CHAPTER 3: GRAPHS OF QUADRATIC RELATIONS

Specific Expectations Addressed in the Chapter

- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with and without the use of technology. **[3.5, Chapter Task]**
- Determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ ($a \neq 0$) can be graphically represented as a parabola, and that the table of values yields a constant second difference. **[3.1]**
- Identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them. **[3.2, 3.3, Chapter Task]**
- Compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2^x$; by applying the exponent rules for multiplication and division). **[3.6]**
- Expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x – y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, [computer algebra systems], paper and pencil) and strategies (e.g., patterning). **[3.4]**
- Determine, through investigation, and describe the connection between the factors of a quadratic expression and the *x*-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form $y = a(x – r)(x – s)$. **[3.3]**
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 – 2x – 8$ using intercepts and symmetry; [sketching $y = x^2 – 12x + 1$ by completing the square and applying transformations;] graphing $h = –4.9t^2 – 50t + 1.5$ using technology. **[3.2, 3.3, Chapter Task]**
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) [or from its defining equation (i.e., by applying algebraic techniques)]. **[3.2, Chapter Task]**
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?). **[3.2, 3.3, 3.5, Chapter Task]**

Prerequisite Skills Needed for the Chapter

- Make a table of values, construct a graph, and write an equation for a linear relation arising from a realistic situation.
- Identify a relation described by a polynomial of degree 1 as linear.
- Substitute into, and evaluate, expressions of degree 1.
- Determine the *x*- and *y*-intercepts of a linear relation.
- Expand and simplify polynomial expressions involving the product of a polynomial and a monomial.
- Create a scatter plot and draw a line or curve of good fit for linear or nonlinear data.
- Interpret the meaning of points on a scatter plot.
- Apply exponent rules in expressions involving one or two variables with positive exponents.

What "big ideas" should students develop in this chapter?

Students who have successfully completed the work of this chapter and who understand the essential concepts and procedures will know the following:

- The second differences for a quadratic relation are constant but not zero, the degree of the relation is 2, and the graph is a U shape, which is called a parabola. The vertex is the highest or lowest point on the curve, and the axis of symmetry passes through the vertex.
- The standard form of a quadratic relation is $y = ax^2 + bx + c$, with y-intercept c.
- The factored form of a quadratic relation is $y = a(x - r)(x - s)$. The x-intercepts are r and s. The equation of the axis of symmetry is $x = \frac{r + s}{2}$. The y-intercept is $a \times r \times s$.
- The product of two binomials can be determined by using algebra tiles, using an area diagram, or multiplying symbolically.
- To determine whether data can be modelled with a quadratic relation, a difference table can show whether the second differences are approximately constant and non-zero.
- To determine an equation of good fit, a scatter plot of the data can be used to estimate the zeros and the value of a. Alternatively, equations can be entered into a graphing calculator to determine a fit. The equation of best fit can be determined using quadratic regression.
- $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$, for $a \neq 0$.

Chapter 3: Planning Chart

Lesson Title	Lesson Goal	Pacing 11 days	Materials/Masters Needed
Getting Started, pp. 130–133	Use concepts and skills developed prior to this chapter.	2 days	grid paper; ruler; Diagnostic Test
Lesson 3.1: Exploring Quadratic Relations, pp. 134–137	Determine the properties of quadratic relations.	1 day	graphing calculator
Lesson 3.2: Properties of Graphs of Quadratic Relations, pp. 138–148	Describe the key features of the graphs of quadratic relations, and use the graphs to solve problems.	1 day	grid paper; ruler; graphing calculator; Lesson 3.2 Extra Practice
Lesson 3.3: Factored Form of a Quadratic Relation, pp. 150–158	Relate the factors of a quadratic relation to the key features of its graph.	1 day	grid paper and ruler, or graphing calculator; Lesson 3.3 Extra Practice
Lesson 3.4: Expanding Quadratic Expressions, pp. 161–168	Determine the product of two binomials using a variety of strategies.	1 day	algebra tiles; Lesson 3.4 Extra Practice
Lesson 3.5: Quadratic Models Using Factored Form, pp. 169–178	Determine the equation of a quadratic model using the factored form of a quadratic relation.	1 day	graphing calculator; grid paper; ruler; Lesson 3.5 Extra Practice
Lesson 3.6: Exploring Quadratic and Exponential Graphs, pp. 179–182	Compare the graphs of $y = x^2$ and $y = 2^x$ to determine the meanings of zero and negative exponents.	1 day	graphing calculator
Mid-Chapter Review, pp. 159–160 **Chapter Review**, pp. 183–186 **Chapter Self-Test**, p. 187	**Curious Math**, p. 149 **Chapter Task**, p. 188	3 days	Mid-Chapter Review Extra Practice; Chapter Review Extra Practice; Chapter Test; Chapters 1–3 Cumulative Review Extra Practice

CHAPTER OPENER

Using the Chapter Opener

Introduce the chapter by discussing the photograph of three hot-air balloons on pages 128 and 129 of the Student Book. Invite students to share their knowledge of the motion of hot-air balloons by relating this motion to the three graphs on page 128. Students might incorrectly view the altitude vs. time graphs as pictures of the paths taken by the balloons. However, the graphs represent changes in altitude vs. time, not the paths of the balloons. Ensure that students realize that altitude describes only height, not horizontal distance.

Ask students to describe the altitude and time for the balloon represented by each graph. Then name pairs of points for each graph, and ask this question: How are the altitude and time changing? Ask questions such as the following to supplement students' descriptions of the graphs:

- Which graph shows a balloon rising only? How do you know?
- Which graph shows a balloon descending first? For how long does it descend?
- What is the maximum height of the balloon whose altitude is represented by the second graph? How long does the balloon take to reach this height?
- What question can you ask about one of these graphs?

GETTING STARTED

Student Book Pages 130–133

Preparation and Planning	
Pacing	
5–10 min	**Words You Need to Know**
40–45 min	**Skills and Concepts You Need**
45–55 min	**Applying What You Know**
Materials	
▪ grid paper	
▪ ruler	
Nelson Website	
http://www.nelson.com/math	

Using the Words You Need to Know

Students might read each term and select the correct diagram or example, look at the examples and search for the matching terms, eliminate choices by matching what they are sure they know, or use a combination of strategies. After students have completed the question, ask them to explain how the arrows in part i) show the distributive property. Also ask them to explain how they would calculate first differences.

Using the Skills and Concepts You Need

Work through each of the examples in the Student Book (or similar examples, if you would like students to see more examples) and answer any questions that students have. For the first example, ask the following questions: Why is the answer an estimate rather than an exact answer? (The curve approximates the data but does not necessarily represent it exactly.) Would other answers be reasonable? (Yes. You could draw different curves to approximate the data, so other estimates are possible.)

For the second example, ask students how the algebra tiles and the distributive property are the same and how they are different. (They are the same because each is a strategy for multiplying binomials. They are different because you cannot use the tiles to multiply binomials with degree greater than 1.) Ask students to look over the Practice questions to see if there are any questions they do not know how to solve. Refer to the Study Aid chart in the Student Book. Allow students to work on the Practice questions in class, and assign any unfinished questions for homework.

Using the Applying What You Know

Arrange students in groups of two or three. Have them read the entire activity before beginning their work. Mention that they may find graphing each data set helpful for answering the questions. After students have completed the questions, ask them to share the strategies they used to determine whether or not the data sets are linear.

Answers to Applying What You Know

A. The balloon is rising in Training Flight 2. The heights in the table of values are increasing. As the time increases from 0 s to 6 s, the heights increase from 20 m to 90 m.
The balloon is descending in Training Flight 1. The heights in the table of values are decreasing. As the times increase from 0 s to 6 s, the heights decrease from 270 m to 210 m.

B. There are three strategies that can be used to determine whether a data set is linear:
Strategy 1: Calculate the slopes of line segments between adjacent points. If the slopes are equal, the data set is linear. If the slopes are not equal, the data set is not linear.

Strategy 2: Calculate the first differences for the data set. If the first differences are the same, the data set is linear. If the first differences are not the same, the data set is not linear.
Strategy 3: Create a scatter plot for the data set. If the points are on a line, the data set is linear. If the points are not on a line, the data set is not linear.

C. Training Flight 1

Strategy 1:

(0, 270), (1, 260)
$$m = \frac{260 - 270}{1 - 0} = -10$$

(1, 260), (2, 250)
$$m = \frac{250 - 260}{2 - 1} = -10$$

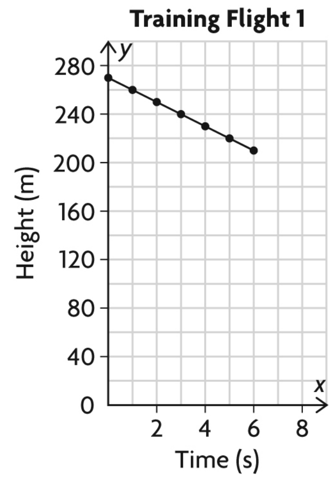
(2, 250), (3, 240)
$$m = \frac{240 - 250}{3 - 2} = -10$$

(3, 240), (4, 230)
$$m = \frac{230 - 240}{4 - 3} = -10$$

(4, 230), (5, 220)
$$m = \frac{220 - 230}{5 - 4} = -10$$

(5, 220), (6, 210)
$$m = \frac{210 - 220}{6 - 5} = -10$$

The slopes are equal. Therefore, the data set is linear.
Strategy 2: The first differences are all –10. Therefore, the data set is linear.
Strategy 3: All the points lie on a line. Therefore, the data set is linear.



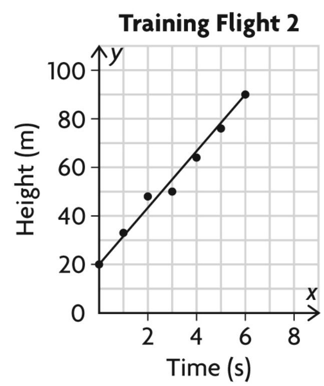
Training Flight 2

Strategy 1:

(0, 20), (1, 33)
$$m = \frac{33 - 20}{1 - 0} = 13$$

(1, 33), (2, 48)
$$m = \frac{48 - 33}{2 - 1} = 15$$

The slopes are not equal. Therefore, the data set is not linear.
Strategy 2: The first differences are 13, 15, 2, 14, 12, and 14. Since they are not the same, the data set is not linear.
Strategy 3: The points do not lie on a line. Therefore, the data set is not linear.



Initial Assessment	What You Will See Students Doing...
When students understand...	**If students misunderstand...**
Students reason whether the balloon is rising or descending by considering the height of the balloon at various times.	Students may not correctly interpret the data in the tables to determine whether heights are increasing or decreasing, or they may not explain an increase or decrease correctly.
Students describe three appropriate strategies to determine whether a data set is linear.	Students may not be able to think of three strategies they could use to determine whether a data set is linear, or they may not correctly describe how each strategy can be used to determine whether a data set is linear.
Students apply the strategies they described to determine correctly whether each data set is linear.	Students may not use the data sets to calculate the rise or the run correctly, or they may not divide the rise by the run to determine the slope. When determining first differences, students may subtract the wrong values or interpret the first differences incorrectly. Students may have difficulty setting up graphs, graphing points, or interpreting graphs.

3.1 EXPLORING QUADRATIC RELATIONS

Lesson at a Glance

GOAL

Determine the properties of quadratic relations.

Prerequisite Skills/Concepts

- Make a table of values, construct a graph, and write an equation for a linear relation.
- Identify a relation described by a polynomial of degree 1 as linear.

Specific Expectation

- Determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and that the table of values yields a constant second difference.

Mathematical Process Focus

- Reasoning and Proving
- Connecting

Student Book Pages 134–137

Preparation and Planning

Pacing
10–20 min **Introduction**
30–40 min **Teaching and Learning**
10 min **Consolidation**

Materials
- graphing calculator

Recommended Practice
Questions 1, 2, 3, 4, 6

New Vocabulary/Symbols
quadratic relation in standard form
second differences
parabola

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students have learned that linear relations have constant first differences and that a straight line represents a linear relation.
- In this lesson, students explore the properties of quadratic relations of the form $y = ax^2 + bx + c$ $(a \neq 0)$, including the first and second differences, the concept of symmetry, and the degree.
- Students use technology to explore how changing the values of a, b, and c affects the position and shape of a parabola.

1 Introducing the Lesson

(10 to 20 min)

Introduce the context of the problem on Student Book page 134 by asking students to explain what a "pop fly" is. Ask students to explain how the height of a pop fly changes over time and how the distance from where it is hit changes over time. Discuss the difference between the change in height and change in distance over time.

2 Teaching and Learning

(30 to 40 min)

Explore the Math

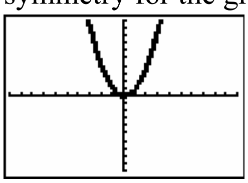
Have students work in pairs. Encourage them to discuss their observations with their partner as they work.

- Students might complete each part with a graphing calculator and then compare their screen with their partner's screen.
- Alternatively, one partner could read each part while the other uses the calculator, exchanging roles for different parts.

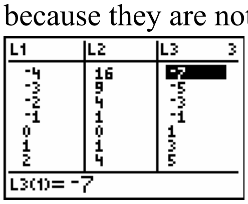
To help students use a graphing calculator, you might demonstrate how to calculate the first and second differences for $y = x^2$, as explained in Appendix B and as noted in the Tech Support in the margin. Circulate throughout the class to ensure that students are not having difficulty working through the activity. Make sure that they know how to calculate first and second differences with a graphing calculator.

Answers to Explore the Math

A. The graph is shaped like the letter U, and the *y*-axis is the line of symmetry for the graph.



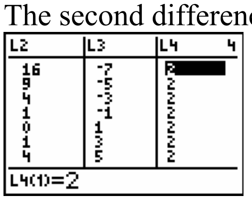
B.–C. The first differences confirm that the graph of the relation is nonlinear because they are not constant.



L1	L2	L3 3
-4	16	-7
-3	9	-5
-2	4	-3
-1	1	-1
0	0	1
1	1	3
2	4	5

L3(1)= -7

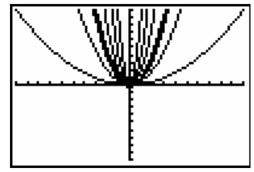
D. The second differences are constant, or the same.



L2	L3	L4 4
16	-7	2
9	-5	2
4	-3	2
1	-1	2
0	1	2
1	3	2
4	5	2

L4(1)=2

E. All the graphs are shaped like the letter U. The *y*-axis is the line of symmetry for all the graphs. Some graphs are wider than others. The first differences are not constant, so all the relations are nonlinear. The second differences are constant for all the relations, but they vary from one relation to another. All the second differences are positive. The value of the second differences for each relation is double the value of *a*.



$y = 2x^2$

L2	L3	L4
32	-14	4
18	-10	4
8	-6	4
2	-2	4
0	2	4
2	6	4
8	10	4

L4(1)=4

$y = 4x^2$

L2	L3	L4
64	-28	8
36	-20	8
16	-12	8
4	-4	8
0	4	8
4	12	8
16	20	8

L4(1)=8

$y = 10x^2$

L2	L3	L4
160	-70	20
90	-50	20
40	-30	20
10	-10	20
0	10	20
10	30	20
40	50	20

L4(1)=20

$y = 0.5x^2$

L2	L3	L4
8	-3.5	1
4.5	-2.5	1
2	-1.5	1
.5	-.5	1
0	.5	1
.5	1.5	1
2	2.5	1

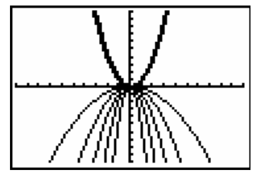
L4(1)=1

$y = 0.1x^2$

L2	L3	L4
1.6	-.7	.2
.9	-.5	.2
.4	-.3	.2
.1	-.1	.2
0	.1	.2
.1	.3	.2
.4	.5	.2

L4(1)=.2

F. All the graphs are shaped like an upside-down letter U. The *y*-axis is the line of symmetry for all the graphs. Some graphs are wider than others. The first differences are not constant, so all the relations are nonlinear. The second differences are constant for all the relations, but they vary from one relation to another. All the second differences are negative. The value of the second differences for each relation is double the value of *a*.



$y = -x^2$

L2	L3	L4
-16	7	-2
-9	5	-2
-4	3	-2
-1	1	-2
0	-1	-2
-1	-3	-2
-4	-5	-2

L4(1)= -2

$y = -2x^2$

L2	L3	L4
-32	14	-4
-18	10	-4
-8	6	-4
-2	2	-4
0	-2	-4
-2	-6	-4
-8	-10	-4

L4(1)= -4

$y = -5x^2$

L2	L3	L4
-80	35	-10
-45	25	-10
-20	15	-10
-5	5	-10
0	-5	-10
-5	-15	-10
-20	-25	-10

L4(1)= -10

$y = -0.5x^2$

L2	L3	L4
-8	3.5	-1
-4.5	2.5	-1
-2	1.5	-1
-.5	.5	-1
0	-.5	-1
-.5	-1.5	-1
-2	-2.5	-1

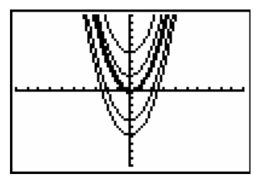
L4(1)= -1

$y = -0.2x^2$

L2	L3	L4
-3.2	1.4	-.4
-1.8	1	-.4
-.8	.6	-.4
-.2	.2	-.4
0	-.2	-.4
-.2	-.6	-.4
-.8	-1	-.4

L4(1)= -.4

G. The value of *c* does not change the shape of a parabola, but it does change the vertical position of the parabola. When *c* is positive, the parabola is shifted up *c* units. When *c* is negative, the parabola is shifted down *c* units. The value of *c* does not affect the line of symmetry, which is the *y*-axis, and it does not affect the direction of opening.



$y = x^2 + 2$

L2	L3	L4
18	-7	2
11	-5	2
6	-3	2
3	-1	2
2	1	2
3	3	2
6	5	2

L4(1)=2

$y = x^2 - 4$

L2	L3	L4
12	-7	2
5	-5	2
0	-3	2
-3	-1	2
-4	1	2
-3	3	2
0	5	2

L4(1)=2

$y = x^2 + 5$

L2	L3	L4
21	-7	2
14	-5	2
9	-3	2
6	-1	2
5	1	2
6	3	2
9	5	2

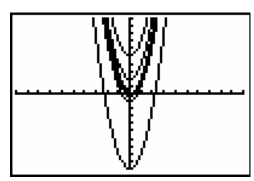
L4(1)=2

$y = x^2 - 6$

L2	L3	L4
10	-7	2
3	-5	2
-2	-3	2
-5	-1	2
-6	1	2
-5	3	2
-2	5	2

L4(1)=2

H. Answers may vary, e.g., I used 2 for the value of *a*. For each parabola, my conjecture is true.



$y = 2x^2$

L2	L3	L4
32	-14	4
18	-10	4
8	-6	4
2	-2	4
0	2	4
2	6	4
8	10	4

L4(1)=4

$y = 2x^2 + 2$

L2	L3	L4
34	-14	4
20	-10	4
10	-6	4
4	-2	4
2	2	4
4	6	4
10	10	4

L4(1)=4

$y = 2x^2 + 5$

L2	L3	L4
37	-14	4
23	-10	4
13	-6	4
7	-2	4
5	2	4
7	6	4
13	10	4

L4(1)=4

$y = 2x^2 - 1$

L2	L3	L4
31	-14	4
17	-10	4
7	-6	4
1	-2	4
-1	2	4
1	6	4
7	10	4

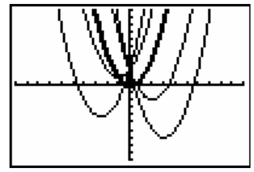
L4(1)=4

$y = 2x^2 - 10$

L2	L3	L4
22	-14	4
8	-10	4
-2	-6	4
-8	-2	4
-10	2	4
-8	6	4
-2	10	4

L4(1)=4

I. Changing the value of b shifts a graph both vertically and horizontally, without changing the shape of the graph or the direction of opening. The line of symmetry shifts by the same horizontal amount and in the same horizontal direction as the parabola.



$y = x^2 + 2x + 2$

L2	L3	L4
10	-5	2
5	-3	2
2	-1	2
1	1	2
2	3	2
5	5	2
10	7	2

L4(1)=2

$y = x^2 - 4x + 2$

L2	L3	L4
34	-11	2
23	-9	2
14	-7	2
7	-5	2
2	-3	2
-1	-1	2
-2	1	2

L4(1)=2

$y = x^2 + 5x + 2$

L2	L3	L4
-2	-2	2
-4	0	2
-4	2	2
-2	4	2
2	6	2
8	8	2
16	10	2

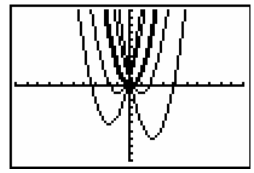
L4(1)=2

$y = x^2 - 6x + 2$

L2	L3	L4
42	-13	2
29	-11	2
18	-9	2
9	-7	2
2	-5	2
-3	-3	2
-6	-1	2

L4(1)=2

J. Answers may vary, e.g., I used 2 for the value of a and 1 for the value of c. For each parabola, my conjecture is true.



$y = 2x^2 + 1$

L2	L3	L4
33	-14	4
19	-10	4
9	-6	4
3	-2	4
1	2	4
3	6	4
9	10	4

L4(1)=4

$y = 2x^2 - 4x + 1$

L2	L3	L4
49	-18	4
31	-14	4
17	-10	4
7	-6	4
1	-2	4
-1	2	4
1	6	4

L4(1)=4

$y = 2x^2 - 8x + 1$

L2	L3	L4
65	-22	4
43	-18	4
25	-14	4
11	-10	4
1	-6	4
-5	-2	4
-7	2	4

L4(1)=4

$y = 2x^2 + 7x + 1$

L2	L3	L4
5	-7	4
-2	-3	4
-5	1	4
-4	5	4
1	9	4
10	13	4
23	17	4

L4(1)=4

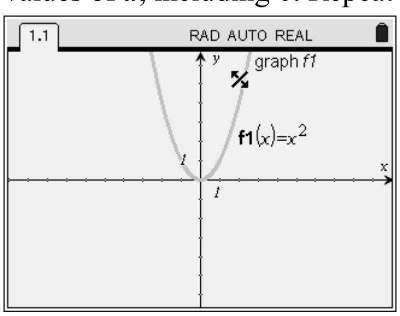
$y = 2x^2 + 4x + 1$

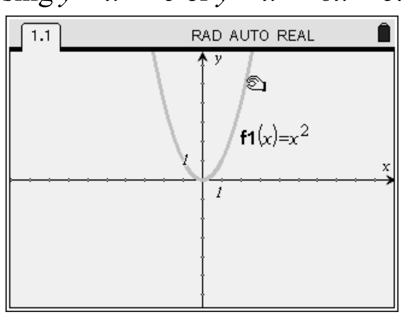
L2	L3	L4
17	-10	4
7	-6	4
1	-2	4
-1	2	4
1	6	4
7	10	4
17	14	4

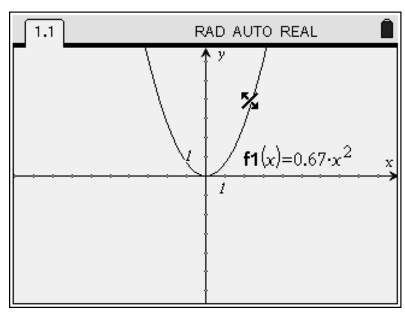
L4(1)=4

Technology-Based Alternative Lesson

If TI-*n*spire calculators are available for Explore the Math, have students enter $y = x^2$ into the Graphs & Geometry application, noting the reference to Appendix B in Tech Support in the Student Book. Then they can use Ctrl G to hide the entry line. (Ctrl G also makes the entry line reappear.) They can change the value of a by moving the cursor to the graph and holding down the Click button. After the closed hand appears, they can use the arrow keys to transform the graph. The equation will change as the graph is transformed. Students will be able to see what happens for a variety of values of a, including 0. Repeat using $y = x^2 + c$ or $y = x^2 + bx + c$.







Students can use the Lists & Spreadsheet application to examine the first and second differences.

1.1 1.2 RAD AUTO REAL

	A	B	C	D
◆		=a[]^2	=δlist(b[])	=δlist(c[])
1	0	0	1	2
2	1	1	3	2
3	2	4	5	2
4	3	9	7	2
5	4	16	9	

D1 =2

Answers to Reflecting

K. **i)** The degree of each equation is 2.

ii) The second differences are constant and non-zero for each relation.

L. **i)** If the value of a is positive, the graph is shaped like the letter U, opening upward. If the value of a is negative, the graph is like an upside-down letter U, opening downward. The non-signed value of a affects the shape of the graph.

ii) The value of the second differences for each quadratic relation is double the value of a. If a is positive, the second differences are positive. If a is negative, the second differences are negative.

M. **i)** Changing the value of b does not change the location of the y-intercept. Changing the value of c does change the location of the y-intercept. The y-intercept is the value of c.

ii) Changing the value of b changes the location of the line of symmetry of a parabola. Changing the value of c does not change the location of the line of symmetry.

3 Consolidation

(10 min)

Students should understand the following features of a quadratic relation:

- The second differences for a quadratic relation are constant but not zero.
- The value of a affects the direction that a parabola opens. The value of the second differences is twice the value of a, positive when a is positive, and negative when a is negative.
- The value of b changes the location of the line of symmetry.
- The constant c is the value of the y-intercept.

Students should be able to complete the Further Your Understanding questions independently.

3.2 PROPERTIES OF GRAPHS OF QUADRATIC RELATIONS

Lesson at a Glance

GOAL

Describe the key features of the graphs of quadratic relations, and use the graphs to solve problems.

Prerequisite Skills/Concepts

- Substitute into, and evaluate, expressions of degree 1.
- Determine the x- and y-intercepts of a linear relation.
- Identify the graph of a parabola.

Specific Expectations

- Identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., [sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = x^2 - 12x + 1$ by completing the square and applying transformations;] graphing $h = -4.9t^2 - 50t + 1.5$ using technology).
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) [or from its defining equation (i.e., by applying algebraic techniques)].
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Selecting Tools and Computational Strategies
- Representing

Student Book Pages 138–149

Preparation and Planning

Pacing

10 min	Introduction
20–25 min	Teaching and Learning
20–25 min	Consolidation

Materials
- grid paper
- ruler
- graphing calculator

Recommended Practice
Questions 4, 7, 9, 10, 11, 13, 14, 16

Key Assessment Question
Question 7

Extra Practice
Lesson 3.2 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students have previously examined the effects of changing the values of a, b, and c on the line of symmetry and the y-intercept.
- In this lesson, students determine key features of a parabola: the vertex, the equation of the axis of symmetry, the x- and y-intercepts, and the maximum or minimum value.
- Students solve problems arising from a realistic situation represented by a quadratic relation, using graphs.

1 Introducing the Lesson

(10 min)

Begin with a brief review of what students know about the general quadratic relation of the form $y = ax^2 + bx + c$. Have students work in small groups to list what they have learned about parabolas and about first and second differences for quadratic relations.

Then ask a student in one group to read one of the features they have listed. Invite comments from the class if there is any disagreement. Continue for other groups. If the following features are not mentioned, pose questions so that students remember them:

- The graph is a parabola.
- The parabola has a line of symmetry.
- The first differences are not constant.
- The second differences are constant but not zero.
- If $a > 0$, the parabola opens upward.
- If $a < 0$, the parabola opens downward.

2 Teaching and Learning

(20 to 25 min)

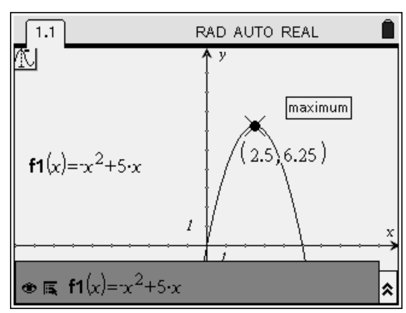
Learn About the Math

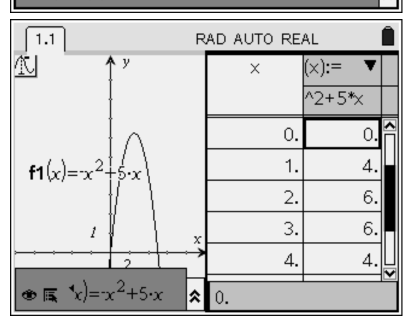
Example 1 presents a situation in which a golf ball is hit out of a sand trap. An equation representing the approximate path of the golf ball is given. Discuss how the graph represents height vs. distance. Lead students to understand how this graph is different from the graph in the introduction to Lesson 1, which represented height vs. time.

Technology-Based Alternative Lesson

For *Example 1*, if TI-*n*spire calculators are available, have students graph the relation $y = -x^2 + 5x$ and then use Trace as described in Appendix B-38 to determine the coordinates of the maximum or zeros. As students move the point along the curve, the coordinates and the appropriate word (maximum or zero) will appear on the screen when the corresponding point has been reached.

Alternatively, a table can be used to identify the maximum, the zeros, or a specific y-value. To add a table, from the menu, select **2: View → 9: Add Function Table**. Ensure that students realize that there must be a relation in the entry line to see the table.





Answers to Reflecting

A. Answers may vary, e.g.,
- The graph was more useful. You can see from the graph that the maximum height of the ball was 6.25 m, which is greater than 6 m and occurred between 2 m and 3 m from where the ball was hit. It is clear from the graph that the x-intercepts are at $x = 0$ and $x = 5$, so I know that the ball landed 5 m from where it was hit.
- The table of values was more useful because the two greatest values in the table are 6 at (2, 6) and (3, 6). Since the y-values increase by 6 from 0 m to 2 m and decrease by 6 from 3 m to 0 m, I know that the maximum height was just over 6 m. The table shows the y-values of 0 for $x = 0$ and $x = 5$, so I know that the ball landed 5 m from where it was hit.

B. The x-value at the ball's maximum height is half the x-value of the point where the ball touches the ground.

C. Yes, it is possible to predict whether a quadratic relation has a maximum value if you know the equation of the relation. If the coefficient of x^2 is less than zero, the graph of the relation opens downward so it has a maximum value but no minimum value. If the coefficient of x^2 is greater than zero, the graph opens upward so it has a minimum value but no maximum value.

3 Consolidation

(20 to 25 min)

Apply the Math

Using the Solved Examples

In *Example 2*, a table of values strategy is used to graph a quadratic relation and identify the equation of the axis of symmetry, coordinates of the vertex, and intercepts. Ask: How do you think Cassandra decided which x-values to use in her table of values? Discuss reasons for using x-values that are greater and less than the x-value of the vertex. For example, if the vertex was at $x = 0$, x-values of -2, -1, 0, 1, and 2 could be used. Then ask what values students would use if the vertex was to the right of $x = 0$ and what values they would use if the vertex was to the left of $x = 0$ (the x-coordinate of the vertex, plus some values greater and some values less than it). Emphasize the relationship between the axis of symmetry and the vertex.

Example 3 focuses on how to determine the minimum value of a quadratic relation. Students could graph the relation with a graphing calculator to verify the minimum, comparing their screens with Cassandra's and noting differences that result from various window settings.

In *Example 4*, a graphing calculator is used to solve a problem arising from a realistic situation represented by a quadratic model. Have students identify points on the parabola, relating each coordinate to the solution. Emphasize how the concluding sentences answer the questions asked.

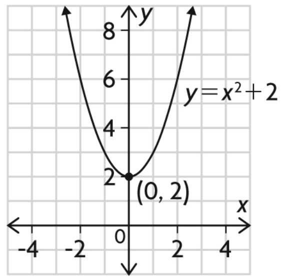
Answer to the Key Assessment Question

After students have completed question 7, ask how features of the quadratic relations that they determined are related. For example, the equation passes through the vertex, and the y-coordinate of the vertex is either the maximum or minimum value.

7. Answers for the table of values may vary, e.g.,

a)

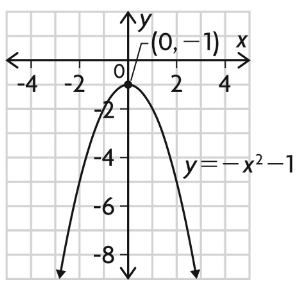
x	y
–2	6
–1	3
0	2
1	3
2	6



i) $x = 0$
ii) (0, 2)
iii) 2
iv) none
v) 2 (minimum)

b)

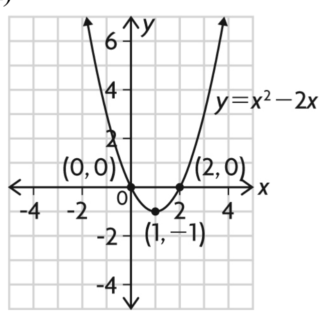
x	y
–2	–5
–1	–2
0	–1
1	–2
2	–5



i) $x = 0$
ii) (0, –1)
iii) –1
iv) none
v) –1 (maximum)

c)

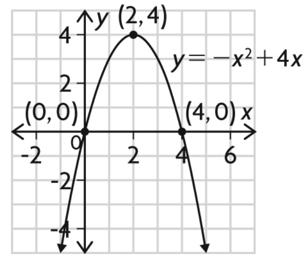
x	y
–2	8
–1	3
0	0
1	–1
2	0



i) $x = 1$
ii) (1, –1)
iii) 0
iv) 0, 2
v) –1 (minimum)

d)

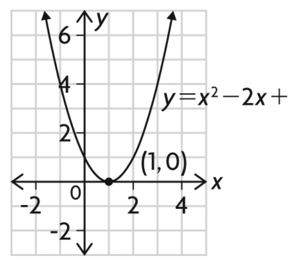
x	y
–1	–5
0	0
1	3
2	4
3	3
4	0



i) $x = 2$
ii) (2, 4)
iii) 0
iv) 0, 4
v) 4 (maximum)

e)

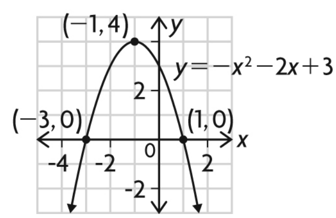
x	y
–1	4
0	1
1	0
2	1
3	4



i) $x = 1$
ii) (1, 0)
iii) 1
iv) 1
v) 0 (minimum)

f)

x	y
–3	0
–2	3
–1	4
0	3
1	0



i) $x = –1$
ii) (–1, 4)
iii) 3
iv) –3, 1
v) 4 (maximum)

Closing

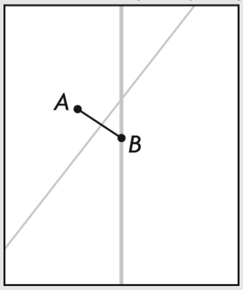
Have students read and answer question 16. Then have students compare their answers with a partner's answers. Discuss the two parts of the question as a class, asking students to read their answers.

Curious Math

This Curious Math feature introduces students to the parabola as one of the ways in which a plane intersects a cone. Students will create a parabola by folding paper. Providing cones may help students visualize the steps and understand the relationships.

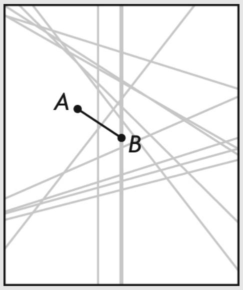
Answers to Curious Math

1.–6. Answers may vary, e.g.,



The crease in the paper intersects *AB* at its midpoint.

7.–9. Answers may vary, e.g.,



The parabola is formed around point *A*, which lies on the axis of symmetry. The parabola is located entirely on one side of the line passing through point *B*.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
Students correctly identify the key features of the graph of a quadratic relation, including the equation of the axis of symmetry, the coordinates of the vertex, and the intercepts.	Students may not remember how to determine the key features, possibly because they do not understand the connections among the features. For example, they may not understand that the axis of symmetry goes through the vertex.
Students solve problems arising from realistic situations that can be modelled by quadratic relations.	Students may have difficulty deciding which key features relate to the information needed, interpreting the *x*- and *y*-coordinates to chose which value gives the answer, or using the appropriate units in their concluding statements.
Key Assessment Question 7	
Students create accurate tables of values and graphs.	Students may have difficulty choosing *x*-values for a table of values, or they may make errors when calculating the *y*-values or when graphing.
Students use appropriate strategies to determine the equation of the axis of symmetry, coordinates of the vertex, *y*-intercept, zeros, and maximum or minimum value for each graph.	Students may have difficulty interpreting the visual representation, using the terms, or relating the terms to the graph.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students are not choosing appropriate *x*-values for tables of values, identify two situations for them. In the first situation, the equation of the quadratic equation should not have an *x* term. The table can include *x*-values from –4 to 4. In the second situation, the equation does have an *x* term. Students can calculate the *y*-coordinates for a few values of *x*, then use these *y*-coordinates to help them make decisions for the table of values.
2. Students can create a plan for determining the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, and the *x*-intercepts. They can use their plan as a reference when needed.

EXTRA CHALLENGE

1. Students can create false statements about key features of quadratic relations and then record why their statements are false. Then they can trade statements to see if others can determine why their statements are false, and compare reasoning.
2. Ask students to explain why relations given in problems in the lesson are reasonable for the situations.

3.3 FACTORED FORM OF A QUADRATIC RELATION

Lesson at a Glance

GOAL

Relate the factors of a quadratic relation to the key features of its graph.

Prerequisite Skills/Concepts

- Expand and simplify polynomial expressions involving the product of a polynomial and a monomial.
- Determine the equation of the axis of symmetry, the coordinates of the vertex, the maximum or minimum value, and the zeros from a graph.

Specific Expectations

- Identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.
- Determine, through investigation, and describe the connection between the factors of a quadratic expression and the *x*-intercepts (i.e., the zeros) of the graph of the corresponding quadratic relation, expressed in the form $y = a(x - r)(x - s)$.
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; [sketching $y = x^2 - 12x + 1$ by completing the square and applying transformations;] graphing $h = -4.9t^2 - 50t + 1.5$ using technology).
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Connecting
- Representing

Student Book Pages 150–158

Preparation and Planning

Pacing
10 min **Introduction**
30–40 min **Teaching and Learning**
10–20 min **Consolidation**

Materials
- grid paper and ruler, or graphing calculator

Recommended Practice
Questions 4, 5, 7, 10, 13, 15, 16

Key Assessment Question
Question 10

Extra Practice
Lesson 3.3 Extra Practice

New Vocabulary/Symbols
factored form of a quadratic relation

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students should be able to determine the equation of a line from information about the line.
- In this lesson, students learn that each factor of a quadratic relation expressed in the factored form, $y = a(x - r)(x - s)$, can be used to determine a zero or *x*-intercept.
- Students determine an equation for a parabola using the zeros and the coordinates of one other point on the parabola.

1 Introducing the Lesson

(10 min)

Remind students about the key features of a quadratic relation: the vertex, the axis of symmetry, the zeros or *x*-intercepts, and the *y*-intercept. Pose questions about the possible number of zeros:

- Is it possible for a parabola to have no zeros? (yes, if the vertex of a parabola that opens upward is above the *x*-axis, or if the vertex for a parabola that opens downward is below the *x*-axis)
- Is it possible for a parabola to have only one zero? (yes, if the vertex is on the *x*-axis)

Next, draw two parabolas, each with two zeros. Draw one parabola so that it opens upward with a vertex below the *x*-axis, and draw the other so that it opens downward with a vertex above the *x*-axis. Ask students to label the vertex (maximum or minimum point), the axis of symmetry, and the *x*- and *y*-intercepts.

2 Teaching and Learning

(30 to 40 min)

Investigate the Math

Introduce the problem about Boris's dog pen. Ask students to explain, in their own words, what they are given in the problem. (The three sides of the pen must total 80 m in length.) Ask: What do you need to determine? (the maximum area for a perimeter of 80 m)

Have students work in pairs to discuss their observations and share their work. Make sure that they understand how to determine an expression for the length of the pen. If students have difficulty deciding what values to use for *x* in the table of values, suggest that they think about values that are convenient for calculating the area. They should choose values that increase from 0 m until the area of the pen is 80 m^2.

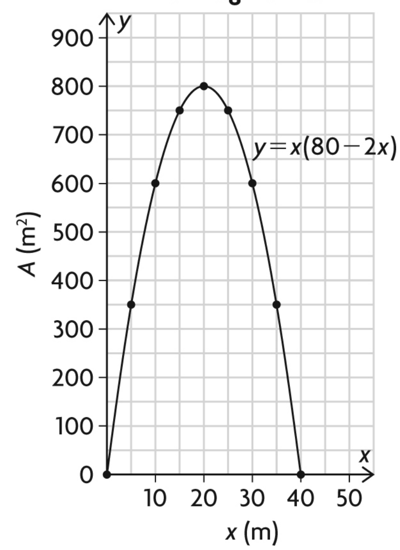
Answers to Investigate the Math

A. An expression for the length is $80 - 2x$.
B. A relation for the area is $A = x(80 - 2x)$. The factors are x and $80 - 2x$.

C.–D. Answers may vary, e.g.,

x	$A = x(80 - 2x)$	First Difference	Second Difference
0	0		
		350	
5	350		–100
		250	
10	600		–100
		150	
15	750		–100
		50	
20	800		–100
		–50	
25	750		–100
		–150	
30	600		–100
		–250	
35	350		–100
		–350	
40	0		

Area of Exercise Pen for Dog Kennel



D. The area relation is quadratic because the second differences are constant and non-zero.

E. The relation has a maximum because the parabola opens downward.

F. The zeros of the parabola are 0 and 40.

G. The equation of the axis of symmetry is $x = 20$.

H. The vertex of the parabola is (20, 800).

I. The maximum area occurs when the width, x, is 20. This means that the length, y, is $80 - 2x = 80 - 40 = 40$. The dimensions that maximize the area of the exercise pen are 20 m by 40 m.

Answers to Reflecting

J. You can determine the zeros of the graph by setting each factor equal to 0 and solving. For example, $x = 0$ is one zero. Set the factor $80 - 2x$ equal to 0 for the other zero, and solve to get $x = 40$.

K. You can use the area relation in factored form, $A = -2(x - 0)(x - 40)$, to determine the zeros, 0 and 40, and then mark these zeros on a graph. You can determine the x-coordinate of the vertex halfway between the zeros to get 20. When $x = 20$, the y-value is $-2(20 - 0)(20 - 40) = 800$, so the y-coordinate of the vertex is 800.

L. The area of the largest exercise pen that Boris can build is 800 m^2.

3 Consolidation

(10 to 20 min)

Apply the Math

Using the Solved Examples

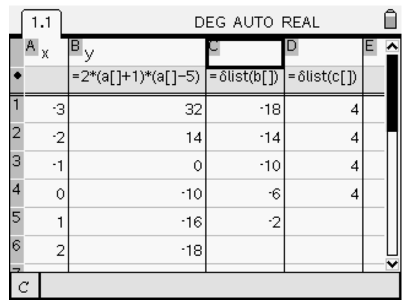
Students have been introduced to the standard form of a quadratic relation, and they have learned how to determine whether a relation is quadratic using first and second differences. *Example 1* gives a quadratic relation in factored form and asks for the direction of opening. Second differences are used to determine the direction of opening.

Example 2 gives a quadratic relation in factored form and asks for the key features of the relation. Make sure that students understand why the factors must be set to 0 to determine the zeros or *x*-intercepts, and why the mean is used to determine the equation of the axis of symmetry.

In *Example 4*, students determine the equation of a parabola, given its graph. Discuss how many different parabolas can have the same *x*-intercepts. Ask: How are the equations of these parabolas different? (The value of *a* is different.)

Technology-Based Alternative Lesson

If TI-*n*spire calculators are available, have students use the Lists & Spreadsheet application as described in Appendix B-43 to create an accurate table of values for *Example 1*. Students can enter *x*-values in column A, the relation in factored form in column B, a list of first differences in column C, and a list of second differences in column D. Label columns A and B by clicking in the cell containing A or B and typing a name, for example, *x* or *y*. Have students change the relation in column B to see what happens.



A x	B y	C	D	E
	=2*(a[]+1)*(a[]−5)	=δlist(b[])	=δlist(c[])	
-3	32	-18	4	
-2	14	-14	4	
-1	0	-10	4	
0	-10	-6	4	
1	-16	-2		
2	-18			

To determine the *y*-intercept, zeros, equation of the axis of symmetry, and vertex of the quadratic relation, add a Graphs & Geometry application. From the menu, select **3: Graph Type → 4: Scatterplot**. Choose the appropriate list for the independent and dependent variables. Then, from the menu, select **4: Window → 9:Zoom – Data** to see the points. It may be necessary to hide the entry line.

Answer to the Key Assessment Question

Question 10 is similar to *Example 4*, except the intercepts in question 10 are given as statements instead of on a graph. Students might use *Example 4* as a reference.

10. a) The equation is $y = 5(x + 3)(x – 5)$.
b) The coordinates of the vertex are (1, –80).

Closing

Have students work on question 16 on their own or in pairs to create a flow chart that summarizes the process used to determine an equation of a parabola from its graph. When finished, students can share their work with the class by posting their flow charts.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
Students correctly determine the *y*-intercept, zeros, equation of the axis of symmetry, and vertex of a quadratic relation expressed in factored form.	When determining the zeros, students may incorrectly use the numbers that appear in each factor rather than setting each factor equal to 0 and solving.
Students correctly graph a quadratic relation expressed in factored form.	Students may have difficulty graphing a quadratic relation in factored form accurately. If they determine incorrect zeros, they will not be able to determine the correct equation of the axis of symmetry and the correct vertex.
Students correctly determine an equation for a parabola using the zeros and one other point on the parabola.	Students may use only the zeros to determine an equation for a parabola. They may not use another point on the graph, and they may neglect to determine the value of *a*.
Key Assessment Question 10	
Students determine an equation for a quadratic relation, given the *x*-intercepts and *y*-intercept. They correctly substitute the zeros into the general equation in factored form, and they use the coordinates of the *y*-intercept to determine the value of *a*.	Students may use the zeros to determine the factors of the general quadratic relation in factored form, but not use the coordinates of the *y*-intercept to determine the value of *a*.
Students use the *x*-intercepts to determine the equation of the axis of symmetry and then use this equation to determine the vertex.	Students may just add together the *x*-intercepts to determine the equation of the axis of symmetry, rather than calculating their mean.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students need help determining the zeros, encourage them to write the equation of the quadratic relation in factored form first. Then they set each factor to 0 and solve. Including all of these steps every time will help to ensure an accurate result. Tell students to make notes about how to determine the zeros and the equation of the axis of symmetry and to refer to their notes.
2. If students have difficulty determining the key features of a quadratic relation, ask them to label the *x*- and *y*-intercepts and the vertex on the graph before determining their values. This will help students make the connection to the algebraic process involved.

EXTRA CHALLENGE

1. Have students use one of the equations in the lesson to create their own problem. Ask them to explain how they chose the equation they used and why this equation fits the problem.

MID-CHAPTER REVIEW

Big Ideas Covered So Far

- The second differences for a quadratic relation are constant but not zero, the degree of the relation is 2, and the graph is a U shape, which is called a parabola. The vertex is the highest or lowest point on the curve, and the axis of symmetry passes through the vertex.
- The standard form of a quadratic relation is $y = ax^2 + bx + c$, with y-intercept c.
- The factored form of a quadratic relation is $y = a(x - r)(x - s)$. The x-intercepts are r and s. The equation of the axis of symmetry is $x = \frac{r + s}{2}$. The y-intercept is $a \times r \times s$.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book page 159. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the Mid-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- Given a table of values representing a quadratic relation, how can you determine which direction the parabola opens?
- Does it matter which two points are used to determine the equation of the axis of symmetry? Explain your reasoning.
- What is the least amount of information that is needed to determine the equation of a parabola in factored form? How would you use this information?

3.4 EXPANDING QUADRATIC EXPRESSIONS

Lesson at a Glance

GOAL

Determine the product of two binomials using a variety of strategies.

Prerequisite Skill/Concept

- Expand and simplify polynomial expressions involving the product of a polynomial and a monomial.

Specific Expectation

- Expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x - y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, [computer algebra systems,] paper and pencil) and strategies (e.g., patterning).

Mathematical Process Focus

- Reasoning and Proving
- Connecting
- Representing

Student Book Pages 161–168

Preparation and Planning

Pacing

5 min	**Introduction**
30–40 min	**Teaching and Learning**
15–20 min	**Consolidation**

Materials
- algebra tiles

Recommended Practice
Questions 5, 6, 7, 8, 9, 11, 12, 13

Key Assessment Question
Question 13

Extra Practice
Lesson 3.4 Extra Practice

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students should be able to multiply a polynomial by a monomial involving the same variable, and to expand and simplify polynomial expressions involving one variable.
- In this lesson, students use the distributive property to expand and simplify quadratic expressions.
- This lesson demonstrates how using algebra tiles or an area diagram to represent the product of two binomials of degree 1 shows the relation between these factors and their product.
- Students make the connection between the factored form and standard form of a quadratic relation.

1 Introducing the Lesson

(5 min)

Sketch two parabolas, one that opens upward and another that opens downward. Draw your parabolas so that both of them cross the x-axis at two points. Write the general equation of a parabola, $y = ax^2 + bx + c$, below the parabolas.

Explain that a parabola in factored form has x-intercepts r and s and that the parabolas you drew have these x-intercepts. Ask students whether they think there is a strategy to take a parabola expressed in factored form and express it in standard form. Encourage discussion about students' ideas.

2 Teaching and Learning

(30 to 40 min)

Learn About the Math

A display with algebra tiles can be used to model the opening problem. Set up the tiles as in the first step of the problem, with $x + 4$ across the top and $x + 2$ along the left side.

Have students work in pairs, and ask them to set up their tiles the same way. After they have worked through the problem, ask one pair to demonstrate their solution on the display. Guide students to arrange the tiles in different ways, as shown in the Communication Tip. Ask: What other ways can the tiles be arranged? Help students understand why the tiles for $x + 4$ could be along the side, with the tiles for $x + 2$ across the top. Emphasize that the result is the same.

Answers to Reflecting

A. Devin used only red tiles in her rectangle model because the numbers and variables in the factors are positive. Blue tiles are used for negative values.

B. Each section in the area diagram corresponds to tiles in Devin's algebra tile model. The top left section of the area diagram represents the x^2 tile in Devin's model. The $4x$ section of the area diagram represents the four x's to the right of the x^2 tile. The $2x$ section of the area diagram corresponds to the two x's below the x^2 tile. The 8 in the area diagram corresponds to the eight single tiles in Devin's model.

C. Yes, the value of a is always the same in factored form and standard form if both relations represent the same parabola. There is only one value of a that will give a parabola specific zeros and y-intercepts. The value of a gives information about the direction of opening and width of the parabola. This information is the same for any form of a relation.

3 Consolidation

(15 to 20 min)

Apply the Math

Using the Solved Examples

Example 2 makes the connection between the product of two binomials and the distributive property. Algebra tiles and an area model are the strategies that are used to make this connection. Students could work in pairs to represent expanding and simplifying. Some pairs could use algebra tiles while other pairs use area models, switching for parts a) and b). Then volunteers could demonstrate their results to the class, relating their results to Lorna's solution.

In *Example 3*, binomial expressions are expanded using the distributive property. Some students may need to continue using algebra tiles or area diagrams to help them understand how to use the distributive property effectively.

In *Example 4*, the graph of a parabola is given, and students are required to determine the equation of the parabola in standard form. First, ask what information the graph provides. Have students develop a plan to solve the problem, given this information and the two forms of a quadratic relation.

Answer to the Key Assessment Question

Sketching area diagrams can provide visual representations for question 13 that show different possible dimensions for the rectangle.

13. I agree with Bill.

$A = (2x + 4)(x + 5)$
$= 2x^2 + 10x + 4x + 20$
$= 2x^2 + 14x + 20$

The rectangle could have the dimensions $(2x + 4)$ and $(x + 5)$, since they form the required area.

$A = (2x + 10)(x + 2)$
$= 2x^2 + 4x + 10x + 20$
$= 2x^2 + 14x + 20$

The rectangle could also have the dimensions $(2x + 10)$ and $(x + 2)$, since they also form the required area.

Closing

Ensure that students understand what is asked in question 16. They need to determine whether the result is always a trinomial when two binomials are multiplied together. If students need help, direct their attention to the results of question 6. These results give students several examples to choose from. All of the results in this question are binomials.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
Students use the distributive property effectively to expand quadratic expressions. Students simplify expanded expressions to get the standard form of a quadratic relation.	Students may have difficulty applying the distributive properly. They may multiply only the first and last terms of the binomials, resulting in a binomial. They may have difficulty identifying like terms or calculating as they simplify.
Students use an area diagram or algebra tiles to show the relation between two binomial factors of degree 1 and their product.	Given a binomial product, students may have difficulty setting up a correct algebra tile model. They may not realize how the tiles need to be arranged.
Students express a quadratic relation in both factored form and standard form.	Students cannot make the connection between the factored form and standard form of a quadratic relation. After expressing the quadratic relation in factored form, students may distribute the value of *a* into each of the two binomial factors.
Key Assessment Question 13	
Students correctly decide that Bill is correct. They justify by expanding each of the two pairs of factors to obtain the same expression for the given area.	Students may not agree with Bill if they expand one or both pairs of factors incorrectly or if they multiply only the first terms and second terms of the binomials.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. Pair a strong student with a student who is experiencing difficulty. Allow the stronger student to help the other student demonstrate the connection between the models and the expanded product.
2. Encourage students to use one of the models if they have difficulty working through the Practising problems. They should eventually be able to make the connection between the models and the distributive property, so they can work without the models.

EXTRA CHALLENGE

1. Give students a quadratic relation in standard form (e.g., $y = x^2 + 2x – 3$ or $y = 2x^2 – 2x – 12$), and ask them to use a model to determine the corresponding relation in standard form.

3.5 QUADRATIC MODELS USING FACTORED FORM

Lesson at a Glance

GOAL

Determine the equation of a quadratic model using the factored form of a quadratic relation.

Prerequisite Skills/Concepts

- Create a scatter plot and draw a line or curve of good fit for linear or nonlinear data.
- Interpret the meaning of points on a scatter plot.

Specific Expectations

- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with and without the use of technology.
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Mathematical Process Focus

- Problem Solving
- Reasoning and Proving
- Selecting Tools and Computational Strategies
- Connecting
- Representing

Student Book Pages 169–178

Preparation and Planning

Pacing
10 min Introduction
30–40 min Teaching and Learning
15–20min Consolidation

Materials
- graphing calculator
- grid paper
- ruler

Recommended Practice
Questions 4, 5, 6, 7, 9, 10

Key Assessment Question
Question 9

Extra Practice
Lesson 3.5 Extra Practice

New Vocabulary/Symbols
curve of good fit
quadratic regression
curve of best fit

Nelson Website
http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students investigate data that produce a graph close to a parabolic shape and approximate the data using a quadratic relation in factored form.
- Students determine the equation of a curve of good fit with and without graphing technology. They determine the equation of a curve of best fit with graphing technology, using quadratic regression.
- Students use estimated or exact x-intercepts to determine the values of r and s in the factored form of a quadratic relation, $y = a(x – r)(x – s)$. Then they calculate the value of a using the coordinates of a point that lies on, or close to, the curve of best fit.
- Students learn how to estimate the value of a graphically with graphing technology.

1 Introducing the Lesson

(10 min)

With the class, discuss real-world situations that could be modelled by quadratic relations. Previous examples include the height of a ball, the altitude of a hot-air balloon, and the possible areas of an exercise pen with a fixed perimeter. Ask how the scatter plot of the data for a quadratic relation would differ from the scatter plot of the data for a linear relation.

Review the information that is needed to determine the equation of a quadratic relation in factored form. Students could take turns providing the information.

2 Teaching and Learning

(30 to 40 min)

Investigate the Math

Have students work in pairs. When students have finished the table of values, discuss the difference between a curve of good fit and the curve of best fit:

- There is more than one possibility for a curve of good fit. It is a curve that approximates, or is close to, points representing the data. Every student may get a different curve of good fit.
- There is only one curve of best fit. It is the curve that best describes the data. In this investigation, students use quadratic regression to determine the curve of best fit.

If necessary, discuss how to determine an appropriate scale for the scatter plot. Depending on students' experience with graphing calculators, you could work on part H with the class.

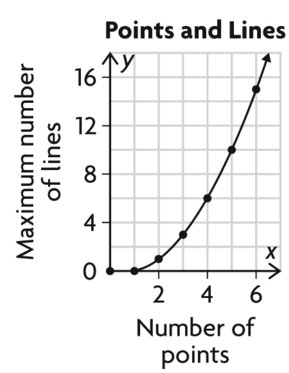
Answers to Investigate the Math

A. No. It would be difficult to count the lines for a lot of points.

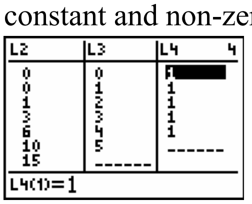
B.

Number of Points, x	0	1	2	3	4	5	6
Maximum Number of Lines, y	0	0	1	3	6	10	15

C.–D. The graph appears to be in the shape of a parabola. Curves of good fit may vary.



E. The first differences are not constant, but the second differences are constant and non-zero. The curve is approximately quadratic.



F. The zeros are 0 and 1. The equation of the relation in factored form is $y = a(x - 0)(x - 1)$.

G. Answers may vary, e.g.,

$1 = a(2 - 0)(2 - 1)$

$1 = 2a$

$a = \frac{1}{2}$

The value of a is $\frac{1}{2}$.

The equation is $y = \frac{1}{2}x(x - 1)$ or $y = \frac{1}{2}x^2 - \frac{1}{2}x$.

H. Using quadratic regression, the equation is $y = 0.5x^2 - 0.5x$.

I. The equations are equivalent.

J. $y = \frac{1}{2}(100)^2 - \frac{1}{2}(100)$

$= 4950$

Therefore, 4950 lines can be drawn using 100 points.

Answers to Reflecting

K. The factored form of a quadratic relation helps you determine the equation of a curve of good fit because the zeros are r and s in $y = a(x - r)(x - s)$.

L. If the data were quadratic and the curve of good fit had only one zero, the two factors would be the same.

M. No. Equations in factored form have zeros.

3 Consolidation

(15 to 20 min)

Apply the Math

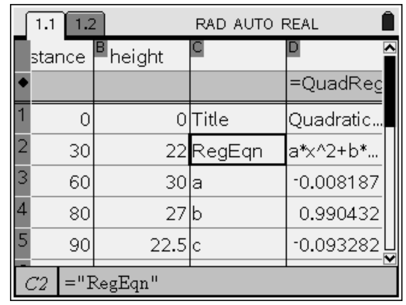
Using the Solved Examples

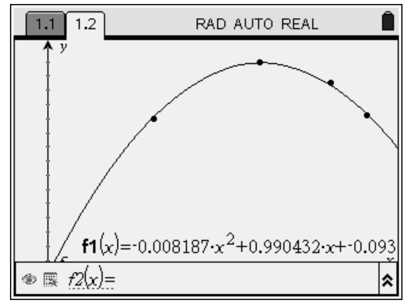
In *Example 1*, the equation of a curve of good fit is determined from a table of values and a given maximum value. Ask students what other information can be determined if they know the maximum value. After the vertex and the equation of the axis of symmetry have been determined, ask students how this information can be used to calculate the second zero. Students could use points other than (60, 30) to find out whether the equation for a curve of good fit would vary.

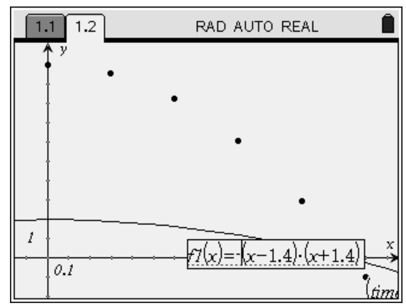
For *Example 2*, present the data and the information provided. You might have students enter the data into a graphing calculator and create a scatter plot. Ask students to use their graph to estimate the two zeros.

In *Example 3*, data collected from a motion detector are used to determine an equation for a curve of good fit. Ask students how they know that a quadratic model might fit the data. Have students use different values from the table to find out whether the equation for a curve of good fit is affected.

Technology-Based Alternative Lesson







There are several ways to determine a quadratic regression using a TI-*n*spire calculator. One way is to enter the data into a Lists & Spreadsheet application as described in Appendix B-46. Students can enter the distance and height from *Example 1* into columns A and B, respectively. Have them label column A “distance” and column B “height.”

Students could also find their own curve of good fit. Have them enter the data from *Example 2* into a Lists & Spreadsheet application and follow these steps:

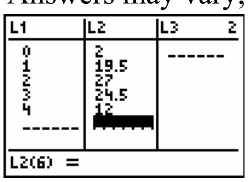
- Add a Graphs & Geometry application to plot the data.
- Change the Graph Type to Scatter Plot.
- Select the appropriate data for x and y.
- Change the Graph Type to Function.

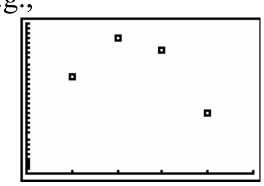
Have students enter what they think the equation might be, in factored form, by estimating the zeros and the value of a. Based on the graph that appears, they can alter the zeros or the value of a to get a better fit. Instead of changing the equation in the entry line, they can click twice on the equation on the screen and then move to the number they want to change.

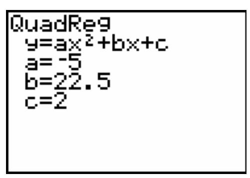
Answer to the Key Assessment Question

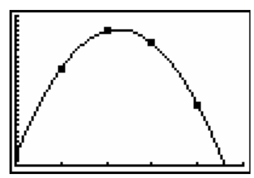
Students might graph the data and calculate an equation of good fit for question 9, or they might use quadratic regression to determine the equation of best fit. If both are used, have students compare answers.

9. a) Answers may vary, e.g.,





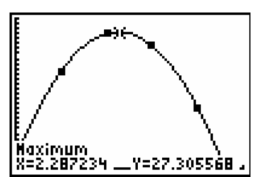




An equation is $y = -5x^2 + 22.5x + 2$, where x is the time elapsed in seconds and y is the height in metres.

b) The rocket was about 15.3 m high after 3.8 s.

c) The maximum height is about 27.3 m at about 2.3 s.



Closing

For question 14, have students work in pairs to create a flow chart that summarizes the steps for determining the equation of a parabola of good fit using the factored form of a quadratic relation. Students can post their flow charts when completed.

Assessment and Differentiating Instruction

What You Will See Students Doing...

When students understand...	If students misunderstand...
Students use given data to determine the factored form of a quadratic curve of good fit that has a parabolic pattern and passes through the horizontal axis.	Students may have difficulty determining the value of *a*. They may simplify incorrectly after substituting the values for the other point on the line.
Students use estimated or actual *x*-intercepts to determine the values of *r* and *s* in the factored form of a quadratic relation. They determine the value of *a* using another point that lies on, or close to, a curve of good fit.	Students may have difficulty determining both *x*-intercepts. This can occur when only one *x*-intercept appears in the data and they must determine the other *x*-intercept after identifying the equation of the axis of symmetry.
Students use graphing technology to estimate the value of *a* graphically or to determine an algebraic model for the curve of best fit.	Students may have difficulty using graphing technology to estimate the value of *a* or determine the equation of the curve of best fit using regression.
Key Assessment Question 9	
Students recognize that the data have a parabolic pattern and are able to determine an equation for the height–time relation.	Students may have difficulty choosing a strategy to determine an equation for the height–time relation.
Students use the equation to determine the approximate height of the rocket at 3.8 s.	Students may make errors in the equation when substituting, calculating, or interpreting the result.
Students determine the maximum height and the time at which it occurs.	Students may switch the values of the coordinates as they substitute into the equation.

Differentiating Instruction | How You Can Respond

EXTRA SUPPORT

1. If students have difficulty deciding on a strategy to solve problems, have them prepare a summary of each strategy used in the lesson and the information they need to be given or need to determine. Each pair could prepare a summary for each strategy to present to other students. They can use the summary to help them decide on a strategy.
2. If students have difficulty with the graphing technology when determining the curve of best fit using quadratic regression, pair them with another student. Working with a partner will help them gain confidence.

EXTRA CHALLENGE

1. Have students collect data that they think could be represented by a quadratic model using secondary sources, such as the Internet or experiments with motion detectors. Then have them graph their data, try to create an equation of good fit or an equation of best fit, and use their results to decide whether a quadratic model is appropriate for the data.

3.6 EXPLORING QUADRATIC AND EXPONENTIAL GRAPHS

Lesson at a Glance

GOAL

Compare the graphs of $y = x^2$ and $y = 2^x$ to determine the meanings of zero and negative exponents.

Prerequisite Skill/Concept

- Apply exponent rules in expressions involving one or two variables with positive exponents.

Specific Expectation

- Compare, through investigation using technology, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent (e.g., by examining patterns in a table of values for $y = 2^x$; by applying the exponent rules for multiplication and division).

Mathematical Process Focus

- Reasoning and Proving
- Connecting
- Representing

Student Book Pages 179–182

Preparation and Planning

Pacing

10 min	**Introduction**
30–40 min	**Teaching and Learning**
10 min	**Consolidation**

Materials

- graphing calculator

Recommended Practice

Questions 1, 2, 3, 4, 5, 6

Nelson Website

http://www.nelson.com/math

MATH BACKGROUND | LESSON OVERVIEW

- Students investigate the patterns in a table of values for $y = 2^x$ to determine the meaning of a^{-n} and a^0 for $a \neq 0$.
- Students compare the properties of the graphs of $y = x^2$ and $y = 2^x$.

1 Introducing the Lesson

(10 min)

Introduce the two relations that will be investigated in the lesson: $y = x^2$ and $y = 2^x$. Discuss how these two relations are similar because they both involve powers but different because of the location of the variable x.

Students will be familiar with the graph of $y = x^2$ but not with the graph of $y = 2^x$. Display the graphs of both relations. Ask the class to decide which relation is quadratic. Then discuss how the graphs are similar and how they are different.

2 Teaching and Learning

(30 to 40 min)

Explore the Math

Introduce the problem that students will explore by demonstrating the first two entries in the table of values. Ask a volunteer to fold the paper once to see the first result. Then ask the volunteer to fold the paper a second time to obtain the second result.

Have students work in pairs to complete the table of values. Circulate throughout the class to ensure that students are not having difficulty with the graphing calculator.

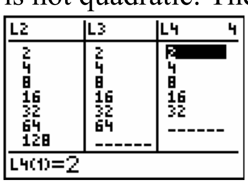
Answers to Explore the Math

A.

Number of Folds	1	2	3	4	5	6	7
Number of Regions	2	4	8	16	32	64	128

The number of folds increases by 1. The number of regions doubles with each fold.

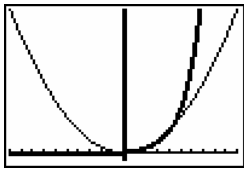
B. The relationship between the number of regions and the number of folds is not quadratic. The second differences are not constant.



C.

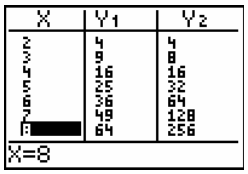
Number of Folds	1	2	3	4	5	6	7
Number of Regions	2	4	8	16	32	64	128
$y = 2^x$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$	$2^7 = 128$

D. $y = x^2$ and $y = 2^x$



E. The quadratic and exponential graphs both increase to the right of zero, but the exponential graph grows more rapidly than the quadratic graph. To the left of zero, the quadratic graph increases as x increases in the negative direction, but the exponential graph does not.

F. As x becomes greater, the exponential relation grows faster than the quadratic relation.



G. **i)** $2^0 = 1$ **ii)** $2^{-1} = 0.5$ **iii)** $2^{-2} = 0.25$ **iv)** $2^{-3} = 0.125$

H. **i)** $1 = 2^0$ **ii)** $0.5 = \frac{1}{2^1}$ **iii)** $0.25 = \frac{1}{2^2}$ **iv)** $0.125 = \frac{1}{2^3}$

I. **i)** $3^0 = 1$ and $5^0 = 1$ **iii)** $3^{-2} = \frac{1}{3^2}$ and $5^{-2} = \frac{1}{5^2}$

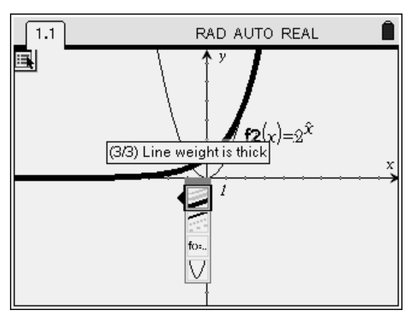
ii) $3^{-1} = \frac{1}{3^1}$ and $5^{-1} = \frac{1}{5^1}$ **iv)** $3^{-3} = \frac{1}{3^3}$ and $5^{-3} = \frac{1}{5^3}$

J. **i)** There is no symmetry in either graph.
ii) There are no x-intercepts for either relation. The y-intercept for both relations is 1.
iii) The y-values are increasing for all x-values in both relations.
iv) The y-values are never decreasing in either relation.
v) The y-values get greater and greater as x gets larger in the positive direction.
vi) The y-values get smaller and smaller as x gets larger in the negative direction.

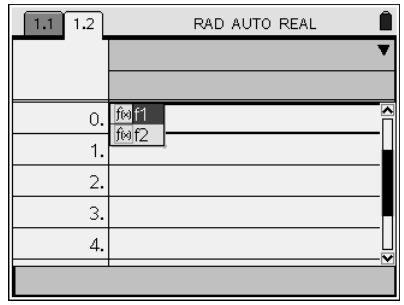
Technology-Based Alternative Lesson

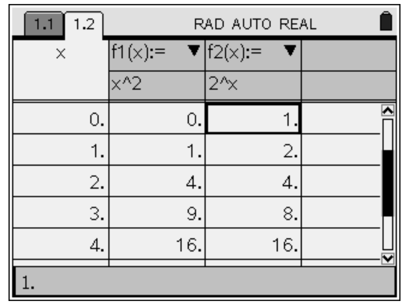
If TI-*n*spire calculators are available for the exploration, have students enter $y = x^2$ and $y = 2^x$ in the entry line of a Graphs & Geometry application. To distinguish between the two relations, students will need to change the attributes of one or both lines by following these steps:

- From the menu, select **1: Actions→ 4: Attributes**.
- Click on the line they would like to change.
- Use the right or left arrow keys to cycle through the options for line weight.
- Press Enter to change the thickness of the line.



Students will need to add a table to see the y-values for each graph. Since it is desirable to view the y-values for both graphs at the same time, have students add a Lists & Spreadsheet application and change it to a table. Since they have a choice of two relations, have them select f(x)f1 and then move over to the next column and select f(x)f2. They can use the up and down arrows to scroll up and down the list.





Answers to Reflecting

K. The graph of $y = 2^x$ will never touch the x-axis. There is no exponent that will give 2^x a value of 0.

L. **i)** a^0 will always equal 1.

ii) To evaluate a^{-1}, take the reciprocal of the base raised to the opposite exponent. The result is $\frac{1}{a^1}$.

iii) To evaluate a^{-2}, take the reciprocal of the base raised to the opposite exponent. The result is $\frac{1}{a^2}$.

iv) To evaluate a^{-n}, take the reciprocal of the base raised to the opposite exponent. The result is $\frac{1}{a^n}$.

M. In the relations $y = x^2$ and $y = 2^x$, the y-values are never negative. For the relation $y = x^2$, the square of any number can never be negative. For the relation $y = 2^x$, 2 raised to any exponent will always be greater than 0. Both positive and negative exponents result in a positive value.

3 Consolidation

(10 min)

Students should be able to compare the graphs of $y = x^2$ and $y = 2^x$ in terms of their symmetry, their x- and y-intercepts, and the characteristics of their y-values for various values of x.

Students should be able to answer the Further Your Understanding questions independently.

CHAPTER REVIEW

Big Ideas Covered So Far

- The second differences for a quadratic relation are constant but not zero, the degree of the relation is 2, and the graph is a U shape, which is called a parabola. The vertex is the highest or lowest point on the curve, and the axis of symmetry passes through the vertex.
- The standard form of a quadratic relation is $y = ax^2 + bx + c$, with y-intercept c.
- The factored form of a quadratic relation is $y = a(x - r)(x - s)$. The x-intercepts are r and s. The equation of the axis of symmetry is $x = \frac{r + s}{2}$. The y-intercept is $a \times r \times s$.
- The product of two binomials can be determined by using algebra tiles, using an area diagram, or multiplying symbolically.
- To determine whether data can be modelled with a quadratic relation, a scatter plot with a curve of good fit or best fit can be drawn. A difference table can show whether the second differences are approximately constant and non-zero.
- To determine an equation of good fit, a scatter plot of the data can be used to estimate the zeros and the value of a. Alternatively, equations can be entered into a graphing calculator to determine a fit. The equation of best fit can be determined using quadratic regression.
- $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$, for $a \neq 0$.

Using the Frequently Asked Questions

Have students keep their Student Books closed. Display the Frequently Asked Questions on a board. Have students discuss the questions and use the discussion to draw out what the class thinks are good answers. Then have students compare the class answers with the answers on Student Book pages 183 and 184. Students can refer to the answers to the Frequently Asked Questions as they work through the Practice Questions.

Using the-Chapter Review

Ask students if they have any questions about any of the topics covered so far in the chapter. Review any topics that students would benefit from considering again. Assign Practice Questions for class work and for homework.

To gain greater insight into students' understanding of the material covered so far in the chapter, you may want to ask questions such as the following:

- How can you convert the equation of a parabola in factored form to standard form?
- How can you use algebra tiles to illustrate expanding quadratic expressions? How can you use an area model?
- What is an important step to remember when you expand a quadratic expression?
- If a data set can be modelled by a quadratic relation, how can you determine the equation in factored form? What is another strategy you can use?

CHAPTER 3 TEST

For further assessment items, please use Nelson's Computerized Assessment Bank.

1. Explain the difference between a quadratic relation and a linear relation.

2. Decide, without graphing, whether each data set can be modelled by a quadratic relation. Justify your decision.

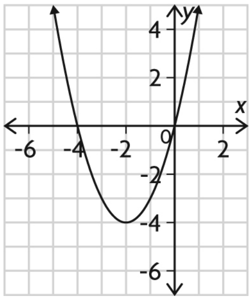
a)

x	–2	–1	0	1	2
y	–3	3	0	3	–3

b)

x	–2	–1	0	1	2
y	–3	3	5	3	–3

3. For the parabola at the right, determine
a) the equation of the axis of symmetry
b) the coordinates of the vertex
c) the zeros



4. Sketch a graph of each relation. Label the zeros and the vertex with their coordinates.
a) $y = x(x + 3)$ **b)** $y = –(x – 2)(x + 3)$

5. The following table represents a quadratic relation.

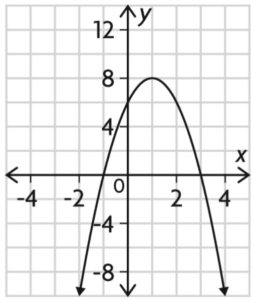
Time (s)	1	2	3	4	5	6
Height (m)	0	10	15	15	10	0

a) Sketch a graph of the relation.
b) Determine the equation of the axis of symmetry.
c) At what time does the maximum height occur? How do you know?
d) Determine an equation for the relation and the coordinates of the vertex.

6. A parabola has zeros at –4 and 6, and a y-intercept of –6. Determine
a) the equation of the axis of symmetry
b) the equation of the parabola
c) the coordinates of the vertex

7. Expand and simplify each expression.
a) $(2x + 1)(x – 4)$ **b)** $(2x + 3y)(3x + 4y)$ **c)** $–3(x – 2)^2$

8. Using the graph below, determine the equation of the parabola. Express the equation in standard form.



9. Ramil has a business cutting lawns. The profit she makes each week depends on the number of lawns she cuts, as shown in the table below.

Number of Lawns	5	10	15	20	25	30
Profit ($)	–106	–6	44	44	–6	–106

a) Do the data in the table represent a quadratic relation? Explain.
b) Create a scatter plot. Then draw a curve of good fit.
c) Determine the equation of the axis of symmetry for the curve.
d) How many lawns per week does Ramil need to cut to make the maximum profit?
e) What do the zeros represent in the context of this situation?

10. Evaluate each power.

a) $(-2)^{-4}$ **b)** $(-7)^{0}$ **c)** -6^{-2} **d)** $-\left(\frac{5}{4}\right)^{-3}$

CHAPTER 3 TEST ANSWERS

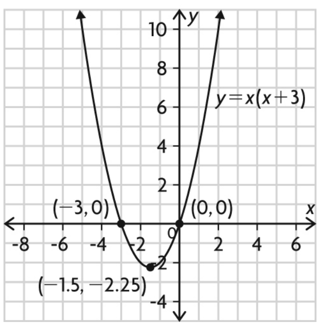
1. A linear relation has constant first differences, but a quadratic relation does not. A quadratic relation has constant second differences, which are not zero. The degree of a linear relation is 1, and the degree of a quadratic relation is 2. The graph of a linear relation is a straight line. The graph of a quadratic relation is a parabola that opens upward or downward.

2. a) No. The first differences are 6, –3, 3, 6, and the second differences are –9, 6, –9. The second differences are not constant.

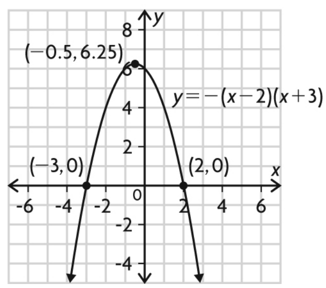
b) Yes. The first differences are 6, 2, –2, –6, and the second differences are all –4. The second differences are constant and non-zero.

3. a) $x = -2$ **b)** $(-2, -4)$ **c)** –4, 0

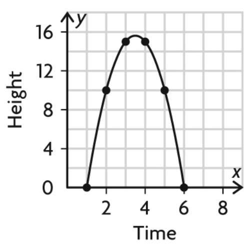
4. a)



b)



5. a)



b) The equation of the axis of symmetry is $x = 3.5$.

c) The maximum height occurs at 3.5 s. It occurs at the vertex, which is on the axis of symmetry.

d) $y = -\frac{5}{2}(x-1)(x-6)$; (3.5, 15.625)

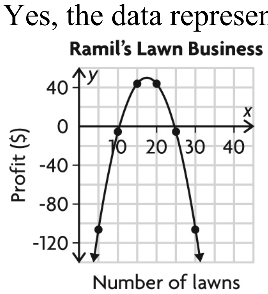
6. a) $x = 1$ **b)** $y = 0.25(x + 4)(x - 6)$ **c)** $(1, -6.25)$

7. a) $2x^2 - 7x - 4$ **b)** $6x^2 + 17xy + 12y^2$ **c)** $-3x^2 + 12x - 12$

8. The equation of the parabola is $y = -2(x + 1)(x - 3)$, or $y = -2x^2 + 4x + 6$ in standard form.

9. a) Yes, the data represent a quadratic relation. The second differences are all –50.

b)



c) $x = 17.5$

d) 17 or 18 lawns

e) The zeros represent the break-even points.

10. a) $\frac{1}{16}$ **b)** 1 **c)** $-\frac{1}{36}$ **d)** $-\frac{64}{125}$

CHAPTER TASK

Comparing the Force of Gravity

Student Book Page 188

Preparation and Planning	
Pacing	
20–25 min	**Introducing the Chapter Task**
35–40 min	**Using the Chapter Task**
Materials ▪ grid paper ▪ ruler ▪ graphing calculator (optional)	
Nelson Website http://www.nelson.com/math	

Specific Expectations

- Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with and without the use of technology.
- Identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.
- Sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; [sketching $y = x^2 - 12x + 1$ by completing the square and applying transformations;] graphing $h = -4.9t^2 - 50t + 1.5$ using technology.
- Determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) [or from its defining equation (i.e., by applying algebraic techniques)].
- Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Introducing the Chapter Task (Whole Class)

Introduce the Chapter Task on page 188 of the Student Book. Invite students to explain what they already know about gravity. Ask: How does gravity affect activities on Earth and on the Moon? The students should have a conceptual idea of how the gravity on the Moon compares with the gravity on Earth.

Using the Chapter Task

Have students work individually. As students work through the task, observe them individually to see how they are interpreting and carrying out the task. Review concepts with them as necessary. They might use a graphing calculator to explore or confirm their solution.

Assessing Students' Work

Use the Assessment of Learning chart as a guide for assessing students' work.

Adapting the Task

You can adapt the task in the Student Book to suit the needs of your students. For example:

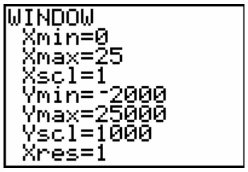
- Have students work in pairs on parts A, B, and C so they can check and review each other's graphs and estimates of the vertex, axis of symmetry, and zeros.
- Have stronger students investigate the gravity on Mars. They can research similar data of an object falling 10 m on Mars and create a scatter plot of the data. They can draw a curve of best fit and compare it with their other two curves.

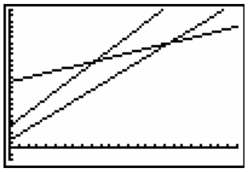
Assessment of Learning—What to Look for in Student Work…				
Assessment Strategy: Interview/Observation and Product Marking				
Level of Performance	**1**	**2**	**3**	**4**
Knowledge and Understanding Knowledge of content	demonstrates **limited** knowledge of content	demonstrates **some** knowledge of content	demonstrates **considerable** knowledge of content	demonstrates **thorough** knowledge of content
Understanding of mathematical concepts	demonstrates **limited** understanding of concepts (e.g., is unable to create a scatter plot or curve of good fit using an appropriate scale)	demonstrates **some** understanding of concepts (e.g., creates a scatter plot and curve of good fit, but is unable to determine the key features of the curve of good fit)	demonstrates **considerable** understanding of concepts (e.g., correctly creates a scatter plot and curve of good fit, and correctly uses the curve of good fit to determine the key features)	demonstrates **thorough** understanding of concepts (e.g., correctly creates a scatter plot and curve of good fit, and uses the curve of good fit to determine the key features; demonstrates the ability to discuss the relationship between height and time)
Thinking Use of planning skills • understanding the problem • making a plan for solving the problem	uses planning skills with **limited** effectiveness	uses planning skills with **some** effectiveness	uses planning skills with **considerable** effectiveness	uses planning skills with a **high degree** of effectiveness
Use of processing skills • carrying out a plan • looking back at the solution	uses processing skills with **limited** effectiveness	uses processing skills with **some** effectiveness	uses processing skills with **considerable** effectiveness	uses processing skills with a **high degree** of effectiveness
Use of critical/creative thinking processes	uses critical/creative thinking processes with **limited** effectiveness	uses critical/creative thinking processes with **some** effectiveness	uses critical/creative thinking processes with **considerable** effectiveness	uses critical/creative thinking processes with a **high degree** of effectiveness

Assessment of Learning—What to Look for in Student Work…				
Assessment Strategy: Interview/Observation and Product Marking				
Level of Performance	**1**	**2**	**3**	**4**
Communication Expression and organization of ideas and mathematical thinking, using oral, visual, and written forms	expresses and organizes mathematical thinking with **limited** effectiveness	expresses and organizes mathematical thinking with **some** effectiveness	expresses and organizes mathematical thinking with **considerable** effectiveness	expresses and organizes mathematical thinking with a **high degree** of effectiveness
Communication for different audiences and purposes in oral, visual, and written forms	communicates for different audiences and purposes with **limited** effectiveness	communicates for different audiences and purposes with **some** effectiveness	communicates for different audiences and purposes with **considerable** effectiveness	communicates for different audiences and purposes with a **high degree** of effectiveness
Use of conventions, vocabulary, and terminology of the discipline in oral, visual, and written forms	uses conventions, vocabulary, and terminology of the discipline with **limited** effectiveness	uses conventions, vocabulary, and terminology of the discipline with **some** effectiveness	uses conventions, vocabulary, and terminology of the discipline with **considerable** effectiveness	uses conventions, vocabulary, and terminology of the discipline with a **high degree** of effectiveness
Application Application of knowledge and skills in familiar contexts	applies knowledge and skills in familiar contexts with **limited** effectiveness (e.g., fails to determine the key features of the curve of good fit or the equation of good fit)	applies knowledge and skills in familiar contexts with **some** effectiveness (e.g., uses a correct strategy for determining the equation of good fit, but makes several errors when determining the equation)	applies knowledge and skills in familiar contexts with **considerable** effectiveness (e.g., chooses a correct strategy for determining the equation of good fit, but makes a minor error)	applies knowledge and skills in familiar contexts with a **high degree** of effectiveness (e.g., correctly determines the equation of good fit in standard form)
Transfer of knowledge and skills to new contexts	transfers knowledge and skills to new contexts with **limited** effectiveness (e.g., is unable to determine the value of *g* on Earth and on the Moon)	transfers knowledge and skills to new contexts with **some** effectiveness (e.g., uses the equation and information incorrectly and determines an incorrect value of *g* on Earth and on the Moon)	transfers knowledge and skills to new contexts with **considerable** effectiveness (e.g., uses the equation and information correctly to determine the value of *g* on Earth and on the Moon, with only a minor error)	transfers knowledge and skills to new contexts with a **high degree** of effectiveness (e.g., uses the equation and information correctly to determine the value of *g* on Earth and on the Moon)
Making connections within and between various contexts	makes connections within and between various contexts with **limited** effectiveness	makes connections within and between various contexts with **some** effectiveness	makes connections within and between various contexts with **considerable** effectiveness	makes connections within and between various contexts with a **high degree** of effectiveness

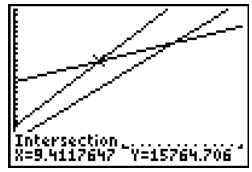
CHAPTERS 1–3 CUMULATIVE REVIEW

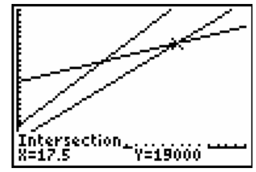
1. C; students who answered incorrectly may need to review solutions for linear relations in Lesson 1.1.
2. B; students who answered incorrectly may need to review solutions for linear systems in Lessons 1.3, 1.4, and 1.6.
3. A; students who answered incorrectly may need to review identifying equivalent equations in Lesson 1.5.
4. D; students who answered incorrectly may need to review representing and solving linear systems in Lessons 1.3, 1.4, and 1.6.
5. C; students who answered incorrectly may need to review identifying equivalent linear systems in Lesson 1.5.
6. B; students who answered incorrectly may need to review representing and solving linear systems in Lessons 1.4 and 1.6.
7. A; students who answered incorrectly may need to review substitution and elimination in Lessons 1.4 and 1.6.
8. C; students who answered incorrectly may need to review determining midpoints in Lesson 2.1.
9. D; students who answered incorrectly may need to review determining midpoints and medians in Lesson 2.1.
10. B; students who answered incorrectly may need to review calculating distances in Lesson 2.2.
11. B; students who answered incorrectly may need to review calculating distances in Lesson 2.2.
12. D; students who answered incorrectly may need to review circle equations in Lesson 2.3.
13. A; students who answered incorrectly may need to review using slopes to classify figures in Lesson 2.4.
14. D; students who answered incorrectly may need to review using slopes and lengths of line segments to classify figures in Lesson 2.4.
15. B; students who answered incorrectly may need to review properties of a circle and chords in Lesson 2.6.
16. A; students who answered incorrectly may need to review properties of graphs of quadratic relations in Lesson 3.2.
17. B; students who answered incorrectly may need to review determining the vertex of quadratic graphs in Lesson 3.2.
18. B; students who answered incorrectly may need to review the factored form of a quadratic relation in Lesson 3.3.
19. C; students who answered incorrectly may need to review the factored form of a quadratic relation in Lesson 3.3.
20. C; students who answered incorrectly may need to review determining the factored form of a quadratic relation in Lesson 3.3.
21. D; students who answered incorrectly may need to review determining the factored form of a quadratic relation in Lesson 3.3.
22. A; students who answered incorrectly may need to review expanding quadratic expressions in Lesson 3.4.
23. B; students who answered incorrectly may need to review negative exponents in Lesson 3.6.
24. D; students who answered incorrectly may need to review the meaning of exponents of zero in Lesson 3.6.
25. a) gas furnace: $C = 4000 + 1250t$; electric baseboard heaters: $C = 1500 + 1000t$; geothermal heat pump: $C = 12\,000 + 400t$





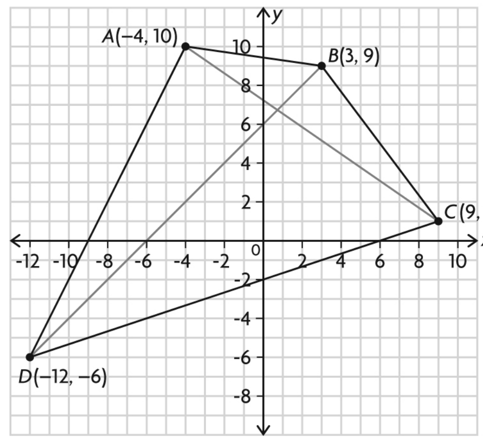
b) Electric is the least expensive for the first 17.5 years. Gas is more expensive than electric, but less expensive than geothermal, for the first 9.4 years. After 17.5 years, geothermal is the least expensive.





c) Answers may vary, e.g., the choice depends on how long Jenny and Oliver plan to live in the home. Another factor is the uncertainty about gas and electricity prices over time. The cost of geothermal heating will remain relatively constant.
Students who answered incorrectly may need to review solving linear systems in Lessons 1.3, 1.4, and 1.6.

26. a) Answers may vary, e.g., if all perpendicular bisectors intersect at the same point, then that point is the centre of a circle through all four vertices. Determine the perpendicular bisector equations; solve the system that is formed by two of these equations. Check that the solution satisfies the other equations.



b) Perpendicular bisector of AD: $y = -0.5x - 2$; perpendicular bisector of DC: $y = -3x - 7$; perpendicular bisector of CB: $y = 0.75x + 0.5$; perpendicular bisector of BA: $y = 7x + 13$. All intersect at $(-2, -1)$, so quadrilateral $ABCD$ is cyclic.

c) $E\left(\frac{8}{11}, \frac{74}{11}\right)$; $AE = \frac{20\sqrt{10}}{11} \doteq 5.749\ 595\ 746$ units;

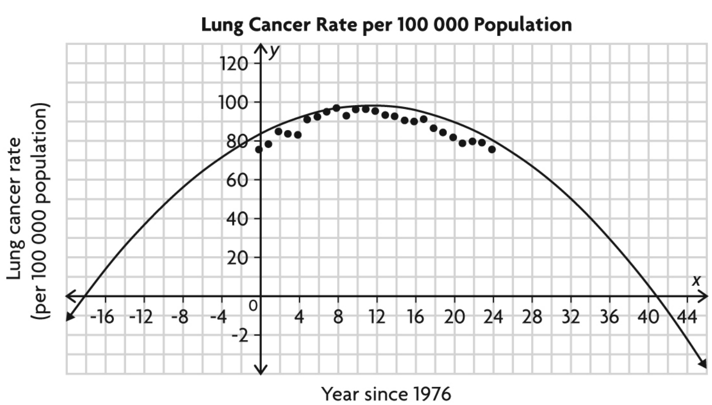
$EC = \frac{35\sqrt{10}}{11} \doteq 10.061\ 792\ 56$ units;

$BE = \frac{25\sqrt{2}}{11} \doteq 3.214\ 121\ 733$ units; $ED = \frac{140\sqrt{2}}{11} \doteq 17.999\ 081\ 7$ units;

$AE \times EC = BE \times ED = \frac{7000}{121} \doteq 57.851\ 239\ 7$ units

d) Any square, rectangle, or isosceles trapezoid could be cyclic. Kites can be cyclic if and only if they have two right angles.
Students who answered incorrectly may need to review verifying properties of geometric figures in Lessons 2.5 and 2.6.

27. a)



b) Answers may vary, e.g., equation for a curve of good fit: $y = -0.113\ 095(x + 18)(x - 41)$

c) equation for the curve of best fit: $y = -0.1358x^2 + 3.0681x + 77.3089$; equation for the curve of good fit: $y = -0.113\ 095x^2 + 2.601\ 185x + 83.464\ 11$

d) If the trend continues, lung cancer rates in Canadian males will continue to decrease.
Students who answered incorrectly may need to review analyzing quadratic models in factored form in Lesson 3.5.