



CCGPS • 4th Grade Math Content Standards Unpacked

This document is an instructional support tool. It is adapted from documents created by the Ohio Department of Education and the North Carolina Department of Public Instruction for the Common Core State Standards in Mathematics.

There are no transition standards for 4th grade mathematics for the 2012-2013 school year.

Frequently asked questions

What is the purpose of this document? To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do.

What is in the document? Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.

How do I send feedback? The explanations and examples in this document are intended to be helpful and specific. As this document is used, however, teachers and educators will find ways in which the unpacking can be improved and made more useful. Please send feedback to lynn.skinner@cowetaschools.org. Your input will be used to refine the unpacking of the standards.

Just want the standards alone? You can find the CCGPS standards for your grade band at www.georgiastandards.org.

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Operations and Algebraic Thinking

CCGPS.4.OA

CCGPS Cluster: Use the four operations with whole numbers to solve problems.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, addition/add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental computation, estimation, rounding.**

Instructional Strategies

Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in [Table 2](#). They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2).

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are

needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Examples of multistep word problems can be accessed from the released questions on the NAEP (National Assessment of Educational Progress) Assessment at <http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx>.

For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, “Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?”

Instructional Resources/Tools

- [Table 2](#): Common multiplication and division situations
- National Assessment of Educational Progress (NAEP) Assessments: <http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx>

Common Misconceptions

Connections – Critical Areas of Focus

This cluster is connected to the first [Critical Area of Focus](#) for Grade 4, **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.**

Connections to Other Grade Levels

Represent and solve problems involving multiplication and division (Grade 3 OA 3).

Solve problems involving the four operations, and identify and explain patterns in arithmetic (Grade 3 OA 8).

CCGPS

What does this standard mean that a student will know and be able to do?

CCGPS.4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative

A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., “*a* is *n* times as much as *b*”). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

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<p>comparisons as multiplication equations.</p>	<p>Examples:</p> <ul style="list-style-type: none"> • $5 \times 8 = 40$: Sally is five years old. Her mom is eight times older. How old is Sally’s Mom? • $5 \times 5 = 25$: Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?
<p>CCGPS.4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</p>	<p>This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Refer Table 2, included at the end of this document, for more examples.</p> <p>Examples:</p> <ul style="list-style-type: none"> • Unknown Product: A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? ($3 \times 6 = p$) • Group Size Unknown: A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? ($18 \div p = 3$ or $3 \times p = 18$) • Number of Groups Unknown: A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? ($18 \div 6 = p$ or $6 \times p = 18$) <p>When distinguishing multiplicative comparison from additive comparison, students should note the following.</p> <ul style="list-style-type: none"> • Additive comparisons focus on the difference between two quantities. <ul style="list-style-type: none"> ○ For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have? ○ A simple way to remember this is, “How many more?” • Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other. <ul style="list-style-type: none"> ○ For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run? ○ A simple way to remember this is “How many times as much?” or “How many times as many?”
<p>CCGPS.4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>	<p>The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.</p> <p>Example 1: On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?</p> <p>Some typical estimation strategies for this problem are shown on the next page.</p>

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Student 1

I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2

I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.

Student 3

I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1

First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.

Student 2

First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. $40 + 20 = 60$. $300 - 60 = 240$, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:

Write different word problems involving $44 \div 6 = ?$ where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7 \frac{2}{6}$

Possible solutions:

- **Problem A: 7.**
Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? $44 \div 6 = p$; $p = 7 \text{ r } 2$. *Mary can fill 7 pouches completely.*
- **Problem B: 7 r 2.**
Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *Mary can fill 7 pouches and have 2 left over.*
- **Problem C: 8.**
Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *Mary can needs 8 pouches to hold all of the pencils.*
- **Problem D: 7 or 8.**
Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? $44 \div 6 = p$; $p = 7 \text{ r } 2$; *Some of her friends received 7 pencils. Two friends received 8 pencils.*
- **Problem E: $7 \frac{2}{6}$.**
Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? $44 \div 6 = p$; $p = 7 \frac{2}{6}$

Example:

There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed?
($128 \div 30 = b$; $b = 4 \text{ R } 8$; *They will need 5 buses because 4 busses would not hold all of the students.*)

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)

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| | <ul style="list-style-type: none">• Clustering around an average (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)• Rounding and adjusting (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)• Using friendly or compatible numbers such as factors (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)• Using benchmark numbers that are easy to compute (Students select close whole numbers for fractions or decimals to determine an estimate.) |
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CCGPS Cluster: Gain familiarity with factors and multiples.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite.**

Instructional Strategies

Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

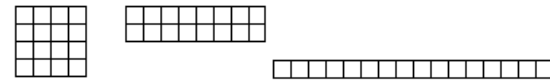
Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12, ... are composite. Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number. They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.

Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers.

Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:



The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example, 5 x 8 is 40; the table below shows the multiples of each factor.

5	10	15	20	25	30	35	40	45
8	16	24	32	40	48	56	64	72

Ask students what they notice about the number 40 in each set of multiples; 40 is the 8th multiple of 5, and the 5th multiple of 8.

Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.

Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

Instructional Resources/Tools

- Calculators
- Counters
- Grid papers
- [Factor Game](#): This activity engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions

Common Misconceptions

- When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.
- Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.
- Some students may need to start with numbers that have only one pair of

<p>of prime and composite numbers.</p> <ul style="list-style-type: none"> • Understanding factoring through geometry: Using square unit tiles, students work with a partner to construct all rectangles whose area is equal to a given number. After several examples, students see that prime numbers are associated with exactly two rectangles, whereas composite numbers are associated with more than two rectangles. • The Product Game: Classifying Numbers. Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the Product Game. • The Product Game: In the Product Game, students start with factors and multiply to find the product. In The Factor Game, students start with a number and find its factors. • Multiplication: It's in the Cards – More Patterns with Products: Activity from NCTM • Sieve of Eratosthenes: This activity relates number patterns with visual patterns. Click on the link for <i>Activities</i> for directions on engaging students in finding all prime numbers 1-100. 	<p>factors, then those with two pairs of factors before finding factors of numbers with several factor pairs.</p>
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Connections – Critical Areas of Focus	Connections to Other Grade Levels
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<p>This cluster is connected to the first Critical Area of Focus for Grade 4, Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.</p>	<ul style="list-style-type: none"> • Understand properties of multiplication and the relationship between multiplication and division (Grade 3 OA 5 – 6). • Geometric measurement: understand concepts of area and relate area to multiplication and to addition (Grade 3 MD 7a). • The concepts of prime, factor and multiple are important in the study of relationships found among the natural numbers. • Compute fluently with multi-digit numbers and find common factors and multiples (Grade 6 NS 4).
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CCGPS	What does this standard mean that a student will know and be able to do?
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<p>CCGPS.4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p>	<p>This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.</p> <p>A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.</p> <p>Prime vs. Composite:</p> <ul style="list-style-type: none"> • A prime number is a number greater than 1 that has only 2 factors, 1 and itself. • Composite numbers have more than 2 factors.
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Students investigate whether numbers are prime or composite by

- Building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, 1×7 and 7×1 , therefore it is a prime number).
- Finding factors of the number.

Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example:

Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5, ... , 24
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
3, 6, 9, 12, 15, 18, 21, 24
4, 8, 12, 16, 20, 24
8, 16, 24
12, 24
24

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- All even numbers are multiples of 2.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4.
- All numbers ending in 0 or 5 are multiples of 5.

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CCGPS Cluster: Generate and analyze patterns.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **pattern (number or shape), pattern rule.**

Instructional Strategies

In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

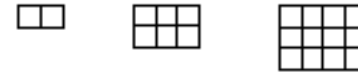
Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to “undo” the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, “Add 4” starting with the number 1, the pattern 1, 5, 9, 13, 17, ... is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, “How is

a number in the sequence related to the one that came before it?”, and “If they started at the end of the pattern, will this relationship be the same?” Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example,



Students may state that the rule is to multiply the previous number of square by 3.

Instructional Resources/Tools

- [Snake Patterns –s-s-s](#): Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.
- [Patterns that Grow: Growing Patterns](#): Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal’s triangle.
- [Patterns that Grow – Exploring Other Number Patterns](#): Students analyze numeric patterns, including Fibonacci numbers. They also describe numeric patterns and then record them in table form.
- [Patterns that Grow – Looking Back and Moving Forward](#): In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.

Common Misconceptions

Connections – Critical Areas of Focus

This cluster goes beyond the Critical Areas of Focus for Grade 4 to address **Analyzing patterns.**

Connections to Other Grade Levels

Solve problems involving the four operations, and identify and explain patterns in arithmetic (CCGPS.3.OA.4).

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CCGPS

What does this standard mean that a student will know and be able to do?

CCGPS.4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23, 28, ...	Start with 3; add 5	The numbers alternately end with a 3 or an 8
5, 10, 15, 20, ...	Start with 5; add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that 3rd with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:

Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers ($3 - 1 = 2$, $9 - 3 = 6$, $27 - 9 = 18$, etc.).

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:

There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	Operation	Beans
0	$3 \times 0 + 4$	4
1	$3 \times 1 + 4$	7
2	$3 \times 2 + 4$	10
3	$3 \times 3 + 4$	13
4	$3 \times 4 + 4$	16
5	$3 \times 5 + 4$	19

CCGPS Cluster: Generalize place value understanding for multi-digit whole numbers.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **place value, greater than, less than, equal to, <, >, =, comparisons/compare, round.**

Instructional Strategies

Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits, e.g., compare 453, 698 and 215; numbers that have the same number in the leading digit position, e.g., compare 45, 495 and 41,223; and numbers that have different numbers of digits and different leading digits, e.g., compare 312, 95, 5245 and 10,002.

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit, e.g., round 23,960 to the nearest hundred. This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of; “Is 48 closer to 40 or 50?” Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit

Instructional Resources/Tools

- Place value boxes
- Place value flip charts
- Number cards

Common Misconceptions

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.

Students need to be aware of the greatest place value. In this example, there is one number with the lead digit in the thousands and another number with its lead digit in the hundreds.

Development of a clear understanding of the value of the digits in a number is critical for the understanding of and using numbers in computations. Helping students build the understanding that 12345 means one ten thousand or 10,000,

¹ Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

	two thousands or 2000, three hundreds or 300, four tens or 40, and 5 ones or 5. Additionally, the answer is the sum of each of these values $10,000 + 2000 + 300 + 40 + 5$.
Connections – Critical Areas of Focus	Connections to Other Grade Levels
This cluster is connected to the first Critical Area of Focus for Grade 4, Developing an understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends.	A strong foundation in whole-number place value and rounding is critical for the expansion to decimal place value and decimal rounding. Understand place value (CCGPS.2.NBT.1, CCGPS.2.NBT.2, CCGPS.2.NBT.3, CCGPS.2.NBT.4). Use place value understanding and properties of operations to perform multi-digit arithmetic (CCGPS.3.NBT.1).
CCGPS	What does this standard mean that a student will know and be able to do? Return to Contents
CCGPS.4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i>	This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with. Example: How is the 2 in the number 582 similar to and different from the 2 in the number 528?
CCGPS.4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.	This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is $285 = 200 + 80 + 5$. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. Students should also be able to compare two multi-digit whole numbers using appropriate symbols.
CCGPS.4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.	This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Example: Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

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Student 1

First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.

Student 2

First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. $40 + 20 = 60$. $300 - 60 = 240$, so we need about 240 more bottles.

Example:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did your family travel?

Some typical estimation strategies for this problem:

Student 1

I first thought about 276 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2

I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.

Student 3

I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

Example:

Round 368 to the nearest hundred.

This will either be 300 or 400, since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400.



CCGPS Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, <, >, =, benchmark fraction.**

Instructional Strategies

A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand.

It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtraction algorithms. Give students many opportunities to talk with classmates about how

they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers, provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

Use a variation of an area model. For example, to multiply 23×36 , arrange the partial products as follows:

	20+3	
30 + 6	600	90
	120	18

Then add the four partial products to get 828.

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients can be viewed at <http://www.teachertube.com>, search for *Outline of partial quotients*. This strategy will help them understand the division algorithm.

² Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Students will have a better understanding of multiplication or division when problems are presented in context.

Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication.

To give students an opportunity to communicate their understandings of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class.

Vocabulary is important. Students should have an understanding of terms such as, sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

Instructional Resources/Tools	Common Misconceptions
<ul style="list-style-type: none"> • Place-value mats • Bound place value flip books (so that the digit in a certain place can be switched) • Base ten blocks • Tens frames • Hundreds flats • Smartboard • Make literature connections by using the resource book, <u>Read Any Good Math Lately?</u>, to identify books related to certain math topics. Books can provide a 'hook' for learning, to activate background knowledge, and to build student interest. 	<p>Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.</p> <p>If students are having difficulty with lining up similar place values in numbers as they are adding and subtracting, it is sometimes helpful to have them write their calculations on grid paper. This assists the student with lining up the numbers more accurately.</p> <p>If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the 'shorthand' of 'carrying,' students could add by place value, moving left to right placing the answers down below the 'equals' line. For example:</p> $ \begin{array}{r} 249 \\ + 372 \\ \hline 500 \\ 110 \\ \hline 11 \\ \hline 621 \end{array} $ <p>(Start with $200 + 300$ to get the 500, then $40 + 70$ to get 110, and $9 + 2$ to get 11.)</p>
Connections – Critical Areas of Focus	Connections to Other Grade Levels
<p>This Cluster is connected to the first Critical Areas of Focus for Grade 4, Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends, and goes beyond to address Adding and subtracting multi-digit whole numbers.</p>	<ul style="list-style-type: none"> • Use place value understanding and properties of operations to perform multi-digit arithmetic (CCGPS.3.NBT.2, CCGPS.3.NBT.3). • Use the four operations with whole numbers to solve problems (CCGPS.4.OA.2, CCGPS.4.OA.3). • Generalize place value understanding for multi-digit whole numbers (CCGPS.4.NBT.1, CCGPS.4.NBT.2).

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CCGPS	What does this standard mean a student will know and be able to do?
<p>CCGPS.4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p>Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.</p> <p>This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.</p> <p>When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.</p> <p>Example: $\begin{array}{r} 3892 \\ + 1567 \\ \hline \end{array}$</p> <p>Student explanation for this problem:</p> <ol style="list-style-type: none"> Two ones plus seven ones is nine ones. Nine tens plus six tens is 15 tens. I am going to write down five tens and think of the 10 tens as one more hundred. (<i>Denotes with a 1 above the hundreds column</i>) Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (<i>Denotes with a 1 above the thousands column</i>) Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand. <p>Example: $\begin{array}{r} 3546 \\ - 928 \\ \hline \end{array}$</p> <p>Student explanations for this problem:</p> <ol style="list-style-type: none"> There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (<i>Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.</i>) Sixteen ones minus 8 ones is 8 ones. (<i>Writes an 8 in the ones column of answer.</i>) Three tens minus 2 tens is one ten. (<i>Writes a 1 in the tens column of answer.</i>) There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (<i>Marks through the 3 and notates with a 2 above it. Writes down a 1 above the hundreds column.</i>) Now I have 2 thousand and 15 hundreds. Fifteen hundreds minus 9 hundreds is 6 hundreds. (<i>Writes a 6 in the hundreds column of the answer.</i>) I have 2 thousands left since I did not have to take away any thousands. (<i>Writes 2 in the thousands place of answer.</i>)

Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

CCGPS.4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. Use of the standard algorithm for multiplication is an expectation in the 5th grade.

This standard calls for students to multiply numbers using a variety of strategies.

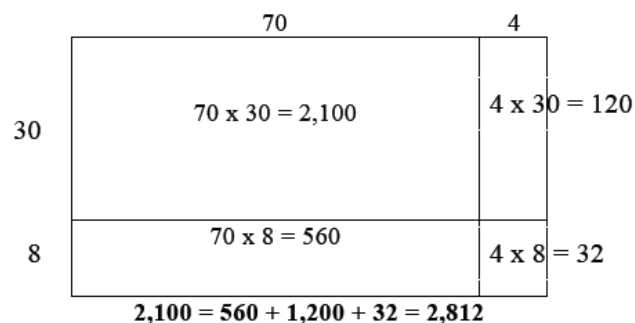
Example:

There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

<p>Student 1</p> 25×12 <p>I broke 12 up into 10 and 2.</p> $25 \times 10 = 250$ $25 \times 2 = 50$ $250 + 50 = 300$	<p>Student 2</p> 25×12 <p>I broke 25 into 5 groups of 5.</p> $5 \times 12 = 60$ <p>I have 5 groups of 5 in 25.</p> $60 \times 5 = 300$	<p>Student 3</p> 25×12 <p>I doubled 25 and cut 12 in half to get 50×6.</p> $50 \times 6 = 300$
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Example:

What would an array area model of 74×38 look like?

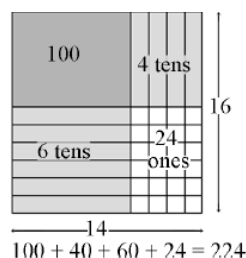


Examples:

To illustrate 154×6 , students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property,

$$\begin{aligned} 154 \times 6 &= (100 + 50 + 4) \times 6 \\ &= (100 \times 6) + (50 \times 6) + (4 \times 6) \\ &= 600 + 300 + 24 = 924. \end{aligned}$$

The area model below shows the partial products for $14 \times 16 = 224$.



Using the area model, students first verbalize their understanding:

- 10×10 is 100
- 4×10 is 40
- 10×6 is 60, and
- 4×6 is 24.

Students use different strategies to record this type of thinking.

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

$$\begin{array}{r} 25 \\ \times 24 \\ \hline 400 \text{ (} 20 \times 20 \text{)} \\ 100 \text{ (} 20 \times 5 \text{)} \\ 80 \text{ (} 4 \times 20 \text{)} \\ \underline{20 \text{ (} 4 \times 5 \text{)}} \\ 600 \end{array}$$

CCGPS.4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example:

A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** $260 \div 4 = (200 \div 4) + (60 \div 4)$
- **Using Multiplication:** $4 \times 50 = 200$, $4 \times 10 = 40$, $4 \times 5 = 20$; $50 + 10 + 5 = 65$; so $260 \div 4 = 65$

This standard calls for students to explore division through various strategies.

Example:

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

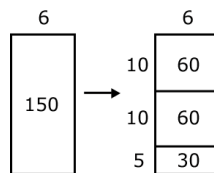
Student 1	Student 2	Student 3														
<p>592 divided by 8</p> <p>There are 70 eights in 560.</p> $592 - 560 = 32$ <p>There are 4 eights in 32.</p> $70 + 4 = 74$	<p>592 divided by 8</p> <p>I know that 10 eights is 80.</p> <p>If I take out 50 eights that is 400.</p> $592 - 400 = 192$ <p>I can take out 20 more eights which is 160.</p> $192 - 160 = 32$ <p>8 goes into 32 four times.</p> <p>I have none left. I took out 50, then 20 more, then 4 more. That's 74.</p> <div style="text-align: right; margin-right: 20px;"> <table style="border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding-right: 5px;">592</td><td style="padding-left: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">-400</td><td style="padding-left: 5px;">50</td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">192</td><td style="padding-left: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">-160</td><td style="padding-left: 5px;">20</td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">32</td><td style="padding-left: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">-32</td><td style="padding-left: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">0</td><td style="padding-left: 5px;"></td></tr> </table> </div>	592		-400	50	192		-160	20	32		-32	4	0		<p>I want to get to 592.</p> $8 \times 25 = 200$ $8 \times 25 = 200$ $8 \times 25 = 200$ $200 + 200 + 200 = 600$ $600 - 8 = 592$ <p>I had 75 groups of 8 and took one away, so there are 74 teams.</p>
592																
-400	50															
192																
-160	20															
32																
-32	4															
0																

Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

1. $150 \div 6$



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, “6 times what number is a number close to 150?” They recognize that 6×10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left, they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6×5 is 30, they write 30 in the bottom area of the rectangle and record 5 as a factor.

4. Student express their calculations in various ways:

a. 150

$$\begin{array}{r} \underline{-60} \\ 90 \end{array} \quad (6 \times 10)$$

$$150 \div 6 = 10 + 10 + 5 = 25$$

$$\begin{array}{r} \underline{-60} \\ 30 \end{array} \quad (6 \times 10)$$

$$\begin{array}{r} \underline{-30} \\ 0 \end{array} \quad (6 \times 5)$$

b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

2. $1917 \div 9$

A student's work showing a rectangular area divided into two sections. The top section is a large rectangle with the number '1800' inside. The bottom section is a smaller rectangle with the number '90' inside. Above the top section is the number '9'. Below the bottom section is a red star and the number '2'.

A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200×9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9×10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. $1917 \div 9 = 213$.

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CCGPS Cluster: Extend understanding of fraction equivalence and ordering.

Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, <, >, =, benchmark fraction.**

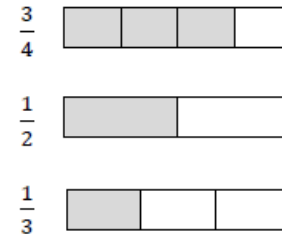
Instructional Strategies

Students’ initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions.

Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators.

Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The


models should be represented in drawings. Students should also use benchmark fractions such as $1/2$ to compare two fractions. The result of the comparisons should be recorded using $>$, $<$ and $=$ symbols.



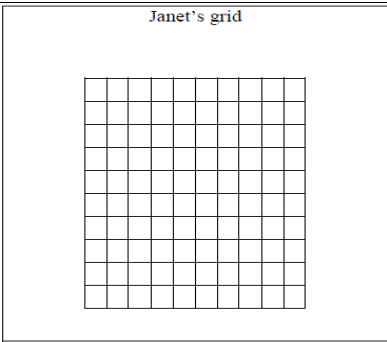
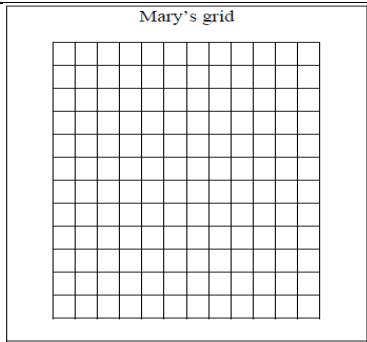
Instructional Resources/Tools	Common Misconceptions
<ul style="list-style-type: none"> • Pattern blocks • Fraction bars or strips 	<p>Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.</p> <p>Students need to use a fraction in the form of one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.</p>
Connections – Critical Areas of Focus	Connections to Other Grade Levels
<p>This cluster is connected to the second Critical Area of Focus for Grade 4, Developing an understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.</p>	<p>Develop understanding of fractions as numbers (CCGPS.3.NF.3).</p>

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³ Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

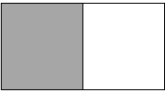
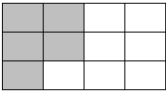
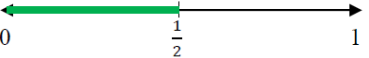
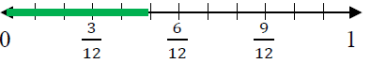
CCGPS	What does this standard mean a student will know and be able to do?
<p>CCGPS.4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>	<p>This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100) This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.</p> <p>Example:</p> <div style="text-align: center;">  <p style="text-align: center;">$\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$</p> </div> <p>Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80</p>
<p>CCGPS.4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>	<p>This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., $1/2$ and $1/8$ of two medium pizzas is very different from $1/2$ of one medium and $1/8$ of one large).</p> <p>Example:</p> <p>Use patterns blocks.</p> <ol style="list-style-type: none"> 1. If a red trapezoid is one whole, which block shows $1/3$? 2. If the blue rhombus is $1/3$, which block shows one whole? 3. If the red trapezoid is one whole, which block shows $2/3$? <p>Example:</p> <p>Mary used a 12×12 grid to represent 1 and Janet used a 10×10 grid to represent 1. Each girl shaded grid squares to show $1/4$. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?</p> <p><i>Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so $1/4$ of each total number is different.</i></p>

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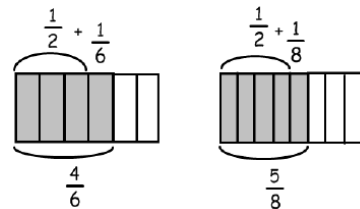
Example:

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

<p>Student 1: Area Model</p> <p>The first cake has more left over. The second cake has $\frac{5}{12}$ left which is smaller than $\frac{1}{2}$.</p>	 
<p>Student 2: Number Line Model</p> <p>The first cake has more left over: $\frac{1}{2}$ is bigger than $\frac{5}{12}$.</p>	 
<p>Student 3: Verbal Explanation</p> <p>I know that $\frac{6}{12}$ equals $\frac{1}{2}$, and $\frac{5}{12}$ is less than $\frac{1}{2}$. Therefore, the second cake has less left over than the first cake. The first cake has more left over.</p>	

Example:

When using the benchmark of $\frac{1}{2}$ to compare to $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

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Number and Operations - Fractions⁴

CCGPS.4.NF

CCGPS Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **operations, addition/joining, subtraction/separating, fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, decomposing, mixed number, rules about how numbers work (properties), multiply, multiple.**

Instructional Strategies

In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fractions and justify with a fraction model. For example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.



Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is

important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

Instructional Resources/Tools

- Fraction tiles/bars
- Circular fraction models
- Rulers with markings of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$
- Number lines

Common Misconceptions

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

Connections – Critical Areas of Focus

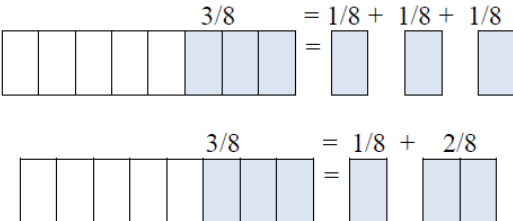
This cluster is connected to the second [Critical Area of Focus](#) for Grade 4, **Developing an understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Connections to Other Grade Levels

Represent and interpret data (CCGPS.4.MD.4).

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⁴ Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

CCGPS	What does this standard mean that a student will know and be able to do?
<p>CCGPS.4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p>	<p>A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as $2/3$, they should be able to join (compose) or separate (decompose) the fractions of the same whole.</p> <p>Example: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$</p> <p>Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.</p> <p>Example: $1\frac{1}{4} - \frac{3}{4} = ? \rightarrow \frac{4}{4} + \frac{1}{4} = \frac{5}{4} \rightarrow \frac{5}{4} - \frac{3}{4} = \frac{2}{4}$ or $\frac{1}{2}$</p> <p>Example of word problem: Mary and Lacey decide to share a pizza. Mary ate $\frac{3}{6}$ and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat together?</p> <p><i>Possible solution:</i> The amount of pizza Mary ate can be thought of a $\frac{3}{6}$ or $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. The amount of pizza Lacey ate can be thought of a $\frac{1}{6} + \frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or $\frac{5}{6}$ of the pizza.</p>
<p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.</p> <p><i>Examples:</i> $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.</p>	<p>Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.</p> <p>Example:</p>  <p>$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$</p> <p>$\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$</p> <p>$2\frac{1}{8} = 1 + 1 + \frac{1}{8}$</p> <p>or</p> <p>$2\frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$</p>

- c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:

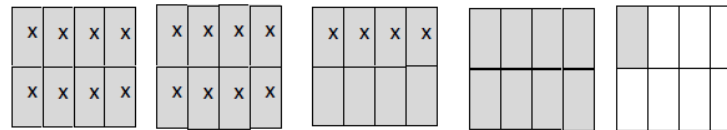
Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. I can write this as $3\frac{1}{8} + 5\frac{3}{8}$. I know they have 8 feet of ribbon by adding the 3 and 5. They also have $\frac{1}{8}$ and $\frac{3}{8}$ which makes a total of $\frac{4}{8}$ more. Altogether they have $8\frac{4}{8}$ feet of ribbon. $8\frac{4}{8}$ is larger than $8\frac{3}{8}$ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left: $\frac{1}{8}$ foot.

Example:

Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

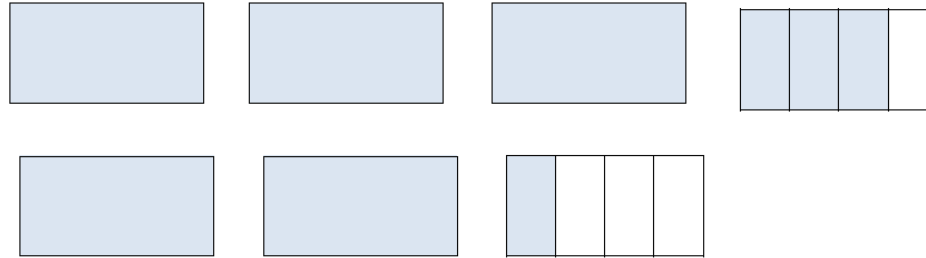
Possible solution: Trevor had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.



Mixed numbers are introduced for the first time in 4th Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions.

Example:

While solving the problem, $3\frac{3}{4} + 2\frac{1}{4}$, students could do the following:



Student 1: $3 + 2 = 5$ and $\frac{3}{4} + \frac{1}{4} = 1$, so $5 + 1 = 6$.

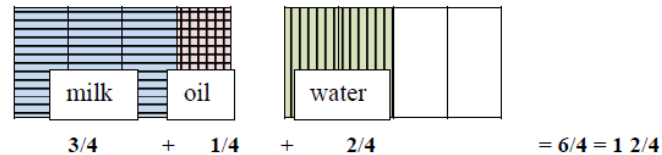
Student 2: $3\frac{3}{4} + 2 = 5\frac{3}{4}$, so $5\frac{3}{4} + \frac{1}{4} = 6$.

Student 3: $3\frac{3}{4} = \frac{15}{4}$ and $2\frac{1}{4} = \frac{9}{4}$, so $\frac{15}{4} + \frac{9}{4} = \frac{24}{4} = 6$.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Example:

A cake recipe calls for you to use $\frac{3}{4}$ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake?



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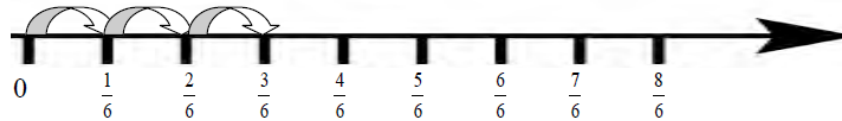
CCGPS.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

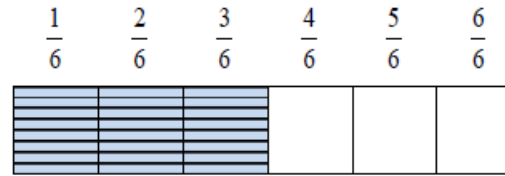
This standard builds on students' work of adding fractions and extending that work into multiplication.

Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

Number line:



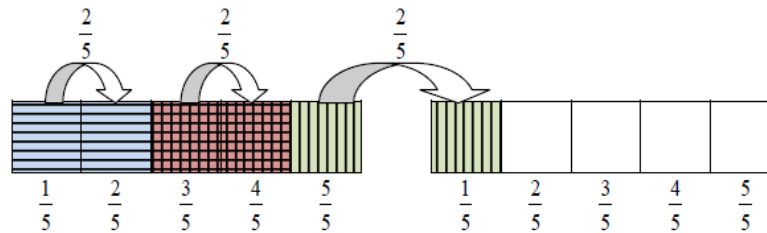
Area model:



b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)

This standard extended the idea of multiplication as repeated addition. For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.

Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

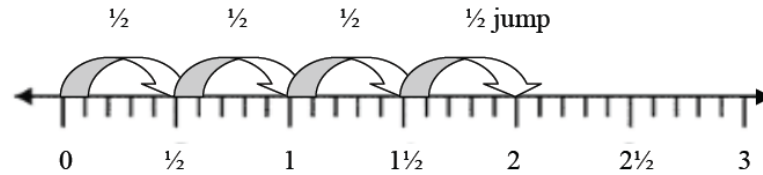
Example:

In a relay race, each runner runs $1/2$ of a lap. If there are 4 team members how long is the race?

(See possible student solutions on the next page.)

answer lie?

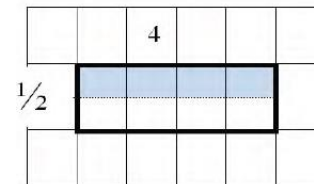
Student 1 – Draws a number line showing 4 jumps of $\frac{1}{2}$:



Student 2 – Draws an area model showing 4 pieces of $\frac{1}{2}$ joined together to equal 2:



Student 3 – Draws an area model representing $4 \times \frac{1}{2}$ on a grid, dividing one row into $\frac{1}{2}$ to represent the multiplier:



Example:

Heather bought 12 plums and ate $\frac{1}{3}$ of them. Paul bought 12 plums and ate $\frac{1}{4}$ of them. Which statement is true?

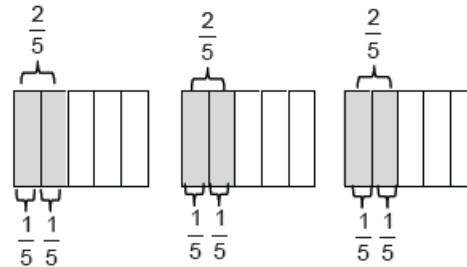
Draw a model to explain your reasoning.

- Heather and Paul ate the same number of plums.
- Heather ate 4 plums and Paul ate 3 plums.
- Heather ate 3 plums and Paul ate 4 plums.
- Heather had 9 plums remaining.

Examples:

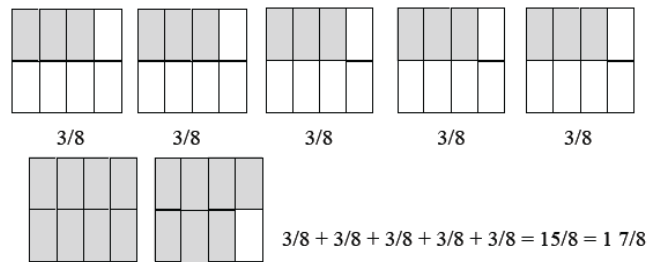
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

1. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$



2. If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:



CCGPS Cluster: Understand decimal notation for fractions, and compare decimal fractions.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, <, >, =.**

Instructional Strategies

The place value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system. Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

Students can use base-ten blocks to represent decimals. A 10 x 10 block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a 10 x 10 grid.

Students need to make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100. Have students say the fraction with denominators of 10 and 100 aloud. For example $\frac{4}{10}$ would be “four tenths” or $\frac{27}{100}$ would be “twenty-seven hundredths.” Also, have students represent decimals in word form with digits and the decimal place value, such as $\frac{4}{10}$ would be 4 tenths.

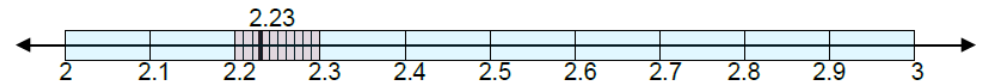
Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions $\frac{3}{10} + \frac{2}{100}$.

Students’ understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

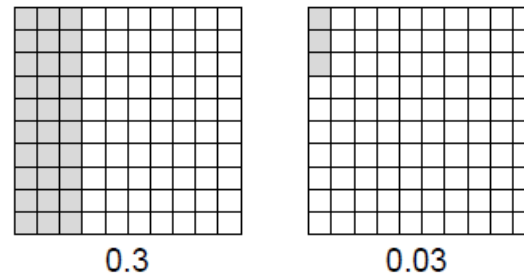
In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations

before they try to do conversions in the metric system. Understanding of the decimal place value system is important prior to the generalization of moving the decimal point when performing operations involving decimals.

Students extend fraction equivalence from Grade 3 with denominators of 2, 3, 4, 6, and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.



When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.



Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

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⁵ Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

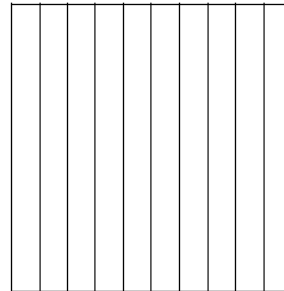
Instructional Resources/Tools		Common Misconceptions
<ul style="list-style-type: none"> Length or area models 10 × 10 square on a grid Decimal place-value mats Base-ten blocks Number lines A Meter of Candy: In this series of three hands-on activities, students develop and reinforce their understanding of hundredths as fractions, decimals, and percentages. Students explore with candy pieces as they physically make and connect a set and linear model (meter) to produce area models (grids and pie graphs). At this time, students are not to do percents. The relationships among fractions, decimals, and percents are developed in Grade 6. 		<p>Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that 0.03 is greater than 0.3.</p>
Connections – Critical Areas of Focus		Connections to Other Grade Levels
<p>This cluster is connected to the second Critical Area of Focus for Grade 4, Developing an understanding of fraction equivalence, addition, and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.</p>		<p>Connect with understanding and generating equivalent fractions (CCGPS.4.NF.1, CCGPS.4.NF.2).</p> <p>Students will perform operations with decimals to hundredths in Grade 5 (CCGPS.5.NBT.5, CCGPS.5.NBT.6, CCGPS.5.NBT.7).</p>
CCGPS	What does this standard mean that a student will know and be able to do?	
<p>CCGPS.4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.⁶ <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i></p>	<p>This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (CCGPS.4.NF.6 and CCGPS.4.NF.7), experiences that allow students to shade decimal grids (10×10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.</p> <p>This work in 4th grade lays the foundation for performing operations with decimal numbers in 5th grade. (See examples for this standard on the next page.)</p>	

⁶ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement for this grade.

Example:

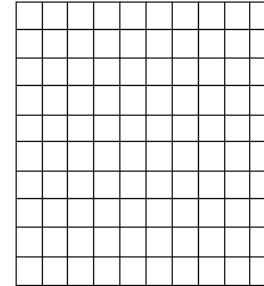
Ones	.	Tenths	Hundredths
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Tenths Grid



$.3 = 3 \text{ tenths} = 3/10$

Hundredths Grid

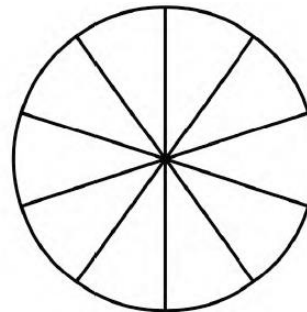


$.30 = 30 \text{ hundredths} = 30/100$

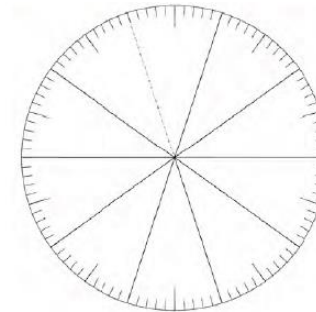
Example:

Represent 3 tenths and 30 hundredths on the models below.

Tenths circle



Hundredths circle



CCGPS.4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

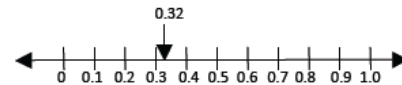
Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown on the following page.

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Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students use the representations explored in CCGPS.4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$. Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.



CCGPS.4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

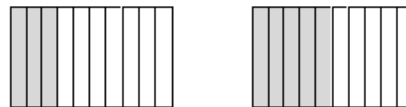
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows $\frac{3}{10}$ but the whole on the right is much bigger than the whole on the left. They are both $\frac{3}{10}$ but the model on the right is a much larger quantity than the model on the left.

When the wholes are the same, the decimals or fractions can be compared.

Example:

Draw a model to show that $0.3 < 0.5$. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.)



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CCGPS Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter.**

Instructional Strategies

In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers and yardsticks to discover these relationships among these units of measurements. Using 12-inch rulers and yardstick, students can see that three of the 12-inch rulers, which is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick. Have students record the relationships in a two column table or t-charts. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.

Present word problems as a source of students’ understanding of the relationships among inches, feet and yards.

Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12 and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

Instructional Resources/Tools

- Yardsticks(meter sticks) and rulers (marked with customary and metric units)
- Teaspoons and tablespoons
- Graduated measuring cups (marked with customary and metric units)

Common Misconceptions

Student believe that larger units will give larger measures. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yard sticks. Students should notice that it takes fewer yard sticks to measure the room than the number of rulers of tiles needed.

Connections – Critical Areas of Focus

This cluster is connected to two Critical Areas of Focus for Grade 4: the first – **Developing understanding and fluency with multi-digit multiplication , and developing understanding of dividing to find quotients involving multi-digit dividends,** and the second – **Developing an understanding of fractions equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.**

Connections to Other Grade Levels

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (Grade 3 MD 8).
 Geometric measurement; understand concepts of area and relate area to multiplication and to addition.
 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Grade 4 NF 3 – 4).

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CCGPS	What does this standard mean that a student will know and be able to do?										
<p>CCGPS.4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p>	<p>The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create.</p> <p>Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.</p> <p>Example: Customary length conversion table</p> <table border="1" data-bbox="1012 496 1549 675"> <thead> <tr> <th>Yards</th> <th>Feet</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> </tr> <tr> <td>2</td> <td>6</td> </tr> <tr> <td>3</td> <td>9</td> </tr> <tr> <td><i>n</i></td> <td><i>n</i> × 3</td> </tr> </tbody> </table> <p>Foundational understandings to help with measure concepts:</p> <ul style="list-style-type: none"> • Understand that larger units can be subdivided into equivalent units (partition). • Understand that the same unit can be repeated to determine the measure (iteration). • Understand the relationship between the size of a unit and the number of units needed (compensatory principle⁷). 	Yards	Feet	1	3	2	6	3	9	<i>n</i>	<i>n</i> × 3
Yards	Feet										
1	3										
2	6										
3	9										
<i>n</i>	<i>n</i> × 3										
<p>CCGPS.4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	<p>This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.</p> <p>Example: Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?</p> <p>Possible solution: Charlie plus 10 friends = 11 total people 11 people × 8 ounces (glass of milk) = 88 total ounces 1 quart = 2 pints = 4 cups = 32 ounces Therefore 1 quart = 2 pints = 4 cups = 32 ounces 2 quarts = 4 pints = 8 cups = 64 ounces 3 quarts = 6 pints = 12 cups = 96 ounces</p>										

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⁷ The compensatory principle states that the smaller the unit used to measure the distance, the more of those units that will be needed. For example, measuring a distance in centimeters will result in a larger number of that unit than measuring the distance in meters.

If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional examples with various operations:

- **Division/fractions:** Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?

Students may record their solutions using fractions or inches. (The answer would be $\frac{2}{3}$ of a foot or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.)

- **Addition:** Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?
- **Subtraction:** A pound of apples costs \$1.20. Rachel bought a pound and a half of apples. If she gave the clerk a \$5.00 bill, how much change will she get back?
- **Multiplication:** Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

Example:

At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.



CCGPS.4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

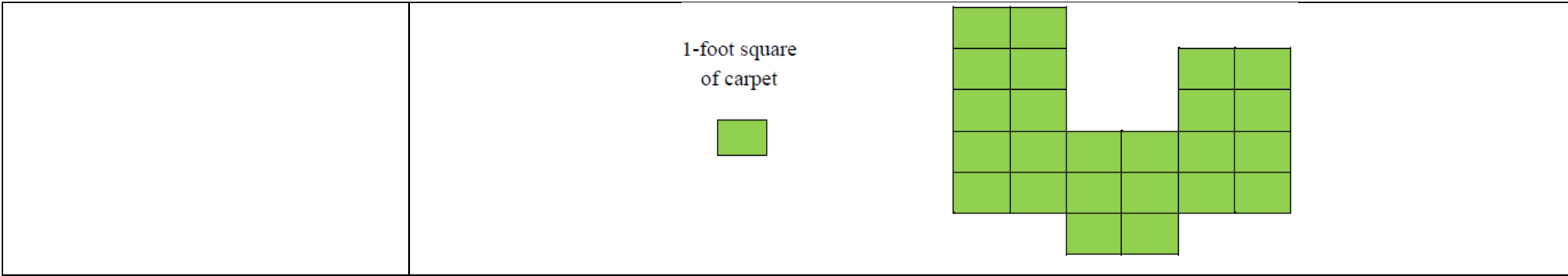
Students developed understanding of area and perimeter in 3rd grade by using visual models.

While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is $l \times w$ and the answer will always be in square units. The formula for perimeter can be $2l + 2w$ or $2(l + w)$ and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:

Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?

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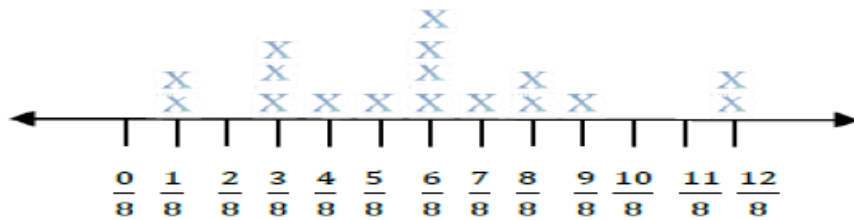
CCGPS Cluster: Represent and interpret data.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **data, line plot, length, fractions.**

Instructional Strategies

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$) and plot data showing multiple data points for each fraction.



Pose questions that students may answer, such as

- “How many one-eighths are shown on the line plot?” Expect “two one-eighths” as the answer. Then ask, “What is the total of these two one-

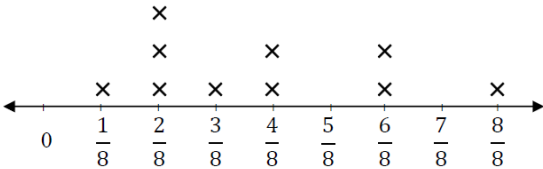
eighths?” Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.

- “What is the total number of inches for insects measuring $\frac{3}{8}$ inches?” Students can use skip counting with fraction names to find the total, such as, “three-eighths, six-eighths, nine-eighths. The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. have them make a statement about the result of adding fractions with the same denominator.
- “What is the total number of insects measuring $\frac{1}{8}$ inch or $\frac{5}{8}$ inches?” Have students write number sentences to represent the problem and solution such as, $\frac{1}{8} + \frac{1}{8} + \frac{5}{8} = \frac{7}{8}$ inches.

Use visual fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Instructional Resources/Tools	Common Misconceptions
<ul style="list-style-type: none"> • Fraction bars or strips 	<p>Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.</p> <p><u>Specific strategies may include:</u> Create number lines with the same denominator without using the equivalent form of a fraction. For example, on a number line using eighths, use 48 instead of 12. This will help students later when they are adding or subtracting fractions with unlike denominators. When representations have unlike denominators, students ignore the denominators and add the numerators only.</p> <p>Have students create stories to solve addition or subtraction problems with fractions to use with student created fraction bars/strips.</p>

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Connections – Critical Areas of Focus		Connections to Other Grade Levels	
<p>This cluster is connected to the second Critical Area of Focus for Grade 4, Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.</p>		<p>Understand a fraction as a number on the number line; represent fractions on a number line diagram (CCGPS.3.NF.2).</p> <p>Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size (CCGPS.3.NF.3).</p> <p>Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters (CCGPS.3.MD.4).</p> <p>Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem (CCGPS.4.NF.3d).</p>	
CCGPS		What does this standard mean that a student will know and be able to do?	
<p>CCGPS.4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p>This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.</p> <p>Example:</p> <p>Students measured objects in their desk to the nearest $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ inch. They displayed their data collected on a line plot. How many objects measured $\frac{1}{4}$ inch? $\frac{1}{2}$ inch? If you put all the objects together end to end what would be the total length of all the objects.</p> <div style="text-align: center;">  </div>		

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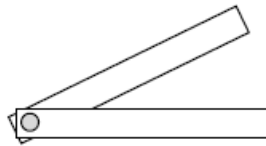
CCGPS Cluster: Geometric measurement – understand concepts of angle and measure angles.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown.**

Instructional Strategies

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.



They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.

Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle and circle which has a measure of 360 degrees. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Instructional Resources/Tools

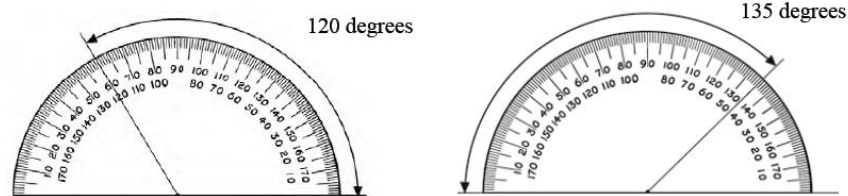
- Cardboard cut in strips to make an angle explorer
- Brass fasteners
- Protractor
- Angle ruler
- Straws
- Transparencies
- Angle explorers
- [Sir Cumference and the Great Knight of Angleland](#): In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest, the successful completion of which will earn him his knighthood. With the help of a family heirloom that functions much like a protractor, he is able to locate the elusive King Lell and restore him to the throne of Angleland. In gratitude, King Lell bestows knighthood on Sir Radius.

Common Misconceptions

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

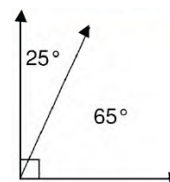
<ul style="list-style-type: none"> • What's My Angle? Math Challenge #10: Students can estimate the measures of the angles between their fingers when they spread out their hand. • 3rd Grade Measuring Game: Identify acute, obtuse and right angles in this online interactive game, • Star Gazing: Determine the correct angle at which to place a telescope in order to see as many stars as possible in this online interactive game. 	
Connections – Critical Areas of Focus	Connections to Other Grade Levels
This cluster is connected to the third Critical Area of Focus for Grade 4, Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.	Connect measuring angle to the Geometry domain in which students draw and identify angles as right, acute and obtuse (CCGPS.4.G.1).
CCGPS	What does this standard mean a student will know and be able to do?
CCGPS.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:	This standard brings up a connection between angles and circular measurement (360 degrees).
a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.	The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same. <div data-bbox="1780 760 1955 976" data-label="Image"> </div>
b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	This standard calls for students to explore an angle as a series of “one-degree turns.” A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?
CCGPS.4.MD.6 Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.	Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360° rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180° . They extend this understanding and recognize and sketch angles that measure approximately 45° and 30° . They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students should measure angles and sketch angles.

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CCGPS.4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.



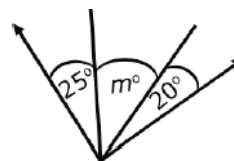
Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses, how many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of m ?



Example:

Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30° . What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

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CCGPS Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **classify shapes/figures, (properties)-rules about how numbers work, point, line, line segment, ray, angle, vertex/vertices, right angle, acute, obtuse, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two dimensional. From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere.**

Instructional Strategies

Angles

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Symmetry

When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Two-dimensional shapes

Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

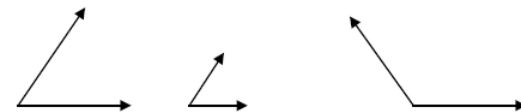
Informal understanding of the characteristics of triangles is developed through angle measures and side length relationships. Triangles are named according to their angle measures (right, acute or obtuse) and side lengths (scalene, isosceles or equilateral). These characteristics are used to draw triangles.

Instructional Resources/Tools

- Mirrors
- Geoboards
- GeoGebra, a free software for learning and teaching which can be accessed at <http://www.geogebra.com>.










Common Misconceptions

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.



Have students draw or trace a shape, then fold in different ways to find all lines of symmetry.

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Connections – Critical Areas of Focus		Connections to Other Grade Levels
<p>This cluster is connected to the third Critical Area of Focus for Grade 4, Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.</p>		<p>Geometric measurement: understand concepts of angles and measure angles (Grade 4 MD 3).</p> <p>Symmetry can be related to experiences in art.</p>
CCGPS	What does this standard mean a student will know and be able to do?	
<p>CCGPS. 4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>	<p>This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily.</p> <p>Students do not easily identify lines and rays because they are more abstract.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>right angle</p>  </div> <div style="text-align: center;"> <p>acute angle</p>  </div> <div style="text-align: center;"> <p>obtuse angle</p>  </div> <div style="text-align: center;"> <p>straight angle</p>  </div> <div style="text-align: center;"> <p>segment</p>  </div> <div style="text-align: center;"> <p>line</p>  </div> <div style="text-align: center;"> <p>ray</p>  </div> <div style="text-align: center;"> <p>parallel lines</p>  </div> <div style="text-align: center;"> <p>perpendicular lines</p>  </div> </div> <p>Example: Draw two different types of quadrilaterals that have two pairs of parallel sides? Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.</p>	

Examples:

How many acute, obtuse and right angles are in this shape?



Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

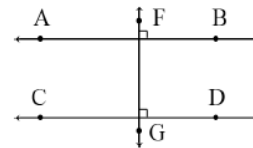
CCGPS.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:

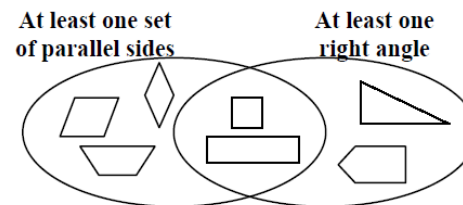
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90°). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:



This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.

Example:



Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:

Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:

For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counterexample.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. (*impossible*)
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

Example:

Identify which of these shapes have perpendicular or parallel sides and justify your selection.



A possible justification that students might give is: “The square has perpendicular lines because the sides meet at a corner, forming right angles.”

Angle Measurement:

This expectation is closely connected to CCGPS.4.MD.5, CCGPS.4.MD.6, and CCGPS.4.G.1. Students’ experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90° , 180° , and 360° to approximate the measurement of angles.

Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

CCGPS.4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

This standard only includes line symmetry, not rotational symmetry.

Example:

For each figure at the right, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.



Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.

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Table 1

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ⁸
Put together/ Take apart⁹	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare¹⁰	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

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⁸ Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

⁹ These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

¹⁰ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2

Common Multiplication and Division Situations

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

	Unknown Product	Group Size Unknown ("How many in each group? Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays¹¹, Area¹²	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

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¹¹ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

¹² Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3

The Properties of Operations

Here a , b , and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

<i>Associative property of addition</i>	$(a + b) + c = a + (b + c)$
<i>Commutative property of addition</i>	$a + b = b + a$
<i>Additive identity property of 0</i>	$a + 0 = 0 + a = a$
<i>Associative property of multiplication</i>	$(a \times b) \times c = a \times (b \times c)$
<i>Commutative property of multiplication</i>	$a \times b = b \times a$
<i>Multiplicative identity property of 1</i>	$a \times 1 = 1 \times a = a$
<i>Distributive property of multiplication over addition</i>	$a \times (b + c) = a \times b + a \times c$

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CCGPS • Critical Areas of Focus

Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

- (1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
- (2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
- (3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

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