Arkansas Council of Teachers of Mathematics
2013 State Contest
Algebra II Exam

In each of the following choose the BEST answer and shade the corresponding letter on the Scantron Sheet. Answer all 25 multiple choice questions before attempting the tie-breaker questions. The tie-breaker questions at the end are to be used to resolve any ties between 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages. The figures are not necessarily drawn to scale. Congratulations for being selected to participate in the ACTM State Contest!

1) Assume *a>0 and b>0. F*actor the following expression completely: $a^2 - b^2 + 6b - 9$

A) $3(a+b)(a-b)(2b-3)$
B) $(a+3)(a-3)b(b-6)$
C) $(a-b+3)(a+b-3)$
D) $(a-b-3)(a+b-3)$
E) Each of the other answers are incorrect.

2) Perform the indicated operations and simplify: $\frac{4x-16}{5x+15} \cdot \frac{4-x}{2x+6}$

A) 0 B) $\frac{-4(x-4)}{5(x+3)^2}$
C) $-\frac{8}{5}$ D) $\frac{3}{10}$
E) Each of the other answers are incorrect.

3) Perform the indicated operations and simplify: $\frac{2}{x^2-9} + \frac{5}{x^2-x-12}$

A) $\frac{7}{(x^2-9)(x^2-x-12)}$ B) $\frac{7x-23}{(x-3)(x-4)(x+3)}$
C) $\frac{7x^2-x-21}{(x^2-9)(x^2-x-12)}$ D) $\frac{7x-7}{(x-3)(x-4)(x+3)}$
E) Each of the other answers are incorrect.

4) Find *a* that will give the interval $[-\frac{1}{4}, \frac{5}{4}]$ as the solution set for the inequality $|ax-1| \le \frac{3}{2}$

A) {-2} B) {2}
C) {4} D) {-5}
E) Each of the other answers are incorrect.

5) Let $i=\sqrt{-1}$. Then $-2i^2+\left(\sqrt{-4}\right)\left(\sqrt{4}\right)-\left(\sqrt{-3}\right)\left(\sqrt{-3}\right)-2i^5=a+bi$ where *a* and *b* are real numbers. **Find the value of (2a+3b).**

A) -108 B) 7
C) 19 D) 16
E) Each of the other answers are incorrect.

6) Assume $1\le y\le x$. Solve for *x*: $9+\sqrt{x^2-y^2}=z$

A) $\pm\sqrt{y^2+z^2-9}$ B) $\pm\sqrt{(z-9)^2+y^2}$
C) $\pm\sqrt{y^2+z^2-9^2}$ D) $+\sqrt{y^2+z^2-9}$
E) Each of the other answers are incorrect.

7) Two hikers, each walking at a constant rate, start walking at constant rates and in the same direction from the city line on the same route. The second hiker whose speed is 5 mph starts 2 hours after the first hiker whose speed is 4 mph. Find the number of hours for which the second hiker walks in order to catch up to the first hiker.

A) 8 hours B) 11 hours
C) 12 hours D) 10 hours
E) Each of the other answers are incorrect.

8) Solve for L . $C=10\log_4\left(\frac{D}{3L}\right)$

A) $L=\frac{D}{3*4^{\frac{C}{10}}}$ B) $L=\frac{3D}{4^{\frac{C}{10}}}$
C) $L=\frac{D}{3*40C}$ D) $L=\frac{D}{12^{\frac{C}{10}}}$
E) Each of the other answers are incorrect.

9) Find the domain of the function *h(x)* for $h(x)=g(f(x))$ if $f(x)=\sqrt{x+5}$ and $g(x)=\sqrt{8-x}$. Write your answer in interval notation.

A) [-5, 59] B) $[-5,\infty)$
C) $(-5,8)\cup(8,\infty)$ D) [-5,0]
E) Each of the other answers are incorrect.

10) Solve the inequality and write your final answer in interval notation: $\frac{x+5}{x\sqrt{x^2-16}} \le 0$

A) $(-\infty,-4)\cup(-4,0)$ B) $(-\infty,-4]\cup[4,\infty)$
C) [-5, 0) D) [-5, -4)
E) Each of the other answers are incorrect.

11) For $f(x) = -a^x + b$, where *a>0,* describe what may happen to f(x) as *x* goes to positive infinity.

I. f(x) goes to positive infinity II. f(x) goes to negative infinity
III. f(x) goes to b IV. f(x) goes to -b V. f(x) = b-1

A) I only B) II only
C) Only II or IV D) Only II, III, or V
E) Only II, IV or V

12) For the quadratic function of the form $y = ax^2 + bx + c$, find the value of *a* for a parabola having its vertex at the point (-2,-9) and passing through the point (1,18).

A) 3 B) 1
C) -1 D) $\frac{3}{2}$
E) Each of the other answers are incorrect.

13) Suppose $12,000 is invested at 7% interest compounded monthly. Approximately how many years will it take for the account to grow to $22,500 if $A = P\left(1+\frac{r}{n}\right)^{nt}$

A) 9 B) 3
C) -9 D) 6.9
E) Each of the other answers are incorrect.

14) Find the value of x : $\frac{1}{4}\log_7 81 + \frac{3}{2}\log_7 4 - \log_7 2 = \log_7 x$

A) 8 B) 12
C) 1 D) No solution
E) Each of the other answers are incorrect.

15) Find all horizontal asymptotes for the graph of *f* if $f(x) = \frac{3 - 2x - 8x^2}{4x^2}$

A) y = -2 B) y = 0
C) none D) x = -2
E) Each of the other answers are incorrect.

16) The area of an ellipse is calculated using the formula $A = \pi * a * b$, where *a* is half the length of the major axis and *b* is half the length of the minor axis of the ellipse. What is the area enclosed by $25x^2 + 16y^2 - 150x + 64y - 111 = 0$?

A) 5π B) 20π
C) 100π D) 200π
E) Each of the other answers are incorrect.

17) Solve for x: $\left(\frac{1}{6}\right)^{5x+2} = 36^{3x-4}$

A) x = -2 B) x = 10
C) x = $-\frac{11}{6}$ D) x = $\frac{6}{11}$
E) Each of the other answers are incorrect.

18) Solve for x exactly: $\log_{?} x + \log_3(x - 8) = 2$. What is the sum of the solution/s?

A) 8 B) 9
C) -8 D) 18
E) Each of the other answers are incorrect.

19) Determine the number of solutions in the system of equations:
$$\begin{aligned} -x + 6y - 3z &= -8 \\ x - 2y + 2z &= 3 \\ 3x + 2y + 4z &= -6 \end{aligned}$$

A) Infinitely many solutions B) one solution
C) three solutions D) No solution
E) Each of the other answers are incorrect.

20) One member of the 5 members of the set {169, 224, $\sqrt{3^2 + 4^2}$, 81+19, 30 minus the sum of the first two prime numbers} is selected at random. Find the probability that the number selected is the square of an integer. Express your answer as a common fraction reduced to lowest terms.

A) $\frac{8}{10}$ B) $\frac{2}{5}$
C) $\frac{3}{5}$ D) $\frac{4}{5}$
E) Each of the other answers are incorrect.

Use for questions 21-22.

A manufacturer of skateboards makes a Youth model and a Super Deluxe model. Each youth board requires 2 labor-hours for fabricating and 3 labor-hours for finishing. Each Super Deluxe board requires 5 labor-hours for fabricating and 4 labor-hours for finishing. The maximum labor-hours available per week in the fabricating and finishing departments are 120 and 75, respectively. Since the company makes a profit of $20 each youth board and $65 on each Super Deluxe board, they want to know how many boards of each type should be manufactured each week in order to maximize total weekly profit. Let x = number of Youth boards and y = number of Super Deluxe boards. Answer the questions below only

21) Write the objective function.

A) P = 120x + 75y B) P = 20x + 65y
C) P = 5x + 4y D) P = 2x + 3y
E) Each of the other answers are incorrect.

22) Write the system of constraints and non-negative constraints.

A) $\begin{cases} 2x+3y \le 120 \\ 3x+4y \le 75 \\ x \ge 0, y \ge 0 \end{cases}$ B) $\begin{cases} 2x+5y \le 120 \\ 3x+4y \le 75 \\ x \ge 0, y \ge 0 \end{cases}$

C) $\begin{cases} 2x+5y \ge 120 \\ 3x+4y \ge 75 \end{cases}$ D) $\begin{cases} 2x+3y \le 120 \\ 3x+4y \le 75 \end{cases}$
E) Each of the other answers are incorrect.

23) If *x* is an integer, find the sum of all distinct values of *x* such that $\frac{x-4}{x-9}-3 \ge 0$

A) 13 B) 4 C) 0 D) 21
E) Each of the other answers are incorrect.

24) For $A=\begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & a & 1 \end{bmatrix}$ and $|A|=2$. Find *a*.

A) $-\frac{8}{3}$ B) 4 C) -4 D) $\frac{8}{3}$ E) Each of the other answers are incorrect.

25) For a daily airline flight between two cities, the number of pieces of checked luggage has a mean of 380 and a standard deviation of 20. Approximately what percentage of the flights would you expect 440 pieces of checked luggage or fewer? Assume normal distribution.

A) 99.7% B) .03% C) 99.85% D) 95% E) Each of the other answers are incorrect.

Tie-Breaker Questions

Name_____ **School_____**
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In each of the following you must show supporting work for your answers to receive credit. The questions will be used in the order given to resolve ties for 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages.

Tie Breaker #1
Using algebra, find the point/s of intersection for $y = -3x$ and $x^2 + y^2 = 3$.

Graph and label all points of intersection.

Tie Breaker #2

Find a function with all the following characteristics. (Answers may vary.):

x - intercepts at (-3,0) and (-1,0)
y - intercept at (0,2)
vertical asymptote: x = 2
horizontal asymptote: y = 1

Tie Breaker #3

Find the product of the zeros of $P(x) = x^4 - 4x^3 + 3x^2 + 8x - 10$.

Algebra II Solutions:

1) C
2) C
3) B
4) B
5) D
6) B
7) A
8) A
9) A
10) D
11) D
12) A
13) A
14) B
15) A
16) B
17) D
18) B
19) D
20) C
21) B
22) B
23) D
24) C
25) C

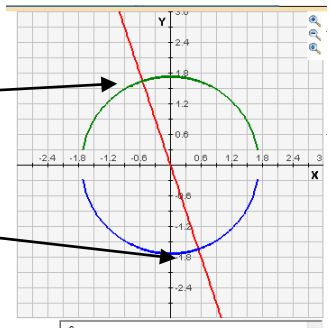
There have been some questions concerning #11:
Number 11 is correct.
The behavior of f(x) as x goes to infinity depends on the value of *a* .
For *a*>0, there are three cases:
Case 1: 0 < a < 1
Case 2: a = 1
Case 3: a > 1
I leave it to the students to explore the answer to the question using the three cases.

Tie Breaker #1:
A. Using algebra, find the point/s of intersection for $y=-3x$ and $x^2+y^2=3$.
SOLUTION:
One method would be to substitute $y=-3x$ into $x^2+y^2=3$.
Solving for *x* : $x=\pm\frac{\sqrt{30}}{10}$
Substituting the two *x* values into $y=-3x$, we get the following solution set: $\left\{\left(\frac{\sqrt{30}}{10},-\frac{3\sqrt{30}}{10}\right),\left(-\frac{\sqrt{30}}{10},\frac{3\sqrt{30}}{10}\right)\right\}$

B. Graph and label all points of intersection.

$(-\frac{\sqrt{30}}{10},\frac{3\sqrt{30}}{10})$

SOLUTION: $(\frac{\sqrt{30}}{10},-\frac{3\sqrt{30}}{10})$



Tie Breaker #2:
Find a function with all the following characteristics. (Answers may vary.):

x - intercepts at (-3,0) and (-1,0)
y - intercept at (0,2)
vertical asymptote: x = 2
horizontal asymptote: y = 1

ONE SOLUTION:
1. Having a horizontal asymptote which is not equal to zero, implies we have a rational function with the numerator and the denominator having the same exponential power for the leading term.
2.From the *x*-intercepts we can use the factors (x+3)(x+1)in the numerator. This implies at a power of 2.
3. The vertical asymptote implies the factor (x-2) is in the denominator.
4. Thus far we have f(x) = $\frac{(x+3)(x+1)}{(x-2)(x+a)}$ and we know the y-intercept is 2.

By substituting x = 0 and f(x) = 2, we have $2=\frac{(3)(1)}{(-2)(a)}$ or $a=-\frac{3}{4}$

One solution would be:

$$f(x)=\frac{(x+3)(x+1)}{(x-2)\left(x-\frac{3}{4}\right)}$$

Tie Breaker #3:
Find the product of the zeros of $P(x) = x^4-4x^3+3x^2+8x-10$.
SOLUTION:
The zeros of P(x) are the values of x which satisfy P(x) = 0.
The first two zeros can be found using a graphing calculator and finding the x-intercepts.
These are $\sqrt{2},-\sqrt{2}$.
$(x-\sqrt{2})(x+\sqrt{2})=x^2-2$
Dividing P(x) by x^2-2 using long division, we get x^2-4x+5.
Using the quadratic formula, we find the other two zeros to be $x=2\pm i$
The product of the zeros is $(\sqrt{2})(-\sqrt{2})(2+i)(2-i)=(-2)(4-i^2)=(-2)(5)=-10$