Chapter 1

Introduction

Holman / Heat Transfer, 10th Edition

Heat transfer is thermal energy transfer that is induced by a temperature difference (or gradient)

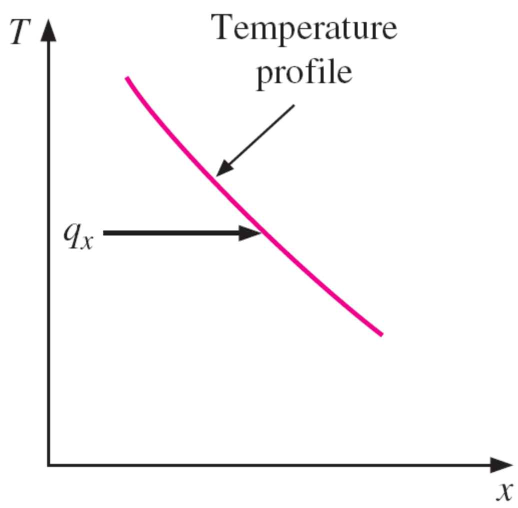
Modes of heat transfer

Conduction heat transfer: Occurs when a temperature gradient exists through a solid or a stationary fluid (liquid or gas).

Convection heat transfer: Occurs within a moving fluid, or between a solid surface and a moving fluid, when they are at different temperatures

Thermal radiation: Heat transfer between two surfaces (that are not in contact), often in the absence of an intervening medium.

1-1. Conduction Heat Transfer



$$\frac{q_x}{A} \sim \frac{\partial T}{\partial x}$$

When the proportionality constant is inserted,

$$q_x = -kA\frac{\partial T}{\partial x} \quad [1\text{-}1]$$

$$\text{Energy conducted in left face} + \text{heat generated within element}$$
$$= \text{change in internal energy} + \text{energy conducted out right face}$$

These energy quantities are given as follows:

$$\text{Energy in left face} = q_x = -kA\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$

$$\text{Change in internal energy} = \rho cA\frac{\partial T}{\partial \tau}dx$$

$$\text{Energy out right face} = q_{x+dx} = -kA\frac{\partial T}{\partial x}\bigg]_{x+dx}$$
$$= -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

where

$\dot{q} =$ energy generated per unit volume, W/m^3

$c =$ specific heat of material, $\text{J/kg}\cdot{}^\circ\text{C}$

$\rho =$ density, $\mathrm{kg/m^3}$

Combining the relations above gives

$$-kA\frac{\partial T}{\partial x} + \dot{q}A\,dx = \rho cA\frac{\partial T}{\partial \tau}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

or $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau} \qquad [1\text{-}2]$$

$$q_x + q_y + q_z + q_{\text{gen}} = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{dE}{d\tau}$$

$$q_x = -k\,dy\,dz\frac{\partial T}{\partial x}$$

$$q_{x+dx} = -\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dy\,dz$$

$$q_y = -k\,dx\,dz\frac{\partial T}{\partial y}$$

$$q_{y+dy} = -\left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dy\right]dx\,dz$$

$$q_z = -k\,dx\,dy\frac{\partial T}{\partial z}$$

$$q_{z+dz} = -\left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dz\right]dx\,dy$$

$$q_{\text{gen}} = \dot{q}\,dx\,dy\,dz$$

$$\frac{dE}{d\tau} = \rho c\,dx\,dy\,dz\frac{\partial T}{\partial \tau}$$

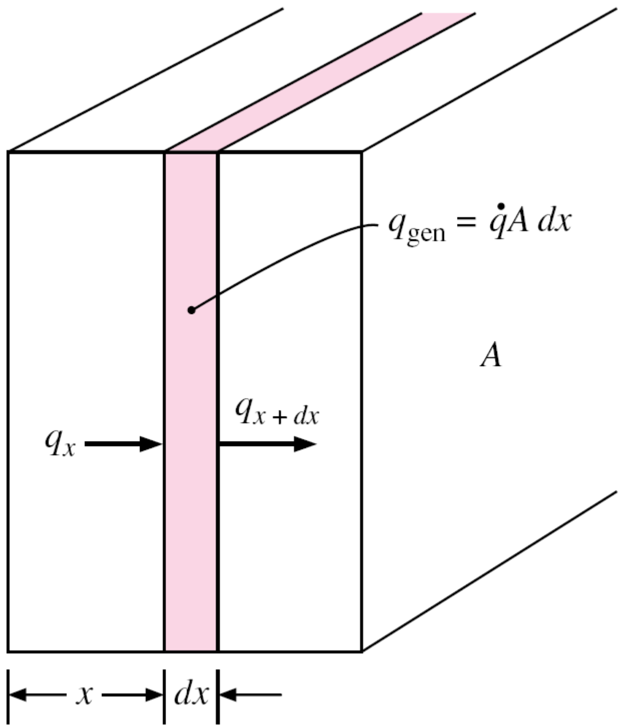


Figure 1-2 Elemental volume for one-dimensional heat conduction analysis.

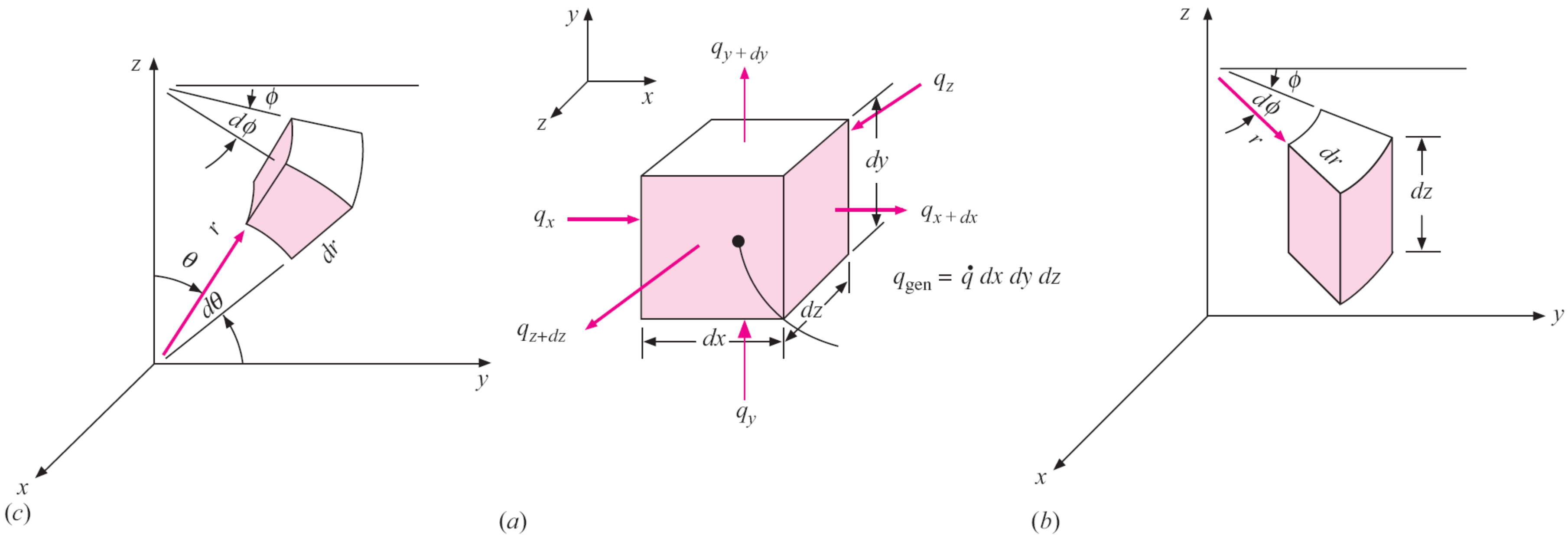


Figure 1-3 Elemental volume for three-dimensional heat-conduction analysis: (*a*) cartesian coordinates; (*b*) cylindrical coordinates; (*c*) spherical coordinates.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c\frac{\partial T}{\partial \tau} \quad [1\text{-}3]$$

For constant thermal conductivity, Equation (1-3) is written

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad [1-3\text{a}]$$

Equation (1-3a) may be transformed into either cylindrical or spherical coordinates by standard calculus techniques. The results are as follows:

Cylindrical coordinates:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad [1-3\text{b}]$$

Spherical coordinates:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial\tau} \qquad [1-3c]$$

Steady-state one-dimensional heat flow (no heat generation):

$$\frac{d^2T}{dx^2}=0 \qquad [1\text{-}4]$$

Steady-state one-dimensional heat flow in cylindrical coordinates (no heat generation):

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}=0 \qquad [1\text{-}5]$$

Steady-state one-dimensional heat flow with heat sources:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad [1\text{-}6]$$

Two-dimensional steady-state conduction without heat sources:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad [1\text{-}7]$$

1-2. Thermal Conductivity

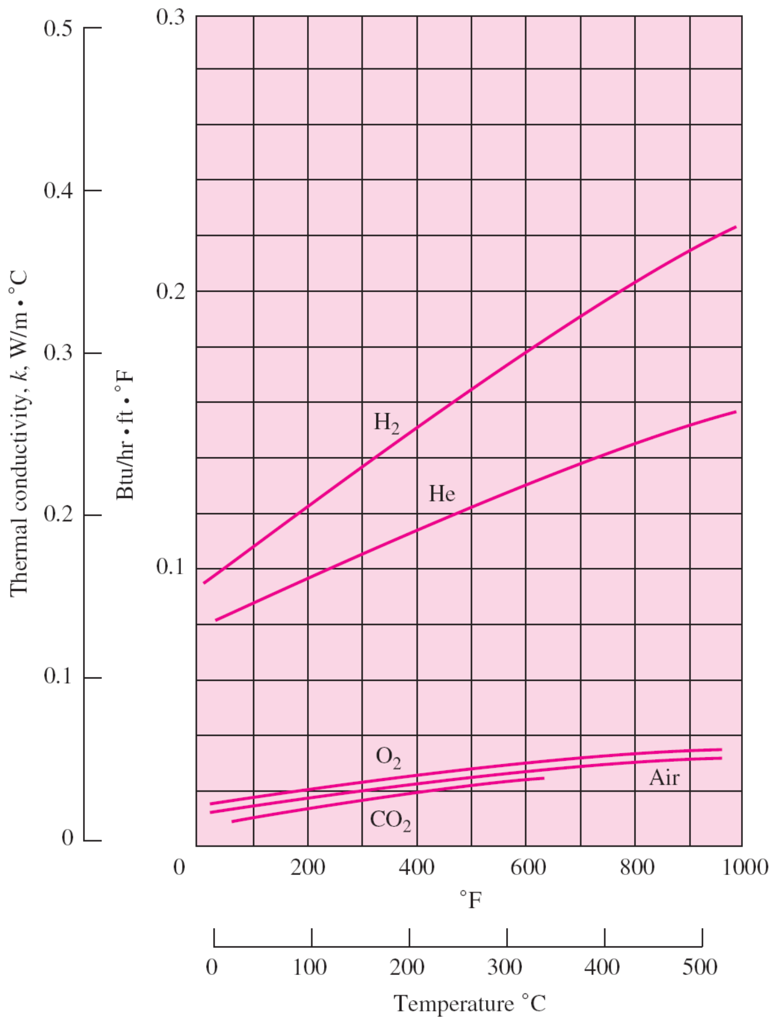


Figure 1-4 Thermal conductivities of some typical gases .

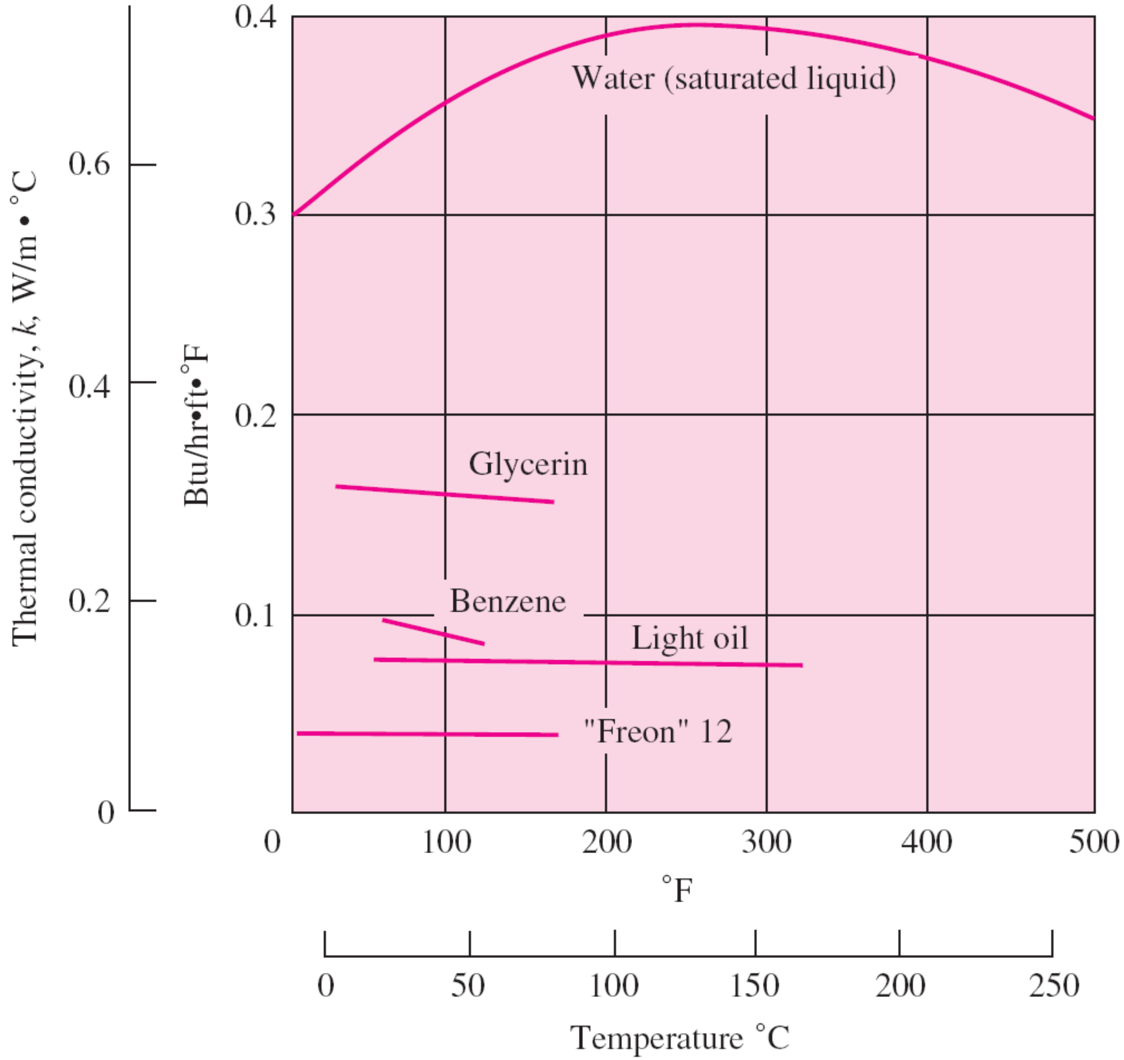


Figure 1-5 Thermal conductivities of some typical liquids.

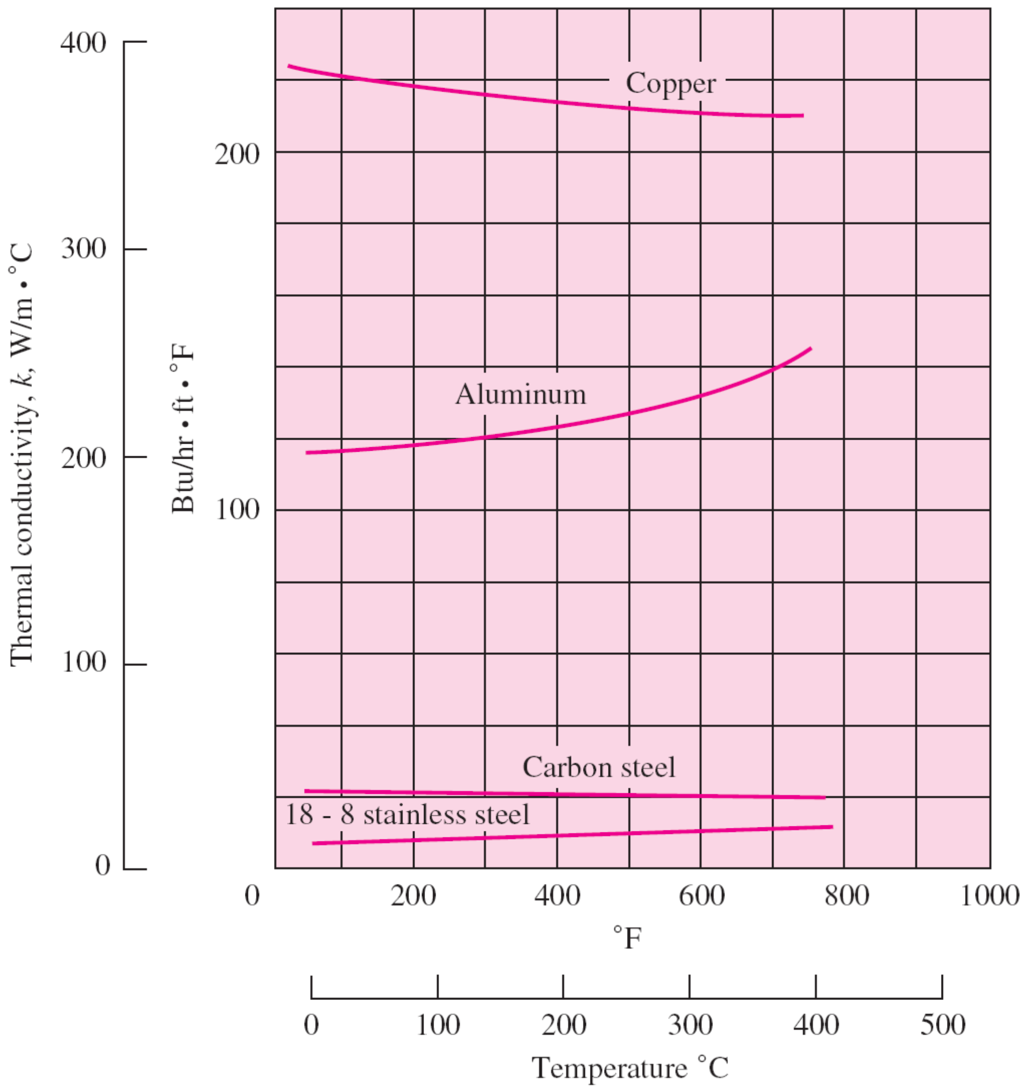


Figure 1-6 Thermal conductivities of some typical solids.

1-3. Convection Heat Transfer

To express the overall effect of convection, we use Newton's law of cooling:

$$q = hA(T_w - T_\infty) \quad [1\text{-}8]$$

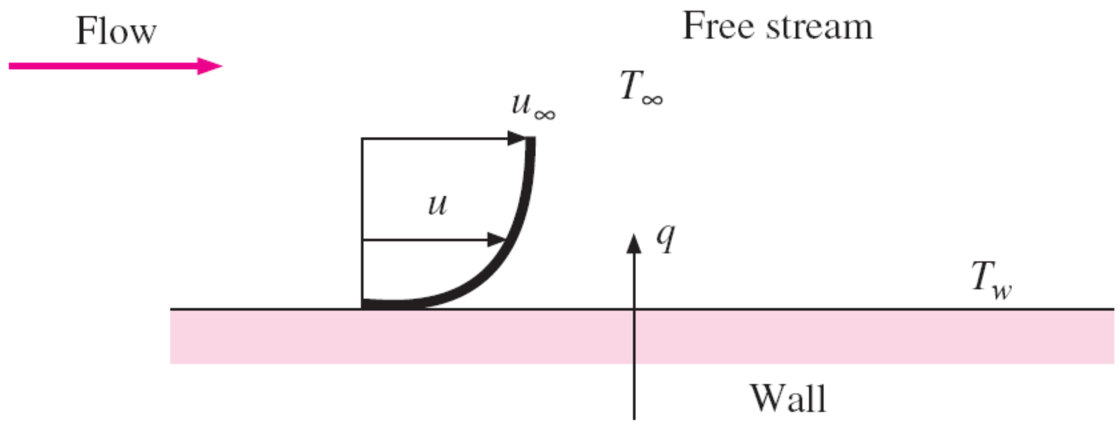


Figure 1-7 Convection heat transfer from a plate.

Convection Energy Balance on a Flow Channel

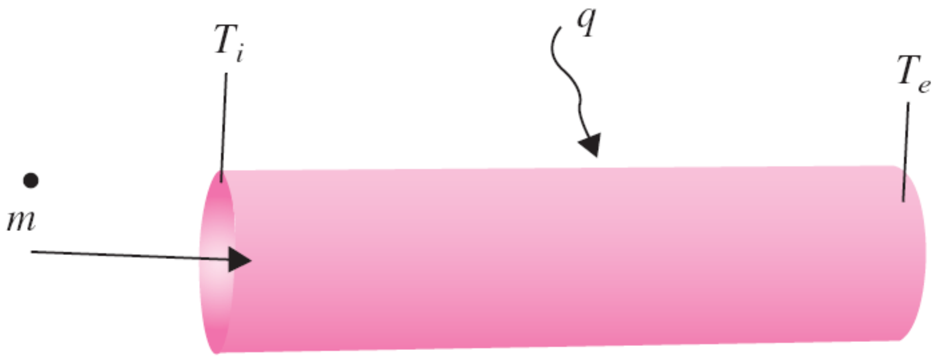


Figure 1-8 Convection in a channel.

$$q = \dot{m}(i_e - i_i)$$

$$q = \dot{m}c_p(T_e - T_i)$$

$$q = \dot{m}c_p(T_e - T_i) = hA(T_{w,\text{avg}} - T_{\text{fluid,avg}}) \qquad [1-8a]$$

$$\dot{m} = \rho u_{\text{mean}} A_c$$

1-4. Radiation Heat Transfer

$$q_{\text{emitted}} = \sigma A T^4 \qquad [1\text{-}9]$$

$$\frac{q_{\text{netexchange}}}{A} \propto \sigma (T_1^4 - T_2^4) \qquad [1\text{-}10]$$

$$q = F_\varepsilon F_G \sigma A (T_1^4 - T_2^4) \; [1\text{-}11]$$

Radiation in an Enclosure

$$q = \varepsilon_1 \sigma A_1 (T_1^4 - T_2^4) \qquad [1\text{-}12]$$

1-5. Dimensions and Units

$$L = \text{length}$$
$$\mathrm{M} = \text{mass}$$
$$\mathrm{F} = \text{force}$$
$$\tau = \text{time}$$
$$\mathrm{T} = \text{temperature}$$

For example, Newton's second law of motion may be written

Force ~ time rate of change of momentum

$$F = k\frac{d(mv)}{d\tau}$$

$$F = kma \qquad [1\text{-}13]$$

$$F = \frac{1}{g_c}ma \qquad [1\text{-}14]$$

Some typical systems of units are .

1. 1-pound force will accelerate a 1-lb mass 32.17 $\mathrm{ft/s^2}$.
2. 1-pound force will accelerate a 1-slug mass 1 $\mathrm{ft/s^2}$.
3. 1-dyne force will accelerate a 1-g mass 1 $\mathrm{cm/s^2}$.
4. 1-newton force will accelerate a 1-kg mass 1 $\mathrm{m/s^2}$.
5. 1-kilogram force will accelerate a 1-kg mass 9.806 $\mathrm{m/s^2}$.

1. $g_c = 32.17\,\mathrm{lb}_m \cdot \mathrm{ft/lb}_f \cdot \mathrm{s}^2$
2. $g_c = 1\,\mathrm{slug} \cdot \mathrm{ft/lb}_f \cdot \mathrm{s}^2$
3. $g_c = 1\,\mathrm{g} \cdot \mathrm{cm/dyn} \cdot \mathrm{s}^2$
4. $g_c = 1\,\mathrm{kg} \cdot \mathrm{m/N} \cdot \mathrm{s}^2$
5. $g_c = 9.806\,\mathrm{kg}_m \cdot \mathrm{m/kg}_f \cdot \mathrm{s}^2$

1. $\mathrm{lb}_f \cdot \mathrm{ft}$
2. $\mathrm{lb}_f \cdot \mathrm{ft}$
3. $\mathrm{dyn} \cdot \mathrm{cm} = 1\mathrm{erg}$
4. $\mathrm{N} \cdot \mathrm{m} = 1\,\mathrm{joule(J)}$
5. $\mathrm{kg}_f \cdot \mathrm{m} = 9.806\mathrm{J}$

In addition, we may use the units of energy that are based on thermal phenomena:

1 Btu will raise 1 lb_m of water $1^\circ\mathrm{F}$ at $68^\circ\mathrm{F}$.

1 cal will raise 1 g of water $1^\circ\mathrm{C}$ at $20^\circ\mathrm{C}$.

1 kcal will raise 1 kg of water $1^\circ\mathrm{C}$ at $20^\circ\mathrm{C}$.

Some conversion factors for the various units of work and energy are

$$1\,\text{Btu} = 778.16\,\text{lb}_f \cdot \text{ft}$$

$$1\,\text{Btu} = 1055\,\text{J}$$

$$1\,\text{kcal} = 4182\,\text{J}$$

$$1\,\text{lb}_f \cdot \text{ft} = 1.356\,\text{J}$$

$$1\,\text{Btu} = 252\,\text{cal}$$

$$W = \frac{g}{g_c} m \qquad [1\text{-}15]$$

$$k \text{ in W/m} \cdot {}^\circ\text{C}$$

$$h \text{ in W/m}^2 \cdot {}^\circ\text{C}$$

$$1\,\text{N} = 1\,\text{kg} \cdot \text{m/s}^2 \qquad [1\text{-}16]$$

Example 1-1 Conduction Through Copper Plate

One face of a copper plate 3 cm thick is maintained at $400^\circ\mathrm{C}$, and the other face is maintained at $100^\circ\mathrm{C}$. How much heat is transferred through the plate?

Solution

From Appendix A, the thermal conductivity for copper is 370 $\mathrm{W/m\cdot{}^\circ C}$ at $250^\circ\mathrm{C}$. From Fourier's law

$$\frac{q}{A} = -k\frac{dT}{dx}$$

Integrating gives

$$\frac{q}{A} = -k\frac{\Delta T}{\Delta x} = \frac{-(370)(100-400)}{3\times10^{-2}} = 3.7\,\mathrm{MW/m^2}[1.173\times10^6\,\mathrm{Btu/h\cdot ft^2}]$$

Example 1-2 Convection Calculation

Air at $20^\circ\mathrm{C}$ blows over a hot plate 50 by 75 cm maintained at $250^\circ\mathrm{C}$. The convection heat-transfer coefficient is 25 $\mathrm{W/m^2\cdot{}^\circ C}$. Calculate the heat transfer.

Solution

From Newton's law of cooling

$$
\begin{aligned}
q &= hA(T_w - T_\infty) \\
&= (25)(0.50)(0.75)(250-20) \\
&= 2.156\mathrm{kW}[7356\mathrm{Btu/h}]
\end{aligned}
$$

Example 1-3 Multimode Heat Transfer

Assuming that the plate in Example 1-2 is made of carbon steel (1%) 2 cm thick and that 300 W is lost from the plate surface by radiation, calculate the inside plate temperature.

Solution

The heat conducted through the plate must be equal to the sum of convection and radiation heat losses:

$$q_{\text{cond}} = q_{\text{conv}} + q_{\text{rad}}$$

$$-kA\frac{\Delta T}{\Delta x}=2.156+0.3=2.456\text{kW}$$

$$\Delta \text{T} = \frac{(-2456)(0.02)}{(0.5)(0.75)(43)} = -3.05^{\circ}\text{C}[-5.49^{\circ}\text{F}]$$

where the value of k is taken from Table 1-1. The inside plate temperature is therefore

$$T_i = 250 + 3.05 = 253.05^{\circ}\text{C}$$

Example 1-4 Heat Source and Convection

An electric current is passed through a wire 1 mm in diameter and 10 cm long. The wire is submerged in liquid water at atmospheric pressure, and the current is increased until the water boils. For this situation $h = 5000\,\mathrm{W/m^2 \cdot {}^\circ C}$, and the water temperature will be $100^\circ\mathrm{C}$. How much electric power must be supplied to the wire to maintain the wire surface at $114^\circ\mathrm{C}$?

Solution

The total convection loss is given by Equation (1-8): $q = hA(T_w - T_\infty)$

For this problem the surface area of the wire is

$$A = \pi dL = \pi(1\times10^{-3})(10\times10^{-2}) = 3.142\times10^{-4}\,\mathrm{m^2}$$

The heat transfer is therefore

$$q = (5000\,\mathrm{W/m^2\cdot{}^\circ C})(3.142\times10^{-4}\,\mathrm{m^2})(114-100) = 21.99\,\mathrm{W}\,[75.03\,\mathrm{Btu/h}]$$ and this is equal to the electric power that must be applied.

Example 1-5 Radiation Heat Transfer

Two infinite black plates at $800^\circ\mathrm{C}$ and $300^\circ\mathrm{C}$ exchange heat by radiation. Calculate the heat transfer per unit area.

Solution

Equation (1-10) may be employed for this problem, so we find immediately

$$q/A = \sigma(T_1^4 - T_2^4)$$

$$= (5.669\times10^{-8})(1073^4 - 573^4)$$

$$= 69.03\mathrm{kW/m^2}[21{,}884\mathrm{Btu/h\cdot ft^2}]$$

Example 1-6 Total Heat Loss by Convection and Radiation

A horizontal steel pipe having a diameter of 5 cm is maintained at a temperature of $50^\circ\mathrm{C}$ in a large room where the air and wall temperature are at $20^\circ\mathrm{C}$. The surface emissivity of the steel may be taken as 0.8. Using the data of Table 1-3, calculate the total heat lost by the pipe per unit length.

Solution

The total heat loss is the sum of convection and radiation. From Table 1-3 we see that an estimate for the heat-transfer coefficient for *free* convection with this geometry and air is $h = 6.5 \mathrm{W/m^2 \cdot {}^\circ C}$. The surface area is πdL, so the convection loss per unit length is

$$q/L]_{\mathrm{conv}} = h(\pi d)(T_w - T_\infty)$$

$$= (6.5)(\pi)(0.05)(50-20) = 30.63 \mathrm{W/m}$$

The pipe is a body surrounded by a large enclosure so the radiation heat transfer can be calculated from Equation (1-12).

With $T_1 = 50^\circ\mathrm{C} = 323^\circ\mathrm{K}$ and $T_2 = 20^\circ\mathrm{C} = 293^\circ\mathrm{K}$, we have

$$
\begin{aligned}
q/L]_{\text{rad}} &= \varepsilon_1(\pi d_1)\sigma(T_1^4 - T_2^4) \\
&= (0.8)(\pi)(0.05)(5.669\times10^{-8})(323^4 - 293^4) \\
&= 25.04\,\mathrm{W/m}
\end{aligned}
$$

The total heat loss is therefore

$$
\begin{aligned}
q/L]_{\text{tot}} &= q/L]_{\text{conv}} + q/L]_{\text{rad}} \\
&= 30.63 + 25.04 = 55.67\,\mathrm{W/m}
\end{aligned}
$$

In this example we see that the convection and radiation are about the same. To neglect either would be a serious mistake.

1-6. Summary

$$-kA\frac{dT}{dy}\bigg]_{\text{wall}} = hA(T_w - T_\infty) + F_\varepsilon F_G \sigma A(T_w^4 - T_s^4)$$

$T_s =$ temperature of surroundings

$T_w =$ surface temperature

$T_\infty =$ fluid temperature

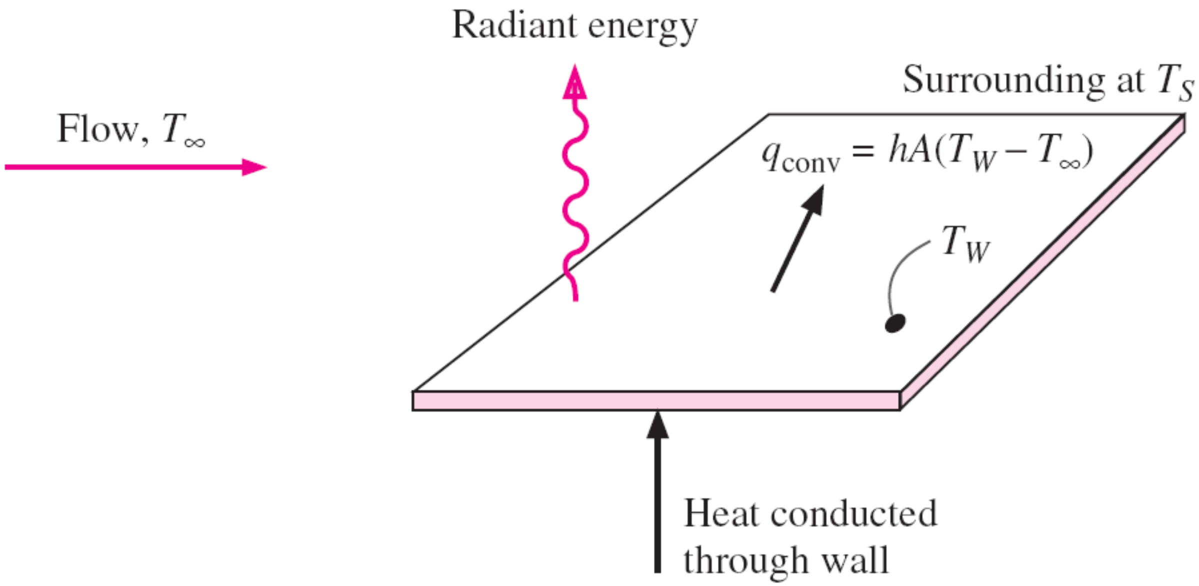


Figure 1-9 Combination of conduction, convection, and radiation heat transfer.