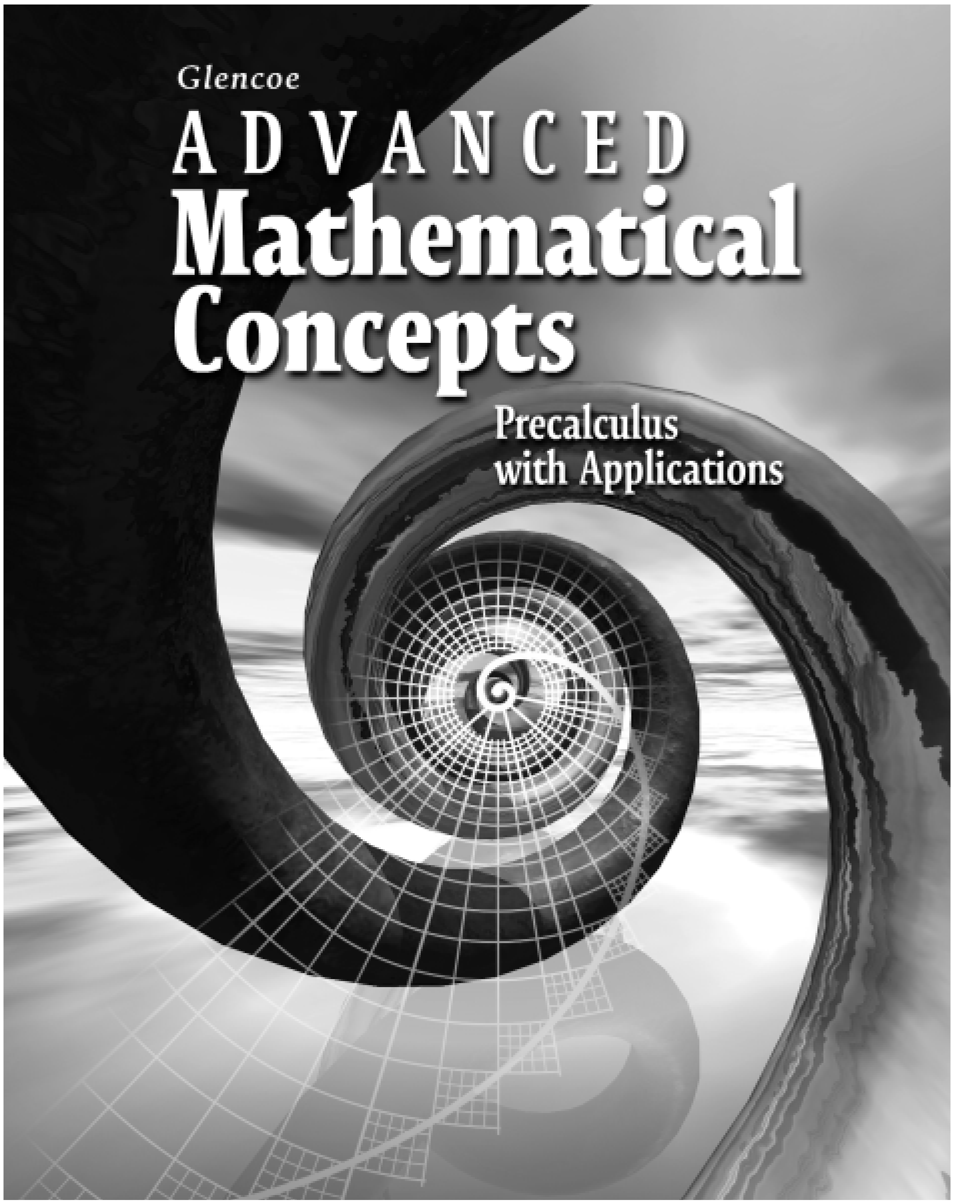
Chapter 4
Resource Masters



Glencoe

New York, New York   Columbus, Ohio   Woodland Hills, California   Peoria, Illinois

StudentWorks™ This CD-ROM includes the entire Student Edition along with the Study Guide, Practice, and Enrichment masters.

TeacherWorks™ All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

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Send all inquiries to:
Glencoe/McGraw-Hill
8787 Orion Place
Columbus, OH 43240-4027

ISBN: 0-07-869131-1

Advanced Mathematical Concepts
Chapter 4 Resource Masters

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Contents

A Teacher's Guide to Using the Chapter 4 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 4 Resource Masters* include the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Advanced Mathematical Concepts TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii-x include a student study tool that presents the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

When to Use Give these pages to students before beginning Lesson 4-1. Remind them to add definitions and examples as they complete each lesson.

Study Guide There is one Study Guide master for each lesson.

When to Use Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for those students who have been absent.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice section of the Student Edition exercises. These exercises are of average difficulty.

When to Use These provide additional practice options or may be used as homework for second day teaching of the lesson.

Enrichment There is one master for each lesson. These activities may extend the concepts in the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

When to Use These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment section of the *Chapter 4 Resources Masters* offers a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessments

Chapter Tests

- *Forms 1A, 1B, and 1C* Form 1 tests contain multiple-choice questions. Form 1A is intended for use with honors-level students, Form 1B is intended for use with average-level students, and Form 1C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.
- *Forms 2A, 2B, and 2C* Form 2 tests are composed of free-response questions. Form 2A is intended for use with honors-level students, Form 2B is intended for use with average-level students, and Form 2C is intended for use with basic-level students. These tests are similar in format to offer comparable testing situations.

  All of the above tests include a challenging Bonus question.
- The **Extended Response Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

Intermediate Assessment

- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of free-response questions.
- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.

Continuing Assessment

- The **SAT and ACT Practice** offers continuing review of concepts in various formats, which may appear on standardized tests that they may encounter. This practice includes multiple-choice, quantitative-comparison, and grid-in questions. Bubble-in and grid-in answer sections are provided on the master.
- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of advanced mathematics. It can also be used as a test. The master includes free-response questions.

Answers

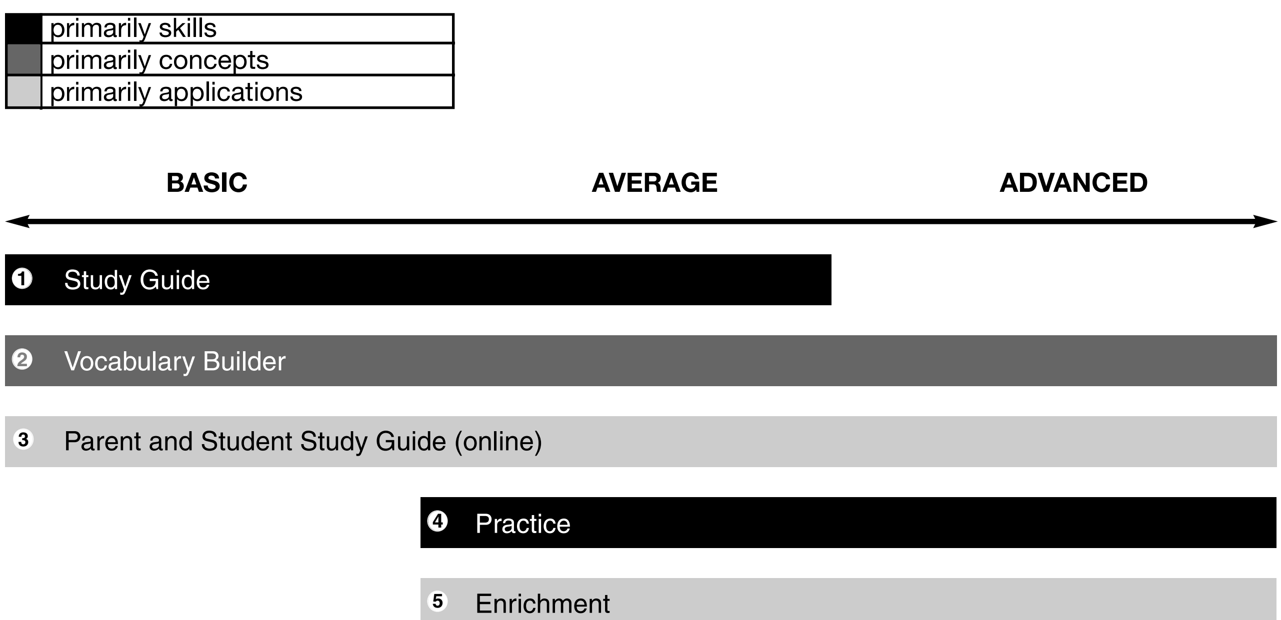
- Page A1 is an answer sheet for the SAT and ACT Practice questions that appear in the Student Edition on page 273. Page A2 is an answer sheet for the SAT and ACT Practice master. These improve students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment options in this booklet.

Chapter 4 Leveled Worksheets

Glencoe's **leveled worksheets** are helpful for meeting the needs of every student in a variety of ways. These worksheets, many of which are found in the **FAST FILE Chapter Resource Masters**, are shown in the chart below.

- **Study Guide** masters provide worked-out examples as well as practice problems.
- Each chapter's **Vocabulary Builder** master provides students the opportunity to write out key concepts and definitions in their own words.
- **Practice** masters provide average-level problems for students who are moving at a regular pace.
- **Enrichment** masters offer students the opportunity to extend their learning.

Five Different Options to Meet the Needs of Every Student in a Variety of Ways



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Chapter 4

Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term.

Vocabulary Term	Found on Page	Definition/Description/Example
completing the square		
complex number		
conjugate		
degree		
depressed polynomial		
Descartes' Rule of Signs		
discriminant		
extraneous solution		
Factor Theorem		
Fundamental Theorem of Algebra		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
imaginary number		
Integral Root Theorem		
leading coefficient		
Location Principle		
lower bound		
lower Bound Theorem		
partial fractions		
polynomial equation		
polynomial function		
polynomial in one variable		
pure imaginary number		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
Quadratic Formula		
radical equation		
radical inequality		
rational equation		
rational inequality		
Rational Root theorem		
Remainder Theorem		
synthetic division		
upper bound		
Upper Bound Theorem		
zero		

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Study Guide

Polynomial Functions

The **degree** of a polynomial in one variable is the greatest exponent of its variable. The coefficient of the variable with the greatest exponent is called the **leading coefficient.** If a function $f(x)$ is defined by a polynomial in one variable, then it is a polynomial function. The values of x for which $f(x) = 0$ are called the **zeros** of the function. Zeros of the function are **roots** of the **polynomial equation** when $f(x) = 0$. A polynomial equation of degree n has exactly n roots in the set of complex numbers.

Example 1 **State the degree and leading coefficient of the polynomial function $f(x) = 6x^5 + 8x^3 - 8x$. Then determine whether $\sqrt{\frac{2}{3}}$ is a zero of $f(x)$.**

$6x^5 + 8x^3 - 8x$ has a degree of 5 and a leading coefficient of 6.

Evaluate the function for $x = \sqrt{\frac{2}{3}}$. That is, find $f\left(\sqrt{\frac{2}{3}}\right)$.

$$f\left(\sqrt{\frac{2}{3}}\right) = 6\left(\sqrt{\frac{2}{3}}\right)^5 + 8\left(\sqrt{\frac{2}{3}}\right)^3 - 8\left(\sqrt{\frac{2}{3}}\right) \qquad x = \sqrt{\frac{2}{3}}$$

$$= \frac{24}{9}\sqrt{\frac{2}{3}} + \frac{16}{3}\sqrt{\frac{2}{3}} - 8\sqrt{\frac{2}{3}}$$

$$= 0$$

Since $f\left(\sqrt{\frac{2}{3}}\right) = 0$, $\sqrt{\frac{2}{3}}$ is a zero of $f(x) = 6x^5 + 8x^3 - 8x$.

Example 2 **Write a polynomial equation of least degree with roots 0, $\sqrt{2}\boldsymbol{i}$, and $-\sqrt{2}\boldsymbol{i}$.**

The linear factors for the polynomial are $x - 0$, $x - \sqrt{2}\boldsymbol{i}$, and $x + \sqrt{2}\boldsymbol{i}$. Find the products of these factors.

$$(x - 0)(x - \sqrt{2}\boldsymbol{i})(x + \sqrt{2}\boldsymbol{i}) = 0$$
$$x(x^2 - 2\boldsymbol{i}^2) = 0$$
$$x(x^2 + 2) = 0 \qquad -2\boldsymbol{i}^2 = -2(-1) \text{ or } 2$$
$$x^3 + 2x = 0$$

Example 3 **State the number of complex roots of the equation $3x^2 + 11x - 4 = 0$. Then find the roots.**

The polynomial has a degree of 2, so there are two complex roots. Factor the equation to find the roots.

$$3x^2 + 11x - 4 = 0$$
$$(3x - 1)(x + 4) = 0$$

To find each root, set each factor equal to zero.

$$3x - 1 = 0 \qquad x + 4 = 0$$
$$3x = 1 \qquad x = -4$$
$$x = \frac{1}{3}$$

The roots are -4 and $\frac{1}{3}$.

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Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial.

1. $6a^4 + a^3 - 2a$

2. $3p^2 - 7p^5 - 2p^3 + 5$

Write a polynomial equation of least degree for each set of roots.

3. $3, -0.5, 1$

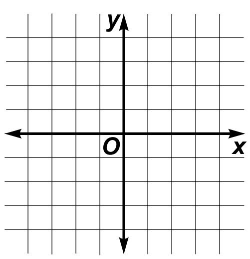
4. $3, 3, 1, 1, -2$

5. $\pm 2\boldsymbol{i}, 3, -3$

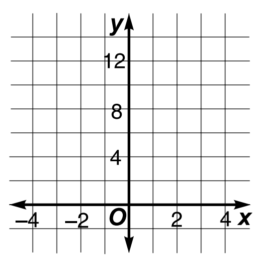
6. $-1, 3 \pm \boldsymbol{i}, 2 \pm 3\boldsymbol{i}$

State the number of complex roots of each equation. Then find the roots and graph the related function.

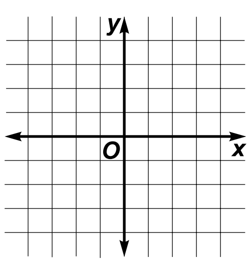
7. $3x - 5 = 0$



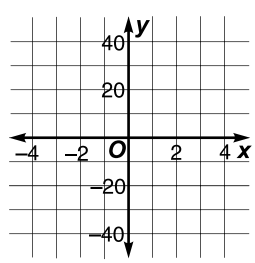
8. $x^2 + 4 = 0$



9. $c^2 + 2c + 1 = 0$



10. $x^3 + 2x^2 - 15x = 0$



11. ***Real Estate*** A developer wants to build homes on a rectangular plot of land 4 kilometers long and 3 kilometers wide. In this part of the city, regulations require a greenbelt of uniform width along two adjacent sides. The greenbelt must be 10 times the area of the development. Find the width of the greenbelt.

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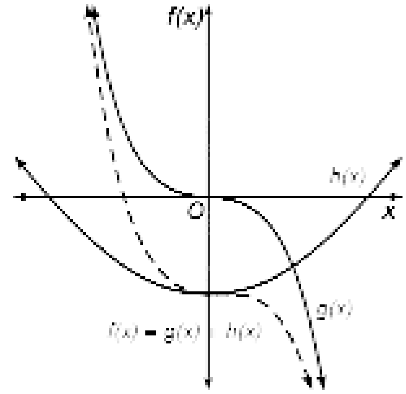
Enrichment

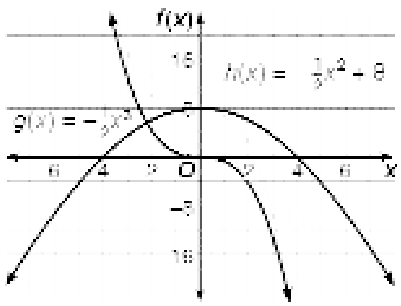
Graphic Addition

One way to sketch the graphs of some polynomial functions is to use *addition of ordinates*. This method is useful when a polynomial function $f(x)$ can be written as the sum of two other functions, $g(x)$ and $h(x)$, that are easier to graph. Then, each $f(x)$ can be found by mentally adding the corresponding $g(x)$ and $h(x)$. The graph at the right shows how to construct the graph of $f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 - 8$ from the graphs of $g(x) = -\frac{1}{2}x^3$ and $h(x) = \frac{1}{2}x^2 - 8$.



In each problem, the graphs of g(x) and h(x) are shown. Use addition of ordinates to graph a new polynomial function f(x), such that f(x) = g(x) + h(x). Then write the equation for f(x).

1.



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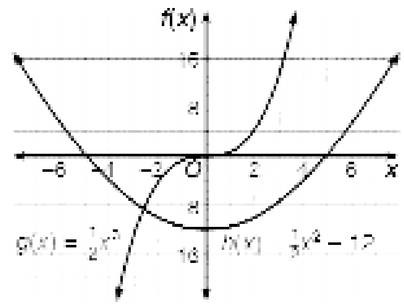

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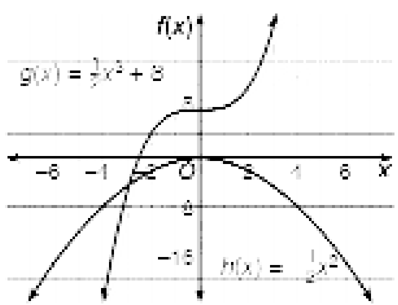

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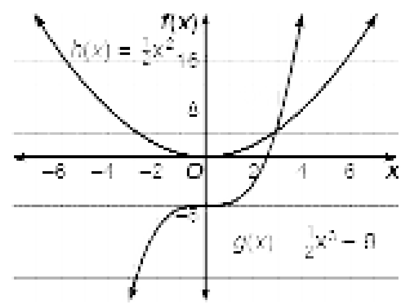

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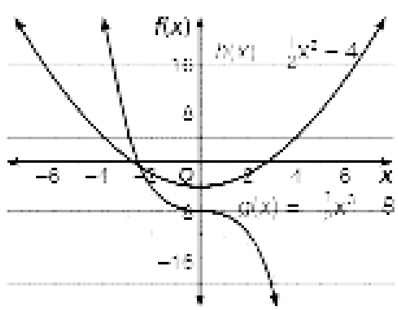

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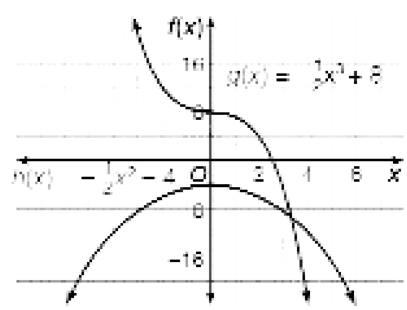

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4-2 Study Guide

Quadratic Equations

A quadratic equation is a polynomial equation with a degree of 2. Solving quadratic equations by graphing usually does not yield exact answers. Also, some quadratic expressions are not factorable. However, solutions can always be obtained by **completing the square.**

Example 1 **Solve $x^2 - 12x + 7 = 0$ by completing the square.**

$x^2 - 12x + 7 = 0$	
$x^2 - 12x = -7$	*Subtract 7 from each side.*
$x^2 - 12x + 36 = -7 + 36$	*Complete the square by adding* $\left[\frac{1}{2}(-12)\right]^2$, *or 36, to each side.*
$(x - 6)^2 = 29$	*Factor the perfect square trinomial.*
$x - 6 = \pm\sqrt{29}$	*Take the square root of each side.*
$x = 6 \pm \sqrt{29}$	*Add 6 to each side.*

The roots of the equation are $6 \pm \sqrt{29}$.

Completing the square can be used to develop a general formula for solving any quadratic equation of the form $ax^2 + bx + c = 0$. This formula is called the **Quadratic Formula** and can be used to find the roots of any quadratic equation.

Quadratic Formula	If $ax^2 + bx + c = 0$ with $a \neq 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the Quadratic Formula, the radicand $b^2 - 4ac$ is called the **discriminant** of the equation. The discriminant tells the nature of the roots of a quadratic equation or the zeros of the related quadratic function.

Example 2 **Find the discriminant of $2x^2 - 3x = 7$ and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.**

Rewrite the equation using the standard form $ax^2 + bx + c = 0$.

$2x^2 - 3x - 7 = 0$ *$a = 2$, $b = -3$, and $c = -7$*

The value of the discriminant $b^2 - 4ac$ is $(-3)^2 - 4(2)(-7)$, or 65.

Since the value of the discriminant is greater than zero, there are two distinct real roots.

Now substitute the coefficients into the quadratic formula and solve.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

The roots are $\frac{3 + \sqrt{65}}{4}$ and $\frac{3 - \sqrt{65}}{4}$.

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4-2 Practice

Quadratic Equations

Solve each equation by completing the square.

1. $x^2 - 5x - \frac{11}{4} = 0$

2. $-4x^2 - 11x = 7$

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

3. $x^2 + x - 6 = 0$

4. $4x^2 - 4x - 15 = 0$

5. $9x^2 - 12x + 4 = 0$

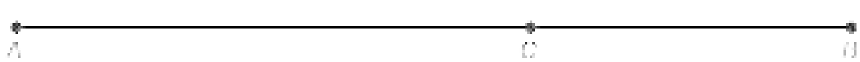
6. $3x^2 + 2x + 5 = 0$

Solve each equation.

7. $2x^2 + 5x - 12 = 0$

8. $5x^2 - 14x + 11 = 0$

9. ***Architecture*** The ancient Greek mathematicians thought that the most pleasing geometric forms, such as the ratio of the height to the width of a doorway, were created using the *golden section*. However, they were surprised to learn that the golden section is not a rational number. One way of expressing the golden section is by using a line segment. In the line segment shown, $\frac{AB}{AC} = \frac{AC}{CB}$. If $AC = 1$ unit, find the ratio $\frac{AB}{AC}$.



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Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$. We can define the absolute value of a complex number as follows.

$$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example **Show that $z^2 = z\bar{z}$ for any complex number z.**

Let $z = x + y\boldsymbol{i}$. Then,

$$\begin{aligned} z\bar{z} &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\ &= x^2 + y^2 \\ &= \left(\sqrt{x^2 + y^2}\right)^2 \\ &= |z|^2 \end{aligned}$$

Example **Show that $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.**

We know that $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$. Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. $2\boldsymbol{i}$

2. $-4 - 3\boldsymbol{i}$

3. $12 - 5\boldsymbol{i}$

4. $5 - 12\boldsymbol{i}$

5. $1 + \boldsymbol{i}$

6. $\sqrt{3} - \boldsymbol{i}$

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$

Study Guide

The Remainder and Factor Theorems

The Remainder Theorem	If a polynomial $P(x)$ is divided by $x - r$, the remainder is a constant $P(r)$, and $P(x) = (x - r) \cdot Q(x) + P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of $P(x)$.

Example 1 **Divide $x^4 - 5x^2 - 17x - 12$ by $x + 3$.**

$$
\begin{array}{rl}
 & x^3 - 3x^2 + 4x - 29 \\
x + 3 \,\big) & x^4 + 0x^3 - 5x^2 - 17x - 12 \\
 & \underline{x^4 + 3x^3} \\
 & -3x^3 - 5x^2 \\
 & \underline{-3x^3 - 9x^2} \\
 & 4x^2 - 17x \\
 & \underline{4x^2 + 12x} \\
 & -29x - 12 \\
 & \underline{-29x - 87} \\
 & 75 \leftarrow \textit{remainder}
\end{array}
$$

Find the value of r in this division.

$x - r = x + 3$
$-r = 3$
$r = -3$

According to the Remainder Theorem, $P(r)$ or $P(-3)$ should equal 75.

Use the Remainder Theorem to check the remainder found by long division.

$$P(x) = x^4 - 5x^2 - 17x - 12$$
$$P(-3) = (-3)^4 - 5(-3)^2 - 17(-3) - 12$$
$$= 81 - 45 + 51 - 12 \text{ or } 75$$

The Factor Theorem is a special case of the Remainder Theorem and can be used to quickly test for factors of a polynomial.

The Factor Theorem	The binomial $x - r$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.

Example 2 **Use the Remainder Theorem to find the remainder when $2x^3 + 5x^2 - 14x - 8$ is divided by $x - 2$. State whether the binomial is a factor of the polynomial. Explain.**

Find $f(2)$ to see if $x - 2$ is a factor.

$$f(x) = 2x^3 + 5x^2 - 14x - 8$$
$$f(2) = 2(2)^3 + 5(2)^2 - 14(2) - 8$$
$$= 16 + 20 - 28 - 8$$
$$= 0$$

Since $f(2) = 0$, the remainder is 0. So the binomial $x - 2$ is a factor of the polynomial by the Factor Theorem.

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Practice

The Remainder and Factor Theorems

Divide using synthetic division.

1. $(3x^2 + 4x - 12) \div (x + 5)$

2. $(x^2 - 5x - 12) \div (x - 3)$

3. $(x^4 - 3x^2 + 12) \div (x + 1)$

4. $(2x^3 + 3x^2 - 8x + 3) \div (x + 3)$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

5. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$

6. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$

7. $(3t^3 - 10t^2 + t - 5) \div (t - 4)$

8. $(10x^3 - 11x^2 - 47x + 30) \div (x + 2)$

9. $(x^4 + 5x^3 - 14x^2) \div (x - 2)$

10. $(2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$

11. $(y^3 + y^2 - 10) \div (y + 3)$

12. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$

13. Use synthetic division to find all the factors of $x^3 + 6x^2 - 9x - 54$ if one of the factors is $x - 3$.

14. ***Manufacturing*** A cylindrical chemical storage tank must have a height 4 meters greater than the radius of the top of the tank. Determine the radius of the top and the height of the tank if the tank must have a volume of 15.71 cubic meters.

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Enrichment

The Secret Cubic Equation

You might have supposed that there existed simple formulas for solving higher-degree equations. After all, there is a simple formula for solving quadratic equations. Might there not be formulas for cubics, quartics, and so forth?

There are formulas for some higher-degree equations, but they are certainly not “simple” formulas!

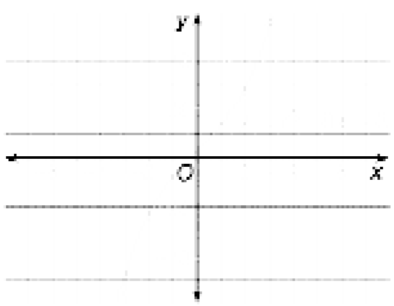
Here is a method for solving a reduced cubic of the form $x^3 + ax + b = 0$ published by Jerome Cardan in 1545. Cardan was given the formula by another mathematician, Tartaglia. Tartaglia made Cardan promise to keep the formula secret, but Cardan published it anyway. He did, however, give Tartaglia the credit for inventing the formula!

Let $R = \left(\frac{1}{2}b\right)^2 + \frac{a^3}{27}$

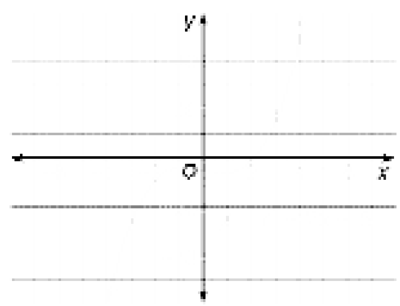
Then, $x = \left[-\frac{1}{2}b + \sqrt{R}\right]^{\frac{1}{3}} + \left[-\frac{1}{2}b - \sqrt{R}\right]^{\frac{1}{3}}$

Use Cardan’s method to find the real root of each cubic equation. Round answers to three decimal places. Then sketch a graph of the corresponding function on the grid provided.

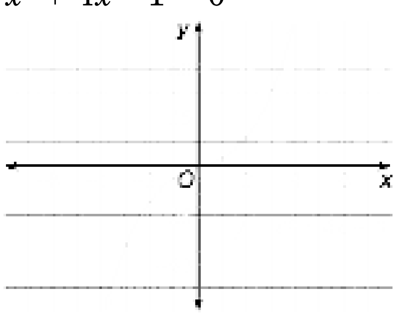
1. $x^3 + 8x + 3 = 0$



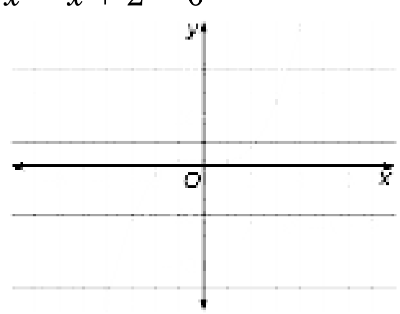
2. $x^3 - 2x - 5 = 0$



3. $x^3 + 4x - 1 = 0$



4. $x^3 - x + 2 = 0$



4-4

Study Guide

The Rational Root Theorem

The **Rational Root Theorem** provides a means of determining possible rational roots of an equation. **Descartes' Rule of Signs** can be used to determine the possible number of positive real zeros and the possible number of negative real zeros.

Rational Root Theorem	Let $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ represent a polynomial equation of degree n with integral coefficients. If a rational number $\frac{p}{q}$, where p and q have no common factors, is a root of the equation, then p is a factor of a_n and q is a factor of a_0.

Example 1 **List the possible rational roots of $x^3 - 5x^2 - 17x - 6 = 0$. Then determine the rational roots.**

p is a factor of 6 and q is a factor of 1
possible values of p: $\pm 1, \pm 2, \pm 3, \pm 6$
possible values of q: ± 1
possible rational roots, $\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6$

Test the possible roots using synthetic division.

r	1	−5	−17	−6
1	1	−4	−21	−27
−1	1	−6	−11	5
2	1	−3	−23	−52
−2	1	−7	−3	0
3	1	−2	−23	−75
−3	1	−8	7	−27
6	1	1	−11	−72
−6	1	−11	49	−300

← *There is a root at $x = -2$.*
The depressed polynomial is $x^2 - 7x - 3$. You can use the Quadratic Formula to find the two irrational roots.

Example 2 **Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = 4x^4 - 13x^3 - 21x^2 + 38x - 8$.**

According to Descartes' Rule of Signs, the number of positive real zeros is the same as the number of sign changes of the coefficients of the terms in descending order or is less than this by an even number. Count the sign changes.

$$f(x) = 4x^4 - 13x^3 - 21x^2 + 38x - 8$$
$$4 \quad -13 \quad -21 \quad 38 \quad -8$$

There are three changes. So, there are 3 or 1 positive real zeros.

The number of negative real zeros is the same as the number of sign changes of the coefficients of the terms of $f(-x)$, or less than this number by an even number.

$$f(-x) = 4(-x)^4 - 13(-x)^3 - 21(-x)^2 + 38(-x) - 8$$
$$4 \quad 13 \quad -21 \quad -38 \quad -8$$

There is one change. So, there is 1 negative real zero

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Practice

The Rational Root Theorem

List the possible rational roots of each equation. Then determine the rational roots.

1. $x^3 - x^2 - 8x + 12 = 0$

2. $2x^3 - 3x^2 - 2x + 3 = 0$

3. $36x^4 - 13x^2 + 1 = 0$

4. $x^3 + 3x^2 - 6x - 8 = 0$

5. $x^4 - 3x^3 - 11x^2 + 3x + 10 = 0$

6. $x^4 + x^2 - 2 = 0$

7. $3x^3 + x^2 - 8x + 6 = 0$

8. $x^3 + 4x^2 - 2x + 15 = 0$

Find the number of possible positive real zeros and the number of possible negative real zeros. Then determine the rational zeros.

9. $f(x) = x^3 - 2x^2 - 19x + 20$

10. $f(x) = x^4 + x^3 - 7x^2 - x + 6$

11. ***Driving*** An automobile moving at 12 meters per second on level ground begins to decelerate at a rate of -1.6 meters per second squared. The formula for the distance an object has traveled is $d(t) = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial velocity and a is the acceleration. For what value(s) of t does $d(t) = 40$ meters?

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4-4

Enrichment

Scrambled Proofs

The proofs on this page have been scrambled. Number the statements in each proof so that they are in a logical order.

The Remainder Theorem

☐ Thus, if a polynomial $f(x)$ is divided by $x - a$, the remainder is $f(a)$.

☐ In any problem of division the following relation holds:
dividend = quotient × divisor + remainder. In symbols, this may be written as:

☐ Equation (2) tells us that the remainder R is equal to the value $f(a)$; that is, $f(x)$ with a substituted for x.

☐ For $x = a$, Equation (1) becomes:

Equation (2) $\quad f(a) = R,$

since the first term on the right in Equation (1) becomes zero.

☐ Equation (1) $\quad f(x) = Q(x)(x - a) + R,$

in which $f(x)$ denotes the original polynomial, $Q(x)$ is the quotient, and R the constant remainder. Equation (1) is true for all values of x, and in particular, it is true if we set $x = a$.

The Rational Root Theorem

☐ Each term on the left side of Equation (2) contains the factor a; hence, a must be a factor of the term on the right, namely, $-c_nb^n$. But by hypothesis, a is not a factor of b unless $a = \pm 1$. Hence, a is a factor of c_n.

☐ $$f\left(\frac{a}{b}\right)^n = c_0\left(\frac{a}{b}\right)^n + c_1\left(\frac{a}{b}\right)^{n-1} + \ldots + c_{n-1}\left(\frac{a}{b}\right) + c_n = 0$$

☐ Thus, in the polynomial equation given in Equation (1), a is a factor of c_n and b is a factor of c_0.

☐ In the same way, we can show that b is a factor of c_0.

☐ A polynomial equation with integral coefficients of the form

Equation (1) $\quad f(x) = c_0x^n + c_1x^{n-1} + \ldots + c_{n-1}x + c_n = 0$

has a rational root $\frac{a}{b}$, where the fraction $\frac{a}{b}$ is reduced to lowest terms. Since $\frac{a}{b}$ is a root of $f(x) = 0$, then

☐ If each side of this equation is multiplied by b^n and the last term is transposed, it becomes

Equation (2) $\quad c_0a^n + c_1a^{n-1}b + \ldots + c_{n-1}ab^{n-1} = -c_nb^n$

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Study Guide

Locating Zeros of a Polynomial Function

A polynomial function may have real zeros that are not rational numbers. The **Location Principle** provides a means of locating and approximating real zeros. For the polynomial function $y = f(x)$, if a and b are two numbers with $f(a)$ positive and $f(b)$ negative, then there must be at least one real zero between a and b. For example, if $f(x) = x^2 - 2$, $f(0) = -2$ and $f(2) = 2$. Thus, a zero exists somewhere between 0 and 2.

The **Upper Bound Theorem** and the **Lower Bound Theorem** are also useful in locating the zeros of a function and in determining whether all the zeros have been found. If a polynomial function $P(x)$ is divided by $x - c$, and the quotient and the remainder have no change in sign, c is an **upper bound** of the zeros of $P(x)$. If c is an upper bound of the zeros of $P(-x)$, then $-c$ is a **lower bound** of the zeros of $P(x)$.

Example 1 **Determine between which consecutive integers the real zeros of $f(x) = x^3 - 2x^2 - 4x + 5$ are located.**

According to Descartes' Rule of Signs, there are two or zero positive real roots and one negative real root. Use synthetic division to evaluate $f(x)$ for consecutive integral values of x.

r	1	−2	−4	5
−4	1	−6	20	−75
−3	1	−5	11	−28
−2	1	−4	4	−3
−1	1	−3	−1	6
0	1	−2	−4	5
1	1	−1	−5	0
2	1	0	−4	−3
3	1	1	−1	2

There is a zero at 1. The changes in sign indicate that there are also zeros between −2 and −1 and between 2 and 3. This result is consistent with Descartes' Rule of Signs.

Example 2 **Use the Upper Bound Theorem to show that 3 is an upper bound and the Lower Bound Theorem to show that −2 is a lower bound of the zeros of $f(x) = x^3 - 3x^2 + x - 1$.**

Synthetic division is the most efficient way to test potential upper and lower bounds. First, test for the upper bound.

r	1	−3	1	−1
3	1	0	1	2

Since there is no change in the signs in the quotient and remainder, 3 is an upper bound.

Now, test for the lower bound of $f(x)$ by showing that 2 is an upper bound of $f(-x)$.

$$f(-x) = (-x)^3 - 3(-x)^2 + (-x) - 1 = -x^3 - 3x^2 - x - 1$$

r	−1	−3	−1	−1
2	−1	−5	−11	−23

Since there is no change in the signs, −2 is a lower bound of $f(x)$.

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Practice

Locating Zeros of a Polynomial Function

Determine between which consecutive integers the real zeros of each function are located.

1. $f(x) = 3x^3 - 10x^2 + 22x - 4$

2. $f(x) = 2x^3 + 5x^2 - 7x - 3$

3. $f(x) = 2x^3 - 13x^2 + 14x - 4$

4. $f(x) = x^3 - 12x^2 + 17x - 9$

5. $f(x) = 4x^4 - 16x^3 - 25x^2 + 196x - 146$

6. $f(x) = x^3 - 9$

Approximate the real zeros of each function to the nearest tenth.

7. $f(x) = 3x^4 + 4x^2 - 1$

8. $f(x) = 3x^3 - x + 2$

9. $f(x) = 4x^4 - 6x^2 + 1$

10. $f(x) = 2x^3 + x^2 - 1$

11. $f(x) = x^3 - 2x^2 - 2x + 3$

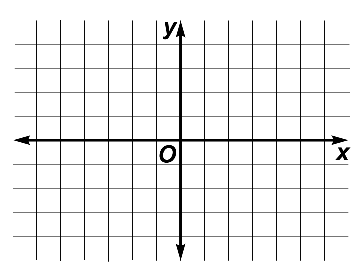
12. $f(x) = x^3 - 5x^2 + 4$

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

13. $f(x) = 3x^4 - x^3 - 8x^2 - 3x - 20$

14. $f(x) = 2x^3 - x^2 + x - 6$

15. For $f(x) = x^3 - 3x^2$, determine the number and type of possible complex zeros. Use the Location Principle to determine the zeros to the nearest tenth. The graph has a relative maximum at $(0, 0)$ and a relative minimum at $(2, -4)$. Sketch the graph.



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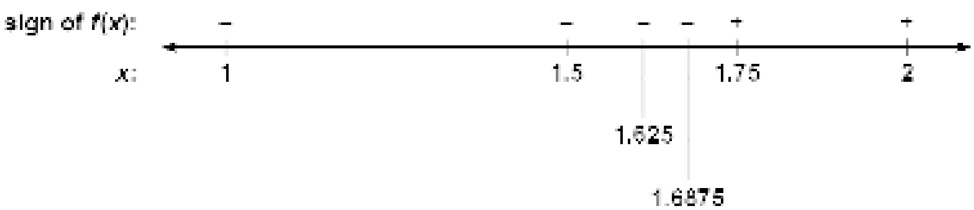
Enrichment

The Bisection Method for Approximating Real Zeros

The bisection method can be used to approximate zeros of polynomial functions like $f(x) = x^3 + x^2 - 3x - 3$. Since $f(1) = -4$ and $f(2) = 3$, there is at least one real zero between 1 and 2. The midpoint of this interval is $\frac{1+2}{2} = 1.5$. Since $f(1.5) = -1.875$, the zero is between 1.5 and 2. The midpoint of this interval is $\frac{1.5+2}{2} = 1.75$. Since $f(1.75) = 0.172$, the zero is between 1.5 and 1.75. $\frac{1.5+1.75}{2} = 1.625$ and $f(1.625) = -0.94$. The zero is between 1.625 and 1.75. The midpoint of this interval is $\frac{1.625+1.75}{2} = 1.6875$. Since $f(1.6875) = -0.41$, the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth. The diagram below summarizes the bisection method.



Using the bisection method, approximate to the nearest tenth the zero between the two integral values of each function.

1. $f(x) = x^3 - 4x^2 - 11x + 2, f(0) = 2, f(1) = -12$

2. $f(x) = 2x^4 + x^2 - 15, f(1) = -12, f(2) = 21$

3. $f(x) = x^5 - 2x^3 - 12, f(1) = -13, f(2) = 4$

4. $f(x) = 4x^3 - 2x + 7, f(-2) = -21, f(-1) = 5$

5. $f(x) = 3x^3 - 14x^2 - 27x + 126, f(4) = -14, f(5) = 16$

4-6

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Study Guide

Rational Equations and Partial Fractions

A **rational equation** consists of one or more rational expressions. One way to solve a rational equation is to multiply each side of the equation by the least common denominator (LCD). Any possible solution that results in a zero in the denominator must be excluded from your list of solutions. In order to find the LCD, it is sometimes necessary to factor the denominators. If a denominator can be factored, the expression can be rewritten as the sum of **partial fractions.**

Example 1 **Solve $\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$.**

$$6(x-2)\left[\frac{x+1}{3(x-2)}\right] = 6(x-2)\left(\frac{5x}{6} + \frac{1}{x-2}\right)$$ *Multiply each side by the LCD, 6(x − 2).*

$$2(x+1) = (x-2)(5x) + 6(1)$$

$$2x + 2 = 5x^2 - 10x + 6$$ *Simplify.*

$$5x^2 - 12x + 4 = 0$$ *Write in standard form.*

$$(5x-2)(x-2) = 0$$ *Factor.*

$$5x - 2 = 0 \qquad x - 2 = 0$$

$$x = \frac{2}{5} \qquad x = 2$$

Since x cannot equal 2 because a zero denominator results, the only solution is $\frac{2}{5}$.

Example 2 **Decompose $\frac{2x-1}{x^2+2x-3}$ into partial fractions.**

Factor the denominator and express the factored form as the sum of two fractions using A and B as numerators and the factors as denominators.

$$x^2 + 2x - 3 = (x-1)(x+3)$$

$$\frac{2x-1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$2x - 1 = A(x+3) + B(x-1)$$

Let $x = 1$.

$$2(1) - 1 = A(1+3)$$

$$1 = 4A$$

$$A = \frac{1}{4}$$

Let $x = -3$.

$$2(-3) - 1 = B(-3-1)$$

$$-7 = -4B$$

$$B = \frac{7}{4}$$

$$\frac{2x-1}{x^2+2x-3} = \frac{\frac{1}{4}}{x-1} + \frac{\frac{7}{4}}{x+3} \text{ or } \frac{1}{4(x-1)} + \frac{7}{4(x+3)}$$

Example 3 **Solve $\frac{1}{2t} + \frac{3}{4t} > 1$.**

Rewrite the inequality as the related function $f(t) = \frac{1}{2t} + \frac{3}{4t} - 1$.

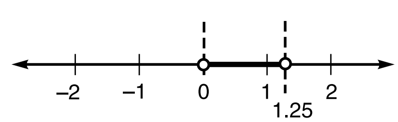
Find the zeros of this function.

$$4t\left(\frac{1}{2t}\right) + 4t\left(\frac{3}{4t}\right) - 4t(1) = 4t(0)$$

$$5 - 4t = 0$$

$$t = 1.25$$

The zero is 1.25. The excluded value is 0. On a number line, mark these values with vertical dashed lines. Testing each interval shows the solution set to be $0 < t < 1.25$.



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Practice

Rational Equations and Partial Fractions

Solve each equation.

1. $\frac{15}{m} - m + 8 = 10$

2. $\frac{4}{b-3} + \frac{3}{b} = \frac{-2b}{b-3}$

3. $\frac{1}{2n} + \frac{6n-9}{3n} = \frac{2}{n}$

4. $t - \frac{4}{t} = 3$

5. $\frac{3a}{2a+1} - \frac{4}{2a-1} = 1$

6. $\frac{2p}{p+1} + \frac{3}{p-1} = \frac{15-p}{p^2-1}$

Decompose each expression into partial fractions.

7. $\frac{-3x-29}{x^2-4x-21}$

8. $\frac{11x-7}{2x^2-3x-2}$

Solve each inequality.

9. $\frac{6}{t} + 3 > \frac{2}{t}$

10. $\frac{2n+1}{3n+1} \leq \frac{n-1}{3n+1}$

11. $1 + \frac{3y}{1-y} > 2$

12. $\frac{2x}{4} - \frac{5x+1}{3} > 3$

13. ***Commuting*** Rosea drives her car 30 kilometers to the train station, where she boards a train to complete her trip. The total trip is 120 kilometers. The average speed of the train is 20 kilometers per hour faster than that of the car. At what speed must she drive her car if the total time for the trip is less than 2.5 hours?

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4-6 Enrichment

Inverses of Conditional Statements

In the study of formal logic, the compound statement "*if p*, *then q*" where p and q represent any statements, is called a *conditional* or an *implication*. The symbolic representation of a conditional is

$$p \rightarrow q.$$

$$\underbrace{\text{If the determinant of a } 2 \times 2 \text{ matrix is 0,}}_{p} \text{ then } \underbrace{\text{the matrix does not have an inverse.}}_{q}$$

If both p and q are negated, the resulting compound statement is called the **inverse** of the original conditional. The symbolic notation for the negation of p is $\sim p$.

Conditional	**Inverse**	
$p \rightarrow q$	$\sim p \rightarrow \sim q$	*If a conditional is true, its inverse may be either true or false.*

Example **Find the inverse of each conditional.**

a. $\boldsymbol{p \rightarrow q}$**: If today is Monday, then tomorrow is Tuesday.** (true)

$\sim p \rightarrow \sim q$ If today is not Monday, then tomorrow is not Tuesday. (true)

b. $\boldsymbol{p \rightarrow q}$ **If *ABCD* is a square, then *ABCD* is a rhombus.** (true)

$\sim p \rightarrow \sim q$ If *ABCD* is not a square, then *ABCD* is not a rhombus. (false)

Write the inverse of each conditional.

1. $q \rightarrow p$ **2.** $\sim p \rightarrow q$ **3.** $\sim q \rightarrow \sim p$

4. If the base angles of a triangle are congruent, then the triangle is isosceles.

5. If the moon is full tonight, then we'll have frost by morning.

Tell whether each conditional is true or false. Then write the inverse of the conditional and tell whether the inverse is true or false.

6. If this is October, then the next month is December.

7. If $x > 5$, then $x > 6$, $x \in R$.

8. If $x = 0$, then $x^{\frac{1}{2}} = 0$, $x \in R$.

9. Make a conjecture about the truth value of an inverse if the conditional is false.

Study Guide

Radical Equations and Inequalities

Equations in which radical expressions include variables are known as **radical equations.** To solve radical equations, first isolate the radical on one side of the equation. Then raise each side of the equation to the proper power to eliminate the radical expression. This process of raising each side of an equation to a power often introduces **extraneous solutions.** Therefore, it is important to check all possible solutions in the original equation to determine if any of them should be eliminated from the solution set. **Radical inequalities** are solved using the same techniques used for solving radical equations.

Example 1 **Solve $3 = \sqrt[3]{x^2 - 2x + 1} - 1$.**

$3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$4 = \sqrt[3]{x^2 - 2x + 1}$ *Isolate the cube root.*

$64 = x^2 - 2x + 1$ *Cube each side.*

$0 = x^2 - 2x - 63$

$0 = (x - 9)(x + 7)$ *Factor.*

$x - 9 = 0 \qquad x + 7 = 0$

$x = 9 \qquad x = -7$

Check both solutions to make sure they are not extraneous.

$\boldsymbol{x = 9}$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(9)^2 - 2(9) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

$\boldsymbol{x = -7}$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(-7)^2 - 2(-7) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

Example 2 **Solve $2\sqrt{3x + 5} > 2$.**

$2\sqrt{3x + 5} > 2$

$4(3x + 5) > 4$ *Square each side.*

$3x + 5 > 1$ *Divide each side by 4.*

$3x > -4$

$x > -1.33$

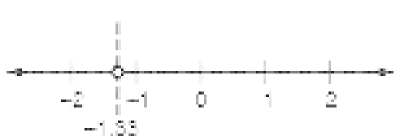
In order for $\sqrt{3x + 5}$ to be a real number, $3x + 5$ must be greater than or equal to zero.

$3x + 5 \geq 0$

$3x \geq -5$

$x \geq -1.67$

Since -1.33 is greater than -1.67, the solution is $x > -1.33$. Check this solution by testing values in the intervals defined by the solution. Then graph the solution on a number line.



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4-7

Practice

Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x-2} = 6$

2. $\sqrt[3]{x^2 - 1} = 3$

3. $\sqrt[3]{7r+5} = -3$

4. $\sqrt{6x+12} - \sqrt{4x+9} = 1$

5. $\sqrt{x-3} - 3\sqrt{x+12} = -11$

6. $\sqrt{6n-3} = \sqrt{4+7n}$

7. $5 + 2x = \sqrt{x^2 - 2x + 1}$

8. $3 - \sqrt{r+1} = \sqrt{4-r}$

Solve each inequality.

9. $\sqrt{3r+5} > 1$

10. $\sqrt{2t-3} < 5$

11. $\sqrt{2m+3} > 5$

12. $\sqrt{3x+5} < 9$

13. ***Engineering*** A team of engineers must design a fuel tank in the shape of a cone. The surface area of a cone (excluding the base) is given by the formula $S = \pi\sqrt{r^2 + h^2}$. Find the radius of a cone with a height of 21 meters and a surface area of 155 meters squared.

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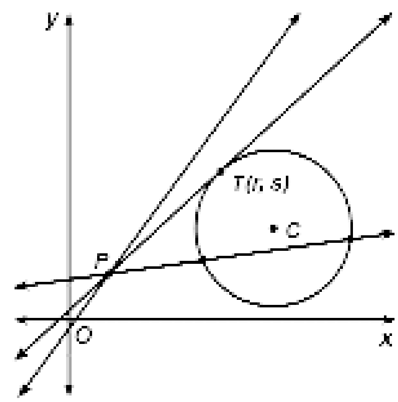
Enrichment

Discriminants and Tangents



The diagram at the right shows that through a point P outside of a circle C, there are lines that do not intersect the circle, lines that intersect the circle in one point (tangents), and lines that intersect the circle in two points (secants).

Given the coordinates for P and an equation for the circle C, how can we find the equation of a line tangent to C that passes through P?

Suppose P has coordinates $P(0, 0)$ and $\odot C$ has equation $(x - 4)^2 + y^2 = 4$. Then a line tangent through P has equation $y = mx$ for some real number m.

Thus, if $T(r, s)$ is a point of tangency, then $s = mr$ and $(r - 4)^2 + s^2 = 4$.
Therefore, $(r - 4)^2 + (mr)^2 = 4$.

$$r^2 - 8r + 16 + m^2r^2 = 4$$
$$(1 + m^2)r^2 - 8r + 16 = 4$$
$$(1 + m^2)r^2 - 8r + 12 = 0$$

The equation above has exactly one real solution for r if the discriminant is 0, that is, when $(-8)^2 - 4(1 + m^2)(12) = 0$. Solve this equation for m and you will find the slopes of the lines through P that are tangent to circle C.

1. a. Refer to the discussion above. Solve $(-8)^2 - 4(1 + m^2)(12) = 0$ to find the slopes of the two lines tangent to circle C through point P.

b. Use the values of m from part **a** to find the coordinates of the two points of tangency.

2. Suppose P has coordinates (0, 0) and circle C has equation $(x + 9)^2 + y^2 = 9$. Let m be the slope of the tangent line to C through P.

a. Find the equations for the lines tangent to circle C through point P.

b. Find the coordinates of the points of tangency.

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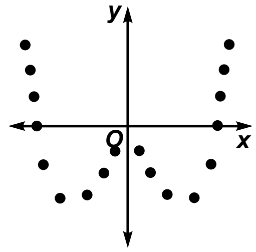
4-8 Study Guide

Modeling Real-World Data with Polynomial Functions

In order to model real-world data using polynomial functions, you must be able to identify the general shape of the graph of each type of polynomial function.

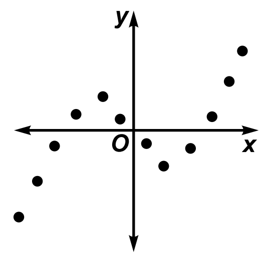
Example 1 **Determine the type of polynomial function that could be used to represent the data in each scatter plot.**

a.



The scatter plot seems to change direction three times, so a quartic function would best fit the scatter plot.

b.

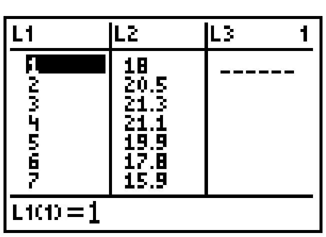


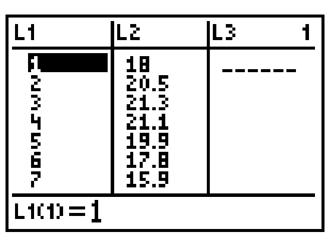
The scatter plot seems to change direction two times, so a cubic function would best fit the scatter plot.

Example 2 **An oil tanker collides with another ship and starts leaking oil. The Coast Guard measures the rate of flow of oil from the tanker and obtains the data shown in the table. Use a graphing calculator to write a polynomial function to model the set of data.**

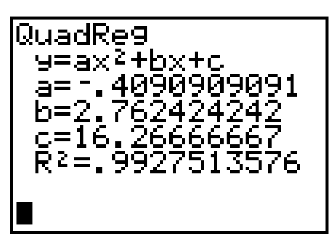
Clear the statistical memory and input the data. Adjust the window to an appropriate setting and graph the statistical data. The data appear to change direction one time, so a quadratic function will fit the scatter plot. Press STAT, highlight **CALC**, and choose **5:QuadReg**. Then enter 2nd **[L1]** , 2nd **[L2]** ENTER. Rounding the coefficients to the nearest tenth, $f(x) = -0.4x^2 + 2.8x + 16.3$ models the data. Since the value of the coefficient of determination r^2 is very close to 1, the polynomial is an excellent fit.

Time (hours)	Flow rate (100s of liters per hour)
1	18.0
2	20.5
3	21.3
4	21.1
5	19.9
6	17.8
7	15.9
8	11.3
9	7.6
10	3.7





[0, 10] scl: 1 by [0.25] scl: 5



4-8

NAME ______________________________ DATE ____________________ PERIOD __________

Practice

Modeling Real-World Data with Polynomial Functions

Write a polynomial function to model each set of data.

1. The farther a planet is from the Sun, the longer it takes to complete an orbit.

Distance (AU)	0.39	0.72	1.00	1.49	5.19	9.51	19.1	30.0	39.3
Period (days)	88	225	365	687	4344	10,775	30,681	60,267	90,582

Source: *Astronomy: Fundamentals and Frontiers*, by Jastrow, Robert, and Malcolm H. Thompson.

2. The amount of food energy produced by farms increases as more energy is expended. The following table shows the amount of energy produced and the amount of energy expended to produce the food.

Energy Input (Calories)	606	970	1121	1227	1318	1455	1636	2030	2182	2242
Energy Output (Calories)	133	144	148	157	171	175	187	193	198	198

Source: *NSTA Energy-Environment Source Book.*

3. The temperature of Earth's atmosphere varies with altitude.

Altitude (km)	0	10	20	30	40	50	60	70	80	90
Temperature (K)	293	228	217	235	254	269	244	207	178	178

Source: *Living in the Environment*, by Miller G. Tyler.

4. Water quality varies with the season. This table shows the average hardness (amount of dissolved minerals) of water in the Missouri River measured at Kansas City, Missouri.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hardness ($CaCO_3$ ppm)	310	250	180	175	230	175	170	180	210	230	295	300

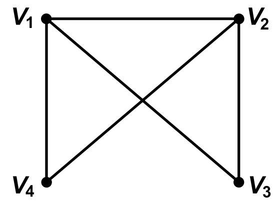
Source: *The Encyclopedia of Environmental Science*, 1974.

NAME ______________________ DATE ______________ PERIOD ________

Enrichment

Number of Paths

For the figure and adjacency matrix shown at the right, the number of paths or circuits of length 2 can be found by computing the following product.



$$A = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$A^2 = AA = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

In row 3 column 4, the entry 2 in the product matrix means that there are 2 paths of length 2 between V_3 and V_4. The paths are $V_3 \rightarrow V_1 \rightarrow V_4$ and $V_3 \rightarrow V_2 \rightarrow V_4$. Similarly, in row 1 column 3, the entry 1 means there is only 1 path of length 2 between V_1 and V_3.

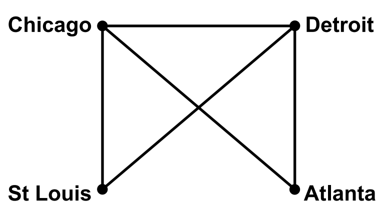
Name the paths of length 2 between the following.

1. V_1 and V_2

2. V_1 and V_3

3. V_1 and V_1

For Exercises 4-6, refer to the figure below.



4. The number of paths of length 3 is given by the product $A \cdot A \cdot A$ or A^3. Find the matrix for paths of length 3.

5. How many paths of length 3 are there between Atlanta and St. Louis? Name them.

6. How would you find the number of paths of length 4 between the cities?

Chapter 4

NAME ______________ DATE ______________ PERIOD ______________

Chapter 4 Test, Form 1A

Write the letter for the correct answer in the blank at the right of each problem.

1. Use the Remainder Theorem to find the remainder when $16x^5 - 32x^4 - 81x + 162$ is divided by $x - 2$. State whether the binomial is a factor of the polynomial. **1.** ________

A. 1348; no **B.** 0; yes **C.** -700; yes **D.** 0; no

2. Solve $3t^2 - 24t = -30$ by completing the square. **2.** ________

A. $4 \pm \sqrt{6}$ **B.** $10, -2$ **C.** $10, 22$ **D.** $4 \pm \sqrt{14}$

3. Use synthetic division to divide $8x^4 - 20x^3 - 14x^2 + 8x + 1$ by $x + 1$. **3.** ________

A. $8x^3 + 28x^2 - 14x$ R11 **B.** $8x^3 - 28x^2 + 14x - 6$ R7

C. $8x^3 + 36x^2 + 18x + 10$ R9 **D.** $8x^3 + 28x^2 - 14x + 8$

4. Solve $\sqrt[3]{x + 2} = \sqrt[6]{9x + 10}$. **4.** ________

A. $-1, 6$ **B.** $\frac{-13 \pm \sqrt{193}}{2}$ **C.** $1, -6$ **D.** $\frac{13 \pm \sqrt{145}}{2}$

5. List the possible rational roots of $2x^3 + 17x^2 + 23x - 42 = 0$. **5.** ________

A. $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$

B. $\pm 1, \pm 2, \pm 6, \pm 7, \pm 21, \pm 42, \pm\frac{1}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$

C. $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm\frac{2}{3}, \pm\frac{7}{2}, \pm\frac{21}{2}$

D. $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm\frac{1}{2}, \pm\frac{2}{7}, \pm\frac{21}{2}$

6. Determine the rational roots of $6x^3 - 25x^2 + 2x + 8 = 0$. **6.** ________

A. $\frac{1}{2}, \frac{2}{3}, -4$ **B.** $-\frac{1}{2}, \frac{2}{3}, 4$ **C.** $-\frac{2}{3}, \frac{1}{2}, 4$ **D.** $-\frac{2}{3}, -\frac{1}{2}, 4$

7. Solve $\frac{2x - 5}{x} + \frac{4x - 1}{x + 2} = -\frac{3x + 8}{x^2 + 2x}$. **7.** ________

A. $\frac{-1 \pm \sqrt{433}}{12}$ **B.** $2, -\frac{3}{2}$ **C.** $-\frac{2}{9}$ **D.** $-\frac{2}{3}, \frac{1}{2}$

8. Find the discriminant of $2x^2 + 9 = 4x$ and describe the nature of the roots of the equation. **8.** ________

A. 56; exactly one real root **B.** 56; two distinct real roots

C. -56; no real roots **D.** -56; two distinct real roots

9. Solve $- 3x^2 + 4 = 0$ by using the Quadratic Formula. **9.** ________

A. $\frac{2 \pm 2\boldsymbol{i}}{3}$ **B.** $\pm\frac{2\sqrt{3}}{3}$ **C.** $\pm\frac{\sqrt{3}}{6}$ **D.** $0, \frac{4}{3}$

10. Determine between which consecutive integers one or more real zeros of $f(x) = 3x^4 + x^3 - 2x^2 + 4$ are located. **10.** ________

A. no real zeros **B.** 0 and 1

C. -2 and -3 **D.** -1 and 0

Chapter 4 Test, Form 1A (continued)

11. Find the value of k so that the remainder of $(-kx^4 - 146x^2 + 32) \div (x - 4)$ is 0. **11.** ________

A. $\frac{37}{4}$ **B.** 9 **C.** $-\frac{37}{4}$ **D.** -9

12. Find the number of possible negative real zeros for $f(x) = 6 + x^4 + 2x^2 - 5x^3 - 12x$. **12.** ________

A. 2 or 0 **B.** 3 or 1 **C.** 0 **D.** 1

13. Solve $\sqrt{7x + 2} < 4$. **13.** ________

A. $x > 1$ **B.** $x > 2$ **C.** $x < 2$ **D.** $-\frac{2}{7} \le x < 2$

14. Decompose $\frac{5x + 1}{x^2 - x - 12}$ into partial fractions. **14.** ________

A. $\frac{2}{x + 3} + \frac{3}{x - 4}$ **B.** $\frac{3}{x + 3} + \frac{2}{x - 4}$

C. $\frac{3}{x + 3} - \frac{2}{x - 4}$ **D.** $\frac{2}{x + 3} - \frac{3}{x - 4}$

15. Solve $\frac{2}{5x} + \frac{3x - 4}{2x} > \frac{x - 2}{3x}$. **15.** ________

A. $0 < x \le \frac{4}{5}$ **B.** $0 \le x \le \frac{4}{5}$ **C.** $x < 0, x > \frac{4}{5}$ **D.** $x \le 0, x \ge \frac{4}{5}$

16. Approximate the real zeros of $f(x) = 2x^4 - 3x^2 - 2$ to the nearest tenth. **16.** ________

A. -2 **B.** ± 1.4 **C.** ± 1.5 **D.** no real zeros

17. Solve $\sqrt{x + 4} = \sqrt{x} + \sqrt{2}$. **17.** ________

A. -2 **B.** $-\frac{1}{2}$ **C.** 2 **D.** $\frac{1}{2}$

18. Which polynomial function best models the set of data below? **18.** ________

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
f(*x*)	4	0	0	0	0	1	1	0	0	1	4

A. $y = 0.02x^4 - 0.25x^2 + 0.11x + 0.84$

B. $y = 0.2x^4 + 0.25x^2 + 0.11x + 0.84$

C. $y = 0.2x^4 - 0.25x^2 + 0.11x + 0.84$

D. $y = 0.02x^4 + 0.25x^2 + 0.11x + 0.84$

19. Solve $\frac{6}{x - 4} + 2 > \frac{1}{3}$. **19.** ________

A. $x > 4$ **B.** $\frac{2}{5} < x < 4$ **C.** $x < \frac{2}{5}, x > 4$ **D.** $x > \frac{2}{5}$

20. Find the polynomial equation of least degree with roots -4, $2\boldsymbol{i}$, and $-2\boldsymbol{i}$. **20.** ________

A. $x^3 - 4x^2 + 4x - 16 = 0$ **B.** $x^3 + 4x^2 - 4x - 16 = 0$

C. $x^3 + 4x^2 + 4x + 16 = 0$ **D.** $x^3 - 4x^2 - 4x + 16 = 0$

Bonus Find the discriminant of $(2 + \sqrt{3})x^2 - (4 - \sqrt{3})x = 1$. **Bonus:** ________

A. $11 - 12\sqrt{3}$ **B.** $-16 - 9\sqrt{3}$ **C.** $-2 + 2\sqrt{3}$ **D.** $27 - 4\sqrt{3}$

Chapter 4

NAME ______________________ DATE ______________ PERIOD ________

Chapter 4 Test, Form 1B

Write the letter for the correct answer in the blank at the right of each problem.

1. Use the Remainder Theorem to find the remainder when $2x^3 + 6x^2 + 3x - 1$ is divided by $x - 1$. State whether the binomial is a factor of the polynomial. **1.** ________

A. 0; yes **B.** -2; no **C.** 10; no **D.** -1; yes

2. Solve $x^2 - 20x = 8$ by completing the square. **2.** ________

A. $5 \pm \sqrt{23}$ **B.** $5 \pm 3\sqrt{3}$ **C.** $10 \pm 2\sqrt{23}$ **D.** $10 \pm 6\sqrt{3}$

3. Use synthetic division to divide $x^3 + 5x^2 + 5x - 2$ by $x + 2$. **3.** ________

A. $x^2 + 7x + 19$ R36 **B.** $x^2 + 3x - 1$
C. $x^2 + 4$ **D.** $x^2 + 7x - 9$ R16

4. Solve $\sqrt[3]{2x + 1} - 4 = -1$. **4.** ________

A. 14 **B.** 13 **C.** -14 **D.** -13

5. List the possible rational roots of $4x^3 + 5x^2 - x + 2 = 0$. **5.** ________

A. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2$ **B.** $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 4$
C. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$ **D.** $\pm 1, \pm\frac{1}{4}, \pm 2$

6. Determine the rational roots of $x^3 + 4x^2 + 6x + 9 = 0$. **6.** ________

A. -3 **B.** ± 3 **C.** 3 **D.** $\pm 3, 9$

7. Solve $\frac{x - 2}{x + 1} + \frac{2x - 7}{x - 6} = \frac{x - 3}{x^2 - 5x - 6}$. **7.** ________

A. $-2 \pm 3\sqrt{3}$ **B.** $\frac{2}{3}, 4$ **C.** $-2 \pm 6\sqrt{3}$ **D.** $-\frac{2}{3}, -4$

8. Find the discriminant of $5x^2 - 8x - 3 = 0$ and describe the nature of the roots of the equation. **8.** ________

A. 4; two distinct real roots **B.** 0; exactly one real root
C. -76; no real roots **D.** 124; two distinct real roots

9. Solve $-3x^2 + 4x - 4 = 0$ by using the Quadratic Formula. **9.** ________

A. $\frac{-2 \pm 2\boldsymbol{i}\sqrt{2}}{3}$ **B.** $\frac{2}{3} \pm 4\boldsymbol{i}\sqrt{2}$ **C.** $\frac{2 \pm 2\boldsymbol{i}\sqrt{2}}{3}$ **D.** $2, -6$

10. Determine between which consecutive integers one or more real zeros of $f(x) = -x^3 + 2x^2 - x - 5$ are located. **10.** ________

A. 0 and 1 **B.** 1 and 2 **C.** -2 and -1 **D.** at -5

11. Find the value of k so that the remainder of $(x^3 + 3x^2 - kx - 24) \div (x + 4)$ is 0. **11.** ________

A. -22 **B.** -10 **C.** 22 **D.** 10

Chapter 4 Test, Form 1B (continued)

12. Find the number of possible negative real zeros for $f(x) = x^3 - 4x^2 - 3x - 9$. **12.** ________

A. 2 or 0 **B.** 3 or 1 **C.** 1 **D.** 0

13. Solve $\sqrt{x + 2} - 2 \geq 7$. **13.** ________

A. $-2 \leq x \leq 79$ **B.** $x \leq -2, x \geq 79$
C. $x \geq -2$ **D.** $x \geq 79$

14. Decompose $\frac{-2x - 23}{2x^2 - 9x - 5}$ into partial fractions. **14.** ________

A. $\frac{4}{2x + 1} - \frac{3}{x - 5}$ **B.** $\frac{4}{2x + 1} + \frac{3}{x - 5}$
C. $\frac{3}{x - 5} - \frac{4}{2x + 1}$ **D.** $\frac{-3}{2x + 1} + \frac{4}{x - 5}$

15. Solve $\frac{3}{x^2 + 3x} + \frac{x + 2}{x + 3} < \frac{1}{x}$. **15.** ________

A. $-3 < x < -1$ **B.** $-1 < x < 0$ **C.** $-3 < x < 0$ **D.** $x > -3$

16. Approximate the real zeros of $f(x) = x^3 - 5x^2 + 2$ to the nearest tenth. **16.** ________

A. $-0.5, 0.7, 4.9$ **B.** $-0.6, 0.7, 4.9$
C. ± 0.6 **D.** $-0.6, 0.7, 5.0$

17. Solve $5 + \sqrt{x + 2} = 8 + \sqrt{x - 7}$. **17.** ________

A. 7 **B.** 0 **C.** -7 **D.** 21

18. Which polynomial function best models the set of data below? **18.** ________

x	−3	−2	−1	0	1	2	3	4	5	6	7
f(x)	−60	−10	10	15	10	0	−5	0	15	50	100

A. $y = 0.9x^3 + 4.9x^2 + 0.5x + 14.4$
B. $y = 0.9x^3 - 4.9x^2 + 0.5x + 14.4$
C. $y = 1.0x^3 - 4.9x^2 - 0.9x + 17.5$
D. $y = 0.9x^3 - 4.8x^2 + 1.2x + 15.0$

19. Solve $\frac{2}{x} > \frac{-1}{x - 1}$. **19.** ________

A. $x > 0$ **B.** $x < \frac{2}{3}, x > 1$ **C.** $0 < x < 1$ **D.** $0 < x < \frac{2}{3}, x > 1$

20. Find the polynomial equation of least degree with roots -1, 3, and $\pm 3\boldsymbol{i}$. **20.** ________

A. $x^4 - 2x^3 - 6x - 9 = 0$
B. $x^4 + 2x^3 + 6x^2 + 18x - 27 = 0$
C. $x^4 - 2x^3 + 6x^2 - 18x - 27 = 0$
D. $x^4 - 2x^3 - 12x^2 + 18x + 27 = 0$

Bonus Solve $x^3 = -1$. **Bonus:** ________

A. $-1, \frac{1 \pm \boldsymbol{i}\sqrt{3}}{2}$ **B.** $1, -1$ **C.** -1 **D.** $-1, \pm \boldsymbol{i}$

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Chapter 4

Chapter 4 Test, Form 1C

Write the letter for the correct answer in the blank at the right of each problem.

1. Use the Remainder Theorem to find the remainder when $2x^3 + x^2 + 3x + 7$ is divided by $x + 2$. State whether the binomial is a factor of the polynomial. **1.** ________

A. 25; yes **B.** -11; no **C.** 33; no **D.** -11; yes

2. Solve $x^2 - 10x = 1575$ by completing the square. **2.** ________

A. 45, -35 **B.** $5 \pm 15\sqrt{7}$ **C.** $-5 \pm 15\sqrt{7}$ **D.** 35, -45

3. Use synthetic division to divide $x^3 - 2x^2 + 5x + 1$ by $x - 1$. **3.** ________

A. $x^2 - 3x + 8$ R-7 **B.** $x^2 - x + 4$ R-3

C. $x^2 + 3x + 8$ R9 **D.** $x^2 - x + 4$ R5

4. Solve $\sqrt[3]{x - 1} = 3$. **4.** ________

A. -26 **B.** 26 **C.** 64 **D.** 28

5. List the possible rational roots of $2x^4 - x^2 - 3 = 0$. **5.** ________

A. $\pm 1, \pm 2, \pm 3, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{2}$

B. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

C. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

D. $\pm 1, \pm 2, \pm 3, \pm\frac{2}{3}, \pm\frac{3}{2}$

6. Determine the rational roots of $3x^3 + 7x^2 + x - 2 = 0$. **6.** ________

A. $2, \frac{1}{3}$ **B.** $-2, -\frac{1}{3}$ **C.** 2 **D.** -2

7. Solve $\frac{1}{x + 4} = \frac{1}{x^2 + 3x - 4} + \frac{4}{x - 1}$. **7.** ________

A. -2 **B.** -6 **C.** 6 **D.** 2

8. Find the discriminant of $16x^2 - 9x + 13 = 0$ and describe the nature of the roots of the equation. **8.** ________

A. 29; two distinct real roots **B.** 0; exactly one real root

C. -751; no real roots **D.** 751; two distinct real roots

9. Solve $2x^2 - 4x + 7 = 0$ by using the Quadratic Formula. **9.** ________

A. $1 \pm \boldsymbol{i}\sqrt{10}$ **B.** $-1 \pm \frac{\boldsymbol{i}\sqrt{10}}{2}$ **C.** $1 \pm \frac{\boldsymbol{i}\sqrt{10}}{2}$ **D.** $1 \pm 5\boldsymbol{i}$

10. Determine between which consecutive integers one or more real zeros of $f(x) = x^3 + x^2 - 5$ are located. **10.** ________

A. 2 and 3 **B.** 1 and 2 **C.** -2 and -1 **D.** -1 and 0

11. Find the value of k so that the remainder of $(x^3 + 5x^2 - 4x + k) \div (x + 5)$ is 0. **11.** ________

A. -230 **B.** -20 **C.** 54 **D.** 20

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Chapter 4 Test, Form 1C (continued)

12. Find the number of possible negative real zeros for $f(x) = x^3 + 2x^2 + x + 1$. **12.** ________

A. 3 **B.** 2 or 0 **C.** 3 or 1 **D.** 1

13. Solve $2 + \sqrt{x + 2} \geq 11$. **13.** ________

A. $x \leq 79$ **B.** $x \geq 79$ **C.** $x \geq 2$ **D.** $2 \leq x \leq 79$

14. Decompose $\frac{8x + 22}{x^2 + 3x - 4}$ into partial fractions. **14.** ________

A. $\frac{6}{x - 1} + \frac{2}{x + 4}$ **B.** $\frac{2}{x + 4} - \frac{6}{x - 1}$

C. $\frac{2}{x - 1} + \frac{6}{x + 4}$ **D.** $\frac{2}{x + 4} - \frac{6}{x + 1}$

15. Solve $\frac{14}{x^2 - 3x} - \frac{8}{x} > \frac{-10}{x - 3}$. **15.** ________

A. $-19 < x < 0$ **B.** $x < 0, x > 3$

C. $-19 < x < 3$ **D.** $-19 < x < 0, x > 3$

16. Approximate the real zeros of $f(x) = 2x^3 + 3x^2 - 1$ to the nearest tenth. **16.** ________

A. -1.0 **B.** $-1.0, 0.5$ **C.** $-1.0, 0.0$ **D.** 0.5

17. Solve $\sqrt{6x - 2} = \sqrt{4x + 4}$. **17.** ________

A. $\frac{1}{2}$ **B.** -1 **C.** 3 **D.** -3

18. Which polynomial function best models the set of data below? **18.** ________

x	0	2	4	6	8	10	12	14	16	18	20
f(*x*)	2	5	5	4	2	0	−2	−2	0	5	14

A. $y = 0.2x^3 - 0.4x^2 + 2.2x + 2.0$

B. $y = 2x^3 - 40x^2 + 217x + 199$

C. $y = 0.02x^3 + 0.40x^2 + 2.17x + 1.99$

D. $y = 0.02x^3 - 0.40x^2 + 2.17x + 1.99$

19. Solve $1 + \frac{5}{x - 1} \geq \frac{7}{6}$. **19.** ________

A. $1 < x \leq 31$ **B.** $x \leq 31$ **C.** $x \leq 1, x \geq 31$ **D.** $x \geq 1$

20. Find a polynomial equation of least degree with roots -3, 0, and 3. **20.** ________

A. $x^3 + x^2 + 3x - 9 = 0$ **B.** $x^3 + x^2 - 3x - 9 = 0$

C. $x^3 + 9x = 0$ **D.** $x^3 - 9x = 0$

Bonus Solve $16x^4 - 16x^3 - 32x^2 + 36x - 9 = 0$. **Bonus:** ________

A. $\pm\frac{1}{2}, \pm\frac{3}{2}$ **B.** $\frac{1}{2}, \pm\frac{3}{2}$ **C.** $\pm\frac{1}{2}, \frac{3}{2}$ **D.** $\pm\frac{3}{2}, 0$

NAME ______________________ DATE ____________ PERIOD ________

Chapter 4

Chapter 4 Test, Form 2A

Solve each equation or inequality.

1. $(3x - 2)^2 = 121$ **1.** ____________

2. $\frac{3}{2}t^2 - 6t = -\frac{15}{2}$ **2.** ____________

3. $4 + \frac{4}{a + 2} \geq \frac{4}{5}$ **3.** ____________

4. $\sqrt{2x + 5} = 2\sqrt{2x} + 1$ **4.** ____________

5. $\sqrt{12b - 3} \leq \sqrt{5b + 2}$ **5.** ____________

6. $\frac{2}{x + 2} + \frac{x}{2 - x} < \frac{13}{4 - x^2}$ **6.** ____________

7. $\sqrt{d - 6} - 3 = \sqrt{d}$ **7.** ____________

8. Use the Remainder Theorem to find the remainder when $x^5 + x^3 + x$ is divided by $x - 3$. State whether the binomial is a factor of the polynomial. **8.** ____________

9. Determine between which consecutive integers the real zeros of $f(x) = 4x^4 - 4x^3 - 25x^2 + x + 6$ are located. **9.** ____________

10. Decompose $\frac{-3x - 19}{2x^2 - 5x - 3}$ into partial fractions. **10.** ____________

11. Find the value of k so that the remainder of $(x^4 - 3x^3 + kx^2 - 10x + 12) \div (x - 3)$ is 0. **11.** ____________

12. Approximate the real zeros of $f(x) = 2x^4 + 3x^2 - 20$ to the nearest tenth. **12.** ____________

Chapter 4 Test, Form 2A (continued)

13. Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of $f(x) = 2x^3 - 4x^2 + 2$. **13.** ______________

14. Write a polynomial function with integral coefficients to model the set of data below. **14.** ______________

x	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9
$f(x)$	7.3	11.2	12.1	11.2	8.0	6.2	3.5	2.5	2.2	5.7	12.0

15. Find the discriminant of $5x - 3x^2 = -2$ and describe the nature of the roots of the equation. **15.** ______________

16. Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = 2x^4 - 7x^3 - 5x^2 + 28x - 12$. **16.** ______________

17. List the possible rational roots of $2x^3 + 3x^2 - 17x + 12 = 0$. **17.** ______________

18. Determine the rational roots of $x^3 - 6x^2 + 12x - 8 = 0$. **18.** ______________

19. Write a polynomial equation of least degree with roots -2, 2, $-3\boldsymbol{i}$, and $3\boldsymbol{i}$. How many times does the graph of the related function intersect the x-axis? **19.** ______________

20. Francesca jumps upward on a trampoline with an initial velocity of 17 feet per second. The distance $d(t)$ traveled by a free-falling object can be modeled by the formula $d(t) = v_0t - \frac{1}{2}gt^2$, where v_0 is the initial velocity and g represents the acceleration due to gravity (32 feet per second squared). Find the maximum height that Francesca will travel above the trampoline on this jump. **20.** ______________

Bonus Find f if f is a cubic polynomial function such that $f(0) = 0$ and $f(x)$ is positive only when $x > 4$. **Bonus:** ______________

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Chapter 4 Test, Form 2B

Solve each equation or inequality.

1. $8x^2 + 5x - 13 = 0$ **1.** ______________

2. $x^2 - 4x = -13$ **2.** ______________

3. $\frac{6}{q} + 4 \geq \frac{3}{q}$ **3.** ______________

4. $\sqrt[3]{10x + 2} - 3 = -5$ **4.** ______________

5. $\sqrt{2n - 5} + 8 \geq 11$ **5.** ______________

6. $\frac{a - 4}{a + 3} < \frac{3a + 2}{a + 3} + \frac{a}{4}$ **6.** ______________

7. $\sqrt{3x + 4} + 7 = 5$ **7.** ______________

8. Use the Remainder Theorem to find the remainder when $x^3 + 5x^2 + 5x - 2$ is divided by $x + 2$. State whether the binomial is a factor of the polynomial. **8.** ______________

9. Determine between which consecutive integers the real zeros of $f(x) = x^3 - x^2 - 4x - 2$ are located. **9.** ______________

10. Decompose $\frac{8x + 17}{x^2 + 3x - 4}$ into partial fractions. **10.** ______________

11. Find the value of k so that the remainder of $(x^3 - 2x^2 + kx + 6) \div (x + 2)$ is 0. **11.** ______________

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Chapter 4 Test, Form 2B (continued)

12. Approximate the real zeros of $f(x) = 2x^4 - x^2 - 3$ to the nearest tenth. **12.** _______________

13. Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of $f(x) = x^3 - 3x^2 + 2$. **13.** _______________

14. Write a polynomial function to model the set of data below. **14.** _______________

x	4	5	6	7	8	9	10	11	12	13	14
f(*x*)	7	9	9	8	6	3	1	1	2	8	17

15. Find the discriminant of $4x^2 + 12x = -9$ and describe the nature of the roots of the equation. **15.** _______________

16. Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = x^3 - 4x^2 - 3x - 9$. **16.** _______________

17. List the possible rational roots of $4x^3 + 5x^2 - x + 2 = 0$. **17.** _______________

18. Determine the rational roots of $x^3 + 4x^2 + 6x + 9 = 0$. **18.** _______________

19. Write a polynomial equation of least degree with roots -2, 2, -1, and $\frac{1}{2}$. How many times does the graph of the related function intersect the *x*-axis? **19.** _______________

20. Belinda is jumping on a trampoline. After 4 jumps, she jumps up with an initial velocity of 17 feet per second. The function $d(t) = 17t - 16t^2$ gives the height in feet of Belinda above the trampoline as a function of time in seconds after the fifth jump. How long after her fifth jump will it take for her to return to the trampoline again? **20.** _______________

Bonus Factor $x^4 - 2x^3 + 2x - 1$. **Bonus:** _______________

Chapter 4

NAME ______________________ DATE ______________ PERIOD ________

Chapter 4 Test, Form 2C

Solve each equation or inequality.

1. $4x^2 - 4x - 17 = 0$ — **1.** ______________

2. $x^2 - 6x = \frac{7}{2}$ — **2.** ______________

3. $\frac{4}{5} > 2 - \frac{3}{x}$ — **3.** ______________

4. $\sqrt[3]{y + 4} = 3$ — **4.** ______________

5. $\sqrt{2x + 5} + 4 \leq 9$ — **5.** ______________

6. $\frac{2}{3y} - 6 \leq \frac{5}{y}$ — **6.** ______________

7. $\sqrt{2x + 5} - 7 = -4$ — **7.** ______________

8. Use the Remainder Theorem to find the remainder when $x^3 - 5x^2 + 6x - 3$ is divided by $x - 1$. State whether the binomial is a factor of the polynomial. — **8.** ______________

9. Determine between which consecutive integers the real zeros of $f(x) = x^3 + x^2 - 5$ are located. — **9.** ______________

10. Decompose $\frac{10x + 4}{x^2 - 4}$ into partial fractions. — **10.** ______________

11. Find the value of k so that the remainder of $(2x^2 - kx - 3) \div (x + 1)$ is zero. — **11.** ______________

12. Approximate the real zeros of $f(x) = x^3 - 2x^2 - 4x - 5$ to the nearest tenth. — **12.** ______________

Chapter 4 Test, Form 2C (continued)

13. Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of $f(x) = -2x^3 + 4x^2 + 1$.

13. ________________

14. Write a polynomial function to model the set of data below.

14. ________________

x	−1	−0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
f(*x*)	0.5	−3.1	−4.2	−4.4	−2.1	0.8	3.4	4.3	3.6	0.9	−5.4

15. Find the discriminant of $2x^2 - x + 7 = 0$ and describe the nature of the roots of the equation.

15. ________________

16. Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = x^3 - 2x^2 - x + 2$.

16. ________________

17. List the possible rational roots of $2x^3 + 3x^2 - 17x + 12 = 0$.

17. ________________

18. Determine the rational roots of $2x^3 + 3x^2 - 17x + 12 = 0$.

18. ________________

19. Write a polynomial equation of least degree with roots −3, −1, and 5. How many times does the graph of the related function intersect the *x*-axis?

19. ________________

20. What type of polynomial function could be the best model for the set of data below?

20. ________________

x	−3	−2	−1	0	1	2	3
f(*x*)	196	25	−2	1	−8	1	130

Bonus Determine the value of k such that $f(x) = kx^3 - x^2 + 7x + 9$ has possible rational roots of $\pm1, \pm3, \pm9, \pm\frac{1}{6}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$.

Bonus: ________________

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Chapter 4 Open-Ended Assessment

Instructions: ***Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.***

1. Use what you have learned about the discriminant to answer the following.

a. Write a polynomial equation with two imaginary roots. Explain your answer.

b. Write a polynomial equation with two real roots. Explain your answer.

c. Write a polynomial equation with one real root. Explain your answer.

2. Given the function $f(x) = 6x^5 + 2x^4 - 5x^3 - 4x^2 + x - 4$, answer the following.

a. How many positive real zeros are possible? Explain.

b. How many negative real zeros are possible? Explain.

c. What are the possible rational zeros? Explain.

d. Is it possible that there are no real zeros? Explain.

e. Are there any real zeros greater than 2? Explain.

f. What term could you add to the above polynomial to increase the number of possible positive real zeros by one? Does the term you added increase the number of possible negative real zeros? How do you know?

g. Write a polynomial equation. Then describe its roots.

3. A 36-foot-tall light pole has a 39-foot-long wire attached to its top. A stake will be driven into the ground to secure the other end of the wire. The distance from the pole to where the stake should be driven is given by the equation $39 = \sqrt{d^2 + 36^2}$, where d represents the distance.

a. Find d.

b. What relationship was used to write the given equation? What do the values 39, 36, and d represent?

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Chapter 4, Mid-Chapter Test (Lessons 4-1 through 4-4)

1. Determine whether -1 is a root of $x^4 - 2x^3 + 5x + 2 = 0$. Explain. **1.** ______________

2. Write a polynomial equation of least degree with roots -4, 1, $\boldsymbol{i}$, and $-\boldsymbol{i}$. How many times does the graph of the related function intersect the x-axis? **2.** ______________

3. Find the complex roots for the equation $x^2 + 20 = 0$. **3.** ______________

4. Solve the equation $x^2 + 6x + 10 = 0$ by completing the square. **4.** ______________

5. Find the discriminant of $6 - 5x = 6x^2$. Then solve the equation by using the Quadratic Formula. **5.** ______________

6. Use the Remainder Theorem to find the remainder when $x^3 + 3x^2 - 4$ is divided by $x + 2$. State whether the binomial is a factor of the polynomial. **6.** ______________

7. Find the value of k so that the remainder of $(x^3 + 5x^2 - kx - 2) \div (x + 2)$ is 0. **7.** ______________

List the possible rational roots of each equation. Then determine the rational roots.

8. $x^4 - 10x^2 + 9 = 0$ **8.** ______________

9. $2x^3 - 7x + 2 = 0$ **9.** ______________

10. Find the number of possible positive real zeros and the number of possible negative real zeros for the function $f(x) = x^3 - x^2 - x + 1$. Then determine the rational zeros. **10.** ______________

Chapter 4

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Chapter 4, Quiz A (Lessons 4-1 and 4-2)

1. Determine whether -3 is a root of $x^3 + 3x^2 + x + 1 = 0$. Explain.

1. ______________

2. Write a polynomial equation of least degree with roots 3, -1, $2\boldsymbol{i}$, and $-2\boldsymbol{i}$. How many times does the graph of the related function intersect the x-axis?

2. ______________

3. Find the complex roots of the equation $-4x^4 + 3x^2 + 1 = 0$.

3. ______________

4. Solve $x^2 + 10x + 35 = 0$ by completing the square.

4. ______________

5. Find the discriminant of $15x^2 = 4x - 1$ and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

5. ______________

Chapter 4

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Chapter 4, Quiz B (Lessons 4-3 and 4-4)

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

1. $(x^3 - 6x + 9) \div (x - 3)$

1. ______________

2. $(x^4 - 6x^2 + 8) \div \left(x - \sqrt{2}\right)$

2. ______________

3. Find the value of k so that the remainder of $(x^3 + 5x^2 - kx - 2) \div (x + 2)$ is 0.

3. ______________

4. List the possible rational roots of $3x^3 - 4x^2 - 5x + 2 = 0$. Then determine the rational roots.

4. ______________

5. Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = 2x^3 - 9x^2 + 3x + 4$. Then determine the rational zeros.

5. ______________

Chapter 4

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Chapter 4, Quiz C (Lessons 4-5 and 4-6)

1. Determine between which consecutive integers the real zeros of $f(x) = x^3 + 4x^2 - 3x - 5$ are located. **1.** ______________

2. Approximate the real zeros of $f(x) = x^4 - 3x^3 + 2x - 1$ to the nearest tenth. **2.** ______________

3. Solve $\frac{a}{a-2} + \frac{6}{a+2} = 2$. **3.** ______________

4. Decompose $\frac{4p^2 + 13p - 12}{p^3 - p^2 - 2p}$ into partial fractions. **4.** ______________

5. Solve $\frac{2}{w} + \frac{6}{w-1} \leq -5$. **5.** ______________

Chapter 4

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Chapter 4, Quiz D (Lessons 4-7 and 4-8)

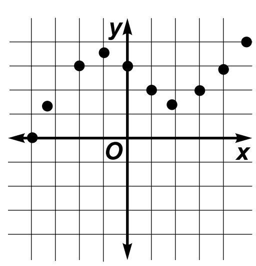
Solve each equation or inequality.

1. $\sqrt{x-3} = 2$ **1.** ______________

2. $\sqrt[3]{2m-1} = -3$ **2.** ______________

3. $\sqrt{3t+7} > 7$ **3.** ______________

4. Determine the type of polynomial function that would best fit the scatter plot shown. **4.** ______________



5. Write a polynomial function with integral coefficients to model the set of data below. **5.** ______________

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	47.6	0.1	-9.3	-0.3	11.6	18.1	14.6	6.4	0.4	12.2	63.8

Chapter 4

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Chapter 4 SAT and ACT Practice

After working each problem, record the correct answer on the answer sheet provided or use your own paper.

Multiple Choice

1. The vertices of a parallelogram are $P(0, 2)$, $Q(3, 0)$, $R(7, 4)$, and $S(4, 6)$. Find the length of the longer sides.

A $4\sqrt{2}$
B $\sqrt{13}$
C $\sqrt{37}$
D $\sqrt{53}$
E None of these

2. A right triangle has vertices $A(-5, -5)$, $B(5 - x, -9)$, and $C(-1, -9)$. Find the value of x.

A -10 **B** -9
C -4 **D** 10
E 15

3. $\frac{2}{3} + \left(-\frac{3}{4}\right) - \left(-\frac{5}{6}\right) + \left(-\frac{7}{8}\right) =$

A $\frac{1}{24}$ **B** $-\frac{1}{7}$
C $-\frac{1}{8}$ **D** -3
E $\frac{1}{8}$

4. $\frac{\sqrt{5}}{\sqrt{5} - 5} =$

A $5 + 5\sqrt{5}$
B $-\frac{1}{4}(1 + 5\sqrt{5})$
C $-\frac{1}{4} + 5\sqrt{5}$
D $\frac{5 + 5\sqrt{5}}{5}$
E None of these

5. Find the slope of a line perpendicular to $3x - 2y = -7$.

A $\frac{3}{2}$ **B** $-\frac{3}{2}$
C $\frac{2}{3}$ **D** $-\frac{2}{3}$
E $-\frac{2}{7}$

6. If the midpoint of the segment joining points $A\left(\frac{1}{2}, 1\frac{1}{5}\right)$ and $B\left(x + \frac{2}{3}, \frac{4}{5}\right)$ has coordinates $\left(\frac{5}{12}, 1\right)$, find the value of x.

A $\frac{1}{3}$
B $-\frac{1}{3}$
C 1
D -1
E None of these

7. If $\frac{6}{x} = 9$, then $\frac{8}{x} =$

A 12 **B** 11
C $\frac{2}{3}$ **D** $\frac{4}{3}$
E $\frac{16}{3}$

8. If a pounds of potatoes serves b adults, how many adults can be served with c pounds of potatoes?

A $\frac{ac}{b}$
B $\frac{bc}{a}$
C $\frac{b}{ac}$
D $\frac{c}{ab}$
E It cannot be determined from the information given.

9. Which are the coordinates of P, Q, R, and S if lines PQ and RS are neither parallel nor perpendicular?

A $P(4, 3)$, $Q(2, 1)$, $R(0, 5)$, $S(-2, 3)$
B $P(6, 0)$, $Q(-1, 0)$, $R(2, 8)$, $S(2, 5)$
C $P(5, 4)$, $Q(7, 2)$, $R(1, 3)$, $S(-1, 1)$
D $P(8, 13)$, $Q(3, 10)$, $R(11, 5)$, $S(6, 3)$
E $P(26, 18)$, $Q(10, 6)$, $R(-13, 25)$, $S(-17, 22)$

10. What is the length of the line segment whose endpoints are represented by the points $C(-6, -9)$ and $D(8, -3)$?

A $2\sqrt{58}$ **B** $4\sqrt{10}$
C $2\sqrt{37}$ **D** $4\sqrt{58}$
E $2\sqrt{10}$

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Chapter 4 SAT and ACT Practice

11. If $x\#$ means $4(x - 2)^2$, find the value of $(3\#)\#$.
A 8 **B** 10
C 12 **D** 16
E 36

12. Each number below is the product of two consecutive positive integers. For which of these is the greater of the two consecutive integers an even integer?
A 6
B 20
C 42
D 56
E 72

13. For which equation is the sum of the roots the greatest?
A $(x - 6)^2 = 4$
B $(x - 2)^2 = 9$
C $(x + 5)^2 = 16$
D $(x + 8)^2 = 25$
E $x^2 = 36$

14. If $\frac{1}{a} + \frac{1}{a} = 12$, then $3a =$
A $\frac{1}{6}$ **B** $\frac{1}{4}$
C $\frac{1}{3}$ **D** $\frac{1}{5}$
E $\frac{1}{2}$

15. For which value of k are the points $A(0, -5)$, $B(6, k)$, and $C(-4, -13)$ collinear?
A 7
B -3
C 1
D $\frac{3}{2}$
E $\frac{2}{3}$

16. Find the midpoint of the segment with endpoints at $(a + b, c)$ and $(2a, -3c)$.
A $\left(\frac{3a + b}{2}, -c\right)$
B $\left(\frac{3a + b}{2}, 2c\right)$
C $\left(\frac{3a}{2}, -c\right)$
D $(4c, b - a)$
E $(a - b, -2c)$

17–18. Quantitative Comparison
A if the quantity in Column A is greater
B if the quantity in Column B is greater
C if the two quantities are equal
D if the relationship cannot be determined from the information given

	Column A	Column B
17.	$0 < y < x < 1$	
	$x - y$	$\frac{1}{x} - \frac{1}{y}$
18.	$x \neq \pm 3$	
	$\frac{x^2 + 6x + 9}{x + 3}$	$\frac{x^2 - 9}{x - 3}$

19. Grid-In If the slope of line AB is $\frac{2}{3}$ and lines AB and CD are parallel, what is the value of x if the coordinates of C and D are $(0, -3)$ and $(x, 1)$, respectively?

20. Grid-In A parallelogram has vertices at $A(1, 3)$, $B(3, 5)$, $C(4, 2)$, and $D(2, 0)$. What is the x-coordinate of the point at which the diagonals bisect each other?

Chapter 4

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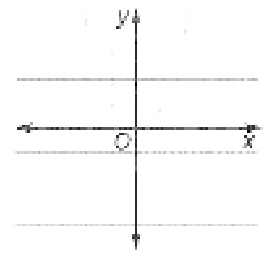
Chapter 4 Cumulative Review (Chapters 1-4)

1. State the domain and range of the relation $\{(-2, 5), (3, -2), (0, 5)\}$. Then state whether the relation is a function. Write *yes* or *no*. **1.** ______________

2. Find $[f \circ g](x)$ if $f(x) = x + 5$ and $g(x) = 3x^2$. **2.** ______________

3. Graph $y > 3|x| - 2$. **3.**



4. Solve the system of equations. **4.** ______________

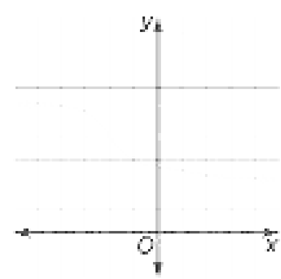
$$\begin{aligned} 2x + y - z &= 0 \\ x - y + z &= 6 \\ x + 2y + z &= 3 \end{aligned}$$

5. The coordinates of the vertices of $\triangle ABC$ are $A(1, -1)$, $B(2, 2)$, and $C(3, 1)$. Find the coordinates of the vertices of the image of $\triangle ABC$ after a 270° counterclockwise rotation about the origin. **5.** ______________

6. Gabriel works no more than 15 hours per week during the school year. He is paid \$12 per hour for tutoring math and \$9 per hour for working at the grocery store. He does not want to tutor for more than 8 hours per week. What are Gabriel's maximum earnings? **6.** ______________

7. Determine whether the graph of $y = \frac{x^2}{4}$ is symmetric to the x-axis, the y-axis, the line $y = x$, the line $y = -x$, or none of these. **7.** ______________

8. Graph $y = 4 - \sqrt[3]{x + 2}$ using the graph of the function $y = x^3$. **8.**



9. Describe the end behavior of $y = -4x^7 + 3x^3 - 5$. **9.** ______________

10. Determine the slant asymptote for $f(x) = \frac{x^2 - 3x - 2}{x + 1}$. **10.** ______________

11. Solve $\frac{-3x}{x^2 - 4x - 32} - \frac{2}{x - 8} = \frac{3}{x + 4}$. **11.** ______________

12. Solve $43x - x^3 + x^4 = 10 + 21x^2$. **12.** ______________

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Unit 1 Review, Chapters 1–4

Given that x is an integer, state the relation representing each equation as a set of ordered pairs. Then, state whether the relation is a function. Write yes or no.

1. $y = 3x + 1$ and $-1 \le x \le 3$
2. $y = |2 - x|$ and $-2 \le x \le 3$

Find [f ∘ g](x) and [g ∘ f](x) for each f(x) and g(x).

3. $f(x) = 3x + 1$
$g(x) = x + 3$
4. $f(x) = 4x^2$
$g(x) = -x^3$
5. $f(x) = x^2 - 25$
$g(x) = 2x - 4$

Find the zero of each function.

6. $f(x) = 4x - 10$
7. $f(x) = 15x$
8. $f(x) = 0.75x + 3$

Write the slope-intercept form of the equation of the line through the points with the given coordinates.

9. $(4, -4), (6, -10)$
10. $(1, 2), (5, 4)$

Write the standard form of the equation of each line described below.

11. parallel to $y = 3x - 1$
passes through $(-1, 4)$
12. perpendicular to $2x - 3y = 6$
x-intercept: 2

The table below shows the number of T-shirts sold per day during the first week of a senior-class fund-raiser.

Day	Number of Shirts Sold
1	12
2	21
3	32
4	43
5	56

13. Use the ordered pairs (2, 21) and (4, 43) to write the equation of a best-fit line.
14. Predict the number of shirts sold on the eighth day of the fund-raiser. Explain whether you think the prediction is reliable.

Graph each function.

15. $f(x) = [\![x + 2]\!]$
16. $f(x) = |2x| - 1$
17. $f(x) = \begin{cases} x - 2 \text{ if } x \le -1 \\ 2x \text{ if } -1 < x < 1 \\ -x \text{ if } x \ge 2 \end{cases}$

Graph each inequality.

18. $x + 3y < 12$
19. $y \ge -\frac{2}{3}x + 5$

Solve each system of equations.

20. $y = -4x$
$x - y = 5$
21. $x + y = 12$
$2x - y = -4$
22. $7x - z = 13$
$y + 3z = 18$
$11x + y = 27$

Use matrices A, B, C, and D to find each sum, difference, or product.

$A = \begin{bmatrix} 6 & 2 \\ 3 & -3 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 6 \\ 5 & 7 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 2 & -1 \\ -5 & -8 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 0 \\ 6 & -3 \\ -5 & -1 \end{bmatrix}$ $E = \begin{bmatrix} -3 & 1 & 5 \\ -1 & -4 & -2 \\ 3 & 2 & -1 \end{bmatrix}$

23. $A + B$
24. $2A - B$
25. CD
26. $AB + CD$

Use matrices A, B, and E above to find the following.

27. Evaluate the determinant of matrix A.
28. Evaluate the determinant of matrix E.
29. Find the inverse of matrix B.

Solve each system of inequalities by graphing. Name the coordinates of the vertices of each polygonal convex set. Then, find the maximum and minimum values for the function f(x, y) = 2y − 2x − 3.

30. $x \ge 0$
$y \ge 0$
$2y + x \le 1$

31. $x \ge 2$
$y \ge -3$
$y \le 5 - x$
$y + 2x \le 8$

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Unit 1 Review, Chapters 1–4 (continued)

Determine whether each function is an even function, an odd function, or neither.

32. $y = -3x^3$
33. $y = 2x^4 - 5$
34. $y = x^3 + 3x^2 - 6x - 8$

Use the graph of $f(x) = x^3$ to sketch a graph for each function. Then, describe the transformations that have taken place in the related graphs.

35. $y = -f(x)$ **36.** $y = f(x - 2)$

Graph each inequality.

37. $y \leq |x + 3|$
38. $y > \sqrt[3]{x + 4}$

Find the inverse of each function. Sketch the function and its inverse. Is the inverse a function? Write yes or no.

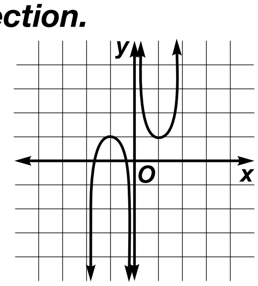
39. $y = \frac{1}{2}x - 5$
40. $y = (x - 1)^3 + 2$

Determine whether each graph has infinite discontinuity, jump discontinuity, point discontinuity, or is continuous. Then, graph each function.

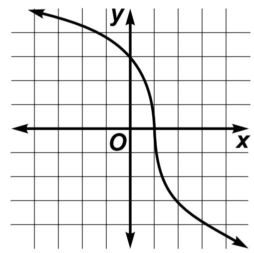
41. $y = \frac{x^2 - 1}{x + 1}$
42. $y = \begin{cases} x - 1 \text{ if } x < 0 \\ x - 3 \text{ if } x \geq 0 \end{cases}$

Find the critical points for the functions graphed in Exercises 43 and 44. Then, determine whether each point is a maximum, a minimum, or a point of inflection.

43.



44.



Determine any horizontal, vertical, or slant asymptotes or point discontinuity in the graph of each function. Then, graph each function.

45. $y = \frac{x}{(2x + 1)(x + 2)}$
46. $y = \frac{x^2 - 9}{x + 3}$

Solve each equation or inequality.

47. $x^2 - 8x + 16 = 0$
48. $4x^2 - 4x - 10 = 0$
49. $\frac{x + 2}{4} + \frac{x - 3}{4} = 6$
50. $2x + \frac{1}{2 - x} > \frac{1}{2}; x \neq 2$
51. $9 + \sqrt{x - 1} = 1$
52. $\sqrt{x + 8} - \sqrt{x + 35} \leq -3$

Use the Remainder Theorem to find the remainder for each division.

53. $(x^2 - x + 4) \div (x - 6)$
54. $(2x^3 - 3x + 1) \div (x - 2)$

Find the number of possible positive real zeros and the number of possible negative real zeros. Determine all of the rational zeros.

55. $f(x) = 3x^2 + x - 2$
56. $f(x) = x^4 + x^3 - 2x^2 + 3x - 1$

Approximate the real zeros of each function to the nearest tenth.

57. $f(x) = x^2 - 2x - 5$
58. $f(x) = x^3 + 4x^2 + x - 2$

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UNIT 1

Unit 1 Test, Chapters 1–4

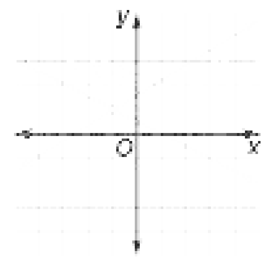
1. Find the maximum and minimum values of $f(x, y) = 3x + y$ for the polygonal convex set determined by $x \geq 1, y \geq 0$, and $x + 0.5y \leq 2$. **1.** ______________

2. Write the polynomial equation of least degree that has the roots $-3\boldsymbol{i}, 3\boldsymbol{i}, \boldsymbol{i}$, and $-\boldsymbol{i}$. **2.** ______________

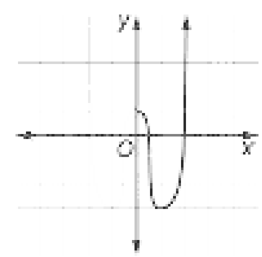
3. Divide $4x^3 + 3x^2 - 2x + 75$ by $x + 3$ by using synthetic division. **3.** ______________

4. Solve the system of equations by graphing. **4.**

$3x - 5y = -8$
$x + 2y = 1$



5. Complete the graph so that it is the graph of an even function. **5.**



6. Solve the system of equations. **6.** ______________

$x - y - z = 2$
$x + 2y - 2z = 3$
$3x - 2y - 4z = 5$

7. Decompose the expression $\frac{17n - 23}{4n^2 + 23n - 6}$ into partial fractions. **7.** ______________

8. Is the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ symmetric with respect to the x-axis, the y-axis, neither axis, or both axes? **8.** ______________

9. Without graphing, describe the end behavior of the graph of $f(x) = -5x^2 - 3x + 1$. **9.** ______________

10. How many solutions does a consistent and dependent system of linear equations have? **10.** ______________

11. Solve $3x^2 - 7x - 6 = 0$. **11.** ______________

12. Solve $3y^2 + 4y - 2 \leq 0$. **12.** ______________

Unit 1 Test, Chapters 1–4 (continued)

13. If $f(x) = -4x^2$ and $g(x) = \frac{2}{x}$, find $[g \circ f](x)$. **13.** ______________

14. Are $f(x) = \frac{1}{2}x + 5$ and $g(x) = 2x - 5$ inverses of each other? **14.** ______________

15. Find the inverse of $y = \frac{x^2}{10}$. Then, state whether the inverse is a function. **15.** ______________

16. Determine if the expression $4m^5 - 6m^8 + m + 3$ is a polynomial in one variable. If so, state the degree. **16.** ______________

17. Describe how the graph of $y = |x - 2|$ is related to its parent graph. **17.** ______________

18. Write the slope-intercept form of the equation of the line that passes through the point $(-5, 4)$ and has a slope of -1. **18.** ______________

19. Determine whether the figure with vertices at $(1, 2)$, $(3, 1)$, $(4, 3)$, and $(2, 4)$ is a parallelogram. **19.** ______________

20. A plane flies with a ground speed of 160 miles per hour if there is no wind. It travels 350 miles with a head wind in the same time it takes to go 450 miles with a tail wind. Find the speed of the wind. **20.** ______________

21. Solve the system of equations algebraically. **21.** ______________

$\frac{1}{3}x + \frac{1}{3}y = 1$

$2x + 2y = 9$

22. Find the value of $\begin{vmatrix} 5 & 3 & -2 \\ 1 & 0 & 4 \\ 4 & -1 & 2 \end{vmatrix}$ by using expansion by minors. **22.** ______________

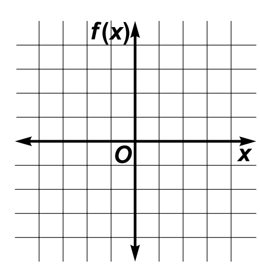
23. Solve the system of equations by using augmented matrices. **23.** ______________

$y = 3x - 10$

$x = 12 - 4y$

24. Approximate the greatest real zero of the function $g(x) = x^3 - 3x + 1$ to the nearest tenth. **24.** ______________

25. Graph $f(x) = \frac{1}{x - 1}$. **25.**



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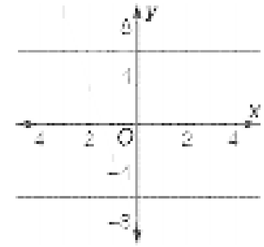
Unit 1 Test, Chapters 1–4 (continued)

26. Write the slope-intercept form of the equation $6x + y + 9 = 0$. Then, graph the equation. **26.** ______________



27. Write the standard form of the equation of the line that passes through $(-3, 7)$ and is perpendicular to the line with equation $y = 3x - 5$. **27.** ______________

28. Use the Remainder Theorem to find the remainder of $(x^3 - 5x^2 + 7x + 3) \div (x - 2)$. State whether the binomial is a factor of the polynomial. **28.** ______________

29. Solve $x - \sqrt{2x + 1} = 7$. **29.** ______________

30. Determine the value of w so that the line whose equation is $5x - 2y = -w$ passes through the point at $(-1, 3)$. **30.** ______________

31. Determine the slant asymptote for $f(x) = \frac{x^2 - 5x - 3}{x}$. **31.** ______________

32. Find the value of $\begin{vmatrix} 3 & 5 \\ 7 & -2 \end{vmatrix}$. **32.** ______________

33. State the domain and range of $\{(-5, 2), (4, 3), (-2, 0), (-5, 1)\}$. Then, state whether the relation is a function. **33.** ______________

34. Determine whether the function $f(x) = [\![x + 1]\!]$ is odd, even, or neither. **34.** ______________

35. Find the least integral upper bound of the zeros of the function $f(x) = x^3 - x^2 + 1$. **35.** ______________

36. Solve $|2 - 3x| \leq 4$. **36.** ______________

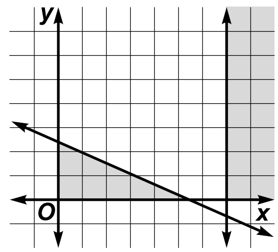
37. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 \\ 1 & 5 \end{bmatrix}$, find AB. **37.** ______________

38. Name all the values of x that are not in the domain of $f(x) = \frac{2 - x^2}{x + 5}$. **38.** ______________

39. Given that x is an integer between -2 and 2, state the relation represented by the equation $y = 2 - |x|$ by listing a set of ordered pairs. Then, state whether the relation is a function. Write *yes* or *no*. **39.** ______________

Unit 1 Test, Chapters 1–4 (continued)

40. Determine whether the system of inequalities graphed at the right is *infeasible*, has *alternate optimal solutions*, or is *unbounded* for the function $f(x, y) = 2x + y$. **40.** ____________



41. Solve $1 = (y + 3)(2y - 2)$. **41.** ____________

42. Determine whether the function $y = -\frac{3}{x^2}$ has *infinite discontinuity, jump discontinuity*, or *point discontinuity*, or is *continuous*. **42.** ____________

43. Find the slope of the line passing through the points at $(a, a + 3)$ and $(4a, a - 5)$. **43.** ____________

44. Together, two printers can print 7500 lines if the first printer prints for 2 minutes and the second prints for 1 minute. If the first printer prints for 1 minute and the second printer prints for 2 minutes, they can print 9000 lines together. Find the number of lines per minute that each printer prints. **44.** ____________

45. A box for shipping roofing nails must have a volume of 84 cubic feet. If the box must be 3 feet wide and its height must be 3 feet less than its length, what should the dimensions of the box be? **45.** ____________

46. Solve the system of equations. **46.** ____________

$-3x - 2y + 3z = -1$
$2x + 5y - 3z = -6$
$4x + 3y + 3z = 22$

47. Solve $4x^2 + 12x - 7 = 0$ by completing the square. **47.** ____________

48. Find the critical point of the function $y = -2(x - 1)^2 - 3$. Then, determine whether the point represents a *maximum*, a *minimum*, or a *point of inflection*. **48.** ____________

49. Solve $\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$. **49.** ____________

50. Write the standard form of the equation of the line that passes through $(5, -2)$ and is parallel to the line with equation $3x + 2y + 4 = 0$. **50.** ____________

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SAT and ACT Practice Answer Sheet

(10 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10

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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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SAT and ACT Practice Answer Sheet

(20 Questions)

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

11 (A) (B) (C) (D) (E)

12 (A) (B) (C) (D) (E)

13 (A) (B) (C) (D) (E)

14 (A) (B) (C) (D) (E)

15 (A) (B) (C) (D) (E)

16 (A) (B) (C) (D) (E)

17 (A) (B) (C) (D) (E)

18 (A) (B) (C) (D) (E)

19

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

20

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

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4-1 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial.

1. $6a^4 + a^3 - 2a$
   4; 6

2. $3p^2 - 7p^5 - 2p^3 + 5$
   5; −7

Write a polynomial equation of least degree for each set of roots.

3. $3, -0.5, 1$
   $2x^3 - 7x^2 + 2x + 3 = 0$

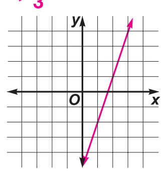
4. $3, 3, 1, 1, -2$
   $x^5 - 6x^4 + 6x^3 + 20x^2 - 39x + 18 = 0$

5. $\pm 2i, 3, -3$
   $x^4 - 5x^2 - 36 = 0$

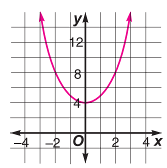
6. $-1, 3 \pm i, 2 \pm 3i$
   $x^5 - 9x^4 + 37x^3 - 71x^2 + 12x + 130 = 0$

State the number of complex roots of each equation. Then find the roots and graph the related function.

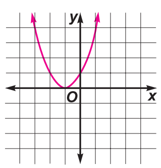
7. $3x - 5 = 0$
   1; $\frac{5}{3}$



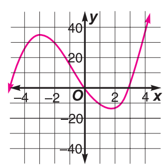
8. $x^2 + 4 = 0$
   2; ±2*i*



9. $c^2 + 2c + 1 = 0$
   2; −1, −1



10. $x^3 + 2x^2 - 15x = 0$
    3; −5, 0, 3



11. ***Real Estate*** A developer wants to build homes on a rectangular plot of land 4 kilometers long and 3 kilometers wide. In this part of the city, regulations require a greenbelt of uniform width along two adjacent sides. The greenbelt must be 10 times the area of the development. Find the width of the greenbelt.
    8 km

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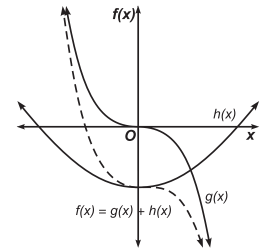
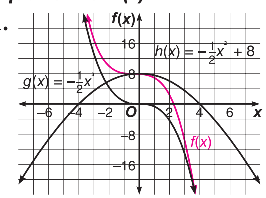
4-1 Enrichment

Graphic Addition

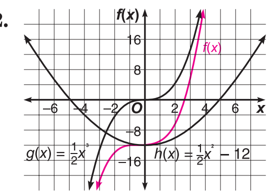
One way to sketch the graphs of some polynomial functions is to use *addition of ordinates*. This method is useful when a polynomial function $f(x)$ can be written as the sum of two other functions, $g(x)$ and $h(x)$, that are easier to graph. Then, each $f(x)$ can be found by mentally adding the corresponding $g(x)$ and $h(x)$. The graph at the right shows how to construct the graph of $f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 - 8$ from the graphs of $g(x) = -\frac{1}{2}x^3$ and $h(x) = \frac{1}{2}x^2 - 8$.



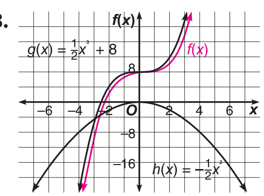
In each problem, the graphs of g(x) and h(x) are shown. Use addition of ordinates to graph a new polynomial function f(x), such that f(x) = g(x) + h(x). Then write the equation for f(x).

1.


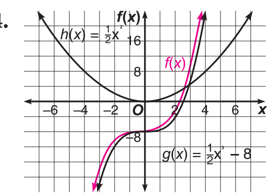
$f(x) = -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 8$

2.


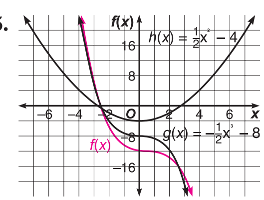
$f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 12$

3.


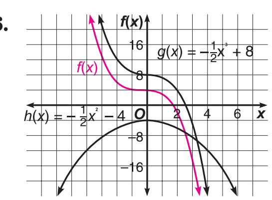
$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 + 8$

4.


$f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 8$

5.


$f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 - 12$

6.


$f(x) = -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4$

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4-2 Practice

Quadratic Equations

Solve each equation by completing the square.

1. $x^2 - 5x - \frac{11}{4} = 0$

   $-\frac{1}{2}, \frac{11}{2}$

2. $-4x^2 - 11x = 7$

   $-1, -\frac{7}{4}$

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

3. $x^2 + x - 6 = 0$

   25; 2 real; −3, 2

4. $4x^2 - 4x - 15 = 0$

   256; 2 real; $\frac{5}{2}, -\frac{3}{2}$

5. $9x^2 - 12x + 4 = 0$

   0; 1 real; $\frac{2}{3}$

6. $3x^2 + 2x + 5 = 0$

   −56; 2 imaginary; $\frac{-1 \pm i\sqrt{14}}{3}$

Solve each equation.

7. $2x^2 + 5x - 12 = 0$

   $-4, \frac{3}{2}$

8. $5x^2 - 14x + 11 = 0$

   $\frac{7 \pm \sqrt{6}i}{5}$

9. ***Architecture*** The ancient Greek mathematicians thought that the most pleasing geometric forms, such as the ratio of the height to the width of a doorway, were created using the *golden section*. However, they were surprised to learn that the golden section is not a rational number. One way of expressing the golden section is by using a line segment. In the line segment shown, $\frac{AB}{AC} = \frac{AC}{CB}$. If $AC = 1$ unit, find the ratio $\frac{AB}{AC}$.

$\frac{AB}{AC} = \frac{1 + \sqrt{5}}{2}$

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4-2 Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$. We can define the absolute value of a complex number as follows.

$$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example **Show that $z^2 = z\bar{z}$ for any complex number z.**

Let $z = x + y\boldsymbol{i}$. Then,

$$\begin{aligned} z\bar{z} &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\ &= x^2 + y^2 \\ &= \left(\sqrt{x^2 + y^2}\right)^2 \\ &= |z|^2 \end{aligned}$$

Example **Show that $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.**

We know that $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$. Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. $2i$

   2; $\frac{-i}{2}$

2. $-4 - 3\boldsymbol{i}$

   5; $\frac{-4 + 3i}{25}$

3. $12 - 5\boldsymbol{i}$

   13; $\frac{12 + 5i}{169}$

4. $5 - 12\boldsymbol{i}$

   13; $\frac{5 + 12i}{169}$

5. $1 + \boldsymbol{i}$

   $\sqrt{2}$**;** $\frac{1 - i}{2}$

6. $\sqrt{3} - \boldsymbol{i}$

   2; $\frac{\sqrt{3} + i}{4}$

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$

   $\frac{\sqrt{6}}{3}$; $\frac{\sqrt{3} - i\sqrt{3}}{2}$

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$

   1; $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$

   1; $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

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4-3 Practice

The Remainder and Factor Theorems

Divide using synthetic division.

1. $(3x^2 + 4x - 12) \div (x + 5)$
$3x - 11$, R43

2. $(x^2 - 5x - 12) \div (x - 3)$
$x - 2$, R−18

3. $(x^4 - 3x^2 + 12) \div (x + 1)$
$x^3 - x^2 - 2x + 2$, R10

4. $(2x^3 + 3x^2 - 8x + 3) \div (x + 3)$
$2x^2 - 3x + 1$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

5. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$
6; no

6. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$
0; yes

7. $(3t^3 - 10t^2 + t - 5) \div (t - 4)$
31; no

8. $(10x^3 - 11x^2 - 47x + 30) \div (x + 2)$
0; yes

9. $(x^4 + 5x^3 - 14x^2) \div (x - 2)$
0; yes

10. $(2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$
0; yes

11. $(y^3 + y^2 - 10) \div (y + 3)$
−28; no

12. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$
0; yes

13. Use synthetic division to find all the factors of $x^3 + 6x^2 - 9x - 54$ if one of the factors is $x - 3$.
$(x - 3)(x + 3)(x + 6)$

14. ***Manufacturing*** A cylindrical chemical storage tank must have a height 4 meters greater than the radius of the top of the tank. Determine the radius of the top and the height of the tank if the tank must have a volume of 15.71 cubic meters.
$r \approx 1$ m, $h \approx 5$ m

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4-3 Enrichment

The Secret Cubic Equation

You might have supposed that there existed simple formulas for solving higher-degree equations. After all, there is a simple formula for solving quadratic equations. Might there not be formulas for cubics, quartics, and so forth?

There are formulas for some higher-degree equations, but they are certainly not "simple" formulas!

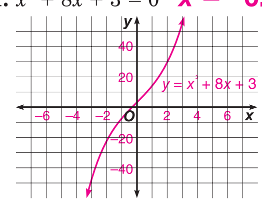
Here is a method for solving a reduced cubic of the form $x^3 + ax + b = 0$ published by Jerome Cardan in 1545. Cardan was given the formula by another mathematician, Tartaglia. Tartaglia made Cardan promise to keep the formula secret, but Cardan published it anyway. He did, however, give Tartaglia the credit for inventing the formula!

Let $R = \left(\frac{1}{2}b\right)^2 + \frac{a^3}{27}$

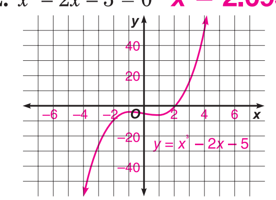
Then, $x = \left[-\frac{1}{2}b + \sqrt{R}\right]^{\frac{1}{3}} + \left[-\frac{1}{2}b - \sqrt{R}\right]^{\frac{1}{3}}$

Use Cardan's method to find the real root of each cubic equation. Round answers to three decimal places. Then sketch a graph of the corresponding function on the grid provided.

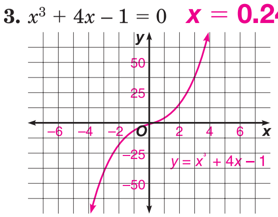
1. $x^3 + 8x + 3 = 0$ **$x = -0.369$**



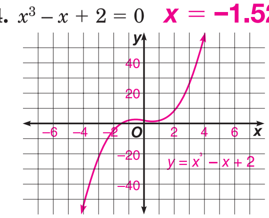
2. $x^3 - 2x - 5 = 0$ **$x = 2.095$**



3. $x^3 + 4x - 1 = 0$ **$x = 0.246$**



4. $x^3 - x + 2 = 0$ **$x = -1.521$**



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4-4

Practice

The Rational Root Theorem

List the possible rational roots of each equation. Then determine the rational roots.

1. $x^3 - x^2 - 8x + 12 = 0$
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12; -3, 2$

2. $2x^3 - 3x^2 - 2x + 3 = 0$
$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}; \pm 1, \frac{3}{2}$

3. $36x^4 - 13x^2 + 1 = 0$
$\pm \frac{1}{36}, \pm \frac{1}{18}, \pm \frac{1}{12}, \pm \frac{1}{9}, \pm \frac{1}{6}, \pm \frac{1}{4}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1; \pm \frac{1}{2}, \pm \frac{1}{3}$

4. $x^3 + 3x^2 - 6x - 8 = 0$
$\pm 1, \pm 2, \pm 4, \pm 8; -4, -1, 2$

5. $x^4 - 3x^3 - 11x^2 + 3x + 10 = 0$
$\pm 1, \pm 2, \pm 5, \pm 10; \pm 1, -2, 5$

6. $x^4 + x^2 - 2 = 0$
$\pm 1, \pm 2; \pm 1$

7. $3x^3 + x^2 - 8x + 6 = 0$
$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$; none

8. $x^3 + 4x^2 - 2x + 15 = 0$
$\pm 1, \pm 3, \pm 5, \pm 15; -5$

Find the number of possible positive real zeros and the number of possible negative real zeros. Then determine the rational zeros.

9. $f(x) = x^3 - 2x^2 - 19x + 20$
2 or 0; 1; $-4, 1, 5$

10. $f(x) = x^4 + x^3 - 7x^2 - x + 6$
2 or 0; 2 or 0; $-3, -1, 1, 2$

11. *Driving* An automobile moving at 12 meters per second on level ground begins to decelerate at a rate of -1.6 meters per second squared. The formula for the distance an object has traveled is $d(t) = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial velocity and a is the acceleration. For what value(s) of t does $d(t) = 40$ meters? **5 s and 10 s**

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4-4

Enrichment

Scrambled Proofs

The proofs on this page have been scrambled. Number the statements in each proof so that they are in a logical order.

The Remainder Theorem

5 Thus, if a polynomial $f(x)$ is divided by $x - a$, the remainder is $f(a)$.

1 In any problem of division the following relation holds: dividend = quotient × divisor + remainder. In symbols, this may be written as:

4 Equation (2) tells us that the remainder R is equal to the value $f(a)$; that is, $f(x)$ with a substituted for x.

3 For $x = a$, Equation (1) becomes:
Equation (2) $\quad f(a) = R,$
since the first term on the right in Equation (1) becomes zero.

2 Equation (1) $\quad f(x) = Q(x)(x - a) + R,$
in which $f(x)$ denotes the original polynomial, $Q(x)$ is the quotient, and R the constant remainder. Equation (1) is true for all values of x, and in particular, it is true if we set $x = a$.

The Rational Root Theorem

4 Each term on the left side of Equation (2) contains the factor a; hence, a must be a factor of the term on the right, namely, $-c_n b^n$. But by hypothesis, a is not a factor of b unless $a = \pm 1$. Hence, a is a factor of c_n.

2 $f\left(\frac{a}{b}\right)^n = c_0\left(\frac{a}{b}\right)^n + c_1\left(\frac{a}{b}\right)^{n-1} + \ldots + c_{n-1}\left(\frac{a}{b}\right) + c_n = 0$

6 Thus, in the polynomial equation given in Equation (1), a is a factor of c_n and b is a factor of c_0.

5 In the same way, we can show that b is a factor of c_0.

1 A polynomial equation with integral coefficients of the form
Equation (1) $\quad f(x) = c_0x^n + c_1x^{n-1} + \ldots + c_{n-1}x + c_n = 0$
has a rational root $\frac{a}{b}$, where the fraction $\frac{a}{b}$ is reduced to lowest terms. Since $\frac{a}{b}$ is a root of $f(x) = 0$, then

3 If each side of this equation is multiplied by b^n and the last term is transposed, it becomes
Equation (2) $\quad c_0a^n + c_1a^{n-1}b + \ldots + c_{n-1}ab^{n-1} = -c_nb^n$

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4-5 Practice

Locating Zeros of a Polynomial Function

Determine between which consecutive integers the real zeros of each function are located.

1. $f(x) = 3x^3 - 10x^2 + 22x - 4$
0 and 1

2. $f(x) = 2x^3 + 5x^2 - 7x - 3$
−4 and −3, −1 and 0, 1 and 2

3. $f(x) = 2x^3 - 13x^2 + 14x - 4$
0 and 1, 5 and 6

4. $f(x) = x^3 - 12x^2 + 17x - 9$
10 and 11

5. $f(x) = 4x^4 - 16x^3 - 25x^2 + 196x - 146$
−4 and −3, 0 and 1

6. $f(x) = x^3 - 9$
2 and 3

Approximate the real zeros of each function to the nearest tenth.

7. $f(x) = 3x^4 + 4x^2 - 1$
±0.5

8. $f(x) = 3x^3 - x + 2$
−1.0

9. $f(x) = 4x^4 - 6x^2 + 1$
±0.4, ±1.1

10. $f(x) = 2x^3 + x^2 - 1$
0.7

11. $f(x) = x^3 - 2x^2 - 2x + 3$
−1.3, 1.0, 2.3

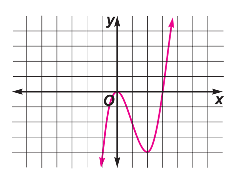
12. $f(x) = x^3 - 5x^2 + 4$
−0.8, 1.0, 4.8

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function. **Sample answers given.**

13. $f(x) = 3x^4 - x^3 - 8x^2 - 3x - 20$
3, −2

14. $f(x) = 2x^3 - x^2 + x - 6$
2, 0

15. For $f(x) = x^3 - 3x^2$, determine the number and type of possible complex zeros. Use the Location Principle to determine the zeros to the nearest tenth. The graph has a relative maximum at (0, 0) and a relative minimum at (2, −4). Sketch the graph.
three real roots; 3, 0, 0



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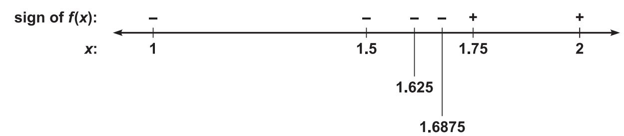
4-5 Enrichment

The Bisection Method for Approximating Real Zeros

The bisection method can be used to approximate zeros of polynomial functions like $f(x) = x^3 + x^2 - 3x - 3$. Since $f(1) = -4$ and $f(2) = 3$, there is at least one real zero between 1 and 2. The midpoint of this interval is $\frac{1+2}{2} = 1.5$. Since $f(1.5) = -1.875$, the zero is between 1.5 and 2. The midpoint of this interval is $\frac{1.5+2}{2} = 1.75$. Since $f(1.75) = 0.172$, the zero is between 1.5 and 1.75. $\frac{1.5+1.75}{2} = 1.625$ and $f(1.625) = -0.94$. The zero is between 1.625 and 1.75. The midpoint of this interval is $\frac{1.625+1.75}{2} = 1.6875$. Since $f(1.6875) = -0.41$, the zero is between 1.6875 and 1.75. Therefore, the zero is 1.7 to the nearest tenth. The diagram below summarizes the bisection method.



Using the bisection method, approximate to the nearest tenth the zero between the two integral values of each function.

1. $f(x) = x^3 - 4x^2 - 11x + 2$, $f(0) = 2$, $f(1) = -12$
0.2

2. $f(x) = 2x^4 + x^2 - 15$, $f(1) = -12$, $f(2) = 21$
1.6

3. $f(x) = x^5 - 2x^3 - 12$, $f(1) = -13$, $f(2) = 4$
1.9

4. $f(x) = 4x^3 - 2x + 7$, $f(-2) = -21$, $f(-1) = 5$
−1.3

5. $f(x) = 3x^3 - 14x^2 - 27x + 126$, $f(4) = -14$, $f(5) = 16$
4.7

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4-6
Practice
Rational Equations and Partial Fractions

Solve each equation.

1. $\frac{15}{m} - m + 8 = 10$ **$-5, 3$**

2. $\frac{4}{b-3} + \frac{3}{b} = \frac{-2b}{b-3}$ **$-\frac{9}{2}, 1$**

3. $\frac{1}{2n} + \frac{6n-9}{3n} = \frac{2}{n}$ **$\frac{9}{4}$**

4. $t - \frac{4}{t} = 3$ **$-1, 4$**

5. $\frac{3a}{2a+1} - \frac{4}{2a-1} = 1$ **$\frac{11 \pm \sqrt{145}}{4}$**

6. $\frac{2p}{p+1} + \frac{3}{p-1} = \frac{15-p}{p^2-1}$ **$-3, 2$**

Decompose each expression into partial fractions.

7. $\frac{-3x-29}{x^2-4x-21}$ **$-\frac{5}{x-7} + \frac{2}{x+3}$**

8. $\frac{11x-7}{2x^2-3x-2}$ **$\frac{3}{x-2} + \frac{5}{2x+1}$**

Solve each inequality.

9. $\frac{6}{t} + 3 > \frac{2}{t}$ **$t < -\frac{4}{3}$ or $t > 0$**

10. $\frac{2n+1}{3n+1} \le \frac{n-1}{3n+1}$ **$-2 \le n < -\frac{1}{3}$**

11. $1 + \frac{3y}{1-y} > 2$ **$\frac{1}{4} < y < 1$**

12. $\frac{2x}{4} - \frac{5x+1}{3} > 3$ **$x < -\frac{20}{7}$**

13. ***Commuting*** Rosea drives her car 30 kilometers to the train station, where she boards a train to complete her trip. The total trip is 120 kilometers. The average speed of the train is 20 kilometers per hour faster than that of the car. At what speed must she drive her car if the total time for the trip is less than 2.5 hours? **at least 35 km/hr**

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4-6
Enrichment
Inverses of Conditional Statements

In the study of formal logic, the compound statement "*if p, then q*" where p and q represent any statements, is called a *conditional* or an *implication*. The symbolic representation of a conditional is

$$p \to q.$$

$$\underbrace{\text{If the determinant of a } 2 \times 2 \text{ matrix is 0,}}_{p} \text{ then } \underbrace{\text{the matrix does not have an inverse.}}_{q}$$

If both p and q are negated, the resulting compound statement is called the **inverse** of the original conditional. The symbolic notation for the negation of p is $\sim p$.

Conditional	**Inverse**	
$p \to q$	$\sim p \to \sim q$	*If a conditional is true, its inverse may be either true or false.*

Example **Find the inverse of each conditional.**

a. $p \to q$: If today is Monday, then tomorrow is Tuesday. (true)

$\sim p \to \sim q$ If today is not Monday, then tomorrow is not Tuesday. (true)

b. $p \to q$ If *ABCD* is a square, then *ABCD* is a rhombus. (true)

$\sim p \to \sim q$ If *ABCD* is not a square, then *ABCD* is not a rhombus. (false)

Write the inverse of each conditional.

1. $q \to p$ **$\sim q \to \sim p$** 2. $\sim p \to q$ **$p \to \sim q$** 3. $\sim q \to \sim p$ **$q \to p$**

4. If the base angles of a triangle are congruent, then the triangle is isosceles. **If the base angles of a triangle are not congruent, then the triangle is not isosceles.**

5. If the moon is full tonight, then we'll have frost by morning. **If the moon is not full tonight, then we won't have frost by morning.**

Tell whether each conditional is true or false. Then write the inverse of the conditional and tell whether the inverse is true or false.

6. If this is October, then the next month is December. **false; If this is not October, then the next month is not December; false**

7. If $x > 5$, then $x > 6$, $x \in R$. **false; if $x \le 5$, then $x \le 6$, $x \in R$; true**

8. If $x = 0$, then $x^{\frac{1}{2}} = 0$, $x \in R$. **true; if $x \ne 0$, then $x^{\frac{1}{2}} \ne 0$, $x \in R$; true**

9. Make a conjecture about the truth value of an inverse if the conditional is false. **The inverse is sometimes true.**

4-7

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Practice

Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x-2} = 6$ **38**
2. $\sqrt[3]{x^2 - 1} = 3$ **$\pm 2\sqrt{7}$**
3. $\sqrt[3]{7r + 5} = -3$ **$-\frac{32}{7}$**
4. $\sqrt{6x + 12} - \sqrt{4x + 9} = 1$ **4**
5. $\sqrt{x-3} - 3\sqrt{x + 12} = -11$ **$4, \frac{97}{16}$**
6. $\sqrt{6n - 3} = \sqrt{4 + 7n}$ **no real solution**
7. $5 + 2x = \sqrt{x^2 - 2x + 1}$ **$-\frac{4}{3}$**
8. $3 - \sqrt{r + 1} = \sqrt{4 - r}$ **0, 3**

Solve each inequality.

9. $\sqrt{3r + 5} > 1$ **$r > -\frac{4}{3}$**
10. $\sqrt{2t - 3} < 5$ **$\frac{3}{2} < t < 14$**
11. $\sqrt{2m + 3} > 5$ **$m > 11$**
12. $\sqrt{3x + 5} < 9$ **$-\frac{5}{3} < x < \frac{76}{3}$**

13. ***Engineering*** A team of engineers must design a fuel tank in the shape of a cone. The surface area of a cone (excluding the base) is given by the formula $S = \pi\sqrt{r^2 + h^2}$. Find the radius of a cone with a height of 21 meters and a surface area of 155 meters squared. **about 44.6 m**

4-7

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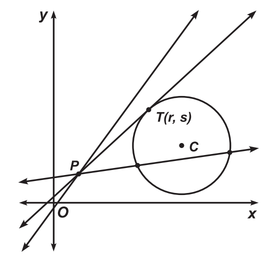
Enrichment

Discriminants and Tangents

The diagram at the right shows that through a point P outside of a circle C, there are lines that do not intersect the circle, lines that intersect the circle in one point (tangents), and lines that intersect the circle in two points (secants).



Given the coordinates for P and an equation for the circle C, how can we find the equation of a line tangent to C that passes through P?

Suppose P has coordinates $P(0, 0)$ and $\odot C$ has equation $(x - 4)^2 + y^2 = 4$. Then a line tangent through P has equation $y = mx$ for some real number m.

Thus, if $T(r, s)$ is a point of tangency, then $s = mr$ and $(r - 4)^2 + s^2 = 4$.

Therefore, $(r - 4)^2 + (mr)^2 = 4$.

$$r^2 - 8r + 16 + m^2r^2 = 4$$
$$(1 + m^2)r^2 - 8r + 16 = 4$$
$$(1 + m^2)r^2 - 8r + 12 = 0$$

The equation above has exactly one real solution for r if the discriminant is 0, that is, when $(-8)^2 - 4(1 + m^2)(12) = 0$. Solve this equation for m and you will find the slopes of the lines through P that are tangent to circle C.

1. **a.** Refer to the discussion above. Solve $(-8)^2 - 4(1 + m^2)(12) = 0$ to find the slopes of the two lines tangent to circle C through point P.
   $\pm\frac{\sqrt{3}}{3}$

   b. Use the values of m from part **a** to find the coordinates of the two points of tangency.
   $(3, \pm\sqrt{3})$

2. Suppose P has coordinates $(0, 0)$ and circle C has equation $(x + 9)^2 + y^2 = 9$. Let m be the slope of the tangent line to C through P.

   a. Find the equations for the lines tangent to circle C through point P.
   $y = \pm\frac{\sqrt{2}}{4}x$

   b. Find the coordinates of the points of tangency.
   $(-8, \pm 2\sqrt{2})$

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4-8

Practice

Modeling Real-World Data with Polynomial Functions

Write a polynomial function to model each set of data.

1. The farther a planet is from the Sun, the longer it takes to complete an orbit.

Distance (AU)	0.39	0.72	1.00	1.49	5.19	9.51	19.1	30.0	39.3
Period (days)	88	225	365	687	4344	10,775	30,681	60,267	90,582

Source: *Astronomy: Fundamentals and Frontiers*, by Jastrow, Robert, and Malcolm H. Thompson.

Sample answer: $f(x) = 35x^2 + 962x - 791$

2. The amount of food energy produced by farms increases as more energy is expended. The following table shows the amount of energy produced and the amount of energy expended to produce the food.

Energy Input (Calories)	606	970	1121	1227	1318	1455	1636	2030	2182	2242
Energy Output (Calories)	133	144	148	157	171	175	187	193	198	198

Source: *NSTA Energy-Environment Source Book.*

Sample answer: $f(x) = -3.9x^3 + 1.5x^2 - 0.1x + 167.0$

3. The temperature of Earth's atmosphere varies with altitude.

Altitude (km)	0	10	20	30	40	50	60	70	80	90
Temperature (K)	293	228	217	235	254	269	244	207	178	178

Source: *Living in the Environment*, by Miller G. Tyler.

Sample answer: $f(x) = -0.0008x^3 + 0.1x^2 - 3.6x + 274.7$

4. Water quality varies with the season. This table shows the average hardness (amount of dissolved minerals) of water in the Missouri River measured at Kansas City, Missouri.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hardness ($CaCO_3$ ppm)	310	250	180	175	230	175	170	180	210	230	295	300

Source: *The Encyclopedia of Environmental Science*, 1974.

Sample answer: $f(x) = 0.1x^4 - 1.6x^3 + 19.7x^2 - 110.0x + 397.7$

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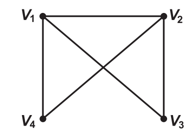
4-8

Enrichment

Number of Paths

For the figure and adjacency matrix shown at the right, the number of paths or circuits of length 2 can be found by computing the following product.



$$A = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$A^2 = AA = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

In row 3 column 4, the entry 2 in the product matrix means that there are 2 paths of length 2 between V_3 and V_4. The paths are $V_3 \to V_1 \to V_4$ and $V_3 \to V_2 \to V_4$. Similarly, in row 1 column 3, the entry 1 means there is only 1 path of length 2 between V_1 and V_3.

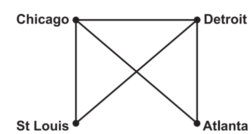
Name the paths of length 2 between the following.

1. V_1 and V_2 **V_1, V_3, V_2; V_1, V_4, V_2**

2. V_1 and V_3 **V_1, V_2, V_3**

3. V_1 and V_1 **V_1, V_2, V_1; V_1, V_3, V_1; V_1, V_4, V_1**

For Exercises 4-6, refer to the figure below.



4. The number of paths of length 3 is given by the product $A \cdot A \cdot A$ or A^3. Find the matrix for paths of length 3. $\begin{bmatrix} \mathbf{4} & \mathbf{5} & \mathbf{5} & \mathbf{5} \\ \mathbf{5} & \mathbf{4} & \mathbf{5} & \mathbf{5} \\ \mathbf{5} & \mathbf{5} & \mathbf{2} & \mathbf{2} \\ \mathbf{5} & \mathbf{5} & \mathbf{2} & \mathbf{2} \end{bmatrix}$

5. How many paths of length 3 are there between Atlanta and St. Louis? Name them. **2; *A, D, C, S* and *A, C, D, S***

6. How would you find the number of paths of length 4 between the cities? **Find Matrix A^4.**

Chapter 4 Answer Key

Form 1A

Page 155

1. B
2. A
3. B
4. A
5. A
6. B
7. D
8. C
9. B
10. A

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11. D
12. C
13. D
14. A
15. C
16. B
17. D
18. A
19. C
20. C

Bonus: D

Form 1B

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1. C
2. D
3. B
4. B
5. A
6. A
7. B
8. D
9. C
10. C
11. D

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12. A
13. D
14. A
15. A
16. B
17. A
18. B
19. D
20. C

Bonus: A

Chapter 4 Answer Key

Form 1C

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1. B
2. A
3. D
4. D
5. C
6. D
7. B
8. C
9. C
10. B
11. B

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12. C
13. B
14. A
15. D
16. B
17. C
18. D
19. A
20. D

Bonus: B

Form 2A

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1. $-3, \frac{13}{3}$
2. $2 \pm i$
3. $a \leq -\frac{13}{4}$, $a > -2$
4. $\frac{2}{9}$
5. $\frac{1}{4} \leq b \leq \frac{5}{7}$
6. $x < -3$; $-2 < x < 2$; $x > 3$
7. no solution
8. 273; no
9. 0 and 1, −1 and 0; at −2 and 3
10. $\frac{5}{2x+1} - \frac{4}{x-3}$
11. 2
12. ± 1.6

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13. lower −1, upper 2
14. Sample answer: $y = x^3 - 19x^2 + 116x - 213$
15. 49; 2 real
16. 3 or 1; 1
17. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$
18. 2
19. $x^4 + 5x^2 - 36 = 0$; 2
20. about 4.5 ft

Bonus: $f(x) = x^3 - 4x^2$

Chapter 4 Answer Key

Form 2B

Page 163

1. $-\frac{13}{8}, 1$
2. $2 \pm 3i$
3. $q \le -\frac{3}{4}, q > 0$
4. -1
5. $n \ge 7$
6. $-8 < a < -3$, $a > -3$
7. no solution
8. 0; yes
9. -1 and 0; 2 and 3; at -1
10. $\frac{3}{x+4} + \frac{5}{x-1}$
11. -5

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12. ± 1.2
13. Sample answer: upper 3, lower -1
14. Sample answer: $y = 0.1x^3 - 3.3x^2 + 23.9x - 45.2$
15. 0; 1 real root
16. 1; 2 or 0
17. $\pm 1, \pm 2, \pm\frac{1}{4}, \pm\frac{1}{2}$
18. -3
19. $2x^4 + x^3 - 9x^2 - 4x + 4 = 0$; 4
20. about 1.06 s

Bonus: $f(x) = (x + 1)(x - 1)^3$

Form 2C

Page 165

1. $\frac{1 \pm 3\sqrt{2}}{2}$
2. $3 \pm \frac{5\sqrt{2}}{2}$
3. $0 < x < \frac{5}{2}$
4. 23
5. $-\frac{5}{2} \le x \le 10$
6. $y \le -\frac{13}{18}, y > 0$
7. 2
8. -1; no
9. 1 and 2
10. $\frac{6}{x-2} + \frac{4}{x+2}$
11. 1
12. 3.5

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13. upper 3; lower -1
14. Sample answer: $y = -1.0x^3 + 4.1x^2 - 0.3x - 4.6$
15. -55; 2 complex roots
16. 2 or 0; 1
17. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$
18. $-4, 1, \frac{3}{2}$
19. $x^3 - x^2 - 17x - 15 = 0$; 3
20. quartic

Bonus: ± 6

Chapter 4 Answer Key

CHAPTER 4 SCORING RUBRIC

Level	Specific Criteria
3 Superior	• Shows thorough understanding of the concepts *positive, negative, and real roots; rational equations;* and *radical equations.* • Uses appropriate strategies to solve problems and finds the number of roots. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
2 Satisfactory, with Minor Flaws	• Shows understanding of the concepts *positive, negative, and real roots; rational equations;* and *radical equations.* • Uses appropriate strategies to solve problems and finds the number of roots. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
1 Nearly Satisfactory, with Serious Flaws	• Shows understanding of most of the concepts *positive, negative, and real roots; rational equations;* and *radical equations.* • May not use appropriate strategies to solve problems and find the number of roots. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies most requirements of problems.
0 Unsatisfactory	• Shows little or no understanding of the concepts *positive, negative, and real roots; rational equations;* and *radical equations.* • May not use appropriate strategies to solve problems and find the number of roots. • Computations are incorrect. • Written explanations are not satisfactory. • Does not satisfy requirements of problems.

Chapter 4 Answer Key

Open-Ended Assessment

Page 167

1a. Sample answer: $x^2 + 2x + 2 = 0$. The discriminant of the equation is -4, which is less than 0. Therefore, the equation has no real roots.

1b. Sample answer: $x^2 + 2x - 2 = 0$. The discriminant of the equation is 12, which is greater than 0. Therefore, the equation has two real roots.

1c. Sample answer: $x^2 + 2x + 1 = 0$. The discriminant of the equation is 0. Therefore, the equation has exactly one real root.

2a. The number of possible positive real zeros is 3 or 1 because there are three sign changes in $f(x)$.

2b. The number of possible negative real zeros is 2 or 0 because there are two sign changes in $f(-x)$.

2c. The possible rational zeros are $\pm\frac{1}{6}$, $\pm\frac{1}{3}$, $\pm\frac{1}{2}$, $\pm\frac{2}{3}$, $\pm\frac{4}{3}$, ±1, ±2, and ±4 .

2d. No, it has five zeros, and complex roots are in pairs.

2e. No, because there are no sign changes in the quotient and remainder when the polynomial is divided by $x - 2$.

2f. Sample answer: add $-x^6$. Adding the term does not change the number of possible negative real zeros because the number of sign changes in $f(-x)$ does not increase.

2g. $y = 2x^2 + 3x + 2$; The equation has no positive real roots. There may be two negative real roots. Possible rational roots are $-\frac{1}{2}$, -1, and -2, because the equation has no positive real roots.

3a. 15 ft

3b. the Pythagorean Theorem; the hypotenuse and the legs of a right triangle

Chapter 4 Answer Key

Mid-Chapter Test
Page 168

1. Yes, because $f(-1) = 0$
2. $x^4 + 3x^3 - 3x^2 + 3x - 4 = 0$; 2
3. $\pm 2\sqrt{5}i$
4. $-3 \pm i$
5. 169; $\frac{2}{3}$, $-\frac{3}{2}$
6. 0; yes
7. -5
8. $\pm 1, \pm 3, \pm 9; \pm 1, \pm 3$
9. $\pm 1, \pm \frac{1}{2}, \pm 2; -2$
10. 2 or 0; 1; ± 1

Quiz A
Page 169

1. No, because $f(-3) = -2$.
2. $x^4 - 2x^3 + x^2 - 8x - 12 = 0$; 2
3. $\pm \frac{1}{2}i$
4. $-5 \pm i\sqrt{10}$
5. -44; 2 imaginary roots; $\frac{2 \pm i\sqrt{11}}{15}$

Quiz B
Page 169

1. 18; no
2. 0; yes
3. -5
4. $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \pm 2; -1, \frac{1}{3}, 2$
5. 2 or 0; 1; $-\frac{1}{2}$, 1, 4

Quiz C
Page 170

1. -5 and -4; -1 and 0; 1 and 2
2. -0.9, 2.8
3. $4 \pm 2\sqrt{3}$
4. $\frac{6}{p} + \frac{5}{p-2} - \frac{7}{p+1}$
5. $-1 \le w < 0$, $\frac{2}{5} \le w < 1$

Quiz D
Page 170

1. 7
2. -13
3. $t > 14$
4. cubic
5. Sample answer: $f(x) = 4x^4 - 24x^3 + 35x^2 + 6x - 9$

Chapter 4 Answer Key

SAT/ACT Practice

Page 171

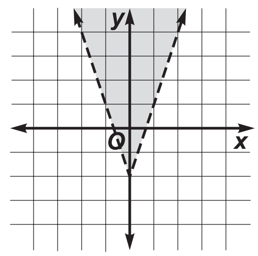
1. A
2. D
3. C
4. E
5. D
6. B
7. A
8. B
9. D
10. A

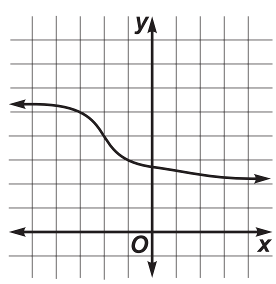
Page 172

11. D
12. D
13. A
14. E
15. A
16. A
17. A
18. C
19. 6
20. 2.5

Cumulative Review

Page 173

1. $D = \{-2, 0, 3\}$, $R = \{-2, 5\}$; yes
2. $3x^2 + 5$
3. 

4. $(2, -1, 3)$
5. $A'(-1, -1)$, $B'(2, -2)$, $C'(1, -3)$
6. $159
7. y-axis
8. 

9. $x \to \infty, y \to -\infty$, $x \to -\infty, y \to \infty$
10. $y = x - 4$
11. 2
12. $-5, 2, 2 \pm \sqrt{3}$

Unit 1 Answer Key

Unit 1 Review

1. {(−1, −2), (0, 1), (1, 4), (2, 7), (3, 10)}; yes

2. {(−2, 4), (−1, 3), (0, 2), (1, 1), (2, 0), (3, 1)}; yes

3. $3x + 10$; $3x + 4$

4. $4x^6$; $-64x^6$

5. $4x^2 - 16x - 9$; $2x^2 - 54$

6. $\frac{5}{2}$ **7.** 0 **8.** −4

9. $y = -3x + 8$

10. $y = \frac{1}{2}x + \frac{3}{2}$

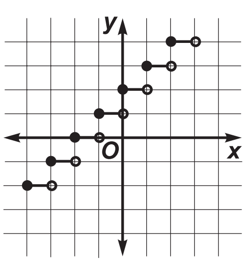
11. $3x - y + 7 = 0$

12. $3x + 2y - 6 = 0$

13. $y = 11x - 1$

14. 87; No, because as the sale continues, fewer students will be left to buy T-shirts. The number of shirts sold will have to decrease eventually.

15.



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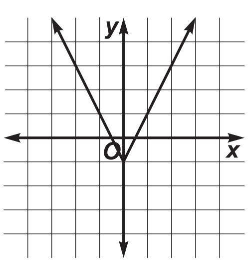
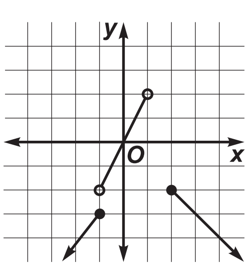
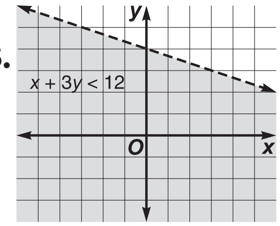


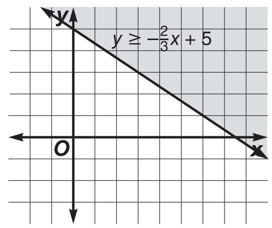
17.



18.



19.



20. (1, −4) **21.** $\left(\frac{8}{3}, \frac{28}{3}\right)$

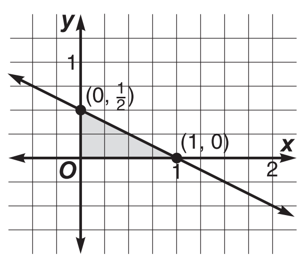
22. (3, −6, 8) **23.** $\begin{bmatrix} 2 & 8 \\ 8 & 4 \end{bmatrix}$

24. $\begin{bmatrix} 16 & -2 \\ 1 & -13 \end{bmatrix}$ **25.** $\begin{bmatrix} 23 & -5 \\ -63 & 23 \end{bmatrix}$

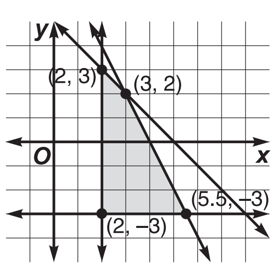
26. $\begin{bmatrix} 9 & 45 \\ -90 & 20 \end{bmatrix}$ **27.** −24

28. 19 **29.** $\begin{bmatrix} -\frac{7}{58} & \frac{3}{29} \\ \frac{5}{58} & \frac{2}{29} \end{bmatrix}$

30. (0, 0), (1, 0), $\left(0, \frac{1}{2}\right)$; −2, −5

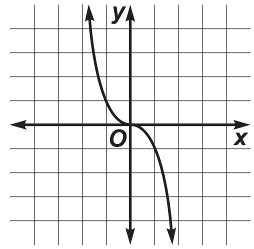


31. (2, −3), (5.5, −3), (3, 2), (2, 3); −1, −20

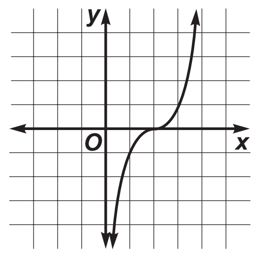


32. odd **33.** even **34.** neither

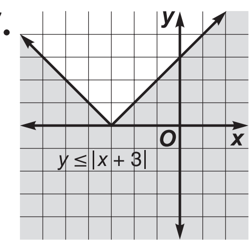
35. reflected over *x*-axis



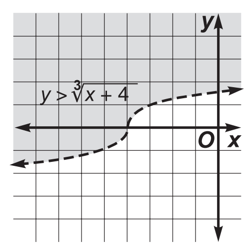
36. translated 2 units right



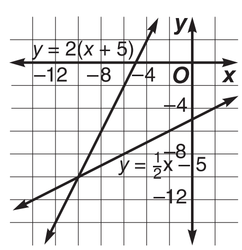
37.



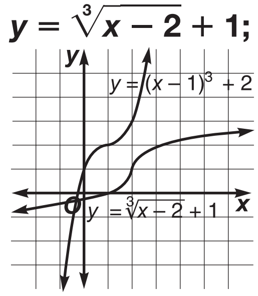
38.



39. $y = 2(x + 5)$; yes

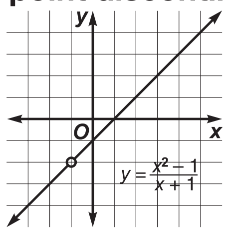


40. $y = \sqrt[3]{x - 2} + 1$; yes

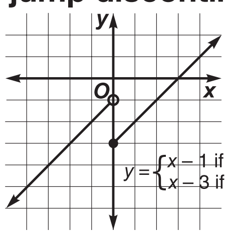


Unit 1 Answer Key (continued)

41. point discontinuity



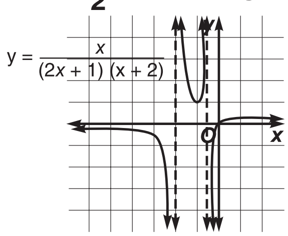
42. jump discontinuity



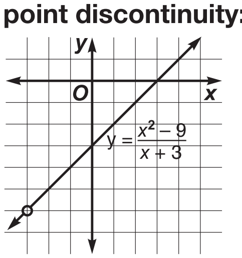
43. max.: $(-1, 1)$; min.: $(1, 1)$

44. pt. of inflection: $(1, 0)$

45. $x = -\frac{1}{2}$, $x = -2$, $y = 0$



46. slant asymptote: $y = x - 3$
point discontinuity: $x = -3$



47. 4 48. $\frac{1 \pm \sqrt{11}}{2}$ 49. $\frac{25}{2}$

50. $0 < x < 2$ or $x > \frac{9}{4}$

51. no real solution

52. $-8 \leq x \leq 1$

53. 34 54. 11

55. 1; 1; -1, $\frac{2}{3}$

56. 3 or 1; 1; none

57. -1.4, 3.4

58. -1, -3.6, 0.6

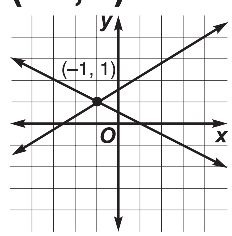
Unit 1 Test

1. max.: 6; min.: 3

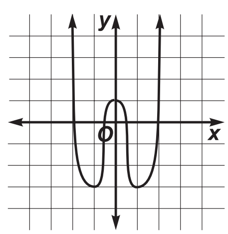
2. $x^4 + 10x^2 + 9 = 0$

3. $4x^2 - 9x + 25$

4. $(-1, 1)$



5.



6. (5, 1, 2)

7. $\frac{5}{n + 6} - \frac{3}{4n - 1}$

8. both axes

9. as $x \to \infty$, $f(x) \to -\infty$;
as $x \to -\infty$, $f(x) \to -\infty$

10. infinitely many

11. $-\frac{2}{3}$, 3

12. $\frac{-2 - \sqrt{10}}{3} \leq y \leq \frac{-2 + \sqrt{10}}{3}$

13. $-\frac{1}{2x^2}$ 14. no

15. no; $y = \pm\sqrt{10x}$ 16. yes; 8

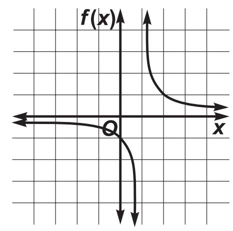
17. translated 2 units to the right

18. $y = -x - 1$ 19. yes

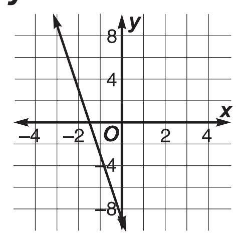
20. 20 mph 21. no solution

22. 64 23. (4, 2) 24. 1.5

25.



26. $y = -6x - 9$



27. $x + 3y - 18 = 0$

28. 5; no 29. 12 30. 11

31. $y = x - 5$ 32. -41

33. $D = \{-5, -2, 4\}$;
$R = \{0, 1, 2, 3\}$; no

34. neither 35. 1

36. $-\frac{2}{3} \leq x \leq 2$

37. $\begin{bmatrix} 3 & 11 \\ 4 & 18 \end{bmatrix}$ 38. -5

39. $\{(-1, 1), (0, 2), (1, 1)\}$; yes

40. infeasible 41. $\frac{-2 \pm 3\sqrt{2}}{2}$

42. infinite discontinuity

43. $-\frac{8}{3a}$ 44. 2000; 3500

45. 3 ft × 7 ft × 4 ft

46. $(4, -1, 3)$ 47. $\frac{1}{2}$, $-\frac{7}{2}$

48. max.: $(1, -3)$ 49. (1, 2)

50. $3x + 2y - 11 = 0$