Chapter 4 Polynomial and Rational Functions

4-1 Polynomial Functions

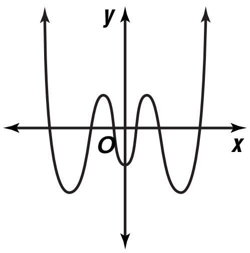
Pages 209–210 Check for Understanding

1. A zero is the value of the variable for which a polynomial function in one variable equals zero. A root is a solution of a polynomial equation in one variable. When a polynomial function is the related function to the polynomial equation, the zeros of the function are the same as the roots of the equation.

2. The ordered pair $(x, 0)$ represents the points on the x-axis. Therefore, the x-intercept of a graph of a function represents the point where $f(x) = 0$.

3. A complex number is any number in the form $a + b\boldsymbol{i}$, where a and b are real numbers and $\boldsymbol{i}$ is the imaginary unit. In a pure imaginary number, $a = 0$ and $b \neq 0$. Examples: $-2\boldsymbol{i}$, $3\boldsymbol{i}$; Nonexamples: 5, $1 + \boldsymbol{i}$

4.



5. 3; 1 **6.** 5; 8

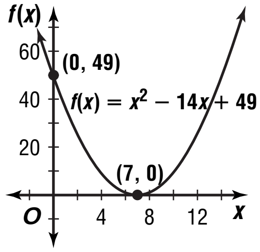
7. no; $f(x) = x^3 - 5x^2 - 3x - 18$
$f(5) = (5)^3 - 5(5)^2 - 3(5) - 18$
$f(5) = 125 - 125 - 15 - 18$
$f(5) = -33$

8. yes; $f(x) = x^3 - 5x^2 - 3x - 18$
$f(6) = (6)^3 - 5(6)^2 - 3(6) - 18$
$f(6) = 216 - 180 - 18 - 18$
$f(6) = 0$

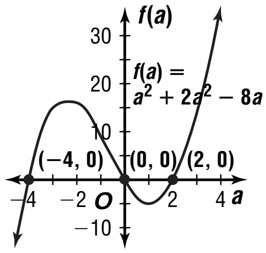
9. $(x - (-5))(x - 7) = 0$
$(x + 5)(x - 7) = 0$
$x^2 - 2x - 35 = 0$; even; 2

10. $(x - 6)(x - 2\boldsymbol{i})(x - (-2\boldsymbol{i}))(x - \boldsymbol{i})(x - (-\boldsymbol{i})) = 0$
$(x - 6)(x - 2\boldsymbol{i})(x + 2\boldsymbol{i})(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$
$(x - 6)(x^2 - 4\boldsymbol{i}^2)(x^2 - \boldsymbol{i}^2) = 0$
$(x - 6)(x^2 + 4)(x^2 + 1) = 0$
$(x^3 - 6x^2 + 4x - 24)(x^2 + 1) = 0$
$x^5 - 6x^4 + 5x^3 - 30x^2 + 4x - 24 = 0$;
odd; 1

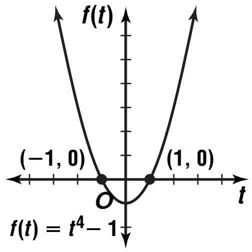
11. 2; $x^2 - 14x + 49 = 0$
$(x - 7)(x - 7) = 0$
$x - 7 = 0$ $\quad$ $x - 7 = 0$
$x = 7$ $\quad$ $x = 7$



12. 3; $a^3 + 2a^2 - 8a = 0$
$a(a^2 + 2a - 8) = 0$
$a(a + 4)(a - 2) = 0$
$a = 0$ $\quad$ $a + 4 = 0$ $\quad$ $a - 2 = 0$
$a = -4$ $\quad$ $a = 2$



13. 4; $t^4 - 1 = 0$
$(t^2 - 1)(t^2 + 1) = 0$
$(t - 1)(t + 1)(t^2 + 1) = 0$
$t - 1 = 0$ $\quad$ $t + 1 = 0$ $\quad$ $t^2 + 1 = 0$
$t = 1$ $\quad$ $t = -1$ $\quad$ $t^2 = -1$
$t = \pm\boldsymbol{i}$



14a. $x^2 + r^2 = 6^2$ $\quad$ $V(x) = Bh$
$r^2 = 36 - x^2$ $\quad$ $V(x) = \pi(36 - x^2)(2x)$

14b. $V(x) = \pi(36 - x^2)(2x)$
$V(x) = (36\pi - \pi x^2)(2x)$
$V(x) = 72\pi x - 2\pi x^3$

14c. $V(x) = 72\pi x - 2\pi x^3$
$V(4) = 72\pi(4) - 2\pi(4)^3$
$V(4) \approx 502.65$ units^3

Pages 210–212 Exercises

15. 4; 5 **16.** 7; 3 **17.** 3; 5 **18.** 5; −25

19. 6; −1 **20.** 2; 1

21. Yes; the coefficients are complex numbers and the exponents of the variable are nonnegative integers.

22. No; $\frac{1}{a} = a^{-1}$, which is a negative exponent.

23. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(0) = (0)^4 - 13(0)^2 + 12(0)$
$f(0) = 0$

24. no; $f(a) = a^4 - 13a^2 + 12a$
$f(-1) = (-1)^4 - 13(-1)^2 + 12(-1)$
$f(-1) = 1 - 13 - 12$
$f(-1) = -24$

25. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(1) = (1)^4 - 13(1)^2 + 12(1)$
$f(1) = 1 - 13 + 12$
$f(1) = 0$

26. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(-4) = (-4)^4 - 13(-4)^2 + 12(-4)$
$f(-4) = 256 - 208 - 48$
$f(-4) = 0$

27. no; $f(a) = a^4 - 13a^2 + 12a$
$f(-3) = (-3)^4 - 13(-3)^2 + 12(-3)$
$f(-3) = 81 - 117 - 36$
$f(-3) = -72$

28. yes; $f(a) = a^4 - 13a^2 + 12a$
$f(3) = (3)^4 - 13(3)^2 + 12(3)$
$f(3) = 81 - 117 + 36$
$f(3) = 0$

29. $f(b) = b^4 - 3b^2 - 2b + 4$
$f(-2) = (-2)^4 - 3(-2)^2 - 2(-2) + 4$
$f(-2) = 16 - 12 + 4 + 4$
$f(-2) = 12$; no

30. $f(x) = x^4 - 4x^3 - x^2 + 4x$
$f(-1) = (-1)^4 - 4(-1)^3 - (-1)^2 + 4(-1)$
$f(-1) = 1 + 4 - 1 - 4$
$f(-1) = 0$; yes

31a. 3; 1 **31b.** 2; 2 **31c.** 4; 2

32. $(x - (-2))(x - 3) = 0$
$(x + 2)(x - 3) = 0$
$x^2 - x - 6 = 0$; even; 2

33. $(x - (-1))(x - 1)(x - 5) = 0$
$(x + 1)(x - 1)(x - 5) = 0$
$(x^2 - 1)(x - 5) = 0$
$x^3 - 5x^2 - x + 5 = 0$; odd; 3

34. $(x - (-2))(x - (-0.5))(x - 4) = 0$
$(x + 2)(x + 0.5)(x - 4) = 0$
$(x^2 + 2.5x + 1)(x - 4) = 0$
$x^3 - 4x^2 + 2.5x^2 - 10x + x - 4 = 0$
$x^3 - 1.5x^2 - 9x - 4 = 0$
$2x^3 - 3x^2 - 18x - 8 = 0$; odd; 3

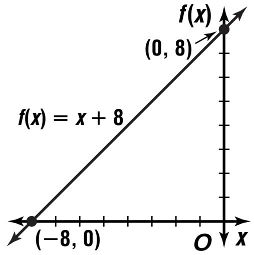
35. $(x - (-3))(x - (-2\boldsymbol{i}))(x - 2\boldsymbol{i}) = 0$
$(x + 3)(x + 2\boldsymbol{i})(x - 2\boldsymbol{i}) = 0$
$(x + 3)(x^2 - 4\boldsymbol{i}^2) = 0$
$(x + 3)(x^2 + 4) = 0$
$x^3 + 3x^2 + 4x + 12 = 0$; odd; 1

36. $(x - (-5\boldsymbol{i}))(x - (-\boldsymbol{i}))(x - \boldsymbol{i})(x - 5\boldsymbol{i}) = 0$
$(x + 5\boldsymbol{i})(x + \boldsymbol{i})(x - \boldsymbol{i})(x - 5\boldsymbol{i}) = 0$
$(x + 5\boldsymbol{i})(x - 5\boldsymbol{i})(x - \boldsymbol{i})(x + \boldsymbol{i}) = 0$
$(x^2 + 25)(x^2 + 1) = 0$
$x^4 + 26x^2 + 25 = 0$; even; 0

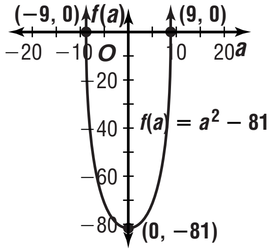
37. $(x - (-1))(x - 1)(x - 4)(x - (-4)(x - 5) = 0$
$(x + 1)(x - 1)(x - 4)(x + 4)(x - 5) = 0$
$(x^2 - 1)(x^2 - 16)(x - 5) = 0$
$(x^4 - 17x^2 + 16)(x - 5) = 0$
$x^5 - 5x^4 - 17x^3 + 85x^2 + 16x - 80 = 0$; odd; 5

38. $(x - (-1))(x - 1)(x - 3)(x - (-3)) = 0$
$(x + 1)(x - 1)(x - 3)(x + 3) = 0$
$(x^2 - 1)(x^2 - 9) = 0$
$x^4 - 10x^2 + 9 = 0$

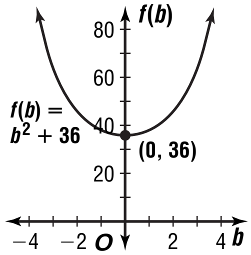
39. 1; $x + 8 = 0$
$x = -8$



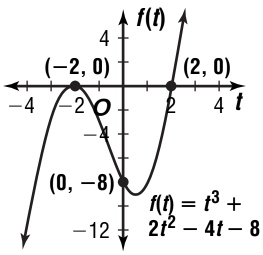
40. 2; $a^2 - 81 = 0$
$(a - 9)(a + 9) = 0$
$a - 9 = 0$ $\quad a + 9 = 0$
$a = 9$ $\quad a = -9$



41. 2; $b^2 + 36 = 0$
$b^2 = -36$
$b = \pm 6\boldsymbol{i}$



42. 3; $t^3 + 2t^2 - 4t - 8 = 0$
$t^2(t + 2) - 4(t + 2) = 0$
$(t + 2)(t^2 - 4) = 0$
$(t + 2)(t + 2)(t - 2) = 0$
$t + 2 = 0$ $\quad t + 2 = 0$ $\quad t - 2 = 0$
$t = -2$ $\quad t = -2$ $\quad t = 2$



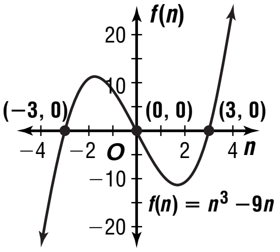
43. 3; $n^3 - 9n = 0$

$n(n^2 - 9) = 0$

$n(n - 3)(n + 3) = 0$

$n = 0 \qquad n - 3 = 0 \qquad n + 3 = 0$

$n = 3 \qquad n = -3$



44. 3; $6c^3 - 3c^2 - 45c = 0$

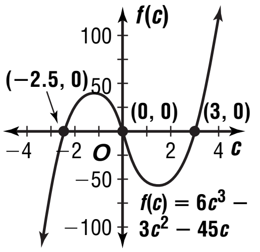
$c(6c^2 - 3c - 45) = 0$

$c(c - 3)(6c + 15) = 0$

$c = 0 \qquad c - 3 = 0 \qquad 6c + 15 = 0$

$c = 3 \qquad c = -\frac{15}{6}$

$c = -2.5$



45. 4; $a^4 + a^2 - 2 = 0$

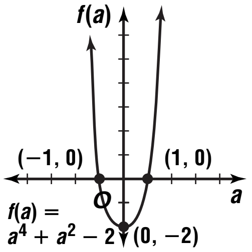
$(a^2 + 2)(a^2 - 1) = 0$

$(a^2 + 2)(a - 1)(a + 1) = 0$

$a^2 + 2 = 0 \qquad a - 1 = 0 \qquad a + 1 = 0$

$a^2 = -2 \qquad a = 1 \qquad a = -1$

$a = \pm\sqrt{2}i$



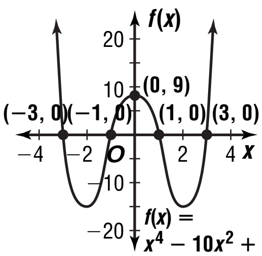
46. 4; $x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$(x - 3)(x + 3)(x - 1)(x + 1) = 0$

$x - 3 = 0 \qquad x + 3 = 0 \qquad x - 1 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -3 \qquad x = 1 \qquad x = -1$



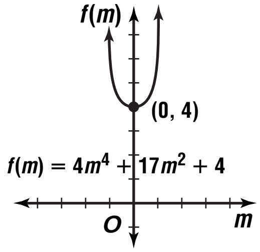
47. 4; $4m^4 + 17m^2 + 4 = 0$

$(4m^2 + 1)(m^2 + 4) = 0$

$4m + 1 = 0 \qquad m^2 + 4 = 0$

$m = \pm\sqrt{\frac{-1}{4}} \qquad m = \pm\sqrt{-4}$

$m = \pm 0.5i \qquad m = \pm 2i$

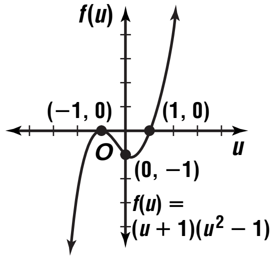


48. $(u + 1)(u^2 - 1) = 0$

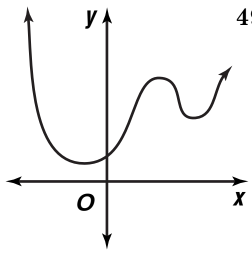
$(u + 1)(u + 1)(u - 1) = 0$

$u + 1 = 0 \qquad u + 1 = 0 \qquad u - 1 = 0$

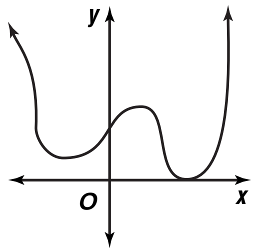
$u = -1 \qquad u = -1 \qquad u = 1$



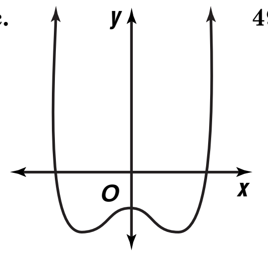
49a.



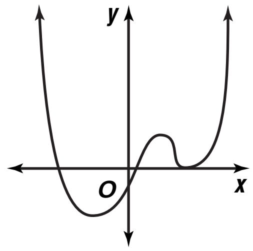
49b.



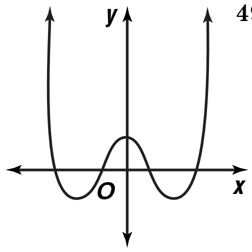
49c.



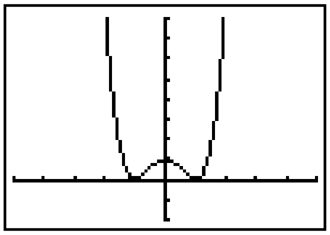
49d.



49e.



49f. not possible

50. 

$[-5, 5]$ sc1:1 by $[-2, 8]$ sc1:1

50a. 4

50b. 2; -1, 1

50c. There are 4 real roots. However, there is a double root at -1 and a double root at 1.

51a. $V(x) = 99{,}000x^3 + 55{,}000x^2 + 65{,}000x$

51b. $r = 0.15$

$x = 1 + r$
$x = 1 + 0.15$
$x = 1.15$

$V(x) = 99{,}000x^3 + 55{,}000x^2 + 65{,}000x$
$V(1.15) = 99{,}000(1.15)^3 + 55{,}000(1.15)^2 + 65{,}000(1.15)$
$V(1.15) = 150{,}566.625 + 72{,}737.5 + 74{,}750$
$V(1.15) = 298{,}054.125$; about \$298,054.13

52. 1 and 3 are two of its zeros.

53a.
$d(t) = \frac{1}{2}at^2$
$d(30) = \frac{1}{2}(16.4)(30)^2$
$d(30) = 7380$ ft

$d(t) = \frac{1}{2}at^2$
$d(60) = \frac{1}{2}(16.4)(60)^2$
$d(60) = 29{,}520$ ft

$d(t) = \frac{1}{2}at^2$
$d(120) = \frac{1}{2}(16.4)(120)^2$
$d(120) = 118{,}080$ ft

53b. It quadruples; $(2t)^2 = 4t^2$.

54. Let x = the width of the sidewalk.
The length of the pool would be $70 - 2x$ feet.
The width of the pool would be $50 - 2x$ feet.
$A = \ell w$
$2400 = (70 - 2x)(50 - 2x)$
$2400 = 3500 - 240x + 4x^2$
$0 = 4x^2 - 240x + 1100$
$0 = x^2 - 60x + 275$
$0 = (x - 55)(x - 5)$
$x - 55 = 0$ $\quad$ $x - 5 = 0$
$x = 55$ $\quad$ $x = 5$
Use $x = 5$ since 55 is an unreasonable solution.
5 ft

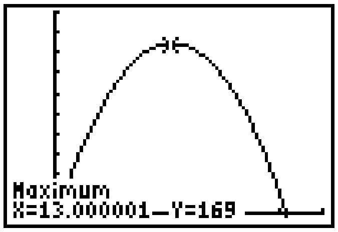
55. Let x = the number of pizzas.
$(160 + 16x)(16 - 0.40x) = 4000$
$-6.4x^2 + 192x + 2560 = 4000$
$-6.4x^2 + 192x - 1440 = 0$
$x^2 - 30x - 225 = 0$
$(x - 15)(x - 15) = 0$
$x - 15 = 0$ $\quad$ $x - 15 = 0$
$x = 15$ $\quad$ $x = 15$
$16 - 0.40x = 16 - 0.40(15)$
$= \$10$

56.
$(x - B)(x - C) = 0$
$x^2 - Cx - Bx + BC = 0$
$x^2 - (C + B)x + BC = 0$
$\left.\begin{array}{l} -C - B = B \\ BC = C \end{array}\right\}$ from $x^2 + Bx + C = 0$
$B = 1$
$-C - 1 = 1$
$-C = 2$
$C = -2$
Sample answer: 1; -2

57. $y = \frac{x - 2}{x(x + 2)(x - 2)}$

58a. Let x = the width. The length $= \frac{1}{2}(52 - 2x)$ or $26 - x$. $A(x) = x(26 - x)$

58b. $A(x) = x(26 - x)$
$A(x) = 26x - x^2$



$[-5, 30]$ sc1:5 by $[-2, 200]$ sc1:20
$x = 13$
$26 - x = 26 - 13$
$= 13$
13 yd by 13 yd

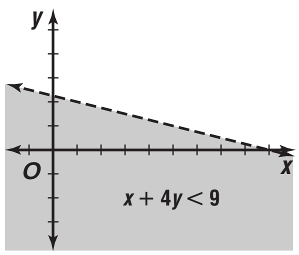
59. The graph of $y = 2x^3 + 1$ is the graph of $y = 2x^3$ shifted 1 unit up.

60. $(-6, 9)$

61. $\begin{vmatrix} -15 & 5 \\ -9 & 3 \end{vmatrix} = -15(3) - (-9)(5)$
$= -45 + 45$ or 0; no

62. $AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$
$= \begin{bmatrix} 2(3) + (-1)(5) & 2(-9) + (-1)(7) & 2(2) + (-1)(-6) \\ 3(3) + 4(5) & 3(-9) + 4(7) & 3(2) + 4(-6) \end{bmatrix}$
$= \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$

63. $x + 4y < 9$
$4y < -x + 9$
$y < -\frac{1}{4}x + \frac{9}{4}$



64. Parallel; the lines have the same slope.

65. $[f \circ g](x) = f(g(x))$
$= f\left(\frac{1}{2}x + 6\right)$
$= \left(\frac{1}{2}x + 6\right)^2 - 4$
$= \frac{1}{4}x^2 + 6x + 36 - 4$
$= \frac{1}{4}x^2 + 6x + 32$

$[g \circ f](x) = g(f(x))$
$= g(x^2 - 4)$
$= \frac{1}{2}(x^2 - 4) + 6$
$= \frac{1}{2}x^2 - 2 + 6$
$= \frac{1}{2}x^2 + 4$

66. The pictograph shows two more small car symbols in the row for 1999 than it does for 2000. These two small cars represent the 270 additional cars that were sold in 1999 compared to 2000. Since the two small cars represent 270 real cars, each small car symbol must represent $\frac{270}{2}$ or 135 real cars.
The correct choice is A.

4-2 Quadratic Equations

Pages 218–219 Check for Understanding

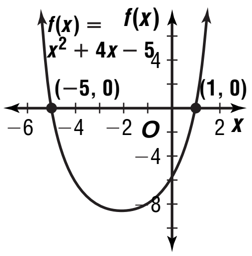
1. Add 4 to each side of the equation to get $t^2 - 6t = 4$. Determine the value needed to make $t^2 - 6t$ a perfect square trinomial. Add this value (9) to each side. Take the square root of each side of the equation and solve the two resulting equations.
$t = 3 \pm \sqrt{13}$

2. Quadratic Formula; Since the leading coefficient does not equal 1 and the discriminant equals 185 which is not a perfect square, the Quadratic Formula would be the best way to get an exact answer. Completing the square can also be used, but errors in arithmetic are more likely. A graph will give only approximate solutions.
$p = \frac{13 \pm \sqrt{(-13)^2 - 4(5)(7)}}{2(5)}$ or $\frac{13 \pm \sqrt{29}}{10}$

3a. equals 0 **3b.** negative number

3c. positive number

4. Graphing



Factoring
$x^2 + 4x - 5 = 0$
$(x + 5)(x - 1) = 0$
$x + 5 = 0$ $\quad$ $x - 1 = 0$
$x = -5$ $\quad$ $x = 1$

Completing the Square
$x^2 + 4x - 5 = 0$
$x^2 + 4x = 5$
$x^2 + 4x + 4 = 5 + 4$
$(x + 2)^2 = 9$
$x + 2 = \pm 3$
$x + 2 = 3$ $\quad$ $x + 2 = -3$
$x = 1$ $\quad$ $x = -5$

Quadratic Formula
$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-5)}}{2(1)}$
$x = \frac{-4 \pm \sqrt{36}}{2}$
$x = \frac{-4 \pm 6}{2}$
$x = -2 \pm 3$
$x = -2 + 3$ $\quad$ $x = -2 - 3$
$x = 1$ $\quad$ $x = -5$
See students' work.

5. $x^2 + 8x - 20 = 0$
$x^2 + 8x = 20$
$x^2 + 8x + 16 = 20 + 16$
$(x + 4)^2 = 36$
$x + 4 = \pm 6$
$x + 4 = 6$ $\quad$ $x + 4 = -6$
$x = 2$ $\quad$ $x = -10$

6. $2a^2 + 11a - 21 = 0$
$a^2 + \frac{11}{2}a - \frac{21}{2} = 0$
$a^2 + \frac{11}{2}a = \frac{21}{2}$
$a^2 + \frac{11}{2}a + \frac{121}{16} = \frac{21}{2} + \frac{121}{16}$
$\left(a + \frac{11}{4}\right)^2 = \frac{289}{16}$
$a + \frac{11}{4} = \pm\frac{17}{4}$
$a + \frac{11}{4} = \frac{17}{4}$ $\quad$ $a + \frac{11}{4} = -\frac{17}{4}$
$a = \frac{3}{2}$ $\quad$ $a = -7$

7. $b^2 - 4ac = 12^2 - 4(1)(36)$
$= 0$; 1 real
$m = -12 \pm \frac{\sqrt{0}}{2(1)}$
$m = -\frac{12}{2}$ or -6

8. $b^2 - 4ac = (-6)^2 - 4(1)(13)$
$= -16$; 2 imaginary
$t = \frac{-(-6) \pm \sqrt{-16}}{2(1)}$
$= \frac{6 \pm 4\boldsymbol{i}}{2}$
$= 3 \pm 2\boldsymbol{i}$

9. $p^2 - 6p + 5 = 0$
$(p - 5)(p - 1) = 0$
$p - 5 = 0$ $\quad$ $p - 1 = 0$
$p = 5$ $\quad$ $p = 1$

10. $r^2 - 4r + 10 = 0$
$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(10)}}{2(1)}$
$r = \frac{4 \pm \sqrt{-24}}{2}$
$r = \frac{4 \pm 2\boldsymbol{i}\sqrt{6}}{2}$
$r = 2 \pm \boldsymbol{i}\sqrt{6}$

11.
$$P = 12I - 0.02I^2$$
$$1600 = 12I - 0.02I^2$$
$$0.02I^2 - 12I + 1600 = 0$$
$$I^2 - 600I + 80{,}000 = 0$$
$$(I - 200)(I - 400) = 0$$
$I - 200 = 0$ $\quad$ $I - 400 = 0$

$I = 200$ amps $\quad$ $I = 400$ amps

Pages 219–221 Exercises

12. $z^2 - 2z - 24 = 0$

$z^2 - 2z = 24$

$z^2 - 2z + 1 = 24 + 1$

$(z - 1)^2 = 25$

$z - 1 = \pm 5$

$z - 1 = 5$ $\quad$ $z - 1 = -5$

$z = 6$ $\quad$ $z = -4$

13. $p^2 - 3p - 88 = 0$

$p^2 - 3p = 88$

$p^2 - 3p + \frac{9}{4} = 88 + \frac{9}{4}$

$\left(p - \frac{3}{2}\right)^2 = \frac{361}{4}$

$p - \frac{3}{2} = \frac{19}{2}$ $\quad$ $p - \frac{3}{2} = -\frac{19}{2}$

$p = 11$ $\quad$ $p = -8$

14. $x^2 - 10x + 21 = 0$

$x^2 - 10x = -21$

$x^2 - 10x + 25 = -21 + 25$

$(x - 5)^2 = 4$

$x - 5 = \pm 2$

$x - 5 = 2$ $\quad$ $x - 5 = -2$

$x = 7$ $\quad$ $x = 3$

15. $d^2 - \frac{3}{4}d + \frac{1}{8} = 0$

$d^2 - \frac{3}{4}d = -\frac{1}{8}$

$d^2 - \frac{3}{4}d + \frac{9}{64} = -\frac{1}{8} + \frac{9}{64}$

$\left(d - \frac{3}{8}\right)^2 = \frac{1}{64}$

$d - \frac{3}{8} = \pm\frac{1}{8}$

$d - \frac{3}{8} = \frac{1}{8}$ $\quad$ $d - \frac{3}{8} = -\frac{1}{8}$

$d = \frac{1}{2}$ $\quad$ $d = \frac{1}{4}$

16. $3g^2 - 12g = -4$

$g^2 - 4g = -\frac{4}{3}$

$g^2 - 4g + 4 = -\frac{4}{3} + 4$

$(g - 2)^2 = \frac{8}{3}$

$g - 2 = \pm\frac{2\sqrt{6}}{3}$

$g = 2 \pm \frac{2\sqrt{6}}{3}$

17. $t^2 - 3t - 7 = 0$

$t^2 - 3t = 7$

$t^2 - 3t + \frac{9}{4} = 7 + \frac{9}{4}$

$\left(t - \frac{3}{2}\right)^2 = \frac{37}{4}$

$t - \frac{3}{2} = \pm\frac{\sqrt{37}}{2}$

$t = \frac{3}{2} \pm \frac{\sqrt{37}}{2}$

18. $\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$

19. $b^2 - 4ac = (6)^2 - 4(4)(25)$

$= -364$

2 imaginary; the discriminant is negative.

20. $b^2 - 4ac = 7^2 - 4(6)(-3)$ or 121; 2 real

$m = \frac{-7 \pm \sqrt{121}}{2(6)}$

$m = \frac{-7 \pm 11}{12}$

$m = -\frac{3}{2}, \frac{1}{3}$

21. $b^2 - 4ac = (-5)^2 - 4(1)(9)$ or -11; 2 imaginary

$s = \frac{5 \pm \sqrt{-11}}{2(1)}$

$s = \frac{5 \pm i\sqrt{11}}{2}$

22. $b^2 - 4ac = (-84)^2 - 4(36)(49)$ or 0; 1 real

$d = \frac{84 \pm \sqrt{0}}{2(36)}$

$d = \frac{84}{72}$ or $\frac{7}{6}$

23. $b^2 - 4ac = (-2)^2 - 4(4)(9)$ or -140; 2 imaginary

$x = \frac{2 \pm \sqrt{-140}}{2(4)}$

$x = \frac{2 \pm 2i\sqrt{35}}{8}$

$x = \frac{1 \pm i\sqrt{35}}{4}$

24. $3p^2 + 4p = 8$

$3p^2 + 4p - 8 = 0$

$b^2 - 4ac = 4^2 - 4(3)(-8)$ or 112; 2 real

$p = \frac{-4 \pm \sqrt{112}}{2(3)}$

$p = \frac{-4 \pm 4\sqrt{7}}{6}$

$p = \frac{-2 \pm 2\sqrt{7}}{3}$

25. $2k^2 + 5k = 9$

$2k^2 + 5k - 9 = 0$

$b^2 - 4ac = 5^2 - 4(2)(-9)$ or 97; 2 real

$k = \frac{-5 \pm \sqrt{97}}{2(2)}$

$k = \frac{-5 \pm \sqrt{97}}{4}$

26. $-7 + i\sqrt{5}$ $\quad$ **27.** $5 + 2i$

28. $s = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$s = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$

$s = \frac{5 \pm \sqrt{-83}}{6}$

$s = \frac{5 \pm i\sqrt{83}}{6}$

29. $x^2 - 3x - 28 = 0$

$(x - 7)(x + 4) = 0$

$x - 7 = 0$ $\quad$ $x + 4 = 0$

$x = 7$ $\quad$ $x = -4$

30. $4w^2 + 19w - 5 = 0$

$(4w - 1)(w - 5) = 0$

$4w - 1 = 0$ $\quad$ $w + 5 = 0$

$4w = 1$ $\quad$ $w = -5$

$w = \frac{1}{4}$

31. $4r^2 - r = 5$

$4r^2 - r - 5 = 0$

$(4r - 5)(r + 1) = 0$

$4r - 5 = 0$ $\quad$ $r + 1 = 0$

$4r = 5$ $\quad$ $r = -1$

$r = \frac{5}{4}$

32. $p = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

$p = \frac{-2 \pm \sqrt{2^2 - 4(1)(8)}}{2(1)}$

$p = \frac{-2 \pm \sqrt{-28}}{2}$

$p = \frac{-2 \pm 2i\sqrt{7}}{2}$

$p = -1 \pm i\sqrt{7}$

33. $x = \frac{2\sqrt{6} \pm \sqrt{(2\sqrt{6})^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{2\sqrt{6} \pm \sqrt{32}}{2}$

$x = \frac{2\sqrt{6} \pm 4\sqrt{2}}{2}$

$x = \sqrt{6} + 2\sqrt{2}$

34a. $P = 0.01A^2 + 0.05A + 107$

$P = 0.01(25)^2 + 0.05(25) + 107$

$P = 6.25 + 1.25 + 107$

$P = 114.5$ mm Hg

34b. $P = 0.01A^2 + 0.05A + 107$

$125 = 0.01A^2 + 0.05A + 107$

$0 = 0.01A^2 + 0.05A - 18$

$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = \frac{-0.05 \pm \sqrt{0.05^2 - 4(0.01)(-18)}}{2(0.01)}$

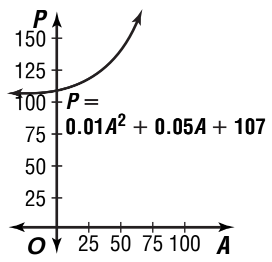
$A = \frac{-0.05 \pm \sqrt{0.7225}}{0.02}$

$A = \frac{-0.05 + \sqrt{0.7225}}{0.02}$ or $A = \frac{-0.05 - \sqrt{0.7225}}{0.02}$

$A = 40$ $\quad$ $A = -45$

40 years old

34c.



As a woman gets older, the normal systolic pressure increases.

35. $b^2 - 4ac < 0$

$8^2 - 4(1)(c) < 0$

$64 - 4c < 0$

$-4c < -64$

$c > 16$

36a. $A = bh$

$A = 12(16)$

$A = 192$

$(12 - 2x)(16 - 2x) = \frac{1}{2}(192)$

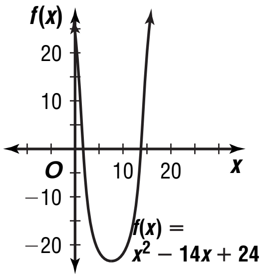
$(12 - 2x)(16 - 2x) = 96$

36b. $(12 - 2x)(16 - 2x) = 96$

$192 - 56x + 4x^2 = 96$

$4x^2 - 56x + 96 = 0$

$x^2 - 14x + 24 = 0$



36c. roots: 2, 12

$12 - 2x = 12 - 2(2)$ or 8

$16 - 2x = 16 - 2(2)$ or 12

8 ft by 12 ft

$12 - 2x = 12 - 2(12)$ or -12

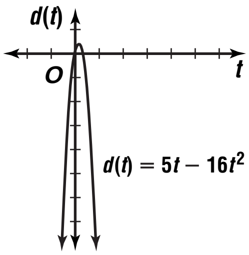
$16 - 2x = 16 - 2(12)$ or -8

$\varnothing$

37a. $d(t) = v_0t - \frac{1}{2}gt^2$

$d(t) = 5t - \frac{1}{2}(32)t^2$

$d(t) = 5t - 16t^2$



37b. 0 and about 0.3

37c. The x-intercepts indicate when the woman is at the same height as the beginning of the jump.

37d. $d(t) = 5t - 16t^2$

$-50 = 5t - 16t^2$

37e. $-50 = 5t - 16t^2$

$16t^2 - 5t - 50 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{5 \pm \sqrt{(-5)^2 - 4(16)(-50)}}{2(16)}$

$t = \frac{5 \pm \sqrt{3225}}{32}$

$t = \frac{5 + \sqrt{3225}}{32}$ $\quad$ $t = \frac{5 - \sqrt{3225}}{32}$

$t \approx 1.93$ s $\quad$ $t \approx -1.62$

about 1.93 s

38. $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$
$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

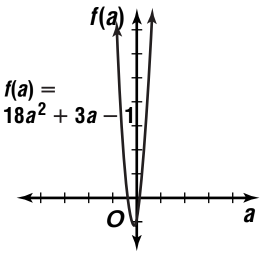
39. 2; $18a^2 + 3a - 1 = 0$

$(3a + 1)(6a - 1) = 0$

$3a + 1 = 0$ $\quad$ $6a - 1 = 0$

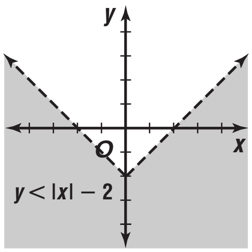
$3a = -1$ $\quad$ $6a = 1$

$a = -\frac{1}{3}$ $\quad$ $a = \frac{1}{6}$



40.

x	y
−2	0
−1	−1
0	−2
1	−1
2	0



41. $f(x) = (x - 9)^2$

$y = (x - 9)^2$

$x = (y - 9)^2$

$\pm\sqrt{x} = y - 9$

$y = \pm\sqrt{x} + 9$

$f^{-1}(x) = \pm\sqrt{x} + 9$

42. $3x + 4y = 375$

$-2(5x + 2y) = -2(345)$ $\rightarrow$

$$\begin{array}{r} 3x + 4y = 375 \\ -10x - 4y = -690 \\ \hline -7x \quad = -315 \\ x = 45 \end{array}$$

$3x + 4y = 375$

$3(45) + 4y = 375$

$y = 60$ $\quad$ (45, 60)

43. $m = \frac{619 - 595}{2.8 - 2.4}$ $\quad$ $60 = \frac{619 - x}{2.8 - 3.2}$

$m = \frac{24}{0.4}$ $\quad$ $619 - x = -24$

$m = 60$ $\quad$ $x = \$643$

44. $3y + 8x = 12$

$3y = -8x + 12$

$y = 3x + 4; -\frac{8}{3}$

45. $x^2 + x - 20 = (x + 5)(x - 4)$

The correct choice is A.

4-3 The Remainder and Factor Theorems

Page 226 Check for Understanding

1. The Remainder Theorem states that if a polynomial $P(x)$ is divided by $x - r$, the remainder is $P(r)$. If a division problem has a remainder of 0, then the divisor is a factor of the dividend. This leads to the Factor Theorem which states that the binomial $x - r$ is a factor if and only if $P(r) = 0$.

2. $(x^3 - 4x^2 - 7x + 8) \div (x - 5)$; $x^2 + x - 2$; -2

3. The degree of a polynomial is one more than the degree of its depressed polynomial.

4. Isabel; if $f(-3) = 0$, then $(x - (-3))$ or $(x + 3)$ is a factor.

5.
$$\begin{array}{r|rrr} 2 & 1 & -1 & 4 \\ & & 2 & 2 \\ \hline & 1 & 1 & 6 \end{array}$$
$x + 1$, R6

6.
$$\begin{array}{r|rrrr} -5 & 1 & 1 & -17 & 15 \\ & & -5 & 20 & -15 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$
$x^2 - 4x + 3$

7. $f(x) = x^2 + 2x - 15$

$f(3) = (3)^2 + 2(3) - 15$

$= 9 + 6 - 15$ or 0; yes

8. $f(x) = x^4 + x^2 + 2$

$f(3) = (3)^4 + (3)^2 + 2$

$= 81 + 9 + 2$ or 92; no

9. $f(x) = x^3 - 5x^2 - x + 5$

$f(1) = (1)^3 - 5(1)^2 - 1 + 5$

$= 1 - 5 - 1 + 5$ or 0

$$\begin{array}{r|rrrr} 1 & 1 & -5 & -1 & 5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$x - 1$ is a factor $\quad$ $x^2 - 4x - 5 = (x - 5)(x + 1)$

$(x - 5), (x + 1), (x - 1)$

10. $f(x) = x^3 - 6x^2 + 11x - 6$

$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

$= 1 - 6 + 11 - 6$ or 0

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$x - 1$ is a factor $\quad$ $x^2 - 5x + 6 = (x - 2)(x - 3)$

$(x - 1), (x - 2), (x - 3)$

11.
$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & k \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 6 + k \end{array}$$

$6 + k = 2$

$k = -4$

12a. 12 $\quad$ **12b.** 12 $\quad$ **12c.** 11

12d. $f(x) = x^7 + x^9 + x^{12} - 2x^2$

$= x^{12} + x^9 + x^7 - 2x^2$

$= x(x^{11} + x^8 + x^6 - 2x)$

$= x^2(x^{10} + x^7 + x^5 - 2)$

$x, x^2, x^{11} + x^8 + x^6 - 2x$, or $x^{10} + x^7 + x^5 - 2$

13. $h = r + 4$

$V = \pi r^2 h$

$V = \pi r^2(r + 4)$

$5\pi = \pi r^2(r + 4)$

$5\pi = \pi r^3 + 4\pi r^2$

$0 = \pi r^3 + 4\pi r^2 - 5\pi$

$0 = \pi(r^3 + 4r^2 - 5)$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 0 & -5 \\ & & 1 & 5 & 5 \\ \hline & 1 & 5 & 5 & 0 \end{array}$$

$r - 1 = 0$ $\quad$ $h = r + 4$

$r = 1$ in. $\quad$ $h = 1 + 4$ or 5 in.

Pages 226–228 Exercises

14. $$\begin{array}{r|rrr} -7 & 1 & 20 & 91 \\ & & -7 & -91 \\ \hline & 1 & 13 & 0 \end{array}$$
$x + 13$

15. $$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -28 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & -1 \end{array}$$
$x^2 - 6x + 9$, R -1

16. $$\begin{array}{r|rrrrr} 2 & 1 & 1 & 0 & 0 & -1 \\ & & 2 & 6 & 12 & 24 \\ \hline & 1 & 3 & 6 & 12 & 23 \end{array}$$
$x^3 + 3x^2 + 6x + 12$, R23

17. $$\begin{array}{r|rrrrr} -2 & 1 & 0 & -8 & 0 & 16 \\ & & -2 & 4 & 8 & -16 \\ \hline & 1 & -2 & -4 & 8 & 0 \end{array}$$
$x^3 - 2x^2 - 4x + 8$

18. $$\begin{array}{r|rrrrr} -1 & 3 & -2 & 5 & -4 & -2 \\ & & -3 & 5 & -10 & 14 \\ \hline & 3 & -5 & 10 & -14 & 12 \end{array}$$
$3x^3 - 5x^2 + 10x - 14$, R12

19. $$\begin{array}{r|rrrr} 1 & 2 & 0 & -2 & -3 \\ & & 2 & 2 & 0 \\ \hline & 2 & 2 & 0 & -3 \end{array}$$
$2x^2 + 2x$, R -3

20. $f(x) = x^2 - 2$
$f(1) = (1)^2 - 2$
$= 1 - 2$ or -1; no

21. $f(x) = x^5 + 32$
$f(-2) = (-2)^5 + 32$
$= -32 + 32$ or 0; yes

22. $f(x) = x^4 - 6x^2 + 8$
$f(\sqrt{2}) = (\sqrt{2})^4 - 6(\sqrt{2})^2 + 8$
$= 4 - 12 + 8$ or 0; yes

23. $f(x) = x^3 - x + 6$
$f(2) = (2)^3 - 2 + 6$
$= 8 - 2 + 6$ or 12; no

24. $f(x) = 4x^3 + 4x^2 + 2x + 3$
$f(1) = 4(1)^3 + 4(1)^2 + 2(1) + 3$
$= 4 + 4 + 2 + 3$ or 13; no

25. $f(x) = 2x^3 - 3x^2 + x$
$f(1) = 2(1)^3 - 3(1)^2 + 1$
$= 2 - 3 + 1$ or 0; yes

26a-d.

r	1	3	−2	−8
1	1	4	2	−6
−1	1	2	−4	−4
2	1	5	8	8
−2	1	1	−4	0

d

27. $(\sqrt{6})^4 - 36 = 36 - 36$ or 0

28. $$\begin{array}{r|rrrr} -1 & 1 & 7 & -1 & -7 \\ & & -1 & -6 & 7 \\ \hline & 1 & 6 & -7 & 0 \end{array}$$
$x^2 + 6x - 7 = (x - 1)(x + 7)$
$(x - 1)(x + 1)(x + 7)$

29. $$\begin{array}{r|rrrr} 2 & 1 & 1 & -4 & -4 \\ & & 2 & 6 & 4 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$
$x^2 + 3x + 2 = (x + 1)(x + 2)$
$(x - 2), (x + 1), (x + 2)$

30. $$\begin{array}{r|rrrr} 1 & 1 & -1 & -49 & 49 \\ & & 1 & 0 & -49 \\ \hline & 1 & 0 & -49 & 0 \end{array}$$
$x^2 - 49 = (x - 7)(x + 7)$
$(x - 1), (x - 7), (x + 7)$

31. $$\begin{array}{r|rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$
$x^2 - x - 2 = (x - 2)(x + 1)$
$(x - 4), (x - 2), (x + 1)$

32. $$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$
$x^2 - 4 = (x - 2)(x + 2)$
$(x - 2), (x - 2), (x + 2)$

33. $$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -4 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$
$x^2 + 5x + 4 = (x + 1)(x + 4)$
$(x - 1), (x + 1), (x + 4)$

34. $$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$
$x^2 + 2x + 1 = (x + 1)(x + 1)$
$(x + 1), (x + 1), (x + 1)$

35. $$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & -9 & 0 & 24 & 0 & -16 \\ & & 2 & 4 & -10 & -20 & 8 & 16 \\ \hline & 1 & 2 & -5 & -10 & 4 & 8 & 0 \end{array}$$
$x^5 + 2x^4 - 5x^3 - 10x^2 + 4x + 8$
$$\begin{array}{r|rrrrrr} 2 & 1 & 2 & -5 & -10 & 4 & 8 \\ & & 2 & 8 & 6 & -8 & -8 \\ \hline & 1 & 4 & 3 & -4 & -4 & 0 \end{array}$$
$x^4 + 4x^3 + 3x^2 - 4x - 4$
$$\begin{array}{r|rrrrr} 2 & 1 & 4 & 3 & -4 & -4 \\ & & 2 & 12 & 30 & -52 \\ \hline & 1 & 6 & 15 & -26 & -56 \end{array}$$
2 times

36. $$\begin{array}{r|rrrr} -1 & 1 & 2 & -1 & -2 \\ & & -1 & -1 & 2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$
$x^2 + x - 2 = (x + 2)(x - 1)$
1 time; -2, 1

37. $f(x) = 2x^3 - x^2 + x + k$
$f(1) = 2(1)^3 - (1)^2 + 1 + k$
$0 = 2 - 1 + 1 + k$
$-2 = k$

38. $f(x) = x^3 - kx^2 + 2x - 4$
$f(2) = (2)^3 - k(2)^2 + 2(2) - 4$
$0 = 8 - 4k + 4 - 4$
$0 = -4k + 8$
$2 = k$

39. $f(x) = x^3 + 18x^2 + kx + 4$
$f(-2) = (-2)^3 + 18(-2)^2 + k(-2) + 4$
$0 = -8 + 72 - 2k + 4$
$0 = -2k + 68$
$34 = k$

40. $f(x) = x^3 + 4x^2 - kx + 1$
$f(-1) = (-1)^3 + 4(-1)^2 - k(-1) + 1$
$0 = -1 + 4 + k + 1$
$0 = k + 4$
$-4 = k$

41. $d(t) = v_0t + \frac{1}{2}at^2$

$25 = 4t + \frac{1}{2}(0.4)t^2$

$0 = 0.2t^2 + 4t - 25$

$$\begin{array}{r|rrr} 5 & 0.2 & 4 & -25 \\ & & 1 & 25 \\ \hline & 0.2 & 5 & 0 \end{array}$$

$t - 5 = 0$

$t = 5$ s

42.

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -7 & a & b \\ & & 1 & 2 & -5 & -5 + a \\ \hline & 1 & 2 & -5 & -5 + a & -5 + a + b \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & -5 & -5 + a & -5 + a + b \\ & & -2 & 0 & 10 & -10 - 2a \\ \hline & 1 & 0 & -5 & 5 + a & -15 - a + b \end{array}$$

$-5 + a + b = 0$
$-15 - a + b = 0$
$-20 \quad + 2b = 0$
$2b = 20$
$b = 10$

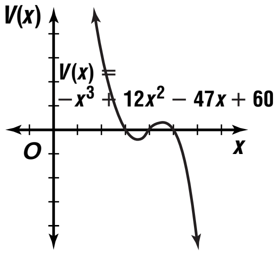
$-5 + a + b = 0$
$-5 + a + 10 = 0$
$a + 5 = 0$
$a = -5$

43a. $V(x) = (3 - x)(4 - x)(5 - x)$

$V(x) = (12 - 7x + x^2)(5 - x)$

$V(x) = -x^3 + 12x^2 - 47x + 60$

43b.



43c. $V = \ell \cdot w \cdot h$

$V = 3 \cdot 4 \cdot 5$ or 60

$\frac{3}{5}V = \frac{3}{5}(60)$

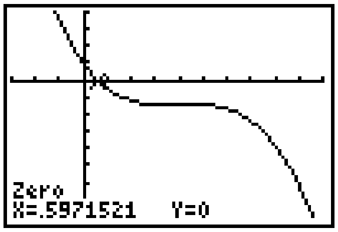
$= 36$

$V(x) = -x^3 + 12x^2 - 47x + 60$

$36 = -x^3 + 12x^2 - 47x + 60$

43d. $36 = -x^3 + 12x^2 - 47x + 60$

$0 = -x^3 + 12x^2 - 47x + 24$



[−3, 10] sc1:1 by [−200, 100] sc1:25

about 0.60 ft

44a. $\ell = \frac{1}{2}(20 - 2x)$ or $10 - x$

$w = 18 - 2x$

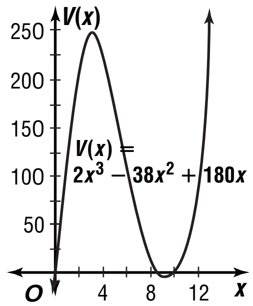
$h = x$

$V(x) = (10 - x)(18 - 2x)(x)$

$V(x) = (180 - 38x + 2x^2)(x)$

$V(x) = 2x^3 - 38x^2 + 180x$

44b.



44c. $V(x) = 2x^3 - 38x^2 + 180x$

$224 = 2x^3 - 38x^2 + 180x$

44d. $224 = 2x^3 - 38x^2 + 180x$

$0 = 2x^3 - 38x^2 + 180x + 224$

$0 = x^3 - 19x^2 + 90x + 112$

$$\begin{array}{r|rrrr} 2 & 1 & -19 & 90 & 112 \\ & & 2 & -34 & -112 \\ \hline & 1 & -17 & 56 & 0 \end{array}$$

2 in.

45. $P(3 + 4\boldsymbol{i}) = 0$ and $P(3 - 4\boldsymbol{i}) = 0$ implies that these are both roots of $ax^2 + bx + c$. Since this polynomial is of degree 2 it has only these two roots.

$x = 3 \pm 4\boldsymbol{i}$

$x - 3 = \pm 4\boldsymbol{i}$

$(x - 3)^2 = -16$

$x^2 - 6x + 9 = 16$

$x^2 - 6x + 25 = 0$

$a = 1, b = -6, c = 25$

46. $r^2 + 5r - 8 = 0$

$r^2 + 5r = 8$

$r^2 + 5r + \frac{25}{4} = 8 + \frac{25}{4}$

$\left(r + \frac{5}{2}\right)^2 = \frac{57}{4}$

$r + \frac{5}{2} = \pm \frac{\sqrt{57}}{2}$

$r = -\frac{5}{2} \pm \frac{\sqrt{57}}{2}$

47a. $f(x) = x^4 - 4x^3 - x^2 + 4x$

$f(2) = (2)^4 - 4(2)^3 - (2)^2 + 4(2)$

$f(2) = 16 - 32 - 4 + 8$ or -12; no

47b. $f(0) = (0)^4 - 4(0)^3 - (0)^2 + 4(0)$

$f(0) = 0 - 0 - 0 + 0$ or 0; yes

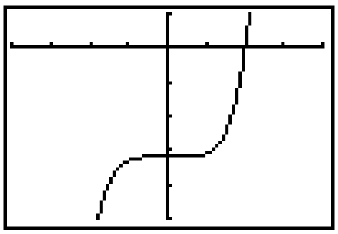
47c. $f(-2) = (-2)^4 - 4(-2)^3 - (-2)^2 + 4(-2)$

$f(-2) = 16 + 32 - 4 - 8$ or 36; no

47d. $f(4) = (4)^4 - 4(4)^3 - (4)^2 + 4(4)$

$f(4) = 256 - 256 - 16 + 16$ or 0; yes

48. $f(x) = x^5 - 32$



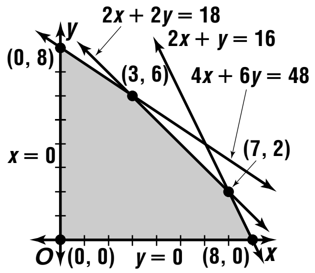
(0, −32); point of inflection

[−4, 4] sc1:1 by [−50, 10] sc1:10

49. wider than parent graph and moved 1 unit left

50. Let x = number of 100 foot units of Pipe A and y = number of 100 foot units of Pipe B.

$4x + 6y \leq 48$
$2x + 2y \leq 18$
$2x + y \leq 16$
$x \geq 0$
$y \geq 0$



$P(x, y) = 34x + 40y$
$P(0, 0) = 34(0) + 40(0)$ or 0
$P(0, 8) = 34(0) + 40(8)$ or 320
$P(3, 6) = 34(3) + 40(6)$ or 342
$P(7, 2) = 34(7) + 40(2)$ or 318
$P(8, 0) = 34(8) + 40(0)$ or 272
3 − 100 foot units of A, or 300 ft of A
6 − 100 foot units of B, or 600 ft of B

51. $4x + 2y + 3z = 6 \qquad\qquad 4x + 2y + 3z = 6$
$2x + 7y = 3z \qquad \rightarrow \qquad 2x + 7y - 3z = 0$
$-3x - 9y + 13 = -2z \qquad -3x - 9y + 2z = -13$

$$\begin{array}{l} 4x + 2y + 3z = 6 \\ \underline{2x + 7y - 3z = 0} \\ 6x + 9y \qquad = 6 \end{array}$$

$2(2x + 7y - 3z) = 2(0)$
$3(-3x - 9y + 2z) = 3(-13)$
$\downarrow$

$$\begin{array}{l} 4x + 14y - 6z = 0 \\ \underline{-9x - 27y + 6z = -39} \\ -5x - 13y \qquad = -39 \end{array}$$

$5(6x + 9y) = 5(6)$
$6(-5x - 13y) = 6(-39)$
$\downarrow$

$$\begin{array}{l} 30x + 45y = 30 \\ \underline{-30x - 78y = -234} \\ \quad -33y = -204 \\ \quad y = \frac{68}{11} \end{array}$$

$6x + 9y = 6 \qquad\qquad 4x + 2y + 3z = 6$
$6x + 9\left(\frac{68}{11}\right) = 6 \qquad 4\left(-\frac{91}{11}\right) + 2\left(\frac{68}{11}\right) + 3z = 6$
$6x = -\frac{546}{11} \qquad\qquad 3z = \frac{294}{11}$
$x = -\frac{91}{11} \qquad\qquad z = \frac{98}{11}$

$\left(-\frac{91}{11}, \frac{68}{11}, \frac{98}{11}\right)$

52. $M\left(\frac{-7-2}{2}, \frac{2+6}{2}\right)$ or $(-4.5, 4)$
$N\left(\frac{-2-2}{2}, \frac{6-3}{2}\right)$ or $(-2, 1.5)$
slope of $\overline{MN} = \frac{4 - 1.5}{-4.5 - (-2)}$ or -1
slope of $\overline{RI} = \frac{2 - (-3)}{-7 - (-2)}$ or -1
Since the slopes are the same, $\overline{MN} \parallel \overline{RI}$.

53. $a > b \qquad\qquad a > b \qquad\qquad a > b$
$ac < bc \qquad a + c > b + c \qquad a - c > b - c$
I. true $\qquad$ II. true $\qquad$ III. false
The correct choice is D.

4-4 The Rational Root Theorem

Page 232 Graphing Calculator Exploration

1. 3; 1, −1, −2
2. 2; −1, 2
3. (1) 1 positive;
$f(-x) = (-x)^4 + 4(-x)^3 + 3(-x)^2 - 4(-x) - 4$
$f(-x) = x^4 - 4x^3 + 3x^2 + 4x - 4$; 3 or 1
(2) 1 positive; $f(-x) = (-x)^3 - 3(-x) - 2$
$f(-x) = -x^3 + 3x - 2$; 2 or 0
4. In the first function, there are 2 negative zeros, but according to Descartes' Rule of Signs, there should be 3 or 1 negative zeros. This is because the −2 is a double zero. In the second function, there is one negative zero, but according to Descartes' Rule of Signs, there should be 2 or 0 zeros. This is because −1 is a double root.
5. One number represents two zeros of the function.

Page 233 Check for Understanding

1. possible values of p: ±1, ±2, ±3, ±6
possible values of q: ±1
possible rational roots: ±1, ±2, ±3, ±6
2. If the leading coefficient is 1, then q must equal 1. Therefore, $\frac{p}{q}$ becomes $\frac{p}{1}$ or p, and p is defined as a factor of a_n.
3. Sample answer: $f(x) = x^3 - x^2 + x - 3$;
$f(-x) = (-x)^3 - (-x)^2 + (-x) - 3$
$f(-x) = -x^3 - x^2 - x - 3$; 0
3 or 1 possible positive zeros and no possible negative zeros
4. Sample answer: You can factor the polynomial, graph the function, complete the square, or use the Quadratic Formula if it is a second-degree function, or use the Factor Theorem and the Rational Root Theorem. I would factor the polynomial if it can be factored easily. If not and it is a second-degree function, I would use the Quadratic Formula. Otherwise, I would graph the function on a graphing utility and use the Rational Root Theorem to find the exact zeros.
5. $\frac{p}{q}$: ±1, ±2

r	1	−4	1	2
1	1	−3	−2	0
−1	1	−5	6	−4
2	1	−2	−3	−4
−2	1	−6	13	−24

rational root: 1

6. p: ±1, ±3
q: ±1, ±2
$\frac{p}{q}$: ±1, ±3, $\pm\frac{1}{2}$, $\pm\frac{3}{2}$

r	2	3	−8	3
1	2	5	−3	0
−1	2	1	−9	12
$\frac{1}{2}$	2	4	−6	0
$-\frac{1}{2}$	2	2	−6	−1.5
3	2	9	19	60
−3	2	−3	1	0

rational roots: $-3, \frac{1}{2}, 1$

7. 2 or 0; $f(-x) = 8(-x)^3 - 6(-x)^2 - 23(-x) + 6$
$f(-x) = -8x^3 - 6x^2 + 23x + 6$; 1
$\frac{p}{q}$: ±1, ±2, ±3, ±6, $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

r	8	−6	−23	6
1	8	2	−21	−15
2	8	10	−3	0

$8x^2 + 10x - 3 = 0$
$(4x - 1)(2x + 3) = 0$
$4x - 1 = 0$ $\quad$ $2x + 3 = 0$
$4x = 1$ $\quad$ $2x = -3$
$x = \frac{1}{4}$ $\quad$ $x = -\frac{3}{2}$ or $-1\frac{1}{2}$
$-1\frac{1}{2}, \frac{1}{4}, 2$

8. 1; $f(-x) = (-x)^3 + 7(-x)^2 + 7(-x) - 15$
$f(-x) = -x^3 + 7x^2 - 7x - 15$; 2 or 0
$\frac{p}{q}$: ±1, ±3, ±5, ±15

r	1	7	7	−15
1	1	8	15	0
−1	1	6	1	−16
−3	1	4	−5	0
−5	1	2	−3	0

−5, −3, 1

9. $r^2 = 15^2 - x^2$

$$V = \frac{1}{3}\pi r^2 h$$
$$1152\pi = \frac{1}{3}\pi(15^2 - x^2)(15 + x)$$
$$3456 = (15^2 - x^2)(15 + x)$$
$$3456 = 3375 + 225x - 15x^2 - x^3$$

$x^3 + 15x^2 - 225x + 81 = 0$
Possible rational roots: ±1, ±9, ±81
$f(x) = x^3 + 15x^2 - 225x + 81 = 0$
$f(1) = -128$ $\quad$ $f(-1) = 320$
$f(9) = 0$ $\quad$ $f(-9) = 2592$
$f(81) = 611{,}712$ $\quad$ $f(-81) = -414{,}720$
x represents 9 cm.

Pages 234–235 Exercises

10. $\frac{p}{q}$: ±1, ±2, ±3, ±6

r	1	2	−5	−6
1	1	3	−2	−8
2	1	4	3	0

$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3, x = -1$
rational roots: −3, −1, 2

11. $\frac{p}{q}$: ±1, ±2, ±3, ±6, ±9, ±18

r	1	−2	1	18
−1	1	−3	4	14
−2	1	−4	9	0

$x^2 - 4x + 9 = 0$
does not factor
rational root: −2

12. $\frac{p}{q}$: ±1, ±2

r	1	−5	9	−7	2
1	1	−4	5	−2	0

$x^3 - 4x^2 + 5x - 2$

r	1	−4	5	−2
2	1	−2	1	0

$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
$x = 1, x = 1$
rational roots: 1, 2

13. $\frac{p}{q}$: ±1, ±2, ±4, ±5, ±10, ±20

r	1	−5	−4	20
1	1	−4	−8	12
−1	1	−6	2	18
2	1	−3	−10	0

$x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$
$x = 5, x = -2$
rational roots: −2, 2, 5

14. p: ±1, ±3
q: ±1, ±2
$\frac{p}{q}$: ±1, ±3, $\pm\frac{1}{2}$, $\pm\frac{3}{2}$

r	2	−1	0	−6	3
$\frac{1}{2}$	2	0	0	−6	0

$2x^3 - 6 = 0$
$x^3 = 3$
$x = \sqrt[3]{3}$
rational root: $\frac{1}{2}$

15. p: ± 1

q: $\pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}$: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

r	6	35	−1	−7	−1
$\frac{1}{2}$	6	38	18	2	0

$6x^3 + 38x^2 + 18x + 2$

r	6	38	18	2
$-\frac{1}{3}$	6	36	6	0

$6x^2 + 36x + 6 = 0$

$x^2 + 6x + 1 = 0$

does not factor

rational roots: $-\frac{1}{3}, \frac{1}{2}$

16. 4; 3 or 1;

$f(-x) = (-x)^4 - 2(-x)^3 + 7(-x) + 4(-x) - 15$

$f(-x) = x^4 + 2x^3 - 7x - 4x - 15$; 1 negative

1 positive

17. $f(-x) = -x^3 + 7x - 6$

0 or 2 negative

r	1	0	−7	−6
1	1	1	−6	−12
−1	1	−1	−6	0

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3, x = -2$

rational zeros: $-2, -1, 3$

18. 1 positive

$f(-x) = -x^3 - 2x^2 + 8$

1 negative

$f(x) = x^3 - 2x^2 - 8x$

$0 = x(x^2 - 2x - 8)$

$= x(x - 4)(x + 2)$

$x = 0, x = 4, x = -2$

rational zeros: $-2, 0, 4$

19. 1 positive

$f(-x) = -x^3 + 3x^2 + 10x - 24$

2 or 0 negative

r	1	3	−10	−24
3	1	6	8	14

$x^2 + 6x + 8 = 0$

$(x + 4)(x + 2) = 0$

$x = -4, x = -2$

rational zeros: $-4, -2, 3$

20. 2 or 0 positive

$f(-x) = -10x^3 - 17x^2 + 7x + 2$

1 negative

r	10	−17	−7	2
$-\frac{1}{2}$	10	−22	4	0

$10x^2 - 22x + 4 = 0$

$5x^2 - 11x + 2 = 0$

$(5x - 1)(x - 2) = 0$

$x = \frac{1}{5}, x = 2$

rational zeros: $-\frac{1}{2}, \frac{1}{5}, 2$

21. 2 or 0 positive

$f(-x) = x^4 - 2x^3 - 9x^2 + 2x + 8$

2 or 0 negative

r	1	2	−9	−2	8
1	1	3	−6	−8	0

$x^3 + 3x^2 - 6x - 8$

r	1	3	−6	−8
−1	1	2	−8	0

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4, x = 2$

rational zeros: $-4, -1, 1, 2$

22. 2 or 0 positive

$f(-x) = x^4 - 5x^2 + 4$

2 or 0 negative

r	1	0	−5	0	4
1	1	1	−4	−4	0

$x^3 + x^2 - 4x - 4$

r	1	1	−4	−4
−1	1	0	−4	0

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$

$x = 2, x = -2$

rational zeros: $-2, -1, 1, 2$

23a. $f(x) = (x - 2)(x + 2)(x + 1)^2$

$0 = (x - 2)(x + 2)(x + 1)^2$

$x - 2 = 0$ $\quad$ $x + 2 = 0$ $\quad$ $(x + 1)^2 = 0$

$x = 2$ $\quad$ $x = -2$ $\quad$ $x + 1 = 0$

$x = -1$

23b. $f(x) = (x - 2)(x + 2)(x + 1)^2$

$f(x) = (x^2 - 4)(x^2 + 2x + 1)$

$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$

23c. 1 positive

$f(-x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

3 or 1 negative

23d. There are 2 negative zeros, but according to Descartes' Rule of Signs, there should be 3 or 1. This is because -1 is actually a zero twice.

24a. Let ℓ = the length.

$w = \ell - 4$

$h = 2\ell - 1$

$V(\ell) = \ell \cdot w \cdot h$

$V(\ell) = \ell(\ell - 4)(2\ell - 1)$

$V(\ell) = (\ell^2 - 4\ell)(2\ell - 1)$

$V(\ell) = 2\ell^3 - 9\ell^2 + 4\ell$

24b. $V(\ell) = 2\ell^3 - 9\ell^2 + 4\ell$
$2208 = 2\ell^3 - 9\ell^2 + 4\ell$

24c. $2208 = 2\ell^3 - 9\ell^2 + 4\ell$
$0 = 2\ell^3 - 9\ell^2 + 4\ell - 2208$

r	2	−9	4	−2208
12	2	15	184	0

$\ell = 12$ $\quad w = \ell - 4$ $\quad h = 2\ell - 1$
$w = 12 - 4$ or 8 $\quad h = 2(12) - 1$ or 23
12 in. × 8 in. × 23 in.

25a. Sample answer: $x^4 + x^3 + x^2 + x + 3 = 0$

25b. Sample answer: $x^3 - x^2 - 2 = 0$

25c. Sample answer: $x^3 - x = 0$

26a. Let ℓ = the length.
$h = \ell - 9$
$V(\ell) = \frac{1}{3}Bh$
$V(\ell) = \frac{1}{3}(\ell^2)(\ell - 9)$
$V(\ell) = \frac{1}{3}\ell^3 - 3\ell^2$

26b. $V(\ell) = \frac{1}{3}\ell^3 - 3\ell^2$
$6300 = \frac{1}{3}\ell^3 - 3\ell^2$

26c. $6300 = \frac{1}{3}\ell^3 - 3\ell^2$
$0 = \frac{1}{3}\ell^3 - 3\ell^2 - 6300$
$0 = \ell^3 - 9\ell^2 - 18{,}900$

r	1	−9	0	−18,900
30	1	21	630	0

$\ell = 30$ $\quad h = \ell - 9$
$h = 30 - 9$ or 21
base: 30 in. by 30 in., height: 21 in.

27. $d = 0.0000008x^2(200 - x)$
$0.8 = 0.0000008x^2(200 - x)$
$0 = 0.00016x^2 - 0.0000008x^3 - 0.8$
$0 = 8x^3 - 1600x^2 - 8{,}000{,}000$
$0 = x^3 - 200x^2 - 1{,}000{,}000$

r	1	−200	0	1,000,000
100	1	−100	−10,000	0

$x = 100$ ft

28. The graphs are reflections of each other over the x-axis. The zeros are the same.

29.

−7	1	−1	−56
		−7	56
	1	−8	0

$x - 8$

30. $b^2 - 4ac = 6^2 - 4(4)(25)$
$= -364$; 2 imaginary

31. $(x - 1)(x - (-1))(x - 2)(x - (-2)) = 0$
$(x - 1)(x + 1)(x - 2)(x + 2) = 0$
$(x^2 - 1)(x^2 - 4) = 0$
$x^4 - 5x^2 + 4 = 0$

32. $y = 4.3x - 8424.3$
$y = 4.3(2008) - 8424.3$
$y = \$210.10$

33. $\frac{2x - 3}{x} = \frac{3 - x}{2}$
$2(2x - 3) = x(3 - x)$
$4x - 6 = 3x - x^2$
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x + 3 = 0 \quad x - 2 = 0$
$x = -3 \quad x = 2$
The correct choice is A.

Page 235 Mid-Chapter Quiz

1. $(x - 1)(x - (-1))(x - 2\boldsymbol{i})(x - (-2\boldsymbol{i})) = 0$
$(x - 1)(x + 1)(x - 2\boldsymbol{i})(x + 2\boldsymbol{i}) = 0$
$(x^2 - 1)(x^2 + 4) = 0$
$x^4 + 3x^2 - 4 = 0$

2. 3; $x^3 - 11x^2 + 30x = 0$
$x(x^2 - 11x + 30) = 0$
$x(x - 6)(x - 5) = 0$
$x = 0 \quad x - 6 = 0 \quad x - 5 = 0$
$x = 6 \quad x = 5$

3. $x^2 + 5x = 150$
$x^2 + 5x + \frac{25}{4} = 150 + \frac{25}{4}$
$\left(x + \frac{5}{2}\right)^2 = \frac{625}{4}$
$x + \frac{5}{2} = \pm\frac{25}{2}$
$x = \frac{-5 \pm 25}{2}$
$x = \frac{-5 + 25}{2} \quad x = \frac{-5 - 25}{2}$
$x = 10 \quad x = -15$

4. $b^2 - 4ac = (-39)^2 - 4(6)(45)$
$= 441$; 2 real roots
$b = \frac{39 \pm \sqrt{441}}{2(6)}$
$b = \frac{39 \pm 21}{12}$
$b = \frac{39 + 21}{12}$ or $b = \frac{39 - 21}{12}$
$b = 5 \quad b = \frac{3}{2}$

5.

−2	1	3	−2	−8
		−2	−2	8
	1	1	−4	0

$x^2 + x - 4$

6.

4	1	−4	2	−6
		4	0	8
	1	0	2	2

2; no

7.

1	1	−2	−5	6
		1	−1	−6
	1	−1	−6	0

$x^2 - x - 6 = (x - 3)(x + 2)$
$(x - 3)(x - 1)(x + 2)$

8. $\frac{p}{q}$: $\pm1, \pm3$

r	1	6	10	3
−3	1	3	1	0

$x^2 + 3x + 1 = 0$
does not factor
rational root: −3

9. 1 positive
$F(-x) = x^4 - 4x^3 + 3x^2 + 4x - 4$
3 or 1 negative

r	1	4	3	−4	−4
1	1	5	8	4	0

$x^3 + 5x^2 + 8x + 4 = 0$

r	1	5	8	4
−1	1	4	4	0

$x^2 + 4x + 4 = 0$
$(x + 2)(x + 2) = 0$
$x = -2, x = -2$
rational zeros: −2, −1, 1

10. Let r = radius.
$h = r + 6$
$V = \frac{1}{3}\pi r^2 h$
$27\pi = \frac{1}{3}\pi r^2(r + 6)$
$0 = \frac{1}{3}\pi r^3 + 2\pi r^2 - 27\pi$
$0 = r^3 + 6r^2 - 81$

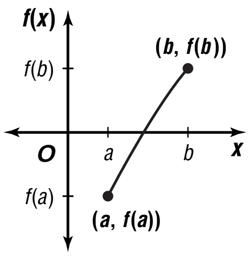
r	1	6	0	−81
3	1	9	27	0

$r = 3$ $\quad h = r + 6$
$h = 3 + 6$ or 9
$r = 3$ cm, $h = 9$ cm

4-5 Locating Zeros of a Polynomial Function

Pages 239–240 Check for Understanding

1. If the function is negative for one value and positive for another value, the function must cross the x-axis in at least one point between the two values.



2. Use synthetic division to find the values of the polynomial function for consecutive integers. When the values of the function change from positive to negative or from negative to positive, there is a zero between the integers.

3. Use synthetic division to find the values of the polynomial function for consecutive integers. An integer that produces no sign change in the quotient and the remainder is an upper bound. To find a lower bound of a function, find an upper bound for the function of $-x$. The lower bound is the negative of the upper bound for the function of $-x$.

4. Nikki; the sign changes between −2 and −1.

5.

r	1	−4	−2
−2	1	−6	10
−1	1	−5	3
0	1	−4	−2
1	1	−3	−5
2	1	−2	−6
3	1	−1	−5
4	1	0	−2
5	1	1	3

4 and 5, −1 and 0

6.

r	1	−3	−2	4
−2	1	−5	8	−12
−1	1	−4	2	2
0	1	−3	−2	4
1	1	−2	−4	0 ←
2	1	−1	−4	−4
3	1	0	−2	−2
4	1	1	2	12

−2 and −1, at 1, 3 and 4

7.

r	2	−4	0	−3
0	2	−4	0	−3
1	2	−2	−2	−5
2	2	0	0	−3
3	2	2	6	15

approximate zero: 2.3

8.

r	1	3	2
−2	1	1	0
−1	1	2	0

zeros: −2, −1

9. Sample answer:

r	1	0	0	−8	2
1	1	1	1	−7	−5
2	1	2	4	0	2

upper bound: 2
$f(-x) = x^4 + 8x + 2$

r	1	0	0	8	2
0	1	0	0	8	2

lower bound: 0

10. Sample answer:

r	1	0	1	0	−3
1	1	1	2	2	−1
2	1	2	5	10	17

upper bound: 2
$f(-x) = x^4 + x^2 - 3$

r	1	0	1	0	−3
1	1	1	2	2	−1
2	1	2	5	10	17

lower bound: −2

11a. Let x = amount of increase.
$V(x) = (25 + x)(30 + x)(5 + x)$
$V(x) = (750 + 55x + x^2)(5 + x)$
$V(x) = x^3 + 60x^2 + 1025x + 3750$

11b. $V = \ell \cdot w \cdot h$ $\quad$ $1.5V = 1.5(3750)$
$V = 25(30)(5)$ $\quad$ $1.5V = 5625$
$V = 3750$
$V(x) = x^3 + 60x^2 + 1025x + 3750$
$5625 = x^3 + 60x^2 + 1025x + 3750$

11c. $5625 = x^3 + 60x^2 + 1025x + 3750$
$0 = x^3 + 60x^2 + 1025x - 1875$

r	1	60	1025	−1875
1	1	61	1086	−789
2	1	62	1149	423

$x = 1.7$
$25 + x = 25 + 1.7 = 26.7$ $\quad$ $30 + x = 30 + 1.7 = 31.7$
$5 + x = 5 + 1.7 = 6.7$
about 26.7 cm by 31.7 cm by 6.7 cm

Pages 240–242 Exercises

12.

r	1	0	0	−2
−1	1	−1	1	−3
0	1	0	0	−2
1	1	1	1	−1
2	1	2	4	6
3	1	3	9	25

1 and 2

13.

r	2	−5	1
−1	2	−7	8
0	2	−5	1
1	2	−3	−2
2	2	−1	−1
3	2	1	4

0 and 1, 2 and 3

14.

r	1	−2	0	1	−2
−2	1	−4	8	−15	28
−1	1	−3	3	−2	0
0	1	−2	0	1	−2
1	1	−1	−1	0	−1
2	1	0	0	1	0

at −1, at 2

15.

r	1	0	−8	0	10
−3	1	−3	1	−3	19
−2	1	−2	−4	8	−6
−1	1	−1	−7	7	3
0	1	0	−8	0	10
1	1	1	−7	−7	3
2	1	2	−4	−8	−6
3	1	3	1	3	19

−3 and −2, −2 and −1, 1 and 2, 2 and 3

16.

r	1	0	−3	1
−2	1	−2	1	−1
−1	1	−1	−2	3
0	1	0	−3	1
1	1	1	−2	−1
2	1	2	1	3

−2 and −1, 0 and 1, 1 and 2

17.

r	2	0	1	−3	3
−3	2	−6	19	−60	183
−2	2	−4	9	−21	45
−1	2	−2	3	−6	9
0	2	0	1	−3	3
1	2	2	3	0	3
2	2	4	9	16	35

no real zeros

18.

r	6	24	−54	−3
−6	6	−12	18	−111
−5	6	−6	−24	117

yes; $f(-6) = -111$, $f(-5) = 117$

19–25. Use the TABLE feature of a graphing calculator.

19. −0.7, 0.7
20. −2.6, −0.4
21. −2.5
22. −0.4, 3.4
23. −1, 1
24. −1.3, 0.9, 7.4
25. −1.24

26. Sample answers:

r	3	−2	5	−1
1	3	1	6	5

upper bound: 1
$f(-x) = -3x^3 - 2x^2 - 5x - 1$

r	−3	−2	−5	−1
0	−3	−2	−5	−1

lower bound: 0

27. Sample answers:

r	1	−1	−1
1	1	0	−1
2	1	1	1

upper bound: 2
$f(-x) = x^2 + x - 1$

r	1	1	−1
1	1	2	1

lower bound: −1

28. Sample answers:

r	1	−6	2	6	−13
1	1	−5	−3	3	−10
2	1	−4	−6	−6	−25
3	1	−3	−7	−15	−58
4	1	−2	−6	−18	−85
5	1	−1	−3	−9	−58
6	1	0	2	18	95

upper bound: 6
$f(-x) = x^4 + 6x^3 + 2x^2 - 6x - 13$

r	1	6	2	−6	−13
1	1	7	9	3	−10
2	1	8	18	30	47

lower bound: −2

29. Sample answers:

r	1	5	−3	−20
1	1	6	3	−17
2	1	7	11	2

upper bound: 2

$f(-x) = -x^3 + 5x^2 + 3x - 20$

r	−1	5	3	−20
1	−1	4	7	−13
2	−1	3	9	−2
3	−1	2	9	7
4	−1	1	7	8
5	−1	0	3	−5
6	−1	−1	−3	−38

lower bound: −6

30. Sample answers:

r	1	−3	−2	3	−5
1	1	−2	−4	−1	−6
2	1	−1	−4	−5	−15
3	1	0	−2	−3	−14
4	1	1	2	11	39

upper bound: 4

$f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$

r	1	3	−2	−3	−5
1	1	4	2	−1	−6
2	1	5	8	13	21

lower bound: −2

31. Sample answers:

r	1	5	−3	20	0	−15
1	1	6	3	23	23	8

upper bound: 1

$f(-x) = -x^5 + 5x^4 + 3x^3 + 20x^2 - 15$

r	−1	5	3	20	0	−15
1	−1	4	7	27	27	8
2	−1	3	9	38	76	137
3	−1	2	9	47	141	408
4	−1	1	7	48	192	753
5	−1	0	3	35	175	860
6	−1	−1	−3	2	12	57
7	−1	−2	−11	−57	−399	−2808

lower bound: −7

32a. 4

32b. ±1, ±5

32c. 3 or 1; $f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$
1 negative real zero

32d.

r	1	−3	−2	3	−5
5	1	2	8	43	210
4	1	1	2	11	39
3	1	0	−2	−3	−14
2	1	−1	−4	−5	−15
1	1	−2	−4	−1	−6
0	1	−3	−2	3	−5
−1	1	−4	2	1	−6
−2	1	−5	8	−13	21
−3	1	−6	16	−45	130

−2 and −1, 3 and 4

32e. Sample answers:
upper bound: 4 (See table in 32d.)
$f(-x) = x^4 + 3x^3 - 2x^2 - 3x - 5$

r	1	3	−2	−3	−5
1	1	4	2	−1	−6
2	1	5	8	13	21

lower bound: −2

32f. −1.4, 3.4 (Use TABLE feature of a graphing calculator.)

33a. 1890: $P(0) = -0.78(0)^4 + 133(0)^3 - 7500(0)^2 + 147{,}500(0) + 1{,}440{,}000 = 1{,}440{,}000$

1910: $P(20) = -0.78(20)^4 + 133(20)^3 - 7500(20)^2 + 147{,}500(20) + 1{,}440{,}000 = 2{,}329{,}200$

1930: $P(40) = -0.78(40)^4 + 133(40)^3 - 7500(40)^2 + 147{,}500(40) + 1{,}440{,}000 = 1{,}855{,}200$

1950: $P(60) = -0.78(60)^4 + 133(60)^3 - 7500(60)^2 + 147{,}500(60) + 1{,}440{,}000 = 1{,}909{,}200$

1970: $P(80) = -0.78(80)^4 + 133(80)^3 - 7500(80)^2 + 147{,}500(80) + 1{,}440{,}000 = 1{,}387{,}200$

The model is fairly close, although it is less accurate at for 1950 and 1970.

33b. $1980 - 1890 = 90$

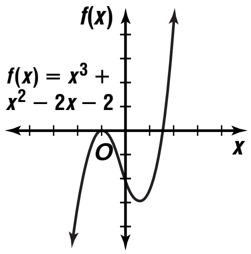
$P(90) = -0.78(90)^4 + 133(90)^3 - 7500(90)^2 + 147{,}500(90) + 1{,}440{,}000$

$P(90) = -253{,}800$

33c. The population becomes 0.

33d. No; there are still many people living in Manhattan.

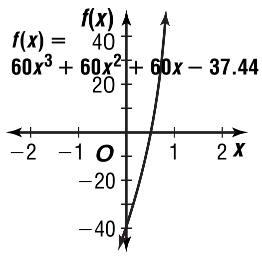
34. Sample answer:
$f(x) = (x - \sqrt{2})(x + \sqrt{2})(x + 1)$
$f(x) = (x^2 - 2)(x + 1)$
$f(x) = x^3 + x^2 - 2x - 2$; $\sqrt{2}$, −1



35a. $37.44 = 60x^3 + 60x^2 + 60x$

35b. $f(x) = 60x^3 + 60x^2 + 60x - 37.44$

35c.

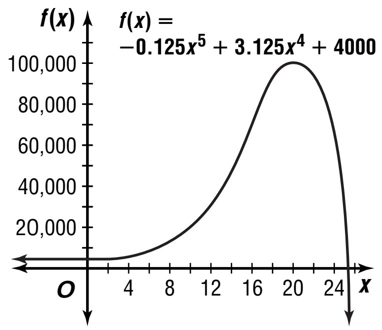


about $\frac{1}{2}$

35d. 0.4 (Use TABLE feature of a graphing calculator.)

36. Sample answer: $f(x) = x^2 - 1$

37a.



37b. $f(0) = -0.125(0)^5 + 3.125(0)^4 + 4000$
$f(0) = 4000$ deer

37c. $1920 - 1905 = 15$
$f(15) = -0.125(15)^5 + 3.125(15)^4 + 4000$
$f(15) = 67{,}281.25$
about 67,281 deer

37d. in 1930

38a. $81.58 = 6x^4 + 18x^3 + 24x^2 + 18x$

38b. $81.58 = 6x^4 + 18x^3 + 24x^2 + 18x$
$0 = 6x^4 + 18x^3 + 24x^2 + 18x - 81.58$
about 1.1 (Use TABLE feature of a graphing calculator.)

38c. $x = \text{rate} + 1$
$1.1 = \text{rate} + 1$
$0.1 = \text{rate}$
about 10%

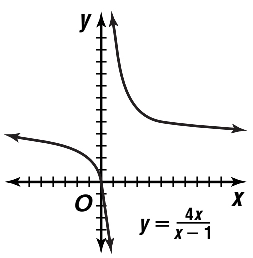
39. 2 or 0; $f(-x) = -2x^3 - 5x^2 + 28x + 15$
1 negative zero

r	2	−5	−28	15
−3	2	−11	5	0

$2x^2 - 11x + 5 = 0$
$(2x + 1)(x - 5) = 0$
$x = 0.5$ $\qquad x = 5$
rational zeros: −3, 0.5, 5

40.
$$d(t) = v_0t - \frac{1}{2}gt^2$$
$$-1750 = 4t - \frac{1}{2}(9.8)t^2$$
$4.9t^2 - 4t - 1750 = 0$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{4 \pm \sqrt{(-4)^2 - 4(4.9)(-1750)}}{2(4.9)}$$
$$t = \frac{4 \pm \sqrt{34{,}316}}{9.8}$$
$$t = \frac{4 + \sqrt{34{,}316}}{9.8} \qquad t = \frac{4 - \sqrt{34{,}316}}{9.8}$$
$t \approx 19.3$ $\qquad t \approx -18.5$
about 19.3 s

41.

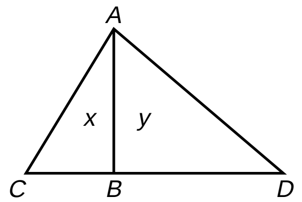


42. $\begin{vmatrix} 7 & 9 \\ 3 & 6 \end{vmatrix} = 7(6) - 3(9)$ or 15

43. $(x, y) = \left(\frac{-3 + 8}{2}, \frac{-2 + 4}{2}\right)$
$= (2.5, 1)$

44. $x - 2y - 4 = 0$
$y = \frac{1}{2}x - 2; \frac{1}{2}; -2$

45.



$y < x$ cannot be true.
The correct choice is B.

4-6 Rational Equations and Partial Fractions

Page 247 Check for Understanding

1. Multiply by the LCD, $6(b - 2)$. Then, solve the resulting equation.
2. If a possible solution causes a denominator to equal 0, it is not a solution of the equation.
3. Decomposing a fraction means to find two fractions whose sum or difference equals the original fraction.
4.
$$x + \frac{2}{x - 2} = 2 + \frac{2}{x - 2}$$
$$\left(x + \frac{2}{x - 2}\right)(x - 2) = \left(2 + \frac{2}{x - 2}\right)(x - 2)$$
$$x(x - 2) + 2 = 2(x - 2) + 2$$
$$x^2 - 2x + 2 = 2x - 4 + 2$$
$$x^2 - 4x + 4 = 0$$
$$(x - 2)(x - 2) = 0$$
$x - 2 = 0$ $\qquad x - 2 = 0$
$x = 2$ $\qquad x = 2$
If you solve the equation, you will get $x = 2$. However, if $x = 2$, the denominators will equal 0.
5.
$$b - \frac{5}{b} = 4$$
$$\left(b - \frac{5}{b}\right)b = (4)b$$
$$b^2 - 5 = 4b$$
$$b^2 - 4b - 5 = 0$$
$$(b - 5)(b + 1) = 0$$
$b - 5 = 0$ $\qquad b + 1 = 0$
$b = 5$ $\qquad b = -1$
6.
$$\frac{9}{b + 5} = \frac{3}{b - 3}$$
$$\left(\frac{9}{b + 5}\right)(b + 5)(b - 3) = \left(\frac{3}{b - 3}\right)(b + 5)(b - 3)$$
$$9(b - 3) = 3(b + 5)$$
$$9b - 27 = 3b + 15$$
$$6b - 42 = 0$$
$$b = 7$$

7. $$\frac{t+4}{t} + \frac{3}{t-4} = \frac{-16}{t^2-4t}$$
$$\left(\frac{t+4}{t} + \frac{3}{t-4}\right)(t)(t-4) = \left(\frac{-16}{t^2-4t}\right)(t)(t-4)$$
$$(t+4)(t-4) + 3(t) = -16$$
$$t^2 - 16 + 3t = -16$$
$$t^2 + 3t = 0$$
$$t(t+3) = 0$$
$t = 0$ $\quad$ $t + 3 = 0$
$t = -3$
But $t \neq 0$, so $t = -3$.

8. $$\frac{3p-1}{p^2-1} = \frac{3p-1}{(p+1)(p-1)}$$
$$\frac{3p-1}{p^2-1} = \frac{A}{p+1} + \frac{B}{p-1}$$
$3p - 1 = A(p-1) + B(p+1)$
Let $p = 1$.
$3(1) - 1 = A(1-1) + B(1+1)$
$2 = 2B$
$1 = B$
Let $p = -1$.
$3(-1) - 1 = A(-1-1) + B(-1+1)$
$-4 = -2A$
$2 = A$
$$\frac{3p-1}{p^2-1} = \frac{2}{p+1} + \frac{1}{p-1}$$

9. $5 + \frac{1}{x} > \frac{16}{x}$; exclude: 0
$5x + 1 = 16$
$5x = 15$
$x = 3$
Test $x = -1$: $5 + \frac{1}{(-1)} > \frac{16}{(-1)}$
$4 > -16$ true
Test $x = 1$: $5 + \frac{1}{1} > \frac{16}{1}$
$6 > 16$ false
Test $x = 4$: $5 + \frac{1}{4} > \frac{16}{4}$
$5\frac{1}{4} > 4$ true
Solution: $x < 0$, $x > 3$

10. $1 + \frac{5}{a-1} \leq \frac{7}{6}$; exclude: 1
$6(a-1) + 30 = 7(a-1)$
$6a - 6 + 30 = 7a - 7$
$31 = a$
Test $a = -1$: $1 + \frac{5}{-1-1} \leq \frac{7}{6}$
$-\frac{3}{2} \leq \frac{7}{6}$ true
Test $a = 2$: $1 + \frac{5}{2-1} \leq \frac{7}{6}$
$6 \leq \frac{7}{6}$ false
Test $a = 36$: $1 + \frac{5}{36-1} \leq \frac{7}{6}$
$1 + \frac{1}{7} \leq \frac{7}{6}$
$\frac{48}{42} \leq \frac{49}{42}$ true
Solution: $a < 1$, $a \geq 31$

11a. $\frac{3 \times 60 + 20}{3 + x} = 57.14$

11b. $\frac{3 \times 60 + 20}{3 + x} = 57.14$
$3 \times 60 + 20 = 57.14(3 + x)$
$200 = 171.42 + 57.14x$
$0.50 \approx x$; 0.50 h

Pages 247–250 Exercises

12. $$\frac{12}{t} + t - 8 = 0$$
$$\left(\frac{12}{t} + t - 8\right)t = (0)t$$
$12 + t^2 - 8t = 0$
$t^2 - 8t + 12 = 0$
$(t-6)(t-2) = 0$
$t - 6 = 0$ $\quad$ $t - 2 = 0$
$t = 6$ $\quad$ $t = 2$

13. $$\frac{1}{m} = \frac{m-34}{2m^2}$$
$$\left(\frac{1}{m}\right)2m^2 = \left(\frac{m-34}{2m^2}\right)2m^2$$
$m^2 - 34m = 2m^2$
$0 = m^2 + 34m$
$0 = m(m + 34)$
$m = 0$ $\quad$ $m + 34 = 0$
$m = -34$
But $m \neq 0$, so $m = -34$.

14. $$\frac{2}{y+2} + \frac{3}{y} = \frac{-y}{y+2}$$
$$\left(\frac{2}{y+2} + \frac{3}{y}\right)(y)(y+2) = \left(\frac{-y}{y+2}\right)(y)(y+2)$$
$2y + 3(y+2) = -y^2$
$5y + 6 = -y^2$
$y^2 + 5y + 6 = 0$
$(y+3)(y+2) = 0$
$y + 3 = 0$ $\quad$ $y + 2 = 0$
$y = -3$ $\quad$ $y = -2$
But $y \neq -2$, so $y = -3$.

15. $$\frac{10}{n^2-1} + \frac{2n-5}{n-1} = \frac{2n+5}{n+1}$$
$$\left(\frac{10}{n^2-1} + \frac{2n-5}{n-1}\right)(n-1)(n+1) = \left(\frac{2n+5}{n+1}\right)(n-1)(n+1)$$
$10 + (2n-5)(n+1) = (2n+5)(n-1)$
$2n^2 - 3n + 5 = 2n^2 + 3n - 5$
$-6n = -10$
$n = \frac{5}{3}$

16. $$\frac{1}{b+2} + \frac{1}{b+2} = \frac{3}{b+1}$$
$$\left(\frac{1}{b+2} + \frac{1}{b+2}\right)(b+2)(b+1) = \left(\frac{3}{b+1}\right)(b+2)(b+1)$$
$b + 1 + b + 1 = 3(b+2)$
$2b + 2 = 3b + 6$
$-4 = b$

17. $$\frac{7a}{3a+3} - \frac{5}{4a-4} = \frac{3a}{2a+2}$$
$$\frac{7a}{3(a+1)} - \frac{5}{4(a-1)} = \frac{3a}{2(a+1)}$$
$$\left(\frac{7a}{3(a+1)} - \frac{5}{4(a-1)}\right)(12)(a-1)(a+1) =$$
$$\left(\frac{3a}{2(a+1)}\right)(12)(a-1)(a+1)$$
$4(a-1)7a - 3(a+1)5 = 6(a-1)3a$
$28a^2 - 28a - 15a - 15 = 18a^2 - 18a$
$10a^2 - 25a - 15 = 0$
$2a^2 - 5a - 3 = 0$
$(2a+1)(a-3) = 0$
$2a + 1 = 0$ $\quad$ $a - 3 = 0$
$a = -\frac{1}{2}$ $\quad$ $a = 3$

18. $1 = \frac{1}{1-a} + \frac{a}{a-1}$

$1(1-a)(a-1) = \left(\frac{1}{1-a} + \frac{a}{a-1}\right)(1-a)(a-1)$

$a - 1 - a^2 + a = a - 1 + a(1-a)$

$-a^2 + 2a - 1 = -a^2 + 2a - 1$

$0 = 0$

all reals except 1

19. $\frac{2q}{2q+3} - \frac{2q}{2q-3} = 1$

$\left(\frac{2q}{2q+3} - \frac{2q}{2q-3}\right)(2q+3)(2q-3) = 1(2q+3)(2q-3)$

$2q(2q-3) - 2q(2q+3) = (2q+3)(2q-3)$

$4q^2 - 6q - 4q^2 - 6q = 4q^2 - 9$

$0 = 4q^2 + 12q - 9$

$q = \frac{-12 \pm \sqrt{144 - 4(4)(-9)}}{2 \cdot 4}$

$= \frac{-12 \pm \sqrt{288}}{8}$

$= \frac{-12 \pm 12\sqrt{2}}{8}$

$= \frac{-3 \pm 3\sqrt{2}}{2}$

20. $\frac{1}{3m} + \frac{6m-9}{3m} = \frac{3m-3}{4m}$

$\left(\frac{1}{3m} + \frac{6m-9}{3m}\right)(12m) = \left(\frac{3m-3}{4m}\right)(12m)$

$4 + 4(6m-9) = 3(3m-3)$

$4 + 24m - 36 = 9m - 9$

$15m = 23$

$m = \frac{23}{15}$

21. $\frac{-4}{x-1} = \frac{7}{2-x} + \frac{3}{x+1}$

$\left(\frac{-4}{x-1}\right)(x-1)(2-x)(x+1) = \left(\frac{7}{2-x} + \frac{3}{x+1}\right)$

$(x-1)(2-x)(x+1)$

$-4(2-x)(x+1) = 7(x-1)(x+1)$

$+ 3(x-1)(2-x)$

$-4(-x^2 + x + 2) = 7(x^2 - 1)$

$+ 3(-x^2 + 3x - 2)$

$4x^2 - 4x - 8 = 4x^2 + 9x - 13$

$5 = 13x$

$\frac{5}{13} = x$

22a. $(n+1)(n-2)$ **22b.** $-1, 2$

22c. $1 + \frac{n+6}{n+1} = \frac{4}{n-2}$

$\left(1 + \frac{n+6}{n+1}\right)(n+1)(n-2) = \left(\frac{4}{n-2}\right)(n+1)(n-2)$

$(n+1)(n-2) + (n-2)(n+6) = 4(n+1)$

$n^2 - n - 2 + n^2 + 4n - 12 = 4n + 4$

$2n^2 - n - 18 = 0$

$n = \frac{1 \pm \sqrt{1 - 4(2)(-18)}}{2 \cdot 2}$

$= \frac{1 \pm \sqrt{145}}{4}$

23. $\frac{x-6}{x^2-2x} = \frac{x-6}{x(x-2)}$

$\frac{x-6}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$

$x - 6 = A(x-2) + B(x)$

Let $x = 2$.

$2 - 6 = A(2-2) + B(2)$

$-4 = 2B$

$-2 = B$

Let $x = 0$.

$0 - 6 = A(0-2) + B(0)$

$-6 = -2A$

$3 = A$

$\frac{x-6}{x^2-2x} = \frac{3}{x} + \frac{-2}{x-2}$

24. $\frac{5m-4}{m^2-4} = \frac{5m-4}{(m+2)(m-2)}$

$\frac{5m-4}{m^2-4} = \frac{A}{m+2} + \frac{B}{m-2}$

$5m - 4 = A(m-2) + B(m+2)$

Let $m = 2$.

$5(2) - 4 = A(2-2) + B(2+2)$

$6 = 4B$

$1.5 = B$

Let $m = -2$.

$5(-2) - 4 = A(-2-2) + B(-2+2)$

$-14 = -4A$

$3.5 = A$

$\frac{5m-4}{m^2-4} = \frac{3.5}{m+2} + \frac{1.5}{m-2}$

25. $\frac{-4y}{3y^2-4y+1} = \frac{-4y}{(3y-1)(y-1)}$

$\frac{-4y}{3y^2-4y+1} = \frac{A}{3y-1} + \frac{B}{y-1}$

$-4y = A(y-1) + B(3y-1)$

Let $y = 1$.

$-4(1) = A(1-1) + B(3(1)-1)$

$-4 = 2B$

$-2 = B$

Let $y = \frac{1}{3}$.

$-4\left(\frac{1}{3}\right) = A\left(\frac{1}{3} - 1\right) + B\left(3\left(\frac{1}{3}\right) - 1\right)$

$-\frac{4}{3} = -\frac{2}{3}A$

$2 = A$

$\frac{-4y}{3y^2-4y+1} = \frac{2}{3y-1} + \frac{-2}{y-1}$

26. $\frac{9-9x}{x^2-9} = \frac{9-9x}{(x+3)(x-3)}$

$\frac{9-9x}{x^2-9} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

Let $x = 3$.

$9 - 9(3) = A(3-3) + B(3+3)$

$-18 = 6B$

$-3 = B$

Let $x = -3$.

$9 - 9(-3) = A(-3-3) + B(-3+3)$

$36 = -6A$

$-6 = A$

$\frac{9-9x}{x^2-9} = \frac{-6}{x+3} + \frac{-3}{x-3}$

$\frac{-6}{x+3}, \frac{-3}{x-3}$

27a. $a(a-6)$

27b. $\frac{a-2}{a} = \frac{a-4}{a-6}$

$\left(\frac{a-2}{a}\right)(a)(a-6) = \left(\frac{a-4}{a-6}\right)(a)(a-6)$

$(a-2)(a-6) = (a-4)(a)$

$a^2 - 8a + 12 = a^2 - 4a$

$12 = 4a$

$3 = a$

27c. 0, 6

27d. Test $a = -1$: $\frac{-1-2}{-1} < \frac{-1-4}{-1-6}$

$3 < \frac{5}{7}$ false

Test $a = 1$: $\frac{1-2}{1} < \frac{1-4}{1-6}$

$-1 < \frac{3}{5}$ true

Test $a = 4$: $\frac{4-2}{4} < \frac{4-4}{4-6}$

$\frac{1}{2} < 0$ false

Test $a = 7$: $\frac{7-2}{7} < \frac{7-4}{7-6}$

$\frac{5}{7} < 3$ true

Solution: $0 < a < 3$, $6 < a$

28. $\frac{2}{w} + 3 > \frac{29}{w}$; exclude: 0

$2 + 3w = 29$

$w = 9$

Test $w = -1$: $\frac{2}{-1} + 3 > \frac{29}{-1}$

$1 > -29$ true

Test $w = 1$: $\frac{2}{1} + 3 > \frac{29}{1}$

$5 > 29$ false

Test $w = 10$: $\frac{2}{(10)} + 3 > \frac{29}{10}$

$\frac{32}{10} > \frac{29}{10}$ true

Solution: $w < 0$, $w > 9$

29. $\frac{(x-3)(x-4)}{(x-5)(x-6)^2} \leq 0$; exclude 5, 6

$(x-3)(x-4) = 0$

$x - 3 = 0$ $\quad$ $x - 4 = 0$

$x = 3$ $\quad$ $x = 4$

Test $x = 0$: $\frac{(0-3)(0-4)}{(0-5)(0-6)^2} \leq 0$

$\frac{12}{-180} \leq 0$ true

Test $x = 3.5$: $\frac{(3.5-3)(3.5-4)}{(3.5-5)(3.5-6)^2} \leq 0$

$\frac{-0.25}{-9.375} \leq 0$ false

Test $x = 4.5$: $\frac{(4.5-3)(4.5-4)}{(4.5-5)(4.5-6)^2} \leq 0$

$\frac{0.75}{-1.125} \leq 0$ true

Test $x = 5.5$: $\frac{(5.5-3)(5.5-4)}{(5.5-5)(5.5-6)^2} \leq 0$

$\frac{3.75}{0.125} \leq 0$ false

Test $x = 6.5$: $(\frac{6.5-3)(6.5-4)}{(6.5-5)(6.5-6)^2} \leq 0$

$\frac{8.75}{0.375} \leq 0$ false

Solution: $x \leq 3$, $4 \leq x < 5$

30. $\frac{x^2-16}{x^2-4x-5} \geq 0$

$\frac{x^2-16}{(x-5)(x+1)} = 0$; exclude 5, -1

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

Test $x = -5$: $\frac{(-5)^2-16}{(-5-5)(-5+1)} \geq 0$

$\frac{9}{40} \geq 0$ true

Test $x = -2$: $\frac{(-2)^2-16}{(-2-5)(-2+1)} \geq 0$

$\frac{-12}{7} \geq 0$ false

Test $x = 0$: $\frac{0-16}{0^2-4(0)-5} \geq 0$

$\frac{-16}{-5} \geq 0$ true

Test $x = 4.5$: $\frac{4.5^2-16}{(4.5-5)(4.5+1)} \geq 0$

$\frac{4.25}{-2.75} \geq 0$ false

Test $x = 6$: $\frac{6^2-16}{6^2-24-5} \geq 0$

$\frac{20}{7} \geq 0$ true

Solution: $x \leq -4$, $-1 < x \leq 4$, $x > 5$

31. $\frac{1}{4a} + \frac{5}{8a} > \frac{1}{2}$; exclude: 0

$\frac{1}{4a} + \frac{5}{8a} = \frac{1}{2}$

$2 + 5 = 4a$

$\frac{7}{4} = a$

Test $a = -1$: $\frac{1}{4(-1)} + \frac{5}{8(-1)} > \frac{1}{2}$

$-\frac{7}{8} > \frac{1}{2}$ false

Test $a = 1$: $\frac{1}{4(1)} + \frac{5}{8(1)} > \frac{1}{2}$

$\frac{7}{8} > \frac{1}{2}$ true

Test $a = 2$: $\frac{1}{4(2)} + \frac{5}{8(2)} > \frac{1}{2}$

$\frac{7}{16} > \frac{1}{2}$ false

Solution: $0 < a < \frac{7}{4}$

32. $\frac{1}{2b+1} + \frac{1}{b+1} > \frac{8}{15}$; exclude: $-\frac{1}{2}, -1$

$\frac{1}{2b+1} + \frac{1}{b+1} = \frac{8}{15}$

$15(b+1) + 15(2b+1) = 8(2b+1)(b+1)$

$45b + 30 = 16b^2 + 24b + 8$

$0 = 16b^2 - 21b - 22$

$0 = (16b + 11)(b - 2)$

$16b + 11 = 0 \qquad b - 2 = 0$

$b = -\frac{11}{16} \qquad b = 2$

Test $b = -2$: $\frac{1}{2(-2)+1} + \frac{1}{-2+1} > \frac{8}{15}$

$-\frac{4}{3} > \frac{8}{15}$ false

Test $b = -0.8$: $\frac{1}{2(-0.8)+1} + \frac{1}{(-0.8)+1} > \frac{8}{15}$

$\frac{10}{3} > \frac{8}{15}$ true

Test $b = -0.6$: $\frac{1}{2(-0.6)+1} + \frac{1}{(-0.6)+1} > \frac{8}{15}$

$-\frac{5}{2} > \frac{8}{15}$ false

Test $b = 0$: $\frac{1}{2(0)+1} + \frac{1}{0+1} > \frac{8}{15}$

$2 > \frac{8}{15}$ true

Test $b = 3$: $\frac{1}{2(3)+1} + \frac{1}{3+1} > \frac{8}{15}$

$\frac{11}{28} > \frac{8}{15}$ false

Solution: $-1 < b < -\frac{11}{16}, -\frac{1}{2} < b < 2$

33. $\frac{7}{y+1} > 7$; exclude -1

$7 = 7(y+1)$

$1 = y + 1$

$0 = y$

Test $y = -2$: $\frac{7}{-2+1} > 7$

$-7 > 7$ false

Test $y = -0.5$: $\frac{7}{-0.5+1} > 7$

$14 > 7$ true

Test $y = 1$: $\frac{7}{1+1} > 7$

$\frac{7}{2} > 7$ false

Solution: $-1 < y < 0$

34. Let x = the number.

$4\left(\frac{1}{x}\right) + x = 10\frac{2}{5}$

$20 + 5x^2 = 52x$

$5x^2 - 52x + 20 = 0$

$(5x - 2)(x - 10) = 0$

$5x - 2 = 0 \qquad x - 10 = 0$

$x = \frac{2}{5} \qquad x = 10$

35. $\frac{x+2}{x-5} > 0.30$

$\frac{x+2}{x-5} = 0.30$; exclude 5

$x + 2 = 0.30(x - 5)$

$x + 2 = 0.30x - 1.5$

$0.7x = -3.5$

$x = -5$

Test $x = -6$: $\frac{-6+2}{-6-5} > 0.30$

$0.36 > 0.30$ true

Test $x = 0$: $\frac{0+2}{0-5} > 0.30$

$-0.4 > 0.30$ false

Test $x = 6$: $\frac{6+2}{6-5} > 0.30$

$8 > 0.30$ true

Solution: $x < -5$ or $x > 5$

36a. $\frac{1}{8} = \frac{1}{d_i} + \frac{1}{32}$

36b. $\frac{1}{8} = \frac{1}{d_i} + \frac{1}{32}$

$\left(\frac{1}{8}\right)(32d_i) = \left(\frac{1}{d_i} + \frac{1}{32}\right)(32d_i)$

$4d_i = 32 + d_i$

$3d_i = 32$

$d_i = 10\frac{2}{3}$ cm

37. Sample answer: $\frac{x}{x-3} = \frac{1}{x+2}$

38. Let x = capacity of larger truck.

$\frac{5}{2} = \frac{x}{x-3}$

$5(x - 3) = 2x$

$5x - 15 = 2x$

$3x = 15$

$x = 5$ tons

39a. $\frac{1}{10} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{20}$

39b. $\frac{1}{10} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{20}$

$\left(\frac{1}{10}\right)(20r) = \left(\frac{1}{2r} + \frac{1}{r} + \frac{1}{20}\right)(20r)$

$2r = 10 + 20 + r$

$r = 30$

$2r = 2(30)$ or 60; 60 ohms, 30 ohms

40. Let x = the number of quiz questions to be answered.

$\frac{11+x}{20+x} = 0.70$

$11 + x = 0.70(20 + x)$

$11 + x = 14 + 0.70x$

$0.3x = 3$

$x = 10$ questions

41. Let x = the speed of the wind.

$\frac{1062}{200+x} = \frac{738}{200-x}$

$1062(200 - x) = 738(200 + x)$

$212{,}400 - 1062x = 147{,}600 + 738x$

$64{,}800 = 1800x$

$36 = x$; 36 mph

42. $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

$$\left(\frac{1}{a} + \frac{1}{b}\right)(a)(b)(c) = \left(\frac{1}{c}\right)(a)(b)(c)$$
$$bc + ac = ab$$
$$bc = ab - ac$$
$$bc = a(b - c)$$
$$\frac{bc}{b - c} = a$$

43a. $\frac{1}{x} = \frac{1}{2}\left(\frac{1}{y} + \frac{1}{z}\right)$

$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{30} + \frac{1}{45}\right)$$

43b.
$$\frac{1}{x} = \frac{1}{2}\left(\frac{1}{30} + \frac{1}{45}\right)$$
$$\frac{1}{x} = \frac{1}{60} + \frac{1}{90}$$
$$\left(\frac{1}{x}\right)(360x) = \left(\frac{1}{60} + \frac{1}{90}\right)(360x)$$
$$360 = 6x + 4x$$
$$360 = 10x$$
$$36 = x$$

44. Let x = number of gallons of gasoline.

$$\frac{20 \text{ m}}{\text{g}} = \frac{15{,}000 \text{ m}}{x \text{ g}}$$
$$20x = 15{,}000$$
$$x = 750 \text{ gallons}$$
$$750 \times \$1.20 = \$900$$
$$x \cdot \$1.20 = \$900 - \$200$$
$$x = 583\tfrac{1}{3} \text{ gallons}$$

Let y = number of miles per gallon.

$$\frac{y \text{ m}}{\text{g}} = \frac{15{,}000 \text{ m}}{583\frac{1}{3} \text{ g}}$$
$$583\tfrac{1}{3}y = 15{,}000$$
$$y = 25.7; \text{ about } 25.7 \text{ mpg}$$

45.
$$T = \frac{d}{s}$$
$$10\tfrac{2}{3} = \frac{26}{s + 5} + \frac{26}{s - 5}$$
$$\left(10\tfrac{2}{3}\right)(3)(s + 5)(s - 5) = \left(\frac{26}{s + 5} + \frac{26}{s - 5}\right)(3)(s + 5)(s - 5)$$
$$32(s + 5)(s - 5) = 26(3)(s - 5) + 26(3)(s + 5)$$
$$32s^2 - 800 = 78s - 390 + 78s + 390$$
$$32s^2 - 156s - 800 = 0$$
$$8s^2 - 39s - 200 = 0$$
$$(8s + 25)(s - 8) = 0$$

$8s + 25 = 0$ $\qquad$ $s - 8 = 0$

$s = -\frac{25}{8}$ $\qquad$ $s = 8$

8 mph

46. $\frac{3x - 5y}{5y} = \frac{3x}{5y} - \frac{5y}{5y}$

$= 11 - 1$

$= 10$

47.

r	1	2	−3	−5
−3	1	−1	0	−5
−2	1	0	−3	1
−1	1	1	−4	−1
0	1	2	−3	−5
1	1	3	0	−5
2	1	4	5	5

−3 and −2, −2 and −1, 1 and 2

48.

$\underline{-5}$	1	0	−30	0
		−5	25	25
	1	−5	−5	25

no

49. 2; $12x^2 + 8x - 15 = 0$

$(6x - 5)(2x + 3) = 0$

$6x - 5 = 0$ $\qquad$ $2x + 3 = 0$

$x = \frac{5}{6}$ $\qquad$ $x = -\frac{3}{2}$

50. $|3x| - 12 = 0$

$|3x| = 12$

$3x = 12$ $\qquad$ $-3x = 12$

$x = 4$ $\qquad$ $x = -4$

51. $y \le \frac{2x + 3}{x}$

$3 \overset{?}{\le} \frac{2(6) + 3}{6}$

$3 \le \frac{5}{2}$ false

no

52. $y^2 = 121x^2 \to b^2 = 121a^2$

52a. $(-b)^2 = 121a^2$
$b^2 = 121a^2$ yes

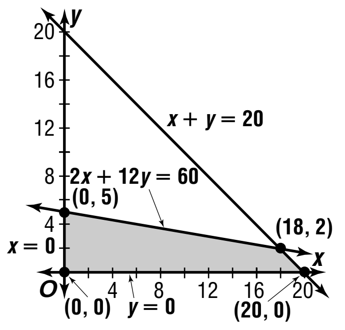
52b. $b^2 = 121(-a)^2$
$b^2 = 121a^2$ yes

52c. $(a)^2 = 121(b)^2$
$a^2 = 121b^2$ no

52d. $(-a)^2 = 121(-b)^2$
$a^2 = 121b^2$ no

53a. Let x = short answer questions and y = essay questions.

$x + y \le 20$
$2x + 12y \le 60$
$x \ge 0$
$y \ge 0$

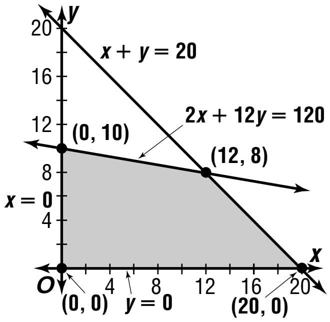


$S(x, y) = 5x + 15y$
$S(0, 0) = 5(0) + 15(0)$ or 0
$S(0, 5) = 5(0) + 15(5)$ or 75
$S(18, 2) = 5(18) + 15(2)$ or 120
$S(20, 0) = 5(20) + 15(0)$ or 100

18 short answer and 2 essay for a score of 120 points

53b. Let x = short answer questions and y = essay questions.

$x + y \le 20$
$2x + 12y \le 120$
$x \ge 0$
$y \ge 0$



$S(x, y) = 5x + 15y$
$S(0, 0) = 5(0) + 15(0)$ or 0
$S(0, 10) = 5(0) + 15(10)$ or 150
$S(12, 8) = 5(12) + 15(8)$ or 180
$S(20, 0) = 5(20) + 15(0)$ or 100
12 short answer and 8 essay for a score of 180 points

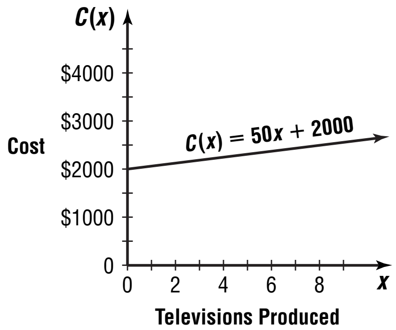
54. $$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -3 & -5 \end{bmatrix} = x$$
$$\begin{bmatrix} 1(3) + 1(-3) & 1(5) + 1(-5) \\ 1(3) + 1(-3) & 1(5) + 1(-5) \end{bmatrix} = x$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = x$$

55. $y - y_1 = m(x - x_1)$
$y - 1 = 2(x - (-3))$
$y - 1 = 2x + 6$
$2x - y + 7 = 0$

56a. $m = \frac{3000 - 5000}{20 - 60}$
$m = 50$
$y - 3000 = 50(x - 20)$
$y = 50x + 2000$
$C(x) = 50x + 2000$

56b. \$2000; \$50

56c.



57. A of $\triangle JKL = \frac{1}{2}(9)(7)$ or 31.5

A of small triangle $= \frac{1}{2}(5)(3)$ or 7.5

A of shaded region $= 31.5 - 7.5$ or 24
The answer is 24.

4-7 Radical Equations and Inequalities

Pages 254–255 Check for Understanding

1. To solve the equation, you need to get rid of the radical by squaring both sides of the equation. If the radical is not isolated first, a radical will remain in the equation.

2. The process of raising to a power sometimes creates a new equation with more solutions than the original equation. These extra or extraneous solutions do not solve the original equation.

3. When solving an equation with one radical, you isolate the radical on one side and then square each side. When there is more than one radical expression in an equation, you isolate one of the radicals and then square each side. Then you isolate the other radical and square each side. In both cases, once you have eliminated all radical signs, you solve for the variable.

4. $\sqrt{1 - 4t} = 2$
$1 - 4t = 4$
$-4t = 3$
$t = -\frac{3}{4}$

Check: $\sqrt{1 - 4t} = 2$
$\sqrt{1 - 4\left(-\frac{3}{4}\right)} \stackrel{?}{=} 2$
$\sqrt{1 + 3} \stackrel{?}{=} 2$
$2 = 2$ ✓

5. $\sqrt[3]{x + 4} + 12 = 3$
$\sqrt[3]{x + 4} = -9$
$x + 4 = -729$
$x = -733$

Check: $\sqrt[3]{x + 4} + 12 = 3$
$\sqrt[3]{-733 + 4} + 12 \stackrel{?}{=} 3$
$\sqrt[3]{-729} + 12 \stackrel{?}{=} 3$
$-9 + 12 \stackrel{?}{=} 3$
$3 = 3$ ✓

6. $5 + \sqrt{x - 4} = 2$
$\sqrt{x - 4} = -3$
$x - 4 = 9$
$x = 13$

Check: $5 + \sqrt{x - 4} = 2$
$5 + \sqrt{13 - 4} \stackrel{?}{=} 2$
$5 + \sqrt{9} \stackrel{?}{=} 2$
$5 + 3 \ne 2$
no real solution

7. $\sqrt{6x - 4} = \sqrt{2x + 10}$
$6x - 4 = 2x + 10$
$4x = 14$
$x = 3.5$

Check: $\sqrt{6x - 4} = \sqrt{2x + 10}$
$\sqrt{6\left(\frac{7}{2}\right) - 4} = \sqrt{2\left(\frac{7}{2}\right) + 10}$
$\sqrt{21 - 4} = \sqrt{7 + 10}$
$\sqrt{17} = \sqrt{17}$ ✓

8. $\sqrt{a + 4} + \sqrt{a - 3} = 7$
$\sqrt{a - 4} = 7 - \sqrt{a - 3}$
$a - 4 = 49 - 14\sqrt{a - 3} + a - 3$
$-42 = -14\sqrt{a - 3}$
$1764 = 196(a - 3)$
$9 = a - 3$
$12 = a$

Check: $\sqrt{a + 4} + \sqrt{a - 3} = 7$
$\sqrt{12 + 4} + \sqrt{12 - 3} \stackrel{?}{=} 7$
$\sqrt{16} + \sqrt{9} \stackrel{?}{=} 7$
$4 + 3 = 7$ ✓

9. $\sqrt{5x+4} \le 8$
$5x + 4 \le 64$
$5x \le 60$
$x \le 12$

$5x + 4 \ge 0$
$5x \ge -4$
$x \ge -0.8$

Test $x = -1$: $\sqrt{5(-1)+4} \le 8$
$\sqrt{-1} \le 8$ meaningless
Test $x = 0$: $\sqrt{5(0)+4} \le 8$
$\sqrt{4} \le 8$ true
Test $x = 13$: $\sqrt{5(13)+4} \le 8$
$\sqrt{69} \le 8$ false
Solution: $-0.8 \le x \le 12$

10. $3 + \sqrt{4a-5} \le 10$
$\sqrt{4a-5} \le 7$
$4a - 5 \le 49$
$4a \le 54$
$a \le 13.5$

$4a - 5 \ge 0$
$4a \ge 5$
$a \ge 1.25$

Test $a = 0$: $3 + \sqrt{4(0)-5} \le 10$
$3 + \sqrt{-5} \le 10$ meaningless
Test $a = 2$: $3 + \sqrt{4(2)-5} \le 10$
$4 + \sqrt{3} \le 10$ true
Test $a = 14$: $3 + \sqrt{4(14)-5} \le 10$
$3 + \sqrt{51} \le 10$ false
Solution: $1.25 \le a \le 13.5$

11a. $v = \sqrt{v_0^2 + 64h}$
$90 = \sqrt{10^2 + 64h}$
$90 = \sqrt{100 + 64h}$

11b. $90 = \sqrt{100 + 64h}$
$8100 = 100 + 64h$
$8000 = 64h$
$1125 = h$; 125 ft

Check: $90 = \sqrt{100 + 64h}$
$90 \stackrel{?}{=} \sqrt{100 + 64(125)}$
$90 \stackrel{?}{=} \sqrt{8100}$
$90 = 90$ ✓

Pages 255–257 Exercises

12. $\sqrt{x+8} = 5$
$\sqrt{x+8} = 25$
$x = 17$

Check: $\sqrt{x+8} = 5$
$\sqrt{17+8} \stackrel{?}{=} 5$
$\sqrt{25} \stackrel{?}{=} 5$
$5 = 5$ ✓

13. $\sqrt[3]{y-7} = 4$
$y - 7 = 64$
$y = 71$

Check: $\sqrt[3]{y-7} = 4$
$\sqrt[3]{71-7} \stackrel{?}{=} 4$
$\sqrt[3]{64} \stackrel{?}{=} 4$
$4 = 4$ ✓

14. $\sqrt{8n-5} - 1 = 2$
$\sqrt{8n-5} = 3$
$8n - 5 = 9$
$8n = 14$
$n = \frac{7}{4}$

Check: $\sqrt{8n-5} - 1 = 2$
$\sqrt{8\left(\frac{7}{4}\right) - 5} - 1 \stackrel{?}{=} 2$
$\sqrt{14-5} - 1 \stackrel{?}{=} 2$
$3 - 1 \stackrel{?}{=} 2$
$2 = 2$ ✓

15. $\sqrt{x+16} = \sqrt{x} + 4$
$x + 16 = x + 8\sqrt{x} + 16$
$0 = 8\sqrt{x}$
$0 = \sqrt{x}$
$0 = x$
Check: $\sqrt{x+16} = \sqrt{x} + 4$
$\sqrt{0+16} \stackrel{?}{=} \sqrt{0} + 4$
$\sqrt{16} \stackrel{?}{=} 4$
$4 = 4$ ✓

16. $4\sqrt{3m^2 - 15} = 4$
$\sqrt{3m^2 - 15} = 1$
$3m^2 - 15 = 1$
$3m^2 = 16$
$m^2 = \frac{16}{3}$
$m = \pm\frac{4}{3}\sqrt{3}$
Check: $4\sqrt{3m^2 - 15} = 4$
$4\sqrt{3\left(\frac{4}{3}\sqrt{3}\right)^2 - 15} \stackrel{?}{=} 4$
$4\sqrt{1} \stackrel{?}{=} 4$
$4 = 4$ ✓
Check: $4\sqrt{3m^2 - 15} = 4$
$4\sqrt{3\left(-\frac{4}{3}\sqrt{3}\right)^2 - 15} \stackrel{?}{=} 4$
$4\sqrt{1} \stackrel{?}{=} 4$
$4 = 4$ ✓

17. $\sqrt{9u-4} = \sqrt{7u-20}$
$9u - 4 = 7u - 20$
$2u = -16$
$u = -8$
Check: $\sqrt{9u-4} = \sqrt{7u-20}$
$\sqrt{9(-8)-4} \stackrel{?}{=} \sqrt{7(-8)-20}$
$\sqrt{-76} = \sqrt{-76}$
no real solution

18. $\sqrt[3]{6u-5} + 2 = -3$
$\sqrt[3]{6u-5} = -5$
$6u - 5 = -125$
$6u = -120$
$u = -20$
Check: $\sqrt[3]{6u-5} + 2 = -3$
$\sqrt[3]{6(-20)-5} + 2 \stackrel{?}{=} -3$
$\sqrt[3]{-125} + 2 \stackrel{?}{=} -3$
$-5 + 2 \stackrel{?}{=} -3$
$-3 = -3$ ✓

19. $\sqrt{4m^2 - 3m + 2} - 2m - 5 = 0$
$\sqrt{4m^2 - 3m + 2} = 2m + 5$
$4m^2 - 3m + 2 = 4m^2 + 20m + 25$
$-23 = 23m$
$-1 = m$
Check: $\sqrt{4m^2 - 3m + 2} - 2m - 5 = 0$
$\sqrt{4(-1)^2 - 3(-1) + 2} - 2(-1) - 5 \stackrel{?}{=} 0$
$\sqrt{9} + 2 - 5 \stackrel{?}{=} 0$
$3 - 3 = 0$
$0 = 0$ ✓

20. $\sqrt{k+9} - \sqrt{k} = \sqrt{3}$
$\sqrt{k+9} = \sqrt{3} + \sqrt{k}$
$k + 9 = 3 + 2\sqrt{3k} + k$
$6 = 2\sqrt{3k}$
$36 = 4(3k)$
$36 = 12k$
$3 = k$
Check: $\sqrt{k+9} - \sqrt{k} = \sqrt{3}$
$\sqrt{3+9} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$
$\sqrt{12} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$
$2\sqrt{3} - \sqrt{3} \stackrel{?}{=} \sqrt{3}$
$\sqrt{3} = \sqrt{3}$ ✓

21. $\sqrt{a+21} - 1 = \sqrt{a+12}$

$a + 21 - 2\sqrt{a+21} + 1 = a + 12$

$-2\sqrt{a+21} = -10$

$4(a + 21) = 100$

$a + 21 = 25$

$a = 4$

Check: $\sqrt{a+21} - 1 = \sqrt{a+12}$

$\sqrt{4+21} - 1 \stackrel{?}{=} \sqrt{4+12}$

$\sqrt{25} - 1 \stackrel{?}{=} \sqrt{16}$

$5 - 1 \stackrel{?}{=} 4$

$4 = 4$ ✓

22. $\sqrt{3x+4} - \sqrt{2x-7} = 3$

$\sqrt{3x+4} = 3 + \sqrt{2x-7}$

$3x + 4 = 9 + 6\sqrt{2x-7} + 2x - 7$

$x + 2 = 6\sqrt{2x-7}$

$x^2 + 4x + 4 = 36(2x - 7)$

$x^2 - 68x + 256 = 0$

$(x - 4)(x - 64) = 0$

$x - 4 = 0$ $\quad$ $x - 64 = 0$

$x = 4$ $\quad$ $x = 64$

Check: $\sqrt{3x+4} - \sqrt{2x-7} = 3$

$\sqrt{3(4)+4} - \sqrt{2(4)-7} \stackrel{?}{=} 3$

$\sqrt{16} - \sqrt{1} \stackrel{?}{=} 3$

$4 - 1 \stackrel{?}{=} 3$

$3 = 3$ ✓

Check: $\sqrt{3x+4} - \sqrt{2x-7} = 3$

$\sqrt{3(64)+4} - \sqrt{2(64)-7} \stackrel{?}{=} 3$

$\sqrt{196} - \sqrt{121} \stackrel{?}{=} 3$

$14 - 11 \stackrel{?}{=} 3$

$3 = 3$ ✓

23. $2\sqrt[3]{7b-1} - 4 = 0$

$2\sqrt[3]{7b-1} = 4$

$8(7b - 1) = 64$

$7b - 1 = 8$

$7b = 9$

$b = \frac{9}{7}$

Check: $2\sqrt[3]{7b-1} - 4 = 0$

$2\sqrt[3]{7\left(\frac{9}{7}\right) - 1} - 4 \stackrel{?}{=} 0$

$2\sqrt[3]{8} - 4 \stackrel{?}{=} 0$

$4 - 4 \stackrel{?}{=} 0$

$0 = 0$ ✓

24. $\sqrt[4]{3t} - 2 = 0$

$\sqrt[4]{3t} = 2$

$3t = 16$

$t = \frac{16}{3}$

Check: $\sqrt[4]{3t} - 2 = 0$

$\sqrt[4]{3\left(\frac{16}{3}\right)} - 2 \stackrel{?}{=} 0$

$\sqrt[4]{16} - 2 \stackrel{?}{=} 0$

$2 - 2 \stackrel{?}{=} 0$

$0 = 0$ ✓

25. $\sqrt{x+2} - 7 = \sqrt{x+9}$

$x + 2 - 14\sqrt{x+2} + 49 = x + 9$

$-14\sqrt{x+2} = -42$

$196(x + 2) = 1764$

$x + 2 = 9$

$x = 7$

Check: $\sqrt{x+2} - 7 = \sqrt{x+9}$

$\sqrt{7+2} - 7 \stackrel{?}{=} \sqrt{7+9}$

$3 - 7 \stackrel{?}{=} 4$

$-4 \neq 4$

no real solution

26. $\sqrt{2x+1} + \sqrt{2x+6} = 5$

$\sqrt{2x+1} = 5 - \sqrt{2x+6}$

$2x + 1 = 25 - 10\sqrt{2x+6} + 2x + 6$

$-30 = -10\sqrt{2x+6}$

$3 = \sqrt{2x+6}$

$9 = 2x + 6$

$3 = 2x$

$\frac{3}{2} = x$

Check: $\sqrt{2x+1} + \sqrt{2x+6} = 5$

$\sqrt{2\left(\frac{3}{2}\right) + 1} + \sqrt{2\left(\frac{3}{2}\right) + 6} \stackrel{?}{=} 5$

$\sqrt{4} + \sqrt{9} \stackrel{?}{=} 5$

$2 + 3 \stackrel{?}{=} 5$

$5 = 5$ ✓

27. $\sqrt{3x+10} = \sqrt{x+11} - 1$

$3x + 10 = x + 11 - 2\sqrt{x+11} + 1$

$2x - 2 = -2\sqrt{x+11}$

$-x + 1 = \sqrt{x+11}$

$x^2 - 2x + 1 = x + 11$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x - 5 = 0$ $\quad$ $x + 2 = 0$

$x = 5$ $\quad$ $x = -2$

Check: $\sqrt{3x+10} = \sqrt{x+11} - 1$

$\sqrt{3(5)+10} \stackrel{?}{=} \sqrt{5+11} - 1$

$\sqrt{25} \stackrel{?}{=} \sqrt{16} - 1$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

Check: $\sqrt{3x+10} = \sqrt{x+11} - 1$

$\sqrt{3(-2)+10} \stackrel{?}{=} \sqrt{-2+11} - 1$

$\sqrt{4} \stackrel{?}{=} \sqrt{9} - 1$

$2 = 3 - 1$

$2 = 2$ ✓

Solution: $x = -2$

28a. $\sqrt{3t-14} + t = 6$

$\sqrt{3t-14} = 6 - t$

$3t - 14 = 36 - 12t + t^2$

$0 = t^2 - 15t + 50$

$0 = (t - 5)(t - 10)$

$t - 5 = 0$ $\quad$ $t - 10 = 0$

$t = 5$ $\quad$ $t = 10$

Check: $\sqrt{3t-14} + t = 6$

$\sqrt{3(5)-14} + 5 \stackrel{?}{=} 6$

$\sqrt{1} + 5 \stackrel{?}{=} 6$

$1 + 5 \stackrel{?}{=} 6$

$6 = 6$ ✓

Check: $\sqrt{3t-14} + t = 6$

$\sqrt{3(10)-14} + 10 \stackrel{?}{=} 6$

$\sqrt{16} + 10 \stackrel{?}{=} 6$

$4 + 10 \stackrel{?}{=} 6$

$14 \neq 6$

10

28b. 5

29. $\sqrt{2x - 7} \geq 5$
$2x - 7 \geq 25$
$2x \geq 32$
$x \geq 16$
$2x - 7 \geq 0$
$2x \geq 7$
$x \geq \frac{7}{2}$

Test $x = 0$: $\sqrt{2(0) - 7} \geq 5$
$\sqrt{-7} \geq 5$
meaningless

Test $x = 4$: $\sqrt{2(4) - 7} \geq 5$
$\sqrt{8 - 7} \geq 5$
$1 \geq 5$
false

Test $x = 17$: $\sqrt{2(17) - 7} \geq 5$
$\sqrt{27} \geq 5$
true

Solution: $x \geq 16$

30. $\sqrt{b + 4} \leq 6$
$b + 4 \leq 36$
$b \leq 32$
$b + 4 \geq 0$
$b \geq -4$

Test $b = -5$: $\sqrt{-5 + 4} \leq 6$
$\sqrt{-1} \leq 6$
meaningless

Test $b = 0$: $\sqrt{0 + 4} \leq 6$
$\sqrt{4} \leq 6$
$2 \leq 6$
true

Test $b = 33$: $\sqrt{33 + 4} \leq 6$
$\sqrt{37} \leq 6$
false

Solution: $-4 \leq b \leq 32$

31. $\sqrt{a - 5} \leq 4$
$a - 5 \leq 16$
$a \leq 21$
$a - 5 \geq 0$
$a \geq 5$

Test $a = 0$: $\sqrt{0 - 5} \leq 4$
$\sqrt{-5} \leq 4$
meaningless

Test $a = 6$: $\sqrt{6 - 5} \leq 4$
$\sqrt{1} \leq 4$
$1 \leq 4$
true

Test $a = 22$: $\sqrt{22 - 5} \leq 4$
$\sqrt{17} \leq 4$
false

Solution: $5 \leq a \leq 21$

32. $\sqrt{2x - 5} \leq 6$
$2x - 5 \leq 36$
$2x \leq 41$
$x \leq 20.5$
$2x - 5 \geq 0$
$2x \geq 5$
$x \geq 2.5$

Test $x = 0$: $\sqrt{2(0) - 5} \leq 6$
$\sqrt{-5} \leq 6$
meaningless

Test $x = 5$: $\sqrt{2(5) - 5} \leq 6$
$\sqrt{5} \leq 6$
true

Test $x = 22$: $\sqrt{2(22) - 5} \leq 6$
$\sqrt{39} \leq 6$
false

Solution: $2.5 \leq x \leq 20.5$

33. $\sqrt[4]{5y - 9} \leq 2$
$5y - 9 \leq 16$
$5y \leq 25$
$y \leq 5$
$5y - 9 \geq 0$
$5y \geq 9$
$y \geq 1.8$

Test $y = 0$: $\sqrt[4]{5(0) - 9} \leq 2$
$\sqrt[4]{-9} \leq 2$
meaningless

Test $y = 2$: $\sqrt[4]{5(0) - 9} \leq 2$
$\sqrt[4]{1} \leq 2$
true

Test $y = 6$: $\sqrt[4]{5(6) - 9} \leq 2$
$\sqrt[4]{21} \leq 2$
false

Solution: $1.8 \leq y \leq 5$

34. $\sqrt{m + 2} \leq \sqrt{3m + 4}$
$m + 2 \leq 3m + 4$
$-2m \leq 2$
$m \geq -1$
$m + 2 \geq 0$
$m \geq -2$
$3m + 4 \geq 0$
$3m \geq -4$
$m \geq -\frac{4}{3}$

Test $m = -3$: $\sqrt{-3 + 2} \leq \sqrt{3(-3) + 4}$
$\sqrt{-1} \leq \sqrt{-5}$ meaningless

Test $m = -1.6$: $\sqrt{-1.6 + 2} \leq \sqrt{3(-1.6) + 4}$
$\sqrt{0.4} \leq \sqrt{-0.8}$
meaningless

Test $m = -1.2$: $\sqrt{-1.2 + 2} \leq \sqrt{3(-1.2) + 4}$
$\sqrt{0.8} \leq \sqrt{0.4}$ false

Test $m = 0$: $\sqrt{0 + 2} \leq \sqrt{3(0) + 4}$
$\sqrt{2} \leq \sqrt{4}$ true

Solution: $m \geq -1$

35. $\sqrt{2c - 5} > 7$
$2c - 5 > 49$
$2c > 54$
$c > 27$
$2c - 5 \geq 0$
$2c \geq 5$
$c \geq 2.5$

Test $c = 0$: $\sqrt{2(0) - 5} \geq 7$
$\sqrt{-5} \geq 7$
meaningless

Test $c = 5$: $\sqrt{2(5) - 5} \geq 7$
$\sqrt{5} \geq 7$
false

Test $c = 28$: $\sqrt{2(28) - 5} \geq 7$
$\sqrt{51} \geq 7$
true

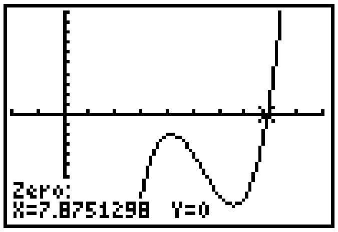
Solution: $c > 27$

36a. $t = \sqrt{\frac{2s}{g}}$
$3 = \sqrt{\frac{2(7.2)}{g}}$
$3 = \sqrt{\frac{14.4}{g}}$

36b. $3 = \sqrt{\frac{14.4}{g}}$
$9 = \frac{14.4}{g}$
$9g = 14.4$
$g = 1.6$ m/s^2

37. $\sqrt{x - 5} = \sqrt[3]{x - 3}$
$x - 5 = \sqrt[3]{(x - 3)^2}$
$x - 5 = \sqrt[3]{x^2 - 6x + 9}$
$(x - 5)^3 = x^2 - 6x + 9$
$x^3 - 15x^2 + 75x - 125 = x^2 - 6x + 9$
$x^3 - 16x^2 + 81x - 134 = 0$

Use a graphing calculator to find the zero.



[−2, 10] sc1:1 by [−10, 10] sc1:1

about 7.88

38a. $s = \sqrt{30fd}$
$s = \sqrt{30(0.6)(25)}$
$s = \sqrt{450}$
$s \approx 21.2$ mph

38b. $s = \sqrt{30fd}$
$35 = \sqrt{30(0.6)d}$
$1225 = 18d$
$68.06 \approx d$; about 68 ft

38c. No; it is not a linear function.

39a. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\pi\sqrt{\frac{1}{9.8}}$

$T \approx 2.01$ s

39b. $t = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\,\pi\sqrt{\frac{1}{8.9}}$

$T \approx 2.11$ s

39c. Let x = the new length of the pendulum.

$$2\left(2\pi\sqrt{\frac{\ell}{g}}\right) = 2\pi\sqrt{\frac{x}{g}}$$
$$4\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{x}{g}}$$
$$2\sqrt{\frac{\ell}{g}} = \sqrt{\frac{x}{g}}$$
$$4\frac{\ell}{g} = \frac{x}{g}$$
$$4\ell = x$$

It must be multiplied by 4.

40.
$$\frac{T_a}{T_b} = \sqrt{\left(\frac{r_a}{r_b}\right)^3}$$
$$\frac{225}{687} = \sqrt{\left(\frac{67{,}200{,}000}{r_b}\right)^3}$$
$$\frac{50{,}625}{471{,}969} = \frac{3.03 \times 10^{23}}{{r_b}^3}$$
$$50{,}625{r_b}^3 = 1.43 \times 10^{29}$$
$${r_b}^3 = 2.83 \times 10^{24}$$
$r = 141{,}433{,}433.8$; about 141,433,434 mi

41. $\sqrt{2x - 9} - a = b$

$\sqrt{2x - 9} = a + b$

$2x + 9 \geq 0$, so $a + b \geq 0$

no real solution when $a + b < 0$

42.
$$T = \frac{t + c}{2} + \sqrt{\left(\frac{t - c}{2}\right)^2 + p^2}$$
$$108 = \frac{t + (-200)}{2} + \sqrt{\left(\frac{t - (-200)}{2}\right)^2 + 50^2}$$
$$108 - \left(\frac{t - 200}{2}\right) = \sqrt{\left(\frac{t + 200}{2}\right)^2 + 2500}$$
$$\frac{-t + 416}{2} = \sqrt{\frac{t^2 - 400t + 40{,}000}{4} + 2500}$$
$$\frac{t^2 - 832t + 173{,}056}{4} = \frac{t^2 + 400t + 50{,}000}{4}$$
$$t^2 - 832t + 173{,}056 = t^2 + 400t + 50{,}000$$
$$123{,}056 = 1232t$$
$99.88 = t$; about 99.88 psi

43. $\frac{a + 2}{2a + 1} = \frac{a}{3} + \frac{3}{4a + 2}$; exclude: $-\frac{1}{2}$

$$\left(\frac{a + 2}{2a + 1}\right)(6)(2a + 1) = \left(\frac{a}{3} + \frac{3}{2(2a + 1)}\right)(6)(2a + 1)$$
$$6(a + 2) = a(2)(2a + 1) + 3(3)$$
$$6a + 12 = 4a^2 + 2a + 9$$
$$0 = 4a^2 - 4a - 3$$
$$0 = (2a - 3)(2a + 1)$$

$2a - 3 = 0$ $\quad$ $2a + 1 = 0$

$a = \frac{3}{2}$ $\quad$ $a = -\frac{1}{2}$

$\frac{3}{2}$

44. $\frac{p}{q}$: $\pm6, \pm3, \pm2, \pm1$

r	1	5	5	−5	−6
1	1	6	11	6	0

$x^3 + 6x^2 + 11x + 6 = 0$

r	1	6	11	6
−1	1	5	6	0

$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x + 3 = 0$ $\quad$ $x + 2 = 0$

$x = -3$ $\quad$ $x = -2$

$-3, -2, -1, 1$

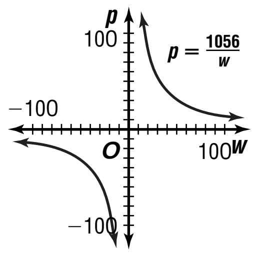
45a. point discontinuity

45b. jump discontinuity

45c. infinite discontinuity

46a. $p = \frac{v}{w}$

$p = \frac{1056}{w}$



46b. x- and y-axes

46c. It increases.

46d. It is halved.

47.
$$\begin{bmatrix} 4 & -1 & 6 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 2 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 4(0) + (-1)(2) + 6(5) & 4(3)(-1)(-2) + 6(1) \\ 4(0) + 0(2) + 2(5) & 4(3) + 0(-2) + 2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 28 & 20 \\ 10 & 14 \end{bmatrix}$$

48. $-4(a + b + c) = -4(6)$ $\quad$ $2a - 3b + 4c = 3$ $\rightarrow$

$$\begin{aligned} -4a - 4b - 4c &= -24 \\ 2a - 3b + 4c &= 3 \\ \hline -2a - 7b &= -21 \end{aligned}$$

$-4(a + b + c) = -4(6)$ $\quad$ $4a - 8b + 4c = 12$ $\rightarrow$

$$\begin{aligned} -4a - 4b - 4c &= -24 \\ 4a - 8b + 4c &= 12 \\ \hline -12b &= -12 \\ b &= 1 \end{aligned}$$

$-2a - 7b = -21$ $\quad$ $a + b + c = 6$

$-2a - 7(1) = -21$ $\quad$ $7 + 1 + c = 6$

$a = 7$ $\quad$ $c = -2$

$(7, 1, -2)$

49. $y = -3.54x + 7125.4$

$y = -3.54(2010) + 7125.4$

$y = 10$ students

50. $7y + 4x - 3 = 0$

$y = -\frac{4}{7}x + \frac{3}{7}$

perpendicular slope: $\frac{7}{4}$

$y - 5 = \frac{7}{4}(x - 2)$

$y = \frac{7}{4}x + \frac{3}{2}$

51. $A = \pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{1}{4}\pi$ $\quad$ $A = \pi r^2 = \pi(1)^2 = 1\pi$

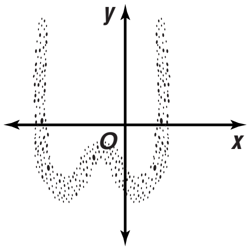
$\frac{1}{4}\pi + 1\pi = \frac{5}{4}\pi$

The correct choice is C.

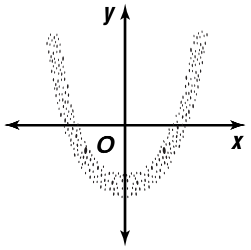
4-8 Modeling Real-World Data with Polynomial Functions

Pages 261-262 Check for Understanding

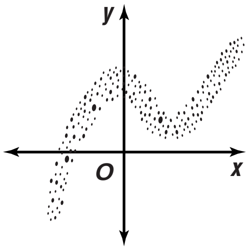
1a. Sample answer:



1b. Sample answer:



1c. Sample answer:



2. You need to recognize the general shape so that you can tell the graphing calculator which type of polynomial function to use as a model.

3. Sample answer: If companies use less packaging materials, consumers keep items longer, and old buildings are restored instead of demolished, the amount of waste will decrease more rapidly. If consumers buy more products, companies package items in larger containers, and many old buildings are destroyed, the amount of waste will increase instead of decrease.

4. quartic

5. Sample answer:
$f(x) = 1.98x^4 + 2.95x^3 - 5.91x^2 + 0.22x + 4.89$

6. Sample answer: $f(x) = 3.007x^2 + 0.001x - 7.896$

7a. Sample answer: $f(x) = 0.48x + 58.0$

7b. Sample answer: $2010 - 1950 = 60$
$f(x) = 0.48x + 58.0$
$= 0.48(60) + 58.0$
$= 86.8\%$

7c. Sample answer: $f(x) = 0.48x + 58.0$
$89 = 0.48x + 58.0$
$64 \approx x$
$1950 + 64 = 2014$

Pages 262-264 Exercises

8. cubic

9. quadratic

10. linear

11. quadratic

12. $f(x) = -1.25x + 5$

13. $f(x) = 8x^2 - 3x - 9$

14. Sample answer:
$f(x) = 1.03x^4 - 5.16x^3 + 6.08x^2 + 0.23x + 0.94$

15. Sample answer:
$f(x) = 0.09x^3 - 2.70x^2 + 24.63x - 65.21$

16. Sample answer:
$f(x) = 4.05x^4 - 0.09x^3 + 6.69x^2 - 222.03x + 2697.74$

17. Sample answer:
$f(x) = -0.02x^3 + 8.79x^2 + 3.35x + 27.43$

18a. Sample answer: $f(x) = 1.99x^2 - 1.74x + 2.76$

18b. Sample answer:
$f(x) = -0.96x^3 + 0.56x^2 + 0.36x + 4.05$

18c. Sample answer: Cubic; the value of r^2 for the cubic function is closer to 1.

19a. Sample answer: $f(x) = 0.126x + 22.732$

19b. Sample answer:
$2010 - 1900 = 110$
$f(x) = 0.126x + 22.732$
$f(110) = 0.126(110) + 22.732$
$f(110) = 36.592$
37

19c. Sample answer:
$2025 - 1900 = 125$
$f(x) = 0.126x + 22.732$
$f(125) = 0.126(125) + 22.732$
$f(125) = 38.482$
38

20. Sample answer:

x	1	2	3	4	5	6	7	8	9
$f(x)$	1	3	6	3	–13	–49	–112	–209	–347

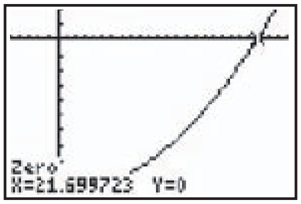
21a. Sample answer:
$f(x) = 0.008x^4 - 0.138x^3 + 0.621x^2 + 0.097x + 18.961$

21b. Sample answer: $1994 - 1992 = 2$
$f(x) = 0.008x^4 - 0.138x^3 + 0.621x^2 + 0.097x + 18.961$
$f(2) = 0.008(2)^4 - 0.138(2)^3 + 0.621(2)^2 + 0.097(2) + 18.961$
$f(2) = 20.663$
about 21%

22. A sixth-degree polynomial; there are 5 changes in direction.

23a. Sample answer:
$f(x) = 0.109x^2 - 0.001x + 48.696$

23b. Sample answer:
$f(x) = 0.109x^2 - 0.001x + 48.696$
$100 = 0.109x^2 - 0.001x + 48.696$
$0 = 0.109x^2 - 0.001x - 51.304$



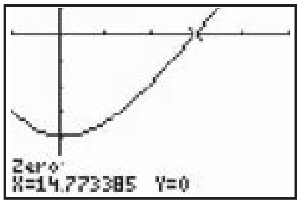
$[-5, 25]$ sc1:1 by $[-50, 10]$ sc1:5
root: (21.7, 0)
$1985 + 22 = 2007$

23c. Sample answer: $1998 - 1985 = 13$
$f(x) = 0.109x^2 - 0.001x + 48.696$
$f(13) = 0.109(13)^2 - 0.001(13) + 48.696$
$f(13) = 67.104$
No; according to the model, there should have been an attendance of only about 67 million. Since the actual attendance was much higher than the projected number, it is likely that the race to break the homerun record increased the attendance.

24a. Sample answer:
$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$

24b. Sample answer: $1996 - 1990 = 6$
$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$
$f(6) = -0.033(6)^3 + 1.471(6)^2 - 1.368(6) + 5.563$
$f(6) = 43.183$
about 43.18 million

24c. Sample answer:
$f(x) = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$
$200 = -0.033x^3 + 1.471x^2 - 1.368x + 5.563$
$0 = -0.033x^3 + 1.471x^2 - 1.368x - 194.437$



$[-5, 25]$ sc1:5 by $[-300, 50]$ sc1:50
root: (14.8, 0)
$14 + 1990 = 2004$
about 2004

25. $5 - \sqrt{b + 2} = 0$
$5 = \sqrt{b + 2}$
$25 = b + 2$
$23 = b$

Check: $5 - \sqrt{b + 2} = 0$
$5 - \sqrt{23 + 2} \stackrel{?}{=} 0$
$5 - \sqrt{25} \stackrel{?}{=} 0$
$5 - 5 \stackrel{?}{=} 0$
$0 = 0$ ✓

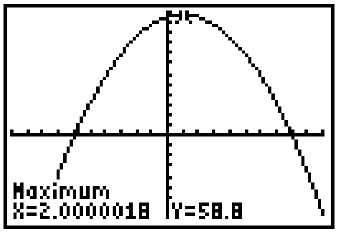
26.
$$\frac{6}{p+3} + \frac{p}{p-3} = 1$$
$$\left(\frac{6}{p+3} + \frac{p}{p-3}\right)(p+3)(p-3) = 1(p+3)(p-3)$$
$$6(p-3) + p(p+3) = (p+3)(p-3)$$
$$6p - 18 + p^2 + 3p = p^2 - 9$$
$$9p = 9$$
$$p = 1$$

27.

r	2	−1	0	1	−2
−1	2	−3	3	−2	0
0	2	−1	0	1	−2
1	2	1	1	2	0

−1, 1

28a. Let x = number of weeks.
$P = (120 + 10x)(0.48 - 0.03x)$
$P = 57.6 + 1.2x - 0.3x^2$



$[-20, 20]$ sc1:2 by $[-40, 60]$ sc1:5
maximum: (2, 58.8)
2 weeks

28b. \$58.80 per tree

29. $x - 0.10x = 0.90x$
$0.90x + 0.10(0.90x) = 0.90x + 0.09x$
$= 0.99x$
The correct choice is B.

4-8B Fitting a Polynomial Function to a Set of Points

Page 266

1. $y = 7x^3 - 4x^2 - 17x + 15$
2. $y = 7x^3 - 4x^2 - 17x + 15$; yes
3. Sample answer: $y = -5x^6 - 2x^5 + 40x^4 - 2x^3 + x^2 + 8x - 4$
4. Infinitely many; suppose that you are given a set of n points in a coordinate plane, no two of which are on the same vertical line. You can pick an infinite number of other points with different x-coordinates. You could find polynomial functions that went through the original n points and any number of the other points.
5. There is no problem with using L1^0 with list L_1 for the example. However, if you are using a different list which happens to have 0 as one of its elements, using L1^0 will result in an error message, since 0^0 is undefined.

Chapter 4 Study Guide and Assessment

Page 267 Understanding and Using the Vocabulary

1. Quadratic Formula
2. Integral Root Theorem
3. zero
4. Factor Theorem
5. polynomial function
6. lower bound

7. Extraneous **8.** complex roots
9. complex numbers **10.** quadratic equation

Pages 268-270 Skills and Concepts

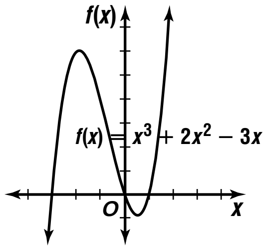
11. no; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(0) = (0)^3 - 3(0)^2 - 3(0) - 4$
$f(0) = -4$

12. yes; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(4) = (4)^3 - 3(4)^2 - 3(4) - 4$
$f(4) = 0$

13. no; $f(a) = a^3 - 3a^2 - 3a - 4$
$f(-2) = (-2)^3 - 3(-2)^2 - 3(-2) - 4$
$f(-2) = -18$

14. $f(t) = t^4 - 2t^2 - 3t + 1$
$f(-3) = (-3)^4 - 2(-3)^2 - 3(-3) + 1$
$f(-3) = 73$
no

15. 3; $x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x + 3)(x - 1) = 0$
$x = 0$ $\quad x + 3 = 0$ $\quad x - 1 = 0$
$x = -3$ $\quad x = 1$



16. $b^2 - 4ac = (-7)^2 - 4(2)(-4)$
$= 81$; 2 real
$x = \frac{7 \pm \sqrt{81}}{2(2)}$
$x = \frac{7 \pm 9}{4}$
$x = \frac{7 + 9}{4}$ $\quad x = \frac{7 - 9}{4}$
$x = 4$ $\quad x = -\frac{1}{2}$

17. $b^2 - 4ac = (-10)^2 - 4(3)(5)$
$= 40$; 2 real
$m = \frac{10 \pm \sqrt{40}}{2(3)}$
$m = \frac{10 \pm 2\sqrt{10}}{6}$
$m = \frac{5 \pm \sqrt{10}}{3}$

18. $b^2 - 4ac = (-1)^2 - 4(1)(6)$
$= -23$; 2 imaginary
$x = \frac{1 \pm \sqrt{-23}}{2(1)}$
$x = \frac{1 \pm i\sqrt{23}}{2}$

19. $b^2 - 4ac = 3^2 - 4(-2)(8)$
$= 73$; 2 real
$y = \frac{-3 \pm \sqrt{73}}{2(2)}$
$y = \frac{3 \pm \sqrt{73}}{4}$

20. $b^2 - 4ac = 4^2 - 4(1)(4)$
$= 0$; 1 real
$a = \frac{-4 \pm \sqrt{0}}{2(1)}$
$a = -\frac{4}{2}$
$a = -2$

21. $b^2 - 4ac = (-1)^2 - 4(5)(10)$
$= -199$; 2 imaginary
$r = \frac{1 \pm \sqrt{-199}}{2(5)}$
$r = \frac{1 \pm i\sqrt{199}}{10}$

22. $f(x) = x^3 - x^2 - 10x - 8$
$f(-2) = (-2)^3 - (-2)^2 - 10(-2) - 8$
$= -8 - 4 + 20 - 8$ or 0; yes

23. $f(x) = 2x^3 - 5x^2 + 7x + 1$
$f(5) = 2(5)^3 - 5(5)^2 + 7(5) + 1$
$= 250 - 125 + 35 + 1$ or 161; no

24. $f(x) = 4x^3 - 7x + 1$
$f\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right) + 1$
$= -\frac{4}{8} + \frac{7}{2} + 1$ or 4; no

25. $f(x) = x^4 - 10x^2 + 9$
$f(3) = (3)^4 - 10(3)^2 + 9$
$= 81 - 90 + 9$ or 0; yes

26. $\frac{p}{q}$: ±1, ±2

r	1	−2	−1	2
1	1	−1	−2	0

$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x - 2 = 0$ $\quad x + 1 = 0$
$x = 2$ $\quad x = -1$
rational roots: −1, 1, 2

27. $\frac{p}{q}$: ±1

r	1	0	−1	1	−1
−1	1	−1	0	1	−2
1	1	1	0	1	0

rational root: 1

28. p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}$: ±1, ±2, ±4; $\pm\frac{1}{2}$

r	2	−2	−2	−4
1	2	0	−2	−6
2	2	2	2	0

$2x^2 + 2x + 2 = 0$
$x^2 + x + 1 = 0$
does not factor
rational root: 2

29. p: ±1, ±3
q: ±1, ±2
$\frac{p}{q}$: ±1, ±3, $\pm\frac{1}{2}$, $\pm\frac{3}{2}$

r	2	3	−6	−11	−3
1	2	5	−1	−12	−15
−1	2	1	−7	−4	1
3	2	9	21	52	153
−3	2	−3	3	−20	57
$\frac{1}{2}$	2	4	−4	−13	$-\frac{19}{2}$
$-\frac{1}{2}$	2	2	−7	$-\frac{15}{2}$	$\frac{3}{4}$
$\frac{3}{2}$	2	6	−3	$-\frac{31}{2}$	$-\frac{105}{4}$
$-\frac{3}{2}$	2	0	−6	−2	0

rational root: $-\frac{3}{2}$

30. $\frac{p}{q}$: ±1, ±2, ±4

r	1	0	−7	1	12	−4
1	1	1	−6	−5	7	3
2	1	2	−3	−5	2	0

$x^4 + 2x^3 - 3x^2 - 5x + 2 = 0$

r	1	2	−3	−5	2
−2	1	0	−3	1	0

$x^3 - 3x + 1 = 0$

r	1	0	−3	1
−2	1	−2	1	−1
−1	1	−1	−2	3
4	1	4	13	53
−4	1	−4	13	−51

rational roots: −2, 2

31. p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}$: ±1, ±2, ±4, ±8, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, $\pm\frac{4}{3}$, $\pm\frac{8}{3}$

r	3	7	−2	−8
1	3	10	8	0

$3x^2 + 10x + 8 = 0$
$(3x + 4)(x + 2) = 0$
$3x + 4 = 0$ $\quad$ $x + 2 = 0$
$x = -\frac{4}{3}$ $\quad$ $x = -2$

rational roots: $-2, -\frac{4}{3}, 1$

32. p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}$: ±1, ±2, $\pm\frac{1}{4}$, $\pm\frac{1}{2}$

r	4	1	8	2
$-\frac{1}{4}$	4	0	8	0

$4x^2 + 8 = 0$
$x^2 + 2 = 0$
does not factor
rational root: $-\frac{1}{4}$

33. $\frac{p}{q}$: ±1, ±5

r	1	0	4	0	−5
1	1	1	5	5	0

$x^3 + x^2 + 5x + 5 = 0$

r	1	1	5	5
−1	1	0	5	0

$x^2 + 5 = 0$
does not factor
rational roots: −1, 1

34. 1 positive
$f(-x) = -x^3 - x^2 + 34x - 56$
2 or 0 negative

r	1	−1	−34	−56
7	1	6	8	0

$x^2 + 6x + 8 = 0$
$(x + 4)(x + 2) = 0$
$x + 4 = 0$ $\quad$ $x + 2 = 0$
$x = -4$ $\quad$ $x = -2$
rational zeros: −4, −2, 7

35. 2 or 0 positive
$f(-x) = -2x^3 - 11x^2 - 12x + 9$
1 negative

r	2	−11	12	9
$-\frac{1}{2}$	2	−12	18	0

$2x^2 - 12x + 18 = 0$
$x^2 - 6x + 9 = 0$
$(x - 3)(x - 3) = 0$
$x - 3 = 0$ $\quad$ $x - 3 = 0$
$x = 3$ $\quad$ $x = 3$
rational zeros: $-\frac{1}{2}$, 3

36. 2 or 0 positive
$f(-x) = x^4 - 13x^2 + 36$
2 or 0 negative

r	1	0	−13	0	36
2	1	2	−9	−18	0

$x^3 + 2x^2 - 9x - 18 = 0$

r	1	2	−9	−18
3	1	5	6	0

$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x + 3 = 0$ $\quad$ $x + 2 = 0$
$x = -3$ $\quad$ $x = -2$
rational zeros: −3, −2, 2, 3

37.

r	3	0	0	1
−2	3	−6	12	−23
−1	3	−3	3	−2
0	3	0	0	1

−1 and 0

38.

r	1	−4	2
0	1	−4	2
1	1	−3	−1
2	1	−2	−2
3	1	−1	−1
4	1	0	2

0 and 1, 3 and 4

39.

r	1	−3	−3
−1	1	−4	1
0	1	−3	−3
1	1	−2	−5
2	1	−1	−5
3	1	0	−3
4	1	1	1

−1 and 0, 3 and 4

40.

r	1	−1	0	1
−2	1	−3	6	−11
−1	1	−2	2	−1
0	1	−1	0	1
1	1	0	0	1

−1 and 0

41.

r	4	2	−11	3
−2	4	−7	3	−3
−1	4	−3	−8	11
0	4	1	−11	3
1	4	5	−6	−3
2	4	9	7	17

−2 and −1, 0 and 1, 1 and 2

42.

r	−9	25	−24	6
−1	−9	34	−58	64
0	−9	25	−24	6
1	−9	16	−8	−2
2	−9	7	−10	−14

0 and 1

43. Use the TABLE feature of a graphing calculator.
−4.9, −1.8, 2.2

44.
$$n - \frac{6}{n} + 5 = 0$$
$$\left(n - \frac{6}{n} + 5\right)(n) = 0(n)$$
$$n^2 + 5n - 6 = 0$$
$$(n + 6)(n - 1) = 0$$
$n + 6 = 0$ $\quad$ $n - 1 = 0$
$n = -6$ $\quad$ $n = 1$

45.
$$\frac{1}{x} = \frac{x + 3}{2x^2}$$
$$\left(\frac{1}{x}\right)(2x^2) = \left(\frac{x + 3}{2x^2}\right)(2x^2)$$
$$2x = x + 3$$
$$x = 3$$

46.
$$\frac{5}{6} = \frac{2m}{2m + 2} - \frac{1}{3m - 3}$$
$$\left(\frac{5}{6}\right)6(m + 1)(m - 1) = \left(\frac{2m}{2(m + 1)} - \frac{1}{3(m - 1)}\right)6(m + 1)(m - 1)$$
$$5(m + 1)(m - 1) = (2m)(3)(m - 1) - 2(m + 1)$$
$$5m^2 - 5 = 6m^2 - 6m - 2m - 2$$
$$0 = m^2 - 8m + 3$$
$$m = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(3)}}{2(1)}$$
$$m = \frac{8 \pm \sqrt{52}}{2}$$
$$m = 4 \pm \sqrt{13}$$

47.
$\frac{3}{y} - 2 < \frac{5}{y}$; exclude: 0
$$\left(\frac{3}{y} - 2\right)y = \left(\frac{5}{y}\right)y$$
$$3 - 2y = 5$$
$$y = -1$$
Test $y = -2$: $\frac{3}{-2} - 2 < \frac{5}{-2}$
$-\frac{7}{2} < -\frac{5}{2}$ true
Test $y = -0.5$: $\frac{3}{-0.5} - 2 < \frac{5}{-0.5}$
$-8 < -10$ false
Test $y = 1$: $\frac{3}{1} - 2 < \frac{5}{1}$
$1 < 5$ true
Solution: $y < -1, y > 0$

48.
$\frac{2}{x + 1} < 1 - \frac{1}{x - 1}$; exclude −1, 1
$$\left(\frac{2}{x + 1}\right)(x + 1)(x - 1) = \left(1 - \frac{1}{x - 1}\right)(x + 1)(x - 1)$$
$$2(x - 1) = (x + 1)(x - 1) - (x + 1)$$
$$2x - 2 = x^2 - x - 2$$
$$0 = x^2 - 3x$$
$$0 = x(x - 3)$$
$x = 0$ $\quad$ $x - 3 = 0$
$x = 3$
Test $x = -2$: $\frac{2}{-2 + 1} < 1 - \frac{1}{-2 - 1}$
$-2 < \frac{4}{3}$ true
Test $x = -0.5$: $\frac{2}{-0.5 + 1} < 1 - \frac{1}{-0.5 + 1}$
$4 < -1$ false
Test $x = 0.5$: $\frac{2}{0.5 + 1} < 1 - \frac{1}{0.5 - 1}$
$\frac{4}{3} < 3$ true
Test $x = 2$: $\frac{2}{2 + 1} < 1 - \frac{1}{2 - 1}$
$\frac{2}{3} < 0$ false
Test $x = 4$: $\frac{2}{4 + 1} < 1 - \frac{1}{4 - 1}$
$\frac{2}{5} < \frac{2}{3}$ true
Solution: $x < -1, 0 < x < 1, x > 3$

49.
$5 - \sqrt{x + 2} = 0$
$5 = \sqrt{x + 2}$
$25 = x + 2$
$23 = x$

Check: $5 - \sqrt{x + 2} = 0$
$5 - \sqrt{23 + 2} \stackrel{?}{=} 0$
$5 - \sqrt{25} \stackrel{?}{=} 0$
$5 - 5 \stackrel{?}{=} 0$
$0 = 0$ ✓

50. $\sqrt[3]{4a-1}+8=5$
$\sqrt[3]{4a-1}=-3$
$4a-1=-27$
$4a=-26$
$a=-6.5$

Check: $\sqrt[3]{4a-1}+8=5$
$\sqrt[3]{4(-6.5)-1}+8 \stackrel{?}{=} 5$
$\sqrt[3]{-27}+8 \stackrel{?}{=} 5$
$-3+8 \stackrel{?}{=} 5$
$5=5$ ✓

51. $3+\sqrt{x+8}=\sqrt{x+35}$
$9+6\sqrt{x+8}+x+8=x+35$
$6\sqrt{x+8}=18$
$\sqrt{x+8}=3$
$x+8=9$
$x=1$

Check: $3+\sqrt{x+8}=\sqrt{x+35}$
$3+\sqrt{1+8} \stackrel{?}{=} \sqrt{1+35}$
$3+\sqrt{9} \stackrel{?}{=} \sqrt{36}$
$3+3=6$
$6=6$ ✓

52. $\sqrt{x-5}<7$ $\quad x-5>0$
$x-5<49$ $\quad x>5$
$x<54$
Test $x=0$: $\sqrt{0-5}<7$
$\sqrt{-5}<7$ meaningless
Test $x=10$: $\sqrt{10-5}<7$
$\sqrt{5}<7$ true
Test $x=60$: $\sqrt{60-5}<7$
$\sqrt{55}<7$ false
Solution: $5<x<54$

53. $4+\sqrt{2a+7}\geq 6$ $\quad 2a+7>0$
$\sqrt{2a+7}\geq 2$ $\quad 2a>-7$
$2a+7\geq 4$ $\quad a>-3.5$
$2a\geq -3$
$a\geq -1.5$
Test $a=-5$: $4+\sqrt{2(-5)+7}\geq 6$
$4+\sqrt{-3}\geq 6$ meaningless
Test $a=-2$: $4+\sqrt{2(-2)+7}\geq 6$
$4+\sqrt{3}\geq 6$ false
Test $a=0$: $4+\sqrt{2(0)+7}\geq 6$
$4+\sqrt{7}\geq 6$ true
Solution: $a\geq -1.5$

54. cubic

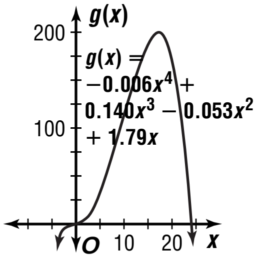
55. $f(x)=2x^2-x+3$

Page 271 Applications and Problem Solving

56. Let x = width of window.
Let $x+6$ = height of window.
$A=\ell w$
$315=x(x+6)$
$315=x^2+6x$
$0=x^2+6x-315$
$0=(x+21)(x-15)$
$x+21=0$ $\quad x-15=0$
$x=-21$ $\quad x=15$
Since distance cannot be negative, $x=15$ and $x+6=21$. the window should be 15 in. by 21 in.

57. Let x = width.
Let $x+6$ = length.
$(x+12)(x+6)-(x+6)(x)=288$
$x^2+18x+72-x^2-6x=288$
$12x+72=288$
$x=18$
$x+6=24$
18 ft by 24 ft

58a.



58b. $g(x)=-0.006x^4+0.140x^3-0.053x^2+1.79x$
$=x(-0.006x^3+0.140x^2-0.053x+1.79)$
$=x(x^3-23.\overline{3}x^2+8.8\overline{3}x-298.\overline{3})$

r	1	−23.333	8.833	−298.333
1	1	−22.333	−13.503	−311.836
5	1	−18.333	−82.835	−712.508
23.5	1	0.167	12.758	≈ 0

rational zeros: 0, about 23.5

59. $T=2\pi\sqrt{\frac{\ell}{g}}$
$1.6=2\pi\sqrt{\frac{\ell}{9.8}}$
$0.25=\sqrt{\frac{\ell}{9.8}}$
$0.06=\frac{\ell}{9.8}$
$0.64\approx\ell$; about 0.64 m

Page 271 Open-Ended Assessment

1. Sample answer: $\frac{x}{x+3}=\frac{2}{2x+1}$
$\left(\frac{x}{x+3}\right)(x+3)(2x+1)=\left(\frac{2}{2x+1}\right)(x+3)(2x+1)$
$x(2x+1)=2(x+3)$
$2x^2+x=2x+6$
$2x^2-x-6=0$
$(2x+3)(x-2)=0$
$2x+3=0$ $\quad x-2=0$
$x=-\frac{3}{2}$ $\quad x=2$

2a. Sample answer: $x-4=\sqrt{x-2}$

2b. Sample answer: $x-4=\sqrt{x-2}$
$(x-4)^2=x-2$
$x^2-8x+16=x-2$
$x^2-9x-18=0$
$(x-6)(x-3)=0$
$x-6=0$ $\quad x-3=0$
$x=6$ $\quad x=3$
Check: $x-4=\sqrt{x-2}$ $\quad x-4=\sqrt{x-2}$
$6-4\stackrel{?}{=}\sqrt{6-2}$ $\quad 3-4\stackrel{?}{=}\sqrt{3-2}$
$2\stackrel{?}{=}\sqrt{4}$ $\quad -1\stackrel{?}{=}\sqrt{1}$
$2=2$ ✓ $\quad -1\neq 1$
The solution is 6. Since $-1\neq 1$, 3 is an extraneous root.

3a. Sample answer:

x	−3	−2	−1	−0.5
$f(x)$	−12	0	2	1.125
x	0	0.5	1	2
$f(x)$	0	−0.625	0	8

3b. Sample answer: $f(x)=x^3+x^2-2x$

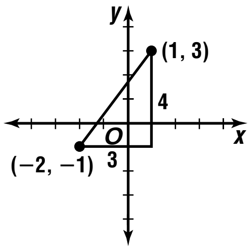
3c. Sample answer: −2, 0, 1

Chapter 4 SAT & ACT Preparation

Page 273 SAT and ACT Practice

1. There are two ways to solve this problem. You can use the distance formula or you can sketch a graph.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - (-2)^2 + (3 - (-1))^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \text{ or } 5 \end{aligned}$$



When you sketch the points and draw a right triangle as shown above, you can see that this is a 3-4-5 right triangle. Using the Pythagorean Theorem, you can calculate that the length of the hypotenuse is 5.

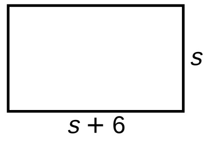
$5^2 = 4^2 + 3^2$ The correct choice is C.

2. Points on the graph of $f(x)$ are of the form $(x, f(x))$. To move the entire graph of $f(x)$ up 2 units, 2 must be added to each of the second coordinates. Points on the translated graph are of the form $(x, f(x) + 2)$. The function which represents the translation of the graph up 2 units is $f(x) + 2$. The correct choice is E.

3. You need to find both the x- and y-coordinates of point C. Use the properties of a parallelogram. First find the y-coordinate. Since opposite sides of a parallelogram are parallel and side AD is on the x-axis, point C must have the same y-coordinate as point B. So the y-coordinate is b. This means you can eliminate answer choices A and B

   Now find the x-coordinate. Since opposite sides of a parallelogram have equal length and side AD has length d, side BC must also have length d. Point B is a units from the y-axis, so point C must be $a + d$ units from the y-axis. The x-coordinate of point C is $a + d$. So point C has coordinates $(a + d, b)$. The correct choice is E.

4. You may want to draw a diagram.



   Use the formula for the perimeter of a rectangle, where ℓ represents the length and w represents the width.

   $2\ell + 2w = P$

   Replace w with s. Replace ℓ with $s + 6$.

   $2(s + 6) + 2s = 60$

   The correct choice is E.

5. First find the slope of the given line. Write the equation in the form $y = mx + b$.

$$\begin{aligned} 3x - 6y &= 12 \\ 6y &= 3x - 12 \\ y &= \tfrac{1}{2}x - 2 \end{aligned}$$

   The slope is $\frac{1}{2}$.

   So the slope of the line perpendicular to this line is the negative reciprocal of this slope. The slope of the perpendicular line is -2. The correct choice is A.

6. Be sure to notice the small piece of given information: x is an integer. You need to find the number, written in scientific notation, that could be x^3. This means that the cube root of the number is an integer.

   Take the cube root of each of the answer choices and see which one is an integer. You can use your calculator or do the calculations by hand. Notice that 2.7 is one-tenth of 27, which is 3^3.

   $2.7 \times 10^{13} = 27 \times 10^{12}$

   $\sqrt[3]{27 \times 10^{12}} = 3 \times 10^4$ or 30,000. 30,000 is an integer.

   When you try the same calculation with each of the other answer choices, the resulting power of 10 has a fractional exponent. So the number cannot be an integer. The correct choice is C.

7. This is a system of equations, but you do not need to solve for x or y. You need to find the value of $6x + 6y$.

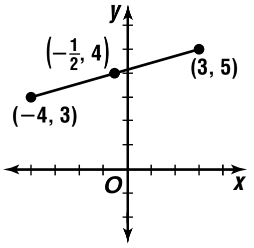
   Notice that the first equation contains $5y$ and the second contains $-1y$. If you subtract the second from the first, you have $6y$. Similarly, subtraction of the x values gives a result of $6x$. Use the same strategy that you would for solving a system. Subtract the second equation from the first.

$$\begin{aligned} 10x + 5y &= 14 \\ -4x + y &= -2 \\ \hline 6x + 6y &= 12 \end{aligned}$$

   The correct choice is C.

8. You can solve this problem using the midpoint formula or by sketching a graph.

The midpoint formula:
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+(-4)}{2}, \frac{5+3}{2}\right) = \left(\frac{-1}{2}, \frac{8}{2}\right) = \left(-\frac{1}{2}, 4\right)$



The correct choice is B.

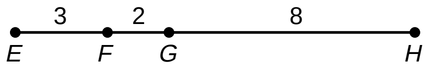
9. The expression $x^2 - 2ax + a^2$ is a perfect square trinomial and can be factored as $(x - a)^2$. The square of a real quantity is never negative.

The correct choice is A.

$(x - a^2 = x^2 - 2ax + a^2$
$= (x^2 + a^2) - 2ax$

So the quantity in Column A equals the quantity in Column B plus the sum of the squares of x and a. Since neither x nor a equal 0, their squares must be greater than 0. So the quantity in Column A is always greater than the quantity in Column B. The correct choice is A.

10. Since the problem does not include a figure, draw one. Label the four points.



One method of solving this problem is to "plug-in" numbers for the segment lengths. Since $EG = \frac{5}{3}EF$, let $EF = 3$. Then $EG = 5$. This means that FG must equal 2, since $EF + FG = EG$.

$HF = 5FG = 5(2) = 10$

$HG = HF - FG = 10 - 2 = 8$

$\frac{EF}{HG} = \frac{3}{8}$

The answer is .375 or 3/8.