac{4}{5}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = \frac{4}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{4}$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}$$

8. $\tan\theta = \frac{2}{3}, \sec\theta = \frac{\sqrt{13}}{3}$

θ is in Quadrant I.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{13}} = \sqrt{\frac{4}{13}} = \frac{2\sqrt{13}}{13}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{13}}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{2}$$

9. $\sin\left(\frac{\pi}{2} - x\right) = \frac{\sqrt{2}}{2} \Rightarrow \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$

$\cot x = \frac{1}{\tan x} = -1$

$\sec x = \frac{1}{\cos x} = \sqrt{2}$

$\csc x = \frac{1}{\sin x} = -\sqrt{2}$

10. $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta = 9, \sin\theta = \frac{4\sqrt{5}}{9}$

θ is in Quadrant I.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{9}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = 4\sqrt{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{9}{4\sqrt{5}} = \frac{9\sqrt{5}}{20}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{4\sqrt{5}} = \frac{\sqrt{5}}{20}$

11. $\frac{1}{\cot^2 x + 1} = \frac{1}{\csc^2 x} = \sin^2 x$

12. $\frac{\tan\theta}{1 - \cos^2\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin^2\theta} = \frac{1}{\sin\theta\cos\theta}$

$= \csc\theta\sec\theta$

13. $\tan^2 x(\csc^2 x - 1) = \tan^2 x(\cot^2 x) = \tan^2 x\left(\frac{1}{\tan^2 x}\right) = 1$

14. $\cot^2 x(\sin^2 x) = \frac{\cos^2 x}{\sin^2 x}\sin^2 x = \cos^2 x$

15. $\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

16. $\frac{\cot\left(\frac{\pi}{2} - u\right)}{\cos u} = \frac{\tan u}{\cos u} = \tan u \sec u$

17. $\cos^2 x + \cos^2 x\cot^2 x = \cos^2 x(1 + \cot^2 x) = \cos^2 x(\csc^2 x)$

$= \cos^2 x\left(\frac{1}{\sin^2 x}\right) = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$

18. $\tan^2\theta\csc^2\theta - \tan^2\theta = \tan^2\theta(\csc^2\theta - 1)$

$= \tan^2\theta\cot^2\theta = 1$

19. $(\tan x + 1)^2\cos x = (\tan^2 x + 2\tan x + 1)\cos x$

$= (\sec^2 x + 2\tan x)\cos x$

$= \sec^2 x\cos x + 2\left(\frac{\sin x}{\cos x}\right)\cos x = \sec x + 2\sin x$

20. $(\sec x - \tan x)^2 = \sec^2 x - 2\sec x\tan x + \tan^2 x$

$= 1 + \tan^2 x - 2\sec x\tan x + \tan^2 x$

$= 1 - 2\sec x\tan x + 2\tan^2 x$

21. $\frac{1}{\csc\theta + 1} - \frac{1}{\csc\theta - 1} = \frac{(\csc\theta - 1) - (\csc\theta + 1)}{(\csc\theta + 1)(\csc\theta - 1)}$

$= \frac{-2}{\csc^2\theta - 1}$

$= \frac{-2}{\cot^2\theta}$

$= -2\tan^2\theta$

22. $\frac{\cos^2 x}{1 - \sin x} = \frac{\cos^2 x}{(1 - \sin x)} \cdot \frac{(1 + \sin x)}{(1 + \sin x)}$

$= \frac{\cos^2 x(1 + \sin x)}{1 - \sin^2 x}$

$= 1 + \sin x$

23. $\csc^2 x - \csc x\cot x = \frac{1}{\sin^2 x} - \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)$

$= \frac{1 - \cos x}{\sin^2 x}$

24. $\sin^{-1/2} x \cos x = \dfrac{1}{\sqrt{\sin x}}(\cos x) = \dfrac{\sqrt{\sin x}}{\sin x}(\cos x)$

$= \sqrt{\sin x}\left(\dfrac{\cos x}{\sin x}\right) = \sqrt{\sin x} \cot x$

25. $\cos x(\tan^2 x + 1) = \cos x \sec^2 x$

$= \dfrac{1}{\sec x} \sec^2 x$

$= \sec x$

26. $\sec^2 x \cot x - \cot x = \cot x(\sec^2 x - 1) = \cot x \tan^2 x$

$= \left(\dfrac{1}{\tan x}\right) \tan^2 x = \tan x$

27. $\cos\left(x + \dfrac{\pi}{2}\right) = \cos x \cos \dfrac{\pi}{2} - \sin x \sin \dfrac{\pi}{2}$

$= (\cos x)(0) - (\sin x)(1)$

$= -\sin x$

28. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$ by the Cofunction Identity

29. $\dfrac{1}{\tan \theta \csc \theta} = \dfrac{1}{\dfrac{\sin \theta}{\cos \theta} \cdot \dfrac{1}{\sin \theta}} = \cos \theta$

30. $\dfrac{1}{\tan x \csc x \sin x} = \dfrac{1}{(\tan x)\left(\dfrac{1}{\sin x}\right)(\sin x)} = \dfrac{1}{\tan x}$

$= \cot x$

31. $\sin^5 x \cos^2 x = \sin^4 x \cos^2 x \sin x$

$= (1 - \cos^2 x)^2 \cos^2 x \sin x$

$= (1 - 2\cos^2 x + \cos^4 x) \cos^2 x \sin x$

$= (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x$

32. $\cos^3 x \sin^2 x = \cos x \cos^2 x \sin^2 x$

$= \cos x(1 - \sin^2 x) \sin^2 x$

$= \cos x(\sin^2 x - \sin^4 x)$

$= (\sin^2 x - \sin^4 x) \cos x$

33. $\sin x = \sqrt{3} - \sin x$

$\sin x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3} + 2\pi n, \dfrac{2\pi}{3} + 2\pi n$

34. $4 \cos \theta = 1 + 2 \cos \theta$

$2 \cos \theta = 1$

$\cos \theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3} + 2n\pi \quad \text{or} \quad \dfrac{5\pi}{3} + 2n\pi$

35. $3\sqrt{3} \tan u = 3$

$\tan u = \dfrac{1}{\sqrt{3}}$

$u = \dfrac{\pi}{6} + n\pi$

36. $\dfrac{1}{2} \sec x - 1 = 0$

$\dfrac{1}{2} \sec x = 1$

$\sec x = 2$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3} + 2n\pi \quad \text{or} \quad \dfrac{5\pi}{3} + 2n\pi$

37. $3 \csc^2 x = 4$

$\csc^2 x = \dfrac{4}{3}$

$\sin x = \pm\dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3} + 2\pi n, \dfrac{2\pi}{3} + 2\pi n, \dfrac{4\pi}{3} + 2\pi n, \dfrac{5\pi}{3} + 2\pi n$

These can be combined as:

$x = \dfrac{\pi}{3} + n\pi \quad \text{or} \quad x = \dfrac{2\pi}{3} + n\pi$

38. $4\tan^2 u - 1 = \tan^2 u$

$3\tan^2 u - 1 = 0$

$\tan^2 u = \frac{1}{3}$

$\tan u = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

$u = \frac{\pi}{6} + n\pi$ or $\frac{5\pi}{6} + n\pi$

39. $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $x = 0$

40. $2\sin^2 x - 3\sin x = -1$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{\pi}{2}$

41. $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x - 1 = 0$

$-\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\quad$ $\sin x - 1 = 0$

$x = 0, \pi$ $\quad$ $\sin x = 1$

$x = \frac{\pi}{2}$

42. $\sin^2 x + 2\cos x = 2$

$1 - \cos^2 x + 2\cos x = 2$

$0 = \cos^2 x - 2\cos x + 1$

$0 = (\cos x - 1)^2$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0$

43. $2\sin 2x - \sqrt{2} = 0$

$\sin 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n$

$x = \frac{\pi}{8} + \pi n, \frac{3\pi}{8} + \pi n$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

44. $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

45. $\cos 4x(\cos x - 1) = 0$

$\cos 4x = 0$ $\quad$ $\cos x - 1 = 0$

$4x = \frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n$ $\quad$ $\cos x = 1$

$x = \frac{\pi}{8} + \frac{\pi}{2}n, \frac{3\pi}{8} + \frac{\pi}{2}n$ $\quad$ $x = 0$

$x = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

46. $3\csc^2 5x = -4$

$\csc^2 5x = -\frac{4}{3}$

$\csc 5x = \pm\sqrt{-\frac{4}{3}}$

No real solution

47. $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad$ $\sin x - 2 = 0$

$x = 0, \pi$ $\quad$ No solution

48. $2\cos^2 x + 3\cos x = 0$

$\cos x(2\cos x + 3) = 0$

$\cos x = 0$ or $2\cos x + 3 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $2\cos x = -3$

$\cos x = -\frac{3}{2}$

No solution

49. $\tan^2\theta + \tan\theta - 12 = 0$

$(\tan\theta + 4)(\tan\theta - 3) = 0$

$\tan\theta + 4 = 0 \qquad \tan\theta - 3 = 0$

$\theta = \arctan(-4) + n\pi \qquad \theta = \arctan 3 + n\pi$

$\theta = \arctan(-4) + \pi, \arctan(-4) + 2\pi, \arctan 3, \arctan 3 + \pi$

50. $\sec^2 x + 6\tan x + 4 = 0$

$1 + \tan^2 x + 6\tan x + 4 = 0$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 5)(\tan x + 1) = 0$

$\tan x + 5 = 0$ or $\tan x + 1 = 0$

$\tan x = -5 \qquad \tan x = -1$

$x = \arctan(-5) + \pi \qquad x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

$x = \arctan(-5) + 2\pi$

51.
$$\begin{aligned}\sin 285^\circ &= \sin(315^\circ - 30^\circ)\\ &= \sin 315^\circ\cos 30^\circ - \cos 315^\circ \sin 30^\circ\\ &= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\\ &= -\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)\end{aligned}$$

$$\begin{aligned}\cos 285^\circ &= \cos(315^\circ - 30^\circ)\\ &= \cos 315^\circ\cos 30^\circ + \sin 315^\circ \sin 30^\circ\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)\end{aligned}$$

$$\begin{aligned}\tan 285^\circ &= \tan(315^\circ - 30^\circ) = \frac{\tan 315^\circ - \tan 30^\circ}{1 + \tan 315^\circ \tan 30^\circ}\\ &= \frac{(-1) - \left(\frac{\sqrt{3}}{3}\right)}{1 + (-1)\left(\frac{\sqrt{3}}{3}\right)} = -2 - \sqrt{3}\end{aligned}$$

52.
$$\begin{aligned}\sin(345^\circ) &= \sin(300^\circ + 45^\circ)\\ &= \sin 300^\circ \cos 45^\circ + \cos 300^\circ \sin 45^\circ\\ &= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{4}\left(-\sqrt{3}+1\right) = \frac{\sqrt{2}}{4}\left(1-\sqrt{3}\right)\end{aligned}$$

$$\begin{aligned}\cos(345^\circ) &= \cos(300^\circ + 45^\circ)\\ &= \cos 300^\circ \cos 45^\circ - \sin 300^\circ \sin 45^\circ\\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{4}\left(1+\sqrt{3}\right)\end{aligned}$$

$$\begin{aligned}\tan(345^\circ) &= \tan(300^\circ + 45^\circ)\\ &= \frac{\tan 300^\circ + \tan 45^\circ}{1 - \tan 300^\circ \tan 45^\circ} = \frac{-\sqrt{3}+1}{1+\sqrt{3}(1)}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}\\ &= \frac{4-2\sqrt{3}}{-2} = -2+\sqrt{3}\end{aligned}$$

53.
$$\begin{aligned}\sin\frac{25\pi}{12} &= \sin\left(\frac{11\pi}{6}+\frac{\pi}{4}\right) = \sin\frac{11\pi}{6}\cos\frac{\pi}{4} + \cos\frac{11\pi}{6}\sin\frac{\pi}{4}\\ &= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)\end{aligned}$$

$$\begin{aligned}\cos\frac{25\pi}{12} &= \cos\left(\frac{11\pi}{6}+\frac{\pi}{4}\right) = \cos\frac{11\pi}{6}\cos\frac{\pi}{4} - \sin\frac{11\pi}{6}\sin\frac{\pi}{4}\\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)\end{aligned}$$

$$\begin{aligned}\tan\frac{25\pi}{12} &= \tan\left(\frac{11\pi}{6}+\frac{\pi}{4}\right) = \frac{\tan\frac{11\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{11\pi}{6}\tan\frac{\pi}{4}}\\ &= \frac{\left(-\frac{\sqrt{3}}{3}\right)+1}{1-\left(-\frac{\sqrt{3}}{3}\right)(1)} = 2-\sqrt{3}\end{aligned}$$

54. $\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$= \sin\frac{11\pi}{6}\cos\frac{\pi}{4} - \cos\frac{11\pi}{6}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos\left(\frac{19\pi}{12}\right) = \cos\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$= \cos\frac{11\pi}{6}\cos\frac{\pi}{4} + \sin\frac{11\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\tan\left(\frac{19\pi}{12}\right) = \tan\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$$= \frac{\tan\frac{11\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{11\pi}{6}\tan\frac{\pi}{4}}$$

$$= \frac{-\frac{\sqrt{3}}{3} - 1}{1 + \left(-\frac{\sqrt{3}}{3}\right)(1)} = \frac{-\sqrt{3} - 3}{3 - \sqrt{3}}\cdot\frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{-\left(12 + 6\sqrt{3}\right)}{6} = -2 - \sqrt{3}$$

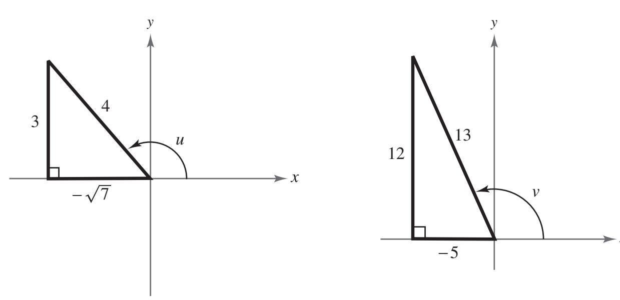
55. $\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ = \sin(60^\circ - 45^\circ) = \sin 15^\circ$

56. $\cos 45^\circ \cos 120^\circ - \sin 45^\circ \sin 120^\circ = \cos(45^\circ + 120^\circ) = \cos 165^\circ$

57. $\frac{\tan 25^\circ + \tan 10^\circ}{1 - \tan 25^\circ \tan 10^\circ} = \tan(25^\circ + 10^\circ)$

$= \tan 35^\circ$

58. $\frac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ} = \tan(68^\circ - 115^\circ) = \tan(-47^\circ)$



Figures for Exercises 59–64

59. $\sin(u + v) = \sin u \cos v + \cos u \sin v$

$= \left(\frac{3}{4}\right)\left(-\frac{5}{13}\right) + \left(-\frac{\sqrt{7}}{4}\right)\left(\frac{12}{13}\right)$

$= -\frac{3}{52}\left(5 + 4\sqrt{7}\right)$

60. $$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\left(-\frac{3}{\sqrt{7}}\right) + \left(-\frac{12}{5}\right)}{1 - \left(-\frac{3}{\sqrt{7}}\right)\left(-\frac{12}{5}\right)}$$

$$= \frac{15 + 12\sqrt{7}}{36 - 5\sqrt{7}}\cdot\frac{36 + 5\sqrt{7}}{36 + 5\sqrt{7}} = \frac{960 + 507\sqrt{7}}{1121}$$

61. $\cos(u - v) = \cos u \cos v + \sin u \sin v$

$$= \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{4}\right)\left(\frac{12}{13}\right)$$

$$= \frac{1}{52}\left(5\sqrt{7} + 36\right)$$

62. $\sin(u - v) = \sin u \cos v - \cos u \sin v$

$$= \left(\frac{3}{4}\right)\left(-\frac{5}{13}\right) - \left(-\frac{\sqrt{7}}{4}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-15 + 12\sqrt{7}}{52} = \frac{12\sqrt{7} - 15}{52}$$

63. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$$= \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{4}\right)\left(\frac{12}{13}\right)$$

$$= \frac{1}{52}\left(5\sqrt{7} - 36\right)$$

64. $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v} = \dfrac{\left(-\frac{3}{\sqrt{7}}\right) - \left(-\frac{12}{5}\right)}{1 + \left(-\frac{3}{\sqrt{7}}\right)\left(-\frac{12}{5}\right)}$

$$= \frac{-15 + 12\sqrt{7}}{36 + 5\sqrt{7}} \cdot \frac{36 - 5\sqrt{7}}{36 - 5\sqrt{7}}$$

$$= \frac{-960 + 507\sqrt{7}}{1121}$$

65. $\cos\left(x + \dfrac{\pi}{2}\right) = \cos x \cos \dfrac{\pi}{2} - \sin x \sin \dfrac{\pi}{2}$

$$= \cos x(0) - \sin x(1)$$

$$= -\sin x$$

66. $\sin\left(x - \dfrac{3\pi}{2}\right) = \sin x \cos \dfrac{3\pi}{2} - \cos x \sin \dfrac{3\pi}{2}$

$$= \sin x(0) - \cos x(-1)$$

$$= \cos x$$

67. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$ by the cofunction identity.

68. $\sin(\pi - x) = \sin \pi \cos x - \cos \pi \sin x$

$$= 0 \cdot \cos x - (-1)\sin x$$

$$= \sin x$$

69. $\cos 3x = \cos(2x + x)$

$$= \cos 2x \cos x - \sin 2x \sin x$$

$$= (\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x$$

$$= \cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x$$

$$= \cos^3 x - 3 \sin^2 x \cos x$$

$$= \cos^3 x - 3(1 - \cos^2 x) \cos x$$

$$= \cos^3 x - 3 \cos x + 3 \cos^3 x$$

$$= 4 \cos^3 x - 3 \cos x$$

70. $\dfrac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \dfrac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}$

$$= \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$= \frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}$$

$$= \tan \alpha + \tan \beta$$

71. $\sin\left(x + \dfrac{\pi}{4}\right) - \sin\left(x - \dfrac{\pi}{4}\right) = 1$

$$2 \cos x \sin \frac{\pi}{4} = 1$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

72.

$$\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) - \left(\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}\right) = 1$$

$$-2 \sin x \sin \frac{\pi}{6} = 1$$

$$-2 \sin x\left(\frac{1}{2}\right) = 1$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

73. $\sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{3}$

$$2 \cos x \sin \frac{\pi}{2} = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

74.

$$\cos\left(x + \frac{3\pi}{4}\right) - \cos\left(x - \frac{3\pi}{4}\right) = 0$$

$$\left(\cos x \cos \frac{3\pi}{4} - \sin x \sin \frac{3\pi}{4}\right) - \left(\cos x \cos \frac{3\pi}{4} + \sin x \sin \frac{3\pi}{4}\right) = 0$$

$$-2 \sin x \sin \frac{3\pi}{4} = 0$$

$$-2 \sin x\left(\frac{\sqrt{2}}{2}\right) = 0$$

$$-\sqrt{2} \sin x = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

75. $\sin u = -\frac{4}{5}, \pi < u < \frac{3\pi}{2}$

$\cos u = -\sqrt{1 - \sin^2 u} = \frac{-3}{5}$

$\tan u = \frac{\sin u}{\cos u} = \frac{4}{3}$

$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = -\frac{7}{25}$

$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2} = -\frac{24}{7}$

76. $\cos u = -\frac{2}{\sqrt{5}}, \frac{\pi}{2} < u < \pi \Rightarrow \sin u = \frac{1}{\sqrt{5}}$ and

$\tan u = -\frac{1}{2}$

$\sin 2u = 2 \sin u \cos u = 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = -\frac{4}{5}$

$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$

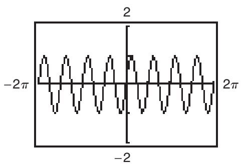
$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{1}{2}\right)}{1 - \left(-\frac{1}{2}\right)^2} = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$

77. $\sin 4x = 2 \sin 2x \cos 2x$

$= 2[2 \sin x \cos x(\cos^2 x - \sin^2 x)]$

$= 4 \sin x \cos x(2 \cos^2 x - 1)$

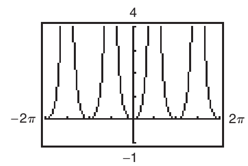
$= 8 \cos^3 x \sin x - 4 \cos x \sin x$



78. $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \dfrac{1 - (1 - 2\sin^2 x)}{1 + (2 \cos x^2 - 1)}$

$= \dfrac{2 \sin^2 x}{2 \cos^2 x}$

$= \tan^2 x$



79. $\tan^2 2x = \dfrac{\sin^2 2x}{\cos^2 2x} = \dfrac{\frac{1 - \cos 4x}{2}}{\frac{1 + \cos 4x}{2}} = \dfrac{1 - \cos 4x}{1 + \cos 4x}$

80. $\cos^2 3x = \dfrac{1 + \cos 6x}{2}$

81. $\sin^2 x \tan^2 x = \sin^2 x\left(\dfrac{\sin^2 x}{\cos^2 x}\right) = \dfrac{\sin^4 x}{\cos^2 x}$

$= \dfrac{\left(\frac{1 - \cos 2x}{2}\right)^2}{\frac{1 + \cos 2x}{2}} = \dfrac{\frac{1 - 2\cos 2x + \cos^2 2x}{4}}{\frac{1 + \cos 2x}{2}}$

$= \dfrac{1 - 2\cos 2x + \frac{1 + \cos 4x}{2}}{2(1 + \cos 2x)}$

$= \dfrac{2 - 4\cos 2x + 1 + \cos 4x}{4(1 + \cos 2x)}$

$= \dfrac{3 - 4\cos 2x + \cos 4x}{4(1 + \cos 2x)}$

82. $\cos^2 x \tan^2 x = \sin^2 x = \dfrac{1 - \cos 2x}{2}$

83. $\sin(-75°) = -\sqrt{\dfrac{1 - \cos 150°}{2}} = -\sqrt{\dfrac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\dfrac{\sqrt{2 + \sqrt{3}}}{2}$

$= -\dfrac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos(-75°) = \sqrt{\dfrac{1 + \cos 150°}{2}} = \sqrt{\dfrac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}} = \dfrac{\sqrt{2 - \sqrt{3}}}{2}$

$= \dfrac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan(-75°) = -\left(\dfrac{1 - \cos 150°}{\sin 150°}\right) = -\left(\dfrac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}\right) = -\left(2 + \sqrt{3}\right)$

$= -2 - \sqrt{3}$

84. $\sin 15° = \sin\left(\frac{30°}{2}\right) = \sqrt{\frac{1 - \cos 30°}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\cos 15° = \cos\left(\frac{30°}{2}\right) = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\tan 15° = \tan\left(\frac{30°}{2}\right) = \frac{1 - \cos 30°}{\sin 30°} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$

85. $\sin\left(\frac{19\pi}{12}\right) = -\sqrt{\frac{1 - \cos\frac{19\pi}{6}}{2}} = -\sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\frac{\sqrt{2 + \sqrt{3}}}{2} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos\left(\frac{19\pi}{12}\right) = \sqrt{\frac{1 + \cos\frac{19\pi}{6}}{2}} = \sqrt{\frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan\left(\frac{19\pi}{12}\right) = \frac{1 - \cos\frac{19\pi}{6}}{\sin\frac{19\pi}{6}} = \frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}} = -2 - \sqrt{3}$

86. $\sin\left(-\frac{17\pi}{12}\right) = \sin\left(-\frac{\frac{17\pi}{6}}{2}\right) = \sqrt{\frac{1 - \cos\left(-\frac{17\pi}{6}\right)}{2}} = \sqrt{\frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos\left(-\frac{17\pi}{12}\right) = \cos\left(\frac{-\frac{17\pi}{6}}{2}\right) = -\sqrt{\frac{1 + \cos\left(-\frac{17\pi}{6}\right)}{2}} = -\sqrt{\frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan\left(-\frac{17\pi}{12}\right) = \tan\left(\frac{-\left(\frac{17\pi}{6}\right)}{2}\right) = \frac{1 - \cos\left(-\frac{17\pi}{6}\right)}{\sin\left(-\frac{17\pi}{6}\right)} = \frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}} = -2 - \sqrt{3}$

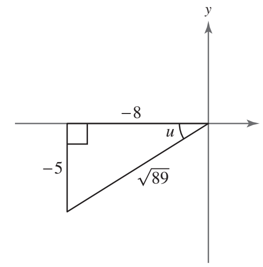
87. Given $\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2} \Longrightarrow \cos u = \frac{4}{5}$ and $\frac{u}{2}$ is in Quadrant I.

$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - 4/5}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$

$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + 4/5}{2}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$

$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 - 4/5}{3/5} = \frac{1}{3}$

88. $\tan u = \frac{5}{8}, \pi < u < \frac{3\pi}{2}$



$\sin u = \frac{-5}{\sqrt{89}}$

$\cos u = \frac{-8}{\sqrt{89}}$

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \left(-8/\sqrt{89}\right)}{2}} = \sqrt{\frac{\sqrt{89} + 8}{2\sqrt{89}}} = \sqrt{\frac{89 + 8\sqrt{89}}{178}}$$

$$\cos \frac{u}{2} = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 + \left(-8/\sqrt{89}\right)}{2}} = -\sqrt{\frac{\sqrt{89} - 8}{2\sqrt{89}}} = -\sqrt{\frac{89 - 8\sqrt{89}}{178}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{-8}{\sqrt{89}}}{\frac{-5}{\sqrt{89}}} = \frac{\sqrt{89} + 8}{-5} = \frac{-8 - \sqrt{89}}{5}$$

89. Given $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi \Longrightarrow \sin u = \frac{3\sqrt{5}}{7}$ and $\frac{u}{2}$ is in Quadrant I.

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - (-2/7)}{2}} = \sqrt{\frac{9}{14}} = \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + (-2/7)}{2}} = \sqrt{\frac{5}{14}} = \frac{\sqrt{70}}{14}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 - (-2/7)}{3\sqrt{5}/7} = \frac{9/7}{3\sqrt{5}/7} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

90. $\sec u = -6, \frac{\pi}{2} < u < \pi, \quad \sin u = \sqrt{1 - \frac{1}{36}} = \frac{\sqrt{35}}{6}, \quad \cos u = -\frac{1}{6}$

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (1/6)}{2}} = \sqrt{\frac{7}{12}} = \frac{\sqrt{21}}{6}$$

$$\cos \frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (1/6)}{2}} = \sqrt{\frac{5}{12}} = \frac{\sqrt{15}}{6}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 + (1/6)}{\sqrt{35}/6} = \frac{7}{6} \cdot \frac{6}{\sqrt{35}} = \frac{7}{\sqrt{35}} = \frac{\sqrt{35}}{5} \quad \text{or}$$

$$\tan \frac{u}{2} = \frac{\sin(u/2)}{\cos(u/2)} = \frac{\sqrt{21}/6}{\sqrt{15}/6} = \frac{\sqrt{21}}{\sqrt{15}} = \frac{\sqrt{35}}{5}$$

91. $-\sqrt{\frac{1 + \cos 10x}{2}} = -\left|\cos \frac{10x}{2}\right| = -|\cos 5x|$

92. $\frac{\sin 6x}{1 + \cos 6x} = \tan 3x$

93. $\cos \frac{\pi}{6} \sin \frac{\pi}{6} = \frac{1}{2}\left[\sin \frac{\pi}{3} - \sin 0\right] = \frac{1}{2} \sin \frac{\pi}{3}$

94. $6 \sin 15° \sin 45° = 6\left(\frac{1}{2}\right)[\cos(15° - 45°) - \cos(15° + 45°)]$

$= 3[\cos(-30°) - \cos 60°]$

$= 3(\cos 30° - \cos 60°)$

95. $\cos 5\theta \cos 3\theta = \frac{1}{2}[\cos 2\theta + \cos 8\theta]$

96. $4 \sin 3\alpha \cos 2\alpha = 4\left(\frac{1}{2}\right)[\sin(3\alpha + 2\alpha) + \sin(3\alpha - 2\alpha)]$

$= 2(\sin 5\alpha + \sin \alpha)$

97. $\sin 4\theta - \sin 2\theta = 2\cos\left(\frac{4\theta + 2\theta}{2}\right)\sin\left(\frac{4\theta - 2\theta}{2}\right)$

$= 2\cos 3\theta \sin\theta$

98. $\cos 3\theta + \cos 2\theta = 2\cos\left(\frac{3\theta + 2\theta}{2}\right)\cos\left(\frac{3\theta - 2\theta}{2}\right)$

$= 2\cos\frac{5\theta}{2}\cos\frac{\theta}{2}$

99. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = -2\sin x \ \sin\frac{\pi}{6}$

100. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 2\cos\left[\frac{\left(x + \frac{\pi}{4}\right) + \left(x - \frac{\pi}{4}\right)}{2}\right]\sin\left[\frac{\left(x + \frac{\pi}{4}\right) - \left(x - \frac{\pi}{4}\right)}{2}\right] = 2\cos x \sin\frac{\pi}{4}$

101. $r = \frac{1}{32}v_0^2 \sin 2\theta$

range $= 100$ feet

$v_0 = 80$ feet per second

$r = \frac{1}{32}(80)^2 \sin 2\theta = 100$

$\sin 2\theta = 0.5$

$2\theta = 30°$

$\theta = 15°$ or $\frac{\pi}{12}$

102. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.

$V = A \cdot l$

(a) $A = \frac{1}{2}bh$

$\cos\frac{\theta}{2} = \frac{h}{0.5} \implies h = 0.5\cos\frac{\theta}{2}$

$\sin\frac{\theta}{2} = \frac{\frac{b}{2}}{0.5} \implies \frac{b}{2} = 0.5\sin\frac{\theta}{2}$

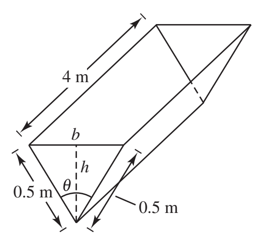
$A = 0.5\sin\frac{\theta}{2}\,0.5\cos\frac{\theta}{2}$

$= (0.5)^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= 0.25\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ square meters

$V = (0.25)(4)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters

$= \sin\frac{\theta}{2}\cos\frac{\theta}{2}$ cubic meters

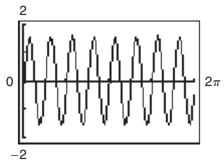


Not drawn to scale

(b) $V = \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{1}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{1}{2}\sin\theta$ cubic meters

Volume is maximum when $\theta = \frac{\pi}{2}$.

103. $y = 1.5 \sin 8t - 0.5 \cos 8t$



104. $y = 1.5 \sin 8t - 0.5 \cos 8t = \frac{1}{2}(3 \sin 8t - 1 \cos 8t)$

Using the identity

$a \sin B\theta + b \cos B\theta = \sqrt{a^2 + b^2} \sin(B\theta + C),$

$C = \arctan \frac{b}{a}, a > 0$

(Exercise 83, Section 5.4), we have

$$y = \frac{1}{2}\sqrt{(3)^2 + (-1)^2} \sin\left(8t + \arctan\left(-\frac{1}{3}\right)\right)$$

$$= \frac{\sqrt{10}}{2} \sin\left(8t - \arctan\left(\frac{1}{3}\right)\right).$$

105. Amplitude $= \frac{\sqrt{10}}{2}$ feet

106. Frequency $= \frac{1}{\frac{2\pi}{8}} = \frac{4}{\pi}$ cycles per second

107. False. If $\frac{\pi}{2} < \theta < \pi$, then $\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$ and $\frac{\theta}{2}$ is in Quadrant I.

$\cos \frac{\theta}{2} > 0$

108. False. The correct identity is

$\sin(x + y) = \sin x \cos y + \cos x \sin y.$

109. True. $4 \sin(-x)\cos(-x) = 4(-\sin x)\cos x$

$$= -4 \sin x \cos x = -2(2 \sin x \cos x)$$

$$= -2 \sin 2x$$

110. True. It can be verified using a product-to-sum identity.

$$4 \sin 45° \cos 15° = 4 \cdot \frac{1}{2}[\sin 60° + \sin 30°]$$

$$= 2\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \sqrt{3} + 1$$

111. Reciprocal Identities: $\sin \theta = \frac{1}{\csc \theta}$ $\quad \csc \theta = \frac{1}{\sin \theta}$

$\cos \theta = \frac{1}{\sec \theta}$ $\quad \sec \theta = \frac{1}{\cos \theta}$

$\tan \theta = \frac{1}{\cot \theta}$ $\quad \cot \theta = \frac{1}{\tan \theta}$

Quotient Identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\quad \cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean Identities: $\sin^2 \theta + \cos^2 \theta = 1$

$1 + \tan^2 \theta = \sec^2 \theta$

$1 + \cot^2 \theta = \csc^2 \theta$

112. No. For an equation to be an identity, the equation must be true for all real numbers. $\sin \theta = \frac{1}{2}$ has an infinite number of solutions but is not an identity.

113. $a \sin x - b = 0$

$\sin x = \frac{b}{a}$

If $|b| > |a|$, then $\left|\frac{b}{a}\right| > 1$ and there is no solution

since $|\sin x| \leq 1$ for all x.

114. $S = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right), \ 0° < \theta \le 90°$

where $h = 2.4$ inches, $s = 0.75$ inch, and θ is the given angle.

(a) For a surface area of 12 square inches,

$$S = 6(2.4)(0.75) + \frac{3}{2}(0.75)^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right) = 12$$

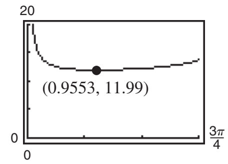
$$= 10.8 + 0.84375\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right) = 12$$

$$0.84375\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right) = 1.2.$$

Using the solve function of a graphing calculator gives

$\theta = 49.91479°$ or $\theta = 59.86118°$.

(b) Using a graphing calculator yields the following graph:



Using the minimum function yields

$\theta = 0.9553$ radians or $\theta = 54.73466°$.

115. The graph of y_1 is a vertical shift of the graph of y_2 one unit upward so $y_1 = y_2 + 1$.

116. $y_1 = \frac{\cos 3x}{\cos x}, \qquad y_2 = (2\sin x)^2$

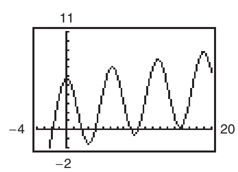
If the graph of y_2 is reflected in the x-axis and then shifted upward by one unit, it coincides with the graph of y_1. Therefore,

$$\frac{\cos 3x}{\cos x} = -(2\sin x)^2 + 1.$$

So, $y_1 = 1 - y_2$.

117. $y = \sqrt{x + 3} + 4\cos x$

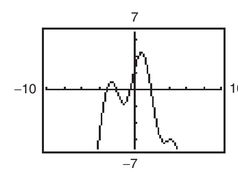
Zeros: $x \approx -1.8431, 2.1758, 3.9903, 8.8935, 9.8820$



118. $y = 2 - \frac{1}{2}x^2 + 3\sin\frac{\pi x}{2}$

Approximate roots:
$-3.1395, -2.0000, -0.4378, 2.0000$

$y = 2 - \frac{1}{2}x^2 + 3\sin\frac{\pi x}{2}$



Problem Solving for Chapter 5

1. (a) Since $\sin^2\theta + \cos^2\theta = 1$ and $\cos^2\theta = 1 - \sin^2\theta$:

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \pm\frac{\sin\theta}{\sqrt{1 - \sin^2\theta}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \pm\frac{\sqrt{1 - \sin^2\theta}}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} = \pm\frac{1}{\sqrt{1 - \sin^2\theta}}$$

$$\cos\theta = \frac{1}{\sin\theta}$$

We also have the following relationships:

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \frac{\sin\theta}{\sin\left(\frac{\pi}{2} - \theta\right)}$$

$$\cot\theta = \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\theta}$$

$$\sec\theta = \frac{1}{\sin\left(\frac{\pi}{2} - \theta\right)}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

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1. —CONTINUED—

(b) $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} = \pm\frac{1}{\sqrt{1-\cos^2\theta}}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$$

We also have the following relationships:

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$$

$$\tan\theta = \frac{\cos[(\pi/2)-\theta]}{\cos\theta}$$

$$\csc\theta = \frac{1}{\cos[(\pi/2)-\theta]}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\cos[(\pi/2)-\theta]}$$

2. $$\cos\left[\frac{(2n+1)\pi}{2}\right] = \cos\left(\frac{2n\pi+\pi}{2}\right)$$
$$= \cos\left(n\pi+\frac{\pi}{2}\right)$$
$$= \cos n\pi\cos\frac{\pi}{2} - \sin n\pi\sin\frac{\pi}{2}$$
$$= (\pm 1)(0) - (0)(1)$$
$$= 0$$

Thus, $\cos\left[\frac{(2n+1)\pi}{2}\right] = 0$ for all integers n.

3. $$\sin\left[\frac{(12n+1)\pi}{6}\right] = \sin\left[\frac{1}{6}(12n\pi+\pi)\right]$$
$$= \sin\left(2n\pi+\frac{\pi}{6}\right)$$
$$= \sin\frac{\pi}{6} = \frac{1}{2}$$

Thus, $\sin\left[\frac{(12n+1)\pi}{6}\right] = \frac{1}{2}$ for all integers n.

4. $p(t) = \frac{1}{4\pi}[p_1(t) + 30p_2(t) + p_3(t) + p_5(t) + 30p_6(t)]$

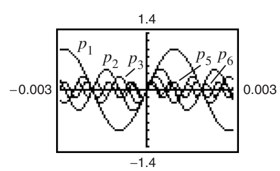
(a) $p_1(t) = \sin(524\pi t)$

$$p_2(t) = \frac{1}{2}\sin(1048\pi t)$$

$$p_3(t) = \frac{1}{3}\sin(1572\pi t)$$

$$p_5(t) = \frac{1}{5}\sin(2620\pi t)$$

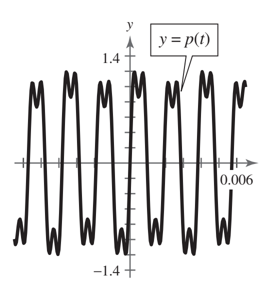
$$p_6(t) = \frac{1}{6}\sin(3144\pi t)$$



The graph of

$$p(t) = \frac{1}{4\pi}\left[\sin(524\pi t) + 15\sin(1048\pi t) + \frac{1}{3}\sin(1572\pi t) + \frac{1}{5}\sin(2620\pi t) + 5\sin(3144\pi t)\right]$$

yields the graph shown in the text and to the right.



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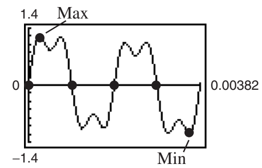
4. —CONTINUED—

(b)

Function	Period
$p_1(t)$	$\frac{2\pi}{524\pi} = \frac{1}{262} \approx 0.0038$
$p_2(t)$	$\frac{2\pi}{1048\pi} = \frac{1}{524} \approx 0.0019$
$p_3(t)$	$\frac{2\pi}{1572\pi} = \frac{1}{786} \approx 0.0013$
$p_5(t)$	$\frac{2\pi}{2620\pi} = \frac{1}{1310} \approx 0.0008$
$p_6(t)$	$\frac{2\pi}{3144\pi} = \frac{1}{1572} \approx 0.0006$

The graph of p appears to be periodic with a period of $\frac{1}{262} \approx 0.0038$.

(c)



Over one cycle, $0 \le t < \frac{1}{262}$, we have four t-intercepts:

$t = 0$, $t \approx 0.00096$, $t \approx 0.00191$, and $t \approx 0.00285$

(d) The absolute maximum value of p over one cycle is $p \approx 1.1952$, and the absolute minimum value of p over one cycle is $p \approx -1.1952$.

5. From the figure, it appears that $u + v = w$. Assume that u, v, and w are all in Quadrant I. From the figure:

$$\tan u = \frac{s}{3s} = \frac{1}{3}$$

$$\tan v = \frac{s}{2s} = \frac{1}{2}$$

$$\tan w = \frac{s}{s} = 1$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{1/3 + 1/2}{1 - (1/3)(1/2)}$$

$$= \frac{5/6}{1 - (1/6)}$$

$$= 1 = \tan w.$$

Thus, $\tan(u + v) = \tan w$. Because u, v, and w are all in Quadrant I, we have

$$\arctan[\tan(u + v)] = \arctan[\tan w]u + v = w.$$

6. $y = -\frac{16}{v_0^2 \cos^2 \theta} x^2 + (\tan \theta)x + h_0$

Let $h_0 = 0$ and take half of the horizontal distance:

$$\frac{1}{2}\left(\frac{1}{32} v_0^2 \sin 2\theta\right) = \frac{1}{64} v_0^2(2 \sin \theta \cos \theta) = \frac{1}{32} v_0^2 \sin \theta \cos \theta$$

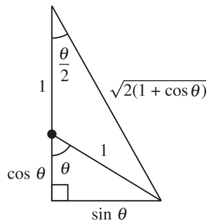
Substitute this expression for x in the model.

$$y = -\frac{16}{v_0^2 \cos^2 \theta}\left(\frac{1}{32} v_0^2 \sin \theta \cos \theta\right)^2 + \left(\frac{\sin \theta}{\cos \theta}\right)\left(\frac{1}{32} v_0^2 \sin \theta \cos \theta\right)$$

$$= -\frac{1}{64} v_0^2 \sin^2 \theta + \frac{1}{32} v_0^2 \sin^2 \theta$$

$$= \frac{1}{64} v_0^2 \sin^2 \theta$$

7.



The hypotenuse of the larger right triangle is:

$$\sqrt{\sin^2\theta + (1 + \cos\theta)^2} = \sqrt{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}$$
$$= \sqrt{2 + 2\cos\theta}$$
$$= \sqrt{2(1 + \cos\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{\sqrt{2(1+\cos\theta)}} = \frac{\sin\theta}{\sqrt{2(1+\cos\theta)}} \cdot \frac{\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}}$$
$$= \frac{\sin\theta\sqrt{1-\cos\theta}}{\sqrt{2(1-\cos^2\theta)}} = \frac{\sin\theta\sqrt{1-\cos\theta}}{\sqrt{2}\sin\theta} = \sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{\sqrt{2(1+\cos\theta)}} = \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{2(1+\cos\theta)}} = \sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$

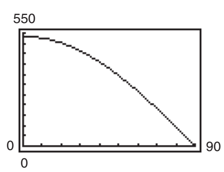
8. $F = \dfrac{0.6W\sin(\theta + 90°)}{\sin 12°}$

(a) $F = \dfrac{0.6W(\sin\theta\cos 90° + \cos\theta\sin 90°)}{\sin 12°}$

$= \dfrac{0.6W[(\sin\theta)(0) + (\cos\theta)(1)]}{\sin 12°}$

$= \dfrac{0.6W\cos\theta}{\sin 12°}$

(b) Let $y_1 = \dfrac{0.6(185)\cos x}{\sin 12°}$.

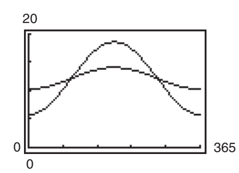


(c) The force is maximum (533.88 pounds) when $\theta = 0°$.
The force is minimum (0 pounds) when $\theta = 90°$.

9. Seward: $D = 12.2 - 6.4\cos\left[\dfrac{\pi(t + 0.2)}{182.6}\right]$

New Orleans: $D = 12.2 - 1.9\cos\left[\dfrac{\pi(t + 0.2)}{182.6}\right]$

(a)



(b) The graphs intersect when $t \approx 91$ and when $t \approx 274$. These values correspond to April 1 and October 1, the spring equinox and the fall equinox.

(c) Seward has the greater variation in the number of daylight hours. This is determined by the amplitudes, 6.4 and 1.9.

(d) Period: $\dfrac{2\pi}{\pi/182.6} = 365.2$ days

10. $d = 35 - 28\cos\dfrac{\pi}{6.2}t$ when $t = 0$ corresponds to 12:00 A.M.

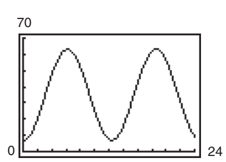
(a) The high tides occur when $\cos\dfrac{\pi}{6.2}t = -1$. Solving yields $t = 6.2$ or $t = 18.6$.

These t-values correspond to 6:12 A.M. and 6:36 P.M.

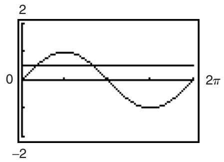
The low tide occurs when $\cos\dfrac{\pi}{6.2}t = 1$. Solving yields $t = 0$ and $t = 12.4$ which corresponds to 12:00 A.M. and 12:24 P.M.

(b) The water depth is never 3.5 feet. At low tide the depth is $d = 35 - 28 = 7$ feet.

(c)

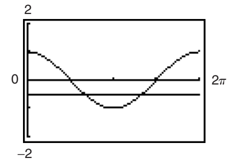


11. (a) Let $y_1 = \sin x$ and $y_2 = 0.5$.



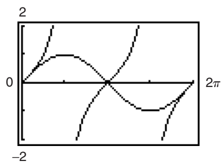
$\sin x \geq 0.5$ on the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

(b) Let $y_1 = \cos x$ and $y_2 = -0.5$.



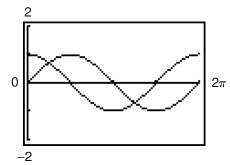
$\cos x \leq -0.5$ on the interval $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$.

(c) Let $y_1 = \tan x$ and $y_2 = \sin x$.



$\tan x < \sin x$ on the intervals $\left(\frac{\pi}{2}, \pi\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

(d) Let $y_1 = \cos x$ and $y_2 = \sin x$.



$\cos x \geq \sin x$ on the intervals $\left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$.

12. (a) $n = \dfrac{\sin\left(\dfrac{\theta}{2} + \dfrac{\alpha}{2}\right)}{\sin\dfrac{\theta}{2}}$

$$= \frac{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\alpha}{2}\right) + \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

$$= \cos\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$

For $\alpha = 60°$, $n = \cos 30° + \cot\left(\frac{\theta}{2}\right)\sin 30°$

$$n = \frac{\sqrt{3}}{2} + \frac{1}{2}\cot\left(\frac{\theta}{2}\right).$$

(b) For glass, $n = 1.50$.

$$1.50 = \frac{\sqrt{3}}{2} + \frac{1}{2}\cot\left(\frac{\theta}{2}\right)$$

$$2\left(1.50 - \frac{\sqrt{3}}{2}\right) = \cot\left(\frac{\theta}{2}\right)$$

$$\frac{1}{3 - \sqrt{3}} = \tan\left(\frac{\theta}{2}\right)$$

$$\theta = 2\tan^{-1}\left(\frac{1}{3 - \sqrt{3}}\right)$$

$$\theta \approx 76.52°$$

13. (a)
$$\begin{aligned}\sin(u + v + w) &= \sin[(u + v) + w]\\ &= \sin(u + v)\cos w + \cos(u + v)\sin w\\ &= [\sin u\cos v + \cos u\sin v]\cos w + [\cos u\cos v - \sin u\sin v]\sin w\\ &= \sin u\cos v\cos w + \cos u\sin v\cos w + \cos u\cos v\sin w - \sin u\sin v\sin w\end{aligned}$$

(b)
$$\begin{aligned}\tan(u + v + w) &= \tan[(u + v) + w]\\ &= \frac{\tan(u + v) + \tan w}{1 - \tan(u + v)\tan w}\\ &= \frac{\left[\dfrac{\tan u + \tan v}{1 - \tan u\tan v}\right] + \tan w}{1 - \left[\dfrac{\tan u + \tan v}{1 - \tan u\tan v}\right]\tan w} \cdot \frac{(1 - \tan u\tan v)}{(1 - \tan u\tan v)}\\ &= \frac{\tan u + \tan v + (1 - \tan u\tan v)\tan w}{(1 - \tan u\tan v) - (\tan u + \tan v)\tan w}\\ &= \frac{\tan u + \tan v + \tan w - \tan u\tan v\tan w}{1 - \tan u\tan v - \tan u\tan w - \tan v\tan w}\end{aligned}$$

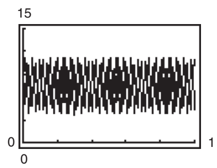
14. (a)
$$\begin{aligned}\cos(3\theta) &= \cos(2\theta + \theta)\\ &= \cos 2\theta \cos\theta - \sin 2\theta \sin \theta\\ &= (1 - 2\sin^2\theta)\cos\theta - (2\sin\theta\cos\theta)\sin\theta\\ &= \cos\theta - 4\sin^2\theta\cos\theta\\ &= \cos\theta(1 - 4\sin^2\theta)\end{aligned}$$

(b)
$$\begin{aligned}\cos(4\theta) &= \cos(2\theta + 2\theta)\\ &= \cos 2\theta\cos 2\theta - \sin 2\theta \sin 2\theta\\ &= \cos^2 2\theta - \sin^2 2\theta\\ &= (1 - \sin^2 2\theta) - \sin^2 2\theta\\ &= 1 - 2\sin^2 2\theta\\ &= 1 - 2(2\sin\theta\cos\theta)^2\\ &= 1 - 8\sin^2\theta\cos^2\theta\end{aligned}$$

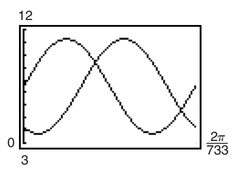
15. $h_1 = 3.75 \sin 733t + 7.5$

$h_2 = 3.75 \sin 733\left(t + \dfrac{4\pi}{3}\right) + 7.5$

(a)



(b) The period for h_1 and h_2 is $\dfrac{2\pi}{733} \approx 0.0086$.



The graphs intersect twice per cycle.

There are $\dfrac{1}{2\pi/733} \approx 116.66$ cycles in the interval $[0, 1]$, so the graphs intersect approximately 233.3 times.

Chapter 5 Practice Test

1. Find the value of the other five trigonometric functions, given $\tan x = \frac{4}{11}$, $\sec x < 0$.

2. Simplify $\dfrac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)}$.

3. Rewrite as a single logarithm and simplify $\ln|\tan \theta| - \ln|\cot \theta|$.

4. True or false:

   $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{1}{\csc x}$

5. Factor and simplify: $\sin^4 x + (\sin^2 x) \cos^2 x$

6. Multiply and simplify: $(\csc x + 1)(\csc x - 1)$

7. Rationalize the denominator and simplify:

   $\dfrac{\cos^2 x}{1 - \sin x}$

8. Verify:

   $\dfrac{1 + \cos \theta}{\sin \theta} + \dfrac{\sin \theta}{1 + \cos \theta} = 2 \csc \theta$

9. Verify:

   $\tan^4 x + 2 \tan^2 x + 1 = \sec^4 x$

10. Use the sum or difference formulas to determine:

    (a) $\sin 105°$ (b) $\tan 15°$

11. Simplify: $(\sin 42°) \cos 38° - (\cos 42°) \sin 38°$

12. Verify $\tan\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan \theta}{1 - \tan \theta}$.

13. Write $\sin(\arcsin x - \arccos x)$ as an algebraic expression in x.

14. Use the double-angle formulas to determine:

    (a) $\cos 120°$ (b) $\tan 300°$

15. Use the half-angle formulas to determine:

    (a) $\sin 22.5°$ (b) $\tan \dfrac{\pi}{12}$

16. Given $\sin = 4/5$, θ lies in Quadrant II, find $\cos(\theta/2)$.

17. Use the power-reducing identities to write $(\sin^2 x) \cos^2 x$ in terms of the first power of cosine.

18. Rewrite as a sum: $6(\sin 5\theta) \cos 2\theta$.

19. Rewrite as a product: $\sin(x + \pi) + \sin(x - \pi)$.

20. Verify $\dfrac{\sin 9x + \sin 5x}{\cos 9x - \cos 5x} = -\cot 2x$.

21. Verify:

    $(\cos u) \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$.

22. Find all solutions in the interval $[0, 2\pi)$:

    $4 \sin^2 x = 1$

23. Find all solutions in the interval $[0, 2\pi)$:

    $\tan^2 \theta + \left(\sqrt{3} - 1\right) \tan \theta - \sqrt{3} = 0$

24. Find all solutions in the interval $[0, 2\pi)$:

    $\sin 2x = \cos x$

25. Use the quadratic formula to find all solutions in the interval $[0, 2\pi)$:

    $\tan^2 x - 6 \tan x + 4 = 0$