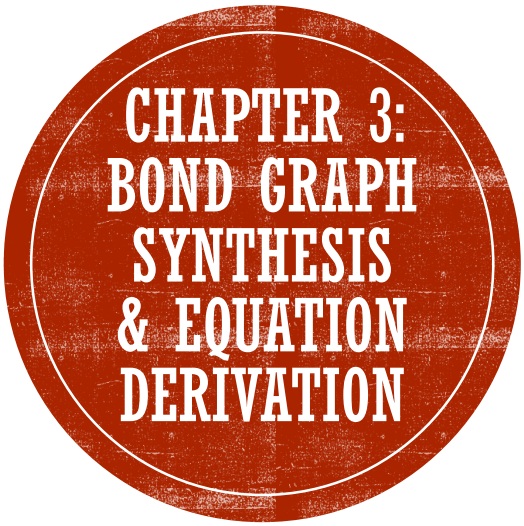


Samantha
Ramirez

PREVIEW QUESTIONS

- What type of mathematical equations are needed?
- How are these equations systematically derived?
- How are the individual constitutive relations of the components connected to generate a mathematical model?

Objectives:

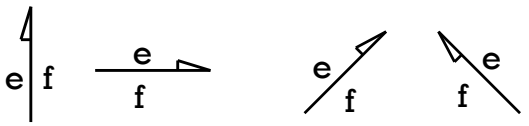
To effectively use bond graphs to formulate models that facilitate deriving mathematical representations of dynamic systems,	To be able to systematically derive mathematical representations using bond graphs, and	To understand the flow of information within a system dynamics model and its relation to mathematical representations.

Outcomes: Upon completion, you should be able to

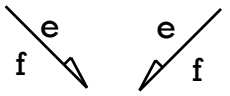
synthesize bond graph models of mechanical, electrical, and hydraulic systems,	annotate bond graphs to indicate appropriate power flow and causality, and	derive mathematical models in the form of differential and algebraic equations using bond graph representations.

RECAP: CHAPTER 2

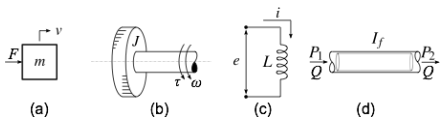
- Power bond labels



- R-Elements
  - Dissipate Energy
  - Direct algebraic relationship between e & f

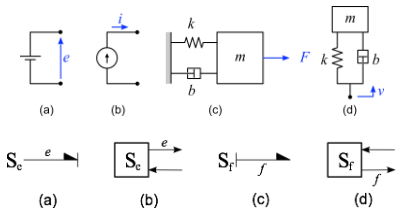


- C-Elements
  - Store Potential Energy
  - Derivative Causality
- I-Elements
  - Store Kinetic Energy
  - Derivative Causality

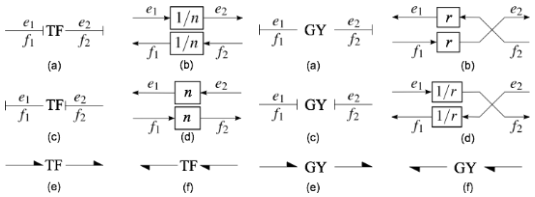


RECAP: CHAPTER 2

- Sources
  - Supply energy



- Transformers
  - Convert energy
  - Power through



- Gyrators
  - Convert energy
  - Power through

$e_1 = ne_2$

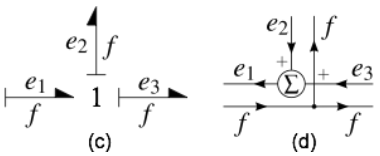
$nf_1 = f_2$

$e_1 = rf_2$

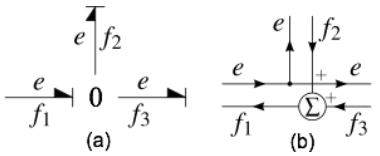
$rf_1 = e_2$

RECAP: CHAPTER 2

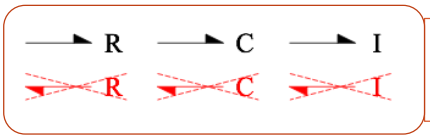
- 1-Junction
  - Common flow
  - Summation of efforts



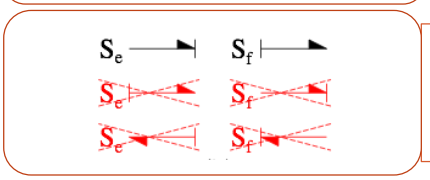
- 0-Junction
  - Common effort
  - Summation of flows



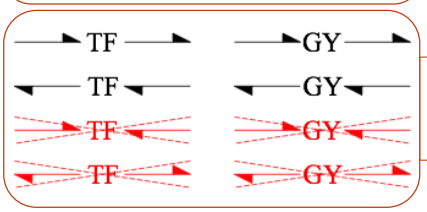
GENERAL GUIDELINES



- Power goes **from the system to** R-, C-, and I-elements



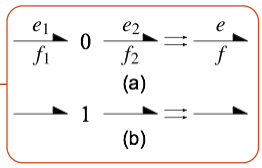
- Sources generally assumed to **supply power to** the system
  - Effort sources specify effort into the system
  - Flow sources specify flow into the system



- 2-ports have a **power through** convention

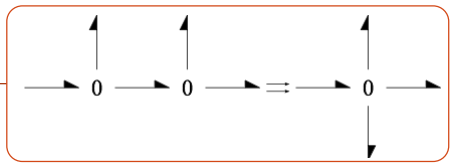
SIMPLIFICATIONS

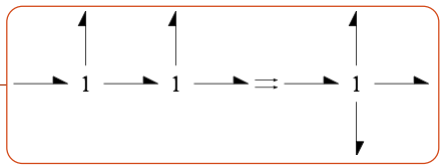
Junctions with two bonds (power in, power out) can be simplified into a single bond



What if power is not showing a power in, power out convention?

Adjacent junctions of the same type are actually the same junction and can be collapsed





BOND GRAPH SYNTHESIS:

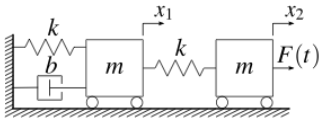
MECHANICAL TRANSLATION & ROTATION

1. **Identify distinct velocities (linear/angular)**
2. **Insert the force/torque-generating 1-ports and the energy-conserving 2-ports**
3. **Assign power directions**
4. **Eliminate zero velocity (linear/angular) sources**
5. **Simplify**
6. Assign causality

MECHANICAL TRANSLATION

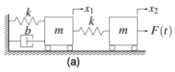
R-Element	Damper or friction
C-Element	Spring
I-Element	Mass
Effort Source	External force
Flow Source	Velocity source or shaker
Transformer	Lever or rocker arm
1-Junction	Common velocity; Sum of forces
0-Junction	Common force; Sum of velocities

Mechanical Translation Example 1



(a)

Figure 3.3



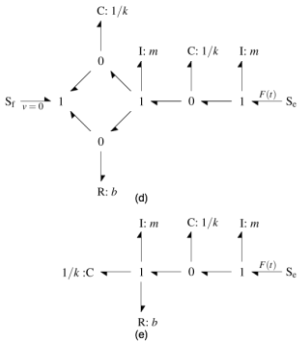
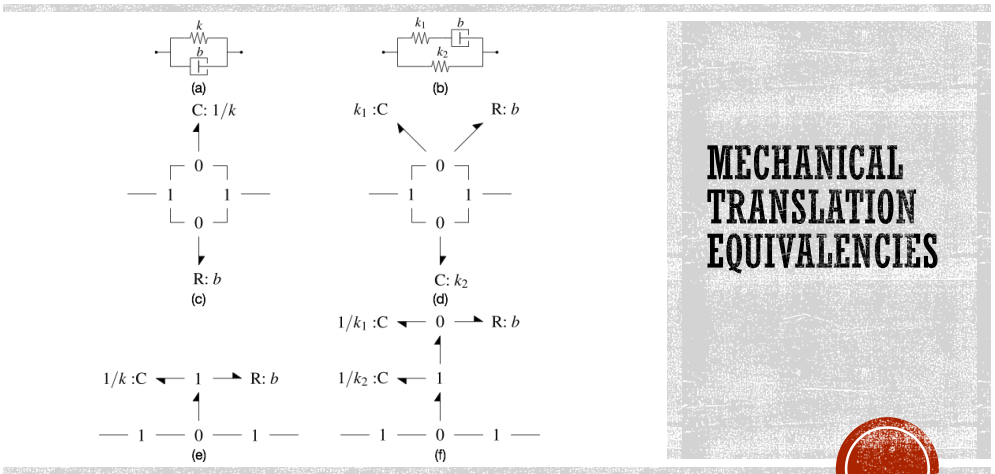
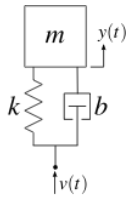


Figure 3.4



MECHANICAL TRANSLATION EQUIVALENCIES

Mechanical Translation Example 2



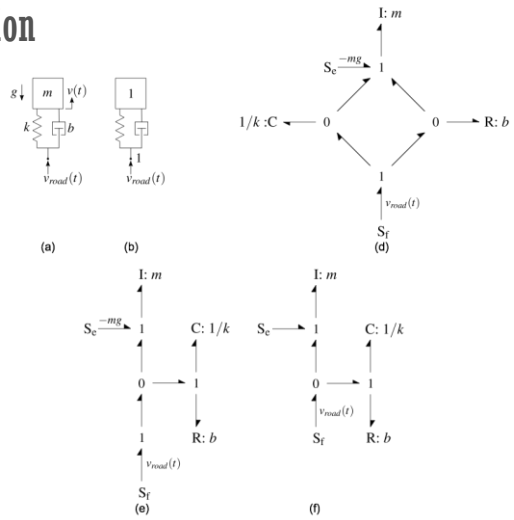
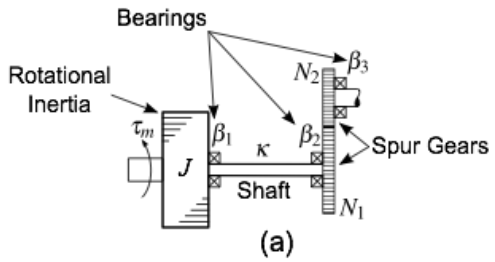


Figure 3.6

MECHANICAL ROTATION

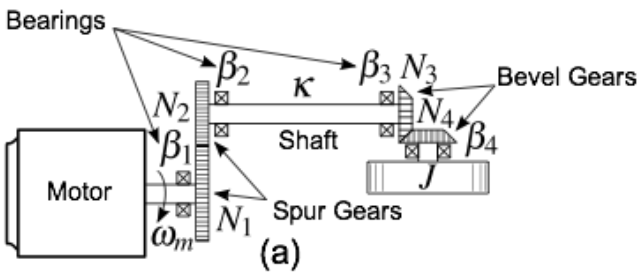
R-Element	Bearing or friction
C-Element	Torsion spring or shaft
I-Element	Rotational inertia
Effort Source	External torque (motor)
Flow Source	Angular velocity source (motor)
Transformer	Gear pair or chain and sprockets
1-Junction	Common angular velocity; Sum of moments (torques)
0-Junction	Common moment (torque); Angular velocity differential

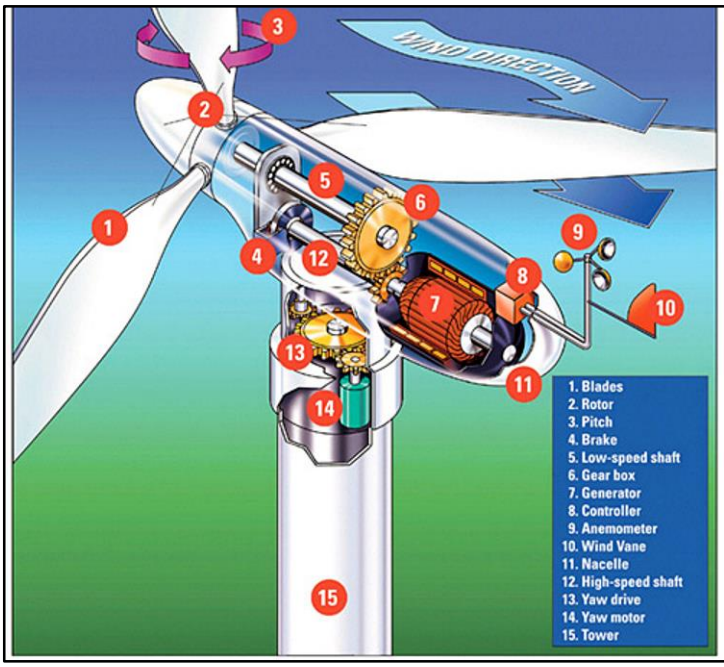
MECHANICAL ROTATION EXAMPLE 1



(a)

MECHANICAL ROTATION EXAMPLE 2





CHALLENGE

- Generate a bond graph to predict the response of the system.

BOND GRAPH SYNTHESIS:

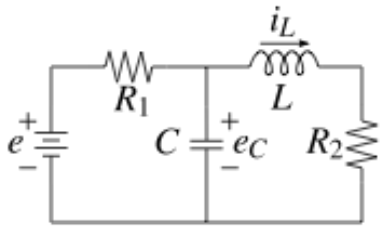
ELECTRIC & HYDRAULIC CIRCUITS

1. **Identify distinct voltages/pressures**
2. **Insert 1-port circuit elements and energy-converting 2-ports**
3. **Assign power directions**
4. **Eliminate explicit ground/atmospheric pressure (or reference pressure)**
5. **Simplify**
6. Assign causality

ELECTRICAL CIRCUITS

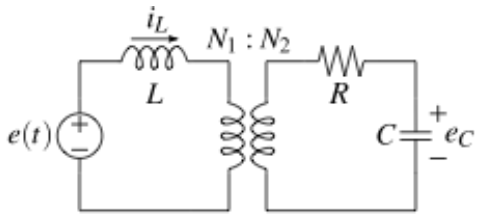
R-Element	Resistor
C-Element	Capacity
I-Element	Inductor
Effort Source	Battery or voltage source
Flow Source	Ideal current source
Transformer	Transformer
1-Junction	Common current; KVL
0-Junction	Common voltage: KCL

ELECTRIC CIRCUIT EXAMPLE 1

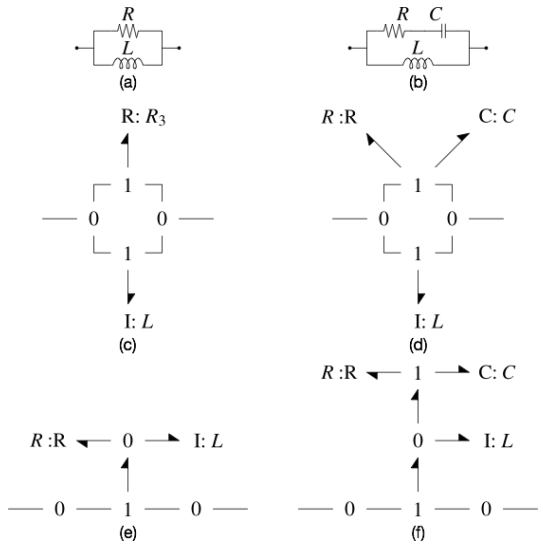


(a)

ELECTRIC CIRCUIT EXAMPLE 2



(a)



ELECTRICAL EQUIVALENCIES

- Electrical elements connected between the same pair of voltages
- Equivalencies can be used to simplify circuit branches connected in parallel
- Circuit elements connected in parallel share a common voltage drop across them

Electric Circuit Example 3

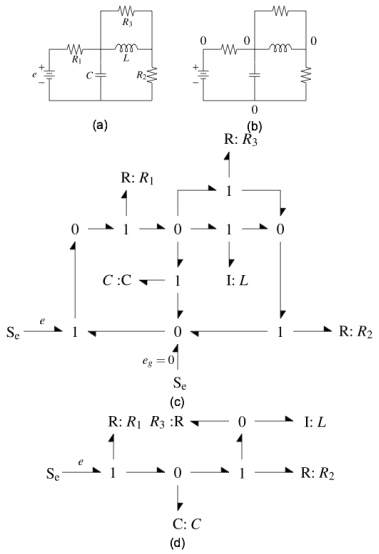


Figure 3.12

HYDRAULIC CIRCUITS

R-Element	Valve or surface roughness
C-Element	Accumulator
I-Element	Slug of fluid
Effort Source	Displacement pump or pressure source
Flow Source	Centrifugal pump or ideal flow source
Transformer	N/A
1-Junction	Common flow; Sum of pressure drops around a loop
0-Junction	Common pressure; Sum of flows into a junction

Hydraulic Circuit Example 1

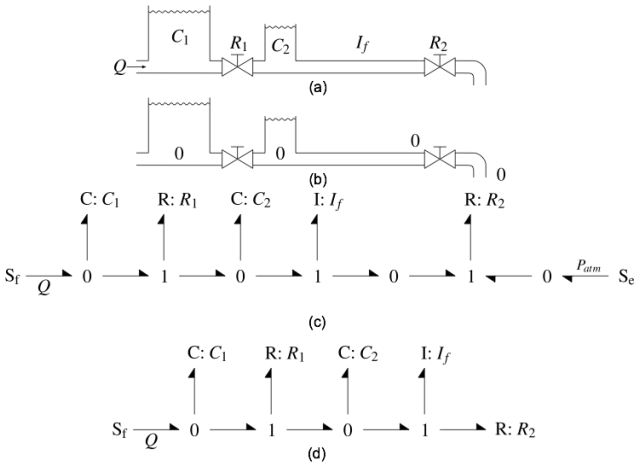


Figure 3.13

Hydraulic Circuit Example 2

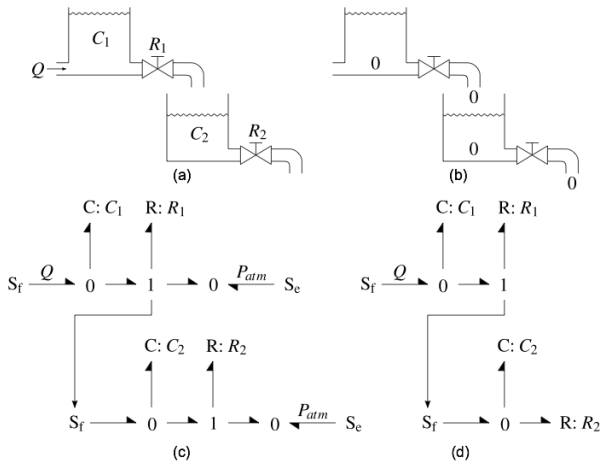
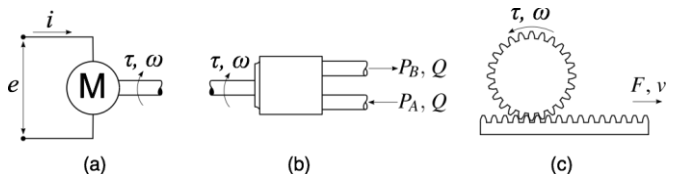


Figure 3.14



MIXED SYSTEMS

- Multiple energy domains that are coupled through transducers
- Procedure
  - Decompose into single energy domain subsystems at the transducers
  - Apply energy specific guidelines to each subsystem
  - Recouple using transducers

A Mixed System Example

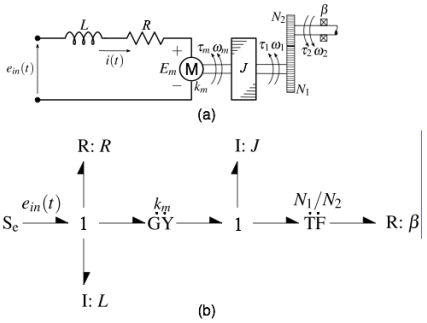
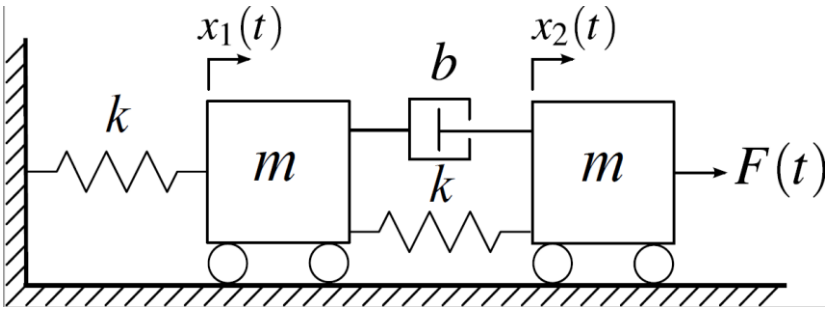


Figure 3.16

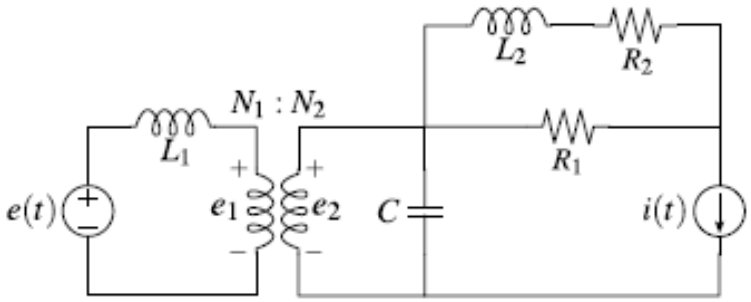
PRACTICE PROBLEM 1

- Synthesize the bond graph for the given system.



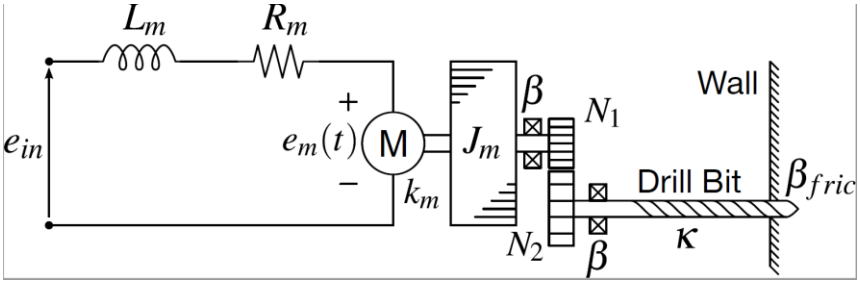
PRACTICE PROBLEM 2

- Synthesize the bond graph for the given system.



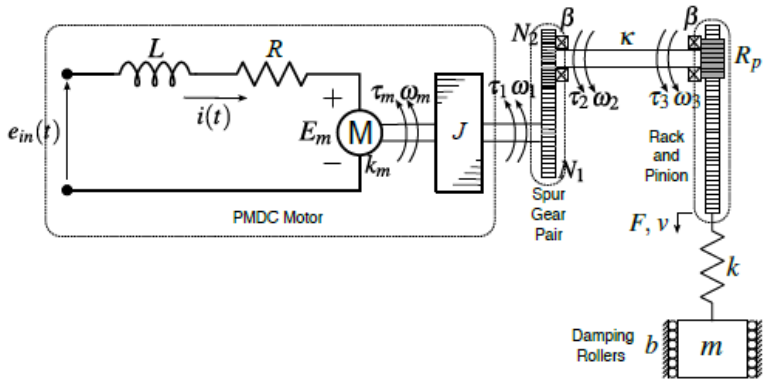
PRACTICE PROBLEM 3

- Synthesize the bond graph for the given system.



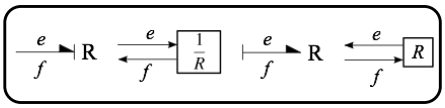
PRACTICE PROBLEM 4

- Synthesize the bond graph for the given system.

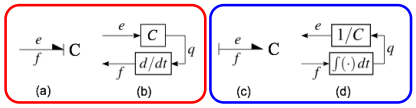


CAUSALITY FOR 1-PORTS

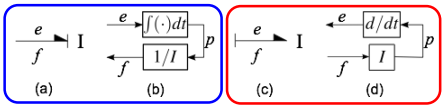
R-Element



C-Element



I-Element



Derivative Causality Integral Causality

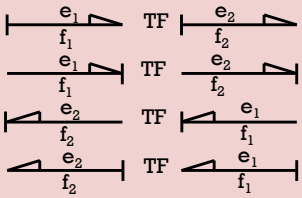
LABELS FOR 1-PORTS AFTER ASSIGNING INTEGRAL CAUSALITY

	Mechanical Translation		Mechanical Rotation		Electric Circuits		Hydraulic Circuits	
	Effort	Flow	Effort	Flow	Effort	Flow	Effort	Flow
C-Elements	kx	$\dot{x}$	$\kappa\Delta\theta$	$\dot{\Delta\theta}$	$\frac{q}{C}$	$\dot{q}$	$\frac{V}{C_f}$	$\dot{V}$
I-Elements	$\dot{p}$	$\frac{p}{m}$	$\dot{h}$	$\frac{h}{J}$	$\dot{\lambda}$	$\frac{\lambda}{L}$	$\dot{\Gamma}$	$\frac{\Gamma}{I_f}$

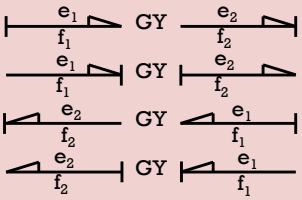
C - Elements: Displacement
I - Elements: Linear Momentum
Angle
Angular Momentum
Charge
Flux Linkage
Volume
Hydraulic Momentum

CAUSALITY FOR 2-PORTS

Transformer



Gyrator



STATE EQUATION DERIVATION

- Synthesize simplified system bond graph
- Assign causality
  - Sources first
  - Then energy-storing elements
  - If unspecified bond remains, select an R-element, assign causality, and propagate
- Label efforts and flows on energy storing elements
- Apply primary conditions
- Apply secondary condition

Mass-Spring-Damper Example

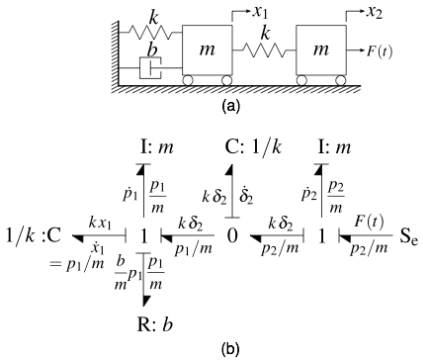
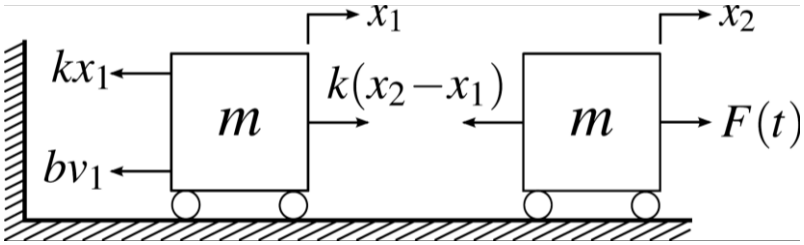


Figure 3.18

FREE BODY DIAGRAM FOR MASS-SPRING-DAMPER EXAMPLE



$\overset{+}{\rightarrow}\Sigma F_x = m\ddot{x}_1 = k(x_2 - x_1) - kx_1 - bv_1$

$\overset{+}{\rightarrow}\Sigma F_x = m\ddot{x}_2 = F(t) - k(x_2 - x_1)$

Mechanical Rotation Example

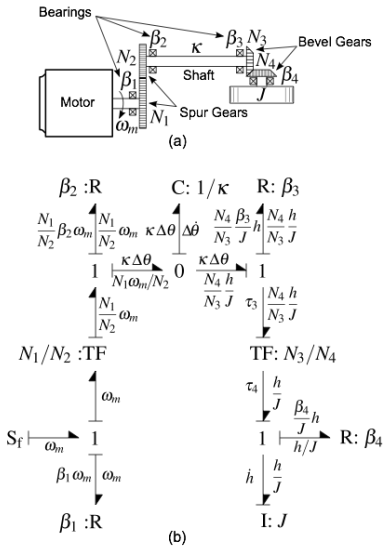
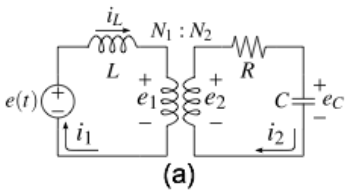
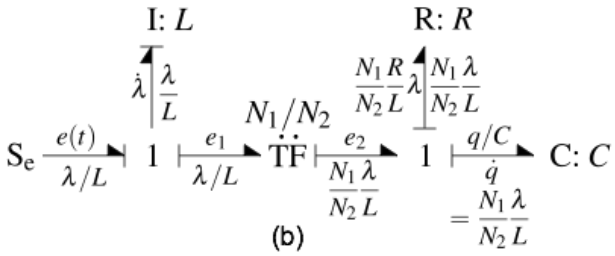


Figure 3.20

Electric Circuit Example



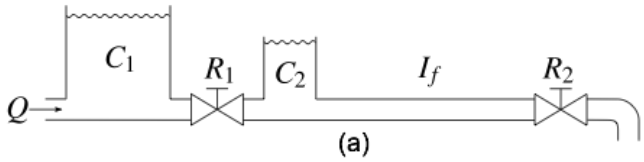
(a)



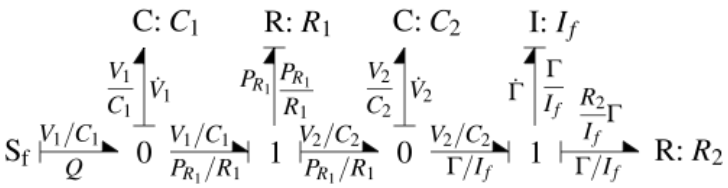
(b)

Figure 3.21

Hydraulic Circuit Example



(a)



(b)

Figure 3.22

Mixed System Example

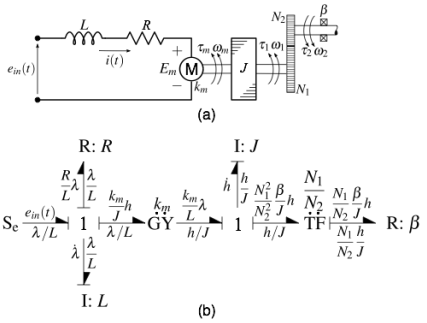


Figure 3.23

Algebraic Loops

- The mass-spring-damper system shown is a model of two railcars being pushed up against a snubber. What if the first railcar was a fully loaded coal car and the second an empty flatbed railcar?

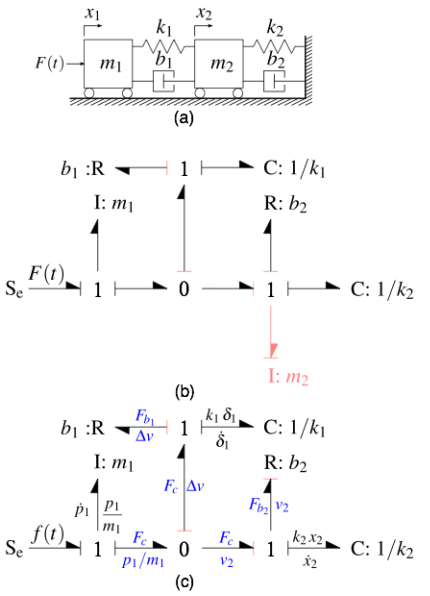
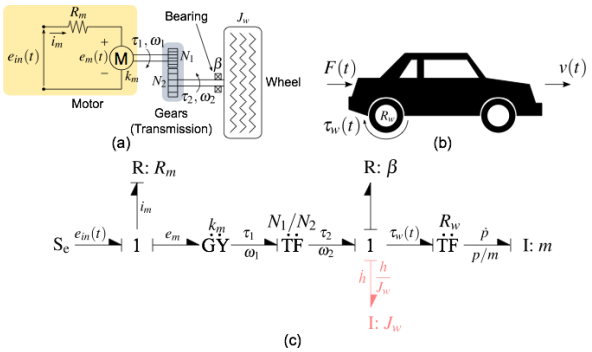
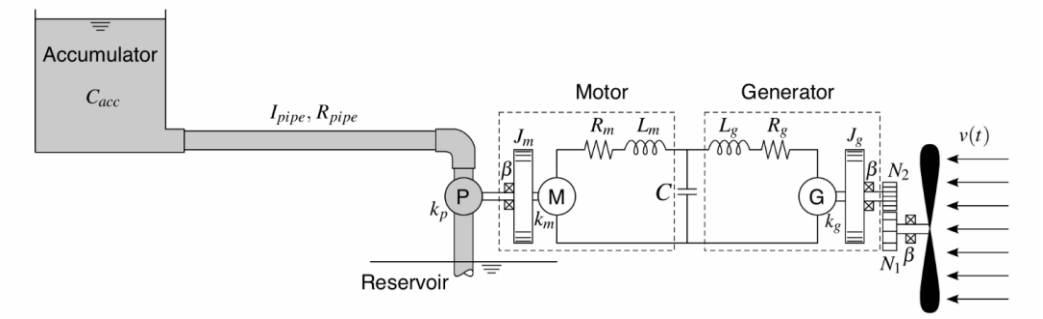


Figure 3.24

DERIVATIVE CAUSALITY

- Energy-storing elements in derivative causality are not dynamically independent, but rather dependent.





CHALLENGE PROBLEM

Synthesize a bond graph and derive the state equations of the following system.

Summary

- As illustrated in Figure 3.1 (a), generally, it is assumed that power flows from the system to energy-storing or dissipating elements.
- Usually, it is assumed that power flows from the source to the system. Moreover, effort sources supply effort as an input and flow sources supply flow inputs (refer to Figure 3.1 (b)).
- Transformers and gyrators have power through convention. As depicted in Figure 3.1 (c), the power goes in one port and out the other.
- Adjacent 0- or 1-junctions can be collapsed into a single junction. Common junction types adjacent to one another are in actuality the same junction and the attached bonds share a common effort or flow (Figure 3.2).
- When synthesizing bond graphs for mechanical systems, we first identify distinct velocities and establish 1-junctions. For each 1-junction we identify elements that are directly associated. For example, inertias are commonly associated with distinct velocities. Then we insert effort-generating 1-ports off of 0-junctions or 2-ports between appropriate pairs of 1-junctions. Next, we eliminate zero-velocity sources and simplify.

Summary Continued

- For circuits (both electric and hydraulic) we first identify distinct potentials (voltages or pressures) and establish 0-junctions. If there are any elements directly associated with these distinct efforts, we place them directly off the associated junction using a bond. We then insert the 1- and 2-ports between pairs of 0-junctions. The 1-ports are placed off of 1-junctions that are inserted between pairs of 0-junctions. Next, we eliminate the ground or reference pressure and simplify.
- Mixed systems can be dissected into subsystems, each of which is of a single energy domain. Each subsystem can be analyzed using the associated guidelines. The subsystems interface at energy-converting transducers which are modeled as either transformers or gyrators. Some examples were provided in Figure 3.15.
- When deriving differential equations from a bond graph one must first assign causality beginning with the sources, then the energy-storying elements, and last, if necessary, the R-elements. At each stage we as- sign the causality to an element and propagate if the causality affects adjacent junctions and/or elements. The process proceeds until all the bonds have an assigned causality. The differential equations result from applying the primary and secondary conditions at the junctions.
- Algebraic loops and derivative causality require extra analysis to derive the differential equations.