

Scottsdale Community College

Arithmetic for College Readiness

Student Workbook

Development Team

Amy Volpe

Jenifer Bohart

Judy Sutor

Donna Slaughter

Martha Gould

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ABOUT THIS WORKBOOK

Mathematics instructors at Scottsdale Community College in Scottsdale, Arizona created this workbook. The included content is designed to lead students through arithmetic, from a multiple representations approach, and to develop a deep understanding of the concepts associated with number and operations. The included curriculum is broken into eleven units. Each unit includes the following components:

MEDIA LESSON

- The **Media Lesson** is the main instructional component for each lesson.
- **Media Examples** can be worked by watching online videos and taking notes/writing down the problem as written by the instructor. Video links can be found at <http://sccmath.wordpress.com> or may be located within the MathAS Online Homework Assessment System.
- **You Try** problems reinforce Lesson concepts and should be worked in the order they appear showing as much work as possible.

PRACTICE PROBLEMS

- This section follows the Lesson. If you are working through this material on your own, the recommendation is to work all practice problems. If you are using this material as part of a formal class, your instructor will provide guidance on which problems to complete. Your instructor will also provide information on accessing answers/solutions for these problems.

END OF UNIT ASSESSMENT

- The last part of each Unit is a short end of lesson assessment. If you are working through this material on your own, use these assessments to test your understanding of the unit concepts. Take the assessments without the use of the book or your notes and then check your answers. If you are using this material as part of a formal class, your instructor will provide instructions for completing these problems and for obtaining solutions to the practice problems.

MATHAS ONLINE HOMEWORK ASSESSMENT SYSTEM

If you are using these materials as part of a formal class and your class utilizes an online homework/assessment system, your instructor will provide information as to how to access and use that system in conjunction with this workbook.

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UNIT 1 – PLACE VALUE AND WHOLE NUMBERS OPERATIONS

INTRODUCTION

We will begin our study of Arithmetic by learning about the number system we use today. The Base-10 Number System or Hindu-Arabic Numeral System began its development in India in approximately 50 BC. By the 10th century, the system had made its way west to the Middle East where it was adopted and adapted by Arab mathematicians. This number system moved further west to Europe in the early 13th century when the Italian mathematician Fibonacci recognized its efficiency and promoted its use. In this lesson, we will learn the basics that make this number system so useful and how to perform basic arithmetic operations in our number system.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson.

Section	Learning Objective	Media Examples	You Try
1.1	Write numbers in place value, extended form, and word form	1	2
1.2	Identify place values for large numbers	3	4
1.3	Write place value form for large numbers	5	6
1.4	Round numbers using place value	7	8
1.5	Identify addition and subtraction application problems	9	
1.5	Use strategies to solve addition and subtraction application problems	10	11
1.6	Identify multiplication and division application problems	12	
1.6	Use strategies to solve multiplication and division application problems	13	14
1.7	Represent and evaluate addition and subtraction applications symbolically that contain more than one operation	15, 16	17
1.8	Represent and evaluate multiplication and division applications symbolically that contain more than one operation	18, 19	20
1.9	Represent and evaluate +, -, ×, ÷ applications symbolically that contain more than one operation	21, 22	23
1.10	Represent and evaluate applications symbolically that use parentheses as a grouping symbol	24, 25	26
1.11	Represent applications using the notation of exponents	27	
1.12	Write the language and symbolism of exponents in multiple ways	28	29
1.13	Use PEMDAS to evaluate expression	30	31

UNIT 1 – MEDIA LESSON

SECTION 1.1: WHOLE NUMBERS AND PLACE VALUE

Whole numbers are often referred to as *the counting numbers plus the number 0*. The first few whole numbers are written as

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ...

There are ten *digits* that we can use to represent *any* whole number. They are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Since we use ten digits, our number system is a **base-10 number system**. This means each place value is 10 times as large as the place value to its right. The first few place values are 1's, 10's, 100's, 1000's, etc... The typical way we write a number is called the **place value form of a number**. It is based on the idea that the placement of each numeral *determines the value* of the quantity.

Consider the whole numbers 264 and 642. They use the same digits, but the digits are in a different order relative to one another. To see how the ordering of the digits makes a difference, we will represent these numbers in multiple ways as shown below.

Place Value Form	Place Value Chart	Expanded Form	Word Name						
264	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">6</td> <td style="text-align: center;">4</td> </tr> </tbody> </table>	Hundreds	Tens	Ones	2	6	4	$200 + 60 + 4$	two hundred sixty-four
Hundreds	Tens	Ones							
2	6	4							
642	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">6</td> <td style="text-align: center;">4</td> <td style="text-align: center;">2</td> </tr> </tbody> </table>	Hundreds	Tens	Ones	6	4	2	$600 + 40 + 2$	six hundred forty-two
Hundreds	Tens	Ones							
6	4	2							

Important Notes on the Word Name for a Number:

- We do not use the word “and” when writing a word name for a whole number. This word will be used later to connect a whole number with a fraction or decimal.
- We use a hyphen to connect the ten's and one's place of a whole number if these digits cannot be written as a single word. (i.e. 35 is written as *thirty-five*, 50 is written as *fifty* (one word so no hyphen), and 13 is written as *thirteen* (one word so no hyphen))



Problem 1

MEDIA EXAMPLE - Writing the Expanded Form, Word Name, and Place Value Form

Write the following numbers in the indicated forms.

a) 423

100's 10's 1's

Place Value Form in a Table:

--	--	--

Expanded Form: _____ Word Name: _____

b) Eight hundred sixteen

Place Value Form in a Table:

100's 10's 1's

--	--	--

Number Form: _____ Expanded Form: _____

c) $900 + 40 + 6$

Place Value Form in a Table:

100's 10's 1's

--	--	--

Number Form: _____ Word Name: _____



Problem 2

YOU-TRY - Writing the Expanded, Word Name and Place Value Form

Write the number in expanded form, place value form (in a chart) and word name form.

736

100's 10's 1's

Place Value Form in a Table:

--	--	--

Expanded Form: _____ Word Name: _____

SECTION 1.2: EXTENDING PLACE VALUE TO LARGER NUMBERS

Our Place Value System is partitioned into groups of three all based on hundreds, tens and ones. Each place value is 10 times as large as the place value to the right of it. In this section, we will identify these place values and represent them as words and numbers.

 Problem 3	MEDIA EXAMPLE – Identifying the Place Value for Larger Numbers
---	---

Place the number 261,942,037,524 in the place value chart below and answer the corresponding questions.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

- a) Determine the place value for the digit 9 and write what it represents as a word and a number.

- b) Determine the digit in the ten thousand’s place and write what it represents as a word and a number.

 Problem 4	YOU-TRY - Identifying the Place Value for Larger Numbers
---	---

Place the number 72,902,635,524 in the place value chart below and answer the corresponding questions.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

- a) Determine the place value for the digit 7 and write what it represents as a word and a number.

- b) Determine the digit in the hundred thousand’s place and write what it represents as a word and a number.

SECTION 1.3: WRITING PLACE VALUE FORMS OF LARGE NUMBERS

In this section, we will continue our study of place value by writing the place value forms of large numbers that are given in words.


Problem 5
MEDIA EXAMPLE - Writing Place Value Forms for Large Numbers

Place the numbers below in the place value chart. Use the chart to assist you in writing the place value form of the number.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

a) twenty-four million

Place Value Form: _____

b) three hundred sixty-seven thousand

Place Value Form: _____

Problem 6
YOU-TRY - Writing Place Value Forms for Large Numbers

Place the numbers below in the place value chart. Use the chart to assist you in writing the place value form of the number.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

a) seventeen billion

Place Value Form: _____

b) one hundred three million

Place Value Form: _____

SECTION 1.4: ROUNDING NUMBERS USING PLACE VALUE

To **round** a number means to approximate that number by replacing it with another number that is “close” in value. Rounding is often used when estimating.

For rounding, we will follow the process below.

1. *Rounding up* when the place value after the digit we are rounding to is *5 or greater*.
2. *Rounding down* when the place value after the digit we are rounding to is *less than 5*.



Problem 7

MEDIA EXAMPLE - Rounding Numbers Using Place Value

Write the given numbers in the place value chart and then round to the indicated place value.

- a) [NASA](#) states that the moon is an average of 238,855 miles away from earth.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Round 238,855 to the following place values.

thousand: _____ ten thousand: _____ hundred thousand: _____

- b) According to the [2010 US Census](#), Chicago was the 3rd largest US city with a population of 2,695,598 people.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Round 2,695,598 to the following place values.

thousand: _____ ten thousand: _____ hundred thousand: _____



Problem 8

YOU-TRY - Rounding Numbers Using Place Value

According to the [2010 US Census](#), Phoenix was the 6th largest US city with a population of 1,445,632 people.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Round 1,445,632 to the following place values.

thousand: _____ ten thousand: _____ hundred thousand: _____

SECTION 1.5: ADDING AND SUBTRACTING WHOLE NUMBERS

In this section, we will look at some simple applications of addition and subtraction. Our goal is to recognize the structure of these operations in context so we can recognize and apply this knowledge to many different problem types.

First let's introduce some general terminology.

Language and Notation of Addition

We call the numbers we are adding, *addends* and the result is called the *sum*. The symbol “+” is the *plus sign* and indicates that we should use the operation of addition.

$$\boxed{5} + \boxed{3} = \boxed{8}$$

In words, we may say any of the following.

5 plus 3 the sum of 5 and 3 3 added to 5 3 more than 5

Language and Notation of Subtraction

We call the first number on the left the *minuend*, the second number the *subtrahend*, and the result is called the *difference*. The symbol “-” is the *minus sign* and indicates that we should use the operation of subtraction.

$$\boxed{8} - \boxed{3} = \boxed{5}$$

In words, we may say any of the following.

8 minus 3 the difference of 8 and 3 8 decreased by 3 3 less than 8 3 subtracted from 8

Note: Notice we have different names for the two numbers in the subtraction problem, but use the same name (addends) for the numbers in the addition problem. This is because changing the order in the subtraction problem changes the difference. This is not the case for addition. In practice, we rarely use the words “minuend” and “subtrahend”. However, it is important for you to know they play different roles in subtraction.



Draw a diagram or picture to represent the addition or subtraction problem. Then write the problem symbolically and find the sum or difference.

1: Action or Change

- a) Glenn has \$3. He earns \$4 mowing his neighbor's lawn. How many dollars does Glenn have now?
- b) Grant has \$7. He spent \$4 on a snack and a drink. How many dollars does Grant have now?

2: Part/Part/Whole

- a) There are 3 boys and 4 girls on the swim team. How many children are on the swim team?
- b) There are 7 children on the swim team. Three of the children are boys. How many girls are on the swim team?

3: Comparison

- a) Lara has 4 more goldfish than Owen. Owen has 3 goldfish. How many goldfish does Lara have?
- b) Lisa has 7 goldfish and Oscar has 3 goldfish. How many more goldfish does Lisa have?

Summary: The addition and subtraction problems we performed fall into three basic categories. We will expand on these further in the next example.

Problem Type	Addition	Subtraction
a) Action/Change	One amount is <i>added on</i> to another	One amount is <i>taken away</i> from another
b) Part/Part/Whole	Part A + Part B = Whole	Whole – Part A = Part B
c) Comparison	Amount A + Difference = Amount B	Amount B – Difference = Amount A Amount B – Amount A = Difference
d) Fact Family Example	$3 + 4 = 7$ $4 + 3 = 7$	$7 - 3 = 4$ $7 - 4 = 3$



Problem 10

MEDIA EXAMPLE – Adding and Subtracting Application Strategies

Underline or highlight the *Givens* and circle the *Goal* for each problem. Determine the structure of the problem by finding whether it is Action, Part/Part/Whole, or Comparison. Set up an appropriate expression to find the *Goal* using words and symbols. Then add or subtract using place value to find the solution. Write your final answer as a complete sentence.

- a) Sara and John were selling candy to benefit the Theatre Club. Sara sold 84 candy bars more than John. Sara sold 279 candy bars. How many candy bars did John sell?
- b) Jessica has been saving for a tablet for college. After six months, she saved enough money and spent \$1300 on the tablet. Jessica has \$127 leftover after the purchase. How much money had Jessica originally saved?
- c) Brianna took the SAT exam twice. The first time, she earned 520 on the math portion. The second time, she prepared for the exam and earned 680 on the math portion. How many more points did she earn on her second try after studying?

Unit 1 – Media Lesson

- d) In 2017, there were 28 member countries in the North Atlantic Treaty Organization (NATO). All 28 of these countries are also members of the United Nations (UN). If there are 193 member countries in the UN, how many of the countries in the UN are not members of NATO?



Problem 11

YOU-TRY – Adding and Subtracting Application Strategies

Underline or highlight the ***Givens*** and circle the ***Goal*** for each problem. Determine the structure of the problem by finding whether it is Action, Part/Part/Whole, or Comparison. Set up an appropriate expression to find the ***Goal*** using words and symbols. Then add or subtract using place value to find the solution. Write your final answer as a complete sentence.

- a) The water level in Don’s pool was low. He added 180 gallons of water to bring it to the proper level of 900 gallons. How much water was in the pool before Don added the water?
- b) Joan and Sandy were counting their Halloween candy. Joan had 32 more pieces than Sandy. Sandy had 173 pieces of candy. How much candy did Joan have?

SECTION 1.6: MULTIPLYING AND DIVIDING WHOLE NUMBERS

We will begin our investigation of multiplication and division by looking at word problems that use these operations in different ways. First let's introduce some general terminology.

Language and Notation of Multiplication

We call the numbers we are multiplying *factors*, and the result is called the *product*.

$$\boxed{4} \times \boxed{3} = \boxed{12}$$

In words, we may say any of the following.

4 times 3 the product of 4 and 3 4 copies of 3 4 multiplied by 3 4 groups of 3

We may use any of the notations below to request this product.

$$4 \times 3 \quad 4 \cdot 3 \quad 4(3) \quad (4)(3) \quad 4 * 3$$

Language and Notation of Division

We call the number we are dividing the *dividend*, the number we are dividing by the *divisor*, and the result is called the *quotient*.

$$\boxed{12} \div \boxed{4} = \boxed{3}$$

In words, we may say any of the following.

12 divided by 4 4 into 12 12 over 4 (fraction form) the quotient of 12 and 4

How many groups of size 4 are in 12? If 12 is broken into 4 equal groups, what is the size of each group?

We may use any of the notations below to request this quotient.

$$12 \div 4 \quad (12) \div (4) \quad (12) \overline{\div} 4 \quad 12 \overline{\div} (4) \quad \frac{12}{4}$$



Draw a diagram or picture to represent the multiplication or division problem. Then write the problem symbolically and find the product or quotient.

1: Equal Grouping or Fair Sharing

- a) Bernadette is having a party. She invites 4 friends over and is going to make 3 cupcakes per friend. How many cupcakes does she need for her friends?
- b) Chester is making apple pies. He wants to make 3 pies and he needs 4 apples per pie. How many apples does he need altogether?
- c) Penny is having a party. She invites 4 friends over and has 12 cupcakes for them. How many cupcakes does each friend receive if they share the cupcakes equally?
- d) Charlie is making apple pies. He has 12 apples and he needs 4 apples per pie. How many pies can Charlie make?

2: Area

- a) Dan is carpeting a utility room in his house that is 4 feet by 3 feet. How many square feet of carpet does he need?
- b) Dave is carpeting a utility room in his house that is 12 square feet. The length of the rectangular room is 4 feet. If the carpet fits perfectly, what is width of the room?

3: Distance, rate, and time

- a) Fannie is walking at a rate of 4 miles per hour for 3 hours. How many miles did she walk?
- b) Dani walked 12 miles at a rate of 4 miles per hour. How many hours was she walking?

4: Multiplicative Comparison

- a) Chuck has three times as much money as Tony. If Tony has \$4, how much does Chuck have?
- b) Chip has 3 times as much money as Timmy. If Chip has \$12, how much does Timmy have?

Summary: The multiplication and division problems we performed fall into three basic categories. We will expand on these further in the next example.

Problem Type	Multiplication	Division
a) Fair Sharing or Equal Grouping	number of groups \times size of group = total size of group \times number of groups = total	Fair Sharing total \div number of groups = size of group Equal Grouping total \div size of group = number of groups
b) Area of a Rectangle	Area = length \times width	Area \div length = width Area \div width = length
c) Distance, rate, and time	distance = rate \times time	distance \div rate = time distance \div time = rate
d) Multiplicative Comparison	multiplier \times quantity A = quantity B	quantity B \div quantity A = multiplier quantity B \div multiplier = quantity A
e) Fact Family Example	$3 \cdot 4 = 12$ $4 \cdot 3 = 12$	$12 \div 3 = 4$ $12 \div 4 = 3$



Underline or highlight the *Givens* and circle the *Goal* for each problem. Determine the structure of the problem by finding whether it is Fair Sharing, Equal Grouping, Area, Distance/rate/time or Multiplicative Comparison. Set up an appropriate expression to find the *Goal* using words and symbols. Then multiply or divide using place value to find the solution. Write your final answer as a complete sentence.

- a) Last month, Larry worked out 4 times as many hours as he did this month. Last month, he worked out 20 hours. How many hours did he work out this month?
- b) Sky drove from Scottsdale to Las Vegas. Her average speed was 65 miles per hour. The trip took 5 hours. How far did Sky drive?
- c) A high school soccer field is approximately 110 yards long and 70 yards wide. How many square yards is the soccer field?

- d) Michael's parents give him \$180 to go out with his friends on his birthday. If he wants to spend \$20 per person, how many people can attend (including Michael)?



Problem 14

YOU TRY – Multiplication and Division Application Strategies

Underline or highlight the ***Givens*** and circle the ***Goal*** for each problem. Determine the structure of the problem by finding whether it is Fair Sharing, Equal Grouping, Area, Distance/rate/time or Multiplicative Comparison. Set up an appropriate expression to find the ***Goal*** using words and symbols. Then multiply or divide using place value to find the solution. Write your final answer as a complete sentence.

- a) Daisy drove from Scottsdale to San Diego. She traveled a total distance of 378 miles and the trip took 6 hours. How fast did Daisy drive on average?

- b) Judy has a rectangular garden that covers 117 square feet. If the garden is 13 feet long, how wide is the garden?

SECTION 1.7: ADDITION, SUBTRACTION AND THE ORDER OF OPERATIONS

Thus far, we have only performed one operation at a time. In the remaining section, we will learn about the order we will use when there is more than one operation to perform in an expression.



Problem 15

MEDIA EXAMPLE – Addition, Subtraction and the Order of Operations

Solve the problem below. Be sure to indicate every step in the process of your solution.

- a) Suppose on the first day of the month you start with \$150 in your bank account. You make a debit transaction on the second day for \$60 and then make a deposit on the third day for \$20. What is the balance in your account on the third day?
- b) What string of operations (written horizontally) can be used to determine the amount in your account?

Rule 1: When we need to add or subtract 2 or more times in one problem, we will perform the operations from left to right



Problem 16

MEDIA EXAMPLE – Addition, Subtraction and the Order of Operations

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a) $4+8-3+6$

of operations ____

b) $12-5+6-2-1+4$

of operations ____



Problem 17

YOU TRY - Addition, Subtraction and the Order of Operations

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a) $4+7-3-2$

operations ____

b) $8-5-1+12+7-3$

operations ____

SECTION 1.8: MULTIPLICATION, DIVISION AND THE ORDER OF OPERATIONS



Problem 18

MEDIA EXAMPLE – Multiplication, Division and the Order of Operations

Solve the problem below. Be sure to indicate every step in the process of your solution.

- a) Suppose you and your three siblings inherit \$40,000. You divide it amongst yourselves equally. You then invest your portion and make 5 times the amount of your portion. How much money do you have? Be sure to indicate every step in your process.
- b) What string of operations (written horizontally) can be used to determine the result?

Rule 2: When we need to multiply or divide 2 or more times in one problem, we will perform the operations from left to right.



Problem 19

MEDIA EXAMPLE – Multiplication, Division and the Order of Operations

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a) $6 \cdot 4 \div 2 \cdot 3$	# of operations ____	b) $24 \div 4 \div 2(3)$	# of operations ____
-------------------------------	----------------------	--------------------------	----------------------



Problem 20

You Try – Multiplication, Division and the Order of Operations

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a) $16(2) \div 4 \div 2$	# of operations ____	b) $36 \div 9(4)(2)$	# operations ____
--------------------------	----------------------	----------------------	-------------------

SECTION 1.9: THE ORDER OF OPERATIONS FOR +, −, ×, ÷



Problem 21

MEDIA EXAMPLE – The Order of Operations for +, −, ×, ÷

Solve the two problems below. Be sure to indicate every step in your process

- a) Bill went to the store and bought 3 six-packs of soda and an additional 2 cans. How many cans did he buy in total?

What string of operations (written horizontally) can be used to represent this problem?

- b) Amber went to the store and bought 3 six-packs of cola and an additional 2 six-packs of diet cola. How many cans did she buy in total?

What string of operations (written horizontally) can be used to represent this problem?

Rule 3: Unless otherwise indicated by parentheses, we perform multiplication and division *before* addition and subtraction. We continue to perform the operations from left to right.



Problem 22

MEDIA EXAMPLE – The Order of Operations for ÷, ×, −, +

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Perform the operations in the appropriate order. Show all intermediary steps.

a) $10 \div 2 \cdot 5 - 3$

b) $28 \div 4 - 2(3)$

c)
$$\frac{5(2) + 4}{5 - 3}$$



Problem 23

YOU TRY – The Order of Operations for ÷, ×, −, +

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Perform the operations in the appropriate order. Show all intermediary steps.


a) $36 \div 9 + 2(3)$

b) $26 \div 2 \cdot 5 - (3)(4)$

c)
$$\frac{6 + 3(3)}{7 - 2}$$

SECTION 1.10: PARENTHESES AS A TOOL FOR CHANGING ORDER

There are cases when we want to perform addition and subtraction before multiplication and division in the order of operations. So we need a method of indicating we want to make such a modification. In the next media problem, we will discuss how to show this change.


	Problem 24	MEDIA EXAMPLE – Parentheses as a Tool for Changing Order
---	------------	--

Solve the problems below.

- a) Howard bought a \$25 comic book and a \$35 belt buckle. He paid with a \$100 bill. How much change will Howard receive? Be sure to indicate every step in your process.

- b) What string of operations (written horizontally) can be used to determine the amount in your account?

Rule 4: If we want to change the order in which we perform operations in an arithmetic expression, we can use parentheses to indicate that we will perform the operation(s) inside the parentheses first.

	Problem 25	MEDIA EXAMPLE – Parentheses as a Tool for Changing Order
--	------------	--

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Perform the operations in the appropriate order. Show all intermediary steps.

a) $8 \div (4 - 2)$ # of operations ____	b) $53 - [6(5 + 2)]$ # of operations ____
--	---

	Problem 26	YOU TRY - Parentheses as a Tool for Changing Order
---	------------	--

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a) $6 \div [7 - 4]$ # of operations ____	b) $13 - [9 - (6 - 2)]$ # operations ____
--	---

SECTION 1.11: EXPONENTS



Problem 27

MEDIA EXAMPLE – Introduction to Exponents

Solve the problem below. Use the rectangle below to represent the problem visually.

- a) Don makes a rectangular 20 square foot cake for the state fair. After he wins his award, he wants to share it with the crowd. First he cuts the cake into 2 pieces. Then he cuts the 2 pieces into 2 pieces each. Then he cuts all of these pieces into two pieces. He continues to do this a total of 5 times. How many pieces of cake does he have to share?



- b) Write a mathematical expression that represents the total number of pieces in which Don cut the cake.

Terminology

We will use *exponential expressions* to represent problems such as the last one. Exponents represent repeated multiplication just like multiplication represents repeated addition as shown below.

$$\text{Multiplication: } 5 \cdot 2 = 2 + 2 + 2 + 2 + 2 = 10$$

$$\text{Exponents: } 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

In the exponential expression, 2^5

2 is called the *base*
5 is called the *exponent*


We will say 2^5 as “2 raised to the fifth power” or “2 to the fifth”

Since exponents represent repeated multiplication, and we call the numbers we multiply factors, we will also use this more meaningful language when discussing exponents.

$$2^5 \text{ means 5 factors of 2}$$

We also have special names for bases raised to the second or third power.

- a) For 3^2 , we say 3 squared or 3 to the second power
b) For 4^3 , we say 4 cubed or 4 to the third power


 Problem 28	MEDIA EXAMPLE – Language and Notation of Exponents
--	--

Represent the given exponential expressions in the four ways indicated.

a) 6^2

b) 2^6

Expanded Form		Expanded Form	
Word Name		Word Name	
Factor Language		Factor Language	
Math Equation		Math Equation	

 Problem 29	YOU TRY – Language and Notation of Exponents
--	--

Represent the given exponential expressions in the four ways indicated.

a) 7^2

Expanded Form	
Word Name	
Factor Language	
Math Equation	

b) 4^3

Expanded Form	
Word Name	
Factor Language	
Math Equation	

SECTION 1.12: PEMDAS AND THE ORDER OF OPERATIONS

Finally, we will consider problems that may contain any combination of parentheses, exponents, multiplication, division, addition and subtraction.



Problem 30

MEDIA EXAMPLE – PEMDAS and the Order of Operations

Rule 5: Exponents are performed before the operations of addition, subtraction, multiplication and division.

P	Simplify items inside Parentheses (), brackets [] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M	Perform Multiplication and Division next
D	(as they appear from Left to Right)
A	Perform Addition and Subtraction on what is left.
S	(as they appear from Left to Right)

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the correct order of operations. Check your results on your calculator.

a) $(8-3)^2 - 4$

b) $2 \cdot 4^2 + 3$

c) $(5)^2 - 4(5) + 2$



Problem 31

YOU TRY – PEMDAS and the Order of Operations

Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the correct order of operations. Check your results on your calculator.

a) $7 - (3 - 2)^2$ # of operations ___

b) $(4)^2 + 5(4) - 6$ # operations ___

UNIT 1 – PRACTICE PROBLEMS

1. Write the following numbers in expanded form, place value form (in a chart) and word name form.

a) 513

Expanded Form: _____

100's
10's
1's

--	--	--

Place Value Form:

Word Name: _____

b) 27

Expanded Form: _____

100's
10's
1's

--	--	--

Place Value Form:

Word Name: _____

c) 801

Expanded Form: _____

100's
10's
1's

--	--	--

Place Value Form:

Word Name: _____

Unit 1 – Practice Problems

2. Write the following numbers in numerical form, expanded form and place value form (in a chart).

a) One hundred eighty-three

Number: _____

Expanded Form: _____

Place Value Form:

100's	10's	1's

b) Four hundred thirty-two

Number: _____

Expanded Form: _____

Place Value Form:

100's	10's	1's

c) Nine hundred one

Number: _____

Expanded Form: _____

Place Value Form:

100's	10's	1's

3. Write the following numbers in place value form, numerical form, and word name form.

a) $600+30+5$

Place Value Form:

100's	10's	1's

Number: _____

Word Name: _____

b) $700+40+0$

Place Value Form:

100's	10's	1's

Number: _____

Word Name: _____

c) $100+20+5$

Place Value Form:

100's	10's	1's

Number: _____

Word Name: _____

Unit 1 – Practice Problems

4. Round using the place value method.

a) Round 283 to the nearest hundred

b) Round 352 to the nearest hundred

c) Round 106 to the nearest ten

d) Round 349 to the nearest hundred

e) Round 52 to the nearest ten

f) Round 819 to the nearest ten

g) Round 437 to the nearest hundred

h) Round 86 to the nearest hundred

i) Round 182 to the nearest hundred

j) Round 23 to the nearest hundred

k) Round 409 to the nearest ten

l) Round 409 to the nearest hundred

For 5 – 8: Round as indicated.

5. The Math Club raised \$127 with their bake sale. Round this to the nearest ten.

6. Kevin earned \$98 delivering pizza. Round this to the nearest ten.

7. The weekend trip cost \$412 per person. Round to the nearest hundred.

8. There were 26,577 tickets sold for the football game. Round this to the nearest hundred.

9. Place the numbers below in the place value chart. Use the chart to assist you in writing the place value form of the number

a) Thirty-eight thousand

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Place Value Form: _____

b) Seven hundred eight million

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Place Value Form: _____

Unit 1 – Practice Problems

c) Thirty-two billion

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

Place Value Form: _____

10. The population of the United States is constantly changing. According the website www.worldometers.info when I last checked, the population of the U.S. was listed as 323,352,941. Round this number to the nearest thousand.

11. The national debt is also constantly changing. The website www.usdebtclock.org shows real time estimates of the national debt. At one point, the estimate of the debt was \$17,882,815,724,883. Round this number to the nearest billion dollars.

12. Scientists don't know exactly how many cells are in the human body, but they estimate that there are about 37,200,000,000,000 cells. What place value are they rounding to?

13. Place the number 413,163,092,107 in the place value chart below and answer the corresponding questions.

BILLIONS			MILLIONS			THOUSANDS			ONES		
100	10	1	100	10	1	100	10	1	100	10	1

a) Determine the place value for the digit 6 and write what it represents as a word and a number.

b) Determine the digit in the ten thousand's place and write what it represents as a word and a number.

14. Melinda had three new pairs of shoes. She bought two more pairs of shoes. How many new pairs of shoes does she have now?

15. Melinda had three new pairs of shoes. She bought some more pairs of shoes. Now she has five new pairs of shoes. How many pairs of shoes did Melinda buy?

16. Melinda had some new pairs of shoes. She bought two more pairs of shoes. Now she has five new pairs of shoes. How many new pairs of shoes did Melinda have before she bought some more?

17. Connie had five pens at the beginning of the semester. She lost two of the pens during the first week. How many pens does Connie have left?

18. Connie had five pens at the beginning of the semester. She lost some of the pens during the first week. Now she has three pens left. How many pens did Connie lose?

19. Connie had some pens at the beginning of the semester. She lost two of the pens during the first week. Now she has three pens. How many pens did Connie have at the beginning of the semester?

20. There are four seniors and three juniors on the debate team. How many students are on the debate team?

21. There are seven students on the debate team. Four of the students are seniors and the rest are juniors. How many juniors are on the debate team?

Unit 1 – Practice Problems

22. Paul and Ed are going to lunch. Paul has \$10 and Ed has \$8. How much more money does Paul have than Ed?
23. Ed has \$8. Paul has \$2 more than Ed. How much money does Paul have?
24. Paul has \$2 more than Ed. Paul has \$10. How much does Ed have?
25. An instructor wanted to give 2 pencils to each student taking the final exam in the Introductory Algebra class. There are 25 students in the class. How many pencils did the instructor need?
26. Linda is taking her relatives on a hiking trip. She has 15 bottles of water that need to be placed into three different coolers. How many bottles will Linda put into each cooler if she wants the same number of bottles in each cooler?
27. A group of 30 incoming freshmen students is going to be divided into teams of five students to go on a campus tour. How many teams are there?
28. Packages of markers cost 75 cents each. How many cents does 4 packages cost?
29. Mary Ellen spent \$100 on four concert tickets. How much did each ticket cost?

30. Sara enrolled at a local college. She spent \$640 dollars on tuition. Each credit hour costs \$80. How many credit hours will Sara be taking?
31. Ronna has to read 2 books for her fine arts class. She has to read 3 times as many for her English class. How many books does Ronna have to read for her English class?
32. Sandy solved 12 word problems. This is 3 times as many as Nancy. How many word problems did Nancy solve?
33. Sandy solved 12 word problems. Nancy solved 4 word problems. How many times greater is the number of word problems Sandy solved compared with the number of word problems Nancy solved?
34. Leon has a rectangular dining room that is tiled. Leon counts 12 tiles along one wall and 10 tiles along an adjacent wall. How many tiles cover the floor of the dining room?
35. There are 24 desks in the classroom. Lisa notices that there are 4 rows. How many desks are in each row?
36. Roberto has four shirts and three pairs of slacks packed for his vacation trip. How many different outfits does Roberto have?
37. The local diner has 15 different ice cream sundaes consisting of one scoop of ice cream and a syrup. The diner has five different ice cream flavors. How many different types of syrup does the diner offer?

Unit 1 – Practice Problems

38. Jorge earned 37 points on a quiz. He makes corrections for extra credit and earns 8 more points. What is the total amount of points Jorge earned? Show all of your work and write your answer in a complete sentence.
39. Lisa tweeted 19 times in January and 33 times in February. How many times did she tweet in total? Show all of your work and write your answer in a complete sentence.
40. Vincent earns \$80 on Monday and he earns \$35 on Tuesday. How much total money does Vincent earn? Show all of your work and write your answer in a complete sentence.
41. On a road trip, Anna drove 420 miles on the first day and 380 miles on the second day. How many miles did she drive in all? Show all of your work and write your answer in a complete sentence.
42. Kathryn is 14 years younger than Joe. If Joe is 48 years old, how old is Kathryn? Show all of your work, and write your answer in a complete sentence.
43. Amy deposited \$650 into her checking account one month and withdrew \$220 to pay bills and expenses. How much money does she have left over after paying her bills? Show all of your work, and write your answer in a complete sentence.

44. It took Alice 45 minutes to drive to work this morning. On the way home, she ran into traffic and it took her 86 minutes. How much longer did the return trip take? Show all of your work, and write your answer in a complete sentence.
45. The temperature was 77°F and it drops 9 degrees. What is the new temperature? Show all of your work, and write your answer in a complete sentence.
46. Tally sprinted 1000 meters in 210 seconds on her first try and in 187 seconds on her second try. How much faster was her second try? Show all of your work, and write your answer in a complete sentence.
47. Sean buys a package of 20 chocolate chip cookies from the bakery and wants to divide them equally to each of the five members of his family. How many cookies will each person get?
48. Amber buys a package of 18 eggs, and wants to make 3-egg omelets. How many 3-egg omelets can she make?
49. Sara hiked uphill for 3 hours. Each hour, her elevation increased by 40 meters. Compute her change in elevation in meters relative to her starting point. Show all of your work, and write your answer in a complete sentence.

Unit 1 – Practice Problems

50. A baby gained 8 ounces per month for 4 months. Find the baby's total change in weight relative to her original weight. Show all of your work, and write your answer in a complete sentence.
51. Tally bought 20 packages of hot dog buns for her fair booth. Each package contained 9 buns. How many hot dog buns is this in total? Show all of your work, and write your answer in a complete sentence.
52. Daphne paid \$66 each month for one year for internet service. How much did she pay in total? Show all of your work, and write your answer in a complete sentence.
53. Together, 6 friends have 30 dollars. If they share the money equally, how much does each friend get? Show all of your work, and write your answer in a complete sentence.
54. Jorge bikes to school each day. If he can travel 36 miles in 4 hours, how fast does he travel in one hour? Show all of your work, and write your answer in a complete sentence.

55. Yvonne bought a total of 880 t-shirts. If there were 8 t-shirts per package. How many packages did Yvonne buy? Show all of your work, and write your answer in a complete sentence.

56. Write the following in symbolic form, then evaluate.

a) The sum of twelve and six	b) The product of twelve and six
b) The quotient of twelve and six	d) Twelve minus six.
e) Twelve divided by 6	f) Six times twelve
g) Six less than twelve	h) The difference between twelve and six

57. Suppose on the first day of the month you start with \$870 in your bank account. You make a debit transaction on the second day for \$130 and then make a deposit on the third day for \$402. What is the balance in your account on the third day?

58. Mark deposited \$450, \$312, \$125, and \$432 in his bank account this month. He also made deductions of \$205 and \$123. If his balance at the beginning of the month was \$1233, what was his balance at the end of the month?

Unit 1 – Practice Problems

59. An airplane took off and reached a cruising altitude of 34,000 feet. Over the next 4 hours due to weather, the plane descended 2,000 feet, rose 5,000 feet, descended 8,000 feet, and rose 12,000 feet. Determine the altitude of the plane at the end of the 4 hours.

60. Jane's monthly gross pay is \$3014. If she has the following deductions, what is her net pay?

Federal Tax: \$450

Savings Plan: \$24

FICA: \$244

State Tax: \$112

Insurance: \$233

61. Suppose you and your two siblings inherit \$90,000. You divide it amongst yourselves equally. You then invest your portion and make 4 times the amount of your portion. How much money do you have? Be sure to indicate every step in your process.

62. Martha works 40 hours per week and earns \$16 per hour. Determine her total pay for working 6 weeks. Be sure to indicate every step in your process.

63. Jenelle just financed a brand new 2015 Chevy Camaro. To pay off the loan, she agreed to make monthly payments of \$673 for the next five years. How much (total) will she end up paying over this five-year time period? Be sure to indicate every step in your process.
64. Amy drives to Costco to buy supplies for an upcoming event. She is responsible for providing breakfast to a large group of Boy Scouts the next weekend. Hashed browns are on her list of supplies to purchase and she needs to buy enough to serve 100 people. The hashed browns are sold in packs of 8 boxes and each box in the pack will serve 4 people.
- How many packs should she buy minimum?
 - How many people will she be able to serve with this purchase?
65. Bill went to the store and bought 4 twelve-packs of soda and an additional 2 six-packs of soda. How many cans of soda did he buy in total?
66. You join a local center in your community that has a swimming pool and a group that swims laps each week. The initial enrollment fee is \$105 and the group membership is \$44 a month. What are your dues for the first year of membership?
67. Tally bought dog food for an animal rescue shelter. She bought 6 bags that weighed 25 pounds each and 19 bags that weighed 7 pounds each. How many pounds of dog food did she buy?

Unit 1 – Practice Problems

68. Sam takes out a \$25,000 student loan to pay his expenses while he is in college. After graduation, he will begin making payments of \$168 per month for the next 20 years to pay off the loan. How much more will Sam end up paying for the loan than the original value of \$25,000?
69. Helen bought a \$19 pair of sunglasses and a \$42 pair of jeans. She paid with a \$100 bill. How much change will she receive? Be sure to indicate every step in your process.
70. Suppose that each semester at a particular community college Jose has to pay \$834 in tuition and \$53 in fees. If Jose has 3 semesters remaining, find the total amount he will need for tuition and fees for all three semesters.
71. Aisha, Jerry and Salma are making confetti for a school parade. Solve the problems below that describe the different strategies they used. Use a diagram to aid your work and write a corresponding mathematical expression using exponents.
- Jerry is making confetti for a school parade. He cuts one piece of paper into two pieces. He then cuts each of the two pieces into two pieces. He performed this process a total of 5 times. Determine how many pieces of paper he had after each cut.

- b. Aisha performed the same process as Jerry, but she cut the paper into 3 pieces each time (instead of two pieces) and only performed the process a total of 4 times. Determine how many pieces of paper she had after each cut.

- c. Salma started with 5 pieces of paper and then cut each piece into two pieces like Jerry did. However, she performed the process a total of 3 times. Determine how many pieces of paper she had after each cut.

- d. Write an expression (horizontally) that represents the total number of pieces of confetti made by the three students combined.

72. Represent the given exponential expressions in the four ways indicated.

a. 2^5

Expanded Form	
Word Name	
Factor Language	
Math Equation	

b. 5^2

Expanded Form	
Word Name	
Factor Language	
Math Equation	

Unit 1 – Practice Problems

c. 3^2

Expanded Form	
Word Name	
Factor Language	
Math Equation	

d. 2^3

Expanded Form	
Word Name	
Factor Language	
Math Equation	

UNIT 1 – END OF UNIT ASSESSMENT

1. Write the following number in numerical form, expanded form, and place value form (in a place value chart):

five hundred twenty-seven

Place value form in table:

100's	10's	1's

Number: _____

Expanded form: _____

2. Determine the place value for the digit 7 and write what it represents as a word and a number

3,516,274,809

3. Write thirty-eight million as a place value number.
4. Round 197,701 to the nearest thousand.
5. The record high temperature in Phoenix on June 19 was 118° . The low temperature for that day was 89° . What was the difference between the high and low temperatures on June 19th?

Unit 1 – Assessment

6. Donna went swimming two days last week. She swam a total of 800 meters. If she swam 350 meters the first day, how far did she swim the second day?
7. Connie has a piece of rope 52 feet long that she needs to cut into 4 pieces of equal length for a craft project.

Draw a picture to represent this situation.

How long is each piece of rope? Write your answer in a complete sentence.

8. Grant collects sports cards as a hobby. He has 3 times as many baseball cards as he has football cards. If he has 51 baseball cards how football cards does he have in total? Write you answer in a complete sentence.

9. Perform the indicated operations.

$$23 - [15 - (12 - 10)]$$

10. Perform the indicated operations.

$$12 - (6 - 3)^2$$

UNIT 2 – INTEGERS

INTRODUCTION

Now that we have discussed the Base-10 number system including whole numbers and place value, we can extend our knowledge of numbers to include integers. The first known reference to the idea of integers occurred in Chinese texts in approximately 200 BC. There is also evidence that the same Indian mathematicians who developed the Hindu-Arabic Numeral System also began to investigate the concept of integers in the 7th century. However, integers did not appear in European writings until the 15th century. After centuries of debate and opinions on the concept of integers, they were accepted as part of our number system and fully integrated into the field of mathematics by the 19th century.

Section	Learning Objective	Media Examples	You Try
2.1	Represent application problems using integers	1	2
2.2	Plot integers on a number line	3	4
2.3	Represent absolute value using number lines	5	6
2.4	Use number lines to find opposites	7	8
2.5	Order integers using number lines	9	10
2.6	Represent integers using the chip model	11	12
2.7	Use words to write integers and opposites using appropriate language	13	14
2.8	Add integers using the chip model	15	16
2.8	Add integers using number lines	17	18
2.9	Use words to write the subtraction of integers in multiple ways	19	20
2.9	Subtract integers using the chip model	21, 22	23
2.10	Rewrite subtraction problems as equivalent addition problems	24	25
2.11	Use patterns to add and subtract integers	26	27
2.12	Multiply integers using the chip model	28	29
2.12	Multiply integers using number lines	30	31
2.13	Divide integers using the chip model	32	33
2.14	Rewrite division problems as missing factor multiplication	34	
2.15	Use patterns to multiply integers	35	36
2.15	Use patterns to divide integers	37	38
2.16	Use correct language to read and evaluate exponents with integers	38	39
2.17	Use the correct order of operations for expressions with integers	40	41


UNIT 2 – MEDIA LESSON

SECTION 2.1: INTEGERS AND THEIR APPLICATIONS

Definition: The *integers* are all positive whole numbers and their opposites and zero.

$$\dots -4, -3, -2, -1, 0, 1, 2, 3, 4 \dots$$

The numbers to the left of 0 are *negative numbers* and the numbers to the right of 0 are *positive numbers*. We denote a negative number by placing a “–” symbol in front of it. For positive numbers, we either leave out a sign altogether or place a “+” symbol in front of it.

	Problem 1	MEDIA EXAMPLE – Integers and their Applications
---	-----------	---

Determine the signed number that best describes the statements below. Circle the word that indicates the sign of the number.

Statement	Signed Number
a) Tom gambled in Vegas and lost \$52	
b) Larry added 25 songs to his playlist.	
c) The airplane descended 500 feet to avoid turbulence.	

	Problem 2	YOU TRY – Integers and their Applications
---	-----------	---

Determine the signed number that best describes the statements below. Circle the word that indicates the sign of the number.

Statement	Signed Number
a) A balloon dropped 59 feet.	
b) The altitude of a plane is 7500 feet	
c) A submarine is 10,000 feet below sea level	

SECTION 2.2: PLOTTING INTEGERS ON A NUMBER LINE

Number lines are very useful tools for visualizing and comparing integers. We separate or “partition” a number line with tick marks into *segments of equal length* so the distance between any two consecutive major tick marks on a number line are equal.

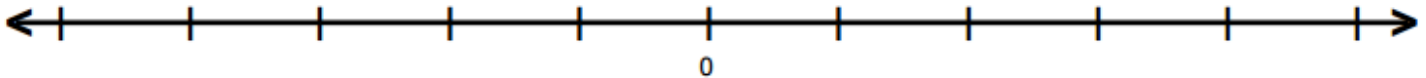


Problem 3

MEDIA EXAMPLE – Plotting Integers on a Number Line

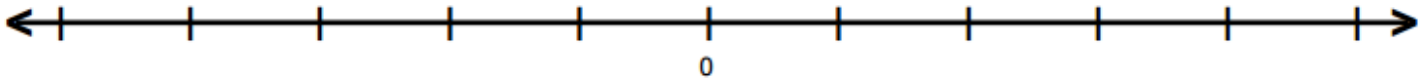
Plot the negative numbers that correspond to the given situations. Use a “•” to mark the correct quantity. Also label all the surrounding tick marks and scale the tick marks appropriately.

- a) The temperature in Greenland yesterday was $-5^{\circ}F$



What does 0 represent in this context?

- b) The altitude of the plane decreased by 60 feet.



What does 0 represent in this context?

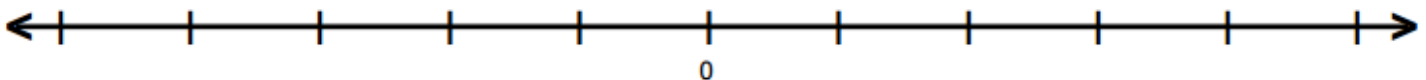


Problem 4

YOU TRY – Plotting Integers on a Number Line

Plot the negative numbers that correspond to the given situations. Use a “•” to mark the correct quantity. Also label all the surrounding tick marks and scale the tick marks appropriately.

Akara snorkeled 30 feet below the surface of the water. What does 0 represent in this context?



What does 0 represent in this context?

SECTION 2.3: ABSOLUTE VALUE AND NUMBER LINES

Definition: The *absolute value* of a number is the positive distance of the number from zero.

Notation: Absolute value is written by placing a straight vertical bar on both sides of the number.

$$|50| = 50 \text{ Read as "the absolute value of 50 equals 50"}$$

$$|-50| = 50 \text{ Read as "the absolute value of -50 equals 50"}$$



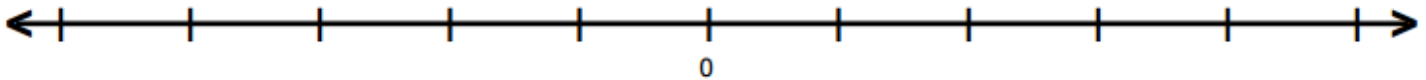
Problem 5

MEDIA EXAMPLE – Absolute Value and Number Lines

Answer the questions below based on the given example.

The submarine dove 15 meters below the surface of the water.

- a) What integer best represents the submarine’s location relative to the surface of the water?
- b) What word indicates the sign of this number?
- c) What does 0 represent in this context?
- d) Plot your number from *part a* and 0 on the number line below.
- e) Draw a line segment that represents this value’s distance from zero on the number line below.



- f) Write the symbolic form of the absolute value representation.



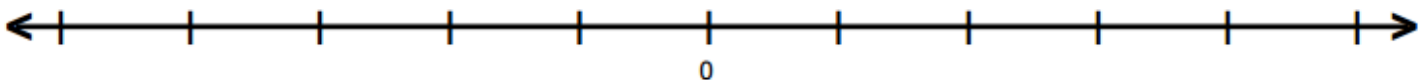
Problem 6

YOU TRY – Absolute Value and Number Lines

Answer the questions below based on the given example.

The temperature dropped 8 degrees overnight.

- a) What integer best represents the change in temperature?
- b) What word indicates the sign of this number?
- c) What does 0 represent in this context?
- d) Plot your number from *part a* and 0 on the number line below.
- e) Draw a line segment that represents this value’s distance from zero on the number line below.



- f) Write the symbolic form of the absolute value representation.

SECTION 2.4: OPPOSITES AND NUMBER LINES

Definition: The *opposite* of a nonzero number is the number that has the same absolute value of the number, but does not equal the number. Another useful way of thinking of opposites is to place a negative sign in front of the number.

The opposite of 4 is $-(4) = -4$

The opposite of -4 is $-(-4) = 4$

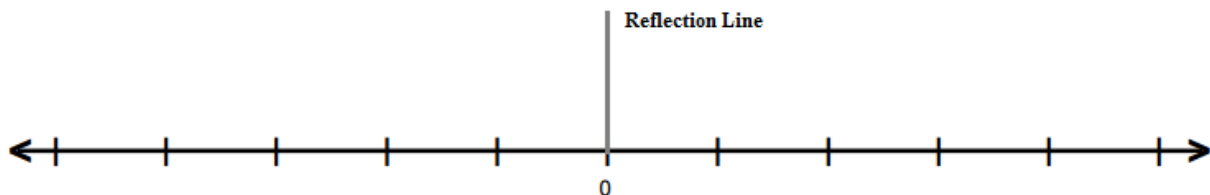


Problem 7

MEDIA EXAMPLE – Opposites and Number Lines

Answer the questions below to use number lines to find the opposite of a number.

- Plot the number 5 on the number line below.
- Draw an arrow that shows the reflection of 5 about the reflection line to find 5's opposite
- The opposite of 5, or $-(5)$ _____
- Draw an arrow that shows the reflection of -5 about the reflection line to find -5 's opposite
- The opposite of -5 , or $-(-5)$ is _____
- Based on the pattern above, what do you think $-(-(-5))$ equals?

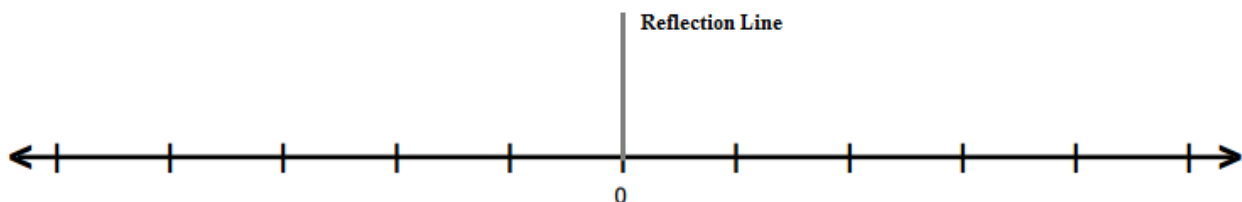


Problem 8

YOU TRY - Opposites and Number Lines

Answer the questions below to use number lines to find the opposite of a number.

- Plot the number -4 on the number line below.
- Draw an arrow that shows the reflection of -4 about the reflection line to find -4 's opposite
- The opposite of -4 , or $-(-4)$ is _____



SECTION 2.5: ORDERING INTEGERS USING NUMBER LINES

Fact: If two numbers are not equal, one must be less than the other. One number is *less than* another if it falls to the left of the other on the number line. Equivalently, if two numbers are not equal, one must be greater than the other. One number is *greater than* another if it falls to the right of the other on the number line.

Notation: We use inequality notation to express this relationship.

$$2 < 5, \text{ read "2 is less than 5"}$$

$$6 > 3, \text{ read "6 is greater than 3"}$$

Although we typically read the “ $<$ ” sign as “less than” and the “ $>$ ” sign as “greater than” because of the equivalency noted above, we can also read them as follows:

$$2 < 5, \text{ is equivalent to "5 is greater than 2"} \quad 6 > 3, \text{ is equivalent to "3 is less than 6"}$$

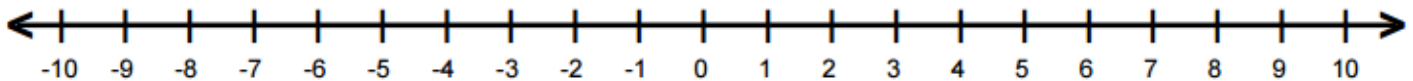


Problem 9

MEDIA EXAMPLE – Ordering Integers Using Number Lines

Plot the given numbers on the number line. Determine which number is greater and insert the correct inequality symbol in the space provided.

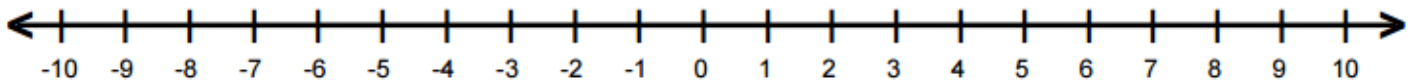
- a) Plot -5 and 3 on the number line below.



Write the number that is further to the right: _____

Insert the correct inequality symbol in the space provided: -5 ____ 3 3 ____ -5

- b) Plot -4 and -7 on the number line below.



Write the number that is further to the right: _____

Insert the correct inequality symbol in the space provided: -4 ____ -7 -7 ____ -4

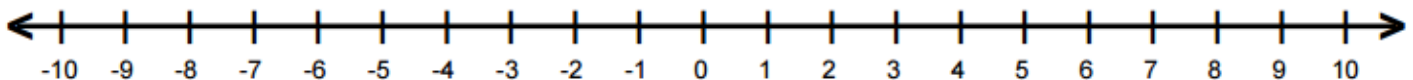


Problem 10

YOU TRY - Ordering Integers Using Number Lines

Plot the given numbers on the number line. Determine which number is greater and insert the correct inequality symbol in the space provided.

- Plot -8 and -2 on the number line below.

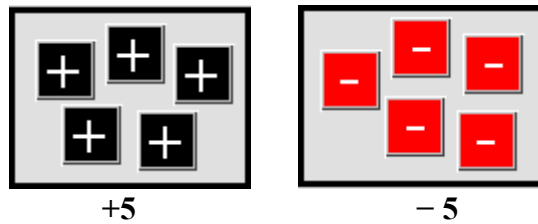



Write the number that is further to the right: _____

Insert the correct inequality symbol in the space provided: -8 ____ -2 -2 ____ -8


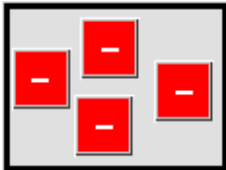
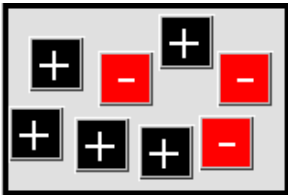
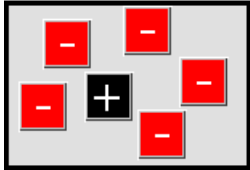
SECTION 2.6: REPRESENTING INTEGERS USING THE CHIP MODEL


Observe the two images below. Although they both have a total of 5 chips, the chips on the left are marked with "+" signs and the chips on the right are marked with "-" signs. This is how we indicate the sign each chip represents.



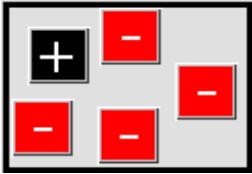
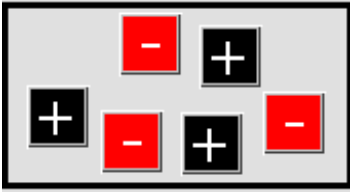
 **Problem 11** **MEDIA EXAMPLE – Representing Integers Using the Chip Model**

Determine the value indicated by the sets of integer chips below.

<p>a)</p>  <p>Number: _____</p>	<p>b)</p>  <p>Number: _____</p>
<p>c)</p>  <p>Number: _____</p>	<p>d)</p>  <p>Number: _____</p>

 **Problem 12** **YOU TRY - Representing Integers Using the Chip Model**

Determine the value indicated by the sets of integer chips below.

<p>a)</p>  <p>Number: _____</p>	<p>b)</p>  <p>Number: _____</p>
--	---


SECTION 2.7: THE LANGUAGE AND NOTATION OF INTEGERS

The + symbol:

1. In the past, you have probably used the symbol + to represent addition. Now it can also represent a positive number such as + 4 read “positive 4”.
2. Let’s agree to say the word “plus” when we mean addition and “positive” when we refer to a number’s sign.

The – symbol:

1. In the past, you have probably used the symbol – to represent subtraction. Now it can also mean a negative number such as –4 read “negative 4” or “the opposite of 4”
2. Let’s agree to say the word “minus” when we mean subtraction and “negative” when we refer a number’s sign.

 Problem 13	MEDIA EXAMPLE – The Language and Notation of Integers
--	---

Write the given numbers or mathematical expressions using correct language using the words “opposite of”, “negative”, “positive”, “plus” or “minus”.

Number or Expression	Written in Words
a) -6	
b) $-(-6)$	
c) $(-3)+2$	
d) $3-(-4)$	

 Problem 14	YOU TRY - The Language and Notation of Integers
--	---

Write the given numbers or mathematical expressions using correct language using the words “opposite of”, “negative”, “positive”, “plus” or “minus”.

Number or Expression	Written in Words
a) -3	
b) $-(-7)$	
c) $(-4)+(-2)$	
d) $1-(-5)$	

SECTION 2.8: ADDING INTEGERS



Problem 15

MEDIA EXAMPLE – Adding Integers Using the Chip Model

We call the numbers we are adding in an addition problem the *addends*. We call the simplified result the *sum*.

a) Using integer chips, represent positive 5 and positive 3. Find their sum by combining them into one group.

Addend	Addend	Sum
		$5 + 3 =$

b) Using integer chips, represent negative 5 and negative 3. Find their sum by combining them into one group.

Addend	Addend	Sum
		$(-5) + (-3) =$

c) Using integer chips, represent positive 5 and negative 3. Find their sum by combining them into one group.


Addend	Addend	Sum
		$5 + (-3) =$

d) Using integer chips, represent negative 5 and positive 3. Find their sum by combining them into one group.

Addend	Addend	Sum
		$(-5) + 3 =$

Summary of the Addition of Integers

<p>When adding two numbers with the same sign,</p> <ol style="list-style-type: none"> 1. Add the absolute values of the numbers 2. Keep the common sign of the numbers 	<p>When adding two numbers with different signs,</p> <ol style="list-style-type: none"> 1. Find the absolute value of the numbers 2. Subtract the smaller absolute value from the larger absolute value 3. Keep the original sign of the number with the larger absolute value.
--	--


	Problem 16	YOU TRY - Adding Integers Using the Chip Model
---	------------	--

a) Using integer chips, represent negative 6 and negative 4. Find their sum by combining them into one group.

Addend	Addend	Sum
		$(-6) + (-4) =$

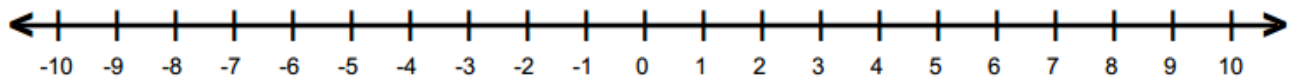
b) Using integer chips, represent negative 6 and positive 4. Find their sum by combining them into one group.

Addend	Addend	Sum
		$(-6) + 4 =$

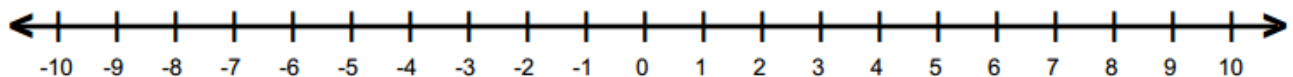
	Problem 17	MEDIA EXAMPLE – Adding Integers Using a Number Line
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Use a number line to represent and find the following sums.

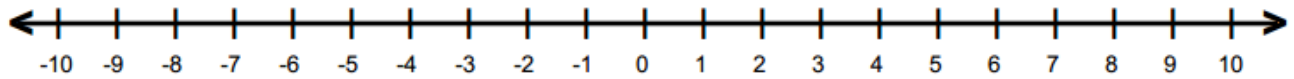
a) $5 + 3 =$



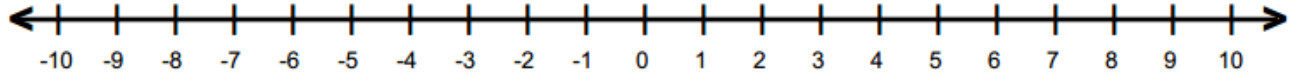
b) $(-5) + (-3) =$



c) $5 + (-3) =$



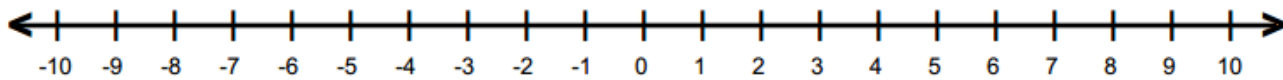
d) $(-5) + 3 =$



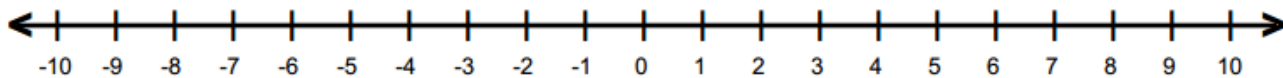
	Problem 18	YOU TRY – Adding Integers Using a Number Line
---	-------------------	--

Use a number line to represent and find the following sums.


a) $(-7) + (-3) =$



b) $(-7) + 3 =$




SECTION 2.9: SUBTRACTING INTEGERS
--


	Problem 19	MEDIA EXAMPLE – The Language of Subtraction
---	-------------------	--

Symbolic	<i>Minus Language</i>	<i>Subtracted from Language</i>	<i>Less than Language</i>	<i>Decreased by Language</i>
$5 - 3$				
$5 - (-3)$				

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 Problem 20	You Try – The Language of Subtraction
--	---------------------------------------

Symbolic	Minus Language	Subtracted from Language	Less than Language	Decreased by Language
$6 - (-5)$				

 Problem 21	MEDIA EXAMPLE – Subtracting Integers with Chips – Part 1
--	--


Using integer chips and the take away method, represent the following numbers and their difference.

a) $5 - 3$

Minuend	Subtrahend	Take Away	Simplified Difference
			$5 - 3 =$

b) $(-5) - (-3)$

Minuend	Subtrahend	Take Away	Simplified Difference
			$(-5) - (-3) =$

 Problem 22	MEDIA EXAMPLE – Subtracting Integers with Chips – Part II
--	---

Using integer chips and *the comparison method*, represent the following numbers and their difference.

a) $3 - 5$

Minuend	Subtrahend	Comparison	Simplified Difference
			$3 - 5 =$

b) $5 - (-3)$

Minuend	Subtrahend	Comparison	Simplified Difference
			$5 - (-3) =$

c) $(-5) - 3$

Minuend	Subtrahend	Comparison	Simplified Difference
			$(-5) - 3 =$

 Problem 23	YOU TRY – Subtracting Integers with Chips
---	---

Using integer chips and the method indicated to represent the following numbers and their difference.

a) $(-6) - (-2)$

Minuend	Subtrahend	Take Away	Simplified Difference
			$(-6) - (-2) =$

b) $3 - 4$

Minuend	Subtrahend	Comparison	Simplified Difference
			$3 - 4 =$

SECTION 2.10: CONNECTING ADDITION AND SUBTRACTION

You may have noticed that we did not write a set of rules for integer subtraction like we did with integer addition. The reason is that the set of rules for subtraction is more complicated than the set of rules for addition and, in general, wouldn't simplify our understanding. However, there is a nice connection between integer addition and subtraction that you may have noticed. We will use this connection to rewrite integer subtraction as integer addition.

Fact: Subtracting an integer from a number is the same as adding the integer's opposite to the number.

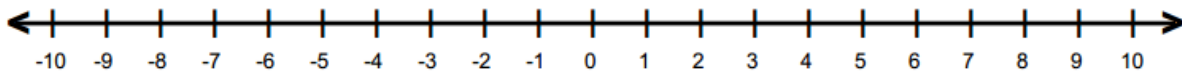


Problem 24

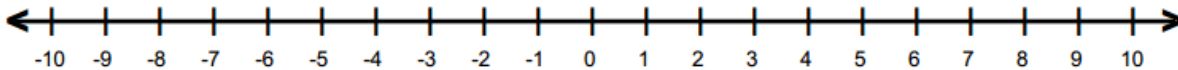
MEDIA EXAMPLE – Rewriting Subtraction as Addition

Rewrite the subtraction problems as equivalent addition problems and use a number line to compute the result.

a) $4 - 7$ Rewrite as addition:



b) $6 - (-2)$ Rewrite as addition:

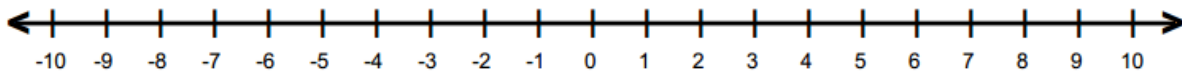


Problem 25

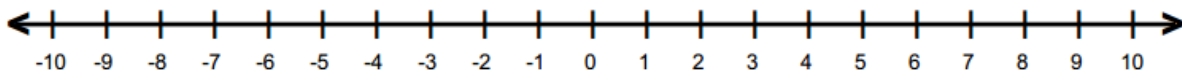
YOU TRY – Rewriting Subtraction as Addition

Rewrite the subtraction problems as equivalent addition problems and use a number line to compute the result.

a) $(-4) - 6$ Rewrite as addition:



b) $3 - (-4)$ Rewrite as addition:



SECTION 2.11: USING PATTERNS TO ADD AND SUBTRACT INTEGERS

Thus far, we have only added and subtracted single digit integers. Now we will use the ideas developed in this lesson to add and subtract larger numbers. We will follow the protocol below.

1. If given a subtraction problem, rewrite it as an addition problem.
2. Use the rules for addition to add the signed numbers as summarized below.

<p>When adding two numbers with the same sign,</p> <ol style="list-style-type: none"> 1. Add the absolute values of the numbers 2. Keep the common sign of the numbers 	<p>When adding two numbers with different signs,</p> <ol style="list-style-type: none"> 1. Find the absolute value of the numbers 2. Subtract the smaller absolute value from the larger absolute value 3. Keep the original sign of the number with the larger absolute value.
--	--


Problem 26
MEDIA EXAMPLE – Using Patterns to Add and Subtract Integers

Solve the addition and subtraction problems below.

a) $308 + 275$

b) $308 - 275$

- c) Use your results from above and your knowledge of integer addition and subtraction to find the following.

$(-275) + (-308) = \underline{\hspace{2cm}}$

$275 - (-308) = \underline{\hspace{2cm}}$

$(-275) + 308 = \underline{\hspace{2cm}}$

$(-275) - (-308) = \underline{\hspace{2cm}}$

$275 + (-308) = \underline{\hspace{2cm}}$

$(-275) - 308 = \underline{\hspace{2cm}}$


Problem 27
YOU TRY – Using Patterns to Add and Subtract Integers

Solve the addition and subtraction problems below.

a) $324 + 137$

b) $324 - 137$

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- c) Use your results from above and your knowledge of integer addition and subtraction to find the following.

$$(-137) + (-324) = \underline{\hspace{2cm}}$$

$$137 - (-324) = \underline{\hspace{2cm}}$$

$$137 + (-324) = \underline{\hspace{2cm}}$$

$$(-137) - 324 = \underline{\hspace{2cm}}$$

SECTION 2.12: MULTIPLYING INTEGERS



Problem 28

MEDIA EXAMPLE – Multiplying Integers Using the Chip Model

- a) Use integer chips to represent and evaluate $3 \cdot 5$

Number of Groups	Number in Each Group	Product	Symbolic Forms Repeated Addition and Multiplication

- b) Use integer chips to represent and evaluate $3(-5)$

Number of Groups	Number in Each Group	Product	Symbolic Forms Repeated Addition and Multiplication

- c) $(-3) \times 5$ can be interpreted as “the opposite of 3 groups of 5”. Use your result from *part a* to fill in the blank below.

$$(-3) \times 5 = \underline{\hspace{2cm}}$$

- d) $(-3)(-5)$ can be interpreted as “the opposite of 3 groups of -5 ”. Use your result from *part b* to fill in the blank below.

$$(-3)(-5) = \underline{\hspace{2cm}}$$


 **Problem 29** **YOU TRY - Multiplying Integers Using the Chip Model**

a) Using integer chips, represent $3(-2)$ and find the resulting product.

Number of Groups	Number in Each Group	Product	Symbolic Forms Repeated Addition and Multiplication

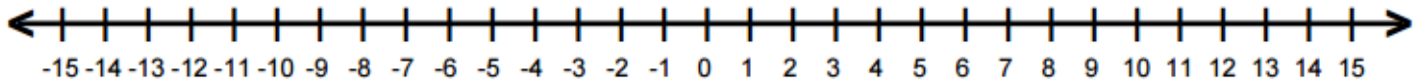
b) $(-3)(-2)$ can be interpreted as “the opposite of 3 groups of -2 ”. Use your result from *part a* to fill in the blank below.

$(-3)(-2) = \underline{\hspace{2cm}}$

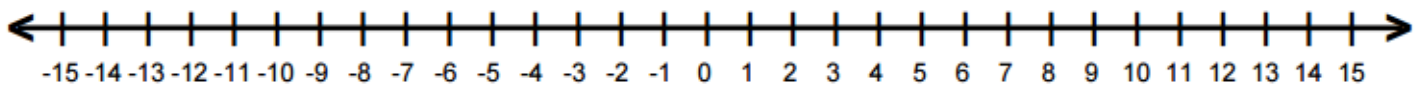
 **Problem 30** **MEDIA EXAMPLE – Multiplying Integers Using a Number Line**

Use a number line to represent and find the following products.

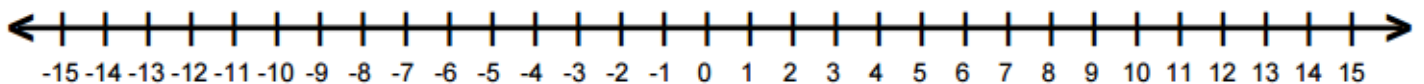
a) $3 \cdot 5$



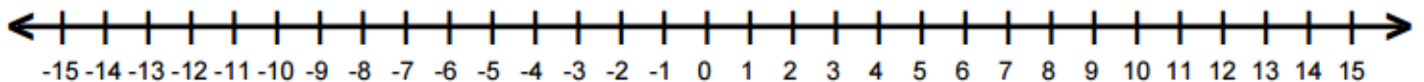
b) $(-3) \cdot 5$



c) $3(-5)$



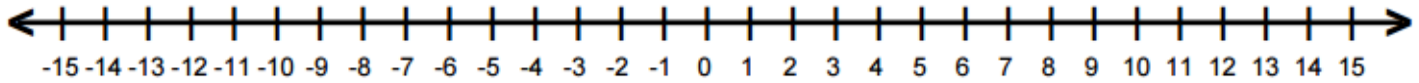
d) $(-3)(-5)$



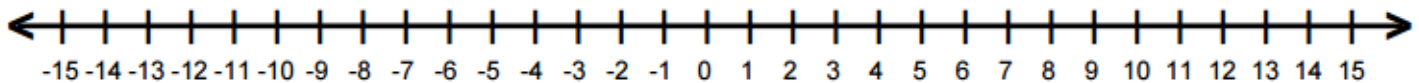
	Problem 31	YOU TRY – Multiplying Integers Using a Number Line
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Use a number line to represent and find the following products.


a) $(-4) \cdot 3$



b) $5(-2)$



SECTION 2.13: DIVIDING INTEGERS

	Problem 32	MEDIA EXAMPLE – Dividing Integers with Chips
---	------------	--

a) Use the chip model to determine $12 \div 4$

Dividend	Divisor (group size)	Represent dividend and circle divisor size groups	Symbolic Forms (Division Symbol and Fraction Symbol)

b) Use the chip model to determine $(-12) \div (-4)$


Dividend	Divisor (group size)	Represent dividend and circle divisor size groups	Symbolic Forms (Division Symbol and Fraction Symbol)

c) $(-12) \div 4$ can be interpreted as “the opposite of $12 \div 4$ ”. Use your result from *part a* to fill in the blank below.

$$(-12) \div 4 = \underline{\hspace{2cm}}$$

d) $12 \div (-4)$ can be interpreted as “the opposite of $(-12) \div (-4)$ ”. Use your result from *part b* to fill in the blank below.

$$12 \div (-4) = \underline{\hspace{2cm}}$$

 Problem 33	YOU TRY - Dividing Integers with Chips
--	--

- a) Use the chip model to determine $(-15) \div (-5)$

Dividend	Divisor (group size)	Represent dividend and circle divisor size groups	Symbolic Forms (Division Symbol and Fraction Symbol)

- b) $15 \div (-5)$ can be interpreted as “the opposite of $(-15) \div (-5)$ ”. Use your result from *part a* to fill in the blank below.

$$15 \div (-5) = \underline{\hspace{2cm}}$$

SECTION 2.14: CONNECTING MULTIPLICATION AND DIVISION

There is a nice connection between integer multiplication and division that you may have noticed. We will use this connection to rewrite integer division as integer multiplication with a missing factor. This will show us a pattern to create a rule for determining the sign when we multiply or divide any integers.

 Problem 34	MEDIA EXAMPLE – Rewriting Division as Multiplication
---	--

- a) Rewrite the following division problems using groups of language and using the missing factor model.

Division Problem	Groups of Language	Missing Factor Model
$12 \div 4 = ?$	How many groups of 4 are in 12?	$? \cdot 4 = 12$
$(-12) \div (-4) = ?$		
$(-12) \div 4 = ?$		
$12 \div (-4) = ?$		

- b) Based on the table, fill in the blanks below that applies to *both* multiplying and dividing integers.

Summary: To multiply or divide two signed numbers

- Multiply or divide the absolute values.
- If both signs are the same, the sign of the result is _____.
- If the signs are different, the sign of the result is _____.

SECTION 2.15: USING PATTERNS TO MULTIPLY AND DIVIDE INTEGERS

Thus far, we have only multiplied and divided single digit integers. In this section, we will use patterns and our knowledge of integer multiplication and division to perform these operations with larger numbers.



Problem 35

MEDIA EXAMPLE – Using Patterns to Multiply Integers
--

a) Find $14 \cdot 23$

b) Use your results from above and your knowledge of integer multiplication to find the following.

$14 \cdot 23 = \underline{\hspace{2cm}}$

$(-14) \cdot 23 = \underline{\hspace{2cm}}$

$14(-23) = \underline{\hspace{2cm}}$

$(-14)(-23) = \underline{\hspace{2cm}}$



Problem 36

YOU TRY – Using Patterns to Multiply Integers
--

a) Find $25 \cdot 32$

b) Use your results from above and your knowledge of integer multiplication to find the following.

$25 \cdot 32 = \underline{\hspace{2cm}}$

$25(-32) = \underline{\hspace{2cm}}$

$(-25) \cdot 32 = \underline{\hspace{2cm}}$

$(-25)(-32) = \underline{\hspace{2cm}}$



Problem 37

MEDIA EXAMPLE – Using Patterns to Divide Integers

- a) Find $564 \div 4$
- b) Use your results from above and your knowledge of integer division to find the following.

$$564 \div (-4) = \underline{\hspace{2cm}} \qquad \frac{-564}{4} = \underline{\hspace{2cm}} \qquad (-564) \div (-4) = \underline{\hspace{2cm}}$$



Problem 38

YOU TRY – Using Patterns to Divide Integers

- a) Find $462 \div 3$
- b) Use your results from above and your knowledge of integer division to find the following.


$$462 \div (-3) = \underline{\hspace{2cm}} \qquad (-462) \div 3 = \underline{\hspace{2cm}} \qquad \frac{-462}{-3} = \underline{\hspace{2cm}}$$

SECTION 2.16: INTEGERS AND EXPONENTS

Recall from Unit 1 that an *exponential expression* represents repeated multiplication. We used the following forms and language that are shown below. We will use similar ideas for integers, but we will touch on a few components of integer notation that aren't obvious extensions of these forms.

Exponent Form	2^5
Expanded Form	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Word Name	Two to the fifth power
Factor Language	5 factors of 2
Math Equation	$2^5 = 32$

Unit 2 – Media Lesson

	Problem 39	MEDIA EXAMPLE – Language and Notation of Integer Exponents
---	------------	--

Represent the given exponential expressions in the four ways indicated.

c) 6^2


b) -6^2

Expanded Form		Expanded Form	
Word Name		Word Name	
Factor Language		Factor Language	
Math Equation		Math Equation	

c) $(-6)^2$

d) $(-5)^3$

Expanded Form		Expanded Form	
Word Name		Word Name	
Factor Language		Factor Language	
Math Equation		Math Equation	

	Problem 40	YOU TRY – Language and Notation of Exponents
---	------------	--

Represent the given exponential expressions in the four ways indicated.

a) -7^2

b) $(-7)^2$

Expanded Form		Expanded Form	
Word Name		Word Name	
Factor Language		Factor Language	
Math Equation		Math Equation	

SECTION 2.17: INTEGERS AND THE ORDER OF OPERATIONS

In this section, we will revisit the order of operations when the expressions contain integers. We follow the same process, PEMDAS, as we did for whole numbers

P	Simplify items inside Parentheses (), brackets [] or other grouping symbols first.
E	Simplify items that are raised to powers (Exponents)
M	Perform Multiplication and Division next
D	(as they appear from Left to Right)
A	Perform Addition and Subtraction on what is left.
S	(as they appear from Left to Right)



Problem 41

MEDIA EXAMPLE – Integers and the Order of Operations

Use a highlighter to highlight the operations in the problem. Then compute the results by using the correct order of operations. Check your results on your calculator.

a) $12 - (-5) + 6 - 2 + (-1)$

b) $(-24) \div 4 \div (-2)(-3)$

c) $26 \div 2 \cdot 5 - (-3)(-4)$

d) $7 - (2 - 3)^2$

e) $(-3)^2 - 4(-3) + 2$

f) $\frac{5(-2) + 4}{3 - 5}$



Problem 42

YOU TRY – Integers and the Order of Operations

Use a highlighter to highlight the operations in the problem. Then compute the results by using the correct order of operations. Check your results on your calculator.

a) $8 + (-5) - 6 - (-2) + 9$

b) $(-36) \div 9(-4)(-1)(2)$

c) $24 \div 4 - 2(-3)$

d) $(8 - 3)^2 - 4$

e) $(-4)^2 + 5(-4) - 6$

f) $\frac{9 - (-6)}{-1 + (-2)}$

UNIT 2 – PRACTICE PROBLEMS

1. Determine the signed number that best describes the statements below.

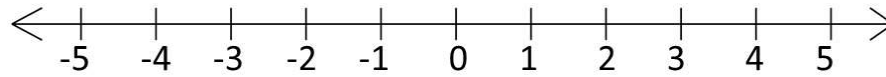
Statement	Signed Number
The boiling point of water is 212°F	
Carlos snorkeled 40 feet below the surface of the water	
Jack lost 32 pounds.	
Jill gained 5 pounds.	
The company suffered a net loss of twelve million dollars.	
The elevation of Death Valley is about 280 feet below sea level	
The elevation of Longs Peak is about 14,000 feet above sea level	

2. A golfer's score is based on the difference between the number of strokes and the predetermined par score for each hole. Complete the table below.

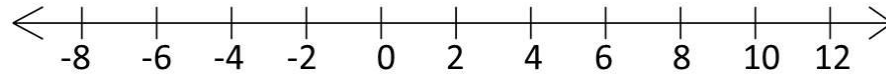
Name	Definition	Signed Number
Triple Bogey	Three strokes over par	3
Double Bogey	Two strokes over par	
Bogey		1
Par	Par	0
Birdie	One stroke under par	-1
Eagle	Two strokes under par	
Albatross (Double Eagle)	Three strokes under par	
Condor		-4

Unit 2 – Practice Problems

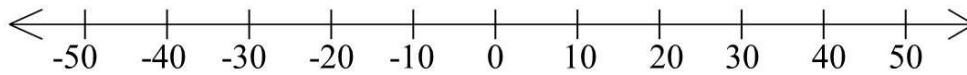
3. Plot the numbers 4 and -1 on the number line below.



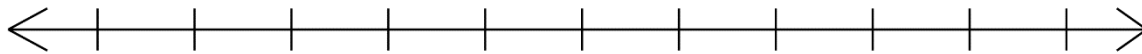
4. Plot the numbers 4 and -1 on the number line below.



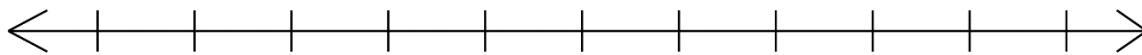
5. Plot the numbers -20 , -5 , and 30 on the number line below.



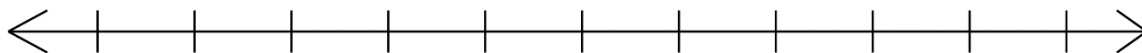
6. Label the following number line so that it includes 0 and the integers from -3 to 7.



7. Label the following number line so that it includes 0 and the integers from -100 to 100.

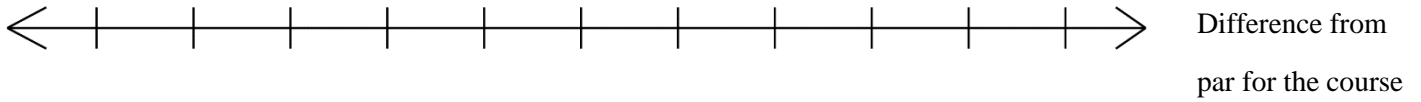


8. Label the following number line so that it includes 0 and the integers from $-8,000$ to 12,000.

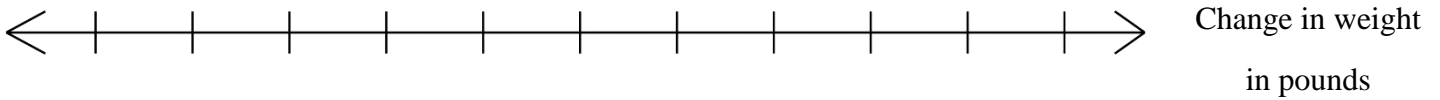


9. Plot the numbers that correspond to the given situations. Use a “•” to mark the correct quantity. Also label all the surrounding tick marks. Make sure to include 0 on your number line and scale the tick marks appropriately.

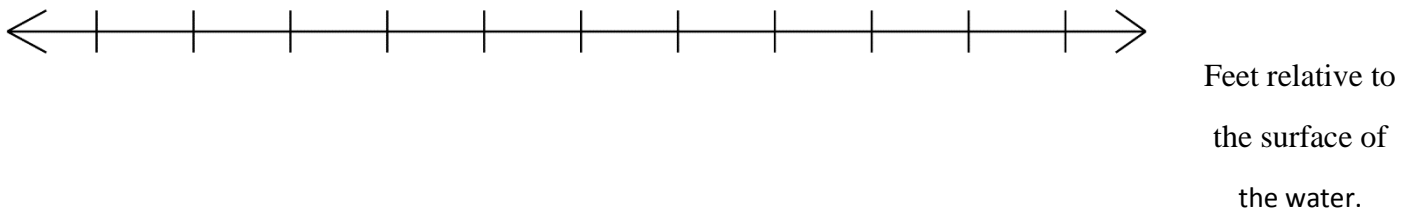
a. In golf, an “eagle” is two strokes under par.



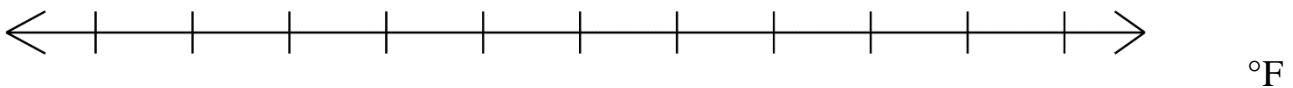
b. Shelby lost 8 pounds



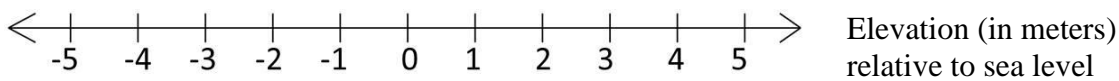
c. Juan snorkeled 25 feet below the surface of the water



d. Liquid nitrogen evaporates at about -300°F .



10. Consider the number line shown below.



a. What does -3 represent in this situation? _____

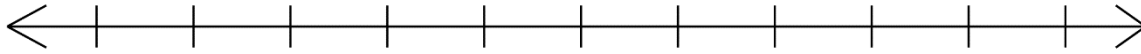
b. What does 2 represent in this situation? _____

c. What does 0 represent in this situation? _____

Unit 2 – Practice Problems

11. Jason snorkeled 30 feet below the surface of the water

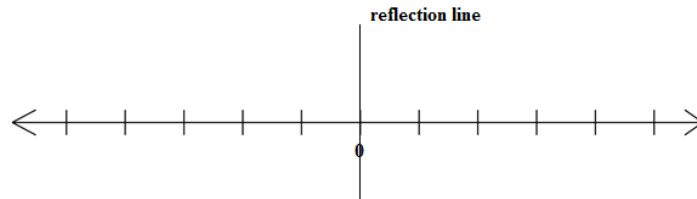
- a. Use a “•” to plot this quantity on the number line below and label all the surrounding tick marks. Make sure to include 0 on your number line and scale the tick marks appropriately.



- b. What does 0 represent in this context?

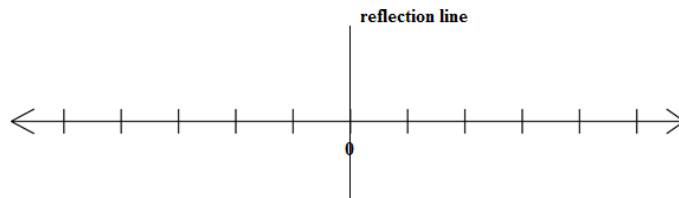
12. Use the number line to plot the given number and use the reflection line to find the opposite.

- a. Plot the number 2. Make sure to scale the tick marks on your number line appropriately.



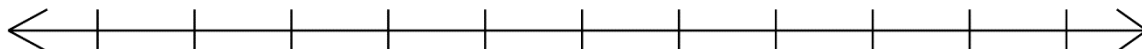
The opposite of 2 is _____

- b. Plot the number -30 . Make sure to scale the tick marks on your number line appropriately.

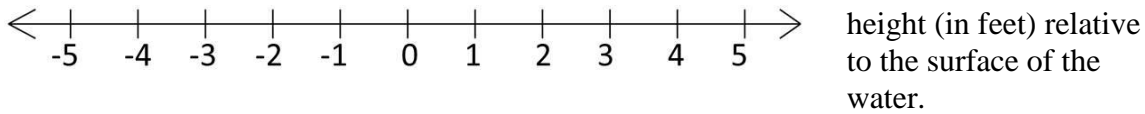


The opposite of -30 is _____

13. Label the following number line so that it includes 0 and the integers from -100 to 100 . Then use a “•” to mark the following values: $-80, -30, 10, 60$

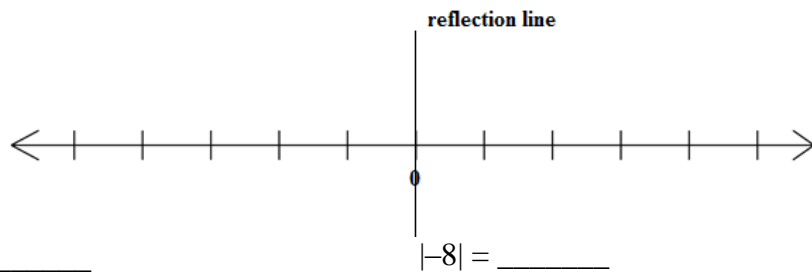


14. Consider the number line shown below.



- a. What does -4 represent in this situation? _____
- b. What does 1 represent in this situation? _____
- c. What does 0 represent in this situation? _____

15. Plot the number -8 . Make sure to scale the tick marks on your number line appropriately.



The opposite of -8 is _____

$|-8| =$ _____

16. Insert the correct inequality symbol in the space provided.

a. 3 _____ 9

g. 390 _____ -400

l. $-|-5|$ _____ $|-5|$

b. -5 _____ 1

h. -23 _____ -487

m. 0 _____ $-|-21|$

c. 0 _____ -8

i. $|-40|$ _____ -40

n. -435 _____ -543

d. 312 _____ 213

j. $|-8|$ _____ $|5|$

o. $1,213$ _____ $1,123$

e. -8 _____ -2

k. $|-4|$ _____ 0

p. $-4,651$ _____ $-4,650$

f. -400 _____ -450

Unit 2 – Practice Problems

17. Write TRUE or FALSE in the space provided.

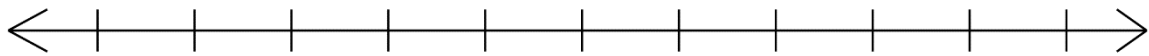
If two numbers are positive, the one that is closest to zero is greater. _____

If two numbers are negative, the one that is closest to zero is greater. _____

If one number is positive and one number is negative, the positive number is greater. _____

18. Camden, SC had a record low temperature of -19°F on Jan 21, 1985, and Monahans, TX had a record low temperature of -23°F on Feb 8, 1933. (*Data Source Wikipedia: http://en.wikipedia.org/wiki/U.S._state_temperature_extremes*)

a. Plot these numbers on the number line below, and label all the surrounding tick marks. Make sure to include 0 on your number line and scale the tick marks appropriately.

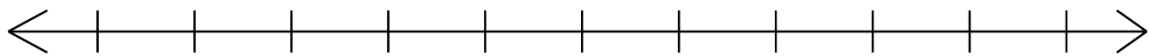


b. Write an inequality statement that compares the two numbers.

c. Which of the two temperatures was colder?

19. Liquid hydrogen evaporates at about -400°F Liquid nitrogen evaporates at about -300°F

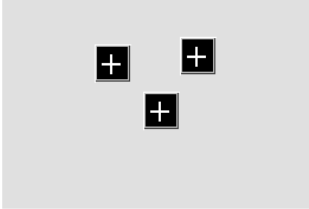
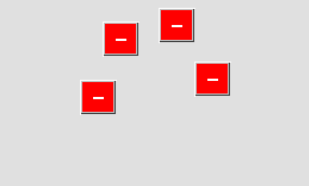
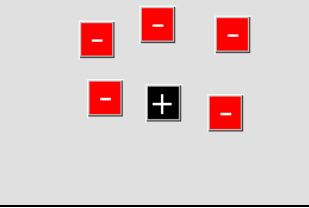
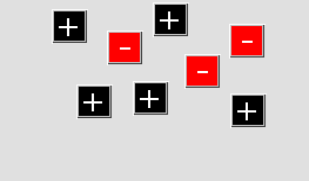
a. Plot these numbers on the number line below, and label all the surrounding tick marks. Make sure to include 0 on your number line and scale the tick marks appropriately.



b. Write an inequality statement that compares the two numbers.

c. Which liquid has the lower evaporating temperature?

20. Determine the value indicated by the sets of integer chips below.

Chip Representation	Number
a) 	
b) 	
c) 	
d) 	

21. Use integer chips to represent -2 in three different ways.

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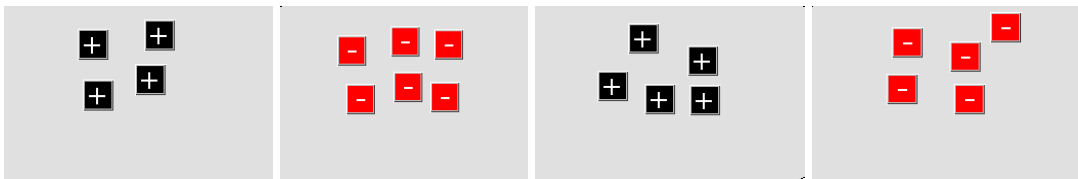
22. Use integer chips to represent 4 in three different ways.

--	--	--

23. Use integer chips to represent 0 in three different ways.

--	--	--

24. Write the following numbers from least to greatest.



Ordering from least to greatest: _____

25. Write the following numbers from least to greatest.



Ordering from least to greatest: _____

26. Write “+” or “-” in the blank next to each of the following words.

___ negative ___ opposite ___ plus ___ positive ___ minus

27. Write the given numbers or mathematical expressions using correct language using the words “opposite of”, “negative”, “positive”, “plus”, or “minus”.

Number or Expression	Written in Words
a. -5	
b. $-(-5)$	
c. $+5$	
d. $5 - 3$	
e. $-(+2)$	
f. $1 + 7$	
g. $-2 + 6$	
h. $4 + (-9)$	
i. $-(5 - 1)$	

Unit 2 – Practice Problems

28. Complete the table.

Symbolic	<i>Minus Language</i>	<i>Subtracted from Language</i>	<i>Less than Language</i>	<i>Decreased by Language</i>
$7-4$				
$7-(-4)$				

29. Using integer chips, represent the expressions and their combined amount. Use the table to show how you did this using + for positive chips and – for negative chips.

a. Using integer chips, represent $4 + 2$ and find the sum.

Addend	Addend	Sum

b. Using integer chips, represent $-4 + (-2)$ and find the sum.

Addend	Addend	Sum

c. Using integer chips, represent $-3 + (-3)$ and find the sum.

Addend	Addend	Sum

- d. Using integer chips, represent $-3 + 5$ and find the sum.

Addend	Addend	Sum

- e. Using integer chips, represent $6 + (-4)$ and find the sum.

Addend	Addend	Combined Sum	Simplified Sum

- f. Using integer chips, represent $-6 + 4$ and find the sum.

Addend	Addend	Combined Sum	Simplified Sum

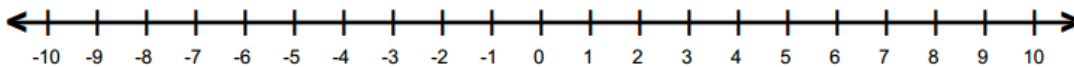
- g. Using integer chips, represent $-5 + 5$ and find the sum.

Addend	Addend	Combined Sum	Simplified Sum

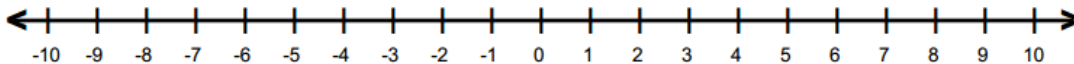
Unit 2 – Practice Problems

30. Use a number line to find the following sums.

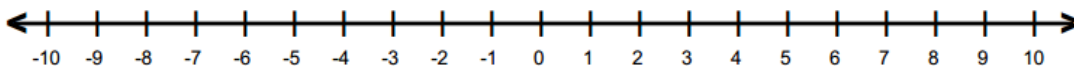
a. $4 + 2$



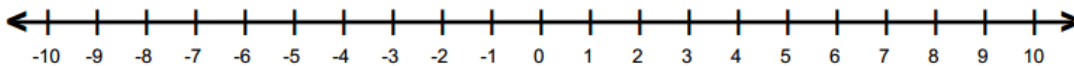
b. $3 + (-1)$



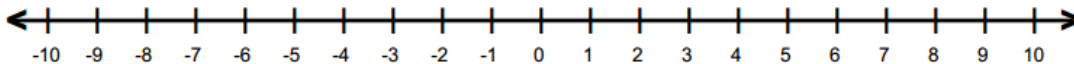
c. $-2 + 7$



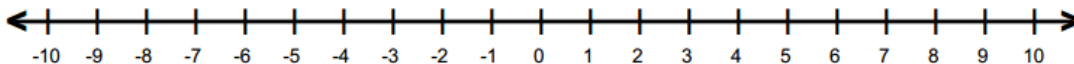
d. $5 + (-5)$



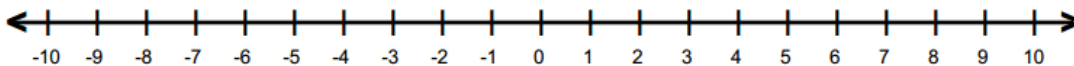
e. $-3 + (-3)$



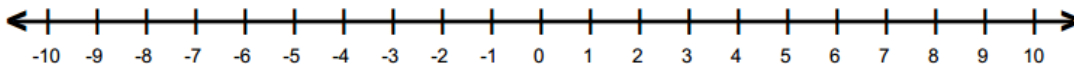
f. $-2 + 2$



g. $8 + (-9)$



h. $-5 + 8$



31. Kathryn is 14 years younger than Joe. If Joe is 48 years old, how old is Kathryn? Show all of your work, and write your answer in a complete sentence.
32. Amy deposited \$650 into her checking account one month and withdrew \$220 to pay bills and expenses. How much money does she have left over after paying her bills? Show all of your work, and write your answer in a complete sentence.
33. It took Alice 45 minutes to drive to work this morning. On the way home, she ran into traffic and it took her 86 minutes. How much longer did the return trip take? Show all of your work, and write your answer in a complete sentence.
34. Tally sprinted 1000 meters in 210 seconds on her first try and in 187 seconds on her second try. How much faster was her second try? Show all of your work, and write your answer in a complete sentence.

Unit 2 – Practice Problems

35. Using integer chips, represent the following numbers and their difference. Use the table to show how you did this using + for positive chips and – for negative chips.

a. $5 - 3$

Minuend	Subtrahend	Circle Subtrahend Taken Away from Minuend	Simplified Difference

b. $(-5) - (-3)$

Minuend	Subtrahend	Circle Subtrahend Taken Away from Minuend	Simplified Difference

c. $2 - 6$

Minuend	Subtrahend	Circle Subtrahend Taken Away from Minuend	Simplified Difference

d. $(-6) - 2$

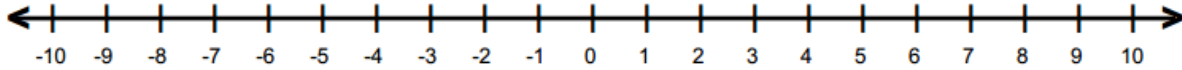
Minuend	Subtrahend	Circle Subtrahend Taken Away from Minuend	Simplified Difference

e. $5 - (-4)$

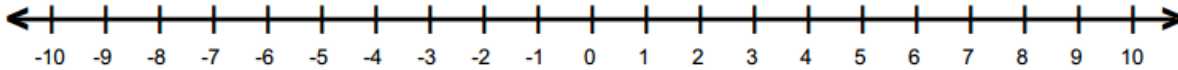
Minuend	Subtrahend	Circle Subtrahend Taken Away from Minuend	Simplified Difference

36. Rewrite the following as equivalent addition problems and use a number line to compute the result.

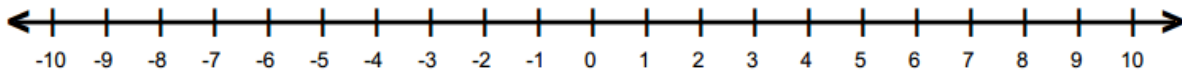
a) $6 - (-4)$ Rewrite as addition:



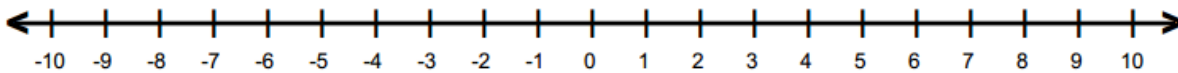
b) $(-5) - (-3)$ Rewrite as addition:



c) $-2 - 4$ Rewrite as addition:



d) $3 - 6$ Rewrite as addition:



Unit 2 – Practice Problems

37. Rewrite the following as addition problems and compute.

Subtraction Problem	Rewrite as Addition	Compute Result
a. $5 - (-2)$		
b. $-5 - (-2)$		
c. $5 - 2$		
d. $-5 - 2$		
e. $2 - 5$		
f. $-2 - 5$		
g. $-2 - (-5)$		
h. $2 - (-5)$		
i. $5 - 5$		
j. $-5 - 5$		
k. $-5 - (-5)$		
l. $5 - (-5)$		

38. Represent the application problem using addition in symbolic form and evaluate. Then write your answer as a complete sentence.
(**Note:** *Make sure to use an addition statement even though a subtraction statement may apply as well*).
- Kayla camped at -9 miles relative to sea level. She then hiked 4 miles upwards. What is her current altitude relative to sea level?
 - Tom gained 10 pounds and then lost 12 pounds. What is his total change in weight relative to his original weight?
 - Sheldon has 140 dollars in his checking account and Penny has -150 dollars in her checking account. How much did they have all together?
 - A plane descended 1400 feet. Twenty minutes later, it descended another 1200 feet. What is the total change in altitude of the plane relative to its original altitude?

40. Perform the indicated operations.

a. $35 - (-22)$

b. $46 - 58$

c. $-140 + (-200)$

d. $-310 + 104$

e. $57 - 18$

f. $-35 - (-35)$

g. $12 - 30$

h. $41 - (-41)$

Unit 2 – Practice Problems

41. Use integer chips to represent and evaluate $5(2)$.

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

42. Use integer chips to represent and evaluate $2(-6)$.

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

43. Use integer chips to represent and evaluate 4×-3 .

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

44. Use integer chips to represent and evaluate $-5 \cdot 2$.

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

45. Use integer chips to represent and evaluate $-2(-6)$.

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

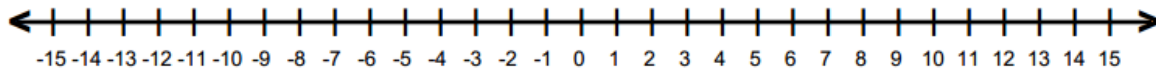
46. Use integer chips to represent and evaluate -4×-3 .

Number of Copies	Number in Each Copy	Product	Symbolic Form Repeated Addition and Multiplication

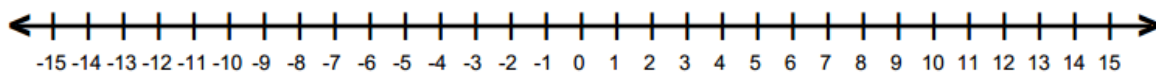
Unit 2 – Practice Problems

47. Use a number line to find the following products.

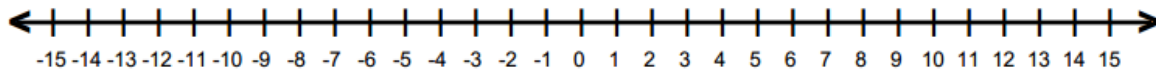
a) $-6 \cdot 2$



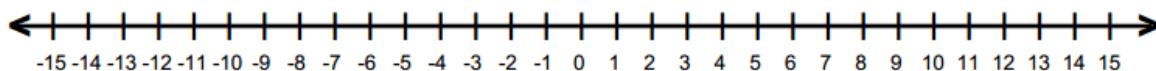
b) $4(-2)$



c) -3×-1



d) $-2(-5)$



48. Use the chip model to determine $30 \div 10$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

49. Use the chip model to determine $(-24) \div (-4)$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

50. Use the chip model to determine $(-9) \div (-9)$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

Unit 2 – Practice Problems

51. Use the chip model to determine $-20 \div 4$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

52. Use the chip model to determine $32 \div (-4)$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

53. Use the chip model to determine $-4 \div 1$

Dividend (Goal)	Divisor (Copy Size)	Circle Number of Copies to Reach Goal	Math Equation in Symbolic Forms (Division Symbol and Fraction Symbol)

54. Rewrite the following division problems using copies of language and using the missing factor model.

a.

Division Problem	Groups Language	Missing Factor Model
$32 \div 8 = ?$	How many groups of 8 are in 32?	$? \cdot 8 = 32$
$-32 \div -8 = ?$		
$-32 \div 8 = ?$		
$32 \div -8 = ?$		

b.

Division Problem	Groups Language	Missing Factor Model
$12 \div 1 = ?$	How many groups of 1 are in 12?	$? \cdot 1 = 12$
$-12 \div -1 = ?$		
$-12 \div 1 = ?$		
$12 \div -1 = ?$		

c.

Division Problem	Groups Language	Missing Factor Model
$5 \div 0 = ?$	How many groups of 0 are in 5?	$? \cdot 0 = 5$
$-5 \div 0 = ?$		

d. Explain why problem c shows that dividing by zero yields an undefined answer.

Unit 2 – Practice Problems

55. Represent the application problem using multiplication in symbolic form and evaluate. Then write your answer as a complete sentence. Make sure to use signed numbers when appropriate based on the context of the problem.

- a. Sara hiked down a mountain for 3 hours. Each hour, her elevation decreased by 30 meters. Compute her change in elevation in meters relative to her starting point.

Symbolic form: _____

Answer as a Complete Sentence:

- b. Joanne lost 3 pounds per month for 6 months. Find Joanne’s total change in weight relative to her original weight.

Symbolic form: _____

Answer as a Complete Sentence:

- c. Leslie bought coffee 8 days this month and charged it to her checking account. She spent 6 dollars each time she visited the store. Determine the change in dollars in her checking account.

Symbolic form: _____

Answer as a Complete Sentence:

56. Represent the application problem using multiplication or division in symbolic form and evaluate. Then write your answer as a complete sentence. Make sure to use signed numbers when appropriate based on the context of the problem.

- a. A total of 10 friends have a debt of -50 dollars. If they share the debt equally, what number represents the change in dollars for each friend?

Symbolic form: _____

Answer as a Complete Sentence:

- b. Morgan bought gas 8 days this month and charged it to her checking account. She spent 12 dollars each time she visited the store. Determine the change in dollars in her checking account.

Symbolic form: _____

Answer as a Complete Sentence:

- c. The temperature in Minneapolis changed by -32 degrees in 8 days. If the temperature changed by the same amount each day, what was the change in temperature per day?

Symbolic form: _____

Answer as a Complete Sentence:

- d. Tally bought 50 packages of printer paper for her business. Each package contained 300 sheets of paper. How many sheets of paper is this in total?

Symbolic form: _____

Answer as a Complete Sentence:

Unit 2 – Practice Problems

57. Perform the indicated operations

$16 \cdot 25 = \underline{\hspace{2cm}}$

$-16 \cdot 25 = \underline{\hspace{2cm}}$

$16 \cdot (-25) = \underline{\hspace{2cm}}$

$-16 \cdot -25 = \underline{\hspace{2cm}}$

58. Perform the following operations

$213 \div (-3) = \underline{\hspace{2cm}}$

$\frac{-213}{3} = \underline{\hspace{2cm}}$

$(-213) \div (-3) = \underline{\hspace{2cm}}$

$635 \div (-5) = \underline{\hspace{2cm}}$

$\frac{-635}{5} = \underline{\hspace{2cm}}$

$(-635) \div (-5) = \underline{\hspace{2cm}}$

59. The chart below displays the weight loss or gain per week of five friends on a 6-week exercise program.

Name	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Total
Carlos	+2	+3	0	-2	-1	0	
Frank	-2	-2	-2	-3	-2	-2	
Jillian	-1	-2	0	-1	0	-1	
Sara	-4	-2	-3	-2	-1	-1	
Raj	+2	+1	-1	-1	-1	-1	
Total							

- Complete the Total Column and Total Row in the table below. (Note: Since the weight loss or gain is per week, each value in the table is only for that given week not the weeks prior.)
- In which week(s) was there the greatest weight loss?
- Which person(s) lost the most weight over the 6 weeks?

60. A golfer's scores relative to par for the first nine holes were -2, +1, +2, -1, +3, +1, 0, +2, -2. Determine the golfer's total score relative to par at the end of the nine holes.

61. Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a. $5+8-10$

Number of Operations Highlighted: _____

b. $6-9+3$

Number of Operations Highlighted: _____

c. $7+(-1)+5$

Number of Operations Highlighted: _____

d. $9-(-8)-1$

Number of Operations Highlighted: _____

e. $1-(-11)-8+(-2)$

Number of Operations Highlighted: _____

f. $-5+(-4)+11+(-15)$

Number of Operations Highlighted: _____

Unit 2 – Practice Problems

62. Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the convention of performing the operations from left to right.

a. $(-12) \cdot 2 \div (-6)$

Number of Operations Highlighted: _____

b. $16 \div 4 \cdot 4$

Number of Operations Highlighted: _____

c. $32 \cdot 5 \div 8 \div 2(-6)$

Number of Operations Highlighted: _____

d. $25 \cdot 2 \div (-10) \cdot 8$

Number of Operations Highlighted: _____

e. $(-12) \div 6 \cdot 7 \div 2$

Number of Operations Highlighted: _____

63. Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Perform the operations in the appropriate order. Show all intermediary steps.

a. $5 + 3 \cdot 4$

Number of Operations Highlighted: _____

b. $8 - 4 \div (-2)$

Number of Operations Highlighted: _____

c. $(-10) \div 2 + 5(-8)$

Number of Operations Highlighted: _____

d. $4 \div (-4) - 8(-2)$

Number of Operations Highlighted: _____

e. $6(2) - 5(10)$

Number of Operations Highlighted: _____

f. $4(-4) + 8(9)$

Number of Operations Highlighted: _____

Unit 2 – Practice Problems

64. Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Perform the operations in the appropriate order. Show all intermediary steps.

a. $36 \div (6 \cdot 2)$

Number of Operations Highlighted: _____

b. $(-3 + 1) \cdot 2$

Number of Operations Highlighted: _____

c. $4 \div (1 - 3)$

Number of Operations Highlighted: _____

d. $3 - 5(-7 + 3)$

Number of Operations Highlighted: _____

e. $4 \div (-1 - 3)$

Number of Operations Highlighted: _____

f. $-3 - 5(7 + 3)$

Number of Operations Highlighted: _____

65. Perform the operations in the appropriate order. Show all steps.

a. $30 \div 5 \cdot 3$

b. $30 \div (5 \cdot 3)$

c. $8 - 6 + 12$

d. $8 - (6 + 12)$

66. Represent the given exponential expressions in the four ways indicated.

a. -5^2

Expanded Form	
Word Name	
Factor Language	
Math Equation	

b. $(-5)^2$

Expanded Form	
Word Name	
Factor Language	
Math Equation	

Unit 2 – Practice Problems

67. Represent the given exponential expressions in the four ways indicated.

a. -2^3

Expanded Form	
Word Name	
Factor Language	
Math Equation	

b. $(-2)^3$

Expanded Form	
Word Name	
Factor Language	
Math Equation	

68. Represent the given exponential expressions in the four ways indicated.

a. $(-5)^4$

Expanded Form	
Word Name	
Factor Language	
Math Equation	

b. -6^2

Expanded Form	
Word Name	
Factor Language	
Math Equation	

c. $(-3)^5$

Expanded Form	
Word Name	
Factor Language	
Math Equation	

d. -5^4

Expanded Form	
Word Name	
Factor Language	
Math Equation	

69. Use a highlighter to highlight the operations in the problem. Determine the number of operations to be performed in the problem. Then compute the results by using the correct order of operations.

a. $5 + 3^2$

Number of Operations Highlighted: _____

b. $(5 + 3)^2$

Number of Operations Highlighted: _____

c. $-5^2 - 5^2$

Number of Operations Highlighted: _____

Unit 2 – Practice Problems

d. $(-5)^2 - (-5)^2$

Number of Operations Highlighted: _____

e. $6 - 3(-2)^2$

Number of Operations Highlighted: _____

f. $4(-5 \cdot 2)^2$

Number of Operations Highlighted: _____

g. $4 - 2(7 + 1 \cdot 6)^2$

Number of Operations Highlighted: _____

70. Use a calculator to check your work from the previous problem. Write the key strokes you used for each one.

a. $5 + 3^2$

Key strokes: _____

Final Answer: _____

b. $(5 + 3)^2$

Key strokes: _____

Final Answer: _____

c. $-5^2 - 5^2$

Key strokes: _____

Final Answer: _____

d. $(-5)^2 - (-5)^2$

Key strokes: _____

Final Answer: _____

e. $6 - 3(-2)^2$

Key strokes: _____

Final Answer: _____

f. $4(-5 \cdot 2)^2$

Key strokes: _____

Final Answer: _____

g. $4 - 2(7 + 1 \cdot 6)^2$

Key strokes: _____

Final Answer: _____

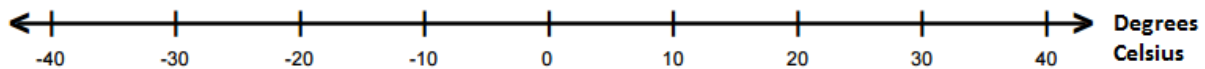
UNIT 2 – END OF UNIT ASSESSMENT

1. Determine the signed number that best describes the statement below.

The NASDAQ Stock Market was down 45 points last Wednesday.

2. Plot the number that corresponds to the given situation. Use a “•” to mark the correct quantity.

The element Chlorine has a boiling point of -30°C .



3. $-|-4| = \underline{\hspace{2cm}}$
4. $-(-4) = \underline{\hspace{2cm}}$
5. Order the following numbers from least to greatest.

$$-|-5| \quad 0 \quad -(-5) \quad -3 \quad |-3|$$

6. Combine the following numbers. Use a number line to help you visualize. Show steps if possible.

a) $4 - 6$

b) $9 - (-5)$

c) $2 + (-7)$

d) $(-12) + 2$

e) $(-1) + (-3)$

f) $(-8) - 6$

7. The average high temperature in Salt Lake City in December is 3°C . The average low temperature is 8°C lower. What is the average low temperature? Show your work. Write your answer in a complete sentence.

8. Determine whether the following statement is true or false:

The sum of two negative numbers is always a positive number.

TRUE FALSE

9. Lee lost \$200 on each of four consecutive days in the stock market. What was his total loss? Show your work. Write your answer in a complete sentence. Make sure to use signed numbers when appropriate based on the context of the problem.

10. Simplify the expression by performing all of the operations.

$$\frac{5 - (6 - 2)^2}{3 - 4}$$

UNIT 3 – DIVISIBILITY, FACTORS, AND MULTIPLES

INTRODUCTION

In this Unit, we will look at *decomposing*, or “breaking down” numbers multiplicatively. This means we will look at different ways to rewrite a number as a product of two or more factors. Specifically, we will want to find all the ways to write a number as a product of exactly 2 factors, and find a special factorization of a number called its prime factorization. These skills can be used to solve application problems, and will be essential in our future work with fractions.

Section	Learning Objective	Media Examples	You Try
3.1	Given a division problem, find the quotient and remainder	1	2
3.2	List all of the factors of a number	3	4
3.3	Determine if a number is prime or composite	5	6
3.4	Find the prime factorization of a number	7	8
3.5	Solve applications involving GCF and LCM	9	
3.5	Find the GCF by comparing lists of factors	10	11
3.6	Find the LCM by comparing lists of multiples	12	13
3.7	Use the prime factorizations of numbers to find their GCF and LCM	14	15

UNIT 3 – MEDIA LESSON

SECTION 3.1: FACTORS, DIVISORS, AND DIVISIBILITY

In this section, we will investigate factors, divisors, and divisibility. Consider the following fact family.

<i>Multiplication</i>	<i>Division</i>
$3 \cdot 4 = 12$	$12 \div 4 = 3$
$4 \cdot 3 = 12$	$12 \div 3 = 4$

In Unit 1, we said that 3 and 4 are the factors of the multiplication statements, and that 12 is the product. Now we will also say the following.

3 and 4 *are factors of* 12.

In Unit 1, we said that in the division statement, $12 \div 4 = 3$, 12 is the dividend, 4 is the divisor, and 3 is the quotient. Similarly, we said that in the division statement, $12 \div 3 = 4$, 12 is the dividend, 3 is the divisor, and 4 is the quotient. Now we will also say the following.

3 and 4 *are divisors of* 12

12 is *divisible by* 3

12 is *divisible by* 4

Notice that these statements are slightly different than our original statements. In the fact family, we were performing operations. Now we are stating a relationship between the numbers 3, 4, and 12. Specifically,

Definitions	Examples
A whole number n <i>is a factor of</i> a whole number p , if there exists another whole number m , so that $n \cdot m = p$	3 <i>is a factor of</i> 12 since $3 \cdot 4 = 12$ (Note this also implies that 4 is a factor of 12)
A whole number n <i>is a divisor of</i> a number p , if dividing p by n gives a quotient that is a whole number (without a remainder).	3 <i>is a divisor of</i> 12 since $12 \div 3 = 4$ (no remainder) (Note this also implies that 4 is a divisor of 12)
A whole number p <i>is divisible by</i> a whole number n , if dividing p by n gives a quotient that is a whole number (without a remainder).	12 <i>is divisible by</i> 3 since $12 \div 3 = 4$ (no remainder) (Note this also implies that 4 is a divisor of 12)

These definitions can be tricky to read. Since the relationships are connected, they seem to be saying the same thing. Review the definitions and their examples carefully, and make sure you can see the subtle differences in wording so you can use these words correctly. For example, *a factor of a number is also a divisor of a number* even though “factor” refers to multiplication and “divisor” refers to division. However, when we use the words “is divisible by” the direction of the relationship reverses. Meaning, *a number is divisible by its factors and divisors*.

Thus far, we have not performed a division problem where the remainder was nonzero. So we will begin by looking at a few examples of division with nonzero remainders.



Problem 1

MEDIA EXAMPLE – Division with Remainders

- a) Solve the following division problem by grouping the dividend in divisor size groups. Write your result symbolically as both multiplication and division equations.

$$29 \div 6 =$$



Division Equation: _____ Multiplication Equation: _____

- b) Solve the following division problem using a calculator. Write your result symbolically as both multiplication and division equations.

$$178 \div 19 =$$

Division Equation: _____ Multiplication Equation: _____



Problem 2

YOU TRY – Division with Remainders

- a) Solve the following division problem by grouping the dividend in divisor size groups. Write your result symbolically as both multiplication and division equations.

$$37 \div 5 =$$



Division Equation: _____ Multiplication Equation: _____

- b) Solve the following division problem using a calculator. Write your result symbolically as both multiplication and division equations.

$$112 \div 12 =$$

Division Equation: _____ Multiplication Equation: _____

SECTION 3.2: FINDING ALL THE FACTORS OF A NUMBER

Our next goal is to find all of the factors of a number. Sometimes when a number is smaller, we can determine its factors by just recalling all of its factor pairs from the multiplication table. However, when a number is larger, we want to have a *systematic process* to ensure we haven't missed any factors.

Method: To determine all of the factors of a whole number, we will find all the pairs of whole numbers whose product is the number. We will begin with the product of 1 and the number itself, and then check to see if the number is divisible by 2, 3, 4, etc.... Frequently we are going to use *division* to determine if a number is a factor. This is why it's important to have a strong understanding of the terms divisibility and factor. Our final concern is knowing when we have checked all the numbers necessary to find all the factors. An example will go a long way to show how we can determine when we've accomplished this task.

**Problem 3****MEDIA EXAMPLE – Finding All of the Factors of a Number**

Find all factors of the given numbers by finding factor pairs. Write your final answer as a list of factors separated by commas.

a) 18

List of Factors: _____

b) 90

List of Factors: _____

c) 154

List of Factors: _____



Problem 4

YOU TRY – Finding All of the Factors of a Number

Find all factors of the given numbers by finding factor pairs. Use the table of perfect squares to see what the largest number you have to check is. Write your final answer as a list of factors separated by commas.

84

List of Factors: _____

SECTION 3.3: PRIME AND COMPOSITE NUMBERS

In this section, we will investigate the concept of prime and composite numbers.

Definitions:

1. A **prime number** is a whole number greater than 1 whose factor pairs are only the number itself and one.

Examples: The smallest prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 ...

Note: There is no end to the list of prime numbers. There are infinitely many prime numbers!

2. A **composite number** is a whole number greater than 1 which has at least one factor other than itself and one.

Examples: Some composite numbers are 4, 9, 12, 51, and 91.

Note: You may not be able to think of a factor of 51 or 91 off the top of your head. Fifty-one and 91 aren't in the multiplication tables we usually memorize. However, 51 can be factored as $3 \cdot 17$ and 91 can be factored as $7 \cdot 13$, so they are both composite.

Goal: We are going to determine whether a number less than 121 is prime or composite.

Fact: A number less than 121 is prime if it is *not* divisible by 2, 3, 5, or 7.

Method: Given a number less than 121, we will check to see if it is divisible by 2, 3, 5, or 7. If the given number is divisible by any of the numbers, it is composite. If it is not divisible by any of these numbers, it is prime.

Note: We will describe in the video why this method works and some shortcuts for divisibility tests for 2, 3, and 5. A similar method can be used to test whether numbers larger than 121 are prime or composite. See if you can figure out how to extend this idea to larger numbers after watching the video.



Problem 5

MEDIA EXAMPLE – Prime and Composite Numbers

Determine whether the numbers are prime or composite. If it is composite, show at least one factor pair of the number besides 1 and itself. If it is prime, show the numbers you tested and the results of your division.

a) 105

b) 71

c) 119



Problem 6

YOU TRY – Prime and Composite Numbers

Determine whether the numbers are prime or composite. If it is composite, show at least one factor pair of the number besides 1 and itself. If it is prime, show the numbers you tested and the results of your division.

a) 73

b) 87

SECTION 3.4: PRIME FACTORIZATION

In this section, we will find the prime factorization of a number. This will be a useful skill for breaking down numbers into smaller factors when we learn about fractions and fraction operations.

Definition: The *prime factorization* of a number is the number written as a product of only prime factors.

Fact: The prime factorization of a number is unique. Meaning, there is exactly one way to write any given number as a product of only prime numbers.

Method: We will use *factor trees* to find the prime factorization. This involves repeatedly dividing the given number by known factors and writing the results as products until we are left with only prime numbers. We will elaborate on this process much more in the media example.



Problem 7	MEDIA EXAMPLE – Prime Factorization
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Find the prime factorizations for the given numbers using factor trees. Write the final result in *exponential form* and *factored form*.

a) 12

b) 75

c) 155

Factored Form:


Factored Form:

Factored Form:

Exponential Form:

Exponential Form:

Exponential Form:

	Problem 8	YOU TRY – Prime Factorization
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Find the prime factorizations for the given numbers using factor trees. Write the final result in *exponential form* and *factored form*.

a) 18

b) 84

Factored Form:

Factored Form:

Exponential Form:

Exponential Form:

SECTION 3.5: GREATEST COMMON FACTOR AND LEAST COMMON MULTIPLE

In this section, we will use our knowledge of factors, divisibility and primes to determine factors and multiples that two or more numbers share. We'll begin with a few definitions, and then look at an application to help us build the concept of the utility of these ideas.

Definitions:

1. **Common Factors** of two numbers are factors that both numbers share.
2. The **Greatest Common Factor (GCF)** of two numbers is the largest of these common factors.
3. **Common Multiples** of two numbers are multiples that both numbers share.
4. The **Least Common Multiple (LCM)** of two numbers is the least of these common multiples



Problem 9

MEDIA EXAMPLE – Intro to Greatest Common Factor and Least Common Multiple

- a) You and your friends are sending care packages to military service members overseas. Each package will contain brownies and cookies. You have 20 brownies and 12 cookies. Every package made needs to be identical. What is the greatest number of packages you can send that meets this requirement?

- b) Judy and Dan are running around a track. Judy can run one lap in 3 minutes while it takes Dan 4 minutes. If they both start at the same time, how many minutes will it take them to meet?



a) Find *all* factors of 36. Write your final answer as a list of factors separated by commas.

List of Factors 36: _____

b) Find *all* factors of 90. Write your final answer as a list of factors separated by commas.

List of Factors of 90: _____

c) List the *common factors* of 36 and 90: _____

d) Identify the Greatest Common Factor (GCF) of 36 and 90: _____



Problem 11

YOU TRY – Finding the GCF of Two Numbers

- a) Find *all* factors of 24. Write your final answer as a list of factors separated by commas.

List of Factors 24: _____

- b) Find *all* factors of 60. Write your final answer as a list of factors separated by commas.

List of Factors of 60: _____

- c) List the *common factors* of 24 and 60: _____

- d) Identify the Greatest Common Factor (GCF) of 24 and 60: _____



Problem 12

MEDIA EXAMPLE – Multiples, Common Multiples, and LCM

- a) The first six multiples of 8 are: _____
- b) The first six multiples of 12 are: _____
- c) Some *common multiples* of 8 and 12 are: _____
- d) The Least Common Multiple (LCM) of 8 and 12 is: _____



Problem 13

YOU TRY – Multiples, Common Multiples, and LCM

- a) The first six multiples of 6 are: _____
- b) The first six multiples of 4 are: _____
- c) Some *common multiples* of 6 and 4 are: _____
- d) The Least Common Multiple (LCM) of 6 and 4 is: _____

Summary of Results

A. To find the GCF of 8 and 12, we use the steps below.

1. Find all the factors of 8. Factors of 8: 1, 2, 4, 8
2. Find all the factors of 12. Factors of 12: 1, 2, 3, 4, 6, 12
3. The GCF of 8 and 12 is the largest factor they have in common. So the GCF is 4.

B. To find the LCM of 8 and 12, we use the steps below.

1. List some multiples of 8. Multiples of 8: 8, 16, 24, 32, 40, 48, ...
2. List some multiples of 12. Multiples of 12: 12, 24, 36, 48, 60, ...
3. The LCM of 8 and 12 is the smallest multiple they have in common. So the LCM is 24.

SECTION 3.6: PRIME FACTORIZATION, GCF, AND LCM

In this section, we are going to use prime factorization to find a more streamlined approach to finding the GCF and LCM of two numbers. There is a nice relationship between the GCF and LCM that will help. Observe the following pattern for the GCF and LCM of 8 and 12.

Product of 8 and 12: $8 \cdot 12 = 96$

GCF of 8 and 12: 4

LCM of 8 and 12: 24

Product of 8 and 12 = Product of GCF of 8 and 12 and LCM of 8 and 12

$$8 \cdot 12 = 4 \cdot 24$$

$$96 = 96$$

Fact: The product of two numbers equals the product of their GCF and LCM.

It is usually easier to find the GCF of number since the list of factors of a number is finite (unlike multiples that go on forever). So we will use prime factorization to find the GCF of two numbers. Then we will rewrite the relationship stated in the fact above to find the LCM.

$$LCM = \frac{\text{Product of the two numbers}}{\text{GCF of the two numbers}}$$

In our example, once we have found that the GCF of 8 and 12 is 4, we can find the LCM as shown below.

$$LCM = \frac{8 \cdot 12}{4} = \frac{96}{4} = 96 \div 4 = 24$$

We will use prime factorization to determine all of the common prime factors of two numbers and find their product to determine the GCF. For example, $8 = 2 \cdot 2 \cdot 2$ and $12 = 2 \cdot 2 \cdot 3$. Since the 8 and 12 share two factors of 2, their GCF is $2 \cdot 2 = 4$.



Problem 14

MEDIA EXAMPLE – Prime Factorization, GCF, and LCM

1. Use the prime factorization method to determine the GCF and LCM of 30 and 24.

- a) Find the prime factorizations of 30 and 24 using factor trees and write the prime factorizations in factored form.

30

24

Factored Form:

Factored Form:

- b) List of common prime factors: _____ (include repeated factors)

- c) The product of the common prime factors of 30 and 24 is their GCF. Find the GCF.

GCF of 30 and 24: _____

- d) The LCM of 30 and 24 is *their product divided by their GCF*. Find the LCM. Show all steps.

LCM of 30 and 24: _____

2. Use the prime factorization method to determine the GCF and LCM of 54 and 90.

- a) Find the prime factorizations of 54 and 90 using factor trees and write the prime factorizations in factored form.

54

90

Factored Form:

Factored Form:

- b) List of common prime factors: _____ (include repeated factors)

- c) The product of the common prime factors of 54 and 90 is their GCF. Find the GCF.

GCF of 54 and 90: _____

- d) The LCM of 54 and 90 is their product divided by their GCF. Find the LCM. Show all steps.

LCM of 54 and 90: _____



Use the prime factorization method to determine the GCF and LCM of 18 and 84.

- a) Find the prime factorizations of 18 and 84 using factor trees and write the prime factorizations in factored form.

Factored Form:

Factored Form:

- b) List the common prime factors of 18 and 84: _____ (include repeated factors)

- c) The product of the common prime factors of 18 and 84 is their GCF. Find the GCF.

GCF of 18 and 84: _____

- d) The LCM of 18 and 84 is their product divided by their GCF. Find the LCM. Show all steps.

LCM of 18 and 84: _____

UNIT 3 – PRACTICE PROBLEMS

1. Solve the following division problems by grouping the dividend in divisor size groups. Write your results as equations.

a. $13 \div 4 =$



Division Equation: _____

Multiplication Equation: _____

b. $19 \div 5 =$

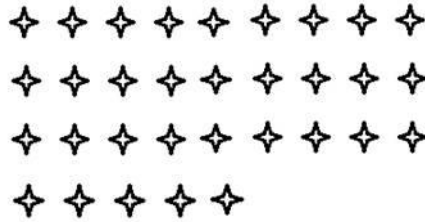


Division Equation: _____

Multiplication Equation: _____

Unit 3 – Practice Problems

c. $32 \div 9 =$



Division Equation: _____

Multiplication Equation: _____

d. $13 \div 2 =$



Division Equation: _____

Multiplication Equation: _____

2. Solve the following division problems using a calculator. Write your results as equations.

a. $122 \div 18 =$

Division Equation: _____

Multiplication Equation: _____

b. $421 \div 37 =$

Division Equation: _____

Multiplication Equation: _____

c. $632 \div 112 =$

Division Equation: _____

Multiplication Equation: _____

3. Solve the following application problems using division with remainders. Make sure to include units in your answers.

- a. Terri is sending care packages to troops overseas. She baked 112 cookies. She wants to share the cookies equally among the 6 different troops.

How many cookies will each troop get?

How many cookies will be leftover?

- b. Sean is biking at a rate of 14 miles per hour. He wants to bike a total of 71 miles.

What is the maximum number of whole hours he will spend biking?

How many miles will he have left to travel after riding the maximum number whole hours?

Unit 3 – Practice Problems

- c. Judy's favorite t-shirts are on sale for \$19. She has \$195 and wants to buy as many t-shirts as possible.

How many t-shirts can Judy buy?

How much money will she have leftover?

4. Find all factors of the given numbers by finding factor pairs. Write your final answer as a list of factors separated by commas.

d) 12

List of Factors: _____

e) 48

List of Factors: _____

f) 185

List of Factors: _____

5. Fill in the blanks:
- A _____ number is a whole number greater than 1 whose factor pairs are only the number itself and one.
 - A _____ number is a whole number greater than 1 which has at least one factor other than itself and one
6. Determine all of the prime numbers less than 50.
7. Determine whether the numbers are prime or composite. If it is composite, show at least one factor pair of the number besides 1 and itself. If it is prime, show the numbers you tested and the results of your division.
- 107
 - 61
 - 81
 - 39

Unit 3 – Practice Problems

8. Fill in the blank: The _____ of a number is the number written as a product of only prime factors.

9. Find the prime factorizations for the given numbers using factor trees. Write the final result in *exponential form* and *factored form*.

b) 32

c) 175

d) 72

e) 280

10. Use two **different** factor trees to determine the prime factorizations of 90. Write the final result in *exponential form* and *factored form*.

11. Fill in the blanks:

- c. *Common factors* of two or more numbers are factors that both numbers _____.
- d. The _____ of two or more numbers is the largest of the two numbers' common factors.

12. Find the GCF of the given numbers.

- a. 8 and 20

- b. 30 and 105

- c. 16 and 18

- d. 22 and 25

- e. 12 and 24

Unit 3 – Practice Problems

13. Fill in the blanks:

- a. A _____ of a number is a product of the number with any whole number.
- b. The _____ is the smallest multiple of 2 or more numbers.

14. Find the LCM for the given numbers.

a. 4 and 6

b. 10 and 8

c. 15 and 9

d. 6 and 15

e. 12 and 24

15. Find the prime factorizations using factor trees for the following pairs of numbers. Then find the LCM and GCF.

a. 4 and 6

b. 10 and 8

c. 15 and 9

d. 12 and 26

Unit 3 – Practice Problems

16. Consider the numbers 30 and 105

a. Determine the Greatest Common Factor (GCF) of 30 and 105.

b. Find the Least Common Multiple (LCM) of 30 and 105 by using the relationship below.

$$\frac{\textit{product of numbers}}{\textit{GCF}} = \textit{LCM}$$

17. Consider the numbers 60 and 48

c. Determine the Least Common Multiple (LCM) of 60 and 48.

d. Find the Greatest Common Factor (GCF) of 60 and 48 by using the relationship below.

$$\frac{\textit{product of numbers}}{\textit{LCM}} = \textit{GCF}$$

18. Penny and Sheldon are assembling hair clips. Penny can assemble a hair clip in 6 minutes and Sheldon can assemble a hair clip in 9 minutes.
- If they start making the hair clips at the same time, what is the least amount of minutes it will take for them finish a hair clip at the same time?
 - After this amount of minutes, how many hair clips will Penny have made?
 - After this amount of minutes, how many hair clips will Sheldon have made?
19. Kathryn is packing bags of food at the local food pantry. She has 24 jars of tomato sauce and 30 cans of soup.
- If she wants each bag to have the same numbers of tomato sauce and soup, what is the greatest number of bags she can pack?
 - How many jars of tomato sauce will each bag have?
 - How many cans of soup will each bag have?
20. Paige is buying hot dogs and buns for a family reunion. Each package of hot dogs contains 8 hot dogs. Each package of buns contains 10 buns.
- What is the least total amount of hot dogs and buns she needs to buy in order for the amounts to be equal?
 - How many packages of hot dogs will she buy?
 - How many packages of buns will she buy?

UNIT 3 – END OF UNIT ASSESSMENT

1. Solve the division problem by grouping the dividend in divisor size groups. Write your result symbolically as equations.

$$17 \div 4$$



Division Equation: _____

Multiplication Equation: _____

2. Solve. Show your work. Write your answer in a complete sentence.

Connie had 18 kindergarten students at the beginning of the school year. She bought 56 pencils for her students. Can Connie divide the pencils equally among her students? If not, how many pencils will be left over?

Unit 3 – Assessment

3. Find all factors of the given numbers by finding factor pairs. Use the table of perfect squares (#5) to see what the largest number you have to check. Write your final answer as a list of factors separated by commas.

a) 24

List of factors: _____

b) 175

List of factors: _____

c) 192

List of factors: _____

4. Determine whether the numbers are prime or composite. If it is composite, show at least one factor pair of numbers beside 1 and itself. If it is prime, show the numbers you tested and the result of your division.

a) 111

b) 97

c) 91

5. Create two different factor trees to determine the prime factorization of 36.

6. Determine whether the statements are true or false.

a) We will arrive at the same prime factorization of a number no matter what original factors we use when creating a factor tree.

TRUE FALSE

b) There is only one unique prime factorization for any given number.

TRUE FALSE

7. Find the LCM of 8 and 28. Show your work.

8. Find the GCF of 15 and 36. Show your work.

Unit 3 – Assessment

9. Find the GCF and LCM of 27 and 52. Show your work.
10. A furniture store has outdoor furniture that it would like to sell in identical sets with no furniture left over. The store has 20 end tables and 30 lounge chairs. What is the greatest number of sets that the furniture store can sell? Show your work. Write your answer in a complete sentence.

UNIT 4 – INTRODUCTION TO FRACTIONS AND DECIMALS

INTRODUCTION

In this Unit, we will investigate fractions and decimals. We have seen fractions before in the context of division. For example, we can think of the division problem $6 \div 3$ as the equivalent fractional expression $\frac{6}{3}$. It will be very useful to use equivalencies such as these when working with fractions. Decimals are in fact fractions and are sometimes even referred to as *decimal fractions*. They are special because they use an extension of our base 10 number system and the place value ideas we used earlier to write fractions in a different form.

Section	Learning Objective	Media Examples	You Try
4.1	Compare and contrast four models of fractions	1	2
4.1	Determine the unit of a fraction in context	3	4
4.2	Represent unit fractions in multiple ways	5	6
4.3	Represent composite fractions on a number line	7	8
4.3	Represent composite fractions using an area model	9	11
4.3	Represent composite fractions using a discrete model	10	11
4.4	Represent improper fractions and mixed numbers	12	13
4.5	Find an equivalent fraction	14	15
4.6	Simplify fractions using repeated division or prime factorization	16	17
4.7	Compare fractions with the same denominator or numerator	18, 19	20
4.8	Write a fraction as a decimal in multiple forms	21, 24	23, 25
4.8	Write decimal fractions in multiple forms	22, 24	23, 25
4.9	Plot decimals on a number line	26	27
4.10	Order and compare decimals using place value	28, 29	30
4.11	Rounding decimals	31, 32	33
4.12	Use the context of an application problem to round	34	36
4.12	Write the place value form of small or large number given a decimal times a power of 10	35	36
4.13	Converting between decimals and fractions	37, 39	38, 40

UNIT 4 – MEDIA LESSON

SECTION 4.1: WHAT IS A FRACTION?

There are many ways to think of a fraction. A fraction can be thought of as one quantity divided by another written by placing a horizontal bar between the two numbers such as $\frac{1}{2}$ where 1 is called the *numerator* and 2 is called the *denominator*. Or we can think of fractions as a part compared to a whole such as 1 out of 2 cookies or $\frac{1}{2}$ of the cookies. In this lesson, we will look at a few other ways to think of fractions as well.

Officially, fractions are *any numbers* that can be written as $\frac{a}{b}$ but in this course, we will consider fractions where the numerator and denominator are integers. These special fractions where the numerator and denominator are both integers are called *rational numbers*. Since rational numbers are indeed fractions, we will frequently refer to them as “fractions” instead of “rational numbers”.

Language and Notation of Fractions

Each of the phrases below show a way we may indicate a fraction with words, and the corresponding fraction word name.

Language	Fraction Representation	Fraction Word Name
20 divided by 6	$\frac{20}{6}$	twenty sixths
8 out of 9	$\frac{8}{9}$	eight ninths
A ratio of 3 to 2	$\frac{3}{2}$	three halves
11 per 5	$\frac{11}{5}$	eleven fifths
2 for every 7	$\frac{2}{7}$	two sevenths

Proper fractions are fractions whose numerator is less than their denominator. *Improper fractions* are fractions whose numerator is greater than or equal to its denominator. For our examples,

$$\text{Proper Fractions: } \frac{8}{9}, \frac{2}{7} \quad \text{Improper Fractions: } \frac{20}{6}, \frac{3}{2}, \frac{11}{5}$$

In the first example, we will look at four different types of fractions to see how they are used in context.

1. **Quotient Model (Division):** Sharing equally into a number of groups
2. **Part-Whole Model:** A part in the numerator a whole in the denominator
3. **Ratio Part to Part Model:** A part in the numerator and a different part in the denominator
4. **Rate Model:** Different types of units in the numerator and denominator (miles and hours)



Problem 1

MEDIA EXAMPLE – Fractions in Context: Four Models

Represent the following as fractions. Determine whether it is a quotient, part-whole, part to part, or rate model.

- Three cookies are shared among 6 friends. How many cookies does each friend get?
- Four out of 6 people in the coffee shop have brown hair. What fraction of people in the coffee shop have brown hair?
- Tia won 6 games of heads or tails and lost 3 games of heads or tails. What is the ratio of games won to games lost?
- A snail travels 3 miles in 6 hours. What fraction of miles to hours does he travel? What fraction of hours to miles does he travel?



Problem 2

YOU-TRY - Examples of Fractions in Context

Represent the following scenarios using fraction. Indicate whether the situation is a *Quotient*, *Part to Whole*, *Ratio Part to Part*, or *Rate*.

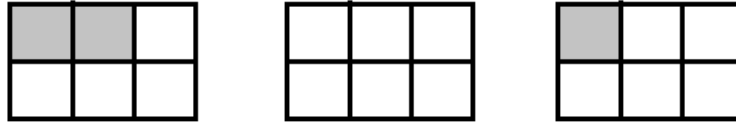
- Jorge bikes 12 miles in 3 hours. What fraction of miles to hours does he travel?
- Callie has 5 pairs of blue socks and 12 pairs of grey socks. What fraction of blue socks to grey socks does she have?
- Callie has 5 pairs of blue socks and 12 pairs of grey socks. What fraction of all of her socks are blue socks?



Problem 3

MEDIA EXAMPLE – The Importance of the Unit When Representing Fractions

Sean’s family made 3 trays of brownies. Sean ate 2 brownies from the first batch and 1 from the 3rd batch and shown in the image below (brownies eaten are shaded).



His family disagreed on the amount of brownies he ate and gave the three answers below. Draw a picture of the unit (the amount that represents 1) that makes each answer true.

Answer 1: 3

Draw a Picture of the Unit:

Answer 2: $\frac{3}{6}$

Draw a Picture of the Unit:

Answer 3: $\frac{3}{18}$

Draw a Picture of the Unit:

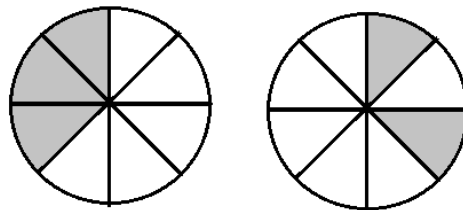


Problem 4

YOU-TRY - The Importance of the Unit When Representing Fractions

Consider the following problem and the given answers to the problem. Determine the unit you would need to use so each answer would be correct.

The picture below shows the pizza Homer ate. Determine the unit that would make each answer below reasonable.



Answer 1: 5

Draw a Picture of the Unit:

Answer 2: $\frac{5}{8}$

Draw a Picture of the Unit:

Answer 3: $\frac{5}{16}$

Draw a Picture of the Unit:

SECTION 4.2: REPRESENTING UNIT FRACTIONS

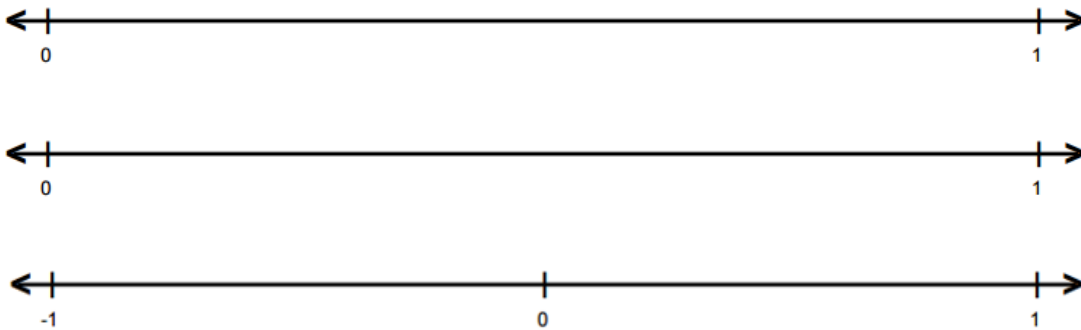
A unit fraction is a fraction with a numerator of 1. In this section we will develop the idea of unit fractions and use multiple representations of unit fractions.



Problem 5

MEDIA EXAMPLE – Multiple Representations of Unit Fractions

- a) Plot the following unit fractions on the number line, $\frac{1}{2}$, $\frac{1}{4}$, $-\frac{1}{5}$ Label your points below the number line.









- b) Represent the fractions using the area model. The unit is labeled in the second row of the table.

$$\frac{1}{5}$$

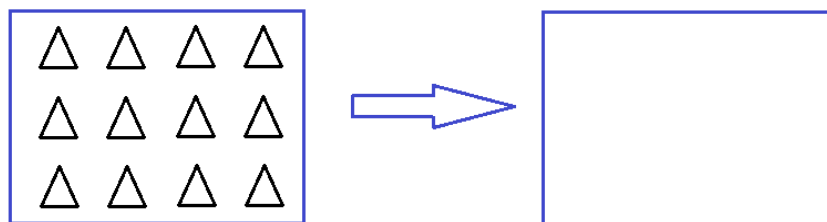
$$\frac{1}{6}$$

$$\frac{1}{4}$$

		
Unit: 	Unit: 	Unit: 

- c) Represent the unit fractions using the discrete objects. The unit is *all* of the triangles in the rectangle.

Represent $\frac{1}{4}$ of the triangles.

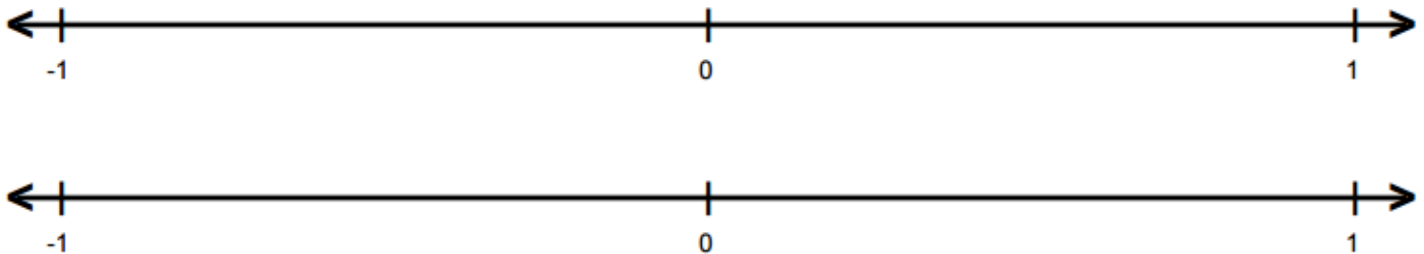




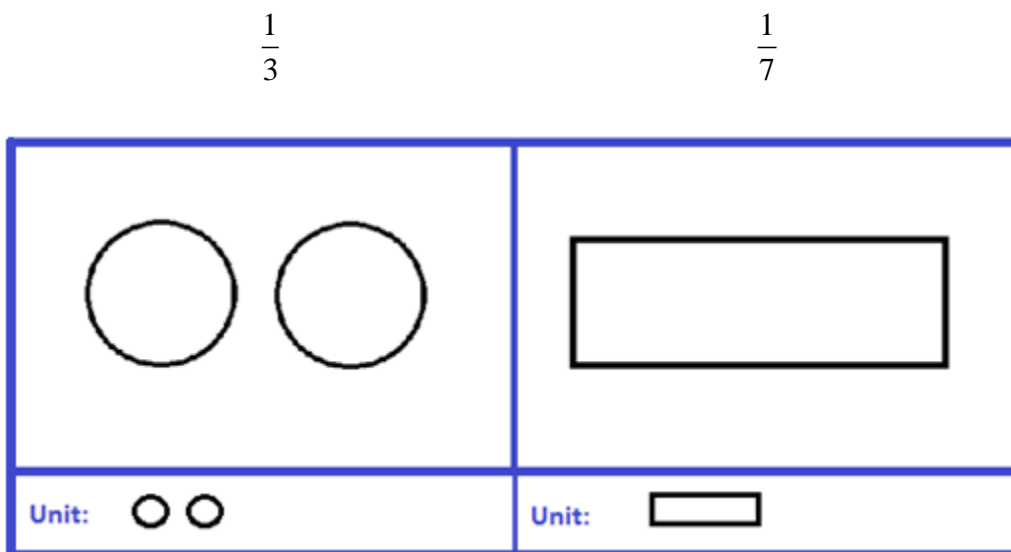
Problem 6

YOU-TRY – Multiple Representations of Unit Fractions

- a) Plot the following unit fractions on the number line $\frac{1}{3}$, $\frac{1}{4}$. Label your points below the number line.

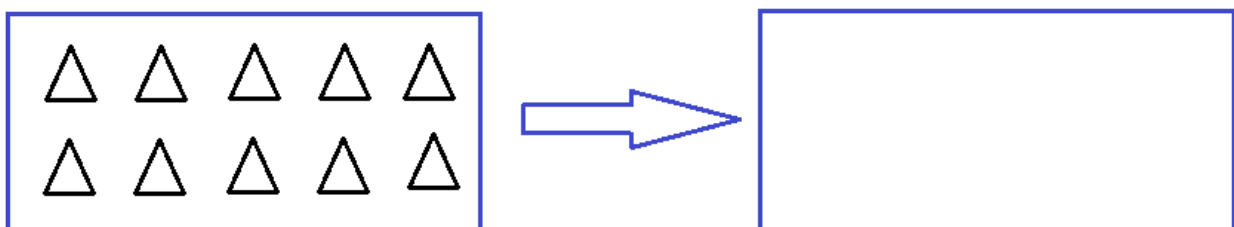


- b) Represent the fractions using the area model. The unit is labeled in the second row of the table.



- c) Represent the unit fractions using the discrete objects. The unit is all of the triangles in the rectangle.

Represent $\frac{1}{5}$ of the triangles.



SECTION 4.3: COMPOSITE FRACTIONS

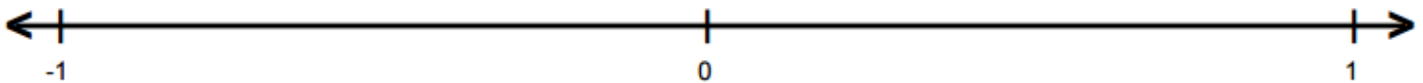
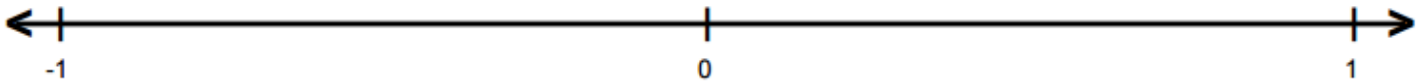
In this section, we will use unit fractions to make composite fractions. Composite fractions are fractions that have a numerator that is an integer that is not 1 or -1 . We will look at both proper and improper fractions.



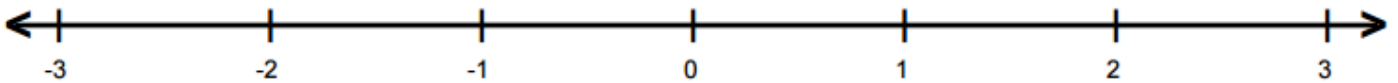
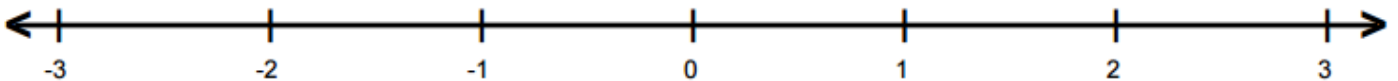
Problem 7

MEDIA EXAMPLE – Cut and Copy: Composite Fractions on the Number Line

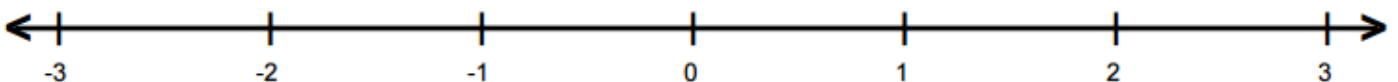
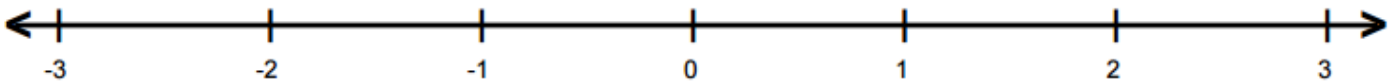
- a) Plot the following composite fractions on the number line $\frac{2}{3}$, $-\frac{4}{5}$. Label your points below the number line.




- b) Plot the following composite fractions on the number line $\frac{5}{2}$, $-\frac{8}{3}$. Label your points below the number line.

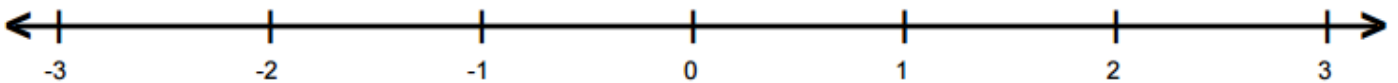
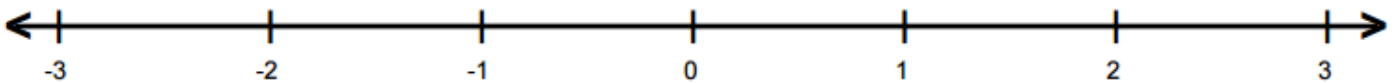
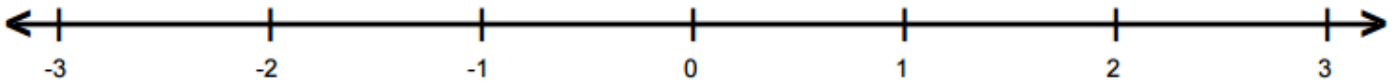
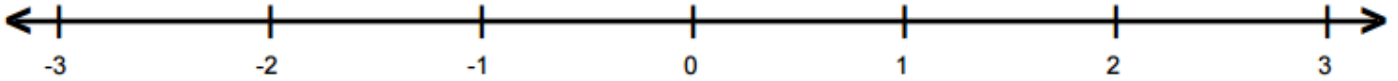


- c) Plot the following composite fractions on the number line $\frac{12}{6}$, $-\frac{8}{4}$. Label your points below the number line.



	Problem 8	YOU-TRY – Cut and Copy: Composite Fractions on the Number Line
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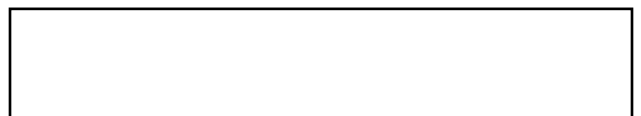
Plot the following composite fractions on the number line $\frac{3}{4}$, $-\frac{5}{4}$, $\frac{5}{2}$, $-\frac{12}{4}$. Label your points below the number line.



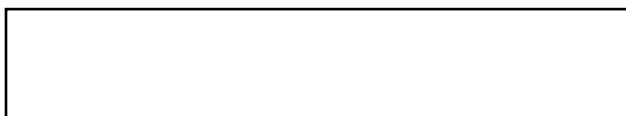
	Problem 9	MEDIA EXAMPLE – Cut and Copy: Composite Fractions and Area Models
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Represent the composite fractions using an area model. A single rectangle is the unit. An additional rectangle is given in each problem for the fractions which may require it.

a) Represent $\frac{3}{4}$ with a rectangle as the unit. _____ copies of _____ (unit fraction)



b) Represent $\frac{7}{4}$ with a rectangle as the unit. _____ copies of _____ (unit fraction)



 Problem 10	MEDIA EXAMPLE – Cut and Copy: Composite Fractions Using Discrete Models
--	---

Represent the composite fractions using the discrete objects. The unit is all of the triangles in the rectangle.

a) Represent $\frac{5}{6}$ of the triangles.



Drawing of associated unit fraction: _____ copies of _____ (unit fraction)

b) Represent $\frac{5}{3}$ of the triangles.

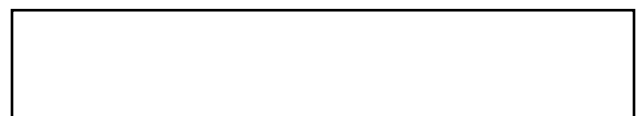


Drawing of associated unit fraction: _____ copies of _____ (unit fraction)

 Problem 11	YOU-TRY - Cut and Copy: Composite Fractions and Area and Discrete Models
--	--

a) Represent the composite fractions using an area model. A single rectangle is the unit. An additional rectangle is given in each problem for the fractions which may require it.

Represent $\frac{8}{5}$ with a rectangle as the unit. _____ copies of _____ (unit fraction)



b) Represent the composite fractions using the discrete objects. The unit is all of the triangles in the rectangle.

Represent $\frac{3}{4}$ of the triangles.



Drawing of associated unit fraction: _____ copies of _____ (unit fraction)

SECTION 4.4: IMPROPER FRACTIONS AND MIXED NUMBERS

Improper fractions are fractions whose numerators are greater or equal to their denominators. You may have noticed that these fractions are greater than equal to 1. We can also represent improper fractions as mixed numbers. A mixed number is the representation of a number as an integer and proper fraction. In this section, we will represent and rewrite improper fractions as mixed numbers and vice versa.



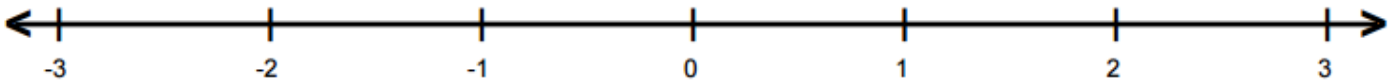
Problem 12

MEDIA EXAMPLE – Improper Fractions and Mixed Numbers

- a) Represent $\frac{7}{5}$ with a rectangle as the unit. Then rewrite it as a mixed number. (A single rectangle is the unit)

Mixed Number: _____

- b) Represent $-\frac{8}{3}$ on the number line. Then rewrite it as a mixed number.



Mixed Number: _____



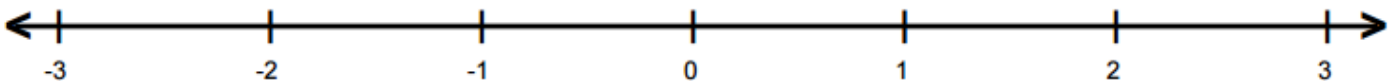
Problem 13

YOU-TRY Improper Fractions and Mixed Numbers

- a) Represent $\frac{8}{7}$ with a rectangle as the unit and then rewrite it as a mixed number. (A single rectangle is the unit)

Mixed Number: _____

- b) Represent $-\frac{7}{5}$ on the number line and then rewrite it as a mixed number.



Mixed Number: _____

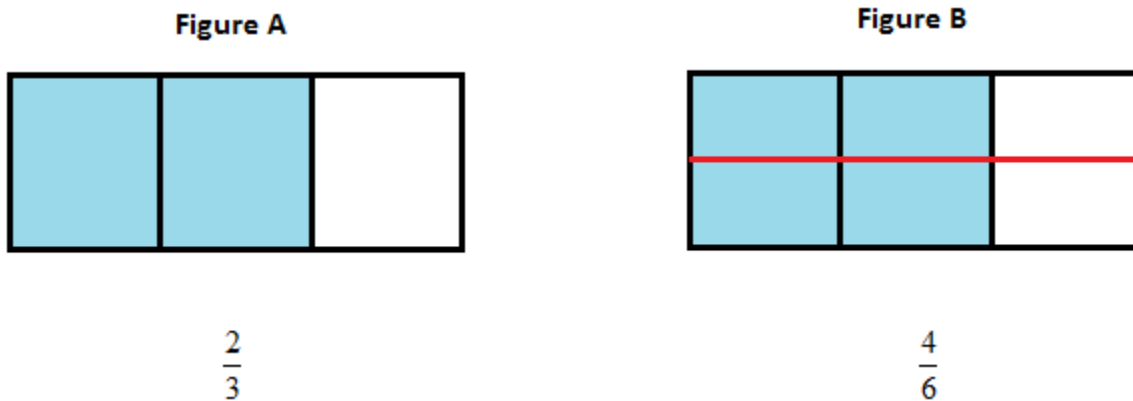
SECTION 4.5: EQUIVALENT FRACTIONS

At some point in time, you have probably eaten half of something, maybe a pizza or a cupcake. There are many ways you can have *half* of some unit. A pizza (the unit) can be cut into 4 equal pieces and you have 2 of these pieces, or $\frac{2}{4}$. Or maybe a really big pizza is cut into 100 equal pieces and you have 50, or $\frac{50}{100}$. In either case, the amount you have is *equivalent to* $\frac{1}{2}$ because you ate one for every two pieces in the unit.

Definition: Two fractions are *equivalent* if they represent the same number.

Example: In Figures A and B below, let one rectangle be the unit.

1. Figure A is cut into 3 pieces and 2 pieces are shaded. This represents the fraction $\frac{2}{3}$.
2. In Figure B, the 3 pieces from Figure A were cut into 2 pieces each making 6 pieces. Now 4 pieces are shaded representing the fraction $\frac{4}{6}$.
3. Since the same area is shaded these fractions are equivalent.



In fact, we could continue to cut the original 3 pieces from Figure A into any whole number of pieces and create an equivalent fraction. We indicate that two fractions are equivalent with an equal's sign.

Below we show a few of the infinite number of fractions that are equivalent to $\frac{2}{3}$. Do you notice any patterns in the numerators or denominators?

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{8} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} \dots$$



Problem 14

MEDIA EXAMPLE – Rewriting Equivalent Fractions with One Value Given

Rewrite the given fractions as equivalent fractions given the indicated numerator or denominator.

a. Rewrite $\frac{3}{7}$ with a denominator of 21.

b. Rewrite $\frac{-12}{10}$ with a numerator of -120.

c. Rewrite $\frac{85}{60}$ with a denominator of 12.

d. Rewrite $\frac{-36}{52}$ with a numerator of -9.



Problem 15

YOU-TRY - Rewriting Equivalent Fractions with One Value Given

Rewrite the given fractions as equivalent fractions given the indicated numerator or denominator.

a. Rewrite $\frac{5}{8}$ with a denominator of 32.

b. Rewrite $\frac{-18}{33}$ with a numerator of -6.

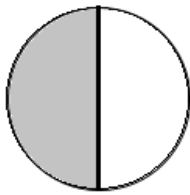
SECTION 4.6: WRITING FRACTIONS IN SIMPLEST FORM

In the last section, we learned there are infinitely many ways to write any fraction as an equivalent fraction. We most often follow the convention of writing a fraction in what we call *simplest form* to have a standard for writing our end results.

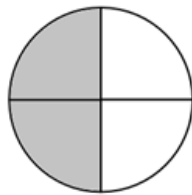
Definition: The *simplest form* of a fraction is the equivalent form of the fraction where the numerator and denominator are written as integers without any common factors besides 1.

Example: In the figure below, one circle is the unit, and each circle is half shaded. Notice that that only the first fraction, $\frac{1}{2}$, has the property that its numerator and denominator share no common factors besides 1. So

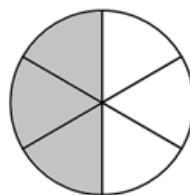
all of the fractions are *equivalent*, but $\frac{1}{2}$ is in the *simplest form*.



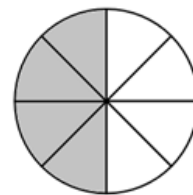
$$\frac{1}{2}$$



$$\frac{2}{4} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{1}{2}$$



$$\frac{3}{6} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{1}{2}$$



$$\frac{4}{8} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{1}{2}$$



Problem 16

MEDIA EXAMPLE – Simplifying Fractions by Repeated Division and Prime Factorization

We can use two different methods to simplify a fraction; *repeated division or prime factorization*.

1. **Repeated Division:** Look for common factors between the numerator and denominator and divide both by the common factor. Continue this process until you are certain the numerator and denominator have no common factors.
2. **Prime Factorization:** Write the prime factorizations of the numerator and denominator and cancel out any common factors.

Simplify the given fractions completely using both the repeated division and prime factorization methods. In each case, state which you think is easier and why.

a) $\frac{10}{24}$

b) $4\frac{12}{27}$

c) $-\frac{84}{63}$



Problem 17

YOU-TRY – Simplifying Fractions by Repeated Division and Prime Factorization

Simplify the given fractions completely using both the repeated division and prime factorization methods. In each case, state which you think is easier and why.

a) $\frac{6}{8}$

b) $6\frac{30}{42}$

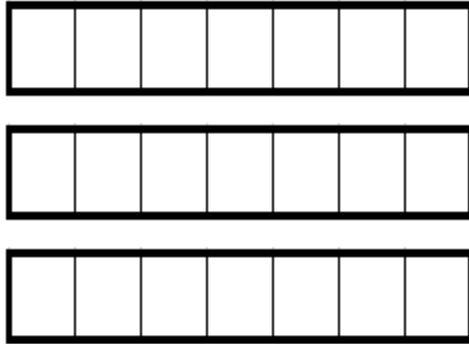
$-\frac{132}{100}$

SECTION 4.7: COMPARING FRACTIONS

In this section, we will learn to compare fractions in numerous ways to determine their relative size.

Problem 18	MEDIA EXAMPLE – Comparing Fractions with Same Denominator of Numerator
------------	--

- a) Shade the following areas representing the fractions using the rectangles below. $\frac{3}{7}, \frac{6}{7}, \frac{1}{7}$



- b) Order the numbers from least to greatest by comparing the amount of the unit area shaded.

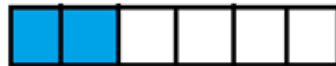
- c) Identify the fractions represented by area shaded in the rectangles below.



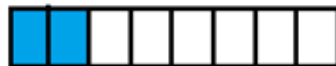
Area of fraction shaded: _____



Area of fraction shaded: _____



Area of fraction shaded: _____



Area of fraction shaded: _____

- d) Order the numbers from least to greatest by comparing the amount of the unit area shaded.

- e) Develop a general rule for ordering fractions.
- If two fractions have the same denominator and the *fractions are positive*, then the fraction with the _____ numerator is greater.
 - If two fractions have the same numerator and the *fractions are positive*, then the fraction with the _____ denominator is greater.
 - If one fraction is positive and the other is negative, then the _____ fraction is greater.
 - If two fractions have neither the same numerator nor denominator, rewrite them as equivalent fractions with the same numerator or denominator so you can compare them.



Problem 19

MEDIA EXAMPLE – Comparing Fractions with Equal Numerators or Denominators

Order the fractions from least to greatest and justify your answer.

a) $\frac{7}{12}, \frac{15}{12}, \frac{0}{12}, \frac{3}{12}$ Ordering:

b) $\frac{3}{65}, \frac{3}{5}, \frac{3}{100}, \frac{3}{1}$ Ordering:

c) $-\frac{5}{8}, -\frac{5}{3}, \frac{2}{5}$ Ordering:

d) $\frac{3}{7}, \frac{2}{7}, \frac{5}{14}$ Ordering:



Problem 20

YOU-TRY – Comparing Fractions with Equal Numerators or Denominators

Order the fractions from least to greatest and justify your answer.

a) $\frac{1}{10}, -\frac{3}{7}, -\frac{4}{7}$ Ordering:

b) $\frac{5}{9}, \frac{5}{12}, \frac{7}{9}$ Ordering:

SECTION 4.8: WHAT IS A DECIMAL?

Decimals are a different way of representing fractions. In fact, each place value of a decimal represents a different fraction whose *denominator is a power of ten*. In this section we will develop the idea of a decimal by writing and representing them in numerous ways.

Grid Representation of Decimals:

In Figure A, the square represents the unit. Vertical lines were drawn to partition the unit into 10 equal pieces. So each long rectangle is one tenth or $\frac{1}{10}$ of the unit. In Figure B, 3 long rectangles are shaded with *orange strips*. So three tenths or $\frac{3}{10}$ of the unit is shaded in Figure B.

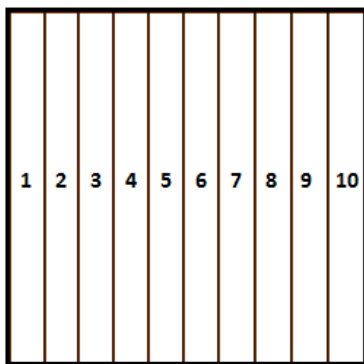


Figure A

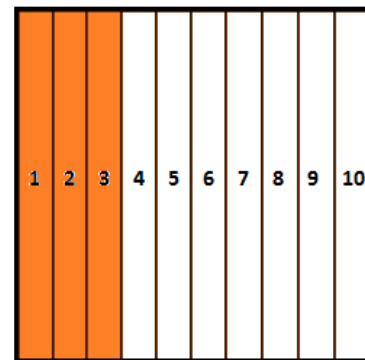


Figure B

In Figure C, we took the diagram from Figure A and cut each of the 10 pieces into 10 pieces using horizontal lines. Now the unit is partitioned into 100 equal pieces. So each small square is one hundredth or $\frac{1}{100}$ of the unit. In Figure D, 30 small squares are shaded yellow. So thirty hundredths or $\frac{30}{100}$ of the unit is shaded in Figure D. Notice the areas in Figures B and D are equivalent, so the fractions $\frac{3}{10}$ and $\frac{30}{100}$ are equivalent.

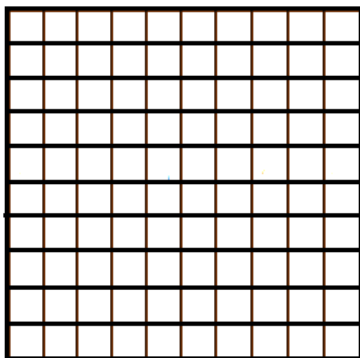


Figure C

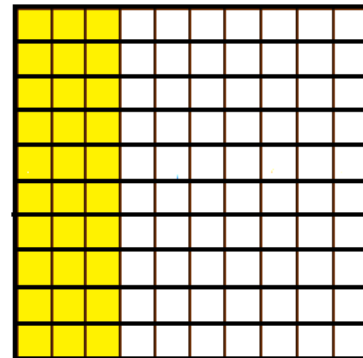
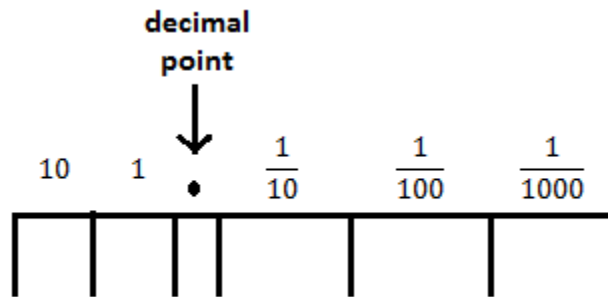


Figure D

Place Value Representation of Decimals:

The diagram below shows how we extend our current place value system to include decimal fractions.



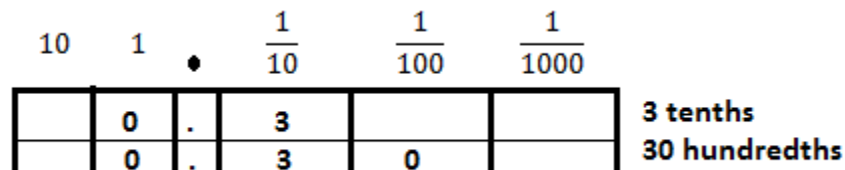
Observe that we insert a decimal point to the right of the one’s place to indicate the digits to the right of the decimal point are decimal fractions. The pattern we use for our base-10 system continue. Each place value is 10 times as large as the place value to its right. Or equivalently, we can divide each place value by 10 to get the place value to its left. We can continue the place values in either direction using this process.

$$1 \text{ tenth} = \frac{1}{10} \text{ of one}$$

$$1 \text{ hundredth} = \frac{1}{10} \text{ of one tenth}$$

$$1 \text{ thousandth} = \frac{1}{10} \text{ of one hundredth}$$

The fractions we found in the decimal grids would be written as shown in the place value chart below.

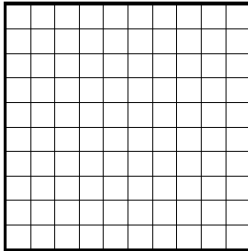


Word Name	Fraction	Place Value	Expanded Form
Three tenths	$\frac{3}{10}$	0.3	$0 \cdot 1 + 3 \cdot \frac{1}{10}$
Thirty hundredths	$\frac{30}{100}$	0.30	$0 \cdot 1 + 3 \cdot \frac{1}{10} + 0 \cdot \frac{1}{100}$

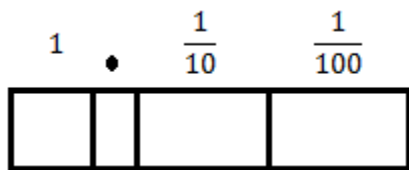


Shade the indicated quantity, fill in the place value chart, and rewrite in the indicated forms.

a) 57 hundredths



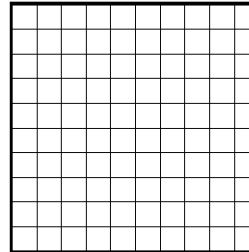
Place Value Chart:



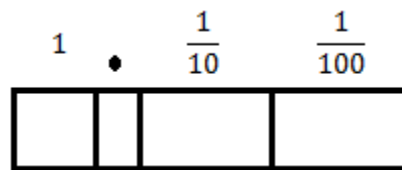
Decimal: _____

Expanded Form: _____

b) $\frac{7}{100}$



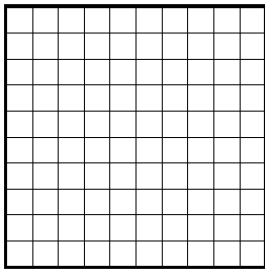
Place Value Chart:



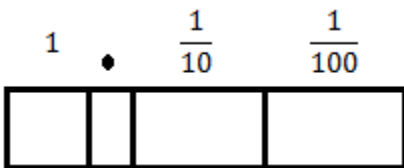
Decimal: _____

Expanded Form: _____

c) 6 tenths and 3 hundredths



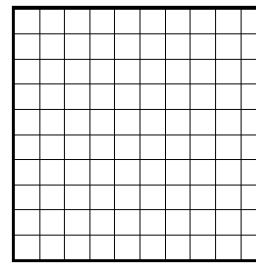
Place Value Chart:



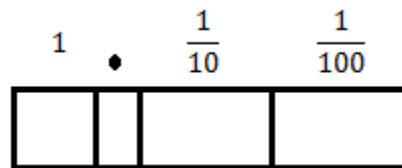
Decimal: _____

Fraction: _____

d) $\frac{3}{10} + \frac{8}{100}$



Place Value Chart:



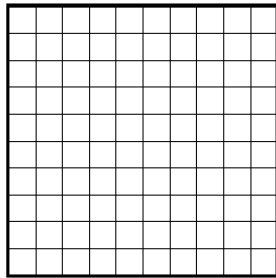
Decimal: _____

Fraction: _____

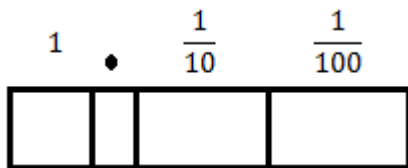
Problem 22 MEDIA EXAMPLE – Writing Decimals in Fraction Form

Shade the indicated quantity and rewrite in the indicated forms.

a) 0.7



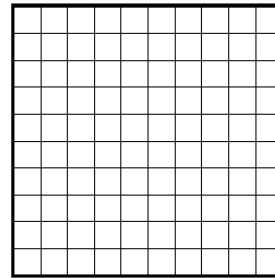
Place Value Chart:



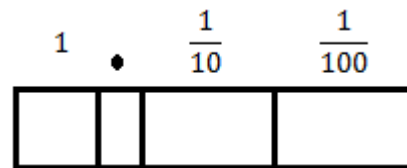
Fraction: _____

Expanded Form: _____

b) 0.60



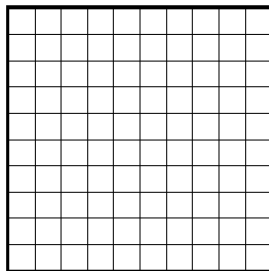
Place Value Chart:



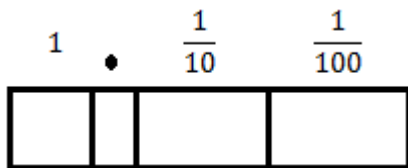
Fraction: _____

Expanded Form: _____

c) 0.47



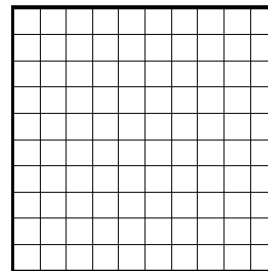
Place Value Chart:



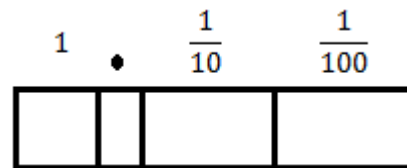
Fraction: _____

Expanded Form: _____

d) 0.06



Place Value Chart:



Fraction: _____

Expanded Form: _____

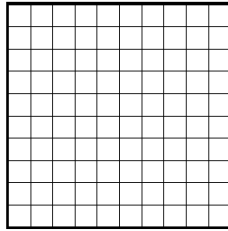


Problem 23

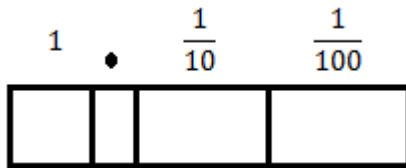
YOU-TRY – Writing Fractions and Decimals in Multiple Forms

Shade the indicated quantity and rewrite in the indicated forms.

a) 0.37



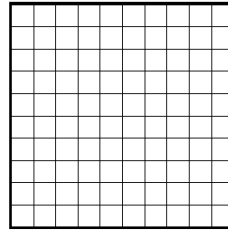
Place Value Chart:



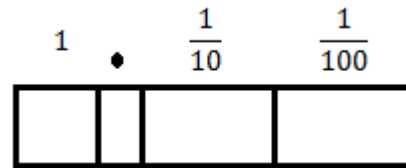
Fraction Name: _____

Expanded Form: _____

b) 8 tenths and 7 hundredths



Place Value Chart:



Fraction Name: _____

Expanded Form: _____

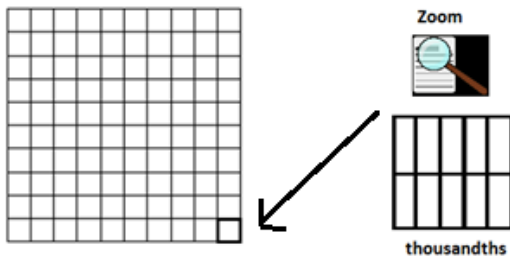


Problem 24

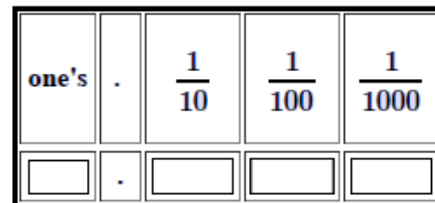
MEDIA EXAMPLE – Writing the Thousandths Place in Multiple Forms

Shade the indicated quantity and write the corresponding decimal number.

a) 5 hundredths and 7 thousandths



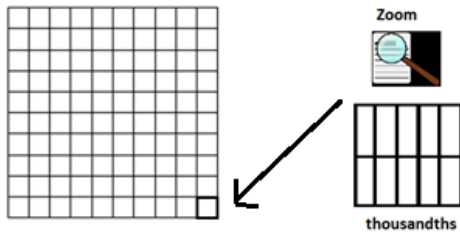
Place Value Chart



Decimal Number: _____

Expanded Form: _____

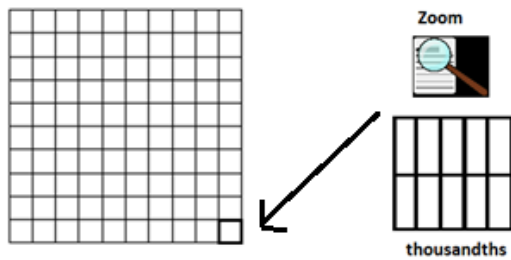
b) 0.536



Place Value Chart				
one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$


Expanded Form: _____ In words: _____

c) 0.603



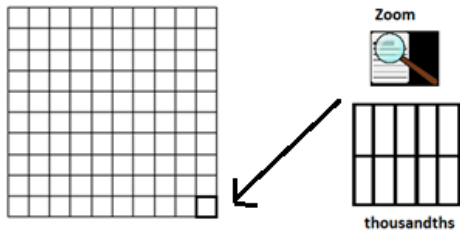
Place Value Chart				
one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Expanded Form: _____ In words: _____

 **Problem 25** YOU-TRY - Writing the Thousandths Place in Multiple Forms

Shade the indicated quantity and rewrite in the indicated forms.

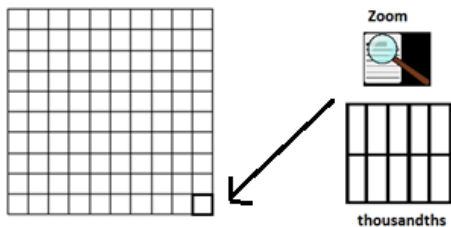
a) 2 hundredths and 9 thousandths



Place Value Chart				
one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimal Number: _____ Expanded Form: _____

b) 0.407



Place Value Chart				
one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Expanded Form: _____ In words: _____

SECTION 4.9: PLOTTING DECIMALS ON THE NUMBER LINE

Like whole number, integers, and fractions, decimal fractions can also be plotted on the number line. In this section, we will plot decimals on the number line.

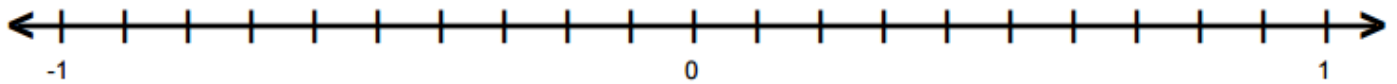


Problem 26

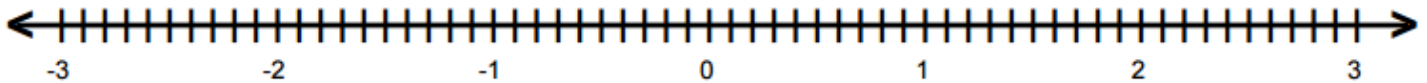
MEDIA EXAMPLE – Plotting Decimals on the Number Line

Use the give number lines to plot the following decimals.

- a) Plot the decimals on the number line below. Label the points underneath the number line.

 $0.4, 0.7, -0.3, -0.9$


- b) Plot the decimals on the number line below. Label the points underneath the number line.

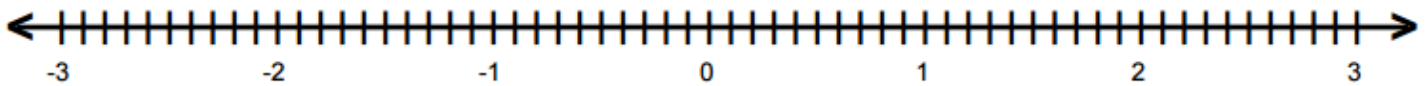
 $2.3, 1.9, -2.6, -1.2$


Problem 27

YOU-TRY - Plotting Decimals on the Number Line

Use the give number lines to plot the following decimals.

- a) Plot the decimals on the number line below. Label the points underneath the number line.

 $1.4, 2.7, -0.8, -1.9$


SECTION 4.10: ORDERING DECIMALS



Problem 28

MEDIA EXAMPLE – Using Place Value to Order Decimals

To order decimals from least to greatest, we use the following procedure. When we find the largest place value where two numbers differ,

- i. The number with the larger digit in this place value is larger.
- ii. The number with the smaller digit in this place value is smaller.

- a) Use the place value chart to order the numbers from least to greatest.

3.555, 3.055, 3.55, 3.5, 3.05

Ones			Decimals		
Hundreds	Ten	One	Tenth	Hundredths	Thousandths
			.		
			.		
			.		
			.		

Ordering: _____


- b) Use your knowledge of negative numbers to order the opposites of the numbers from part a.

-3.555, -3.055, -3.55, -3.5, -3.05

Ordering: _____


- c) Explain in words how you can determine whether one negative number is greater than another negative number.

Unit 4 – Media Lesson

	Problem 29	MEDIA EXAMPLE – Comparing Decimals Using Inequality Symbols
---	------------	---

Order the signed decimals below using the symbols, $<$, $=$, or $>$.

- a) 0.53 ____ 0.62 b) -0.01 ____ -0.09 c) -0.13 ____ 0.99
 d) 3.42 ____ -5.67 e) -2.4 ____ -1.7 f) -6.17 ____ 0.03

	Problem 30	YOU-TRY – Ordering Decimals
---	------------	-----------------------------

- a) Use the place value chart to order the numbers from least to greatest.

4.25, 0.425, 4.05, 4.2, 4.5

Ones			Decimals		
Hundreds	Ten	One	Tenth	Hundredths	Thousandths
			.		
			.		
			.		
			.		

Ordering: _____

- b) Order the signed decimals below using the symbols, $<$, $=$, or $>$.

- 0.54 ____ 0.504 -0.12 ____ -0.2 -0.98 ____ 0.1
 4.19 ____ -6.21 -3.07 ____ -3.7 -0.07 ____ -0.06

SECTION 4.11: ROUNDING DECIMALS

Frequently, we will have decimals that have more decimal places than we need to compute. For example, you probably know your weight in pounds. Do you think you know your *exact* weight? My digital scale approximates my weight to the nearest half of a pound. So it rounds my weight to the half of a pound closest to my weight. So it may say I weigh 123.5 pounds when I really weigh 123.33247 pounds.

To **round a decimal** means to give an *approximation* of the number to a given decimal place.

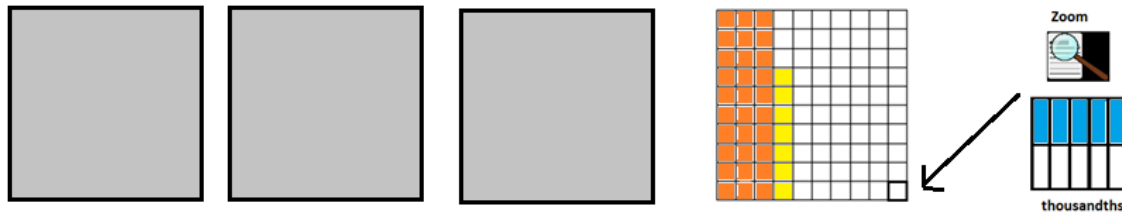
Except in certain application problems, we follow the convention of

- a) “Rounding up” when the place value after the digit we are rounding to is 5 or greater (5, 6, 7, 8, 9)
- b) “Rounding down” when the place value after the digit we are rounding to is less than 5 (0, 1, 2, 3, 4)

Round to the...	Alternative language	Example: 23.5471
One’s place	Whole number	24
Tenth’s place	One decimal place	23.5
Hundredth’s place	Two decimal places	23.55
Thousandth’s place	Three decimal places	23.547

Problem 31 **MEDIA EXAMPLE – Visualizing Rounding Decimals**

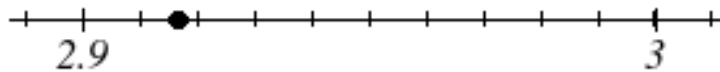
- a) Round the number represented below to the nearest one’s place, tenth’s place and hundredth’s place.
(Note: The big square is the unit. Gray shading represents a whole.)



Given number: _____ Rounded to the nearest one’s place: _____

Rounded to the tenth’s place: _____ Rounded to the hundredth’s place: _____

- b) Round the number represented below to the nearest whole number, one decimal place, and two decimal places.



Rounded to the nearest whole number: _____

Rounded to one decimal place: _____

Rounded to two decimal places: _____

 Problem 32	MEDIA EXAMPLE – Rounding Decimals Using Place Value
--	---


To round a number using the place value method,

1. Locate the place value in which you need to round.
2. Determine the digit one place value to the right of this place value.
3. If the digit is 0,1,2,3 or 4, drop all the digits to the right of place value you are rounding.
4. If the digit is 5,6,7,8 or 9, add one to the place value in which you are rounding and drop all the digits to the right of place value you are rounding.

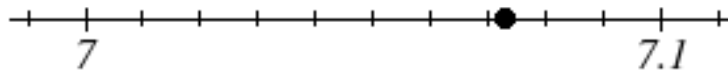
Put the numbers in the place value chart. Use the place value chart as an aid to round the number to the indicated place value.

- a) Round 3.24 to the nearest tenth.
- b) Round 23.56 to the nearest whole number.
- c) Round 0.073 to the nearest hundredth.
- d) Round 5.043 to the nearest tenth.
- e) Round 22.296 to the nearest hundredth

Ones			.	Decimals		
Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
			.			
			.			
			.			
			.			
			.			
			.			

 Problem 33	YOU-TRY - Rounding Decimals
--	-----------------------------

- a) Round the number represented below to the nearest whole number, one decimal place, and two decimal places.



Rounded to the nearest whole number: _____

Rounded to the nearest tenth: _____


Rounded to two decimal places: _____

- b) Put the numbers in the place value chart. Use the place value chart as an aid to round the number to the indicated place value.
- Round 5.32 to the nearest tenth.
 - Round 37.09 to the nearest whole number.
 - Round 0.054 to the nearest hundredth.
 - Round 6.032 to one decimal place.
 - Round 17.497 to two decimal places

Ones			.	Decimals		
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	
			.			
			.			
			.			
			.			
			.			
			.			

SECTION 4.12: WRITING AND ROUNDING DECIMALS IN APPLICATIONS

In this section, we will look at a few application where we may round counter the standard convention. Also, we will look at applications that use rounded decimals to represent very large and very small numbers to approximate numbers.

 Problem 34	MEDIA EXAMPLE – Applications and Rounding
---	--

Round the results of the application problems so that it makes sense in the context of the problem.

- Lara runs her own plant business. She computes that she needs to sell 72.38 plants per week to make a profit. Since she can only sell a whole number of plants, how many does she need to sell to make a profit?
- Tia is making a work bench for her art studio. She measures the space and needs 3.42 meters of plywood. The store only sells plywood by the tenth of a meter. How many meters should Tia buy?
- Crystal is buying Halloween candy at the store. She has \$20 and wants to buy as many bags of candy as possible. She computes that she has enough to buy 4.87 bags of candy. How many bags of candy can she buy?

 **Problem 35** **MEDIA EXAMPLE – Writing Large and Small Numbers with Rounded Decimals**

Write the decimal approximations for the given numbers as place value numbers. Use the place value chart below to aid your work.

- a) Mount Kilimanjaro is approximately 19.3 thousand feet.
- b) In 2013, the population of China was approximately 1.357 billion people.
- c) A dollar bill is approximately 1.1 hundredths of a centimeter thick.

Billions			Millions			Thousands			Ones			Decimals		
Hundreds	Tens	One	Hundreds	Tens	One	Hundreds	Ten	One	Hundreds	Ten	One	Tenth	Hundredths	Thousandths

 **Problem 36** **YOU-TRY – Applications of Rounded Decimals**

- a) Jamie is running a booth at the local fair. She computes that she needs to sell 73.246 snow cones that day to make a profit. Since she can only sell a whole number of snow cones, how many does she need to sell to make a profit?
- b) Write the decimal approximations for the given numbers as place value numbers. Use the place value chart below to aid your work.
 - i. The Empire State building is approximately 17.4 thousand inches tall.
 - ii. The diameter of a grain of sand is approximately 6.3 hundredths of a millimeter.

Billions			Millions			Thousands			Ones			Decimals		
Hundreds	Tens	One	Hundreds	Tens	One	Hundreds	Ten	One	Hundreds	Ten	One	Tenth	Hundredths	Thousandths

SECTION 4.13: CONVERTING BETWEEN DECIMALS AND FRACTIONS



We can use a calculator to divide a fraction's numerator by its denominator to convert a fraction to a decimal. When the corresponding decimal for a fraction doesn't terminate, we will approximate the decimal by dividing and then rounding.

Many calculators have functions that will convert a decimal to a simplified fraction. We will use the TI 30XS Multiview which also has the capability to convert improper fractions to mixed numbers and simplify the results. If you use a different calculator, you can look it up in the manual or you should google how to convert decimal to fractions and the calculator type. There are many videos online to assist you.



Problem 37

MEDIA EXAMPLE – Approximating Fractions as Decimals with a Calculator

Approximate the following fractions with decimals by dividing on your calculator. Give approximations to one, two, and three decimal places.

a) $\frac{8}{21}$	b) $-\frac{13}{17}$	c) $-4\frac{5}{7}$
one decimal place: _____	one decimal place: _____	one decimal place: _____
two decimal places: _____	two decimal places: _____	two decimal places: _____
three decimal places: _____	three decimal places: _____	three decimal places: _____



Problem 38

YOU-TRY — Approximating Fractions as Decimals with a Calculator

Approximate the following fraction with a decimal by dividing on your calculator. Give approximations to one, two, and three decimal places.


$$-3\frac{4}{13}$$

one decimal place: _____

two decimal places: _____


three decimal places: _____

Unit 4 – Media Lesson

	Problem 39	MEDIA EXAMPLE –Writing Decimals as Simplified Fractions or Mixed Numbers
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Complete the table below. First perform the work by hand. Then check your results on your calculator.

	Decimal	Fraction or Mixed Number	Simplify Fraction	Final Answer
a)	- 0.4			
b)	3.25			
c)	6.008			
d)	-7.024			

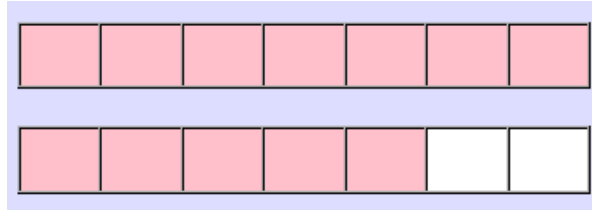
	Problem 40	YOU-TRY — Writing Decimals as Simplified Fractions or Mixed Numbers
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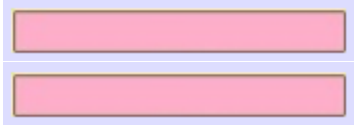
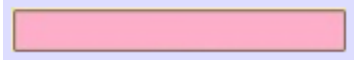

Complete the table below. Show all of your work for simplifying the fraction.

	Decimal	Fraction or Mixed Number	Simplify Fraction	Final Answer
a)	- 0.12			
b)	6.45			
c)	-7.016			

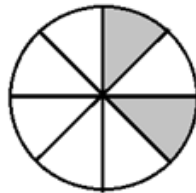
UNIT 4 – PRACTICE PROBLEMS

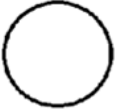


1. Determine the fraction represented by the area shaded pink using the given unit.



Unit	Fraction
	
	
	

2. Determine the fraction represented by the shaded area using the given unit.

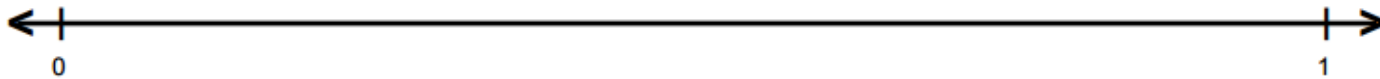


Unit	Fraction
	
	
	







Unit 4 – Practice Problems

3. Represent the unit fraction $\frac{1}{8}$ using each of the representations below.

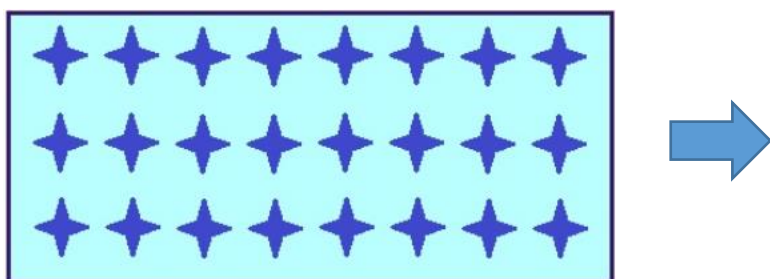
a) Number line



b) Area models. Use the unit labeled in the second row of the table.

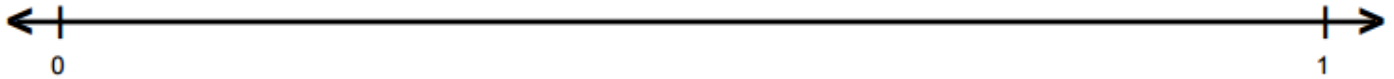
		
Unit: 	Unit: 	Unit: 

c) Discrete objects.









4. Represent the unit fraction $\frac{1}{3}$ using each of the representations below.

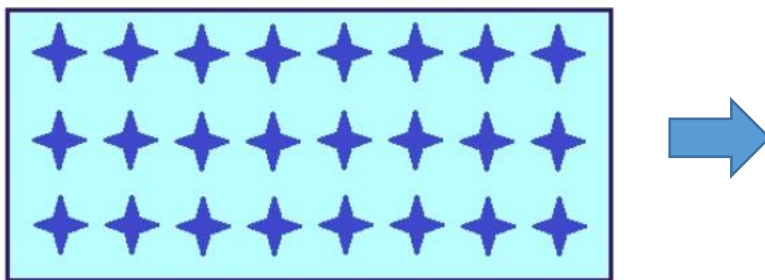
a) Number line



b) Area models. Use the unit labeled in the second row of the table.

		
Unit: 	Unit: 	Unit: 

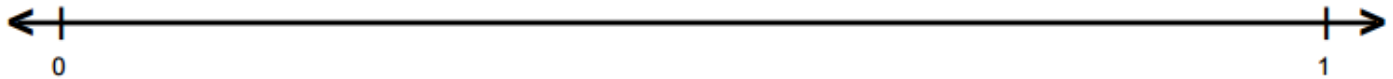
c) Discrete objects.









Unit 4 – Practice Problems

5. Represent the unit fraction $\frac{3}{8}$ using each of the representations below.

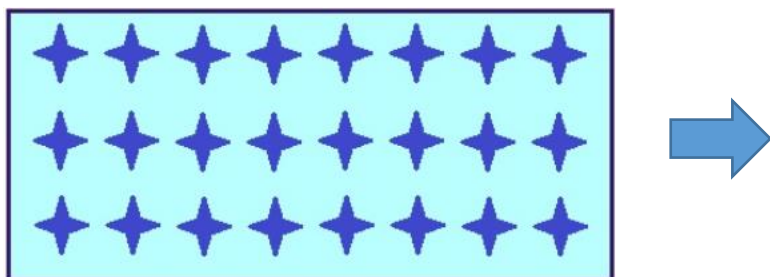
a) Number line



b) Area models. Use the unit labeled in the second row of the table.

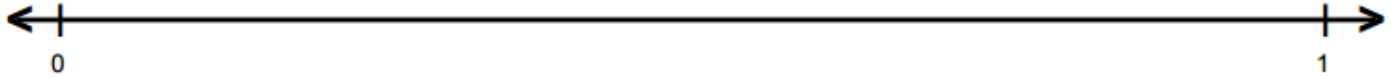
		
Unit: 	Unit: 	Unit: 

c) Discrete objects.









6. Represent the unit fraction $\frac{5}{6}$ using each of the representations below.

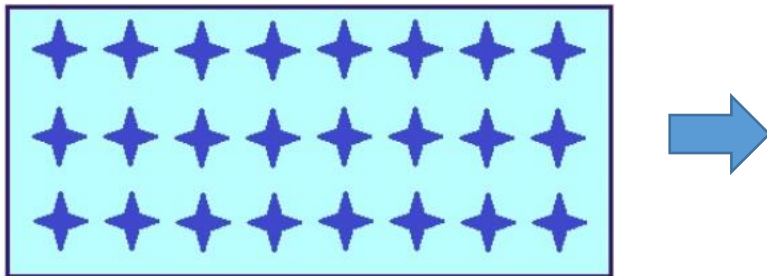
a) Number line



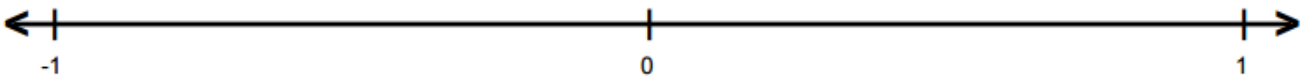
b) Area models. Use the unit labeled in the second row of the table.

		
Unit: 	Unit: 	Unit: 

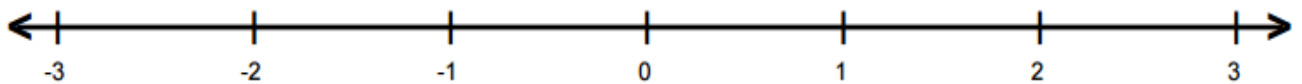
c) Discrete objects.



7. Plot the following fractions on the number line $\frac{1}{8}$, $-\frac{3}{8}$, $\frac{1}{2}$, $-\frac{3}{4}$. Label your points.



8. Plot the following improper fractions on the number line $-\frac{5}{2}$, $\frac{11}{4}$, $-\frac{5}{4}$, $\frac{0}{4}$, $\frac{4}{2}$. Label your points.



Unit 4 – Practice Problems

9. Represent the composite fractions using an area model. A single rectangle is the unit. An additional rectangle is given in each problem for the fractions which may require it.

c) Represent $\frac{5}{8}$ with a rectangle as the unit. _____ copies of _____ (unit fraction)

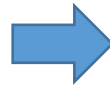
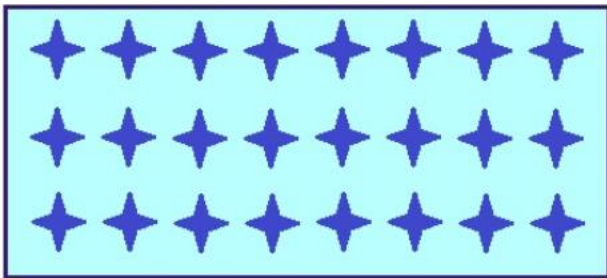


d) Represent $\frac{8}{5}$ with a rectangle as the unit. _____ copies of _____ (unit fraction)

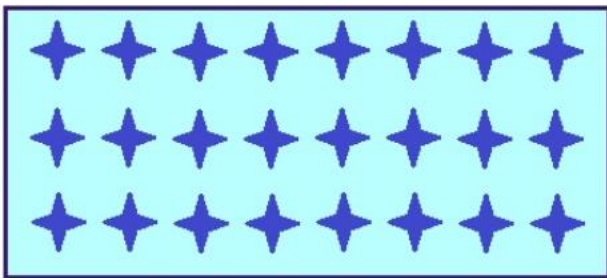


10. Represent the composite fractions using the discrete objects. The unit is all of the stars in the rectangle.

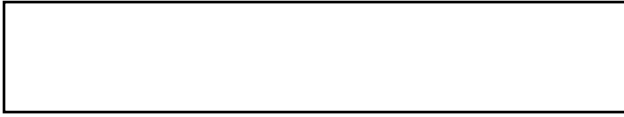
a) Represent $\frac{5}{6}$ of the stars. _____ copies of _____ (unit fraction)



b) Represent $\frac{5}{4}$ of the stars. _____ copies of _____ (unit fraction)

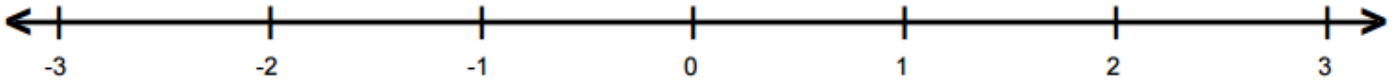


11. Represent $\frac{5}{3}$ with a rectangle as the unit. Then rewrite it as a mixed number. (A single rectangle is the unit)



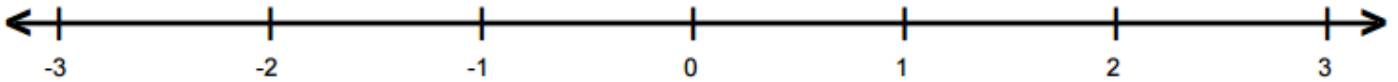
Mixed Number: _____

12. Represent $\frac{5}{3}$ on the number line. Then rewrite it as a mixed number.



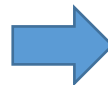
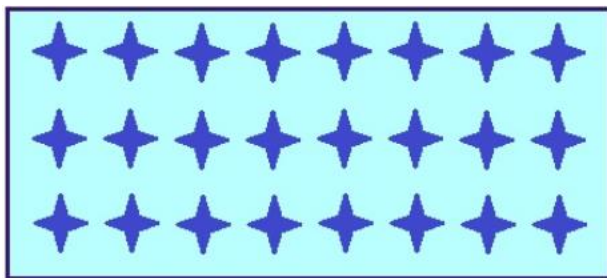
Mixed Number: _____

13. Represent $-\frac{7}{4}$ on the number line. Then rewrite it as a mixed number.



Mixed Number: _____

14. Represent $\frac{5}{3}$ of the stars. Then rewrite it as a mixed number.



Mixed Number: _____

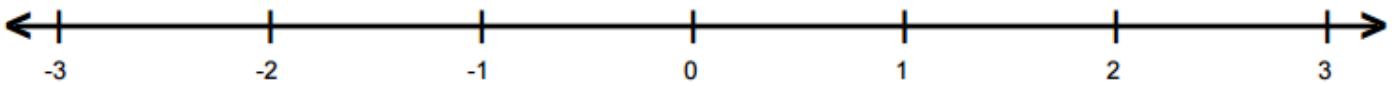
Unit 4 – Practice Problems

15. Represent $1\frac{3}{8}$ with a rectangle as the unit. Then rewrite it as an improper fraction. (A single rectangle is the unit)



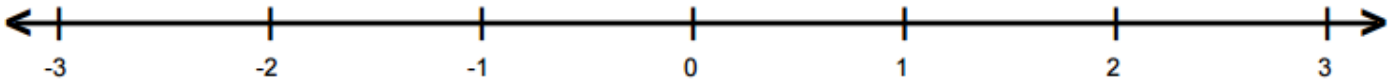
Improper fraction: _____

16. Represent $2\frac{3}{4}$ on the number line. Then rewrite it as an improper fraction.



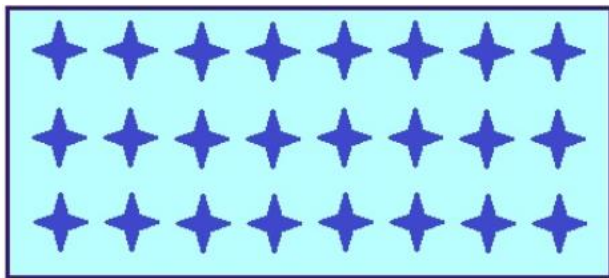
Improper fraction: _____

17. Represent $-1\frac{1}{3}$ on the number line. Then rewrite it as an improper fraction.



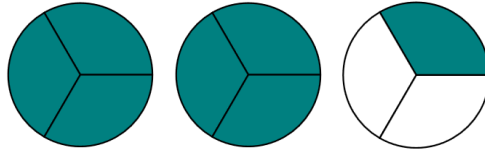
Improper fraction: _____

18. Represent $1\frac{1}{12}$ of the stars. Then rewrite it as an improper fraction.



Improper fraction: _____

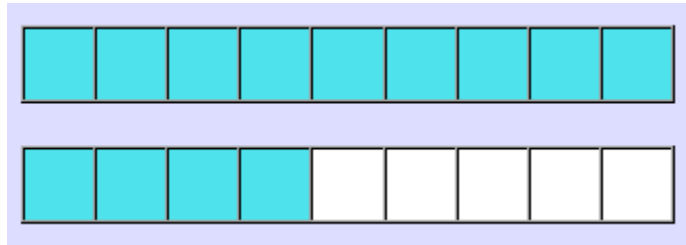
19. Use the image below to answer the following questions. The unit is one circle.



a) Determine the *improper fraction* that represents the shaded portion of the circles.

b) Determine the *mixed number* that represents the shaded portion of the circles.

20. Use the image below to answer the following questions. The unit is one rectangle.



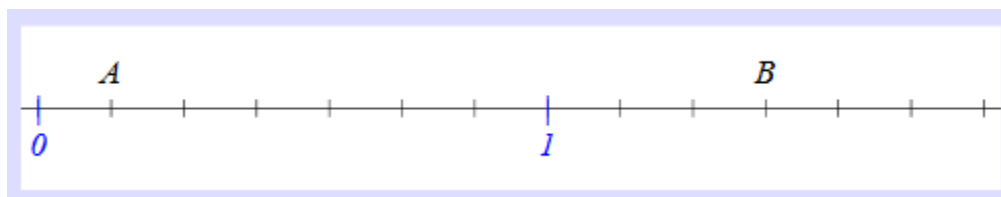
a) Determine the *improper fraction* that represents the shaded portion.

b) Determine the *mixed number* that represents the shaded portion.

21. Complete the table below.

Improper Fraction	Mixed Number
$\frac{19}{5}$	
$-\frac{33}{10}$	
$\frac{52}{7}$	
	$1\frac{3}{8}$
	$-5\frac{1}{6}$
	$10\frac{5}{9}$

22. Identify the fractions labeled with the letters A and B on the scale below. If appropriate, write your answers as *both* an improper fraction *and* a mixed number.



a) Letter A represents the fraction: _____

b) Letter B represents the fraction: _____

23. Rewrite the given fractions as equivalent fractions given the indicated numerator or denominator.

a. Rewrite $\frac{2}{3}$ with a denominator of 21.

b. Rewrite $\frac{6}{15}$ with a numerator of 30.

c. Rewrite $\frac{-5}{8}$ with a numerator of -20.

d. Rewrite $\frac{60}{6}$ with a denominator of 2.

e. Rewrite $\frac{-36}{48}$ with a numerator of -9.

f. Rewrite 2 with a denominator of 5.

g. Rewrite -3 with a denominator of 4.

h. Rewrite -1 with a numerator of 7.

Unit 4 – Practice Problems

24. For each problem below, write the fraction that best describes the situation. Be sure to reduce your final result.
- John had 12 marbles in his collection. Three of the marbles were Comet marbles. What fraction of the marbles were Comet marbles? What fraction were NOT Comet marbles?
 - Jorge's family has visited 38 of the 50 states in America. What fraction of the states have they visited?
 - In a given bag of M & M's, 14 were yellow, 12 were green, and 20 were brown. What fraction were yellow? Green? Brown?
 - Donna is going to swim 28 laps. She has completed 8 laps. What fraction of laps has she completed? What fraction of her swim remains?
 - Last night you ordered a pizza to eat while watching the football game. The pizza had 12 pieces of which you ate 6. Today, two of your friends come over to help you finish the pizza and watch another game. What is the fraction of the LEFTOVER pizza that each of you gets to eat (assuming equally divided). What is the fraction of the ORIGINAL pizza that each of you gets to eat (also assuming equally divided).

25. Which of the following CANNOT be written as a mixed number and why?

a. $\frac{8}{3}$

b. $\frac{15}{8}$

c. $\frac{21}{25}$

d. $\frac{34}{27}$

e. $\frac{11}{12}$

26. Write two equivalent fractions for each of the fractions below.

a. $\frac{3}{7}$

b. $\frac{4}{5}$

c. $\frac{2}{9}$

d. $\frac{5}{8}$

e. $\frac{11}{12}$

27. Write each fraction in simplest form.

a. $\frac{3}{6}$

b. $\frac{15}{5}$

c. $\frac{12}{36}$

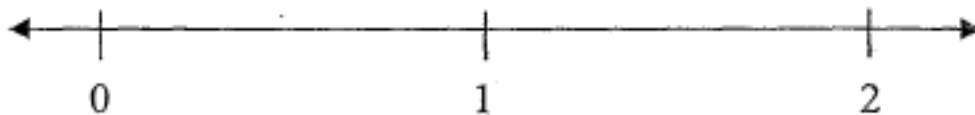
d. $\frac{120}{164}$

e. $\frac{11}{11}$

f. $\frac{0}{21}$

28. Using equally spaced tick marks, plot the following numbers on the number line.

$$\frac{1}{8}, \frac{8}{8}, \frac{11}{8}, \frac{0}{8}, \frac{3}{4}$$



29. Simplify each of the following fractions if possible.

a) $\frac{5}{1}$

b) $\frac{6}{6}$

c) $\frac{0}{4}$

d) $\frac{1}{6}$

e) $\frac{1}{1}$

f) $\frac{1}{0}$

Unit 4 – Practice Problems

30. Order the fractions from least to greatest and justify your answer.

a) $\frac{7}{19}, \frac{7}{8}, \frac{7}{14}, \frac{7}{5}, \frac{7}{7}$ Ordering:

b) $\frac{8}{7}, \frac{5}{7}, \frac{1}{7}, \frac{7}{7}, \frac{12}{7}$ Ordering:

c) $\frac{3}{8}, -\frac{8}{3}, -\frac{3}{8}, \frac{8}{3}, \frac{8}{8}$ Ordering:

d) $\frac{7}{8}, \frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{8}{9}$ Ordering:

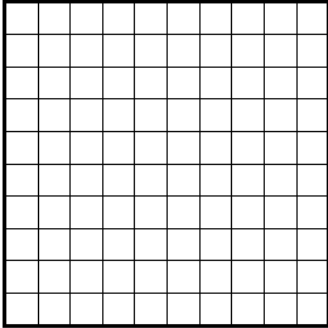
31. Simplify the given fractions completely using **both the repeated division and prime factorization methods**. In each case, state which you think is easier and why.

Fraction	Repeated Division Method	Prime Factorization Method
a) $\frac{12}{42}$		
b) $-\frac{16}{100}$		
c) $5\frac{27}{63}$		

Unit 4 – Practice Problems

32. Shade the indicated quantity and rewrite in the indicated forms.

a) 38 hundredths

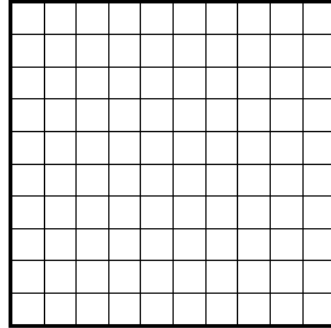


Decimal: _____

Expanded Form: _____

Fraction Form: _____

b) $\frac{15}{100}$

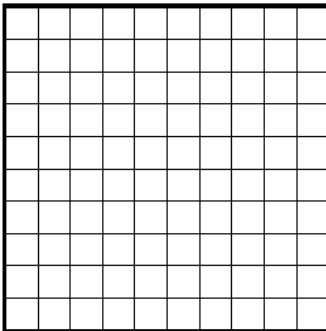


Decimal: _____

Expanded Form: _____

Word Name: _____

c) 2 tenths and 2 hundredths

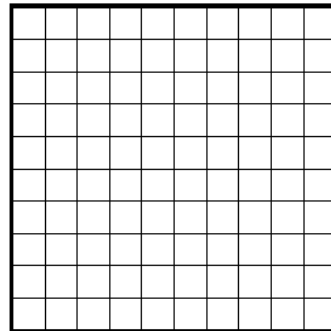


Decimal: _____

Expanded Form: _____

Fraction Form: _____

d) $\frac{5}{10} + \frac{1}{100}$



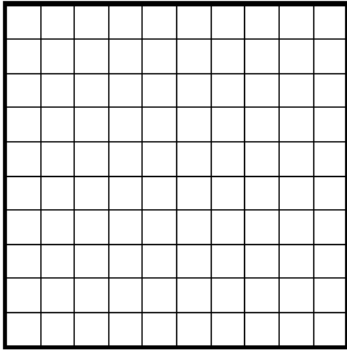
Decimal: _____

Expanded Form: _____

Word Name: _____

33. Shade the indicated quantity and rewrite in the indicated forms.

a) 0.4

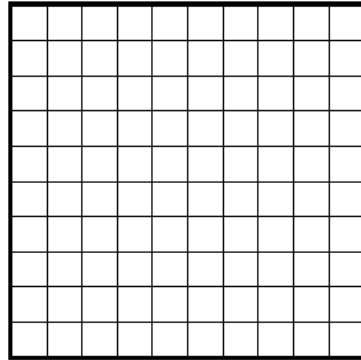


Fraction Name: _____

Word Name: _____

Expanded Form: _____

b) 0.80

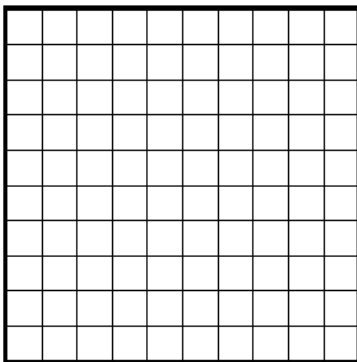


Fraction Name: _____

Word Name: _____

Expanded Form: _____

c) 0.91

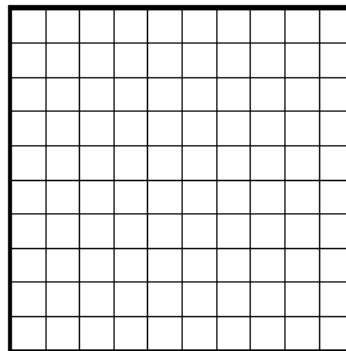


Fraction Name: _____

Word Name: _____

Expanded Form: _____

d) 0.03



Fraction Name: _____

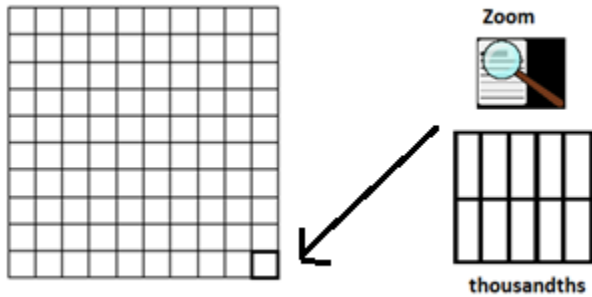
Word Name: _____

Expanded Form: _____

Unit 4 – Practice Problems

34. Shade the indicated quantity and write the corresponding decimal number.

a) 6 tenths and 3 hundredths and 5 thousandths

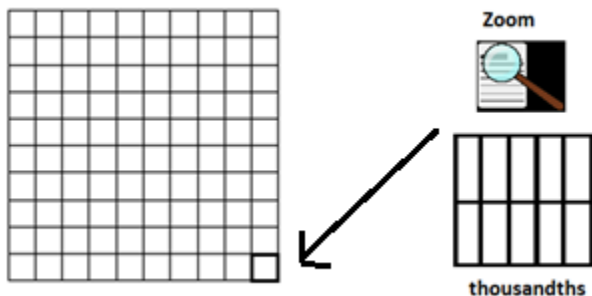


Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimal Number: _____ Expanded Form: _____

b) 3 hundredths and 2 thousandths

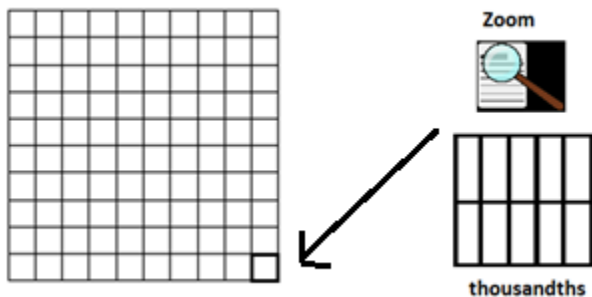


Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimal Number: _____ Expanded Form: _____

c) 452 thousandths



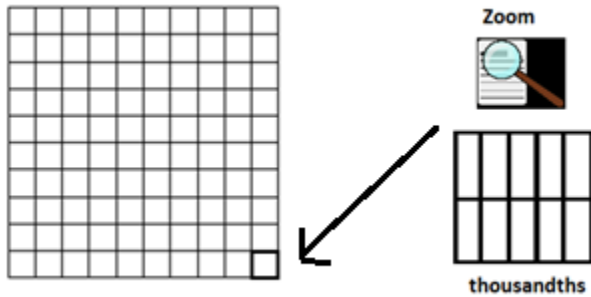
Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimal Number: _____ Expanded Form: _____

35. Shade the indicated quantity. Then write the number in words and expanded form.

a) 0.123



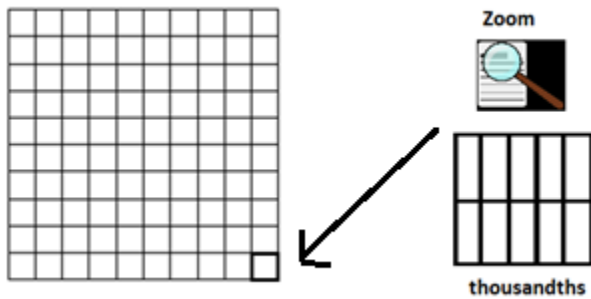
Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	.			

Expanded Form: _____

In words: _____

b) 0.016



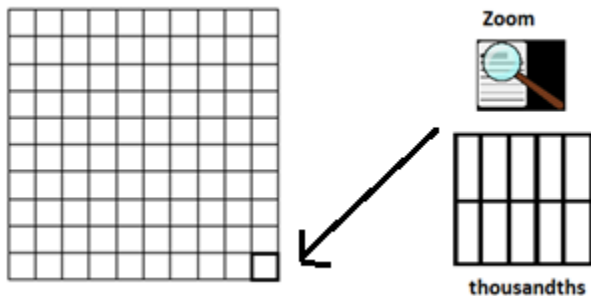
Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	.			

Expanded Form: _____

In words: _____

c) 0.502



Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	.			

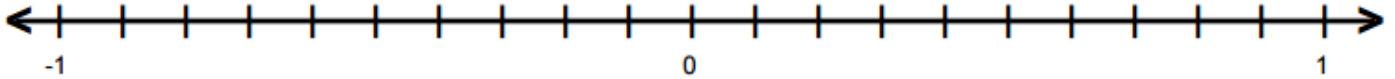
Expanded Form: _____

In words: _____

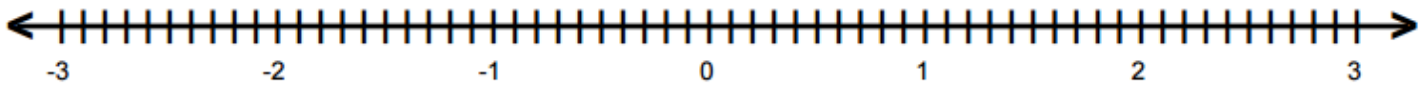
Unit 4 – Practice Problems

36. Use the given number lines to plot the following decimals.

a) Plot 0.3, 0.8, -0.5, and -0.9 on the number line below. Label the points underneath the number line.



b) Plot 1.8, 0.2, -1.1, and -2.7 on the number line below. Label the points underneath the number line.



37. Use the place value chart to order the numbers from least to greatest

2.8, 2.08, 2.88, 2.088, 2.008, 2.808, 0.28

Ones			.	Decimals		
Hundred	Ten	One	.	Tenth	Hundredth	Thousandth
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			

38. Use your knowledge of negative numbers to order the numbers below

-2.8, -2.08, -2.88, -2.088, -2.008, -2.808, -0.28

39. Place the following numbers in order from smallest to largest.

0.2, 0.25, 0.74, 0.7, 0.40, 0.08

40. Order the signed decimals below using the symbols $<$, $=$, or $>$.

a) 0.45 _____ 0.54

b) 0.308 _____ 0.038

c) 0.32 _____ -0.99

d) 3.005 _____ 3.05

e) 0.33 _____ 0.3

f) -0.48 _____ -0.048

g) 5.09 _____ 5.1

h) 19.321 _____ 19.32

i) -12.403 _____ 1.002

j) 3.42 _____ 3.402

k) -5.96 _____ -6

l) -8.19 _____ -8.2

41. Round the number represented below. (Note: The big square is the unit. Gray shading represents a whole.)



Given number: _____

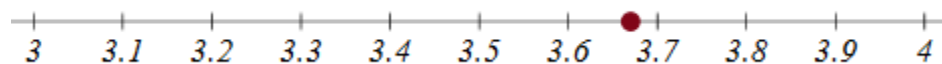
Rounded to the one's place: _____

Rounded to the tenth's place: _____

Rounded to the nearest whole number: _____

Rounded to one decimal place: _____

42. Round the number represented below.



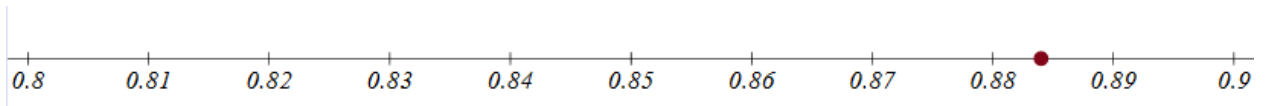
Rounded to the nearest whole number: _____

Rounded to one decimal place: _____

Rounded to the nearest tenth: _____

Unit 4 – Practice Problems

43. Round the number represented below.



Rounded to the nearest whole number: _____

Rounded to one decimal place: _____

Rounded to the nearest hundredth: _____

44. Put the numbers in the place value chart. Use the place value chart as an aid to round the number to the indicated place value.

a) Round 8.53 to the nearest tenth.

b) Round 186.485 to the nearest whole number.

c) Round 5.283 to the nearest hundredth.

d) Round 139.081 to the nearest tenth.

e) Round 78.165 to two decimal places

f) Round 8.53 to the ones place.

g) Round 186.485 to the nearest tenth.

h) Round 5.283 to one decimal place.

i) Round 139.081 to the nearest ten.

Ones			.	Decimals		
Hundred	Ten	One	Tenth	Hundredth	Thousandth	
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			

45. Round the results of the application problems so that it makes sense in the context of the problem.
- a) Amy is buying ribbon for an art project. She estimates that she will need 3.34 meters of ribbon. The store only sells ribbon by the tenth of a meter. How many meters should she buy?

 - b) John is catering a luncheon and needs 12.37 pounds of sugar. If sugar is only sold in one pound bags, how many bags should John buy?

 - c) Shelly is buying shoes online and computes that she has enough money to buy 2.78 pairs of shoes. How many pairs of shoes can she buy?

 - d) Tia is making a work bench for her art studio. She measures the space and needs 8.24 meters of plywood. The store only sells plywood by the tenth of a meter. How many meters should Tia buy?

 - e) Jamie is running a booth at the local fair. She computes that she needs to sell 86.25 snow cones that day to make a profit. Since she can only sell a whole number of snow cones, how many does she need to sell to make a profit?

 - f) Crystal is buying Halloween candy at the store. She has \$20 and wants to buy as many bags of candy as possible. She computes that she has enough to buy 6.91 bags of candy. How many bags of candy can she buy?

Unit 4 – Practice Problems

46. Write the decimal approximations for the given numbers as place value numbers.

- a) In 2015, the population of Tallyville was approximately 8.82 million people.

- b) The tallest building in Tallyville is approximately 22.4 thousand feet.

- c) The smallest bug in Tallyville has a radius of approximately 5.6 hundredths of an inch.

- d) The width of a piece of paper in Tallyville is approximately 1.81 tenths of an inch.

47. Write each fraction in decimal form.

a) $\frac{33}{100}$

b) $\frac{308}{10}$

Decimal: _____

Decimal: _____

c) $\frac{81}{10}$

d) $\frac{5}{100}$

Decimal: _____

Decimal: _____

e) $\frac{400}{100}$

f) $\frac{3}{1000}$

Decimal: _____

Decimal: _____

48. Write the following fractions in decimal form. Round to the nearest thousandth as needed.

a) $\frac{1}{9}$

b) $\frac{2}{3}$

c) $\frac{2}{11}$

49. Approximate the following fraction with a decimal by dividing on your calculator. Round to the indicated place value.

a) $\frac{5}{9}$	Tenth: _____ Whole Number: _____	Hundredth: _____ One decimal place: _____
b) $-\frac{11}{13}$	Two decimal places: _____ Thousandth: _____	Four decimal places: _____ Integer: _____
c) $2\frac{1}{7}$	Four decimals: _____ Whole Number: _____	Tenth: _____ Hundredth: _____
d) $-3\frac{4}{11}$	Tenth: _____ Integer: _____	Hundredth: _____ Thousandth: _____
e) $24\frac{8}{9}$	Tenth: _____ Whole Number: _____	Hundredth: _____ Ten: _____
f) $-\frac{8}{3}$	Two decimal places: _____ Thousandth: _____	Four decimal places: _____ Integer: _____
g) $548\frac{3}{7}$	Ten: _____ Hundred: _____	Tenth: _____ Hundredth: _____

Unit 4 – Practice Problems

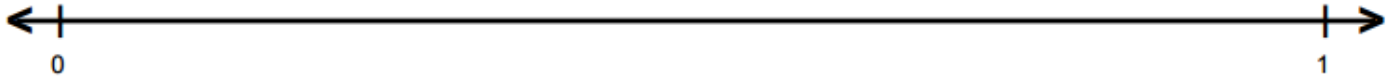
50. Complete the table below. Show all of your work for simplifying the fraction.

	Decimal	Fraction	Simplified Fraction
a)	0.5		
b)	- 0.46		
c)	0.42		
d)	0.008		
e)	-0.2		
f)	7.05		
g)	11.012		
h)	- 8.004		







UNIT 4 – END OF UNIT ASSESSMENT

1. Represent the unit fraction $\frac{1}{8}$ using each of the representations below:

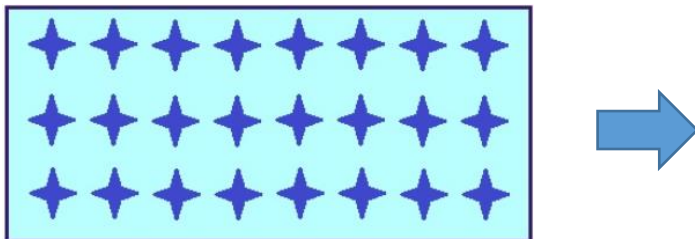
a) Number line



b) Area models. Use the unit labeled in the second row of the table.

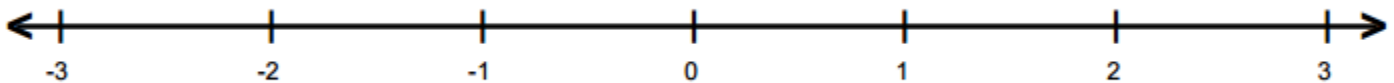
		
Unit: 	Unit: 	Unit: 

c) Discrete objects.



2. Plot the following fractions on the number line:

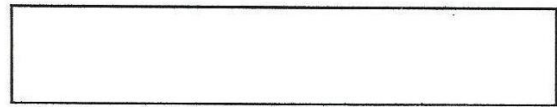
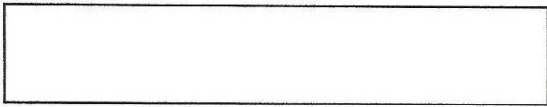
$$-\frac{5}{6}, \frac{1}{6}, \frac{7}{6}, -\frac{1}{6}, \frac{4}{6}, -\frac{12}{6}$$



Unit 4 – Assessment

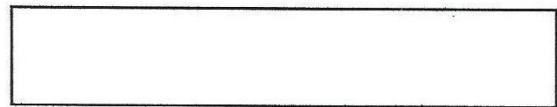
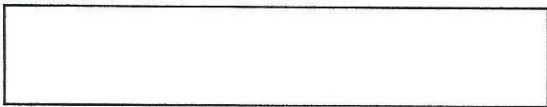
3. Represent the fractions using an area model. A single rectangle is the unit. An additional rectangle is given in each problem for the fractions which may require it.

a) Represent $\frac{2}{3}$ with a rectangle as the unit.



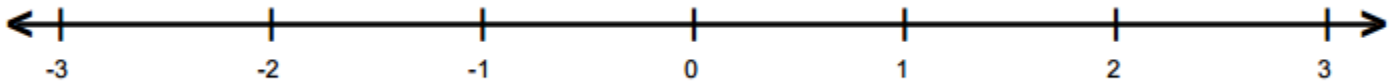
_____ copies of _____ (unit fraction)

b) Represent $\frac{3}{2}$ with a rectangle as a unit.



_____ copies of _____ (unit fraction)

4. Plot $\frac{5}{4}$ on the number line. Then rewrite it as a mixed number.



Mixed number: _____

5. Complete the table below:

Improper Fraction	Mixed Number
$\frac{12}{7}$	
	$1\frac{1}{5}$
$-\frac{9}{8}$	
	$-2\frac{1}{3}$

6. Rewrite the given fractions as equivalent fractions given the indicated numerator or denominator. Show your work.

a) Rewrite $\frac{2}{7}$ with a denominator of 21.

b) Rewrite $\frac{-8}{3}$ with a numerator of -32.

c) Rewrite 3 with a numerator of 12.

7. Write two equivalent fractions for each of the fractions below. Show your work.

a) $\frac{7}{11}$

b) $\frac{3}{10}$

c) $\frac{2}{5}$

Unit 4 – Assessment

8. Write each fraction in simplest form. Show your work.

a) $-\frac{12}{18}$

b) $\frac{30}{16}$

c) $\frac{14}{7}$

9. Order the fractions from least to greatest. Show your work.

$$\frac{3}{4} \quad \frac{3}{8} \quad \frac{9}{8} \quad \frac{5}{8} \quad \frac{1}{4}$$

10. Find the fraction that best describes the situation. Write your answer in a complete sentence making sure that the fraction is in simplest form.

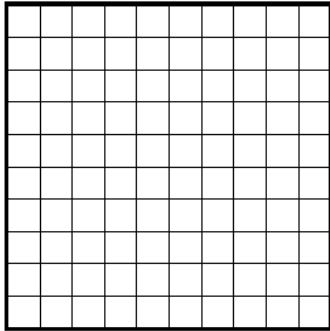
There are 15 freshmen in the Introductory Algebra class of 35 students. What fraction of the students are freshmen?

11. Find the fraction that best describes the situation. Write your answer in a complete sentence making sure that the fraction is in simplest form.

Jan has read 6 out of the 8 required books for her American Literature class. What fraction of the required books has Jan read?

12. Shade indicated quantity and rewrite in the indicated forms.

a) 43 hundredths

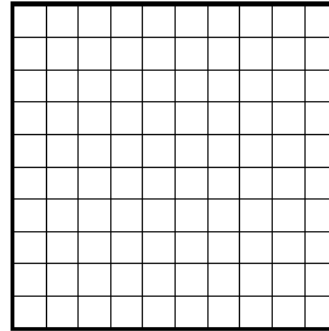


Decimal: _____

Expanded Form: _____

Fraction Form: _____

b) $\frac{7}{10}$



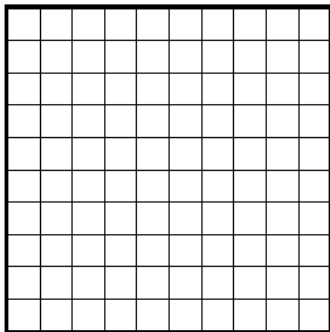
Decimal: _____

Expanded Form: _____

Word Name: _____

13. Shade the indicated quantity and rewrite in the indicated forms.

a) 0.2

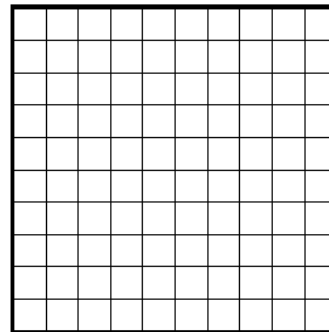


Fraction: _____

Expanded Form: _____

Word Name: _____

b) 0.02



Fraction: _____

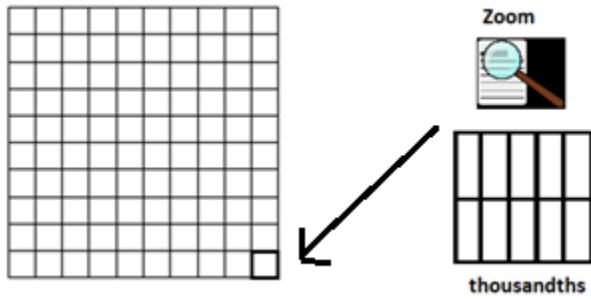
Expanded Form: _____

Word Name: _____

Unit 4 – Assessment

14. Shade the indicated quantity and rewrite in the indicated forms.

a) 0.006



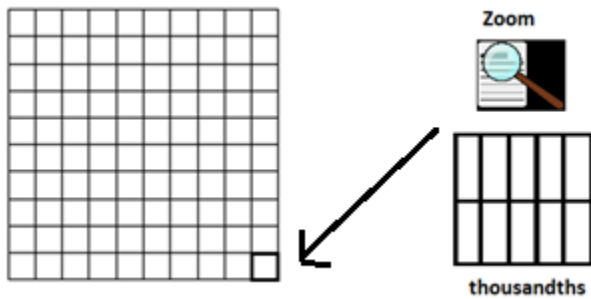
Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Expanded Form: _____

In words: _____

b) 0.435



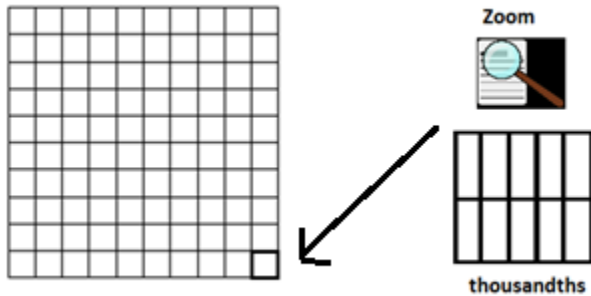
Place Value Chart

one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Expanded Form: _____

In words: _____

c) 0.053



Place Value Chart

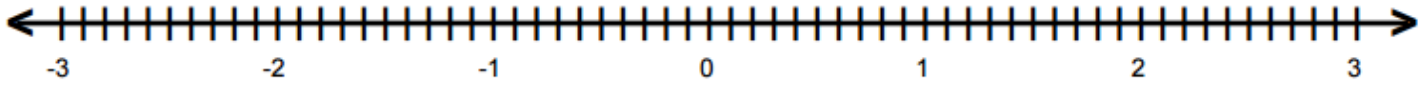
one's	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Expanded Form: _____

In words: _____

15. Plot the following decimals on the number line below. Label the points underneath the number line.

0.5, -1.9, 2.3, -0.3, 1.2



16. Use the place value chart to order the numbers from least to greatest.

1.7 1.07 1.77 1.077 1.007 1.707 0.17

Ones			.	Decimals		
Hundred	Ten	One	.	Tenth	Hundredth	Thousandth
			.			
			.			
			.			
			.			
			.			
			.			
			.			
			.			

17. Use your knowledge of negative numbers to order the numbers below:

-1.7 -1.07 -1.77 -1.077 -1.007 -1.707 -0.17

18. Order the signed decimals below using the symbols $<$, $=$ or $>$.

a) 0.03 _____ 0.3

b) -0.52 _____ -0.5

c) 0.4 _____ 0.40

Unit 4 – Assessment

19. Round each number to the indicated place value.

- a) Round 0.064 to the nearest hundredth.
- b) Round 7.078 to the nearest tenth.
- c) Round 3.15 to the nearest whole number.

20. Round the result of the application problem so that it makes sense in the context of the problem. Show work. Write your answer in a complete sentence.

Donna is planning a barbeque and needs 12.25 pounds of potato salad. If potato salad is sold in one pound containers. How many one pound containers should Donna buy?

21. Round the result of the application problem so that it makes sense in the context of the problem. Show work. Write your answer in a complete sentence.

Linda owns a clothing store and needs to order designer blouses for her customers. She realizes that she has enough money to buy 18.75 blouses from the designer. Linda has 19 customers who each want one of the blouses. Will Linda be able to sell a blouse to each of these customers?

UNIT 5 – OPERATIONS WITH FRACTIONS

INTRODUCTION

In this Unit, we will use the multiple meanings and representations of fractions that we studied in the previous unit to develop understanding and processes for performing operations with fractions. You are likely familiar with algorithms for computing fraction operations. Please accept the challenge of really thinking through the meaning of operations and the different contexts of fractions so you understand *why* we perform operations as we do. If you accept this challenge, this unit will help you see the meaning of these processes not just the steps.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Learning Objective	Media Examples	You Try
Add fractions with like denominators	1	4
Add fractions with unlike denominators	2	4
Add improper fractions and mixed numbers	3	4
Subtract fractions with like denominators	5	8
Subtract fractions with unlike denominators	6	8
Subtract improper fractions and mixed numbers	7	8
Multiply a unit fraction times a whole number	9	11
Multiply a composite fraction times a whole number	10	11
Multiply a whole number times a fraction	12	15
Multiply two fractions	13	15
Multiply mixed numbers	14	15
Divide a whole number by a fraction	16	19
Divide fraction with common denominators	17	19
Divide fraction with uncommon denominators	18	19
Divide mixed numbers	20	21
Perform +, -, \times , \div on signed fraction	22	24
Use the order of operations with signed fractions	23	24

UNIT 5 – MEDIA LESSON

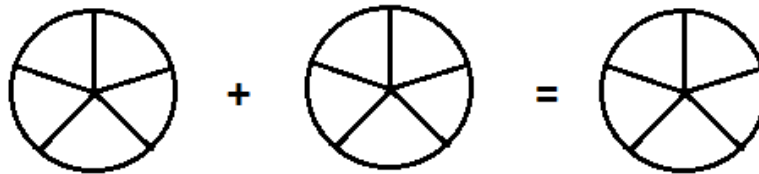
SECTION 5.1: ADDING FRACTIONS

In this section, we will learn to visualize the addition of fractions using an area model and number lines. Recall that the operation of addition is combining two amounts or adding one amount *on to* another. We will achieve this with fractions by ensuring we have a common unit fraction (common denominator) for the numbers we are combining. We can then add the number of copies of each fraction while retaining the common denominator.

Problem 1	MEDIA EXAMPLE – Adding Fractions with Like Denominators
-----------	---

Use the diagrams given to represent the values in the addition problem and find the sum. Then perform the operation and represent the sum using the symbolic representation of the algorithm.

1. Jon had $\frac{1}{5}$ of a pepperoni pizza and $\frac{2}{5}$ of a mushroom pizza. How much of one whole pizza did Jon have?

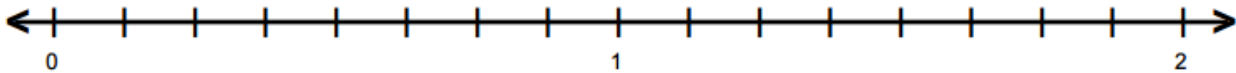


$\frac{1}{5}$ is _____ copies of _____ $\frac{2}{5}$ is _____ copies of _____

Combined, we have a total of _____ copies of _____ or the fraction _____.

Symbolic Representation of Algorithm:

2. Christianne sprinted $\frac{3}{8}$ of a mile and then jogged another $\frac{7}{8}$ of a mile. How far did she run in total?



$\frac{3}{8}$ is _____ copies of _____ $\frac{7}{8}$ is _____ copies of _____

Combined, we have a total of _____ copies of _____ or the fraction _____.

Simplified result as an improper fraction: _____ Simplified result as a mixed number: _____

Symbolic Representation of Algorithm:

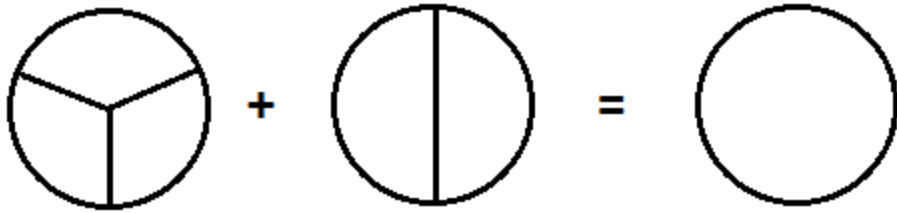


Problem 2

MEDIA EXAMPLE – Adding Fractions with Unlike Denominators

Use the diagrams given to represent the values in the addition problem and find the sum. Then perform the operation and represent the sum using the symbolic representation of the algorithm.

a) $\frac{1}{3} + \frac{1}{2}$



Step 1: Rewrite the given fractions with a common denominator.

$\frac{1}{3}$ is equivalent to _____ $\frac{1}{2}$ is equivalent to _____

Step 2: Rewrite the equivalent fractions with the *common denominator* using copies of language using a common unit fraction.

$\frac{1}{3}$: _____ is _____ copies of _____ $\frac{1}{2}$: _____ is _____ copies of _____

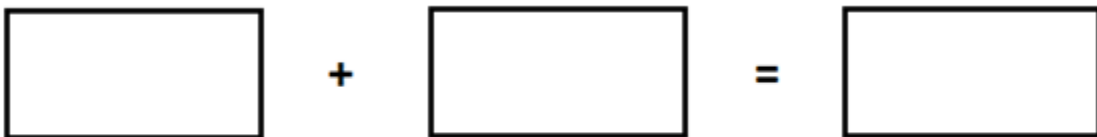
Step 3: Combine (add).

Combined, we have a total of _____ copies of _____ or the fraction _____.

Step 4: Simplify if necessary.

Symbolic Representation of Algorithm:

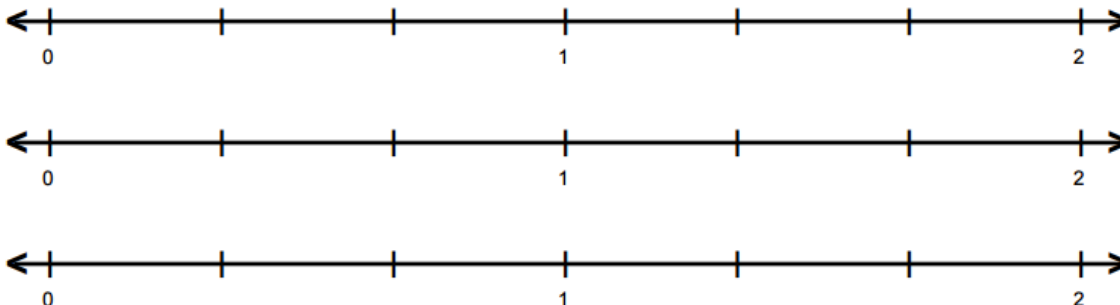
b) $\frac{1}{4} + \frac{3}{8}$




Symbolic Representation of Algorithm:

Unit 5 – Media Lesson

c) $\frac{5}{6} + \frac{4}{9}$

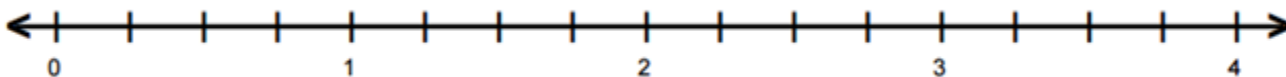


Symbolic Representation of Algorithm:

	Problem 3	MEDIA EXAMPLE – Adding Improper Fractions and Mixed Numbers
---	------------------	--

Use the diagrams given to represent the values in the addition problem and find the sum. Then perform the operation and represent the sum using the symbolic representation of the algorithm.

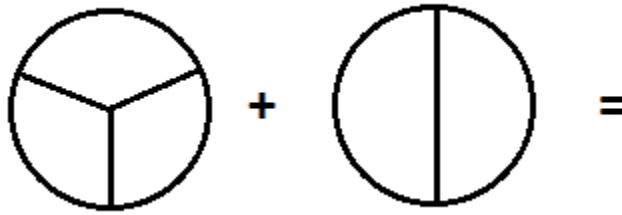
a) $\frac{7}{4} + \frac{5}{4}$



Symbolic Representation of Algorithm:

b) $4\frac{2}{3} + 3\frac{1}{2}$

Step 1: Rewrite the fractional parts of the mixed numbers with a common denominator



Step 2: Rewrite the equivalent fractions with the common denominator using copies of language using a common unit fraction.

$\frac{2}{3}$: _____ is _____ copies of _____ $\frac{1}{2}$: _____ is _____ copies of _____

Step 3: Combine the fractional parts and the whole number parts.

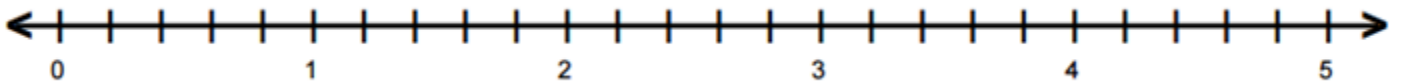
Fractional Parts: Combined, we have a total of _____ copies of _____ or the fraction _____.

Whole Numbers plus fractional parts: Combined we have a total of _____

Step 4: Simplify if necessary.

Symbolic Representation of Algorithm:

c) $2\frac{3}{5} + \frac{9}{5}$

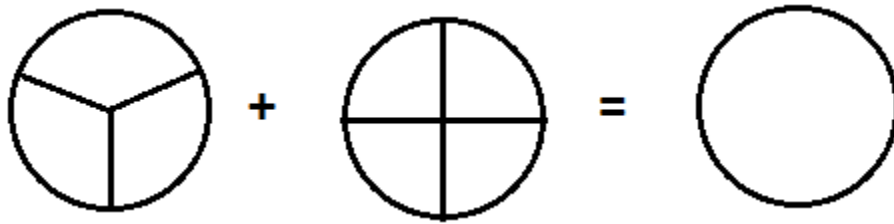


Symbolic Representation of Algorithm:

Problem 4	YOU TRY - Adding Numbers with Fractions
-----------	---

Use the diagrams given to represent the values in the addition problem and find the sum. Then perform the operation and represent the sum using the symbolic representation of the algorithm.

a) $\frac{2}{3} + \frac{1}{4}$



Step 1: Rewrite the given fractions with a common denominator.

$\frac{2}{3}$ is equivalent to _____ $\frac{1}{4}$ is equivalent to _____

Step 2: Rewrite the equivalent fractions with the common denominator using copies of language using a common unit fraction.

$\frac{2}{3}$: _____ is _____ copies of _____ $\frac{1}{4}$: _____ is _____ copies of _____

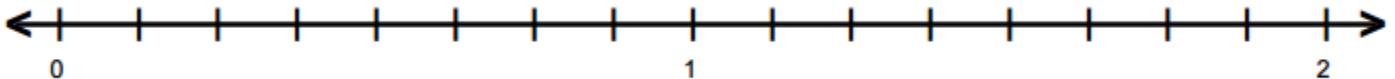
Step 3: Combine (add).

Combined, we have a total of _____ copies of _____ or the fraction _____.

Step 4: Simplify if necessary.

Symbolic Representation of Algorithm:

b) $1\frac{1}{4} + \frac{3}{8}$



Symbolic Representation of Algorithm:

SECTION 5.2: SUBTRACTING FRACTIONS

In this section, we will learn to visualize subtracting fractions using an area model and number lines. We will investigate this idea using our two models of subtraction; subtraction as taking away a part of a whole, and subtraction as comparing two quantities.

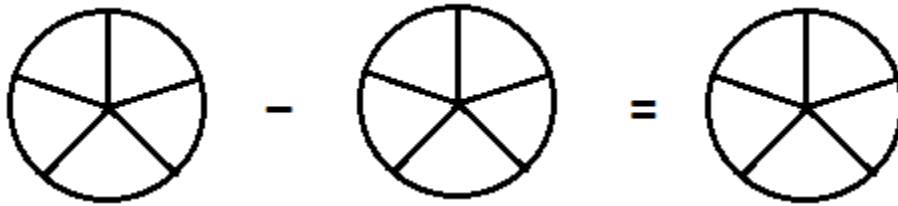


Problem 5

MEDIA EXAMPLE – Subtracting Fractions with Like Denominators

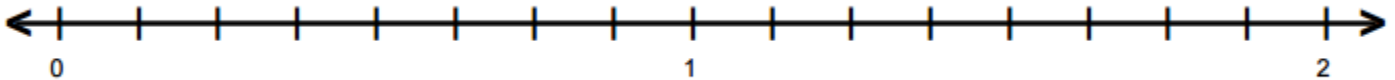
Use the diagrams given to represent the values in the subtraction problem and find the difference. Then perform the operation and represent the difference using the symbolic representation of the algorithm.

- a) The day after Thanksgiving, there was $\frac{3}{5}$ of a pumpkin pie remaining. Lara ate $\frac{1}{5}$ of a pie for breakfast. How much pie is leftover now?



Symbolic Representation of Algorithm:

- b) Billy ran $\frac{5}{8}$ of a mile. Roberta ran $\frac{11}{8}$ of a mile. How much further did Roberta run?



Symbolic Representation of Algorithm:

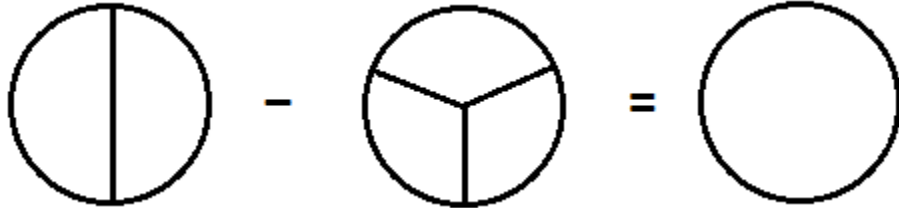


Problem 6

MEDIA EXAMPLE – Subtracting Fractions with Unlike Denominators

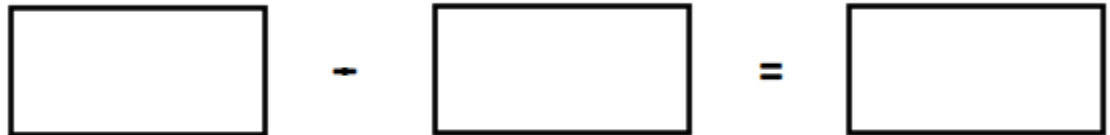
Use the diagrams given to represent the values in the subtraction problem and find the difference. Then perform the operation and represent the difference using the symbolic representation of the algorithm.

a) $\frac{1}{2} - \frac{1}{3}$



Symbolic Representation of Algorithm:

b) $\frac{5}{6} - \frac{2}{3}$



Symbolic Representation of Algorithm:

c) Perform the operation and represent the difference using the symbolic representation of the algorithm.

$$\frac{7}{12} - \frac{3}{8}$$

Symbolic Representation of Algorithm:

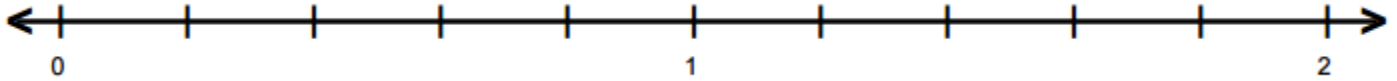


Problem 7

MEDIA EXAMPLE – Subtracting Improper Fractions and Mixed Numbers

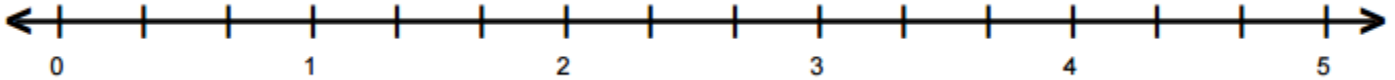
Use the diagrams given to represent the values in the subtraction problem and find the difference. Then perform the operation and represent the difference using the symbolic representation of the algorithm.

a) $\frac{8}{5} - \frac{6}{5}$



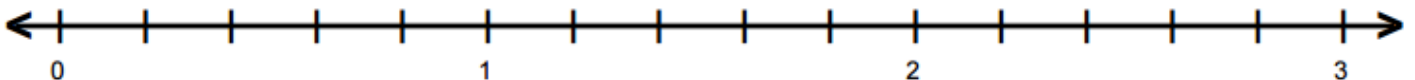
Symbolic Representation of Algorithm:

b) $4\frac{1}{3} - 3\frac{1}{2}$



Symbolic Representation of Algorithm:

c) $2\frac{3}{5} - \frac{11}{10}$



Symbolic Representation of Algorithm:

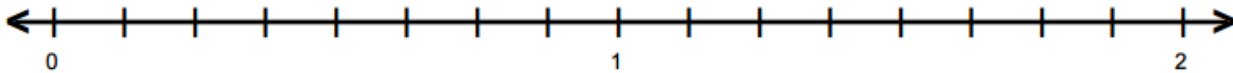


Problem 8

YOU TRY - Subtracting Numbers with Fractions

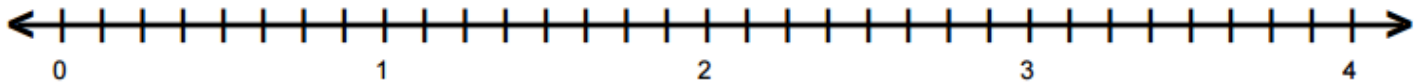
Use the diagrams given to represent the values in the subtraction problem and find the difference. Then perform the operation and represent the difference using the symbolic representation of the algorithm.

- a) Michael lives $1\frac{5}{8}$ of a mile from school. Rachel lives $\frac{7}{8}$ of a mile from school. How much closer to school does Rachel live?



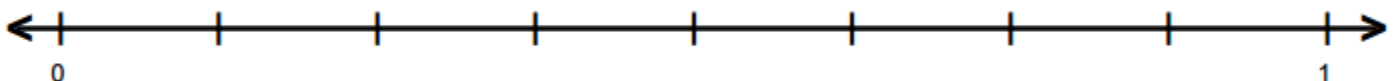
Symbolic Representation of Algorithm:

b) $3\frac{1}{8} - \frac{9}{8}$



Symbolic Representation of Algorithm:

- c) Barney had $\frac{7}{8}$ of his weekly salary left after paying his bills. He then spent $\frac{1}{4}$ of his weekly salary on a weekend trip. What fraction of his weekly salary remains?



Symbolic Representation of Algorithm:

SECTION 5.3: MULTIPLYING FRACTIONS

In this section, we will examine multiplying fractions using the idea that $a \times b$ or $a \cdot b$ is equivalent to a copies of b .

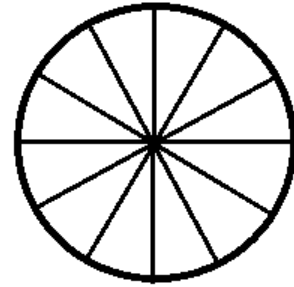


Problem 9

MEDIA EXAMPLE – Multiplying a Unit Fraction and a Whole Number

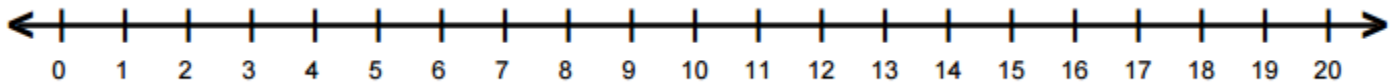
Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Ebony ate $\frac{1}{3}$ of a pizza. The pizza had 12 slices. How many slices did she eat?



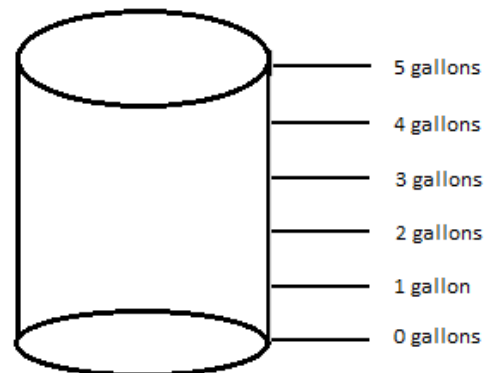
Symbolic Representation of Algorithm:

- b) Kimber was 20 miles from home. She travelled $\frac{1}{5}$ of this distance while listening to her favorite song. How many miles did she travel while listening to her favorite song?




Symbolic Representation of Algorithm:

- c) Logan has a 5 gallon bucket. He fills it $\frac{1}{4}$ of the way to the top. How much water is in the bucket?

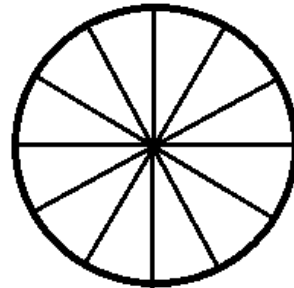


Symbolic Representation of Algorithm:

 Problem 10 MEDIA EXAMPLE – Multiplying a Composite Fraction and a Whole Number

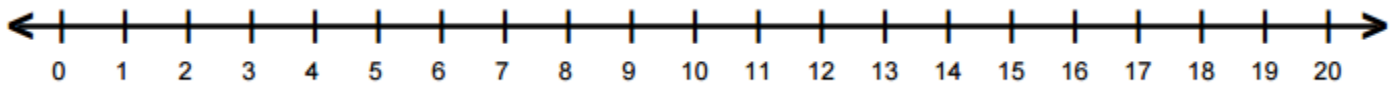
Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Ebony ate $\frac{2}{3}$ of a pizza. The pizza had 12 slices. How many slices did she eat?



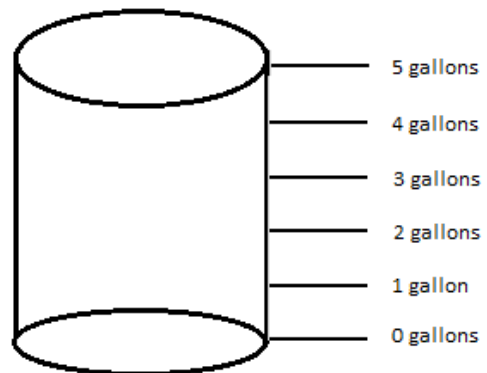
Symbolic Representation of Algorithm:

- b) Kimber was 20 miles from home. She travelled $\frac{3}{5}$ of this distance while listening to her favorite song. How many miles did she travel while listening to her favorite song?



Symbolic Representation of Algorithm:

- c) Logan has a 5 gallon bucket. He fills it $\frac{3}{4}$ of the way to the top. How much water is in the bucket?



Symbolic Representation of Algorithm:

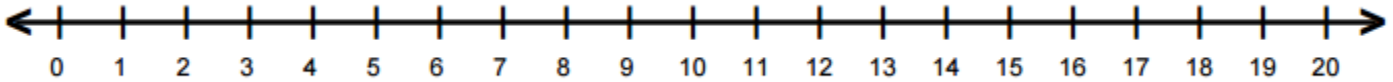


Problem 11

YOU TRY – Multiplying Unit or Composite Fractions with Whole Numbers

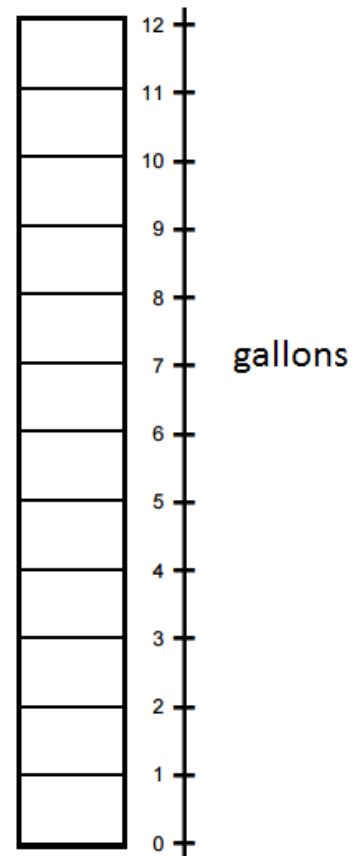
Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Carese was running a 10 kilometer race. She ran $\frac{1}{5}$ of the race in 12 minutes. How many kilometers did she run in 12 minutes?



Symbolic Representation of Algorithm:

- b) Casey's gas tank holds 12 gallons. The gas gage says it is $\frac{2}{3}$ full. How many gallons of gas are in her tank?



Symbolic Representation of Algorithm:



Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Ashley has 3 packs of cupcakes. There are 4 cupcakes per pack. How many cupcakes does Ashley have?

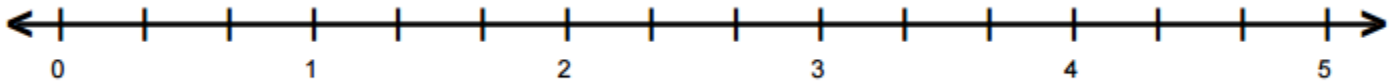
Picture:

Copies of Language:

Symbolic Representation of Algorithm:

- b) Anderson jogged around a lake 5 times this week. The distance around the lake is $\frac{1}{3}$ of a mile. How far did Anderson jog in total over the week?

Picture:

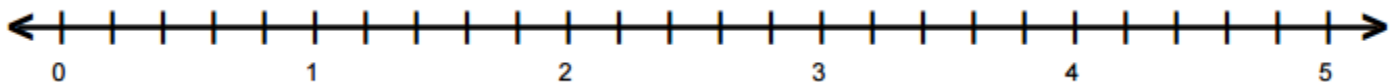


Copies of Language:

Symbolic Representation of Algorithm:

- c) Knia is having a dinner party with a total of 7 people. She bought $\frac{2}{5}$ of a pound of cold cuts per person. How many pounds of cold cuts did she buy in total?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:



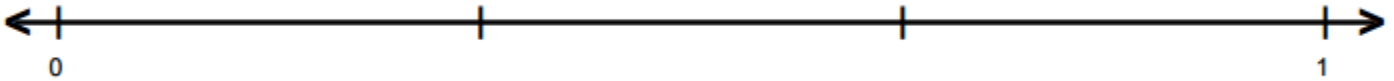
Problem 13

MEDIA EXAMPLE – Multiplying Two Fractions

Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Yesterday, Cameron walked $\frac{1}{3}$ of a mile to school. Today, Cameron's friend picked him up after he had walked $\frac{1}{2}$ of the way to school. How far to Cameron walk today?

Picture:

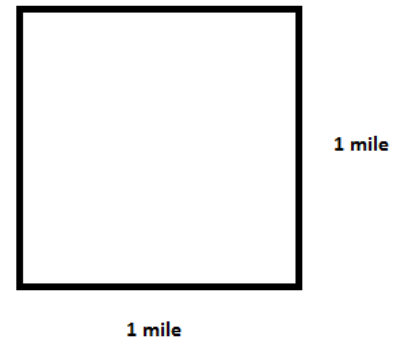


Copies of Language:

Symbolic Representation of Algorithm:

- b) Jassey bought a rectangular piece of land to grow vegetables. The land is $\frac{2}{3}$ of a mile long and $\frac{3}{4}$ of a mile wide. How many square miles of land did Jassey buy?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:

- c) Ray made a tray of brownies. He ate $\frac{1}{5}$ of the tray after they had cooled. The next day, he ate $\frac{3}{4}$ of what was left over in the tray. How much of the whole tray did Ray eat the next day?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:



Create a diagram to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) According to the Bureau of Labor Statistics, the buying power of the dollar is $1\frac{4}{5}$ times larger in 2016 when compared to 1991. Determine the comparable buying power in 2016 of \$10 in 1991.

Picture:

Copies of Language:

Symbolic Representation of Algorithm:

- b) J’Von bought a rectangular piece of land in Alaska. The land was $2\frac{3}{4}$ miles wide and $3\frac{5}{8}$ miles long. How many square miles of land did he buy?

Picture:

Copies of Language:

Symbolic Representation of Algorithm:



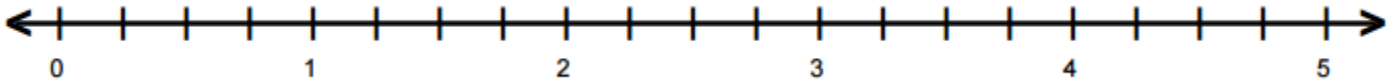
Problem 15

YOU TRY – Multiplying Mixed Numbers, Fractions, and Whole Numbers

Create a diagram to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

- a) Rick walked around a track 6 times this month. The distance around the track is $\frac{3}{4}$ of a mile. How far did Rick walk in total over the month?

Picture:

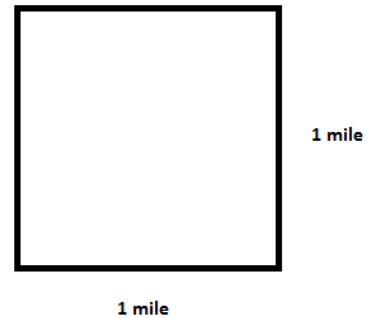


Copies of Language:

Symbolic Representation of Algorithm:

- b) Kate has a rectangular piece a land to raise horses. The land is $\frac{2}{5}$ of a mile long and $\frac{5}{8}$ of a mile wide. How many square miles of land does Kate own?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:

- c) Javier makes $2\frac{1}{3}$ times as much an hour as when he first started his job. If he made \$9 an hour when he first started, how much does Javier make now?

Picture:

Copies of Language:

Symbolic Representation of Algorithm:

SECTION 5.4: DIVIDING FRACTIONS

In this section, we will learn to visualize dividing fractions using an area model and number lines. We will investigate this idea using two models of division; dividing as partitioning the dividend into a known number of copies, and dividing as determining how many copies of a given size are in the dividend.

In general, we do not usually find a common denominator to divide fractions. However, we will begin with examples by finding common denominators to illustrate a general process for dividing fractions.

	Problem 16	MEDIA EXAMPLE – Dividing Whole Numbers by Fractions
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Use the diagrams given to represent the division problem and find the quotient.

- a) Tia made 3 cakes for her guests. If each guest receives $\frac{1}{5}$ of a cake, how many guests can Tia serve?

Picture:

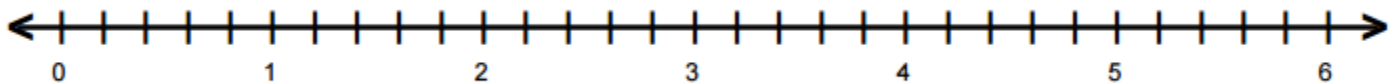


Copies of Language:

Symbolic Representation:

- b) Elaine ran 6 miles this month. If she ran $\frac{3}{5}$ of a mile every day she ran, how many days did Elaine run?

Picture:



Copies of Language:

Symbolic Representation:

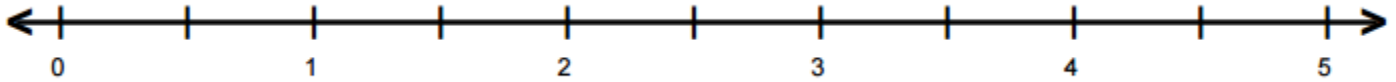


Problem 17

MEDIA EXAMPLE – Dividing Fractions Using Common Denominators

Use the diagrams given to represent the division problem and find the quotient.

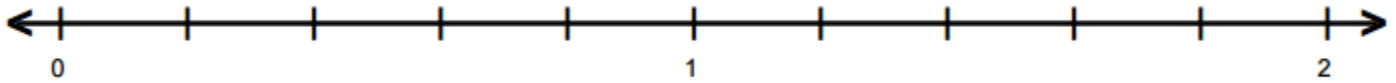
a) $\frac{9}{2} \div \frac{1}{2}$



Copies of Language:

Symbolic Representation:

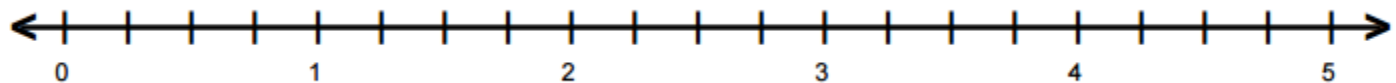
b) $\frac{8}{5} \div \frac{2}{5}$



Copies of Language:

Symbolic Representation:

c) $\frac{18}{4} \div \frac{3}{2}$



Copies of Language:

Symbolic Representation:

d) What patterns do you observe in problems *a* through *c*?

Unit 5 – Media Lesson

Results: Based on the previous examples, we can divide two fractions by dividing their numerators (left to right) and their denominators (left to right) just like we multiply the numerators and denominators of fractions to multiply them.

The last two problems illustrate this fact below.

$$\frac{8}{5} \div \frac{2}{5} = \frac{8 \div 2}{5 \div 5} = \frac{4}{1} = 4 \quad \text{or} \quad \frac{18}{4} \div \frac{3}{2} = \frac{18 \div 3}{4 \div 2} = \frac{6}{2} = 3$$

Problem: The previous two problems worked out nicely because dividing the numerators and denominators resulted in an integer in both the resulting numerator and denominator.

Consider this problem that doesn't work out as nicely.

$$\frac{7}{5} \div \frac{2}{3} = \frac{7 \div 2}{5 \div 3} = \frac{7/2}{5/3}$$

Using the division method actually made the problem worse! We want the result as a fraction of one integer over another integer, not a fraction with fractions as numerators and denominators.

Solution: We will find a common denominator for the two fractions we are dividing and perform the division like we did for the first problems. Then we will check for patterns to simplify the process.

1. Original Problem $\frac{7}{5} \div \frac{2}{3}$
2. Rewrite the two fractions with a common denominator of 15.

$$\frac{7 \cdot 3}{5 \cdot 3} \div \frac{2 \cdot 5}{3 \cdot 5} = \frac{21}{15} \div \frac{10}{15}$$

3. Divide the numerators and the denominators.

$$\frac{21 \div 10}{15 \div 15} = \frac{21 \div 10}{1} = 21 \div 10$$

4. Rewrite the division in the numerator as a fraction.

$$21 \div 10 = \frac{21}{10}$$

Pattern: Let's look at the original problem and the final answer.

$$\text{Final Solution: } \frac{7}{5} \div \frac{2}{3} = \frac{21}{10} \quad \text{Comparable Method: } \frac{7}{5} \div \frac{2}{3} = \frac{7}{5} \times \frac{3}{2} = \frac{21}{10}$$

Rule: To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

(Note: The *reciprocal* of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. In this example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.)



Problem 18

MEDIA EXAMPLE – Dividing Fractions with Unlike Denominators

Divide the fractions by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary. Write any answers greater than 1 as both an improper fraction and a mixed number.

a) $\frac{12}{13} \div \frac{3}{5}$

b) $\frac{18}{5} \div \frac{3}{10}$

c) $\frac{2}{3} \div \frac{12}{5}$

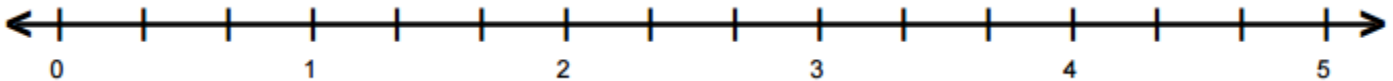


Problem 19

YOU TRY – Dividing Fractions

- a) A snail crawled 5 meters this week. If he crawled $\frac{5}{3}$ of a meter every day he crawled, how many days did he crawl?

Picture:



Copies of Language:

Symbolic Representation:

- b) Divide the fractions by dividing the numerators and the denominators. Simplify your result if necessary.

i. $\frac{18}{11} \div \frac{9}{11}$

ii. $\frac{12}{14} \div \frac{3}{7}$

iii. $\frac{15}{8} \div \frac{17}{8}$

- c) Divide the fractions by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary.

i. $\frac{19}{3} \div \frac{5}{2}$

ii. $\frac{16}{3} \div \frac{4}{6}$

iii. $\frac{4}{7} \div \frac{8}{3}$



Problem 20

MEDIA EXAMPLE – Dividing Mixed Numbers

Rewrite any mixed numbers as improper fractions. Then perform the division by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary and rewrite your result as a mixed number when possible.

a) $2\frac{1}{2} \div \frac{1}{2}$

b) $3\frac{4}{5} \div 2\frac{3}{8}$

c) $1\frac{2}{5} \div 3\frac{3}{5}$



Problem 21

YOU TRY – Dividing Mixed Numbers

Rewrite any mixed numbers as improper fractions. Then perform the division by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary and rewrite your result as a mixed number when possible.

a) $5\frac{2}{3} \div \frac{2}{3}$

b) $6\frac{3}{5} \div 3\frac{1}{6}$

c) $2\frac{2}{5} \div 4\frac{3}{5}$

SECTION 5.5: SIGNED FRACTIONS AND THE ORDER OF OPERATIONS
--

In this section, we will use our knowledge of operations on integers and the order of operations to perform operations with signed fractions and the order of operations with fractions.



Problem 22

MEDIA EXAMPLE – Operations on Signed Fractions

Perform the indicated operations on the fractions and/or mixed numbers using your knowledge of signed numbers.

a) $\frac{3}{7} - \frac{2}{3}$

b) $-\frac{2}{5} + \frac{7}{5}$

c) $-2\frac{4}{5} - 4\frac{3}{5}$

d) $-\frac{3}{5} \cdot \frac{1}{2}$

e) $-\frac{13}{8} \div -\frac{5}{8}$

f) $4\frac{4}{5} \div -8\frac{2}{3}$



Problem 23

MEDIA EXAMPLE – The Order of Operations and Signed Fractions

Perform the indicated operations on the fractions and/or mixed numbers using your knowledge of signed numbers and the order of operations.

a) $\frac{1}{2} - \left(\frac{7}{9} - \frac{8}{9} \right)$

b) $\frac{9}{5} \div \frac{3}{5} \times \frac{7}{2}$

c) $\frac{5}{3} - \left(\frac{1}{2} \right)^2 \cdot \frac{3}{4}$



Perform the indicated operations on the fractions and/or mixed numbers using your knowledge of signed numbers and the order of operations.

a) $\frac{5}{4} - \left(\frac{1}{3} + \frac{1}{4} \right)$

b) $\frac{9}{4} \times \frac{1}{3} \div \frac{3}{2}$

c) $\left(\frac{2}{3} \right)^2 - 4 \left(\frac{2}{3} \right)$

UNIT 5 – PRACTICE PROBLEMS

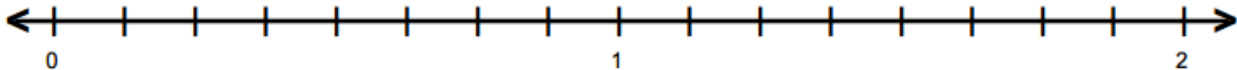
1–8: Use the diagrams given to represent the values in the addition problem and find the sum. Then perform the operation and represent the sum using the symbolic representation of the algorithm.

1. Tom had $\frac{1}{6}$ of a carrot cake last night and $\frac{2}{6}$ of a carrot cake today. How much of one whole carrot cake did Tom have?



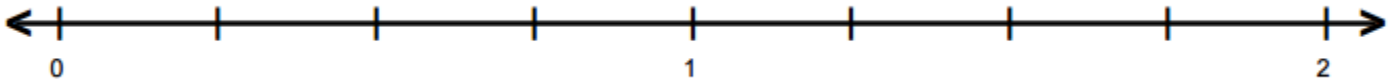
Symbolic Representation of Algorithm:

2. Ava walked $\frac{3}{8}$ of a mile to the store and then ran another $\frac{7}{8}$ of a mile to school. How far did she travel in total?



Symbolic Representation of Algorithm:

3. $\frac{3}{4} + \frac{2}{4}$



Symbolic Representation of Algorithm:

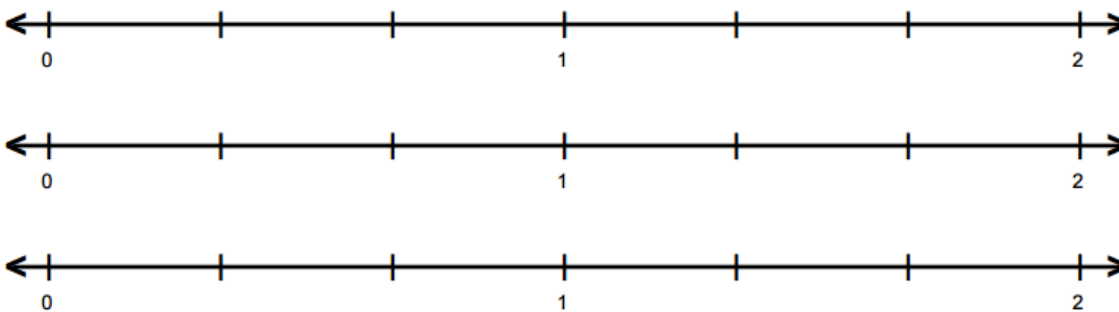
Unit 5 – Practice Problems

4. $\frac{1}{3} + \frac{3}{6}$



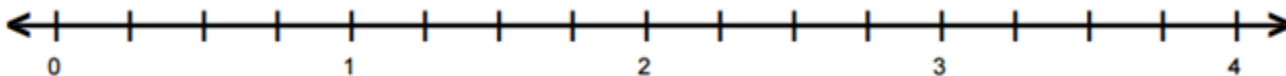
Symbolic Representation of Algorithm:

5. $\frac{1}{6} + \frac{2}{9}$



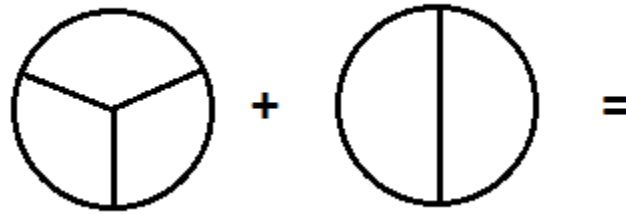
Symbolic Representation of Algorithm:

6. $\frac{6}{4} + \frac{8}{4}$



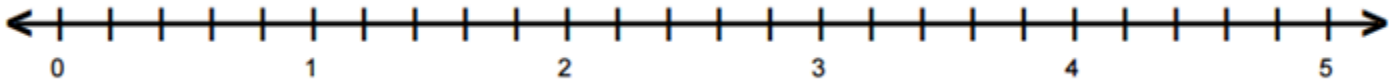
Symbolic Representation of Algorithm:

7. $2\frac{1}{3} + 4\frac{1}{2}$



Symbolic Representation of Algorithm:

8. $1\frac{4}{5} + \frac{12}{5}$



Symbolic Representation of Algorithm:

9– 15: Use the diagrams given to represent the values in the subtraction problem and find the difference. Then perform the operation and represent the difference using the symbolic representation of the algorithm.

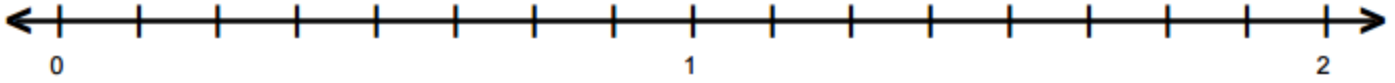
9. There was $\frac{3}{5}$ of a cake left after a party. Joey ate $\frac{2}{5}$ of the cake the next afternoon. How much of a cake is leftover now?



Symbolic Representation of Algorithm:

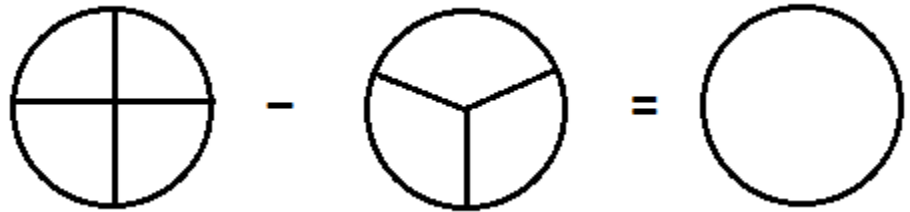
Unit 5 – Practice Problems

10. Sara lives $\frac{12}{8}$ of a mile from school. Ann lives $\frac{3}{8}$ of a mile from school. How much further does Sara live from school?



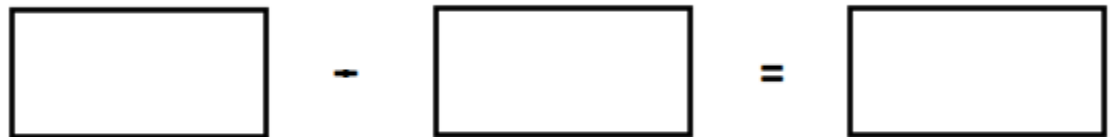
Symbolic Representation of Algorithm:

11. $\frac{3}{4} - \frac{2}{3}$



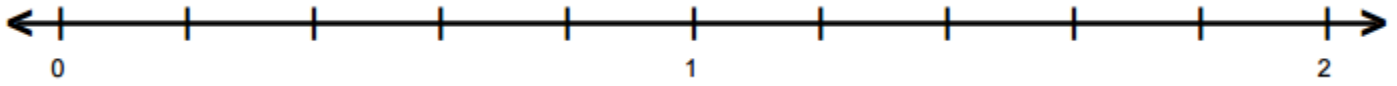
Symbolic Representation of Algorithm:

12. $\frac{5}{8} - \frac{1}{4}$



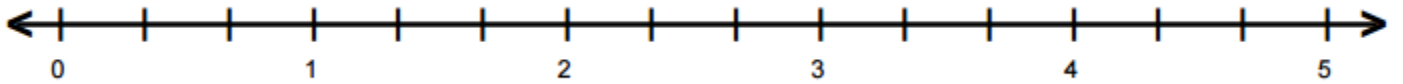
Symbolic Representation of Algorithm:

13. $\frac{9}{5} - \frac{2}{5}$



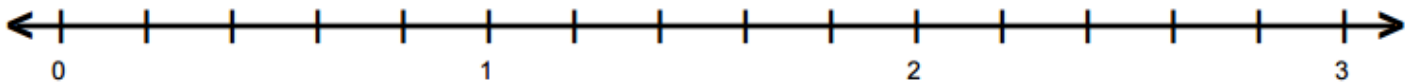
Symbolic Representation of Algorithm:

14. $4\frac{2}{3} - 2\frac{1}{2}$



Symbolic Representation of Algorithm:

15. $2\frac{3}{5} - \frac{7}{5}$



Symbolic Representation of Algorithm:

Unit 5 – Practice Problems

16. Add or subtract each of the following. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{5}{8} + \frac{4}{8}$

b. $\frac{4}{3} - \frac{1}{3}$

c. $\frac{2}{10} + \frac{3}{10}$

d. $\frac{7}{22} + \frac{5}{22}$

e. $\frac{12}{17} - \frac{3}{17}$

17. Add or subtract each of the following. State clearly what the common denominator is. Be sure to leave your answer in simplest (reduced) form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{5}{7} + \frac{4}{9}$

b. $\frac{4}{5} - \frac{1}{3}$

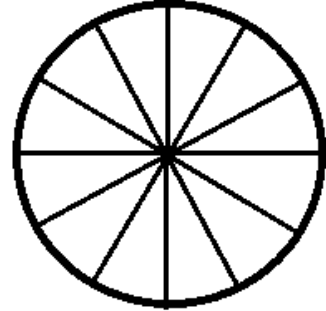
c. $\frac{2}{3} + \frac{3}{5}$

d. $\frac{7}{12} + \frac{5}{24}$

e. $\frac{4}{5} - \frac{3}{7}$

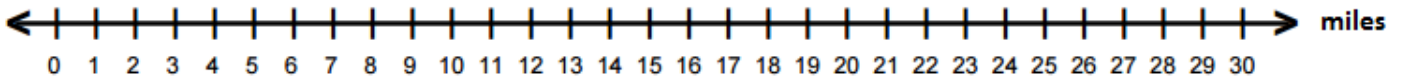
For 18 – 29: Use the diagrams given to represent the multiplication problem and find the product. Then perform the operation and represent the product using the symbolic representation of the algorithm.

18. April’s guests ate $\frac{2}{3}$ of an apple pie. The pie had 12 slices. How many slices did they eat?



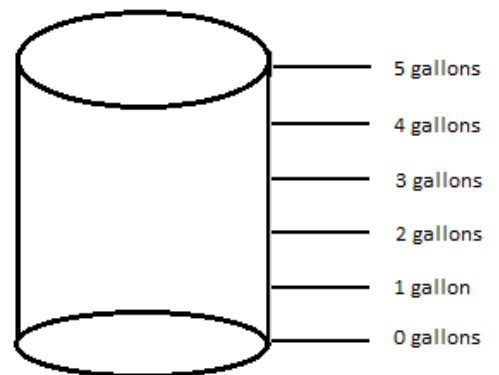
Symbolic Representation of Algorithm:

19. Riley was 30 miles from home. He travelled $\frac{1}{4}$ of this distance before stopping for gas. How many miles did he travel before stopping for gas?



Symbolic Representation of Algorithm:

20. Javier has a 5 gallon bucket. If he fills it $\frac{1}{3}$ of the way to the top. How much water is in the bucket?

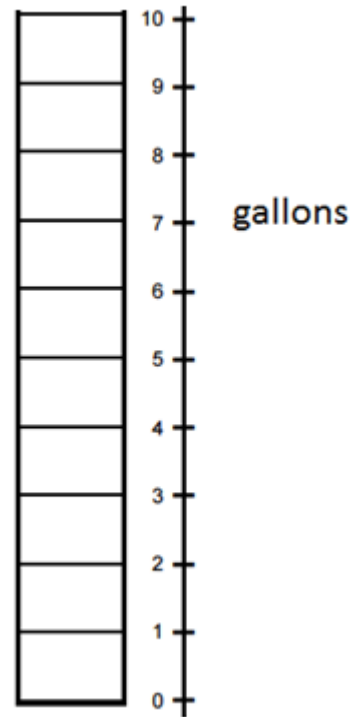


21. How much water is in the bucket if he fills it $\frac{2}{3}$ of the way to the top?

Unit 5 – Practice Problems

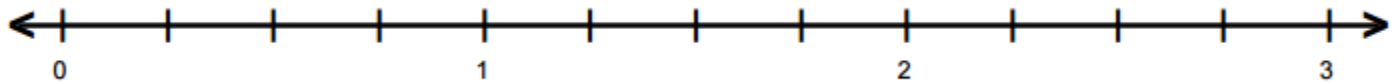
22. Johnny's gas tank holds 10 gallons. The gas gage says it is $\frac{2}{5}$ full. How many gallons of gas are in the tank?

Symbolic Representation of Algorithm:



23. Phil skated around a track 7 times. The distance around the track is $\frac{1}{4}$ of a mile. How far did Phil skate?

Picture:

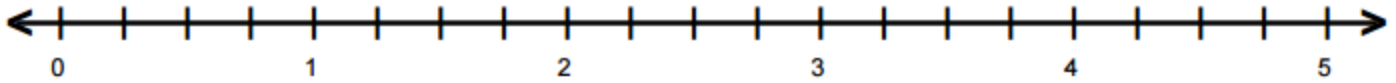


Copies of Language:

Symbolic Representation of Algorithm:

24. Chris is having a party with a total of 8 people. He bought enough cake for each person to have $\frac{2}{4}$ of a cake. How many cakes did he buy?

Picture:

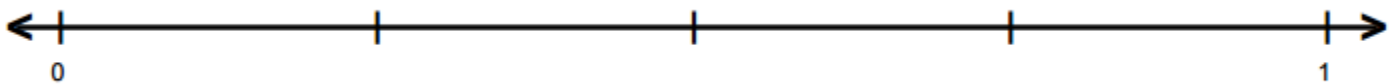


Copies of Language:

Symbolic Representation of Algorithm:

25. Yesterday, Sharon walked $\frac{1}{4}$ of a mile to school. Today, Sharon’s friend picked her up after she had walked $\frac{1}{2}$ of the way to school. How far to Sharon walk today?

Picture:

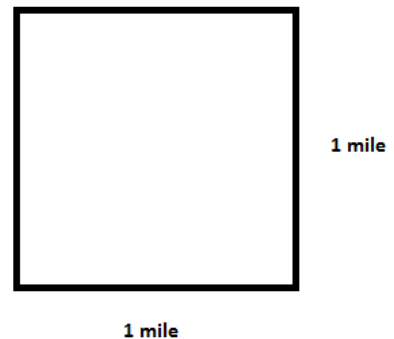


Copies of Language:

Symbolic Representation of Algorithm:

26. Maureen bought a rectangular piece of land to build a vacation home. The land is $\frac{2}{5}$ of a mile long and $\frac{1}{3}$ of a mile wide. How many square miles of land did Maureen buy?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:

Unit 5 – Practice Problems

27. Todd ordered a pizza. He ate $\frac{2}{5}$ of the pizza that night. The next day, he ate $\frac{3}{4}$ of what was left over.

How much of the whole pizza did Todd eat the next day?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:

28. According to the Bureau of Labor Statistics, the buying power of the dollar is $1\frac{4}{5}$ times larger in 2016 when compared to 1991. Determine the comparable buying power in 2016 of \$20 in 1991.

Picture:

Copies of Language:

Symbolic Representation of Algorithm:

29. Kevin made a rectangular drink coaster. The coaster was $2\frac{1}{4}$ inches wide and $3\frac{1}{2}$ inches long. How many square inches was the coaster?

Picture:



Copies of Language:

Symbolic Representation of Algorithm:

30. Multiply and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a) $\frac{1}{6} \cdot \frac{3}{5}$

b) $\frac{8}{9} \cdot \frac{9}{12}$

c) $\frac{3}{4} \cdot 0$

d) $1\frac{1}{2} \cdot \frac{1}{2}$

e) $3\frac{1}{3} \cdot 2\frac{2}{5}$

For 31 – 35: Use the diagrams given to represent the division problem and find the quotient.

31. Tia made 3 cakes for her guests. If each guest receives $\frac{1}{4}$ of a cake, how many guests can Tia serve?

Picture:



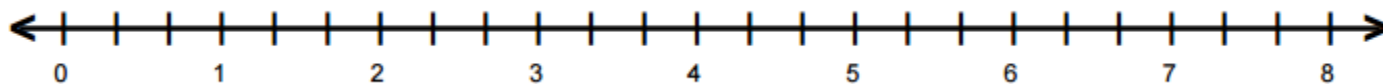
Copies of Language:

Symbolic Representation:

Unit 5 – Practice Problems

32. Greg ran 8 miles this month. If he ran $\frac{4}{3}$ of a mile every day he ran, how many days did Greg run?

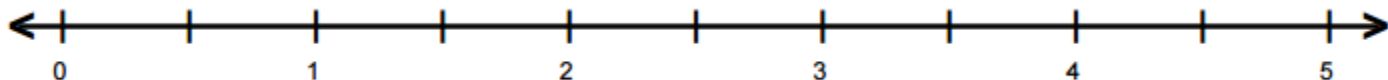
Picture:



Copies of Language:

Symbolic Representation:

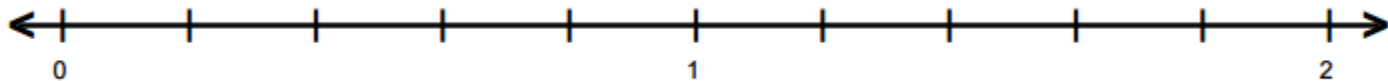
33. $\frac{7}{2} \div \frac{1}{2}$



Copies of Language:

Symbolic Representation:

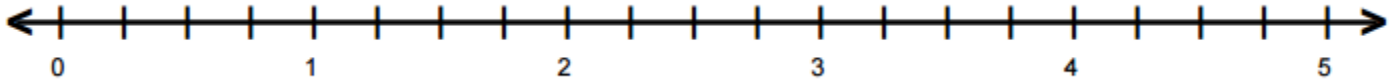
34. $\frac{9}{5} \div \frac{3}{5}$



Copies of Language:

Symbolic Representation:

35. $\frac{15}{4} \div \frac{3}{2}$



Copies of Language:

Symbolic Representation:

36. Divide the fractions by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary. Write any answers greater than 1 as both an improper fraction and a mixed number.

a) $\frac{11}{12} \div \frac{22}{7}$

b) $\frac{18}{5} \div \frac{9}{15}$

c) $\frac{12}{5} \div \frac{5}{6}$

37. Rewrite any mixed numbers as improper fractions. Then perform the division by multiplying the first fraction by the reciprocal of the second fraction. Simplify your result if necessary and rewrite your result as a mixed number when possible.

a) $4\frac{1}{3} \div \frac{5}{7}$

b) $5\frac{1}{3} \div 4\frac{3}{7}$

c) $1\frac{1}{3} \div 4\frac{2}{3}$

Unit 5 – Practice Problems

38. Perform the indicated operations. Write your answer in simplest form. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{1}{2} - \frac{1}{3} + \frac{1}{4}$

b. $2 - \frac{8}{5}$

c. $\frac{2}{3} + \frac{1}{3} - \frac{1}{4}$

d. $\frac{1}{3} + \frac{5}{3} - \left(-\frac{7}{3}\right)$

39. Perform the indicated operations and simplify. If applicable, write your answer as *both* an improper fraction *and* a mixed number.

a. $\frac{3}{4} \div \frac{4}{5} \cdot \frac{5}{6}$

b. $\frac{1}{2} - \frac{1}{3} \cdot \frac{1}{4}$

c. $\left(2 - \frac{8}{5}\right)^2$

d. $1 - \left(\frac{1}{2}\right)^2$

For 40 – 51: Solve the following problems. Show all of your work and write your final answer as a complete sentence. When necessary, write your final answers as both mixed numbers and improper fractions.

40. If Josh ate $\frac{1}{4}$ of a pizza, what fraction of the pizza is left?

41. If I drove $10\frac{2}{3}$ miles one day and $12\frac{1}{4}$ miles the second day and $8\frac{1}{5}$ miles the third day, how far did I drive?

42. Melody bought a 2-liter bottle of soda at the store. If she drank $\frac{1}{8}$ of the bottle and her brother drank $\frac{2}{7}$ of the bottle, how much of the bottle is left?

43. James brought a small bag of carrots for lunch. There are 6 carrots in the bag. Is it possible for him to eat $\frac{2}{6}$ of the bag for a morning snack and $\frac{5}{6}$ of the bag at lunch? Why or why not?

Unit 5 – Practice Problems

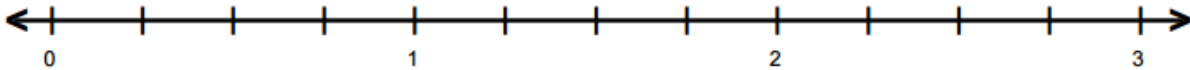
44. Suppose that David is able to tile $\frac{1}{4}$ of his floor in 3 hours. How long would it take him to tile the rest of the floor?
45. Maureen went on a 3 day, 50 mile biking trip. The first day she biked $21\frac{2}{3}$ miles. The second day she biked $17\frac{3}{8}$ miles. How many miles did she bike on the 3rd day?
46. Scott bought a 5 lb. bag of cookies at the bakery. He ate $\frac{2}{5}$ of a bag and his sister ate $\frac{2}{9}$ of a bag. What fraction of the bag did they eat? What fraction of the bag remains?
47. Suppose your school costs for this term were \$2500 and financial aid covered $\frac{3}{4}$ of that amount. How much did financial aid cover?

48. If, on average, about $\frac{4}{7}$ of the human body is water weight how much water weight is present in a person weighing 182 pounds?
49. If, while training for a marathon, you ran 920 miles in $3\frac{1}{2}$ months, how many miles did you run each month? (Assume you ran the same amount each month)
50. On your first math test, you earned 75 points. On your second math test, you earned $\frac{6}{5}$ as many points as your first test. How many points did you earn on your second math test?
51. You are serving cake at a party at your home. There are 12 people in total and $2\frac{3}{4}$ cakes. (You ate some before they got there!). If the cakes are shared equally among the 12 guests, what fraction of a cake will each guest receive?

UNIT 5 – END OF UNIT ASSESSMENT

Answer the questions below. For any problem with a diagram given, represent the problem using the diagram as well as symbolically.

1. Conner waterskied $1\frac{3}{4}$ of a mile without falling. Then he skied another $\frac{3}{4}$ of a mile. How far did he waterski in total?

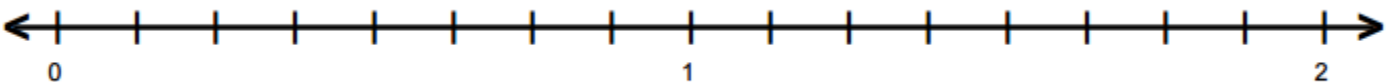


Symbolic Representation of Algorithm:

2. Add. Show all intermediary steps. Write your final answer as an improper fraction and a mixed number.

$$\frac{7}{6} + \frac{5}{9}$$

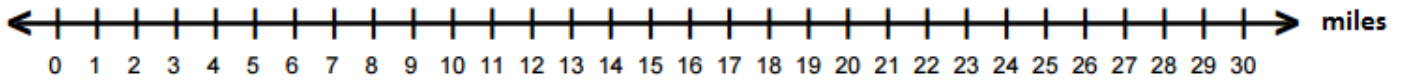
3. Sara lives $\frac{13}{8}$ of a mile from work. She stops at a coffee house on the way that is $\frac{7}{8}$ of a mile from her home. How far is the coffee house from work?



Symbolic Representation of Algorithm:

Unit 5 – Assessment

4. Noah was 30 miles from home. He travelled $\frac{3}{5}$ of this distance before stopping to pick up a friend. How many miles did he travel before picking up his friend?



Symbolic Representation of Algorithm:

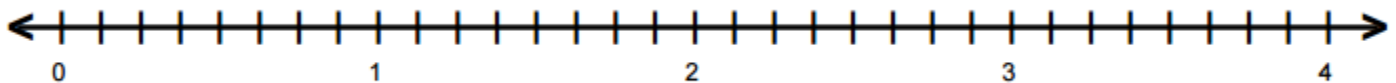
5. Central Park in New York City has a rectangular shape and is approximately $2\frac{1}{2}$ miles long and $\frac{1}{2}$ a mile wide. Using these approximations, about how many square miles of land is Central Park?

Picture: Use the diagram below to represent the square miles of the park.



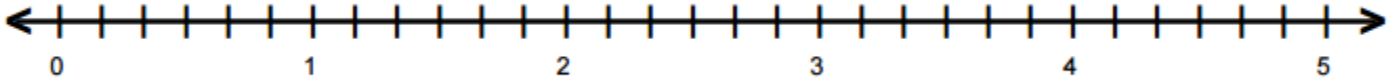
Symbolic Representation of Algorithm:

6. Lois is making headbands. She needs each headband to be $\frac{7}{8}$ of a foot in length and she is making 4 headbands. How many feet of elastic does she need for all 4 headbands?



Symbolic Representation of Algorithm:

7. Callie is cutting a 5 foot tree for firewood. She wants each piece to be $\frac{5}{6}$ of a foot long. How many pieces can she cut of this length from the 5 foot tree?



Symbolic Representation of Algorithm:

8. Divide. Show all intermediary steps. Write your final answer as an improper fraction and a mixed number.

$$5\frac{6}{7} \div 2\frac{3}{4}$$

9. Use the order of operations to evaluate the expression.

$$\frac{4}{9} - \frac{2}{3} + \left(-\frac{7}{9}\right)$$

10. Use the order of operations to evaluate the expression.

$$\left(\frac{2}{3}\right)^2 \div \frac{2}{9} \times \left(-\frac{3}{8}\right)$$

UNIT 6 – OPERATIONS WITH DECIMALS

INTRODUCTION

In this Unit, we will use our understanding of operations, decimals, and place value to perform operations with decimals.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Learning Objective	Media Examples	You Try
Add decimals in the tenths and hundreds place using decimal grids	1	2
Add decimals using a place value chart	3	5
Use an algorithm to add decimals	4	5
Subtract decimals in the tenths and hundreds place using decimal grids	6	7
Subtract decimals using a place value chart	8	10
Use an algorithm to subtract decimals	9	10
Add and subtract signed decimals	11	12
Multiply a whole number times a decimal using decimal grids	13	15
Multiply two decimals using a decimal grid	14	15
Multiply decimal using place value	16	17
Divide decimals using a decimal grid	18	19
Divide decimals using place value	20	21
Multiply decimals by powers of ten	22	24
Divide decimals by powers of 10	23	24
Perform decimal operations on a calculator	25	26
Solve application problems with decimals	27	28

UNIT 6 – MEDIA LESSON

SECTION 6.1: ADDING DECIMALS USING THE AREA MODEL

In this section, we will learn to visualize the addition of decimals using the area model with the 10 by 10 grid.

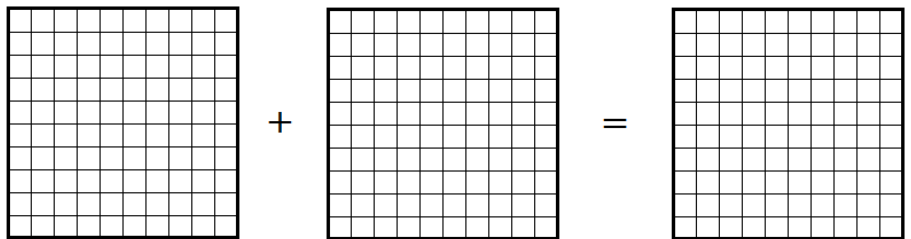


Problem 1

MEDIA EXAMPLE – Adding Decimals in the Tenths and Hundredths Place

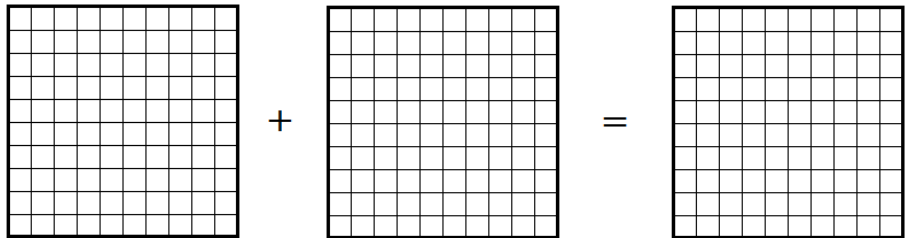
Use the decimal grids to shade the addends of the addition problem. Then combine your addends in a new grid to find the sum. (Note: We call the numbers we are adding in an addition problem the *addends*. We call the simplified result the *sum*.)

a) $0.3 + 0.5$



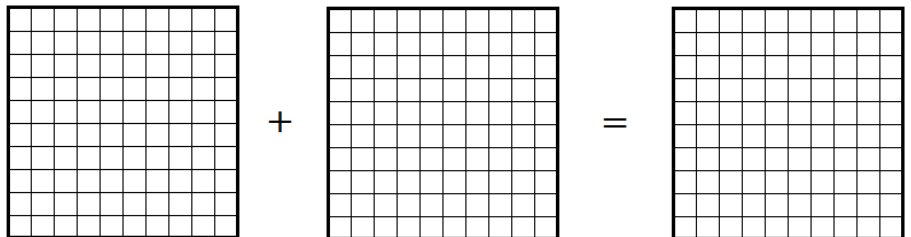
Sum: _____

b) $0.04 + 0.07$



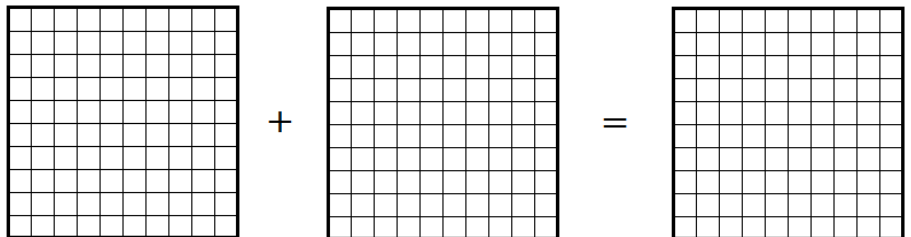
Sum: _____

c) $0.3 + 0.06$



Sum: _____

d) $0.35 + 0.18$



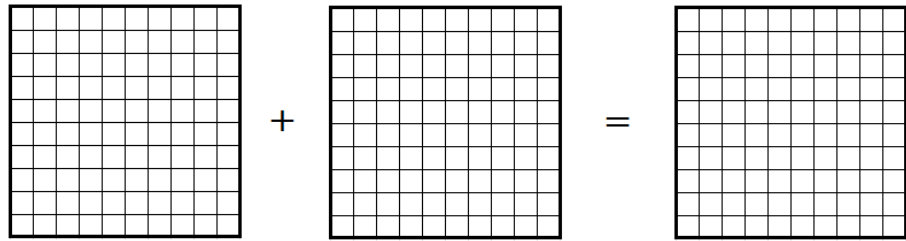
Sum: _____

 **Problem 2** **YOU TRY - Adding Decimals Using the Area Model**

Use the decimal grids to shade the decimal portions of the addends of the addition problem. Then combine your addends in a new grid to find the sum.

a) $0.47 + 0.29$

Sum: _____



SECTION 6.2: ADDING DECIMALS USING PLACE VALUE

In the last section, we were actually using place value to add decimals by grouping according to the place value of the decimals. In this section, we will streamline this process, by adding using a place value chart and then learning how to add without the place value chart.

 **Problem 3** **MEDIA EXAMPLE – Adding Decimals Using a Place Value Chart**

Place the numbers in the place value chart and then use the chart as an aid to add the numbers.

$32.456 + 7.98$

Ones			.	Decimals		
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
			.			
			.			
			.			
			.			
			.			

Sum: _____

 **Problem 4** **MEDIA EXAMPLE – Adding Decimals Using Place Value**

Add the decimals without a place value chart by aligning the decimal points and adding.

$5.09 + 62.784$

	Problem 5	You Try – Adding Decimals Using Place Value
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In the first problem, add the decimals using the place value chart. In the second problem, align the decimal points to add.

a) $15.397 + 6.91$

b) $437.9 + 52.438$

Ones			.	Decimals		
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

+

SECTION 6.3: SUBTRACTING DECIMALS USING THE AREA MODEL

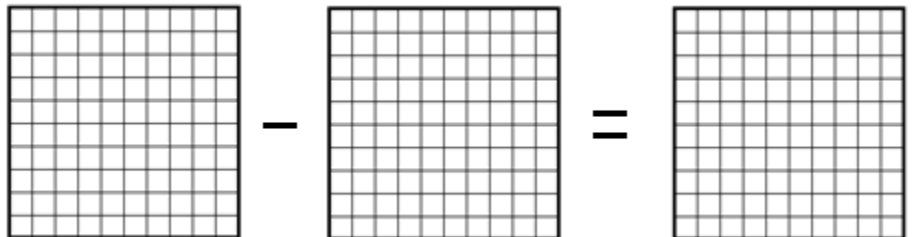
In this section, we will learn to visualize the subtraction of decimals using the 10 by 10 grid.

	Problem 6	MEDIA EXAMPLE – Subtracting Decimals in the Tenths and Hundredths Place
---	------------------	--

Use the decimal grids to shade the given decimals in the subtraction problem. Then find the difference by taking away the second quantity from the first quantity.

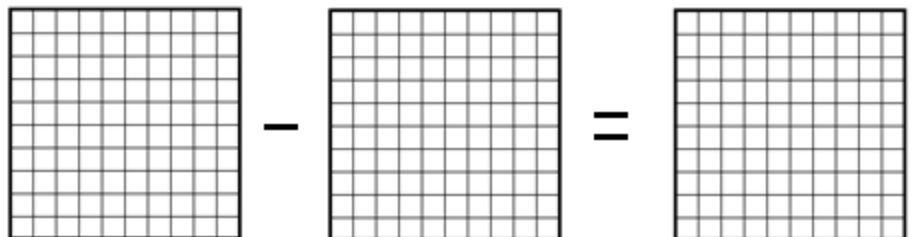
a) $0.7 - 0.4$

Difference: _____

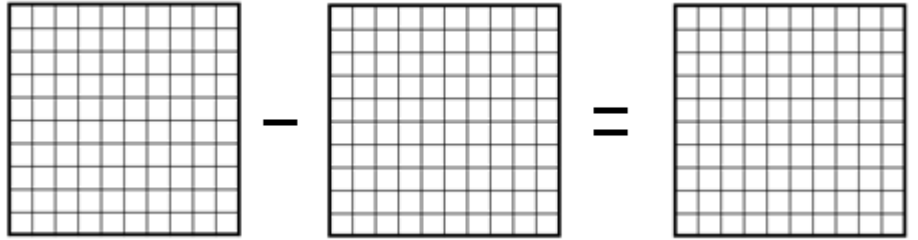


b) $0.09 - 0.06$

Difference: _____

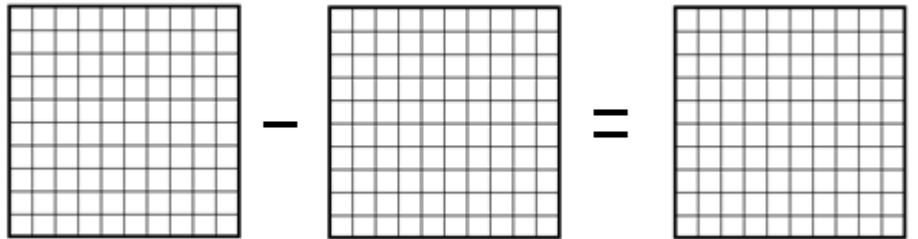


c) $0.3 - 0.06$



Difference: _____

d) $0.47 - 0.28$



Difference: _____

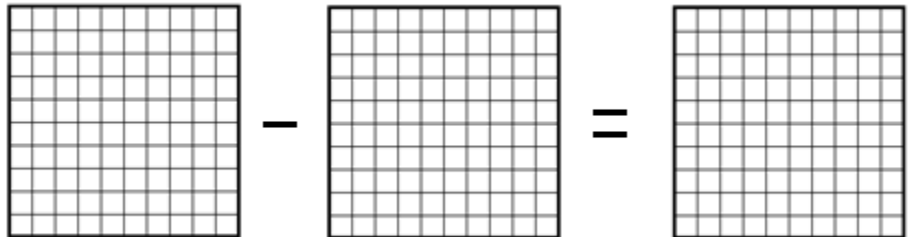


Problem 7

YOU TRY - Subtracting Decimals Using the Area Model

Use the decimal grids to shade the given decimals in the subtraction problem. Then find the difference by taking away the second quantity from the first quantity.


$0.56 - 0.24$



Difference: _____

SECTION 6.4: SUBTRACTING DECIMALS USING PLACE VALUE

In the last section, we were actually using place value to subtract decimals by grouping according to the place value of the decimals. In this section, we will streamline this process, by subtracting using a place value chart and then learning how to subtract without the place value chart.


 **Problem 8** **MEDIA EXAMPLE – Subtracting Decimals Using a Place Value Chart**

Place the numbers in the place value chart and then use the chart as an aid to subtract the numbers.

$21.456 - 8.89$


Ones			.	Decimals		
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

Difference: _____

 **Problem 9** **MEDIA EXAMPLE – Subtracting Decimals Using Place Value**

Subtract the decimals without a place value chart by aligning the decimals points and subtracting.

$52.634 - 7.09$

 **Problem 10** **You Try – Subtracting Decimals Using Place Value**

In the first problem, subtract the decimals using the place value chart. In the second problem, align the decimal points to subtract.

a) $18.547 - 6.82$

b) $371.9 - 342.5$

Ones			.	Decimals		
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

SECTION 6.5: ADDING AND SUBTRACTING SIGNED DECIMALS

In this section, we will add and subtract signed decimals. The same rules that apply to these processes on integers can be extended to decimals. These procedures are summarized below.

- A. When adding two or more numbers, all with the same sign,
3. Add the absolute values of the numbers
 4. Keep the common sign of the numbers
- B. When adding two numbers with different signs.
4. Find the absolute value of the numbers
 5. Subtract the smaller absolute value from the larger absolute value
 6. Keep the original sign of the number with the larger absolute value.
- C. When subtracting two decimals, we can use following fact.
- Fact:** Subtracting a decimal from a number is the same as adding the decimal's opposite to the number.
3. If given a subtraction problem, rewrite it as an addition problem.
 4. Use the rules for addition to add the signed numbers as summarized above.



Problem 11

MEDIA EXAMPLE – Adding and Subtracting Signed Decimals

Use the rules for signed numbers to add or subtract the decimals.

a) $-0.14 + (-0.27)$

b) $5.63 + (-7.24)$

c) $-4.2 - (-3.8)$



Problem 12

You Try – Adding and Subtracting Signed Decimals

Use the rules for signed numbers to add or subtract the decimals.

a) $0.7 + (-0.14)$

b) $-4.63 + 2.61$

c) $5.2 - (-2.7)$

SECTION 6.6: MULTIPLYING DECIMALS USING THE AREA MODEL

In this section, we will learn to visualize the multiplication of decimals using the area model with the 10 by 10 grid.



Problem 13

MEDIA EXAMPLE – Multiplying a Whole Number Times a Decimal

Rewrite the multiplication statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

a) $3(4)$

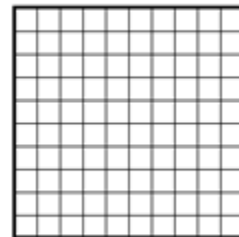
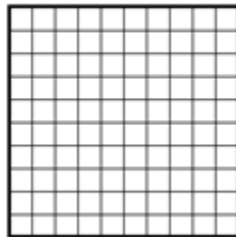
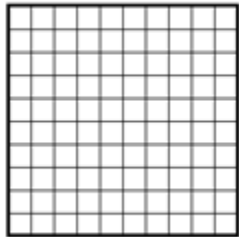
Copies Language:

Picture:

Product: _____

b) $3(0.4)$

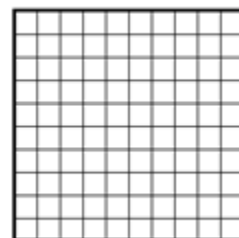
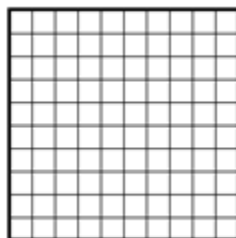
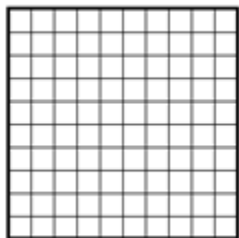
Copies Language:



Product: _____

c) $3(0.04)$

Copies Language:



Product: _____

d) Describe the pattern that you see in a through c.



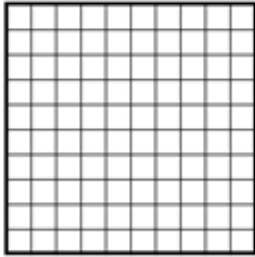
Problem 14

MEDIA EXAMPLE – Multiplying Two Decimals

Rewrite the multiplication statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

a) $0.3 \cdot 0.4$

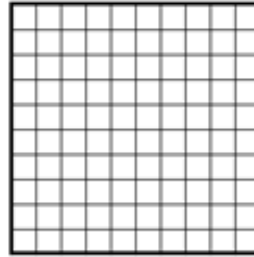
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Product: _____

b) $0.6 \cdot 0.2$

Copies Language:



Product: _____

c) Describe the pattern that you see.



Problem 15

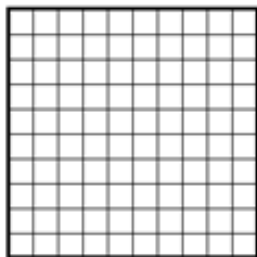
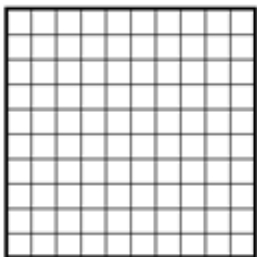
You Try – Multiplying Two Decimals

Rewrite the multiplication statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

a) $2 \cdot 0.08$

Copies Language:

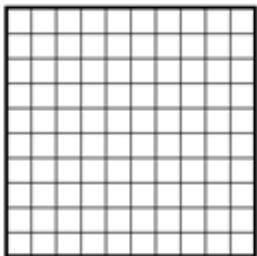
Product: _____



b) $0.2 \cdot 0.8$

Copies Language:

Product: _____



SECTION 6.7: MULTIPLYING DECIMALS USING PLACE VALUE

In this section, we will multiply decimals by using the patterns we saw in Section 4.1. In particular, we will use the strategy below.

To multiply two decimals:

1. Multiply the two numbers as if they were whole numbers (disregard the decimals for now).
2. Determine the total number of digits that were to the right of the decimal points in your two original factors and add them.
3. Take your product from step one. Starting from the right, count as many place values as you found in step 2 and place the decimal point in this spot.

**Problem 16****MEDIA EXAMPLE – Multiplying Decimals Using Place Value**

Multiply the decimals.

a) $1.4 \cdot 3 =$

b) $1.4 \cdot 0.3 =$

c) $0.14 \cdot 3 =$

d) $0.3 \cdot 0.8 =$

e) $0.3 \cdot 0.08 =$

f) $0.03 \cdot 0.8 =$

g) $4 \cdot 2.1 =$

h) $0.4 \cdot 2.1 =$

i) $0.4 \cdot 0.21 =$

**Problem 17****You Try – Multiplying Decimals Using Place Value**

Multiply the decimals.

a) $1.2 \cdot 6 =$

b) $1.2 \cdot 0.6 =$

c) $0.12 \cdot 0.6 =$

SECTION 6.8: DIVIDING DECIMALS USING THE AREA MODEL



Problem 18

MEDIA EXAMPLE – Dividing Decimals using the Area Model

Rewrite the division statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

a) $12 \div 3$

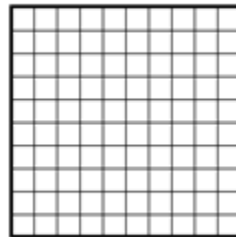
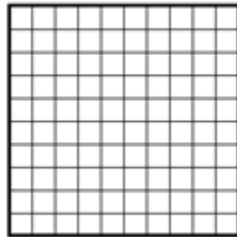
Copies Language:

Picture:

Quotient _____

b) $1.2 \div 0.3$

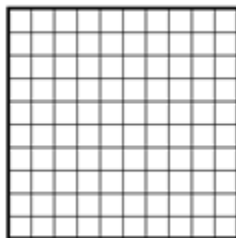
Copies Language:



Quotient: _____

c) $0.12 \div 0.03$

Copies Language:



Quotient: _____

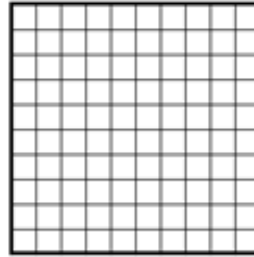
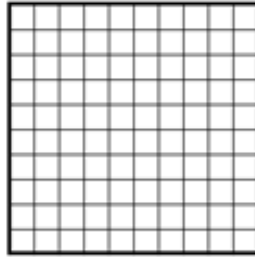
	Problem 19	You Try – Dividing Decimals Using the Area Model
---	-------------------	---

Rewrite the division statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

$1.6 \div 0.8$

Copies Language:

Quotient: _____



SECTION 6.9: DIVIDING DECIMALS USING PLACE VALUE

In this section, we will look at quotients that are not whole numbers. We will use the patterns developed to create a general method for dividing numbers involving decimals.

	Problem 20	MEDIA EXAMPLE – Dividing Decimals Using Place Value
--	-------------------	--

Divide the decimals.

a) $24 \div 8 =$

b) $2.4 \div 0.8 =$

c) $0.24 \div 0.8 =$

d) $0.42 \div 0.07 =$

e) $4.2 \div 0.7 =$

c) $0.42 \div 0.7 =$

	Problem 21	You Try – Dividing Decimals Using Place Value
---	-------------------	--

Divide the decimals.

a) $56 \div 8 =$

b) $5.6 \div 0.8 =$

c) $0.56 \div 8 =$

SECTION 6.10: MULTIPLYING AND DIVIDING DECIMALS BY POWERS OF 10
--

In this section, we will investigate patterns when multiplying or dividing by powers of ten. Some examples of powers of ten are $10^1 = 1$, $10^2 = 100$, and $10^3 = 1000$.



Problem 22

MEDIA EXAMPLE – Multiplying by Powers of Ten

Multiply the numbers by the given powers of 10 by moving the decimal point the appropriate number of places.

a) $4.23 \cdot 10 =$ _____ b) $0.037 \cdot 1000 =$ _____ c) $29.5 \cdot 100 =$ _____

d) $3.1415 \cdot 1000 =$ _____ e) $5.24 \cdot 10 =$ _____ f) $0.076 \cdot 100 =$ _____



Problem 23

MEDIA EXAMPLE – Dividing by Powers of Ten

Divide the numbers by the given powers of 10 on your calculator then look for patterns to make a general strategy.

a) $4.23 \div 10 =$ _____ b) $3.7 \div 1000 =$ _____ c) $29.5 \div 100 =$ _____

d) $3.1415 \div 1000 =$ _____ e) $5.24 \div 10 =$ _____ f) $0.67 \div 100 =$ _____

g) Look for patterns in the examples above and complete the statement below.

To divide a decimal number by a power of 10, you move the decimal place



Problem 24

YOU TRY - Multiplying and Dividing by Powers of Ten

Multiply the numbers by the given powers of 10 by moving the decimal point the appropriate number of places.

a) $1.126 \cdot 100 =$ _____ b) $0.049 \cdot 1000 =$ _____ c) $5.7 \cdot 10 =$ _____

d) $1.126 \div 100 =$ _____ e) $4.9 \div 1000 =$ _____ f) $5.7 \div 10 =$ _____

SECTION 6.11: DECIMAL OPERATIONS ON THE CALCULATOR



When performing the mathematical operations of addition, subtraction, multiplication, and division using decimals, our calculator is a great support tool. Once the given numbers are combined, *rounding* often comes into play when presenting the final result.



Problem 25

MEDIA EXAMPLE – Decimal Operations on the Calculator

Use your calculator to compute each of the following. Round as indicated.

- a) Multiply $4.32 \cdot 3.17$ then round the result to the nearest tenth.

- b) Divide $523.14 \div 23.56$ then round the result to the nearest thousandth.

- c) Evaluate $(0.1)^2$. Write your result first in decimal form. Then, convert to a simplified fraction.

- d) Combine the numbers below. Round your final result to the nearest whole number.

$$3.721 + 4.35 \cdot 21.72 - 0.03$$



Problem 26

YOU TRY - Decimal Operations on the Calculator

Use your calculator to combine the numbers below. Round your final result to the nearest hundredth. When computing, try to enter the entire expression all at once.

$$(6.41)^2 - 5.883 \div 2.17$$

SECTION 6.12: APPLICATIONS WITH DECIMALS



Problem 27

MEDIA EXAMPLE – Applications with Decimals

In preparation for mailing a package, you place the item on your digital scale and obtain the following readings: 6.51 ounces, 6.52 ounces, and 6.60 ounces. What is the average of these weights? Round to the nearest hundredth of an ounce.

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:



Problem 28

YOU TRY - Applications with Decimals

Rally went to Target with \$40 in his wallet. He bought items that totaled \$1.45, \$2.15, \$7.34, and \$14.22. If the tax comes to \$2.26, how much of his \$40 would he have left over? Round to the nearest cent (hundredths place).

GIVEN:

GOAL:

MATH WORK:

CHECK:

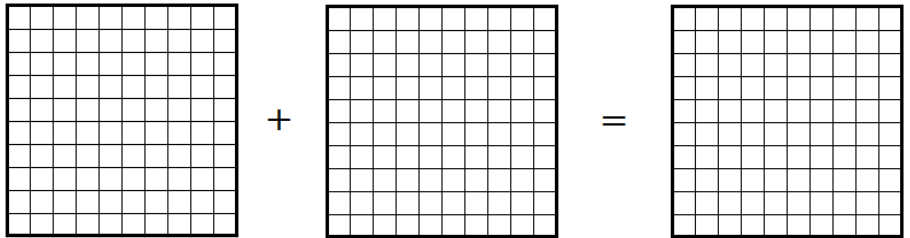
FINAL ANSWER AS A COMPLETE SENTENCE:

UNIT 6 – PRACTICE PROBLEMS

1. Use the decimal grids to shade the addends of the addition problem. Then combine your addends in a new grid to find the sum. (Note: We call the numbers we are adding in an addition problem the *addends*. We call the simplified result the *sum*.)

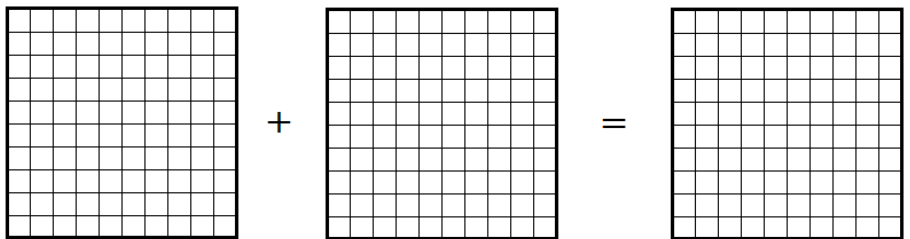
a) $0.4 + 0.3$

Sum: _____



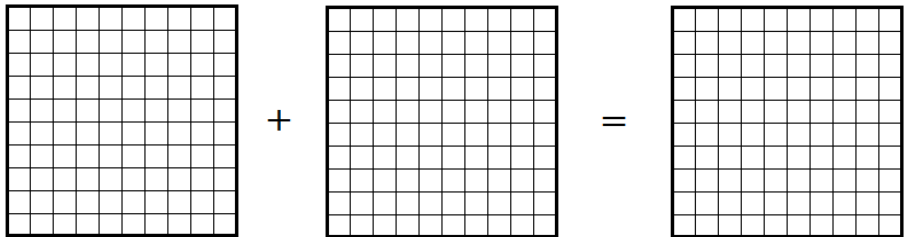
b) $0.04 + 0.03$

Sum: _____



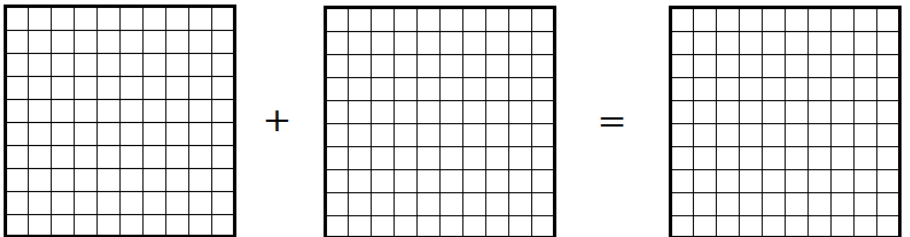
c) $0.5 + 0.05$

Sum: _____



d) $0.25 + 0.38$

Sum: _____



Unit 6 – Practice Problems

2. Place the numbers in the place value chart and then use the chart as an aid to add the numbers.

$$512.305 + 31.68$$

Ones			.	Decimals		
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	
			.			
			.			
			.			
			.			

Sum: _____

3. Place the numbers in the place value chart and then use the chart as an aid to add the numbers.

$$35.795 + 82.457$$

Ones			.	Decimals		
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	
			.			
			.			
			.			
			.			

Sum: _____

4. Add the decimals without a place value chart by aligning the decimal points and adding.

b) $43.136 + 21.823$

b) $536.209 + 497.055$

5. Place the numbers in the place value chart and then use the chart as an aid to subtract the numbers.

$$37.528 - 23.106$$

Ones			.	Decimals		
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	

Difference: _____

6. Place the numbers in the place value chart and then use the chart as an aid to subtract the numbers.

$$254.023 - 88.58$$

Ones			.	Decimals		
Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	

Difference: _____

7. Subtract the decimals without a place value chart by aligning the decimal points and subtracting.

b) $279.381 - 102.16$

b) $520.408 - 39.866$

Unit 6 – Practice Problems

8. Use the rules for signed numbers to add or subtract the decimals.

a) $0.8 + (-1.23)$

b) $(-5.61) + 7.61$

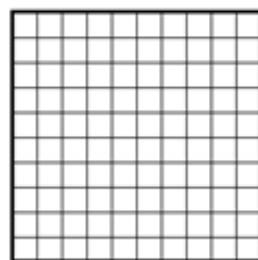
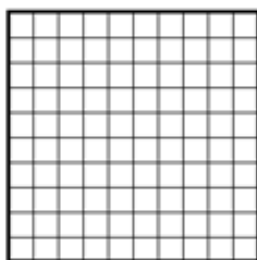
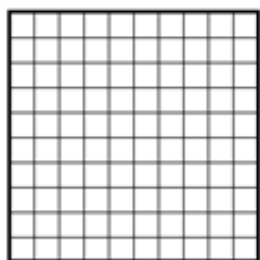
c) $8.91 - (-3.07)$

9. Rewrite the multiplication statements using *copies of language* and word names. Then represent the decimal problems using the decimal grids.

a) $3 \cdot 0.23$

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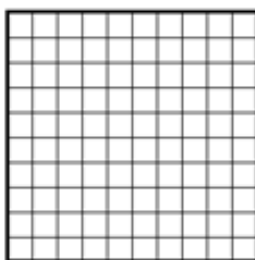
Product: _____



b) $0.4 \cdot 0.6$

Copies Language:

Product: _____



10. Multiply the decimals.

a) $2.1 \cdot 4 =$

b) $2.1 \cdot 0.4 =$

c) $0.21 \cdot 0.4 =$

d) $0.5 \cdot 0.9 =$

e) $5 \cdot 0.09 =$

f) $0.05 \cdot 0.09 =$

h) $2 \cdot 5.4 =$

h) $0.2 \cdot 54 =$

i) $0.02 \cdot 0.54 =$

j) $1.4 \cdot (-3) =$

k) $-1.4 \cdot (-3) =$

l) $-0.14 \cdot (0.03) =$

m) $-0.4(0.8) =$

n) $0.4(-0.08) =$

o) $(-0.04)(-0.8) =$

p) $(-5)(2.6)$

q) $0.5(-26)$

r) $(-0.5)(-0.26)$

Unit 6 – Practice Problems

11. Rewrite the division statements using *copies of* language and word names. Then represent the decimal problems using the decimal grids.

a) $15 \div 3$

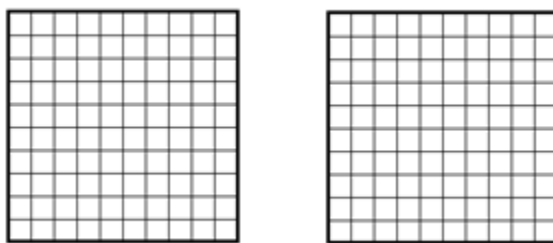
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Picture:

Quotient _____

b) $1.5 \div 0.3$

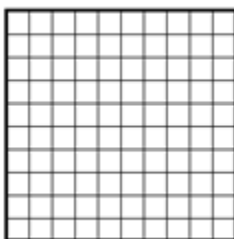
Copies Language:



Quotient: _____

c) $0.15 \div 0.03$

Copies Language:



Quotient: _____

12. Divide the decimals.

a) $32 \div 8 =$

b) $3.2 \div 0.8 =$

c) $0.32 \div 0.8 =$

d) $0.42 \div 0.21 =$

e) $4.2 \div 2.1 =$

f) $0.42 \div 21 =$

g) $24 \div 2 =$

h) $2.4 \div 0.02 =$

i) $0.24 \div 2 =$

j) $(-0.45) \div 0.09 =$

k) $4.5 \div (-0.9) =$

l) $(-0.45) \div (-9) =$

m) $24 \div (-0.24) =$

n) $2.4 \div (0.24) =$

o) $(-0.24) \div (-0.24) =$

Unit 6 – Practice Problems

13. Multiply or divide the numbers by the given powers of 10 by moving the decimal point the appropriate number of places.

a) $5.327 \cdot 100 =$ _____ b) $1.002 \cdot 1000 =$ _____ c) $3.14 \cdot 10 =$ _____

e) $32.81 \div 100 =$ _____ e) $5 \div 1000 =$ _____ f) $53.91 \div 10 =$ _____

14 Use your calculator to combine the numbers below. Round your final result to the nearest hundredth. When computing, try to enter the entire expression all at once.

a. $(4.01)^2 - 2.25 \times 3.85$

b. $(3.523 - 1.20)^2 - (-4.0) + (-2.14)$

c. $12.82 \times 6.238 + 3.457 + 5.02(-6.83712)$

d. $0.256 \div 0.34 \times 7.813 - (-0.214)^2$

e. $(2.1)^3 - (0.15 + 0.19)^2$

15. Travis receives ten cents off per gallon on gas for every \$100 he spends at the grocery store during a given month. During the month of October, he spent \$45.23, \$102.34, \$13.67, \$34.56, \$48.72, and \$52.12. What will Travis' gas discount be for October?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

16. Sylvia just received her monthly water usage data from her local water department. For the past 6 months, her water used (in thousands of gallons) was 19.9, 25.6, 28.8, 22.5, 20.3, and 19.2. What was her average usage during this time? (Round to the nearest tenth)

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

Unit 6 – Practice Problems

17. Marty is standing in line at the store with his friend Danny. Marty says that he can estimate his purchase, without using a calculator, within 50 cents of the actual amount. Danny did not believe him. Marty bought items in the amounts of \$1.25, \$2.04, \$5.62, \$8.81, \$6.12, and \$12.99. Marty estimated his items at \$37. First of all, was he within the 50 cent limit for his estimation and second, how might he have accomplished this?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

18. Glenn normally earns \$8.50 per hour in a given 40-hour work-week. If he works overtime, he earns time and a half pay per hour. During the month of October, he worked 40 hours, 50 hours, 45 hours, and 42 hours for the four weeks. How much did he earn total for October?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

19. Dave is making a gazebo for his yard. He has a piece of wood that is 13 feet long and he needs to cut it into pieces of length 5.3 inches. How many pieces of this size can he cut from the 13 foot piece of wood?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

20. Callie ordered 4 items online. She is charged \$2.37 per pound per shipping. The items weighed 3.2 lbs., 4.6 lbs., 9.2 lbs. and 1.5 lbs. How much will be charged for shipping? (Round to the nearest cent).

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

Unit 6 – Practice Problems

21. Penny is making barrettes for her online business. Each barrette needs 2.3 inches of ribbon. If Penny has 4 feet of ribbon, how many barrettes can she make?

GIVEN:

GOAL:

MATH WORK:

CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

22. Mark visits the grocery store once a week for groceries. The amount he spent on five separate visits was \$52.35, \$36.93, \$44.79, \$88.98, \$55.22. What is the average amount Mark spent per week over these five weeks?

GIVEN:

GOAL:

MATH WORK:

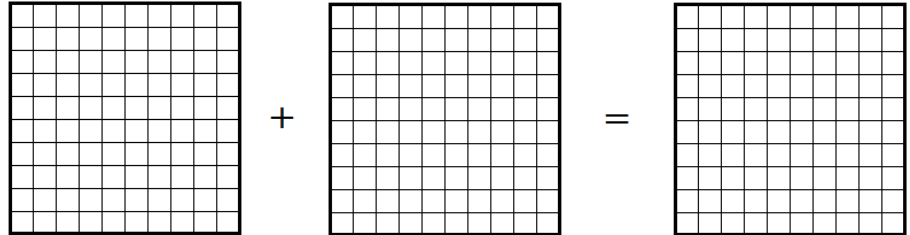
CHECK:

FINAL ANSWER AS A COMPLETE SENTENCE:

UNIT 6 – END OF UNIT ASSESSMENT

1. Use the decimal grid to shade the addends of the addition problem. Then combine your addends in a new grid to find the sum.

$$0.47 + 0.09$$



Sum: _____

2. Add the decimals by aligning the decimal points and adding.

$$35.13 + 245.672$$

3. Place the numbers in the Place Value chart and then use the chart as an aid to subtract the numbers.

$$45.216 - 14.78$$

Difference: _____

Ones			.	Decimals		
Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths

4. Subtract the decimals by aligning the decimal points and subtracting.

$$168.2 - 40.977$$

Unit 6 – Assessment

5. Use the rules for signed numbers to add or subtract the decimals.

a) $0.9 + (-2.34)$

b) $-6.72 + 8.73$

c) $7.81 - (-4.18)$

6. Multiply the decimals.

a) $0.4 \cdot 0.9$

b) $0.5 \cdot 3.2$

c) $-2.3(0.04)$

d) $(-0.6)(-0.27)$

7. Divide the decimals

a) $3.2 \div 0.02$

b) $0.56 \div 0.28$

c) $(-0.54) \div (0.06)$

d) $(-7.2) \div (-0.009)$

8. Multiply or divide the numbers by the given powers of 10 by moving the decimal point the appropriate number of places.

a) $4.218 \cdot 10$

b) $21.73 \div 10$

c) $3.25 \cdot 1000$

d) $6.1 \div 100$

9. Simplify. Show work. Round your final result to the nearest hundredth.

a) $4.25 + (0.8)^2 \div 10$

b) $1.5(2.03 - 1.8)$

c) $\frac{6.006 - 0.064}{0.8}$

10. Joe's eyeglasses cost a total of \$457.99. The frames of the glasses cost \$129.25. How much do the lenses of Joe's eyeglasses cost? Write your final answer as a complete sentence.
11. Denice works 40 hours per week as an administrative assistant at the local pet clinic. What is her total weekly pay if her hourly wage is \$17.75? Write your final answer as a complete sentence.
12. A bag of grass seed covers 142.5 square feet of lawn. The hotel's front lawn measures 15,500 square feet. How many bags of grass seed does the hotel's landscaping manager need to buy if only whole bags can be purchased? Write your final answer as a complete sentence.

UNIT 7 – MULTIPLICATIVE AND PROPORTIONAL REASONING

INTRODUCTION

In this Unit, we will learn about the concepts of multiplicative and proportional reasoning. Some of the ideas will seem familiar such as ratio, rate, fraction forms, and equivalent fractions. We will extend these ideas to focus on using these constructs to compare numbers through multiplication and division (versus addition and subtraction) and find unknown quantities using the relationships between ratios.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Learning Objective	Media Examples	You Try
Compare ratios additively and multiplicatively	1, 2	3
Represent ratios in multiple ways	4	5
Use ratios and double number lines to solve proportional problems	6, 7	8
Find rates and unit rates that correspond to a contextual problem	9	11
Use unit rates to compare two rates	10	11
Use unit rates to solve proportional problems	12, 13	14
Verify that two figures are similar by finding scale factors	15	18
Use scale factors to determine missing sides in similar figures	16	18
Use similarity to solve proportional application problems	17	18

UNIT 7 – MEDIA LESSON

SECTION 7.1: ADDITIVE VERSUS MULTIPLICATIVE COMPARISONS

In this section, we will look at two different ways of comparing quantities; additive comparisons and multiplicative comparisons.

1. When we **compare two numbers additively**, we are finding the absolute difference between the two numbers via subtraction. For example, if Tom is 7 years old and Fred is 9 years old, Fred is 2 years older than Tom because $9 - 7 = 2$ or equivalently, $7 + 2 = 9$ because adding 2 more to 7 is 9.
2. When we **compare two numbers multiplicatively**, we are finding **the ratio** or quotient between the two numbers via division. For example, if Sally is 3 years old and Tara is 6 years old, Tara is 2 times as old as Sally because $\frac{6}{3} = 6 \div 3 = 2$ or equivalently, $2 \times 3 = 6$ because multiplying 3 by 2 means 6 is 2 times as large as 3.

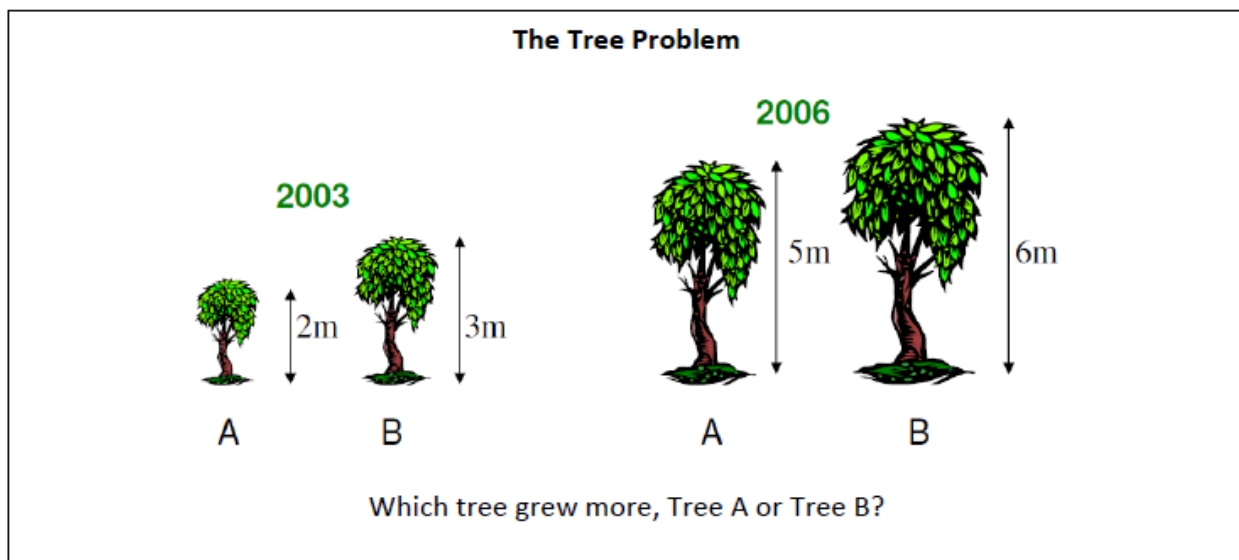
In this section, we will explore these ideas further and compare and contrast these two types of comparisons.



Problem 1

MEDIA EXAMPLE – Additive and Multiplicative Comparisons: Tree Problem

Mike plants two trees in his backyard in 2003 and measures their height. Three years later, he measures the trees again and records their new height. The information on the year and height of the trees is given below.



1. Mike and his family are debating which tree grew more. Which tree do you think grew more and why?

2. Mike's son John says that neither tree grew more than the other because both trees grew exactly 3 meters. How did John determine this mathematically? Write the computations he might have made below.
- a) Tree A growth:

 - b) Tree B growth:

 - c) Is John making an additive or multiplicative comparison? Explain your reasoning.
3. Mike's daughter Danielle says that Tree A grew more than Tree B. She says that even though they both grew 1 meter, since Tree A was shorter than Tree B in 2003, *Tree A grew more relative to its original height*.
- a) Write a ratio that compares Tree A's height in 2006 to Tree A's height in 2003.

 - b) In 2006, Tree A's height is _____ times as large as Tree A's height in 2003.

 - c) Write a ratio that compares Tree B's height in 2006 to Tree B's height in 2003.

 - d) In 2006, Tree B's height is _____ times as large as Tree B's height in 2003.

 - e) Use your answers from parts a – d to determine which tree grew more using a *multiplicative comparison*. Explain your reasoning.

“The Broomstick Problem” by [Dr. Ted Coe](#) is licensed under [CC BY-SA 4.0](#)

You have three broomsticks:
 The RED broomstick is three feet long
 The YELLOW broomstick is four feet long
 The GREEN broomstick is six feet long



a) How much longer is the GREEN broomstick than the RED broomstick?

Additive Comparison	Multiplicative Comparison

b) How much longer is the YELLOW broomstick than the RED broomstick?

Additive Comparison	Multiplicative Comparison

c) The GREEN broomstick is _____ times as long as the YELLOW broomstick.

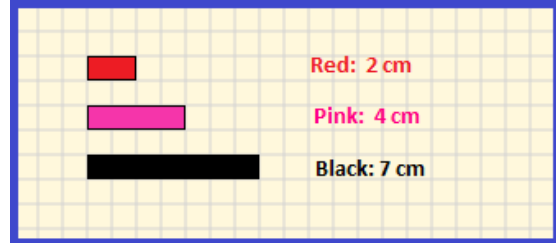
d) The YELLOW broomstick is _____ times as long as the GREEN broomstick.

e) The YELLOW broomstick is _____ times as long as the RED broomstick.

f) The RED broomstick is _____ times as long as the YELLOW broomstick.

 Problem 3 YOU TRY – Additive and Multiplicative Comparisons

You have three toothpicks:
 The RED toothpick is 2 cm long
 The PINK toothpick is 4 cm long
 The BLACK toothpick is 7 cm long



a) How much longer is the PINK toothpick than the RED toothpick?

Additive Comparison	Multiplicative Comparison
---------------------	---------------------------

b) How much longer is the BLACK toothpick than the RED toothpick?

Additive Comparison	Multiplicative Comparison
---------------------	---------------------------

c) The PINK toothpick is _____ times as long as the RED toothpick.

d) The RED toothpick is _____ times as long as the PINK toothpick.

e) The BLACK toothpick is _____ times as long as the RED toothpick.

f) The PINK toothpick is _____ times as long as the BLACK toothpick.

SECTION 7.2: RATIOS AND THEIR APPLICATIONS

In this section, we will investigate ratios and their applications. A *ratio* is *multiplicative comparison* of two quantities. For example, $\frac{6 \text{ miles}}{3 \text{ miles}}$ is a ratio since we are comparing two quantities multiplicatively by division (often written as a fraction). We may write ratios in any of the following forms.


Fraction: $\frac{6 \text{ miles}}{3 \text{ miles}}$ **Colon:** 6 miles: 3 miles **“a to b” language:** 6 miles to 3 miles

In addition, ratios may represent part to part situations or part to whole situations.

Example: Kate is traveling 100 miles to visit Rick. So far she has traveled 40 miles.

Part – Whole Comparison: The ratio of miles Kate has traveled to the total number of miles is $\frac{40 \text{ miles}}{100 \text{ miles}}$

Part – Part Comparison: The ratio of miles Kate has traveled to the miles she still needs to travel is $\frac{40 \text{ miles}}{60 \text{ miles}}$

	Problem 4	MEDIA EXAMPLE – Representing Ratios in Multiple Ways
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Represent the following scenarios as ratios in the indicated ways. Then determine if the comparison is part to part or part to whole.

- a) In Martha’s math class, there were 8 students that passed a test for every 2 students that failed a test. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Students Who Passed to Students who Failed	Ratio of Students Who Failed to Students who Passed
Fraction		
Colon		
“a to b” language		

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain.

- b) In Cedric’s fish tank, there were 6 blue fish and 9 yellow fish. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Blue Fish to Total Fish	Ratio of Yellow Fish to Total Fish
Fraction		
Colon		
<i>“a to b” language</i>		

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain

	Problem 5	YOU TRY – Representing Ratios in Multiple Ways
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Represent the following scenarios as ratios in the indicated ways. Then determine if the comparison is part to part or part to whole.

Bernie’s swim team has 12 girl members and 8 boy members. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Girls to Boys	Ratio of Boys to Girls	Ratio of Girls to Total Members	Ratio of Boys to Total Members
Fraction				
Colon				
<i>“a to b” language</i>				

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain

SECTION 7.3: RATIOS AND PROPORTIONAL REASONING

In this section, we will solve application problems using proportional reasoning. A *proportion* is a statement that two ratios are equal.

Example: You take a test and get 20 out of 25 questions correct. However, each question is worth 2 points.

Since you got 20 questions correct, the points you earned is given below.

$$20 \text{ questions correct} \times 2 \text{ points per correct question} = 40 \text{ points}$$

The total number of possible points you can earn is given below.

$$25 \text{ total questions} \times 2 \text{ points per question} = 50 \text{ total points}$$

The ratios representing these two quantities are

Ratio of Correct Questions to Total Questions: $\frac{20 \text{ correct questions}}{25 \text{ total questions}}$

Ratio of Points Earned to Total Points: $\frac{40 \text{ points earned}}{50 \text{ total points}}$

Since a proportion is a statement that two ratios are equal, the equation below represents this proportion.

Corresponding Proportional Statement: $\frac{20 \text{ correct questions}}{25 \text{ total questions}} = \frac{40 \text{ points earned}}{50 \text{ total points}}$

Observe that if you view these ratios without the units, you can see the ratios are also equivalent fractions.

$$\frac{20}{25} = \frac{40}{50}$$

You can verify this by simplifying each of the fractions $\frac{20}{25}$ and $\frac{40}{50}$ completely. You will see they both are equivalent to $\frac{4}{5}$.



Problem 6

MEDIA EXAMPLE – Using Ratios to Solve Application Problems: Part 1

Represent the following scenarios as ratios in the indicated ways. Then use this information to answer the corresponding questions.

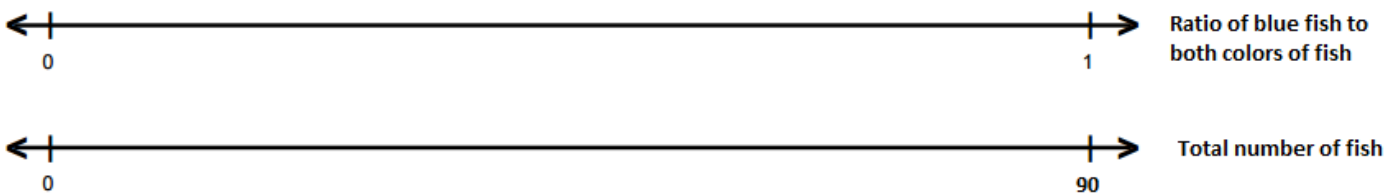
- a) Maureen went to the aquarium. There was a giant fish tank holding only blue and orange fish. A sign on the tank said there were 2 blue fish for every 3 orange fish.

Write the following ratios in fraction form. Include units in your answers.

Ratio of blue fish to orange fish	
Ratio of orange fish to blue fish	
Ratio of blue fish to both colors of fish	
Ratio of orange fish to both colors of fish	

- b) Maureen asked the tour guide how many blue and orange fish there were in total. The tour guide said there were approximately 90 of these fish. Use this information and the double number lines below to represent this scenario. Then approximate how many blue fish are in the tank and how many orange fish are in the tank.

Diagram for Blue Fish:



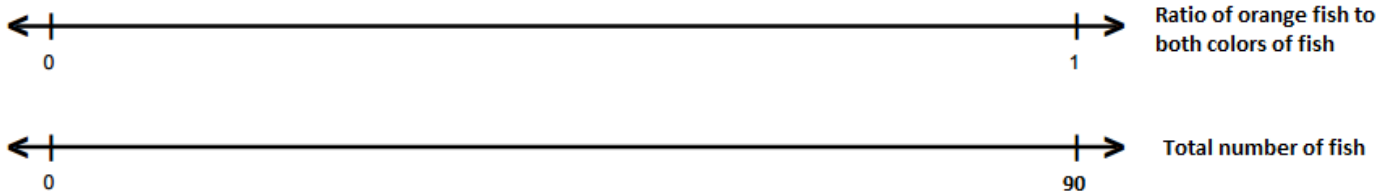
Symbolic Representation:

Approximate number of blue fish in the tank:

Corresponding Proportional Statement:

Unit 7 – Media Lesson


Diagram for Orange Fish:



Symbolic Representation:

Approximate number of orange fish in the tank:

Corresponding Proportional Statement:

 Problem 7	MEDIA EXAMPLE – Using Ratios to Solve Application Problems: Part 2
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Represent the following scenarios as ratios in the indicated ways. Then use this information to answer the corresponding questions.

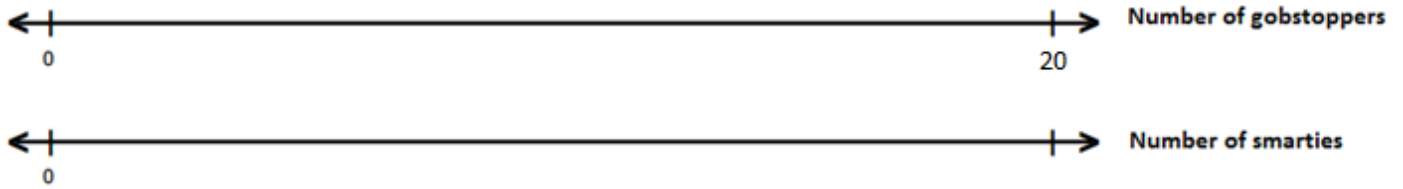
- a) Amy and Jennifer were counting up their candy after trick or treating. Amy’s favorite is smarties candies and Jen’s favorite is gobstopper candies. They decide to make a trade. Amy says she will give Jen 4 gobstopper candies for every 7 smarties candies Jen gives her. Jen agrees.

Write the following ratios in fraction form. Include units in your fractions.

The ratio of the trade of smarties to gobstoppers:

The ratio of the trade of gobstoppers to smarties:

- b) Suppose Amy has 20 gobstoppers. How many smarties would Jen have to give Amy in trade? Use this information and the double number lines below to represent this scenario and find the result.

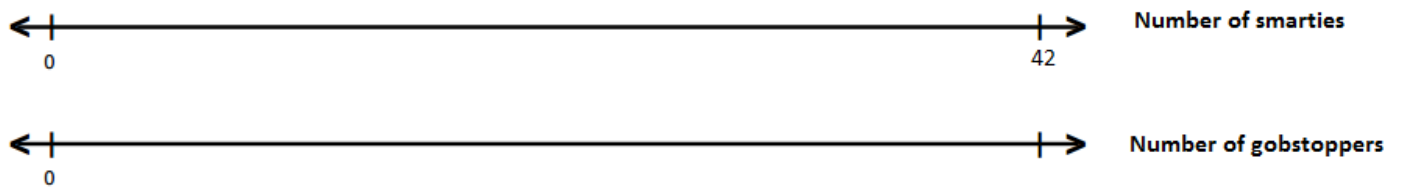


Symbolic Representation:

Number of smarties for 20 gobstoppers:

Corresponding Proportional Statement:

- c) Suppose Jen has 42 smarties. How many gobstoppers would Amy have to give Jen in trade? Use this information and the double number lines below to represent this scenario and find the result.



Symbolic Representation:

Number of gobstoppers for 42 smarties:

Corresponding Proportional Statement:

Problem 8	YOU TRY – Using Ratios to Solve Application Problems
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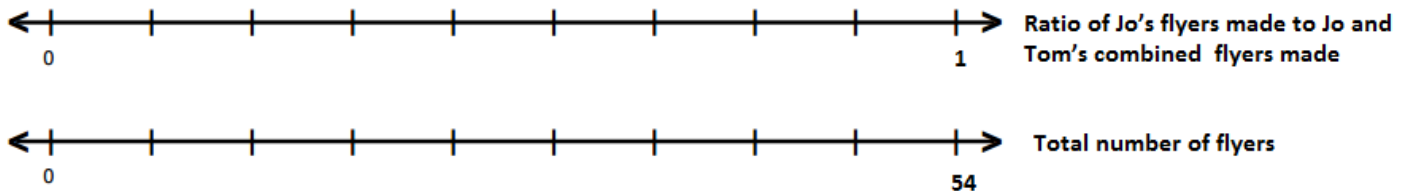
Use the following information to answer the questions below.

Jo and Tom made flyers for a fundraiser. For every 5 flyers Jo made, Tom made 4 flyers.

a) Write the following ratios in fraction form. Include units in your answers.

Ratio of Jo’s flyers made to Tom’s flyers made	
Ratio of Tom’s flyers made to Jo’s flyers made	
Ratio of Jo’s flyers made to Jo and Tom’s combined flyers made	
Ratio of Tom’s flyers made to Jo and Tom’s combined flyers made	

b) If Jo and Tom made 54 flyers in total, how many flyers did Jo make? Use this information and the double number lines below to represent this scenario and find the result.



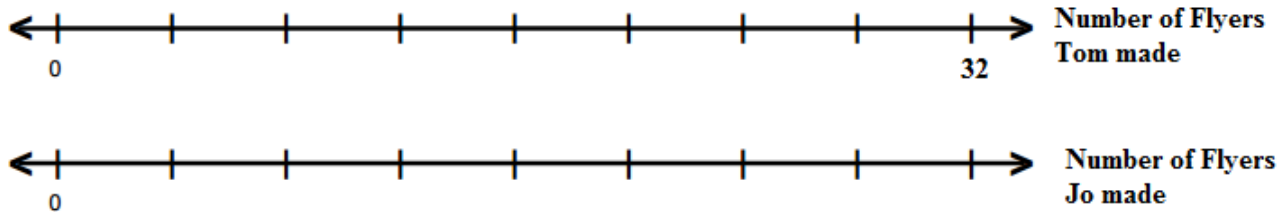
Symbolic Representation:

Number of Flyers Jo made:

Corresponding Proportional Statement:

Based on your previous answer, how many flyers did Tom make?

- c) If Tom made 32 flyers, how many flyers did Jo make? Use this information and the double number lines below to represent this scenario and find the result.



Symbolic Representation:

Number of flyers Jo made:

Corresponding Proportional Statement:

SECTION 7.4: RATES, UNIT RATES, AND THEIR APPLICATIONS

In this section, we will look at a special type of ratio called a rate. A **rate** is a ratio where the quantities we are comparing are measuring different types of attributes. First notice, that a rate is considered a type of ratio so a rate is also a multiplicative comparison of two quantities. However, the two quantities measure different things. For example,

1. miles per hour (distance over time, which we may also call speed)
2. dollars per hour (money over time, which we may also call rate of pay)
3. number of people per square mile (population over land area, which we may also call population density).

A special type of rate is called a unit rate. A **unit rate** is a rate where the quantity of the measurement in the denominator of the rate is 1. For example, suppose you are offered a new job after graduation, and your new employer says that you will be paid at a rate of \$805 per 25 hours or $\frac{\$805}{25 \text{ hours}}$. This is indeed a rate of pay,

but it is difficult to conceptualize this rate. It may be more useful to know how much you will be paid *per 1 hour* instead of *per 25 hours*. This unit rate of pay can be found as shown below.

$$\frac{\$805}{25 \text{ hours}} = \frac{\$805 \div 25}{25 \text{ hours} \div 25} = \frac{\$34}{1 \text{ hour}} \text{ or } \$34 \text{ per hour}$$

In this section, we will learn to write these rates and unit rates in multiple ways and use them to solve application problems.



Problem 9

MEDIA EXAMPLE – Representing Rates and Unit Rates in Multiple Ways

Represent the following scenarios as rates and unit rates in the indicated ways.

- a) Lanie ate 4 cookies for a total of 200 calories.


Rate in calories per cookies	Unit rate in calories per cookie	Rate in cookies per calories	Unit rate in cookies per calorie

- b) Alexis went on a road trip to California. She traveled at a constant speed and drove 434 miles in 7 hours.

Rate in miles per hours	Unit rate in miles per hour	Rate in hours per miles	Unit rate in hours per mile

- c) April bought a bottle of ibuprofen at the store. She bought 300 pills for \$6.30.

Rate in pills per dollars	Unit rate in pills per dollar	Rate in dollars per pills	Unit rate in dollars per pill

 Problem 10	MEDIA EXAMPLE – Using Unit Rates for Comparison
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Callie is buying cereal at the grocery store. A 12.2 ounce box costs \$4.39. A 27.5 ounce box costs \$10.19.

- a) Determine the following unit rates for the small 12.2 ounce box and large 27.5 ounce box. Write your unit rates as decimals rounded to four decimal places.

Small Box Unit rate in ounces per dollar	Large Box Unit rate in ounces per dollar	Small Box Unit rate in dollars per ounce	Large Box Unit rate in dollars per ounce

Based on the information in the table above, complete the following statements.

- b) The _____ box is a better buy because it costs _____ dollars per ounce.
- c) The _____ box is a better buy because you get _____ ounces per dollar.

 Problem 11	YOU TRY – Using Unit Rates for Comparison
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Hector is buying cookies for a party. A regular sized bag has 34 cookies and costs \$2.46. The family size bag has 48 cookies and costs \$3.39 a bag.

- a) Determine the following unit rates for the small 12.2 ounce box and large 27.5 ounce box. Write your unit rates as decimals rounded to four decimal places.

Regular Sized Unit rate in cookies per dollar	Family Sized Unit rate in cookies per dollar	Regular Sized Unit rate in dollars per cookie	Family Sized Unit rate in dollars per cookie

Based on the information in the table above, complete the following statements.

- b) The _____ sized bag is a better buy because it costs _____ dollars per cookie.
- c) The _____ sized bag is a better buy because you get _____ cookies per dollar.

SECTION 7.5: RATES AND PROPORTIONAL REASONING

In this section, we will use the ideas of rate and proportional reasoning to solve application problems involving rates.



Problem 12

MEDIA EXAMPLE – Using Unit Rates to Solve Application Problems: Part 1

Represent the following scenarios as unit rates in the indicated ways. Then use this information to answer the corresponding questions.

- a) Stephanie can walk 5 miles in 2 hours. Use this information to fill in the chart below. Use decimals when needed.

Hours	1	2	3	4	5	6
Miles						

- b) What is Stephanie's unit rate of speed in miles per hour? How can you determine this from the table?
- c) Using the unit rate of miles per hour, how far will Stephanie walk in 8 hours? Also write the corresponding proportion.
- d) Using the unit rate of miles per hour, how far will Stephanie walk in 3.75 hours? Also write the corresponding proportion.
- e) What is Stephanie's unit rate of hours per mile?
- f) Using the unit rate of hours per mile, how long will it take Stephanie to walk in 20 miles? Also write the corresponding proportion.
- g) Using the unit rate of hours per mile, how long will it take Stephanie to walk in 26.2 miles? Also write the corresponding proportion.



Problem 13

MEDIA EXAMPLE – Using Unit Rates to Solve Application Problems: Part 2

Represent the following scenarios as unit rates in the indicated ways. Then use this information to answer the corresponding questions.

- a) The valve on Ray’s washing machine is leaking. He puts a bucket under the leak to catch the water. The next day, after 24 hours, Ray checks the bucket and it has 8 gallons of water in it. Use this information to complete the table below.

Hours	1	3	6	12	24
Gallons					

- b) What is leak’s unit rate of in gallons per hour? How can you determine this from the table?
- c) Using the unit rate of gallons per hour, how much water will leak in 9 hours? Also write the corresponding proportion.
- d) Using the unit rate of gallons per hour, how much water will leak in 13.5 hours? Also write the corresponding proportion.
- e) What is leak’s unit rate in hours per gallon?
- f) Using the unit rate of hours per gallon, how long will it take for the bucket to contain 3 gallons of water? Also write the corresponding proportion.
- g) If the bucket holds 10 gallons of water, how long can Ray go without emptying the bucket without the water overflowing? Also write the corresponding proportion.



Problem 14

YOU TRY – Using Unit Rates to Solve Application Problems

Represent the following scenarios as unit rates in the indicated ways. Then use this information to answer the corresponding questions.

- a) Last week you worked 16 hours and earned \$224. Use this information to complete the table below.



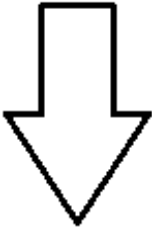
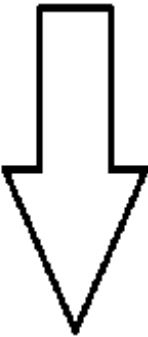
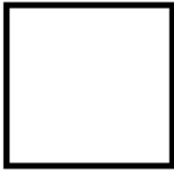

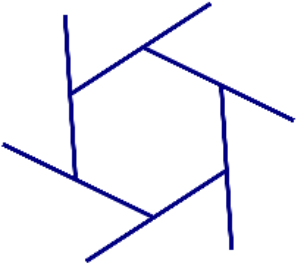
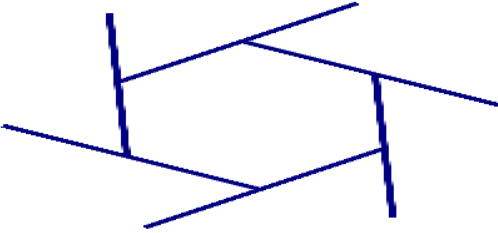
Hours	1	2	4	8	16	32
Dollars						

- b) What is your unit pay rate in dollars per hour? How can you determine this from the table?
- c) Using your unit pay rate in dollars per hour, how much would you earn in 12 hours? Also write the corresponding proportion.
- d) Using your unit pay rate in dollars per hour, how much would you earn in 33.5 hours? Also write the corresponding proportion.
- e) What is your unit pay rate in hours per dollar?
- f) Using your unit pay rate in hours per dollar, how many hours will you need to work to earn \$378? Also write the corresponding proportion.
- g) If you need \$545 to pay your rent, how many hours do you need to work to cover your rent? Round up to the nearest hour.

SECTION 7.6: SIMILARITY AND SCALE FACTORS

In this section, we will study similar figures and scale factors. Two figures are *similar* if they have the exact same shape and their corresponding sides are proportional. The corresponding side lengths of the two figures are related by a scale factor. A *scale factor* is the constant number you can multiply any side length in one figure by to find the corresponding side length of the similar figure.

You probably already have a good intuition about whether two figures are similar. Observe the pairs of figures below and use your judgment and the definition to determine if the figures are similar.

Figure 1	Figure 2	Similar or not similar?
		Yes they are similar. Same shape. I scaled each side by a factor of $1\frac{3}{4}$. Each side in figure 2 is $1\frac{3}{4}$ times the length of the corresponding side in Figure 1.
		No they are not similar. They have the same general arrow shape, but I made the arrow longer and <i>not wider</i> . I scaled in the vertical direction by a factor of $1\frac{1}{2}$ but I left the horizontal scaling the same.
		No they are not similar. Although the bottom side length is the same and they have the same number of sides, they are different shapes.
		No they are not similar. They have the same general shape, but I made the shape wider and <i>not longer</i> in Figure 2. I scaled in the horizontal direction by a factor of 2, but I left the vertical scaling the same.

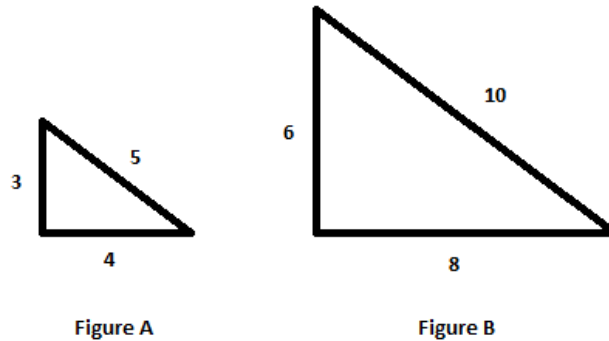


Problem 15

MEDIA EXAMPLE – Verifying Similarity and Finding Scale Factors

Verify that the following figures are similar by finding the indicated scale factor between each corresponding pair of sides.

- a) Complete the table by finding the indicated ratios to determine the scale factors between the figures.



Ratio of the shortest side of Figure B to the shortest side of Figure A	
Ratio of the longest side of Figure B to the longest side of Figure A	
Ratio of the medium side of Figure B to the medium side of Figure A	

- b) Figure B is _____ times as large as Figure A.
- c) To scale Figure A to the size of Figure B, multiply the length of each side of Figure A by the scale factor of _____.
- d) Complete the table by finding the indicated ratios to determine the scale factors between the figures.

Ratio of the shortest side of Figure A to the shortest side of Figure B	
Ratio of the longest side of Figure A to the longest side of Figure B	
Ratio of the medium side of Figure A to the medium side of Figure B	

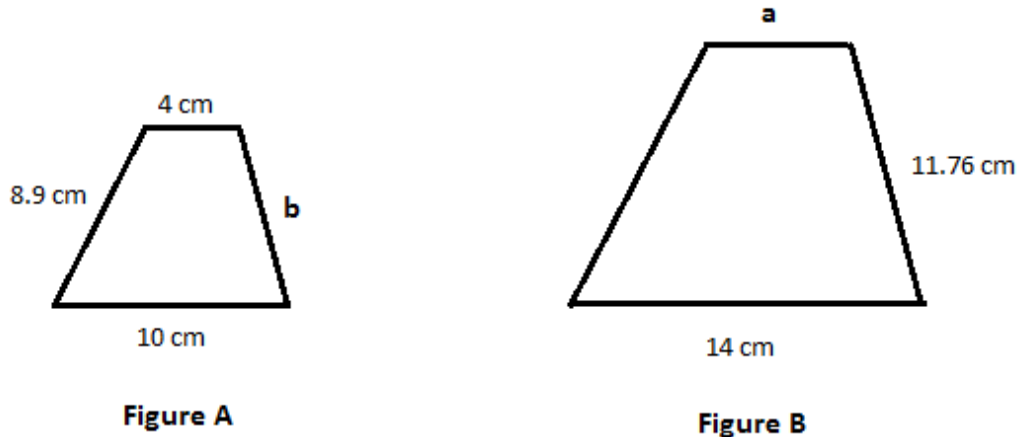
- e) Figure A is _____ times as large as Figure B.
- f) To scale Figure B to the size of Figure A, multiply the length of each side of Figure B by the scale factor of _____.



Problem 16

MEDIA EXAMPLE – Finding Missing Sides in Similar Figures

The following pair of figures are similar. Find the indicate scale factors and use the information to determine the lengths of the missing sides.



- a) Find the scale factor from Figure A to Figure B and complete the sentence below.

To scale Figure A to the size of Figure B, multiply the length of each side of Figure A by the scale factor of _____.

- b) Find the scale factor from Figure B to Figure A and complete the sentence below.

To scale Figure B to the size of Figure A, multiply the length of each side of Figure B by the scale factor of _____.

- c) Use a scale factor to find the length of **side a**. Show your work.

- d) Use a scale factor to find the length of **side b**. Show your work.

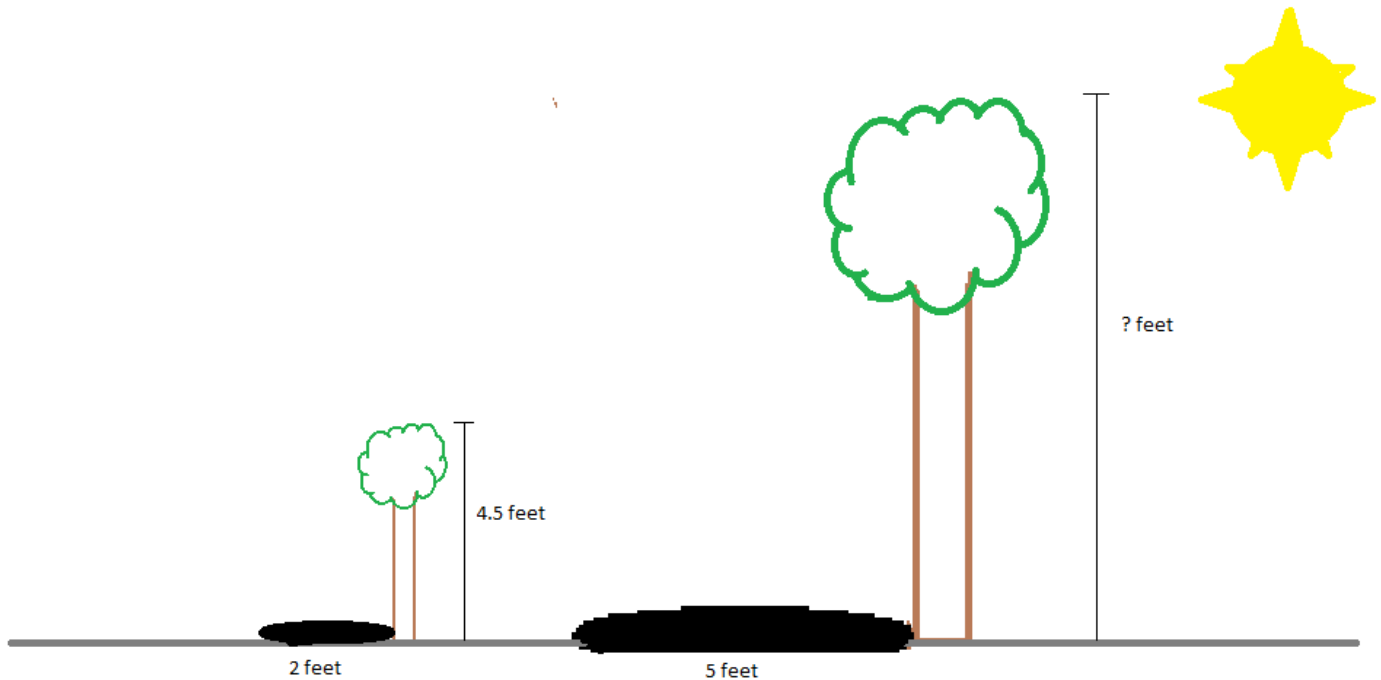


Problem 17


MEDIA EXAMPLE – Using Similarity to Solve Application Problems

Solve the following application problem by determining and using scale factors.

Christianne has a full size tree and a young tree in her backyard. She wants to know how tall the full size tree is, but doesn't have a way of measuring it because it is too tall. She notices the shadows of the tree and realizes the ratios of the shadow height to tree height are proportional. She measures the shadows and the smaller tree and makes the sketch of the information below.



- For which type of measurement, shadow height or tree height, do we have information on both of the trees?
- Using the information from the diagram, find the scale factor from the young tree to the full sized tree.
- Use the scale factor and the height of the young tree to find the height of the full sized tree. Write your answer as a complete sentence.

 Problem 18 | YOU TRY – Similarity and Scale Factors

- a) Verify that the following figures are similar by finding the indicated scale factor between each corresponding pair of sides.

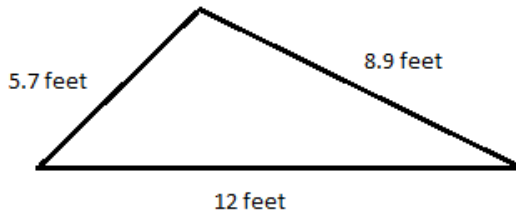


Figure A

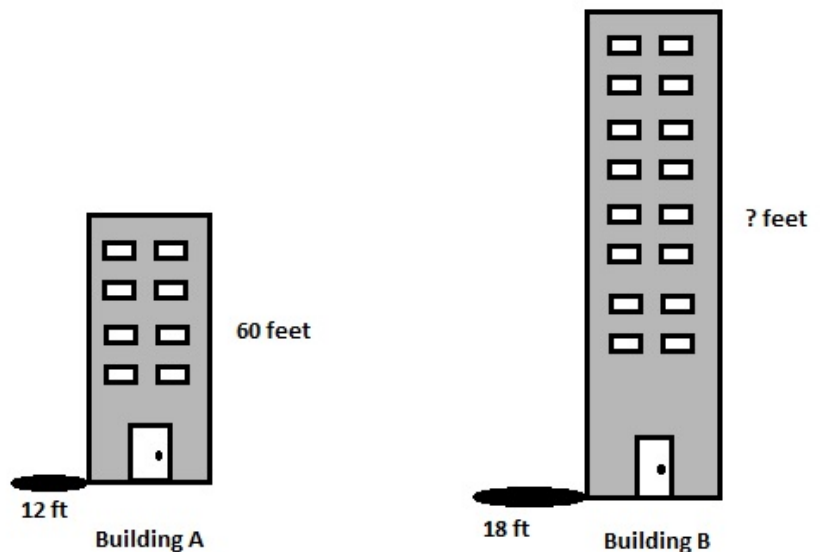


Figure B

Ratio of the shortest side of Figure B to the shortest side of Figure A	
Ratio of the longest side of Figure B to the longest side of Figure A	
Ratio of medium side of Figure B to the medium side of Figure A	

Figure B is _____ times as large as Figure A. Figure A is _____ times as large as Figure B.

- b) The diagram below shows two buildings and their shadows. The ratios of the shadow height to the building height are proportional. Use a scale factor between the shadow lengths and the height of the smaller building to find the height of the larger building. Write your answer as a complete sentence.



UNIT 7 – PRACTICE PROBLEMS

1 – 3: Represent the following scenarios as ratios in the indicated ways. Then determine if the comparison is part to part or part to whole.

- In Kate’s yoga class, there were 15 women for every 4 men. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Women to Men	Ratio of Men to Women
Fraction		
Colon		
<i>“a to b” language</i>		

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain.

- Theo has 18 pairs of sneakers. Twelve pairs are for running and 6 pairs are for tennis. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Running Sneakers to Total Sneakers	Ratio of Tennis Sneakers to Total Sneakers
Fraction		
Colon		
<i>“a to b” language</i>		

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain

Unit 7 – Practice Problems

3. Fran’s drama club has 16 adult members and 12 high school members. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Adult Members to High School Members	Ratio of High School Members to Adult Members	Ratio of Adult Members to Total Members	Ratio of High School Members to Total Members
Fraction				
Colon				
<i>“a to b” language</i>				

Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain

4 – 6: Represent the following scenarios as ratios in the indicated ways. Then use this information to answer the corresponding questions.

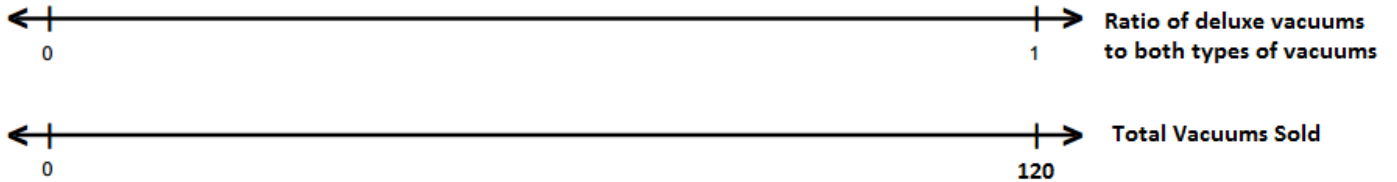
4. Lucas sells vacuums. There are two types of vacuums for sale; deluxe and economy. He sells 2 deluxe versions for every 6 economy versions.

Write the following ratios in fraction form. Include units in your answers.

Ratio of deluxe vacuums to economy vacuums	
Ratio of economy vacuums to deluxe vacuums	
Ratio of deluxe vacuums to both types of vacuums	
Ratio of economy vacuums to both types of vacuums	

5. Lucas sold 120 vacuums this month. Use this information and the double number lines below to represent this scenario. Then approximate how many deluxe vacuums he sold and how many economy vacuums he sold.

a) Diagram for Deluxe Vacuums:

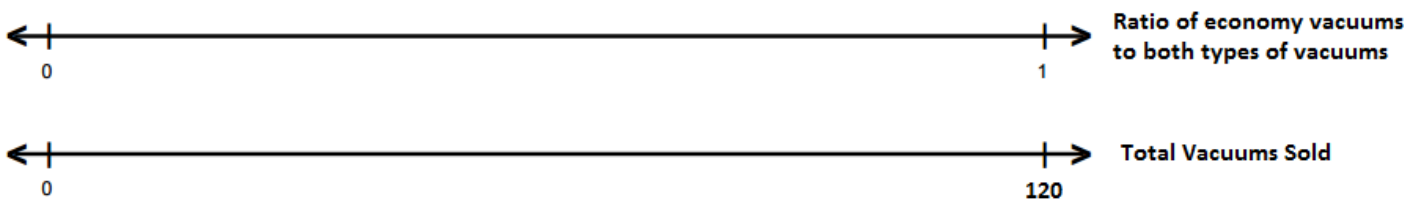


b) Symbolic Representation:

c) Approximate number of deluxe vacuums sold:

d) Corresponding Proportional Statement:

e) Diagram for Economy Vacuums:



f) Symbolic Representation:

g) Approximate number of economy vacuums sold:

h) Corresponding Proportional Statement:

Unit 7 – Practice Problems

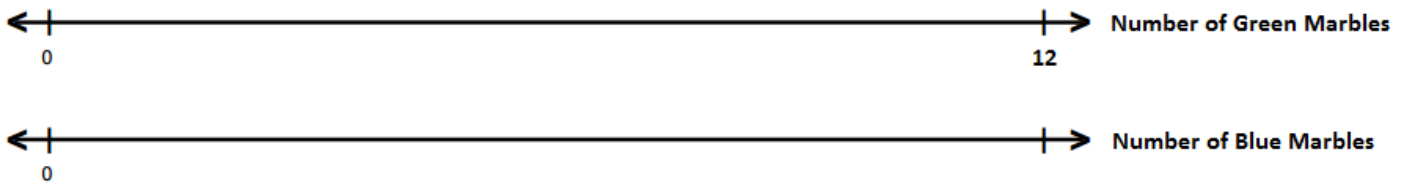
6. Fred and Barney like to collect marbles. Fred’s favorite color marble is blue. Barney’s favorite color marble is green. They decide to make a trade. Fred will give Barney 2 green marbles for every 3 blue marbles Barney gives Fred.

Write the following ratios in fraction form. Include units in your fractions.

a) The ratio of the trade of green marbles to blue marbles:

b) The ratio of the trade of blue marbles to green marbles:

- c) Suppose Fred has 12 green marbles. How many blue marbles would Barney have to give Fred in trade? Use this information and the double number lines below to represent this scenario and find the result.

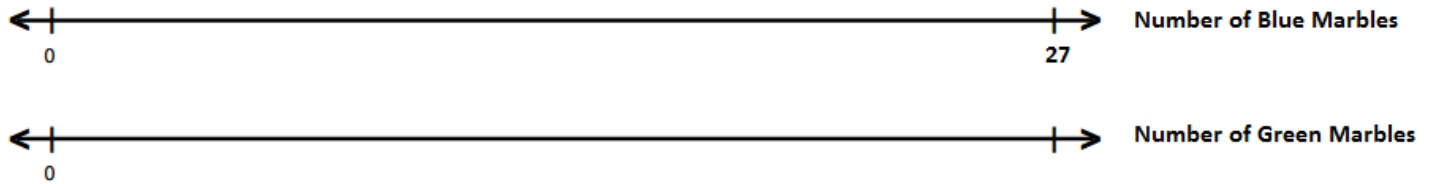


d) Symbolic Representation:

e) Number of blue marbles for 12 green marbles:

f) Corresponding Proportional Statement:

g) Suppose Barney has 27 blue marbles. How many green marbles would Fred have to give Barney in trade? Use this information and the double number lines below to represent this scenario and find the result.



h) Symbolic Representation:

i) Number of green marbles for 27 blue marbles:

j) Corresponding Proportional Statement:

7 – 10: Represent the following scenarios as rates and unit rates in the indicated ways.

7. Meri ate 5 cookies for a total of 175 calories.

Rate in calories per cookies	Unit rate in calories per cookie	Rate in cookies per calories	Unit rate in cookies per calorie

Unit 7 – Practice Problems

8. James went on a road trip. He traveled at a constant speed and drove 315 miles in 5 hours.

Rate in miles per hours	Unit rate in miles per hour	Rate in hours per miles	Unit rate in hours per mile

9. May bought a bottle of aspirin at the store. She bought 250 pills for \$4.25.

Rate in pills per dollars	Unit rate in pills per dollar	Rate in dollars per pills	Unit rate in dollars per pill

10. Callie is buying detergent at the grocery store. A 150 ounce box costs \$18.87. A 100 ounce box costs \$12.73.

a) Determine the following unit rates for the small 100 ounce box and large 150 ounce box. Write your unit rates as decimals rounded to four decimal places.

Small Box Unit rate in ounces per dollar	Large Box Unit rate in ounces per dollar	Small Box Unit rate in dollars per ounce	Large Box Unit rate in dollars per ounce

Based on the information in the table above, complete the following statements.

b) The _____ box is a better buy because it costs _____ dollars per ounce.

c) The _____ box is a better buy because you get _____ ounces per dollar.

11 – 13: Represent the following scenarios as unit rates in the indicated ways. Then use this information to answer the corresponding questions.

11. Daniel can jog 8 miles in 2 hours.

a) Use this information to fill in the chart below. Use decimals when needed.

Hours	1	2	3	4	5	6
Miles						

b) What is Daniel's unit rate of speed in miles per hour? How can you determine this from the table?

c) Using the unit rate of miles per hour, how far will Daniel jog in 8 hours? Also write the corresponding proportion.

d) Using the unit rate of miles per hour, how far will Daniel jog in 3.75 hours? Also write the corresponding proportion.

e) What is Daniel's unit rate of hours per mile?

f) Using the unit rate of hours per mile, how long will it take Daniel to jog in 20 miles? Also write the corresponding proportion.

g) Using the unit rate of hours per mile, how long will it take Daniel to jog in 26.2 miles? Also write the corresponding proportion.

Unit 7 – Practice Problems

12. Samantha’s bathtub has a leaking faucet. She puts a bucket under the leak to catch the water so she can measure the leak. Three hours later, Samantha checks the bucket and it has 4.5 gallons of water in it.

a) Use this information to complete the table below.

Hours	1	3	6	12	24
Gallons					

b) What is leak’s unit rate of in gallons per hour? How can you determine this from the table?

c) Using the unit rate of gallons per hour, how much water will leak in 9 hours? Also write the corresponding proportion.

d) Using the unit rate of gallons per hour, how much water will leak in 13.5 hours? Also write the corresponding proportion.

e) What is leak’s unit rate in hours per gallon?

f) Using the unit rate of hours per gallon, how long will it take for the bucket to contain 3 gallons of water? Also write the corresponding proportion.

13. Last week you worked 16 hours and earned \$192.

a) Use this information to complete the table below.

Hours	1	2	4	8	16	32
Dollars						

b) What is your unit pay rate in dollars per hour? How can you determine this from the table?

c) Using your unit pay rate in dollars per hour, how much would you earn in 12 hours? Also write the corresponding proportion.

d) Using your unit pay rate in dollars per hour, how much would you earn in 33.5 hours? Also write the corresponding proportion.

e) What is your unit pay rate in hours per dollar?

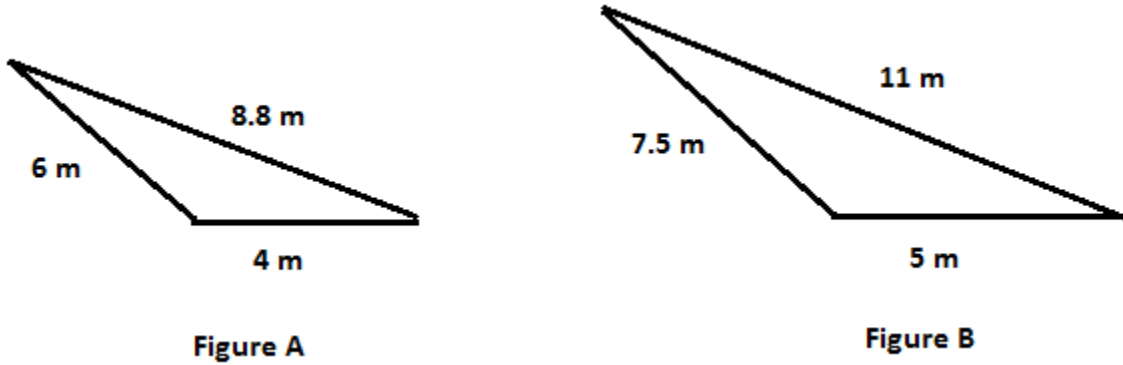
f) Using your unit pay rate in hours per dollar, how many hours will you need to work to earn \$522? Also write the corresponding proportion.

g) You need \$165 to pay your electric bill. How many hours do you need to work to cover your electric bill? Round up to the nearest hour.

Unit 7 – Practice Problems

14. Verify that the following figures are similar by finding the indicated scale factor between each corresponding pair of sides.

a) Complete the table by finding the indicated ratios to determine the scale factors between the figures.



Ratio of the shortest side of Figure B to the shortest side of Figure A	
Ratio of the longest side of Figure B to the longest side of Figure A	
Ratio of the medium side of Figure B to the medium side of Figure A	

- b) Figure B is _____ times as large as Figure A.
- c) To scale Figure A to the size of Figure B, multiply the length of each side of Figure A by the scale factor of _____.
- d) Complete the table by finding the indicated ratios to determine the scale factors between the figures.

Ratio of the shortest side of Figure A to the shortest side of Figure B	
Ratio of the longest side of Figure A to the longest side of Figure B	
Ratio of the medium side of Figure A to the medium side of Figure B	

- e) Figure A is _____ times as large as Figure B.
- f) To scale Figure B to the size of Figure A, multiply the length of each side of Figure B by the scale factor of _____.

15. Use the following figures to answer the questions.

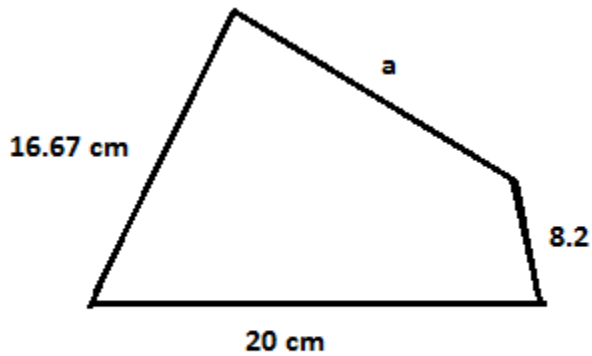


Figure A

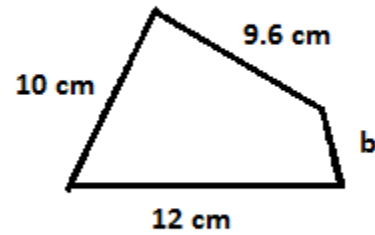


Figure B

a) Find the scale factor from Figure A to Figure B and complete the sentence below.

To scale Figure A to the size of Figure B, multiply the length of each side of Figure A by the scale factor of _____.

b) Find the scale factor from Figure B to Figure A and complete the sentence below.

c) To scale Figure B to the size of Figure A, multiply the length of each side of Figure B by the scale factor of _____.

d) Use a scale factor to find the length of **side a**. Show your work.

e) Use a scale factor to find the length of **side b**. Show your work.

Unit 7 – Practice Problems

16. Write each ratio as a fraction in simplest form.

a. 3 to 9

b. 4:12

c. $\frac{12 \text{ inches}}{24 \text{ inches}}$

d. 14 to 42

e. $\frac{16}{42}$

17. Write each rate as a fractions in simplest form. Include units in your answer.

a. 30 miles in 4 hours

b. 24 inches to 4 feet

c. 12 boys to 18 girls

d. 18 cars to 32 bicycles

18. . Write the unit rate for each of the following. Round to two decimals.

a. 150 miles in 3 hours

b. 24 minutes to 2 feet

c. \$18.25 for 4 gallons

d. \$1.45 for 6 ounces

e. 74 pounds per 12 square inches

19. If the scale on a map is 1 inch to 20 miles, what is the actual distance between two towns that are 3 inches apart on the map?
20. In November 2012 President Obama visited Phnom Penh, Cambodia as part of a summit of Asian leaders. Traffic in the city came to almost a complete standstill with cars moving at a rate of 2 miles in 4 hours. At this rate, how long would it take to travel a distance of 3.5 miles?
21. Ryan works a part-time job mowing lawns and can easily mow 3 lawns in 5 hours. If he got very busy one day and mowed 7 lawns, how long did it take him?
22. The director of a day care center can feed 7 children lunch for a week with 4 pounds of macaroni and cheese. If she has 16 pounds of macaroni and cheese, how many children can she feed lunch for a week?

UNIT 7 – END OF UNIT ASSESSMENT

1. Taylor’s playlist has 12 dance songs and 8 ballads. Write the following ratios for this situation using the given numbers and then write a simplified ratio. Include units in each of your answers.

Form	Ratio of Dance Songs to Ballads	Ratio of Ballads to Dance Songs	Ratio of Dance Songs to Total Songs	Ratio of Ballads to Total Songs
Fraction				
Colon				
<i>“a to b” language</i>				

2. Are the ratios in the table a Part-Whole comparison or a Part-Part comparison? Explain

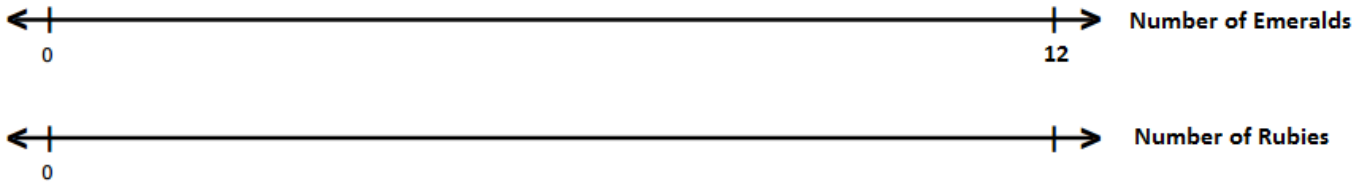
For 3 – 5: Wilma and Betty like to collect gemstones. Wilma’s favorite gemstone is emeralds. Betty’s favorite gemstone is rubies. They decide to make a trade. Wilma will give Betty 2 emeralds for every 3 rubies Betty gives Wilma.

Write the following ratios in fraction form. Include units in your fractions.

3. The ratio of the trade of emeralds to rubies:
4. The ratio of the trade of rubies to emeralds:

Unit 7 – Assessment

5. Suppose Wilma has 12 emeralds. How many rubies would Betty have to give Wilma in trade? Use this information and the double number lines below to represent this scenario and find the result.



Betty will give Wilma _____ rubies for 12 emeralds

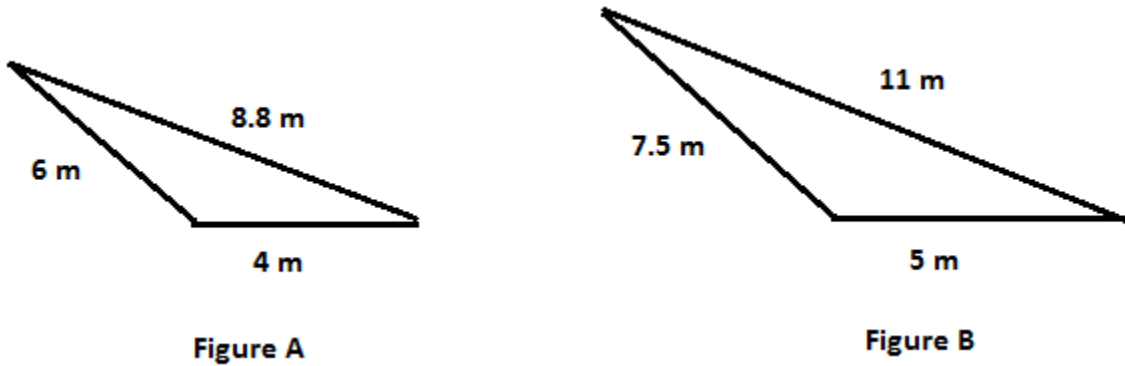
6. Sharon went on a road trip. She traveled at a constant speed and drove 268 miles in 4 hours. Complete the table below using this information.

Rate in miles per hours	Unit rate in miles per hour	Rate in hours per miles	Unit rate in hours per mile

For 7 – 10: Last week your worked 16 hours and earned \$288.

7. What is your unit pay rate in dollars per hour?
8. Using your unit pay rate in dollars per hour, how much would you earn in 33.5 hours?
9. What is you unit pay rate in hours per dollar?
10. Using your unit pay rate in hours per dollar, how many hours will you need to work to earn \$522?

For 11 – 14: Use the similar figures below to answer the questions.



11. Figure B is _____ times as large as Figure A.
12. To scale Figure A to the size of Figure B, multiply the length of each side of Figure A by the scale factor of _____.
13. Figure A is _____ times as large as Figure B.
14. To scale Figure B to the size of Figure A, multiply the length of each side of Figure B by the scale factor of _____.

UNIT 8 – PERCENTS

INTRODUCTION

In this Unit, we will learn about percents and their applications. Percents are a special type of multiplicative relationship and we'll connect the ideas of percent to our prior knowledge of fractions, decimals, ratios, and rates.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Learning Objective	Media Examples	You Try
Identify the usefulness of percents in context	1	
Represent equivalent percents, fractions, and decimals using percent grids	2	4
Represent equivalent percents, fractions, and decimals using triple number lines	3	4
Use algorithms to change forms between decimals, fractions, and percents	5	6
Find a percent that corresponds to a given amount of a whole in context	7	8
Find common percents of a given whole using double number lines	9	
Find common percents of a given whole using algorithms	10	
Find an amount given a percent and a whole	11	12
Find a whole of a given a percent and amount using double number lines	13	
Find a whole of a given a percent and amount using algorithms	14	16
Find a whole of a given a percent and amount in context	15	16
Determine a new percent of a whole and multiplicative factor given a percent increase or decrease	17	
Find a new amount given a whole and a percent increase or decrease	18	19

UNIT 8 – MEDIA LESSON

SECTION 8.1: INTRODUCTION TO PERCENTS

A **percent** represents a ratio with a denominator of 100. Notice that if we think of percent as two words, “**per cent**” we can think of our study of rates that used the word “**per**”, and “**cent**” meaning 100 such as 100 **cents** in a dollar or 100 years in a **century**.

In this section, we will introduce percents and learn why they are useful. We will represent percents in multiple ways to connect the idea of percent with other representations we have learned such as ratio, fraction, and decimal.



Problem 1

MEDIA EXAMPLE – Why Percents?

Sylvia has taken 3 tests in her math class this semester. The table below shows the number of points she earned out of the total number of possible points.

- a) Based on the information in the first two rows, on which test do you think Sylvia earned her best score? Which test do you think was her worst score? Explain.
- b) Complete the missing rows in the table.

	Test 1	Test 2	Test 3
Points Earned	18	16	39
Total Points Possible	25	20	50
Ratio of Points Earned to Total Points Possible			
Equivalent Ratio out of 100			
Equivalent Percent			

- c) Based on the information in the last two rows, on which test do you think Sylvia earned her best score? Which test do you think was her worst score? Is this different from your original analysis in part a? Explain.
- d) Why do you think it is useful to use percents to compare ratios? Explain.

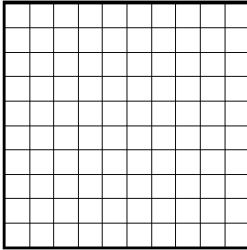


Problem 2

MEDIA EXAMPLE – Percents, Decimals, and Fractions with Grids

Shade the indicated quantity and rewrite in the indicated forms. Write the fraction with a denominator of 100 and also a simplified fraction when appropriate.

a) 57 hundredths

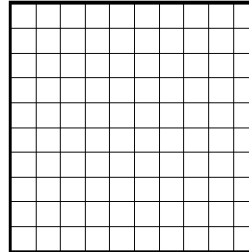


Decimal: _____

Fraction: _____

Percent: _____

b) 7 for every 20



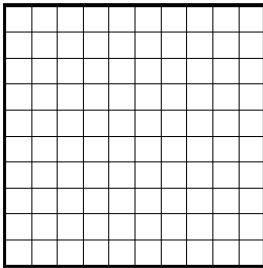
Decimal: _____

Simplified Fraction: _____

Fraction out of 100: _____

Percent: _____

c) 6 tenths and 5 hundredths



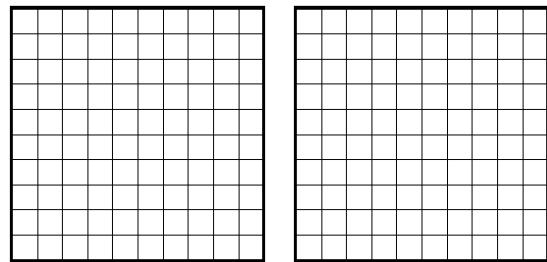
Decimal: _____

Fraction out of 100: _____

Simplified Fraction: _____

Percent: _____

d) 120 per 100



Decimal: _____

Fraction out of 100: _____

Simplified Fraction: _____

Percent: _____

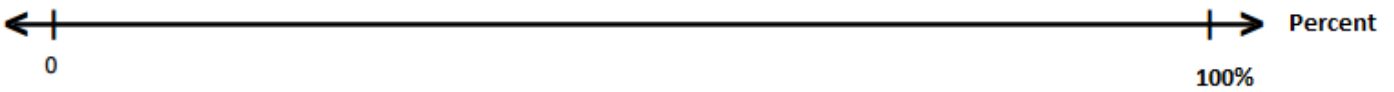
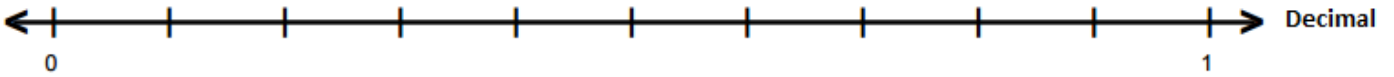
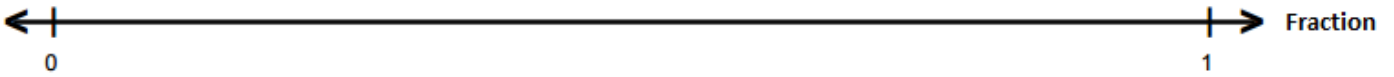


Problem 3

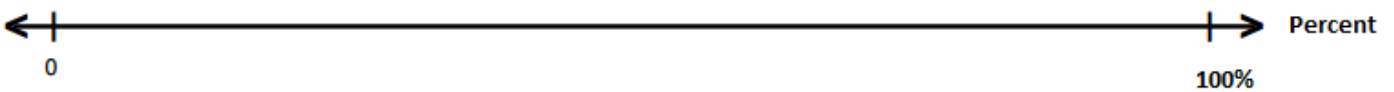
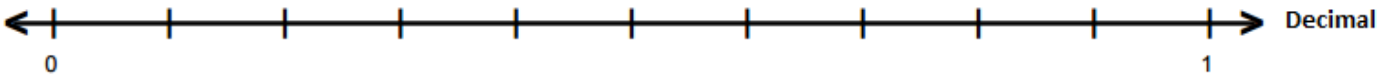
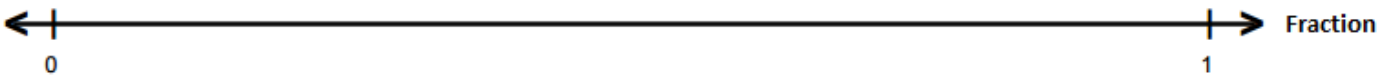
MEDIA EXAMPLE – Percents, Decimals, and Fractions with Number Lines

Plot the fraction, decimal, and percent on the triple number lines. Label the amounts in each form.

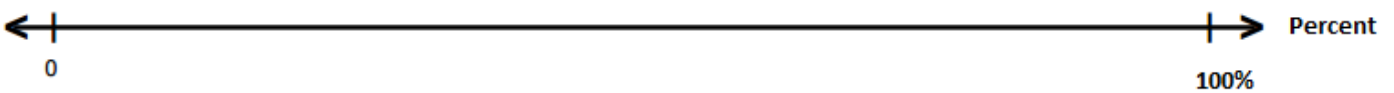
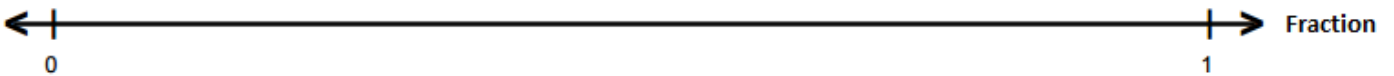
a) $\frac{1}{2}$



b) 0.2



c) 75%



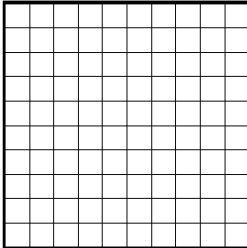


Problem 4

YOU TRY – Percents, Decimals, and Fractions with Grids

Shade the indicated quantity and rewrite in the indicated forms. Write the fraction with a denominator of 100 and also a simplified fraction when appropriate.

a) 3 for every 25



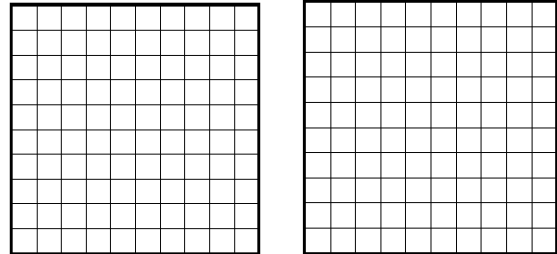
Decimal: _____

Fraction out of 100: _____

Simplified Fraction: _____

Percent: _____

b) 150 per 100



Decimal: _____

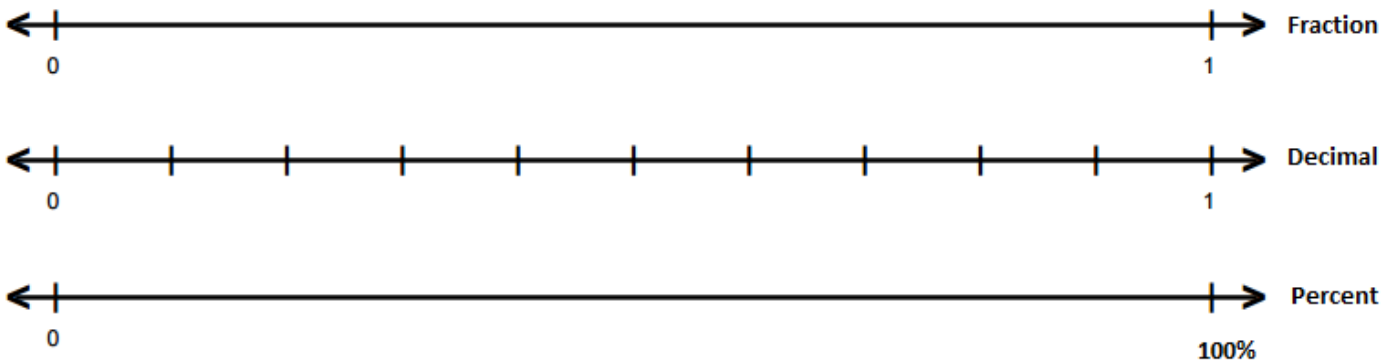
Fraction out of 100: _____

Simplified Fraction: _____

Percent: _____

c) Plot the fraction, decimal, and percent on the triple number lines. Label the amounts in each form.

40%



RESULTS – Changing Forms Between Decimals, Fractions, and Percents

Below is an overview of our results on transforming numbers between varying forms of ratio, fraction, decimal, and percent.

FACT: Since 100% means 100 per 100,

$$100\% = \frac{100}{100} = 1$$

Recall that multiplying or dividing by 1 does not change the value of a number. *So we can multiply or divide by 100% to create an equivalent form of the number.* We will use this idea to change ratios, fractions, or decimals to percents or vice versa.

RULES:

1. To change a ratio, fraction, or decimal to a percent, multiply by 100%.
2. To change a percent to a ratio, fraction, or decimal, divide by 100%

EXAMPLES:

1. Rewrite $\frac{2}{5}$ as a percent.

$$\frac{2}{5} \times 100\% = \frac{2 \times 100}{5} \% = \frac{200}{5} \% = 40\%$$

2. Rewrite 0.76 as a percent.

$$0.76 \times 100\% = 76\%$$

3. Rewrite 80% as a fraction.

$$80\% = \frac{80\%}{100\%} = \frac{80}{100} = \frac{4}{5}$$

4. Rewrite 37% as a decimal.

$$37\% = \frac{37\%}{100\%} = \frac{37}{100} = 0.37$$




Problem 5

MEDIA EXAMPLE – Changing Forms between Decimals, Fractions, and Percents

Complete the table below by writing each given value in the indicated equivalent form.

Fraction	Decimal	Percent
	0.85	
		32%
$\frac{17}{25}$		
	1.237	
		64.25%
$\frac{2}{3}$		
		$17\frac{3}{5}\%$
		0.42%

 Problem 6	YOU TRY – Changing Forms between Decimals, Fractions, and Percents
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Complete the table below by writing each given value in the indicated equivalent form.

Fraction	Decimal	Percent
		23.5%
$\frac{13}{20}$		
	0.783	
		126%
	1	
		200%

SECTION 8.2: FINDING PERCENTS GIVEN AN AMOUNT AND A WHOLE

In the last section, we wrote equivalent forms of ratios, fractions, decimals, and percents in multiple ways. In this section, we will look at application problems where we need to interpret the given information to find a ratio and write the ratio as a percent. In general, we found that

$$\frac{\text{amount}}{\text{whole}} \cdot 100 = \text{percentage}$$

We will need to use the context of the question to determine what given values are the amount and the whole and transform the result into a percent.



Problem 7

MEDIA EXAMPLE – Finding a Percent Given an Amount and a Whole

Write the corresponding scenario as a ratio of an amount multiplicatively compared to a whole. Then write the corresponding percent. Round any percents to two decimal places as needed.

- a) Chanelle is driving to Washington on a 20 hour road trip. So far, she has driven for 8 hours. What percent of the hours has Chanelle already driven?

Write the corresponding ratio for this situation. Include units in your ratio.

$$\text{Ratio: } \frac{\textit{amount}}{\textit{whole}} =$$

Write the percent that corresponds to this ratio:

Write your answer as a complete sentence:

What percent of the trip remains? Explain.

- b) Christian bought a \$60 sweater. The tax on the sweater was \$4.95 for a total cost of \$64.95. What percent of the cost of the sweater was the tax?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

$$\text{Ratio: } \frac{\textit{amount}}{\textit{whole}} =$$

Percent:

Write your answer as a complete sentence:

What is the ratio and percent of the total cost including tax to the cost of the sweater? Include units in your ratio.

$$\text{Ratio: } \frac{\textit{amount}}{\textit{whole}} =$$

Percent:

How does this percent compare to the percent of tax? What relationship do you notice?

Unit 8 – Media Lesson

- c) Carol went shopping for a cell phone. The price was listed as \$400. She had a coupon for \$50 off. What percent of the original price is the coupon savings amount?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

Write your answer as a complete sentence:

What is the ratio and percent of the reduced cost of the phone to the original cost of the phone? Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

How does this percent compare to the percent of the coupon? What relationship do you notice?



Problem 8

YOU TRY – Finding a Percent Given an Amount and a Whole

Write the corresponding scenario as a ratio of an amount multiplicatively compared to a whole. Then write the corresponding percent. Round any percents to two decimal places as needed.

- a) Travis bought 60 cans of soda for a party. He bought 24 cans of diet cola and 36 cans of regular cola. What percent of the soda that Travis bought is diet cola?

Write the corresponding ratio for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Write the percent that corresponds to this ratio:

Write your answer as a complete sentence:

What percent of the soda is regular cola? Explain.

- b) Faith was selling her old math book online. The book originally cost her \$150. Based on her research, she can sell the book for \$67.50. What percent of the original cost of the book can Faith earn back by selling her book?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

Write your answer as a complete sentence:

What percent of the original cost of the book will Faith *lose* by selling her book? Explain.

SECTION 8.3: FINDING AN AMOUNT GIVEN A PERCENT AND A WHOLE

In the previous sections, we were given ratios, fractions, or decimals and wrote them as percentages. In this section, we will learn how to find *a percent of a quantity*. A percent is always referring to a percent of *something*. We typically call this something *the whole*. For example,

1. You earned 83% of the points on a test. The whole refers to the total possible points on the test.
2. You gained 3% of your body weight last year. The whole refers to your body weight last year.
3. You are charged 7% tax on a purchase. The whole refers to the cost of your purchase.
4. The interest rate on your mortgage is 4.35%. The whole is how much you owe on your mortgage.

Recall,

$$\frac{\textit{amount}}{\textit{whole}} \cdot 100 = \textit{percentage}$$

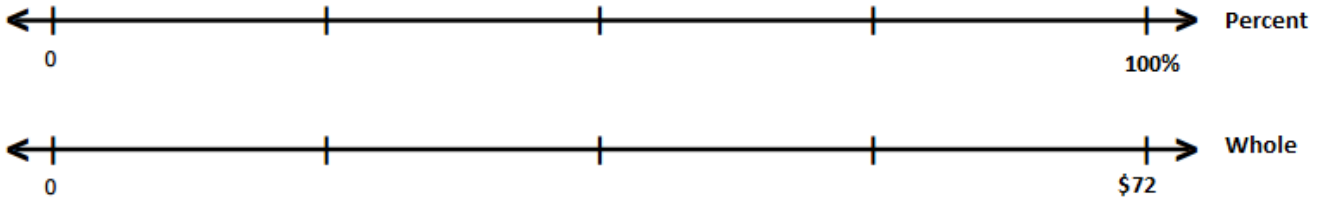
As you work through this section, make certain to focus on which quantities represents the whole, the amount multiplicatively compared to the whole, and the percent.

Problem 9	MEDIA EXAMPLE – Finding Common Percents of a Number
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Complete the following problems by finding common percents of the given wholes.

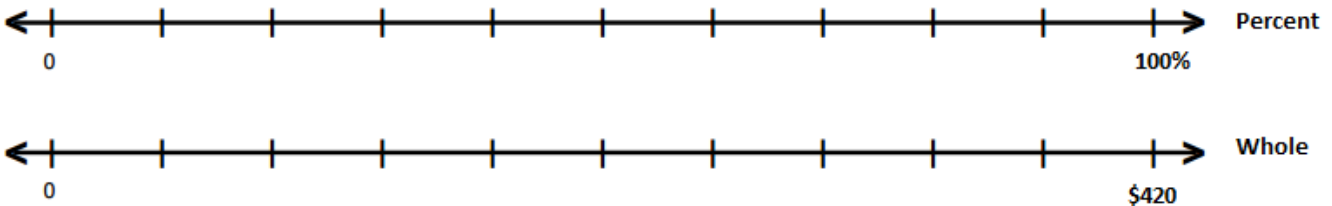
- a) Miguel is saving up for a birthday present for his sister. The gift costs \$72 and her birthday is in four weeks. He decides to save an equal amount each week.

Label the tick marks below to indicate the different percentages and the corresponding amount of money saved for each percent value.



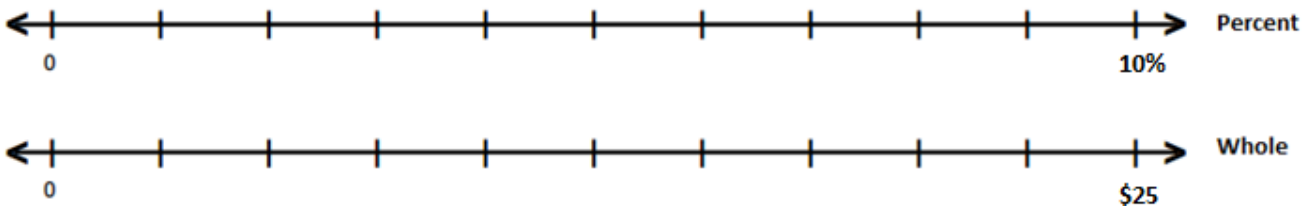
- b) Josh is a server at a local restaurant. He waits on a party of 10 people and their bill is approximately \$420. He wants to figure out how much he'll be tipped if they leave him 10%, 15%, 20% or 25% of their total bill.

Label the tick marks on the *percent* number line to indicate 10%, 15%, 20% and 25%. Then use the *whole* number line to determine the corresponding amounts of money in dollars that Josh may be tipped.



- c) Robert's parents are charging him 1% interest per month on a \$250 loan. The loan is for four months. He wants to know how much he will be charged in interest over the four month period. **He starts by finding that 10% of \$250 is \$25.**

Label the tick marks on the *percent* number line to indicate 1%, 2%, 3% and 4%. Then use the *whole* number line to determine the corresponding amounts of interest Robert will pay his parents.



- d) Marissa works at a clothing store. They are having a sale. Each rack is labeled with the percentage off for items on the rack. She needs to make a chart to show customers the corresponding dollar amount off for certain percentages off.

Fill in the chart below that Marissa is making for customers.

Amount of Discount Based on Item Price and Percent Off		Regular Item Price				
		\$10	\$20	\$30	\$40	\$50
Percent Off	10% off					
	20% off					
	30% off					
	40% off					

RESULTS – Finding an Amount Given a Percent and a Whole

Below is an overview of our results on finding a percent of a whole.

FACT: $n\%$ means n per 100 or n for every 100.

When we find $n\%$ of a number, we can think of **cutting the whole into 100 equal pieces** (each of size 1%) and then **taking n copies** of 1% to attain $n\%$. Cutting into 100 pieces is equivalent to dividing by 100. Taking n copies is equivalent to multiplying by n .

For example, 16% means 16 per 100 or 16 for every 100.

$$16\% = \frac{16}{100} = 0.16$$

We can think of $\frac{16}{100}$ as 16 copies of $\frac{1}{100}$ or 16 copies of 1%.

1. The 100 in the denominator **cuts the whole into 100 pieces** of size $\frac{1}{100}$ or 1%.
2. The 16 in the numerator **takes 16 copies** of these pieces of size $\frac{1}{100}$ or 1%.
3. So multiplying the whole by $\frac{16}{100}$ or equivalently, 0.16, finds 16% of the whole.

RULE: To find a percent of a whole (the amount),

1. Divide the percent by 100.
2. Multiply the whole by the equivalent fraction or decimal.
3. A general formula is $whole \cdot \frac{percent}{100} = amount$



Problem 10

MEDIA EXAMPLE – Finding an Amount Given a Percent and a Whole

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

- a) Find 63% of 270.
- b) What is 23.4% of 18?
- c) $4\frac{7}{8}\%$ of \$32,000 is what number?
- d) Find 137% of 2.83.
- e) What is 0.87% of 92?
- f) 27% of $\frac{3}{4}$ is what number?
- g) What is $9\frac{2}{3}\%$ of $38\frac{1}{3}$?



Problem 11

MEDIA EXAMPLE – Finding an Amount Given a Percent and a Whole Applications

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

- a) Joey is taking a road trip from New York to Washington D.C. The trip is 226 miles. So far, he has driven 43% of the trip.

How many miles has Joey driven so far? Write your answer as a complete sentence.

How many miles does Joey have left to travel?

What percent of miles does Joey have left to travel?

- b) Erica went shopping in Tempe and spent \$213.53 on new work clothes. The sales tax rate in Tempe is 8.1%.

How much tax will Erica have to pay? Write your answer as a complete sentence.

What is the total cost of her purchase including tax?

What percent is the total cost of her purchase compared to the total cost without tax?

- c) Ahmed went shopping for a tablet. The regular price was listed as \$370. The store was having a 20% off sale.

How much will Ahmed save because of the sale? Write your answer as a complete sentence.

What is the reduced price of the tablet after the discount?

What percent is the reduced price of his purchase compared to the original price?



Problem 12

YOU TRY – Finding an Amount Given a Percent and a Whole

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

a) Find 27% of 302.

b) What is 6.7% of 78?

c) Find 114% of 4.9.

d) 87% of $\frac{7}{5}$ is what number?

e) Taylor wants to buy a new Fender guitar. The regular price is listed as \$1200. The online merchant is having a sale for 35% off all purchases over \$1000.

How much will Taylor save because of the sale? Write your answer as a complete sentence.

What is the reduced price of the guitar after the discount?

What percent is the reduced price of the guitar compared to the original price of the guitar?

SECTION 8.4: FINDING THE WHOLE GIVEN A PERCENT AND AN AMOUNT

In this Unit, we have used the idea that

$$\frac{\text{amount}}{\text{whole}} \cdot 100 = \text{percentage}$$

We have solved problems where we were given the amount and the whole and we found the corresponding percentage. We have also solved problems where we knew the whole and percentage and found the amount.

In this section, we will be given a percent and the amount and will need to find the whole. As you work through this section, make certain to focus on which quantities represents the whole, the amount multiplicatively compared to the whole, and the percent.

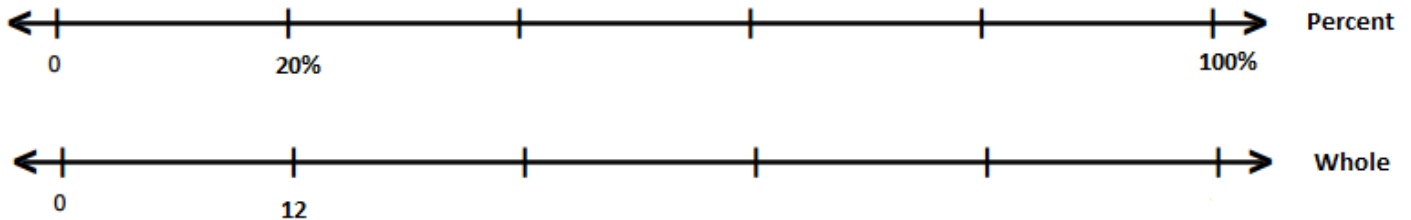


Problem 13

MEDIA EXAMPLE – Finding the Whole Given a Common Percent and an Amount

- a) The Geology Club is taking a trip to Hawaii to explore volcanos. Twenty percent of the club can make the trip which is a total of 12 students. How many students are in the Geology club in total?

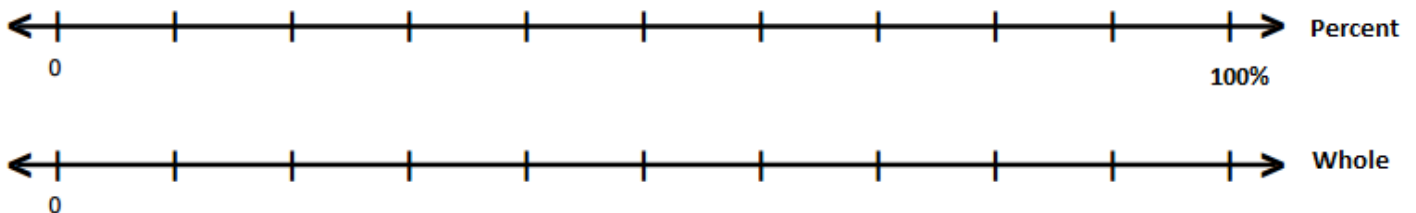
Use the double number line below to find the total number of members in the Geology Club.



Symbolic Representation:

- b) Finn bought a mountain bike for 30% off. If he paid \$245 for the bike, what was the original price before the sale?

Use the double number line below to find the original price of the bike.

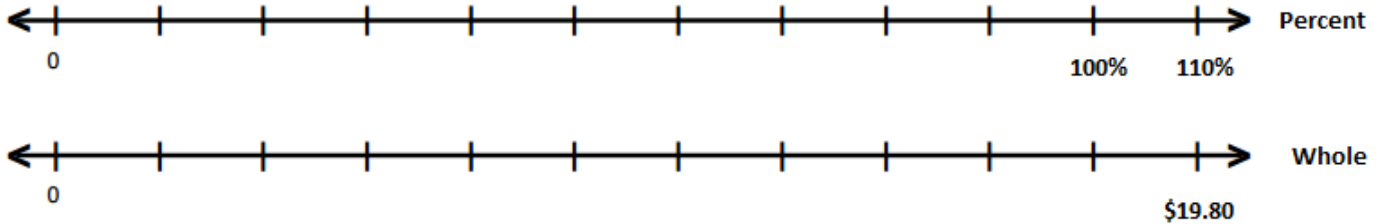


Symbolic Representation:

Unit 8 – Media Lesson

- c) Don went skiing and rented skis and boots. The total cost for the rental including tax was \$19.80. If the tax rate was 10%, how much did the rental cost before tax?

Use the double number line below to find the rental cost before tax.



Symbolic Representation:

RESULTS – Finding the Whole Given a Percent and an Amount

Below is an overview of our results on finding the whole given a percent and an amount.

We are given an amount and its corresponding percentage of the whole. For example,

15 is 30% of what number?

15 is the amount

30% is the percentage 15 is of the whole

The whole, or 100% is unknown.

When we want to find the whole, we want to find 100% of the known amount. In this example, we can think of **cutting the amount 15 into 30 equal pieces** (each of size 1%) and then **taking 100 copies** of 1% to attain 100%. Cutting into 30 pieces is equivalent to dividing by 30. Taking 100 copies is equivalent to multiplying by 100.

So for this example,

$$\text{The whole is } 15 \times \frac{100}{30} = \frac{15 \times 100}{30} = \frac{1500}{30} = 50$$

1. The 30 in the denominator **cuts the amount into 30 pieces** of size $\frac{1}{30}$ or 1%.
2. The 100 in the numerator **takes 100 copies** of these pieces of size $\frac{1}{30}$ or 1%.
3. So multiplying the amount by $\frac{100}{30}$ finds the whole or 100%.

RULE: To find the whole given an amount and its corresponding percent, $n\%$,

$$\text{whole} = \text{amount} \times \frac{100}{n}$$



Problem 14

MEDIA EXAMPLE – Finding the Whole Given a Percent and an Amount

Find the indicated wholes. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

a) 45 is 35% of what number?

b) 4.32% of what number is 7.5?

c) $\frac{11}{7}$ is 22% of what number?

d) 134.7% of what number is 2300?



Problem 15

MEDIA EXAMPLE – Finding the Whole Given a Percent and an Amount Application

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

Dave has worked for the same employer for 5 years. His current salary is \$73,500 which is 122.5% of his starting salary.

a) What was Dave's starting salary?

b) If Dave received equal increases in pay every year, what was his raise per year?



Problem 16

YOU TRY – Finding an Amount Given a Percent and a Whole

Find the indicated wholes. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.


- a) 18.7% of what number is 29.17?
- b) $\frac{46}{13}$ is 23% of what number?
- c) Amelia earned a scholarship and only needed to pay 47.3% of her tuition. If she paid \$638.55, what was the full cost of her tuition before the scholarship?

SECTION 8.5: PERCENT INCREASE AND DECREASE

In this last section, we will learn about percent increase and percent decrease. We have already seen some problems that can be considered to fall in this category. For example,


1. A sale of 20% at a store is a **percent decrease**. The original price is 100%, we *subtract off* 20% of the original price, and the sale price is 80% of the original price.
2. The total amount of an item including 7% tax is a **percent increase**. The amount without tax is 100%, we *add on* 7% of the amount for tax, and the total price with tax is 107% of the amount without tax.

It is important to distinguish between the percent you are adding on (such as tax) or subtracting off (such as a discount) with the value *after* you have made these adjustments. A 50% increase means the *new value* is $100\% + 50\% = 150\%$ of the original value or 1.5 times as large as the original value. You are not only finding 50% of the whole. You are *increasing* the whole by this 50%. We call 1.5 in this example the **multiplicative factor** since it is the number we multiply the original value by to obtain the new value. Keep this idea in mind when you solve percent increase and decrease problems as compared to problems where you are only finding a percent of a number.

 Problem 17	MEDIA EXAMPLE – Multiplicative Factors and Percent Increase and Decrease
--	--

Complete the table below. Write the multiplicative factor as a ratio over 100 and a decimal.

Percent Change	New Percent of Whole	Multiplicative Factor
25% increase		
13% decrease		
4.25% increase		
12.2% decrease		
115% increase		
99% decrease		

 Problem 18	MEDIA EXAMPLE – Percent Increase and Decrease
--	---

Determine the new amounts given the percent change. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

- 150 is increased by 12%. What is the new amount?
- 3000 is decreased by 27.5%. What is the new amount?
- 1000 is decreased by 50%. The resulting amount is then increased by 50%. What is the new amount?
- 600 is doubled. What is the new amount? What is the corresponding percent increase?
- 500 is decreased by half. What is the new amount? What is the corresponding percent decrease?



Problem 19

YOU TRY – Percent Increase and Decrease

- a) Complete the table below. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

Percent Change	New Percent of Whole	Multiplicative Factor
8.75% increase		
27.4% decrease		
132% increase		

- b) 37 is increased by 43%. What is the new amount?

- c) 3000 is decreased by 65.4%. What is the new amount?



Problem 20

MEDIA EXAMPLE – Percent Increase and Decrease Applications

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator

- a) Julianna changed careers. Her old salary was \$53,000 a year. Her new salary is 26% more per year. What is her new salary?
- b) Jordan lost 17% of his body weight over the school year. If he originally weighed 247 pounds, what is his new weight?

- c) The CPI Inflation Calculator measures the buying power of a dollar relative to different years. According to the [Bureau of Labor Statistics](#), \$1.00 in 1985 has the same buying power as \$2.23 in 2016.

What is the multiplicative factor and percent increase in buying power between 1985 and 2016?

- d) If Joe's salary in 1985 was \$25,000 a year, how much would he need to make now just to keep up with inflation?
- e) If Joe's salary is \$62,000 a year in 2016, how much more is he making in addition to the inflation adjustment?



Problem 21

YOU TRY – Percent Increase and Decrease

Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator

In 1970, the minimum wage was \$1.60 per hour. According to the [Bureau of Labor Statistics](#), \$1.00 in 1970 has the same buying power as \$6.19 in 2016.

- a) What is the multiplicative factor and percent increase in buying power between 1970 and 2016?
- b) What should the minimum wage be in 2016 adjusted for inflation to be comparable to the minimum wage in 1970?

Summary of Percentage Relationships

There are three quantities in percent problems. Two are given and one you are trying to find. They are the amount, the whole, and the percent.

1. The **percent** is labeled with a percent symbol, %, or the word percent.
2. The **whole** usually follows the word “of”. This is because you are finding a *percent of something*, namely, the whole.
3. The **amount** doesn’t necessarily have any of these easy to identify indicators. So it’s simplest to find the percent and whole, and then you know the remaining quantity is the amount.

Once you know what you are trying to find, use the appropriate formula. Remember why these **relationships** work from the media lessons to help you remember them!

$$1. \text{ percent} = \frac{\text{amount}}{\text{whole}} \cdot 100$$

$$2. \text{ amount} = \text{whole} \cdot \frac{\text{percent}}{100}$$

$$3. \text{ whole} = \text{amount} \cdot \frac{100}{\text{percent}}$$

Percent Increase: You are given a quantity and a percent of the quantity that it has increased by. You want to find the new amount.

$$\text{new amount} = \text{original amount} \cdot \frac{100 + \% \text{ increase}}{100}$$

Percent Decrease: You are given a quantity and a percent of the quantity that it has decreased by. You want to find the new amount.

$$\text{new amount} = \text{original amount} \cdot \frac{100 - \% \text{ increase}}{100}$$

Percent Change: You are given an original amount and a new amount. You want to find the percent change between the original and new amounts.

$$\text{percent change} = \frac{\text{new amount}}{\text{original amount}} \cdot 100 - 100$$

If the result is positive, the percent change is an increase.

If the result is negative, the percent change is a decrease. (Don’t include the negative in your answer.)

UNIT 8 – PRACTICE PROBLEMS

For 1 – 3: Brad is on the basketball team and is practicing free throws. He records his total number of attempts and his number of successful free throws for 3 days. The results are in the table below.

- Based on the information in the first two rows, on which day do you think Brad performed best? Which day do you think was his worst day? Explain.

- Complete the missing rows in the table.

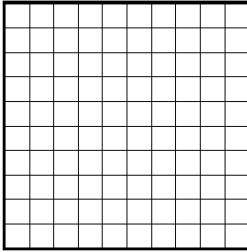
	Day 1	Day 2	Day 3
Successful Throws	24	15	28
Total Attempts	30	20	40
Ratio of Successful Throws to Total Attempts			
Simplified Ratio			
Equivalent Ratio out of 100			
Equivalent Percent			

- Based on the information in the last two rows, on which day do you think Brad performed best? Which day do you think was his worst day? Explain. Is this different from your original analysis in part *a*? Explain.

Unit 8 – Practice Problems

4. Shade the indicated quantity and rewrite in the indicated forms. Write the fraction with a denominator of 100 and also a simplified fraction when appropriate.

b) 63 hundredths

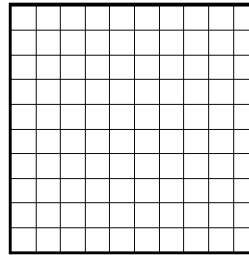


Decimal: _____

Fraction: _____

Percent: _____

b) 11 for every 25



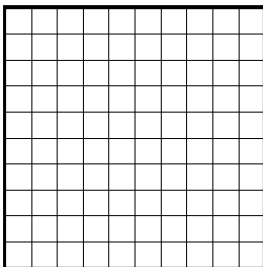
Decimal: _____

Simplified Fraction: _____

Fraction out of 100: _____

Percent: _____

d) 3 tenths and 2 hundredths



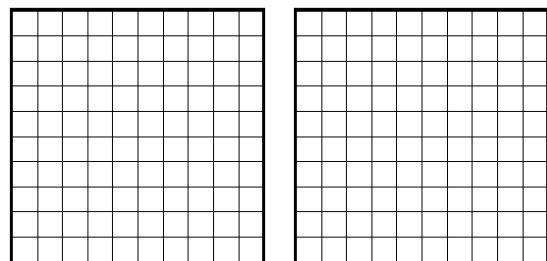
Decimal: _____

Fraction out of 100: _____

Simplified Fraction: _____

Percent: _____

d) 150 per 100



Decimal: _____

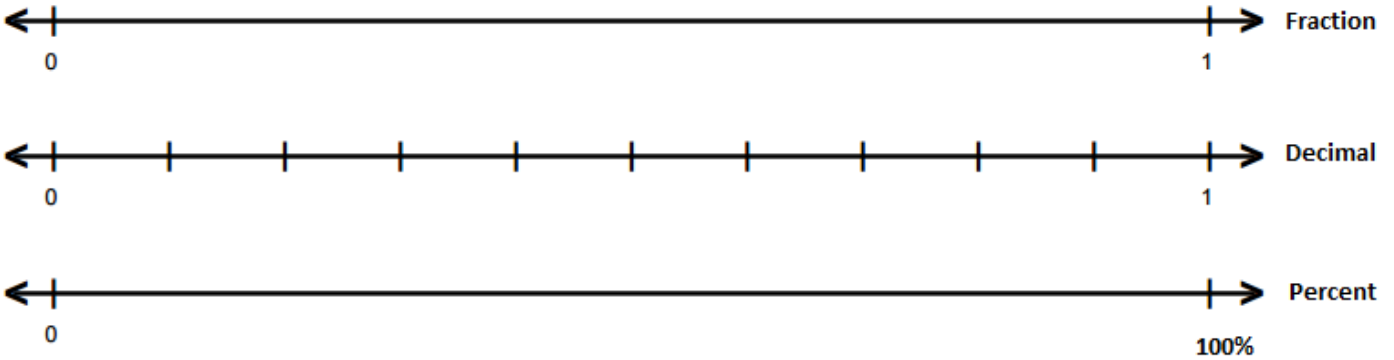
Fraction out of 100: _____

Simplified Fraction: _____

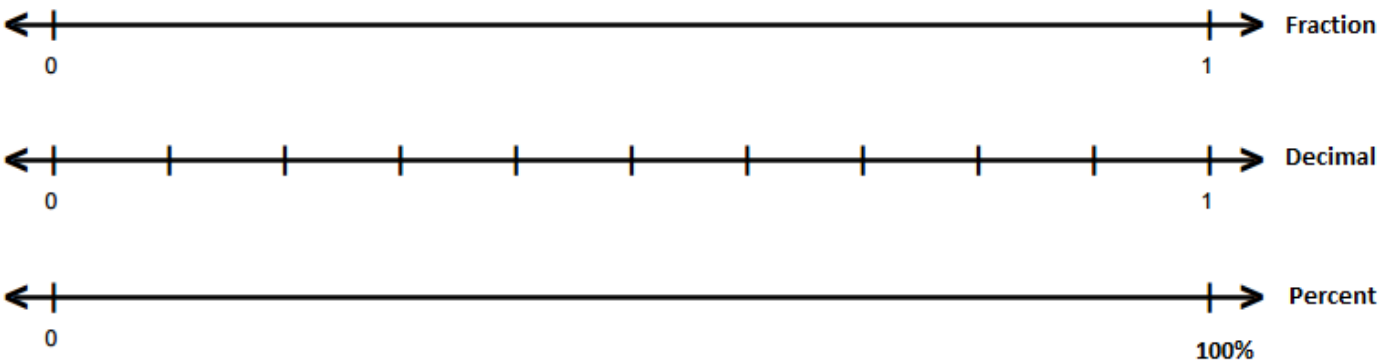
Percent: _____

5. Plot the fraction, decimal, and percent on the triple number lines. Label the amounts in each form.

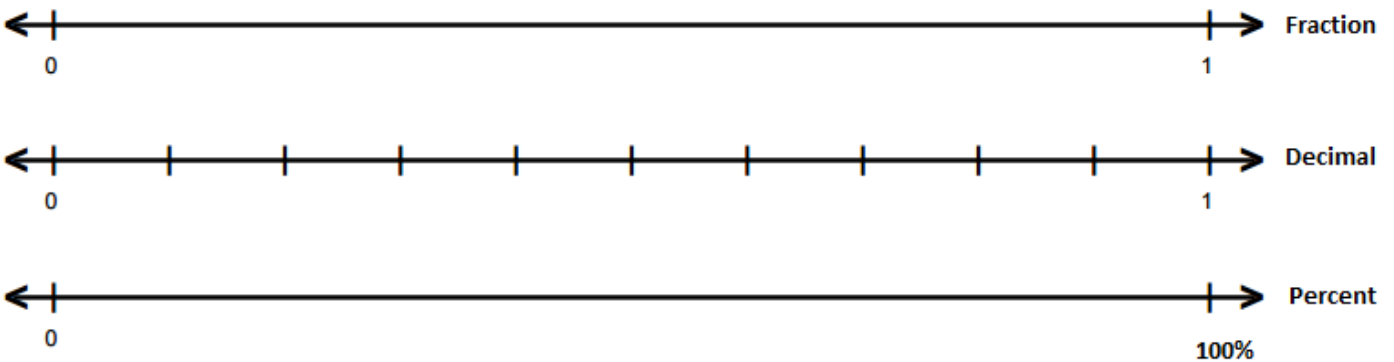
a) $\frac{3}{5}$



b) 0.45



c) 35%



Unit 8 – Practice Problems

6. Complete the table below by writing each given value in the indicated equivalent form.

Fraction	Decimal	Percent
	0.42	
		44%
$\frac{13}{20}$		
	2.34	
		32.7%
$\frac{4}{9}$		
		$8\frac{1}{2}\%$
		0.27%

For 7 – 11: Write the corresponding scenario as a ratio of an amount multiplicatively compared to a whole. Then write the corresponding percent. Round any percents to two decimal places as needed.

7. Maxine plans to exercise for 75 minutes. So far, she has exercised for 45 minutes. What percent of the minutes has Maxine already exercised?

Write the corresponding ratio for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Write the percent that corresponds to this ratio:

Write your answer as a complete sentence:

What percent of her exercising session remains? Explain.

8. John bought an \$80 book. The tax on the book was \$6.64 for a total cost of \$86.64. What percent of the cost of the book was the tax?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

Write your answer as a complete sentence:

What is the ratio and percent of the total cost including tax to the cost of the book without tax? Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

How does this percent compare to the percent of tax? What relationship do you notice?

Unit 8 – Practice Problems

9. Francisco bought a new skateboard. The price was listed as \$700. The website was having a deal for \$105 off the listed price. What percent of the original price is the discount?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

Write your answer as a complete sentence:

What is the ratio and percent of the reduced cost of the skateboard to the original cost of the skateboard? Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

How does this percent compare to the percent of the discount? What relationship do you notice?

10. Kirsten bought 15 bags of chips for a party. She bought 5 bags of low fat chips and 10 bags of regular chips. What percent of the bags of chips that Kirsten bought were low fat?

Write the corresponding ratio for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Write the percent that corresponds to this ratio:

Write your answer as a complete sentence:

What percent of the bags of chips that Kirsten bought were regular? Explain.

11. Hope was selling a pair of shoes online that she never wore. The shoes originally cost her \$75. Based on her research, she can sell the shoes for \$30. What percent of the original cost of the shoes can Hope earn back by selling her shoes?

Write the corresponding ratio and percent for this situation. Include units in your ratio.

Ratio: $\frac{\textit{amount}}{\textit{whole}} =$

Percent:

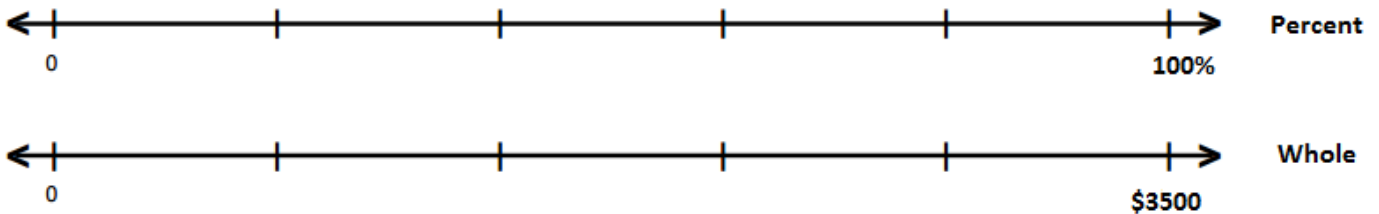
Write your answer as a complete sentence:

What percent of the original cost of the shoes will Hope *lose* by selling her shoes? Explain.

For 12 – 13: Complete the following problems by finding common percents of the given wholes.

12. Molly is saving up for a car. The used car she wants is \$3500. She decides to save an equal amount each month for 5 months.

Label the tick marks below to indicate the different percentages and the corresponding amount of money saved for each percent value over 5 months.



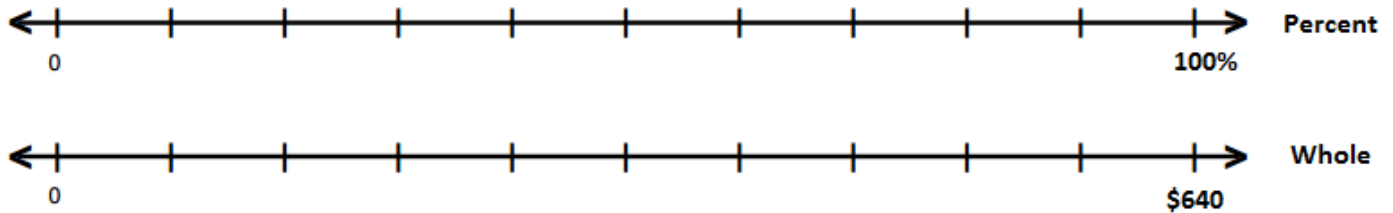
Fill in the table.

Months Saved	Percent	Total Amount Saved
Month 0	0%	\$0
Month 1		
Month 2		
Month 3		
Month 4		
Month 5		

Unit 8 – Practice Problems

13. Noni is a server for a catering company. She waits on a big party and their bill is approximately \$640. She wants to figure out how much she'll be tipped if they leave her 10%, 15%, 20% or 25% of their total bill.

Label the tick marks on the *percent* number line to indicate 10%, 15%, 20% and 25%. Then use the *whole* number line to determine the corresponding amounts of money in dollars that Noni may be tipped.



For 14 – 20: Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

14. Find 34% of 892.

15. What is 16.8% of 39?

16. $3\frac{5}{8}\%$ of \$43,000 is what number?

17. Find 168% of 5.72.

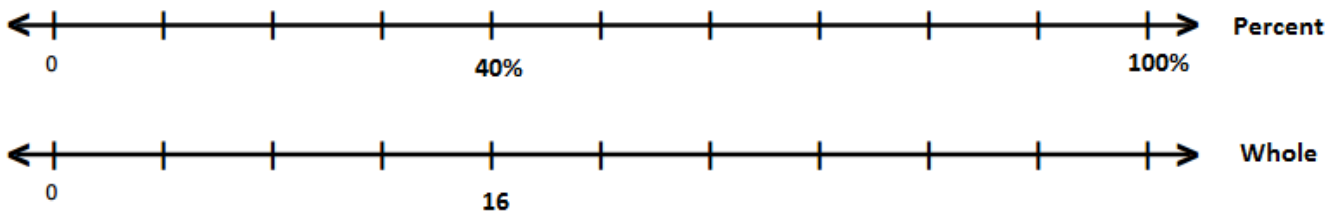
18. What is 0.26% of 345?

19. 25% of $\frac{12}{5}$ is what number?

20. What is $4\frac{1}{8}\%$ of $77\frac{5}{9}$?

21. The Surf Club is taking a trip to Bali. Forty percent of the club can make the trip which is a total of 16 members. How many members are in the Surf club in total?

Use the double number line below to find the total number of members in the Surf Club.

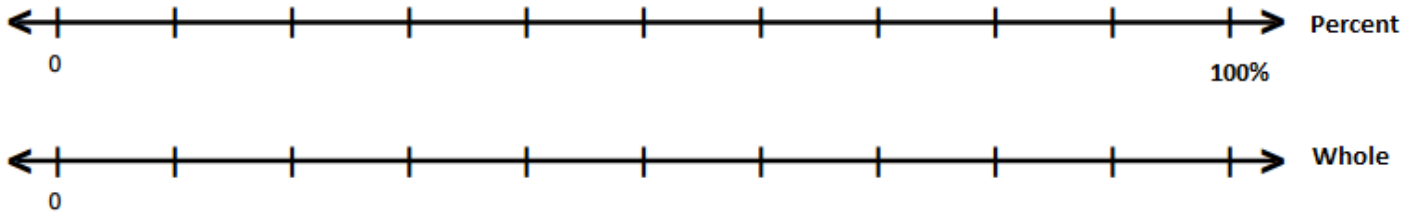


Symbolic Representation:

Unit 8 – Practice Problems

22. Cedric bought an airplane ticket for 20% off. If he paid \$440 for the ticket, what was the original price before the discount?

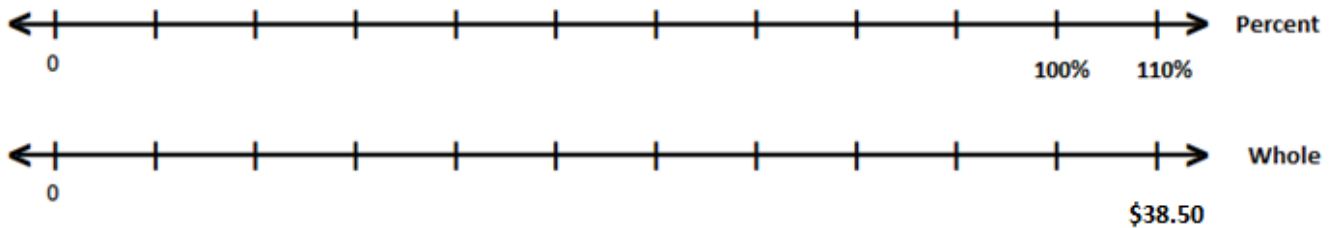
Use the double number line below to find the original price of the bike.



Symbolic Representation:

23. Nancy and her friend went out for dinner. The total cost for dinner including tax was \$38.50. If the tax rate was 10%, how much did dinner cost before tax?

Use the double number line below to find the rental cost before tax.



Symbolic Representation:

For 24 – 27: Find the indicated wholes. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

24. 65 is 45% of what number?

25. 6.17% of what number is 12.3?

26. $\frac{12}{13}$ is 24% of what number?

27. 254.2% of what number is 1650?

28. Complete the table below. Write the multiplicative factor as a ratio over 100 and a decimal.

Percent Change	New Percent of Whole	Multiplicative Factor
35% increase		
25% decrease		
6.21% increase		
9.7% decrease		
135% increase		
1% decrease		
100% increase		
200% increase		

Unit 8 – Practice Problems

29 – 33: Determine the new amounts given the percent change. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator.

29. 87 is increased by 14%. What is the new amount?

30. 2000 is decreased by 33.7%. What is the new amount?

31. 1000 is decreased by 40%. The resulting amount is then increased by 60%. What is the new amount?

32. 800 is tripled. What is the new amount? What is the corresponding percent increase?

33. 500 is decreased by one quarter. What is the new amount? What is the corresponding percent decrease?

For 34 – 35: Find the indicated amounts. Round your final answer to two decimal places as needed. Feel free to use your calculator for your computations. However, make sure to write down the expression that you put in the calculator

34. Hannah changed careers. Her old salary was \$62,000 a year. Her new salary is 7.9% more per year. What is her new salary?

35. Scott lost 6.3% of his body weight over the school year. If he originally weighed 212 pounds, what is his new weight?

For 36 – 38: The CPI Inflation Calculator measures the buying power of a dollar relative to different years. According to the [Bureau of Labor Statistics](#), \$1.00 in 1960 has the same buying power as \$8.12 in 2016.

36. What is the multiplicative factor and percent increase in buying power between 1960 and 2016?
37. If Joe's salary in 1960 was \$9,000 a year, how much would he need to make in 2016 just to keep up with inflation?
38. If Joe's salary is \$78,000 a year in 2016, how much more is he making in addition to the inflation adjustment?
39. Determine the missing number in each of the following. Round to two decimals.
- a) 6% of what number is 12?
 - b) 82% of what number is 116?
 - c) 123% of what number is 25?
 - d) 20 is 0.18% of what number?
 - e) 120 is 125% what number?
40. Determine the missing number in each of the following. Round to two decimals.
- a) What is 5% of 25?
 - b) 0.01% of 12 is what number?
 - c) 123% of 100 is what number?
 - d) 12.56% of 72 is what number?
 - e) 50% of 127 is what number?

Unit 8 – Practice Problems

41. Determine the missing number in each of the following. Round to two decimals.

a) What % of 25 is 5?

b) 12 is what percent of 40?

c) What percent of 32 is 48?

d) 15 is what percent of 23?

e) 0.25 is what percent of 3?

42. Determine the percent increase or decrease for the change for each of the following:

a) 12 to 15

b) 22 to 18

c) 30 to 60

d) 120 to 90

e) 90 to 100

For 43 – 47: Solve each of the following application problems using the methods from this unit.

43. In a recent poll, 28% of the 750 individuals polled indicated that they would vote purely Democratic in the next election. How many of the individuals would vote a straight Democratic ticket?

44. If you decrease your daily intake of calories from 2500 to 1750, by what percent do your daily calories decrease?
45. On a recent trip to the store, you bought \$75.25 worth of goods and paid a total of \$82.02. What was the rate of sales tax that you paid?
46. If you invest \$5000 at simple interest of 8% per year for 6 years, how much money will you earn from interest? How much money will you have at the end of 6 years?
47. In the U.S. Civil War, 750,000 people were estimated to have died. If that number represented 2.5% of the U.S. population of the day, how many people lived in the U.S. during the Civil War? If a war of that scale happened today and the same percentage of people died, how many people would be killed (assume U.S. population of 314,721,724 people). [Source: Smithsonian Magazine, November 2012, page 48]

UNIT 8 – END OF UNIT ASSESSMENT

1. Complete the missing parts of the table. Round to THREE decimal places as need. Simplify all fractions. Show all work.

Fraction	Decimal	Percent
$\frac{3}{5}$		
	1.24	
		16%

2. Determine the missing number. Round to two decimals as needed.

26% of what number is 15?

3. Determine the missing number. Round to two decimals as needed.

0.23% of 37 is what number?

4. Determine the missing number. Round to two decimals as needed.

25 is what percent of 13?

UNIT 9 – SYSTEMS OF MEASURE AND UNIT CONVERSIONS

INTRODUCTION

In this Unit, we will begin our study of Geometry by investigating what it means to measure an object, and what attributes of an object we can measure. We will learn to measure objects in various ways, compare measurements, and convert between different units and systems of measure.

The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

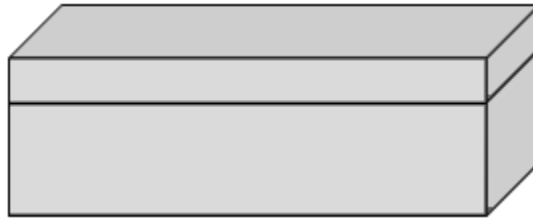
Learning Objective	Media Examples	You Try
Distinguish between 1, 2, and 3 dimensional measures	1	2
Measure length with a ruler or a given unit of length measurement	3	4
Measure area with a given unit of area measurement or gridded object	5	6
Measure volume with a given unit of volume measurement or cubed diagram of an object	7	8
Convert U.S. measurements using a double number line	9	10
Convert simple U.S. measurements using dimensional analysis	11	12
Convert multi-unit U.S. measurements	13	15
Convert multi-step U.S. measurements	14	15
Convert metric measurements using a double number line	16	18
Convert simple metric measurements using a table or dimensional analysis	17	18
Convert between U.S. and metric systems using dimensional analysis	19	20

UNIT 9 – MEDIA LESSON

WHAT IS MEASUREMENT?

When we want to communicate the size of an object, we talk about its *measure*. Most objects have many different attributes that we can measure. For example, a 1-dimensional attribute of an object is its *length* (the distance between two points). A 2-dimensional attribute of an object is its *area* (the size of the surface of the object). In 3-dimensions, we talk about an object's *volume* (the holding capacity of the object, or how much space it takes up).

So what does it mean to measure an object? First we need to know what attribute that we plan to measure. For example, suppose that you are designing shelves in your garage to hold storage boxes. What attributes of the box would we want to measure to help with the design?



In order to plan for the depth of our shelves, we would need to know the length of the box. If we want to know how many boxes will fit on the shelf, it would be helpful to know how much space the base of the box takes up, so we'd want to measure the area of the box's base. If we're thinking about how much we can store in each of the boxes, then we might want to know the volume of the boxes. Once we know what attribute that we want to measure, we can compare the attribute of the object to a known quantity of the same attribute.

SECTION 9.1: UNDERSTANDING DIMENSION

We say that an object is **1-dimensional** if at each location, there is only 1 *independent* direction to move within the object. For example, in a 1-dimensional world, a creature could only move forward/backward. Some examples of 1-dimensional objects are: line segments, the outer edge of a circle, the line segments making up a rectangle, or the edge where two walls meet.

We say that an object is **2-dimensional** if at each location in the object, there are 2 independent directions along which to move within the object. For example, in a 2-dimensional world, a creature could move forward/backward, or right/left. Some examples of 2-dimensional objects are: a piece of paper, the inside of a circle, the inside of a rectangle, the surface of a wall, or the surface of the base of a box.

We say an object is **3-dimensional** if at each location, there are 3 independent directions along which to move within the object. For example, our world is 3-dimensional. We can move forward forward/backward, right/left, or up/down. Some examples of 3-dimensional objects are: the earth, the inside of a box, the feathers that fill a pillow, the contents of a soda bottle.



Problem 1

MEDIA EXAMPLE – Understanding Dimension

Determine whether the following describe a 1-dimensional, 2-dimensional, or 3-dimensional measure.

- The amount of tile needed for the bathroom floor: _____
- The amount of baseboard needed for the bathroom: _____
- The amount of paint needed for the bathroom walls: _____
- The depth of a bathtub: _____
- A footprint on a bathtub mat: _____
- The amount of water that a bathtub will hold: _____



Problem 2

YOU TRY – Understanding Dimension

Determine whether the following describe a 1-dimensional, 2-dimensional, or 3-dimensional measure.

- The distance from home to campus: _____
- The height of a ketchup bottle: _____
- The top surface of a ketchup bottle cap: _____
- The amount of ketchup that a bottle will hold: _____
- Describe one-dimensional, two dimensional, and three-dimensional aspects of a swimming pool. What are some practical reasons for wanting to know these measurements?

SECTION 9.2: MEASURING LENGTH

Length can be thought of as the distance between two points. We measure length to answer the question “how long”, “how far”, or “how wide”? In order to measure the length of our box, we simply compare it to some known length. There are many tools that can be used to measure length; the most common tool is a ruler. Some standard units of length that we might use for comparison are inches, feet, or centimeters. These are units of length that are understood by everyone. But we really could measure our box by comparing it to any known length. Once we choose our measurement unit, then we need to determine how many times as large the length of the box is compared to the known length that we are using for comparison. The most direct way to measure a length is to count how many of the units are in the quantity to be measured.

A **system of measurement** is a collection of standard units. In the U.S. there are two systems of measurement that are commonly used: U.S. Customary system and the Metric System. The U.S. Customary System is derived from the British system of measure and will be familiar to you. The Metric system is more commonly used around the world, and is much easier to understand and to convert between units since it is based on the decimal system of numbers.

In **the metric system** units are created in a uniform way. For any quantity to be measured, there is a base unit (meter, liter, gram), then the base unit is paired with a prefix that indicates the unit’s relationship to the base unit. For example, the prefix *kilo* means *thousand*, so a *kilometer* is a thousand meters. Many of the metric prefixes are only used in scientific contexts. The table below lists some of the commonly used metric prefixes.

Metric Prefixes

Prefix	Meaning
Nano	Billionth
Micro	Millionth
Milli	Thousandth
Centi	Hundredth
Deci	Tenth
Base Unit	One
Deka	Ten
Hecto	Hundred
Kilo	Thousand
Mega	Million
Giga	Billion

Standard Units of Length

U.S. Customary System		
Unit	Abbreviation	Relationships
inch	in	
foot	ft	1 ft = 12 in
yard	yd	1 yd = 3 ft
mile	mi	1 mi = 5280 ft

Metric System		
Unit	Abbreviation	Relationships
millimeter	mm	1000 mm = 1m
centimeter	cm	1 cm = 10 mm
meter	m	1 m = 100 cm
kilometer	km	1 km = 1000 m

You are likely familiar with the size of the units in the US Customary System, but it good to have some sense of the size of the common metric measures. For example, a millimeter is about the size of the width of a dime. A centimeter is about the width of a small fingernail (there are approximately two and a half cm in an inch). A meter is about a yard. A kilometer is 0.6 mi – so a little more than half of a mile.



Problem 3

MEDIA EXAMPLE – Measuring Length

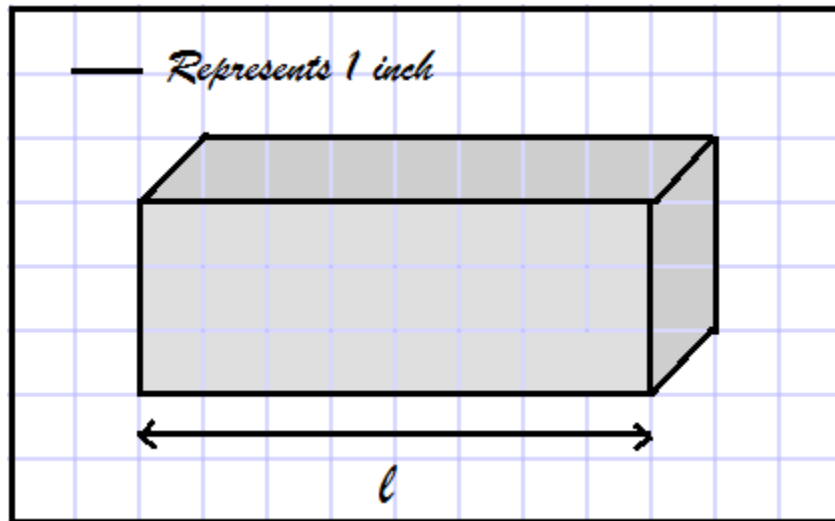
Measure the following lengths. (Link to online ruler: <http://iruler.net/>)

- a) Measure the length of line segment AB using centimeters as the unit of comparison.



Length: _____

- b) Measure the length ℓ of the base of the box using inches as the unit of comparison.



Length: _____

- c) Determine what units would be appropriate to use to measure the following lengths

Item	U.S. customary unit	Metric unit
The distance from home to campus		
The height of a water bottle		
The length of an ant		



Problem 4

YOU TRY – Measuring Length

a) Determine what units would be appropriate to use to measure the following lengths

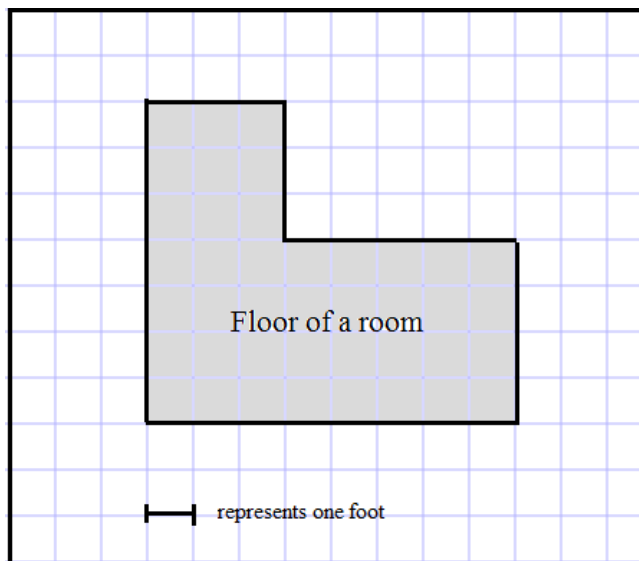
Item	U.S. customary unit	Metric unit
The length of a football field		
The width of a swimming pool		
The height of a citrus tree		

b) Measure the line segment AB using inches as the unit of comparison. (Online ruler: <http://iruler.net/>)



Length: _____

c) Measure the distance around the edge of the room to determine the length of baseboard required.



Length: _____

SECTION 9.3: MEASURING AREA

Area can be thought of as the amount of space within the boundaries of a 2-dimensional shape. We measure area when we are trying to answer questions like, “how much material will it take to make this”, or “how much space do I need on my shelf to fit this”? In order to measure area, we must compare our object to a known unit of area, and we determine how many units (including partial units) it would take to cover the object without gaps or overlaps.


Some standard units of area are square inches (in^2 - a square that has 1-in long sides), square feet (ft^2 - a square that has 1-foot long sides), and square centimeters (cm^2 - a square that has 1-cm long sides). Once we decide on the unit area that we will use, we need to determine how many times larger our object’s area is than the unit area is. More simply, we could count how many of the units it takes to completely cover our object.

Units of Area

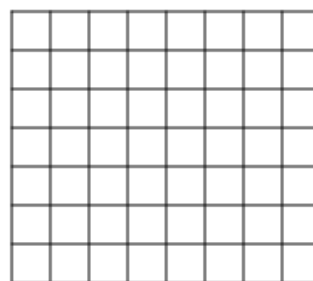
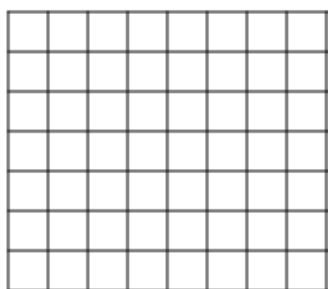
U.S. Customary System		
Unit	Abbreviation	Relationships
square inch	in^2	
square foot	ft^2	$1 \text{ ft}^2 = 12^2 \text{ in}^2 = 144 \text{ in}^2$
square yard	yd^2	$1 \text{ yd}^2 = 3^2 \text{ ft}^2 = 9 \text{ ft}^2$
square mile	mi^2	
acre		$1 \text{ acre} = 43,560 \text{ ft}^2$

Metric System		
Unit	Abbreviation	Relationships
square millimeter	mm^2	
square centimeter	cm^2	$1 \text{ cm}^2 = 10^2 \text{ mm}^2 = 100 \text{ mm}^2$
square meter	m^2	$1 \text{ m}^2 = 100^2 \text{ cm}^2 = 10,000 \text{ cm}^2$
square kilometer	km^2	$1 \text{ km}^2 = 1000^2 \text{ m}^2 = 1,000,000 \text{ m}^2$

Notice that each unit of length has an associated unit of area. The area unit is the square with the given side length. For example, a square inch looks like a square whose side lengths are 1 inch long.

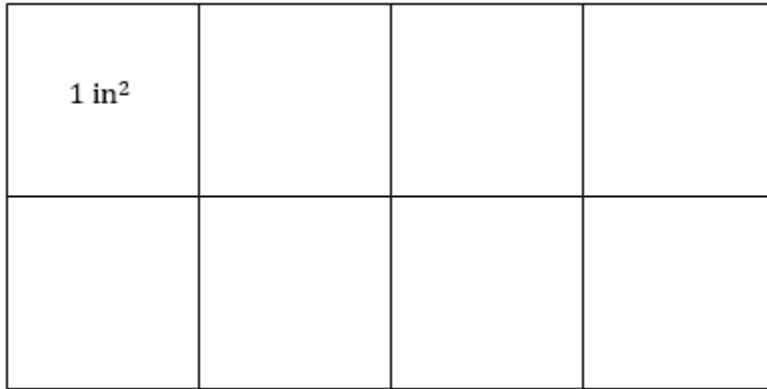
	Problem 5	MEDIA EXAMPLE – Measuring Area
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- a) Create and shade two different shapes in the grids below that cover 9 square units.



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b) Find the area of the shape in square inches.



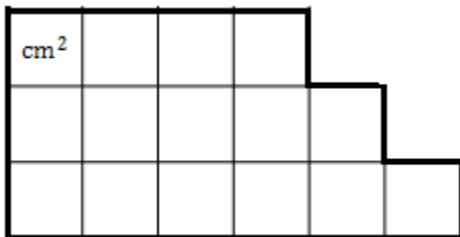
Area: _____

	Problem 6	YOU TRY – Measuring Area
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a) What units would be appropriate to use to measure the following?

Item	U.S. customary unit	Metric unit
The floor of your living room		
The area of a sheet of paper		
The area of a post-it note		
The lot size of a house in Scottsdale		
The area of your kitchen table		

b) Find the area of the figure square centimeters.



Area: _____

SECTION 9.4: MEASURING VOLUME

Volume is the space taken up by a 3-dimensional object. We measure volume when we want to answer questions like “how many sugar cubes would it take to fill this box”, “how much air is in this room”, or “how much water will it take to fill the pool”? In order to measure volume, we must compare our object to a known unit of volume, and we determine how many units (including partial units) it would take to completely fill the object.

Some standard units of volume are cubic inches (in^3 - a cube that has 1-in long sides), cubic feet (ft^3 - a cube that has 1-foot long sides), and cubic centimeters (cm^3 - a cube that has 1-cm long sides). Once we decide on the unit of volume that we will use, we need to determine how many times larger our object’s volume is than the unit volume is. More simply, we could count how many of the units it takes to completely fill our object.

Units of Volume and Capacity (liquid volume)

U.S. Customary System		
Unit	Abbreviation	Relationships
cubic inch	in^3	
cubic foot	ft^3	$1 \text{ ft}^3 = 12^3 \text{ in}^3 = 1728 \text{ in}^3$
cubic yard	yd^3	$1 \text{ yd}^3 = 3^3 \text{ ft}^3 = 27 \text{ ft}^3$

U.S. Customary System		
Unit	Abbreviation	Relationships
teaspoon	tsp	
tablespoon	T or tbsp	$1 \text{ T} = 3 \text{ tsp}$
fluid ounce	fl oz	$1 \text{ fl oz} = 2 \text{ T}$
cup	c	$1 \text{ c} = 8 \text{ fl oz}$
pint	pt	$1 \text{ pt} = 2 \text{ c}$
quart	qt	$2 \text{ qt} = 2 \text{ pt}$
gallon	gal	$1 \text{ gal} = 4 \text{ qt}$

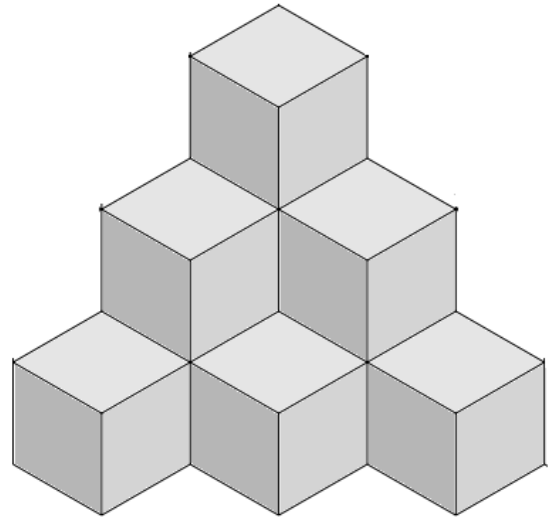
Metric System		
Unit	Abbreviation	Relationships
cubic millimeter	mm^3	
cubic centimeter	cm^3	$1 \text{ cm}^3 = 10^3 \text{ mm}^3 = 1000 \text{ mm}^3$
cubic meter	m^3	$1 \text{ m}^3 = 100^3 \text{ cm}^3 = 1,000,000 \text{ cm}^3$
cubic kilometer	km^3	$1 \text{ km}^3 = 1000^3 \text{ m}^3 = 1,000,000,000 \text{ m}^3$
milliliter	mL	$1 \text{ mL} = 1 \text{ cm}^3$
centiliter	cL	$1 \text{ cL} = 10 \text{ mL} = 10 \text{ cm}^3$
liter	L	$1 \text{ L} = 100 \text{ cL} = 1000 \text{ mL} = 1000 \text{ cm}^3$
kiloliter	kL	$1 \text{ kL} = 1000 \text{ L} = 100,000 \text{ cL} = 1,000,000 \text{ mL}$



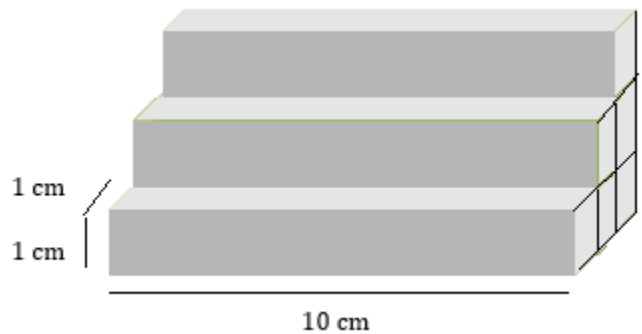
Problem 7

MEDIA EXAMPLE – Measuring Volume

- a) The figure below is the front view of a 3 dimensional object made up of stacked cubes. How many cubes make up the volume of this figure including the ones we cannot see?




- b) Determine the volume of the toy staircase shown by imagining that it is filled with centimeter cubes.



- c) What units would be appropriate to use to measure the following?

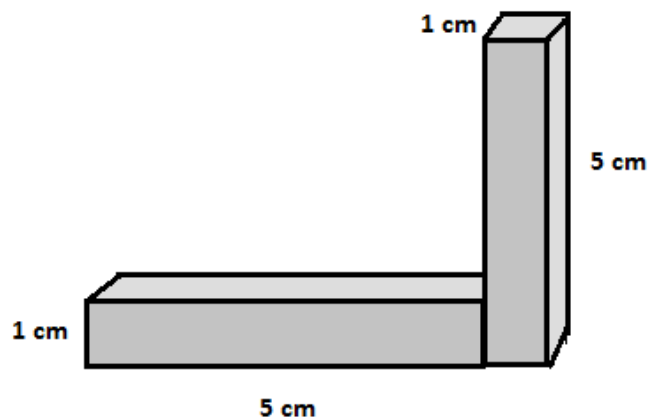
Item	U.S. customary unit	Metric unit
The amount of water in a bathtub		
The amount of coffee in a mug		
The amount of helium in a balloon		
The amount of fluid in single tear of joy		

	Problem 8	YOU TRY – Measuring Volume
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a) What units would be appropriate to use to measure the following?

Item	U.S. customary unit	Metric unit
The amount of water in a pool		
The amount of water in a bottle		
The amount of air in a room		
The amount of fluid in an allergy shot		

b) Determine the volume of the following shape by imagining it is filled with centimeter cubes.



SECTION 9.5: INTRODUCTION TO CONVERTING MEASURES

Recall that measurement is just a comparison between the attribute of an object that we want to measure, and a known quantity with the same attribute. For example, if we want to measure the length of a pencil, we compare the length of the pencil with the length of an inch. We ask ourselves the question, “how many copies of an inch would it take to make the length of this pencil”, or, “how many times larger than an inch is this pencil”? But we could have chosen to compare the length of the pencil with the length of a centimeter. Either approach is valid.

Sometimes we know a measurement in a particular unit, but we are interested in the value of the measurement in a different unit. Suppose we know that the length of a table is 7ft, but we want to know what the value of the measurement is in inches. This process of converting a measurement from one unit to another is called **unit conversion**. We can convert between units within a measurement system or between measurement systems.

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Below is a table showing the primary units of measure in the US Customary system of measurement along with conversions between units. This table is a convenient tool when you need to convert between units.

US Units/Conversions							
Length		Mass/Weight		Area			
Units: Inches (in) Feet (ft) Yards (yd) Miles (mi)		Units: Ounces (oz.) Pounds (lb.) Tons		Units: Square Inches (in ²) Square Feet (ft ²) Square Yards (yd ²)			
Conversions: 1 ft = 12 in 1 yd = 3 ft 1 mi = 5280 ft		Conversions: 1 lb. = 16 oz. 1 ton = 2000 lb.		Conversions: 144 in ² = 1 ft ² 9 ft ² = 1 yd ²			
Volume			Time				
Units: Ounces (oz.) Cup (c) Pint (pt.) Quart (qt) Gallon (gal) Cubic Feet (ft ³) Cubic Yard (yd ³)		Conversions: 1 c = 8 oz. 1 pt. = 2 c 1 qt = 2 pt. 1 qt = 32 oz. 1 gal = 4 qt 1728 cubic in = 1 cubic ft 27 cubic ft = 1 cubic yd		Units: Seconds (sec) Minutes (min) Hours (hr.) Days Weeks (wk.) Months (mo.) Years (yr.)		Conversions: 1 min = 60 sec 1 hr. = 60 min 1 day = 24 hr. 1 wk. = 7 day 1 yr. = 52 wk. 1 yr. = 12 mo. 1 yr. = 365 days	

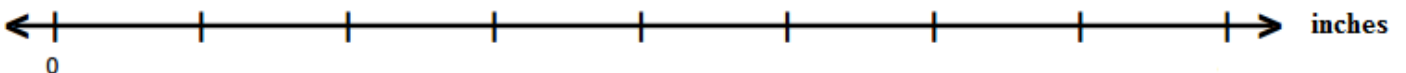


Problem 9

MEDIA EXAMPLE – Using Double Number Lines to Convert Between U.S. Units

Use the number lines to write the corresponding values for each unit of measure and find the indicated conversion.

- a) Complete the missing values in the double number line and find the conversions below.

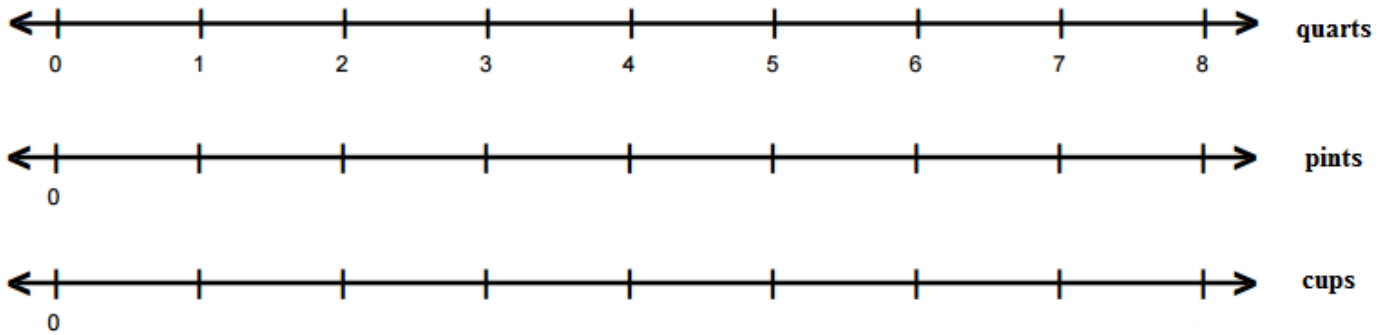


5 feet = _____ inches

36 inches = _____ feet

30 inches = _____ feet

b) Complete the missing values in the triple number line and find the conversions below.



4 quarts = _____ pints

6 pints = _____ cups

3 quarts = _____ cups

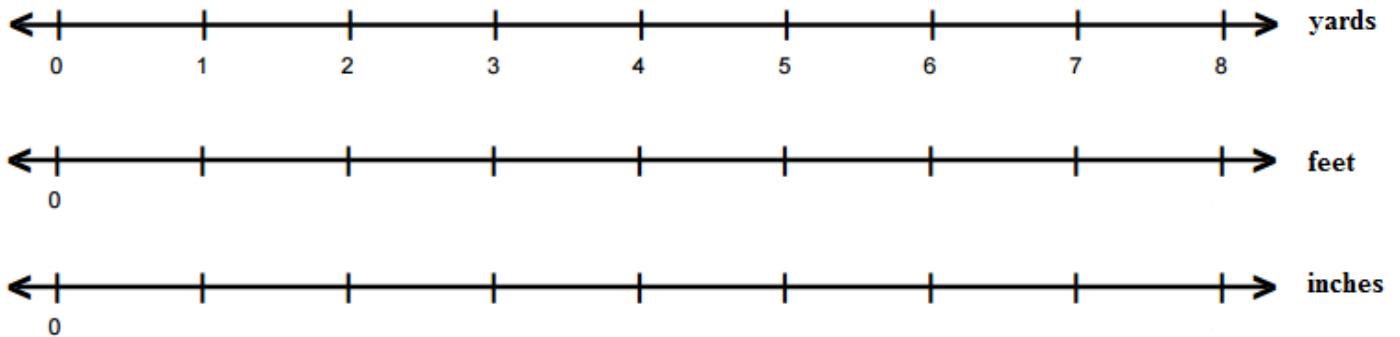


Problem 10

YOU TRY – Using Double Number Lines to Convert Between U.S. Units

Use the number lines to write the corresponding values for each unit of measure and find the indicated conversions.

Complete the missing values in the triple number line and find the conversions below.



6 yards = _____ feet

5 feet = _____ inches

2 yards = _____ inches

SECTION 9.6: DIMENSIONAL ANALYSIS AND U.S. CONVERSIONS

One question that students often ask is whether they should multiply or divide to convert between two units of measure. We will use a method called *dimensional analysis* where we *always multiply by a conversion factor* written in fraction form.

When you multiply by a fraction, you can think of the numerator of the fraction as making copies or multiplying and the denominator of the fraction as cutting into groups or dividing. So multiplying by a fraction is equivalent to the idea of multiplying or dividing to convert between units. However, when we use a conversion factor that is a fraction with our units labeled, we can use dimensional analysis to be certain we are operating in the appropriate way.

Consider the following conversion questions.

How many inches are in 3 feet? How many feet are in 18 inches?

Conversion Equation: 1 foot = 12 inches

Conversion Factors: $\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{12 \text{ inches}}{1 \text{ foot}} = 1$

Notice that the conversion factors are fractions that are both equal to 1. This may seem odd because there are different numbers in the numerator and denominator. However, since 12 inches = 1 foot, dividing one by the other equals 1 when we include the units of measure.

Recall that multiplying by 1 does not change the value of a number, but creates an equivalent form. So we can multiply the given numbers by the appropriate conversion factors to change our units.

$$3 \text{ feet} = \frac{3 \cancel{\text{ feet}}}{1} \cdot \frac{12 \text{ inches}}{1 \cancel{\text{ foot}}} = \frac{3 \cdot 12}{1 \cdot 1} \text{ inches} = 36 \text{ inches}$$

Notice we drew a line crossing out feet in the numerator and foot in the denominator leaving only inches in the numerator. Dimensional analysis helps keep track of units until we have the correct unit remaining. For the second conversion, we will use the other conversion factor to make inches cancel to 1 (instead of division) and the units of feet remain.

$$18 \text{ inches} = \frac{18 \cancel{\text{ inches}}}{1} \cdot \frac{1 \text{ foot}}{12 \cancel{\text{ inches}}} = \frac{18 \cdot 1}{1 \cdot 12} \text{ feet} = \frac{18}{12} \text{ feet} = 1.5 \text{ feet}$$

It is true that to change from feet to inches, we multiply by 12 and to change from inches to feet we divide by 12. When you are very comfortable with the units of measure, it is fine to use this process. However, to be certain you are converting correctly, it is highly recommended that you use dimensional analysis to ensure the correct conversion.

 Problem 11	MEDIA EXAMPLE – Simple U.S. Unit Conversions
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For each problem, write the conversion equation, conversion factors, and conversion multiplication to convert the unit of measure.

Convert	Conversion Equation	Possible Conversion Factors	Conversion Process	Result
a) 4 lbs. to oz.				
b) 10 yds. to ft.				
c) 2.4 pts. to cups				

d) Sarah needs 1.5 cups of ketchup to make her famous meatloaf recipe. She has a brand new, 20-oz bottle of ketchup in her cupboard. How many ounces of ketchup will she need for her meatloaf?

e) Your new truck weighs 8000 lbs. How many tons is this?



Problem 12

YOU TRY – Simple U.S. Unit Conversions

For each problem, write the conversion equation, conversion factors, and conversion multiplication to convert the unit of measure.

Convert	Conversion Equation	Possible Conversion Factors	Conversion Process	Result
a) 7 qt. to gal.				
b) 330 minutes to hours				

c) Your friend Sara writes to you saying that she will be away for 156 weeks. How many years will she be gone?

d) Carlton ran $4\frac{1}{2}$ miles this morning. How many feet did he run?

e) Shari is counting the hours until her vacation. She just realized that she has 219 hours to go! How many days before she goes?



Problem 13

MEDIA EXAMPLE – Multi-Unit U.S. Conversions

The following examples illustrate additional basic conversions within the U.S. System. A modified form of the conversion process will be used for these problems.

- a) Write 26 inches in feet and inches.

- b) Write 5 lbs., 6 oz. in ounces.

- c) Write 30 months in months and years.

- d) Write 1 min, 20 sec in seconds.



Problem 14

MEDIA EXAMPLE – Multi-Step U.S. Conversions

Some conversions require more than one step. See how the single-step conversion process is expanded in each of the following problems.

- a) How many minutes are in a week?

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- b) Bryan needs 10 cups of fruit juice to make Sangria. How many quarts of juice should he buy at the grocery store?
- c) Rick measured a room at 9 ft. long by 10 ft. wide to get an area measurement of 90 square feet (area of a rectangle is length times width). He wants to carpet the room with new carpet, which is measured in square yards. Rick knows that 1 yd is equivalent to 3 ft. so he ordered 30 square yards of carpet. Did he order the correct amount?



Problem 15

YOU TRY – Multi-Unit and Multi-Step Conversions

Perform each of the following conversions within the U.S. system. Round to tenths as needed. Show complete work.

- a) A young girl paced off the length of her room as approximately 8 feet. How many inches would that be?
- b) 18 oz. = _____ lb.

- c) 100 yd = _____ ft.
- d) 10,235 lb. = _____ tons
- e) How many inches are in 6 feet, 8 inches?
- f) How many square inches are in 10 square feet?

SECTION 9.7: UNIT CONVERSIONS IN THE METRIC SYSTEM

The strength of the *metric system* is that it is based on powers of ten as you can see in the chart below. Prefixes are the same for each power of ten above or below the base unit. This also makes conversions easy in the metric system.

Metric Chart						
KILO	HECTO	DEKA		DECI	CENTI	MILLI
$1000 \times \text{Base}$	$100 \times \text{Base}$	$10 \times \text{Base}$	Base Unit	$0.10 \times \text{Base}$	$0.01 \times \text{Base}$	$0.001 \times \text{Base}$
Kilometer (km)	Hectometer (hm)	Dekameter (dam)	Meter (m)	Decimeter (dm)	Centimeter (cm)	Millimeter (mm)
Kiloliter (kl)	Hectoliter (hl)	Dekaliter (dal)	Liter (l)	Deciliter (dl)	Centiliter (cl)	Milliliter (ml)
Kilogram (kg)	Hectogram (hg)	Dekagram (dag)	Gram (g)	Decigram (dg)	Centigram (cg)	Milligram (mg)

Some Common Metric Conversions

1 centimeter (cm) = 10 millimeters (mm)

1 meter (m) = 100 centimeters (cm)

1 kilometer (km) = 1000 meters (m)

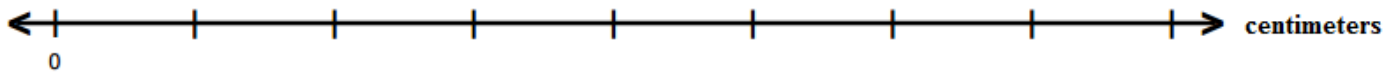


Problem 16

MEDIA EXAMPLE – Using Double Number Lines to Convert Between Metric Units

Use the number lines to write the corresponding values for each unit of measure and find the indicated conversions.

a) Complete the missing values in the triple number line and find the conversions below.

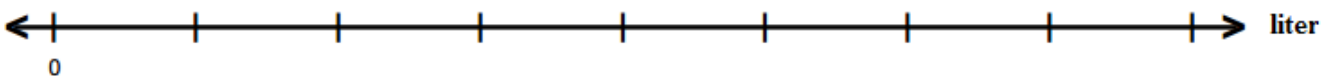
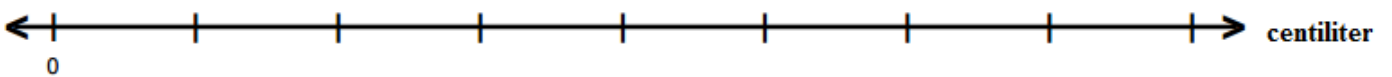
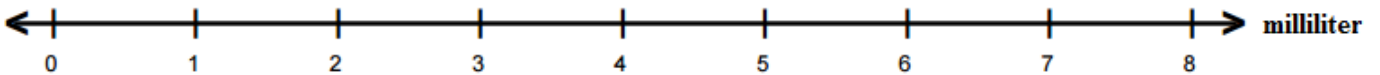


3 m = _____ cm

5 cm = _____ mm

7 m = _____ mm

b) Complete the missing values in the triple number line and find the conversions below.



3 ml = _____ cl

0.4 cl = _____ l

7 ml = _____ l



Problem 17

MEDIA EXAMPLE – Simple Metric Unit Conversions

Use the metric chart given below to convert the metric units.

Metric Chart						
KILO	HECTO	DECA	BASE UNIT	DECI	CENTI	MILLI
$1000 \times \text{Base}$	$100 \times \text{Base}$	$10 \times \text{Base}$	$1 \times \text{Base}$	$0.1 \times \text{Base}$	$0.01 \times \text{Base}$	$0.001 \times \text{Base}$

a) $4200 \text{ g} = \underline{\hspace{2cm}} \text{ mg}$

b) $45 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$

c) $7,236,137 \text{ ml} = \underline{\hspace{2cm}} \text{ kl}$

d) If a person's pupillary distance (from one pupil to the other) is 61 mm and the distance from their pupil to the middle of their upper lip is 7 cm, which distance is longer?



Problem 18

YOU TRY – Simple Metric Unit Conversions

Use a metric chart to convert the metric units. Show all of your work.

a) $1510 \text{ m} = \underline{\hspace{2cm}} \text{ mm}$

b) $13.50 \text{ ml} = \underline{\hspace{2cm}} \text{ l}$

c) $5 \text{ km} = \underline{\hspace{2cm}} \text{ m}$

UNIT 9 – PRACTICE PROBLEMS

1. Describe one-dimensional, two-dimensional, and three-dimensional parts or aspects of a packing box. In each case, name an appropriate U.S. customary unit and an appropriate metric unit for measuring or describing the size of that part or aspect of the packing box. What are practical reasons for wanting to know the sizes of these parts or aspects of the packing box?

	Aspect of bottle	US Customary Unit	Metric unit	Practical reason for wanting to know
1- dimensional				
2- dimensional				
3- dimensional				

2. For each of the following items, state which U.S. Customary units and which metric units would be the most appropriate for describing the size.
- a. The volume of water in a bathtub
 - b. The weight of a dog
 - c. The distance from Phoenix to Los Angeles
 - d. The area of foundation of a house
 - e. The length of a lady bug

Unit 9 – Practice Problems

3. Perform the following conversions:

- a. How many meters are in 2378 feet?

- b. How many seconds are in 768 days?

- c. A car traveled 7.2 miles. How many inches did the car travel?

- d. If a truck has a mass of 23,456 kg, what is its mass in milligrams?

- e. How many liters are in 5 gallons?

- f. How many hours are there in 6.5 decades?

- g. How many miles are there in 34,823 centimeters?

4. A car is 16ft 3in long. How long is this in meters?

5. In Germany, cars typically travel 130 km per hour. How fast are they going in miles per hour?

6. A room has a floor area of 48 square yards. What is the area of the room in square feet? Draw a picture of that could represent the room to help you solve the problem.

7. A house has a floor area of 225 square meters. What is the floor area in square feet?

8. A penny is $\frac{1}{16}$ an inch thick. If you made a stack of 1000 pennies, how tall would it be? Give your answer in feet and inches.

9. Given that there are 3 ft. per 1 yard, explain why there are 9 square feet in a square yard. Draw a picture to aid your explanation.

10. If a horse weighs 1125 lbs., what is its weight in milligrams?

Unit 9 – Practice Problems

11. Complete each of the following showing as much work as possible.

a. Does it take more cups or gallons to measure the amount of water in a large pot? Explain.

b. The lifespan of a common housefly is about 8 days. How many hours are in 8 full days?

c. A 10k running race is about 6.2 miles. How many feet is this? Assuming that the average person's step is 3 feet long, how many steps are traveled when covering a 10k?

d. Tally the cat is 10.5 pounds. How many ounces is this?

e. Fredericka's house gate is 45 inches. How many feet is this? (Use decimals)

12. Complete each of the following showing as much work as possible.

a. If you were born on January 1, 1980 at 12:00 am and measured time until January 1, 2013 at 12:00 pm, how many minutes would you have been alive?

b. How many centuries are in 164,240 days? (1 century = 100 years)

c. A container measures 16 inches in length by 2 feet in width by 1 yard in height. If volume is found by multiplying length times width times height, find the volume of the container in cubic feet.

d. Jose's company measures their gains in \$1000's of dollars. If his company earned 6.2 million in gains, how many \$1000's of dollars is this?

e. Tara's pool is 50 yards in length and 20 feet in width. How many square feet is the pool? How many square yards is the pool?

Unit 9 – Practice Problems

13. Complete each of the following showing as much work as possible.

a. Write 32 months in months and years.

b. 10 years, 6 months is how many months?

c. If your final exam time is 110 minutes, write that time span in hours and minutes.

d. Amy is 14,964 days old today. How old is this in years and days? (Assume 365 days in a year and no leap years). How many days until Amy's birthday?

e. Joseph spent 6.45 hours working on his English paper. How much time is this in hours and minutes?

14. Complete each of the following showing as much work as possible.

a. Add 2 lb. 10 oz. plus 4 lb. 8 oz. Leave your answer in lb., oz.

b. Suppose you took two final exams on a given day. Each final exam allows 110 minutes. You took 1 hour and 5 minutes on the first exam and 50 minutes on the second. How long were you taking exams on that day? How much exam time did you not use on that day?

c. How much greater is 3 gallons than 2 gallons 1 qt?

d. Maria's pool holds 2962.27 gallons of water when filled to the recommended height. She needs to add 57.63 more gallons to reach this height. How many gallons of water are in the pool? How many quarts of water need to be added?

e. Graham ate 9 ounces of protein, 6 ounces of vegetables and 5 ounces of dairy. How many ounces did he eat in total? What is the equivalent weight in pounds?

Unit 9 – Practice Problems

15. Complete each of the following showing as much work as possible.

a. Which is the best estimate for the capacity of a bottle of olive oil? Choose from 500L, 500ml, 500 g, 500mg and explain your choice.

b. Complete the table. Show your work.

	Centimeters	Meters	Kilometers
Distance from Scottsdale to Las Vegas			421

c. If a tractor-trailer has a mass of 18,245 kg, what is its mass in grams?

d. Which measurement would be closest to the length of a newborn baby? 50 mm, 50 cm, 50 dm, or 50 m?

e. Which measurement would be closest to the weight of a penny? 2.5 mg, 2.5 g, 2.5 kg?

16. Complete each of the following showing as much work as possible.

a. George was riding his bike downhill on a street in a Canadian town. The street-side speed sensor clocked him at 30 km per hour. His bike speedometer was set up in U.S. units of mph. What would the readout have been?

b. A short-course meter pool is 25 meters long. A short-course yard pool is 25 yards long. Which one is longer and by how much (in feet)? Round to two decimal places.

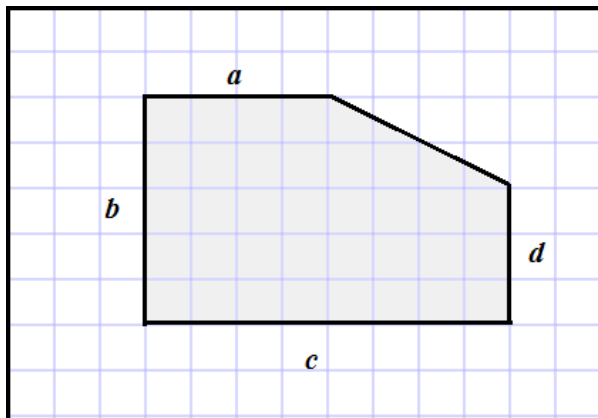
c. In swimming events, a mile in the pool is considered to be 1600 meters. How many meters separate a swimmers mile from an actual mile?

d. At its closest point, the distance from the Moon to the Earth is 225,622 miles. The circumference of the earth is 24,901 miles. How many times would you have to travel around the circumference of the Earth to equal the distance from the Earth to the Moon? (Round to two decimal places)

e. Johanna just returned from a trip to South Africa. She has 7342 rands, the currency of South Africa. She looks up the exchange rate and finds that 1 South African rand = 0.1125 U.S. dollars. What is the value of her money in U.S. dollars?

UNIT 9 – END OF UNIT ASSESSMENT

- Determine whether the following describe a 1-dimensional, 2-dimensional, or 3-dimensional measure.
 - The diagonal of a television: _____
 - The screen of a television: _____
 - The bottom surface of a mug: _____
 - The amount of coffee that a mug will hold: _____
 - Describe one-dimensional, two dimensional, and three-dimensional aspects of a house. What are some practical reasons for wanting to know these measurements?
- Measure and label the lengths of sides a , b , c , and d in the figure below.

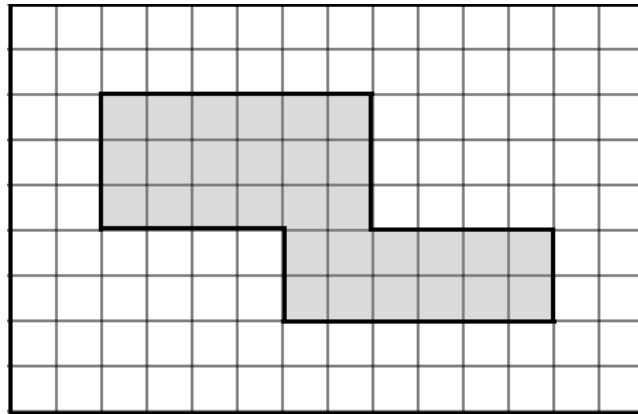


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1 unit

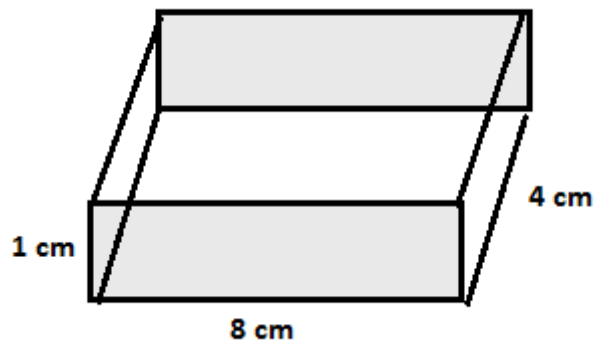
3. Determine what units would be appropriate to use to measure the following lengths

Item	U.S. customary unit	Metric unit
The distance from Phoenix to LA		
The height of a textbook		
The length of an eyelash		

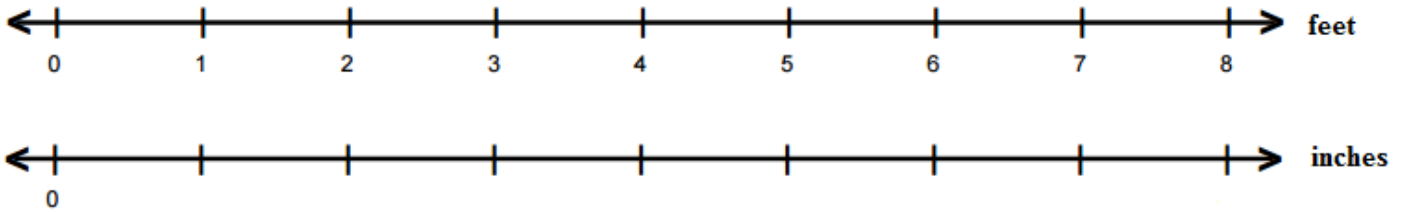
4. Determine the number of square units shaded in the figure.



5. Determine the number of cubic centimeters that would fill up the box below.



6. Complete the missing values in the double number line and find the conversions below.



4 feet = _____ inches

72 inches = _____ feet

6 inches = _____ feet

7. A truck load of coffee weighs 6500 lbs. How many tons is this?

8. Bill ran $3\frac{2}{3}$ miles this morning. How many feet did he run?

9. Convert 1,234,567 milliliters to both centiliters and liters.

10. The distance from Scottsdale to Glendale is approximately 26.56 km. What is this distance in miles?

UNIT 10 – PERIMETER AND AREA

INTRODUCTION

In this Unit, we will define basic geometric shapes and use definitions to categorize geometric figures. Then we will use the ideas of measuring length and area that we studied to find the perimeter, circumference, or area of various geometric figures.

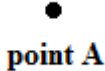
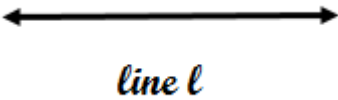
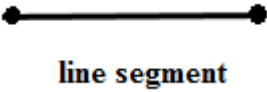
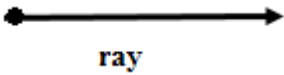
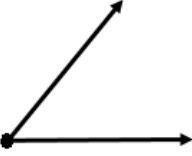
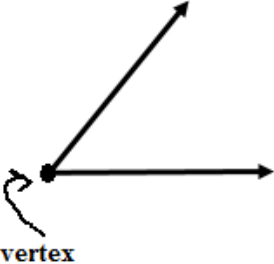
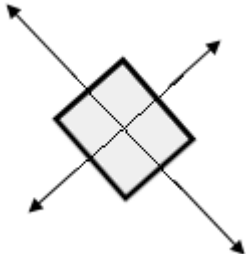
The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

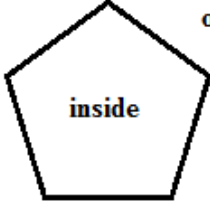

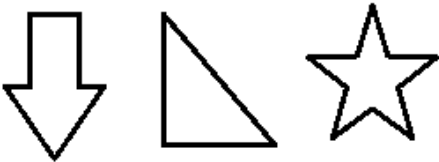
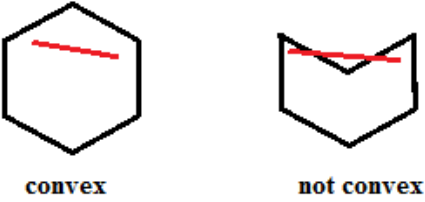
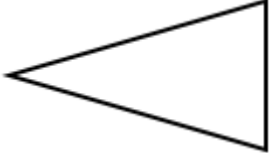

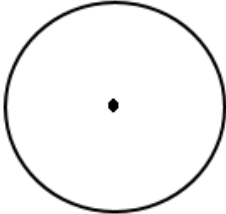
Learning Objective	Media Examples	You Try
Categorize geometric figures using definitions	1	
Model a context as a geometric shape and find its perimeter	2	
Find the perimeters of various shapes	3	4
Find the circumference of circles in various contexts using a formula	5	6
Develop strategies for finding area	7	8
Find the formula for the area of a parallelogram	9	
Apply the formula for the area of a parallelogram	10	11
Find the formula for the area of a triangle	12	
Apply the formula for the area of a triangle	13	14
Find the formula for the area of a trapezoid	15	
Apply the formula for the area of a trapezoid	16	17
Find the formula for the area of a circle	18	
Apply the formula for the area of a circle	19	20
Find the area of nonstandard shapes	21	22

UNIT 10 – MEDIA LESSON

SECTION 10.1: GEOMETRIC SHAPES AND DEFINITIONS

We will begin by defining some elemental shapes and characteristics of geometric figures. The famous mathematician Euclid set out to define basic geometry terms in his book *The Elements* in approximately 300 B.C. in Alexandria, Egypt. We will refer to some of his work below to show the difficulty in defining some of the simplest terms in geometry.

Geometric Definitions		
Name	Definition	Picture/Types
point	A location in space. Euclid defined a point as “ <i>that which has no part</i> ”. A point is dimensionless, and has no width, length, or height.	
line	A collection of points that extend along a straight path in two directions without end. Euclid defined a line as “ <i>a line is breadthless length</i> ”. A line is one-dimensional.	
line segment	A part of a line that has two endpoints. Line segments can be measured and have a finite length.	
ray	A part of a line that has one endpoint.	
angle	Two rays that have a common endpoint. We measure an angle by the amount of rotation from one ray to the other ray.	
vertex	The common endpoint of two rays or two line segments	
plane	A flat two dimensional surface that extends infinitely in its two dimensions	

<p>closed figure</p>	<p>A figure that has an inside and outside. You cannot reach the inside from the outside without crossing the figure's boundary.</p>	
<p>open figure</p>	<p>A figure that is not closed. It does not have an inside and outside.</p>	
<p>polygon</p>	<p>A closed two-dimensional figure with line segments as sides.</p>	
<p>Convex polygon</p>	<p>A polygon in which any line segment drawn between two points within the figure does not cross a boundary.</p>	
<p>triangle</p>	<p>A three sided polygon</p>	
<p>quadrilateral</p>	<p>A four sided polygon</p>	
<p>circle</p>	<p>A two dimensional figure that is the set of all points equidistant from a point called the center.</p>	

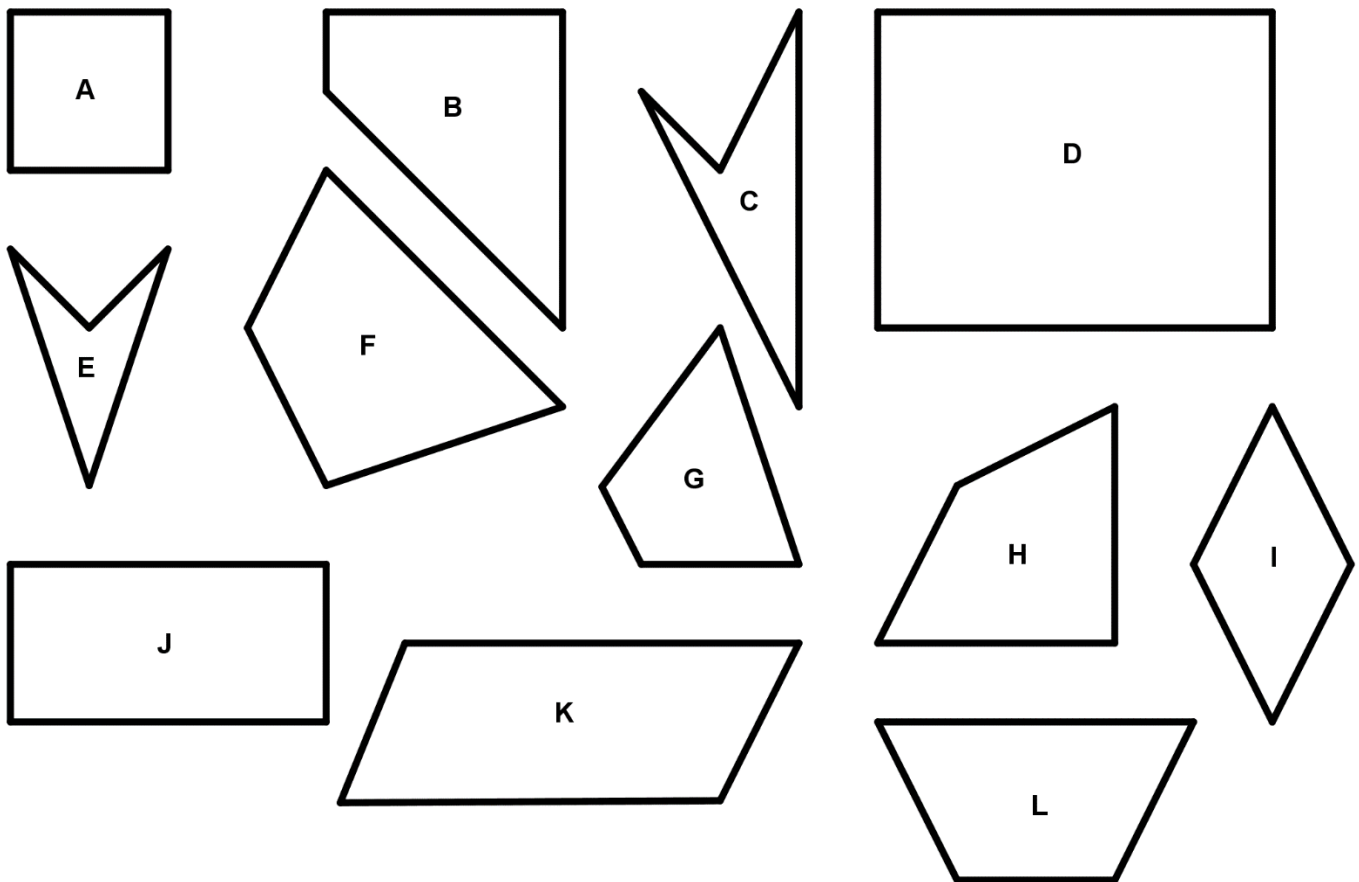


Below are six definitions of categories of quadrilaterals and twelve geometric shapes. For each shape, determine all of the categories in which the shape belongs.

Categories

1. Quadrilateral – a closed shape in a plane consisting of 4 line segments that do not cross each other
2. Square – quadrilateral with 4 right angles whose sides all have the same length
3. Rectangle – quadrilateral with 4 right angles
4. Rhombus (diamond) – quadrilateral whose sides all have the same length
5. Parallelogram – quadrilateral for which opposite sides are parallel
6. Trapezoid – quadrilateral for which at least one pair of opposite sides are parallel


Geometric Shapes



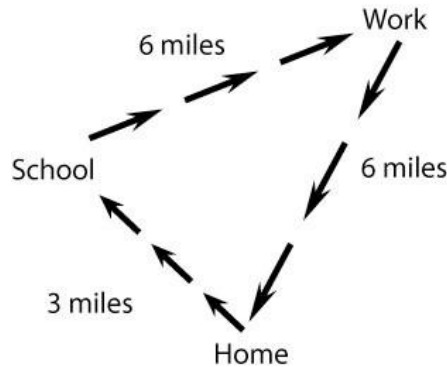
SECTION 10.2: PERIMETER

You may have heard the term perimeter in crime shows. The police will often “surround the perimeter”. This means they are guarding the outside of a building or shape so the suspect cannot escape. In mathematics, the *perimeter* of a two dimensional figure is the one dimensional total distance around the edge of the figure. We want to measure the distance around a figure, building, or shape and determine its length. Since the perimeter refers to the *distance around a closed figure or shape*, we compute it by combining all the lengths of the sides that enclose the shape.

In this section, we will introduce the concept of perimeter and learn why it is useful. We will find the perimeters of many different types of shapes and develop a general strategy for finding the perimeter so we don’t have to rely on formulas.

	Problem 2	MEDIA EXAMPLE – The Perimeter in Context
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Joseph does not own a car so he bikes everywhere he goes. On Mondays, he must get to school, to work, and back home again. His route is pictured below.



- a) Joseph starts his day at home. Complete the chart below by determining how far he has biked between each location and the total amount he has biked that day at each point. Include units in your answers.

Location	Starts at Home	Arrives at School	Arrives at Work	Returns Home
Miles Traveled from Previous Location	0 miles			
Total Miles Traveled since Leaving Home	0 miles			

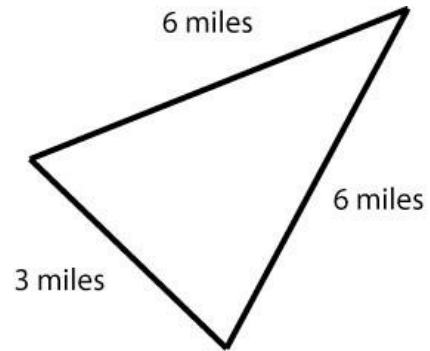
- b) Based on the information in your chart. What is the total distance Joseph Biked on Monday? Write your answer as a complete sentence.

Unit 10 – Media Lesson

- c) Another way to work with this situation is to draw a shape that represents Joseph’s travel route and label it with the distance from one location to the next as shown below. Find the perimeter of this shape.

Computation:

The perimeter is _____ miles



Result: The perimeter of the geometric figure is equivalent to the distance Joseph traveled. However, in part c, we modeled the situation with a geometric shape and then applied a specific geometric concept (*perimeter*) to compute how far Joseph traveled.

Notes on *Perimeter*:

- *Perimeter* is a one-dimensional measurement that represents the distance around a closed geometric figure or shape (no gaps).
- To find *perimeter*, add the lengths of each side of the shape.
- If there are units, include units in your final result. Units will always be of single dimension (i.e. feet, inches, yards, centimeters, etc...)



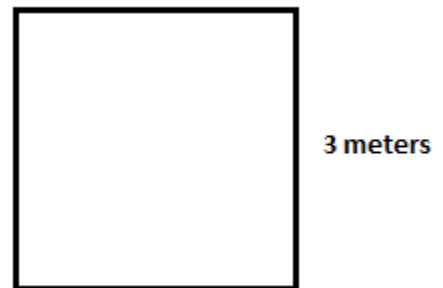
Problem 3

MEDIA EXAMPLE – Finding the Perimeter of a Figure

Find the perimeter for each of the shapes below. Label any sides that aren’t labeled and justify your reasoning. Show all of your work and include units in your answer.

- a) Keith bought a square board for a school project. What is the perimeter of the board?

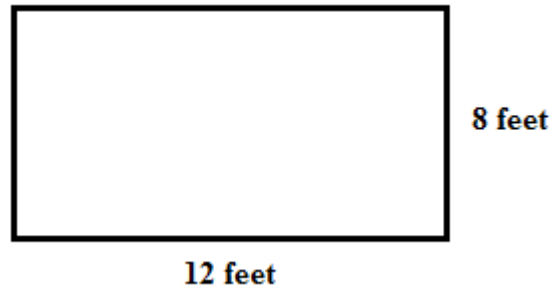
Computation:



The perimeter of the board is _____

- b) Judy is planting flowers in a rectangular garden. How many feet of fence does she need to fence in the garden?

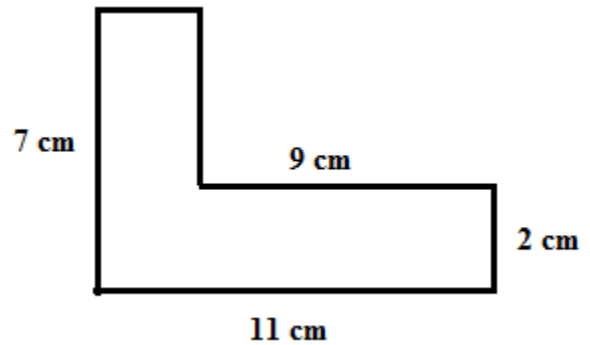
Computation:



The perimeter is _____

- c) Dana cut out the figure to the right from cardboard for an art project. What is the perimeter of the figure?

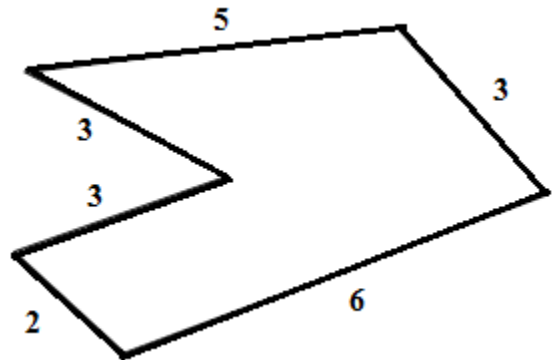
Computation:



The perimeter is _____

- d) Sheldon set up a toy train track in the shape given to the right. Each length is measured in feet. How far would the train travel around the track from start to finish?

Computation:



The perimeter is _____



Problem 4

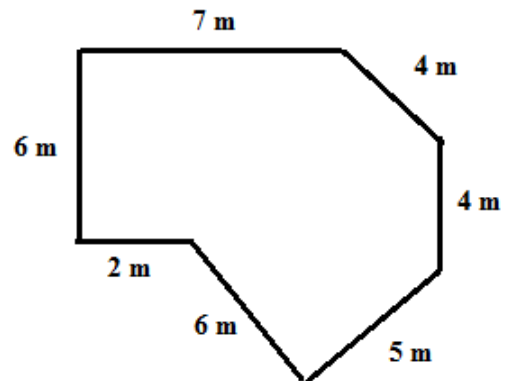
YOU TRY – Finding the Perimeter of a Figure

Find the perimeter for each of the problems below. Draw any figures if the shapes are not given. Label any sides that aren't labeled. Show all of your work and include units in your answer.

a) Find the perimeter of a square with side length 2.17 feet.

b) Find the perimeter of a triangle with sides of length 2, 5, 7.

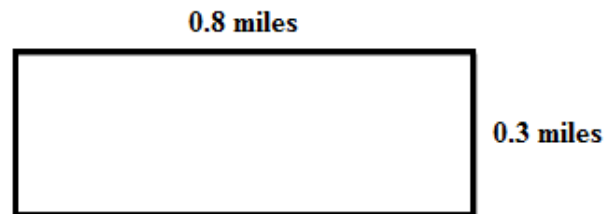
c) Jaik's band was playing at the club The Bitter End in New York City. A diagram of the stage is given below. What was the perimeter of the stage?



Final Answer as a Complete Sentence:

d) Steve works at the mall as a security guard. He is required to walk the perimeter of the mall every shift. The mall is rectangular in shape and the length of each side is labeled in the figure below. How far does Steve need to walk to complete this task?

Computation:

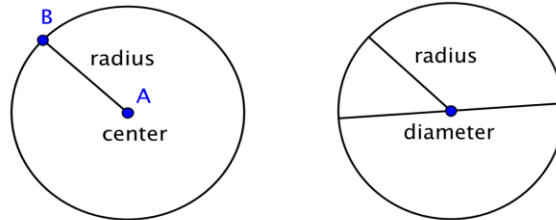


Final Answer as a Complete Sentence:

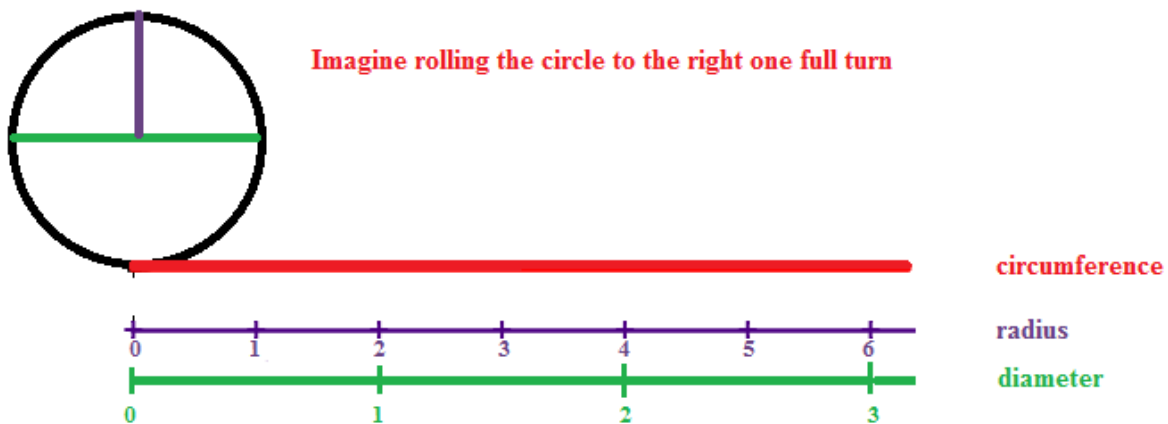
SECTION 10.3: CIRCUMFERENCE

The distance around a circle has a special name called the *circumference*. Since a circle doesn't have line segments as sides, we can't think of the circumference as adding up the sides of a circle. Before we find the formula for the circumference of a circle, we will first need to define a few attributes of a circle.

Mathematically, *a circle* is defined as the set of all points equidistant to its *center*. The *diameter* is the distance across the circle (passing through the center). The *radius* is the distance from the center of a circle to its edge. Notice that the diameter of the circle is two times as long as the radius of the circle.



Imagine a circle as a wheel. Now in your mind's eye, roll the wheel one complete turn. The distance the wheel covered in one rotation equals the distance around the circle, or the circumference.



You can probably imagine that the length of the radius or diameter is related to the circumference. The larger the circle, the larger the radius or diameter, the larger the distance that is covered in one rotation. In fact, the circumference of a circle is a *constant multiple* of its radius or diameter. Observing the number lines in the diagram below the circumference we can see that,


1. If we use the circle's radius as a measuring unit to measure the distance around the circle, we find that it takes just a little more than six copies of the radius to complete the circle.
2. If we use the circle's diameter as a measuring unit to measure the distance around the circle, we find that it takes just a little more than three copies of the diameter to complete the circle.
3. Since the diameter is twice as large as the radius, it makes sense that the number of diameter length segments to cover the distance is half the size of the number of radius length segments.
4. The *constant factor* between the diameter and circumference is a special number in mathematics called *pi*, pronounced "pie", and written with Greek letter π .

Result: The formula for finding the circumference of a circle can be written in terms of either the circle’s radius or diameter. These formulas are given below.

$$C = \pi \times d \quad \text{or} \quad C = \pi \times 2 \times r \quad \text{or} \quad C \approx 6.28r$$

$$C = \pi d \quad \text{or} \quad C = 2\pi r \quad \text{or} \quad C \approx 3.14d$$

Where C is the circumference, d is the diameter, and r is the radius.

 Problem 5	MEDIA EXAMPLE – Finding the Circumference of a Circle
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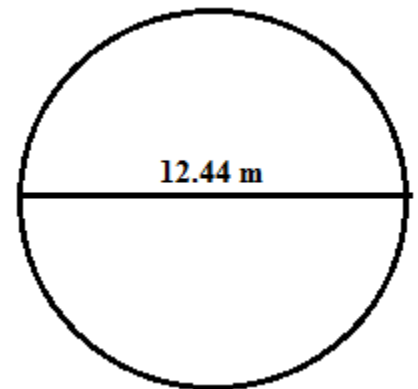
Use the given information to solve the problems. Show all of your work and include units in your answer. Write your answers in *exact form* and in *rounded form* (to the hundredths place).

a) Anderson rollerbladed around a circular lake with a radius of 3 kilometers. How far did Anderson rollerblade?

b) Liz bought a 14 – inch pizza. The server said the 14 inch measurement referred to the diameter of the pizza. What is the circumference of the pizza?

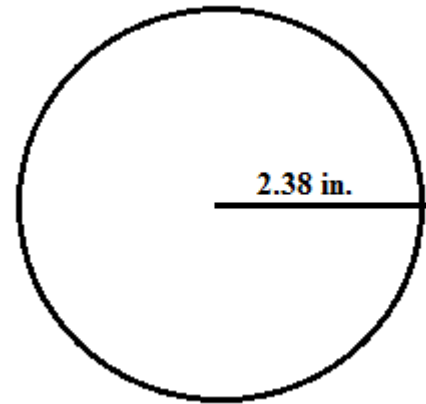
c) Use the diagram of the circle to answer the questions.

i. Are you given the radius or diameter of the circle? How do you know?




ii. Find the circumference of the circle in exact and rounded form.

d) Use the diagram of the circle to answer the questions.



- i. Are you given the radius or diameter of the circle? How do you know?
- ii. Find the circumference of the circle in exact and rounded form.

	Problem 6	YOU TRY – Finding the Circumference of a Circle
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Use the given information to solve the problems. Draw a diagram for each problem labeling either the radius or diameter (as given). Show all of your work and include units in your answer.

- a) The Earth's equator is the circle around the Earth that is equidistant to the North and South Poles, splitting the Earth into what we call the Northern and Southern Hemispheres. The radius of the Earth is approximately 3958.75 miles.

What is circumference of the equator? Write your answers in *exact form* and in *rounded form* (to the hundredths place).

- b) The diameter of a penny is 0.75 inches.

What is circumference of a penny? Write your answers in *exact form* and in *rounded form* (to the hundredths place).

SECTION 10.4: STRATEGIES FOR FINDING AREA

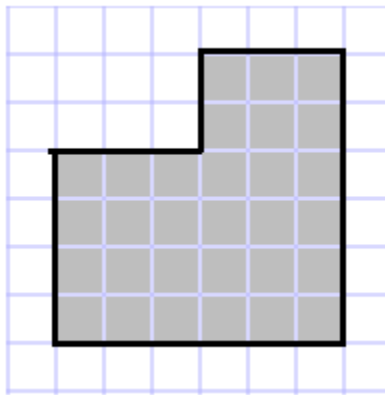
In this section, we will learn to find the area of a two dimensional figure. When we studied perimeter, we found the *one dimensional or linear distance* of the boundary of a two dimensional figure. To find the *area* of a two dimensional figure, we want to find the *two dimensional space inside the figure's boundaries*. Since we are measuring a two dimensional space when we find area, we need a two dimensional measure. Typically, we use *square units* (as opposed to linear units) to measure area. Our goal is to find how many non-overlapping square units *fill up or cover* the inside of the figure. In this section, we will begin our study of area by investigating some common strategies for finding area.



Problem 7

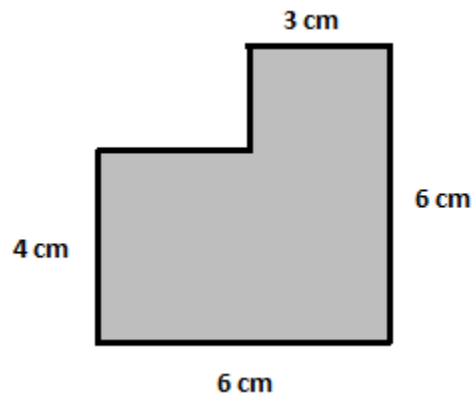
MEDIA EXAMPLE – Strategies for Finding Areas

- a) Find the area of the given shape by counting the square units that cover the interior of the shape. Assume the side of each small square is 1 cm.

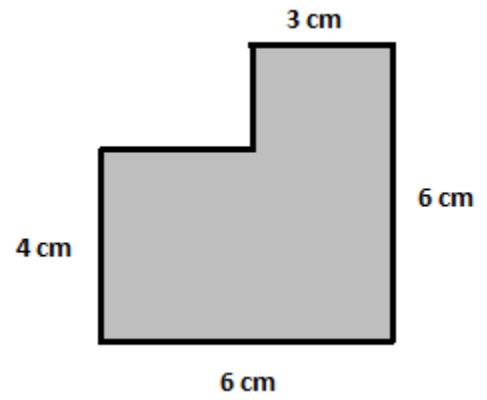


Carrie, Shari, Gary and Larry were given the task of finding the area of the shape, but their teacher didn't give them the grid with the squares to count. Each student knew how to find the area of a rectangle, but they each came up with a different strategy for finding the area of this shape.

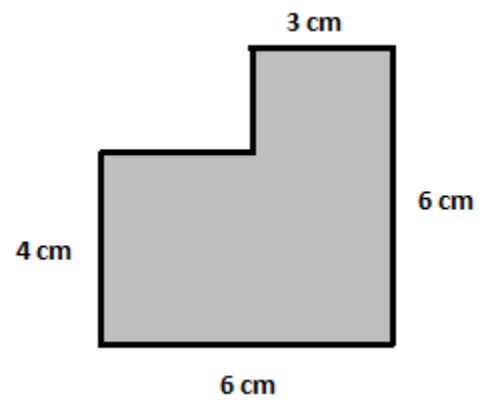
- b) Carrie's Strategy



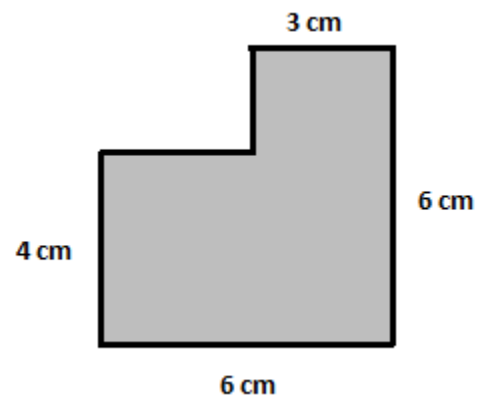
c) Shari's Strategy



d) Gary's Strategy



e) Larry's Strategy



RESULTS – Strategy Types for Finding Areas
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In every example, we used the fact that area of a rectangle can be found by multiplying its length times its width, or equivalently,

For a Rectangle: $Area = length \times width$ or $A = l \cdot w$

The list below contains the specific strategies each student used.


1. Carrie used an *adding strategy* to find the area of shape.
2. Shari used a *subtraction strategy* to find the area of the shape.
3. Gary used a *move and reattach strategy* to find the area of the shape.
4. Larry used a *double and half strategy* to find the area of the shape.

Each of these strategies is valid, and each of the strategies can be helpful when you need to find the area of a shape. When trying to find area, there are two fundamental principles that you need to follow:

The moving principle – you can move a shape and its area doesn't change

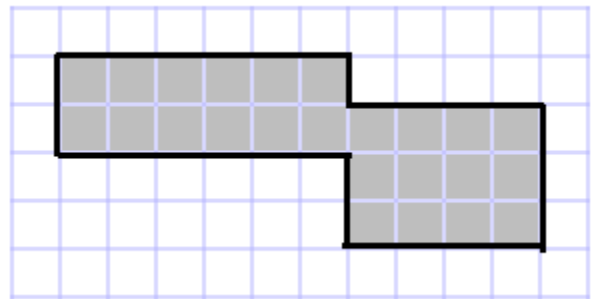
The additivity principle – if you combine shapes without stretching or overlapping them, the area of the new shape is the sum of the area of the smaller shapes.

These two principles allow us to find the area of unusual shapes, because we can divide them into pieces and sum the areas of each piece. We can find the area of a rectangle that surrounds our shape, then we can subtract off the area of pieces that are not part of the rectangle. Or we can reattach the pieces to create shapes that we know how to find the area of. All of the strategies that were used in the example are valid because of the moving and additivity principles.

	Problem 8	YOU TRY – Strategies for Finding Areas
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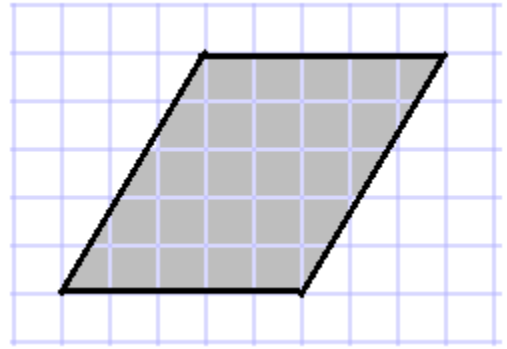
Find the area of the shaded region of the figures using one of the four strategies above. Note which strategy that you used. Show all of your work and include units in your answers. The length of each square in the grid is 1 cm.

- a) Show your work below and in the diagram when needed.



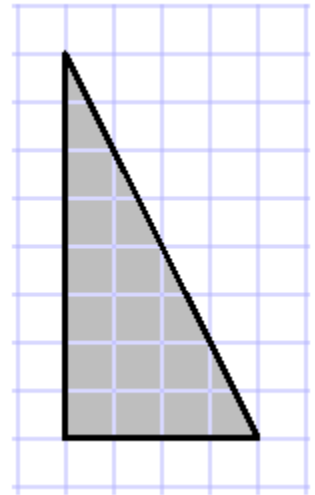
Strategy:

b) Show your work below and in the diagram when needed.



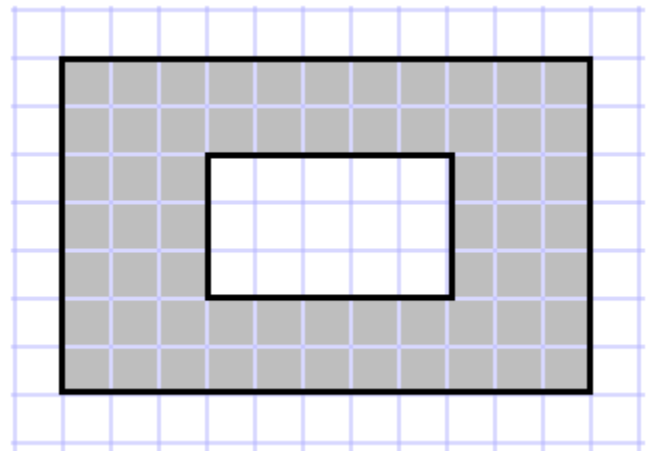
Strategy:

c) Show your work below and in the diagram when needed.



Strategy:

d) Show your work below and in the diagram when needed.



Strategy:

SECTION 10.5: FORMULAS FOR FINDING AREA

For simple shapes, we can often find a formula that will allow us to calculate the area of the shape if we know some measurements of the shape. In this section, we will use the strategies we have learned to develop formulas for some common shapes.

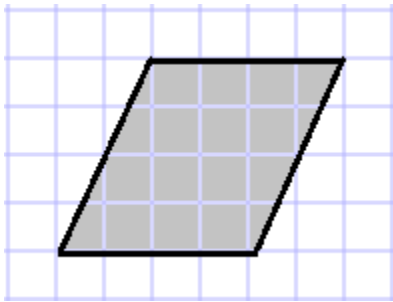


Problem 9

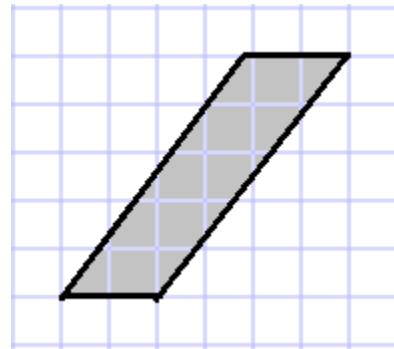
MEDIA EXAMPLE – Finding the Formula for The Area of a Parallelogram

Use the moving and additivity principles to find the areas of the parallelograms. Then use patterns to find a general formula for parallelograms.

a)



b)



Formula for the Area of a Parallelogram:

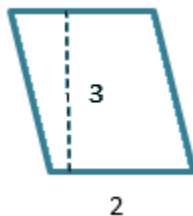


Problem 10

MEDIA EXAMPLE – Applying the Formula for The Area of a Parallelogram

Use the formula for the area of a parallelogram to find the areas. Make sure to indicate which value is the base and which value is the height. Assume that all measures are given in centimeters. Your answer must include units.

a)

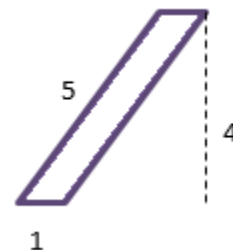


Base:

Height:

Area:


b)



Base:

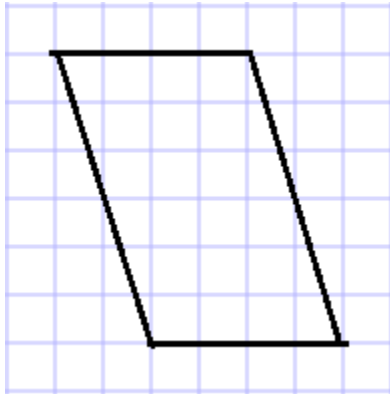
Height:

Area:

	Problem 11	YOU TRY – Applying the Formula for The Area of a Parallelogram
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Use the formula for the area of a parallelogram to find the areas. Make sure to indicate which value is the base and which value is the height. Assume that all measures are given in inches. Your answer must include units.

a)

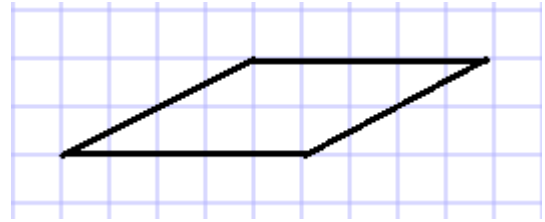


Base:

Height:

Area:


b)



Base:

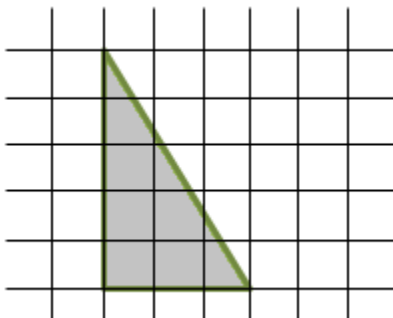
Height:

Area:

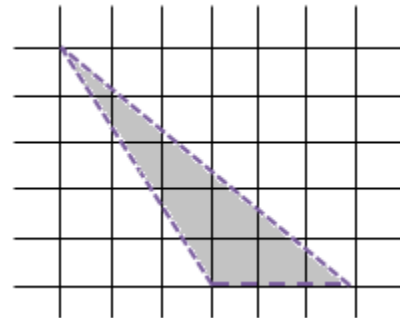
	Problem 12	MEDIA EXAMPLE – Finding the Formula for The Area of a Triangle
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Use the moving and additivity principles to find the areas of the triangles. Then use patterns to find a general formula for triangles.

a)



b)



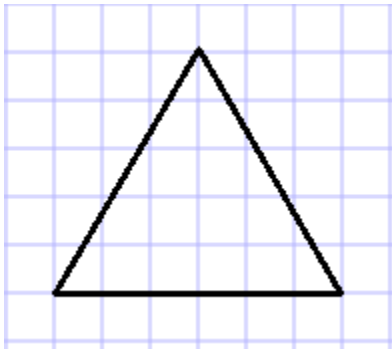
Formula for the Area of a Triangle:



Problem 13

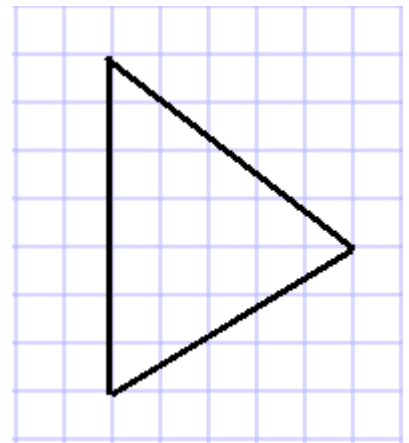
MEDIA EXAMPLE – Applying the Formula for The Area of a Triangle

Use the formula for the area of a triangle to find the areas. Make sure to indicate which value is the base and which value is the height. Assume that all measures are given in centimeters. Your answer must include units.



a)

b)



Base:

Height:

Area:

Base:

Height:

Area:



Problem 14

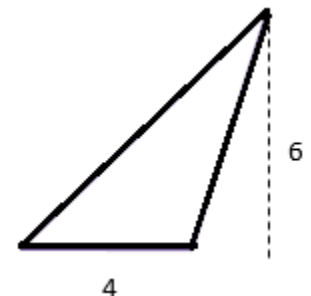
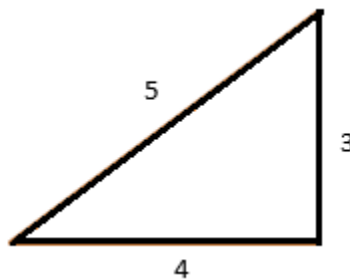
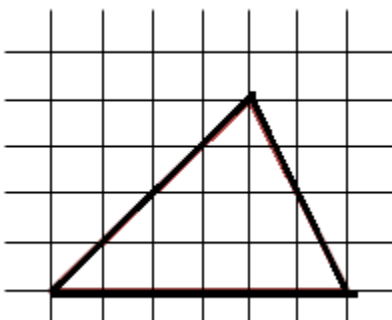
YOU TRY – Applying the Formula for The Area of a Triangle

Use the formula for the area of a triangle to find the areas. Make sure to indicate which value is the base and which value is the height. Assume that all measures are given in feet. Your answer must include units.

a)

b)

c)



Base:

Height:

Area:

Base:

Height:

Area:

Base:

Height:

Area:

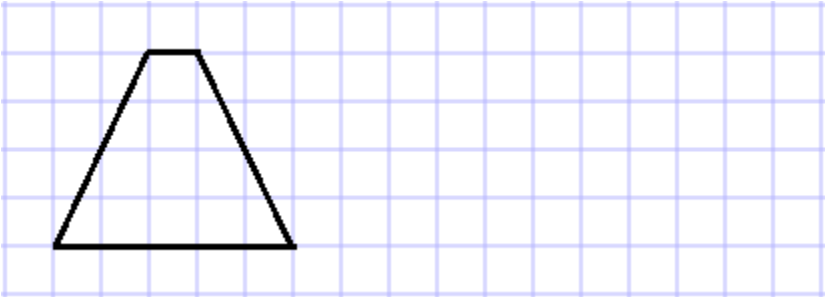


Problem 15

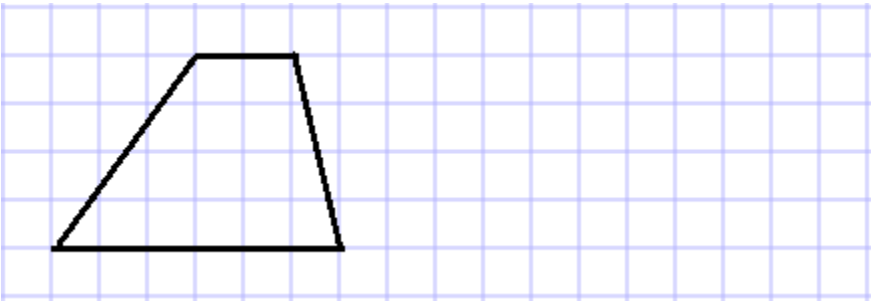
MEDIA EXAMPLE – Finding the Formula for The Area of a Trapezoid

Use the moving and additivity principles to find the areas of the trapezoids. Then use patterns to find a general formula for trapezoids.

a)



b)



c) Formula for the Area of a Trapezoid:

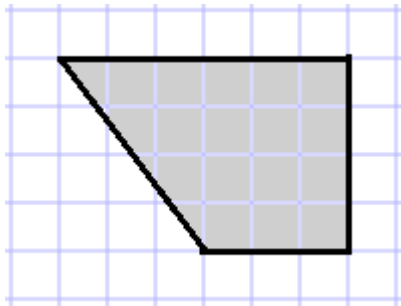


Problem 16

MEDIA EXAMPLE – Applying the Formula for The Area of a Trapezoid

Use the formula for the area of a trapezoid to find the areas. Make sure to indicate the base lengths and the height. Assume that all measures are given in feet. Your answer must include units.

a)



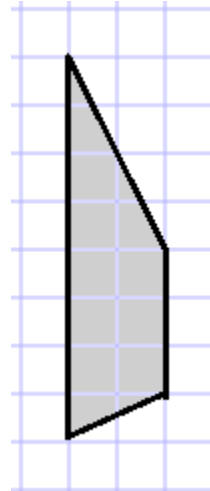
Base 1:

Base 2

Height:

Area:

b)



Base 1:

Base 2:

Height:

Area:



Problem 17

YOU TRY – Applying the Formula for The Area of a Trapezoid

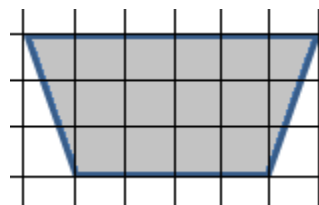
Use the formula for the area of a trapezoid to find the area. Make sure to indicate the base lengths and the height. Assume that all measures are given in kilometers. Your answer must include units.

Base 1:

Base 2

Height:

Area:





Problem 18

MEDIA EXAMPLE – Finding the Formula for The Area of a Circle

Even though a circle looks quite different than the shapes we have been talking about, we can use the move and reattach strategy to derive the formula for finding the area contained within the circle.

- a) Figure A is a circle cut into 8 pieces. Figure B is a rearrangement of these pieces. Approximate the lengths of the two line segments labeled with question marks in Figure B in relation to the radius and circumference of Figure A.

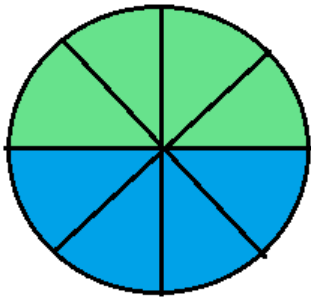


Figure A

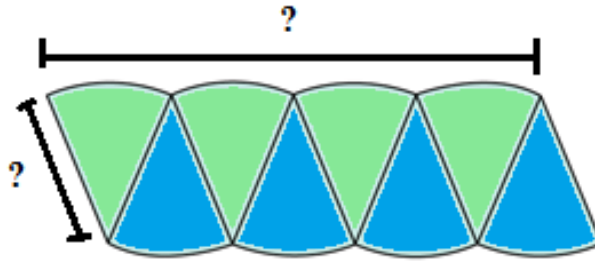


Figure B

If we continue to cut the circle in Figure A into more pieces, we would get the diagrams below. From left to right, the circle is cut into an increasing number of pieces.



- b) Describe the change in shape of the resulting figures as they are cut into more pieces.
- c) If the last figure is equivalent to the area of the original circle after cutting the circle into really small pieces, what is the area of the circle in terms of its radius and circumference?
- d) Write a general formula for the area of a circle in terms of π and the circle's radius.

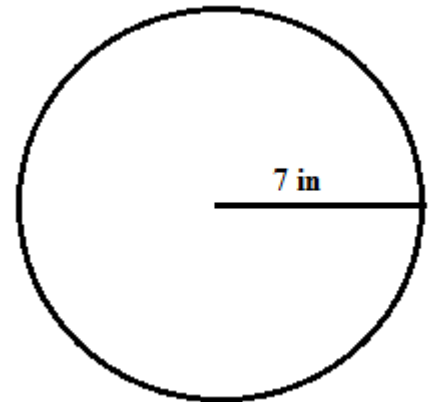


Use the given information to solve the problems. Show all of your work and include units in your answer. Write your answers in *exact form* and in *rounded form* (to the hundredths place).

- a) Liz bought a 14 – inch pizza. The server said the 14 inch measurement referred to the diameter of the pizza. What is the area of the pizza?

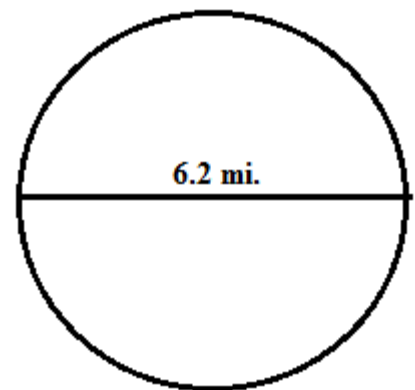
- b) Use the diagram of the circle to answer the questions.

- i. Are you given the radius or diameter of the circle? How do you know?
- ii. Find the area of the circle in exact and rounded form.



- c) Use the diagram of the circle to answer the questions.

- i. Are you given the radius or diameter of the circle? How do you know?
- ii. Find the circumference of the circle in exact and rounded form.





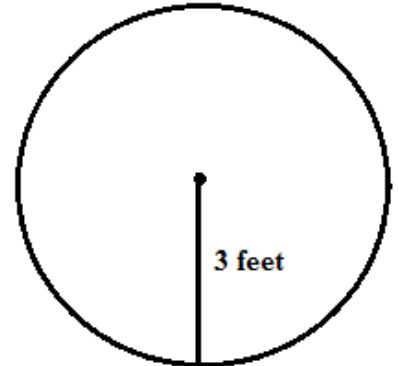
Problem 20

YOU TRY – Applying the Formula for The Area of a Circle

Use the given information to solve the problems. Show all of your work and include units in your answer. Write your answers in *exact form* and in *rounded form* (to the hundredths place).

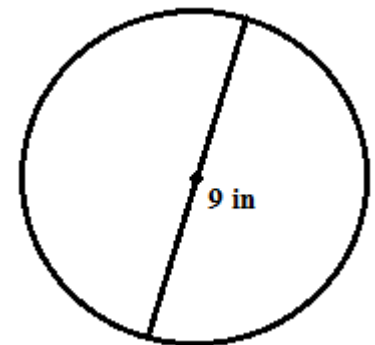
a) Use the diagram of the circle to answer the questions.

1. Are you given the radius or diameter of the circle? How do you know?
2. Find the area of the circle in exact and rounded form.



b) Use the diagram of the circle to answer the questions.

- i. Are you given the radius or diameter of the circle? How do you know?
- ii. Find the circumference of the circle in exact and rounded form.



- c) A circular kiddie pool has a diameter of 4.5 feet. What is the area of the bottom of the pool? Use 3.14 for π and round your answer to two decimal places.

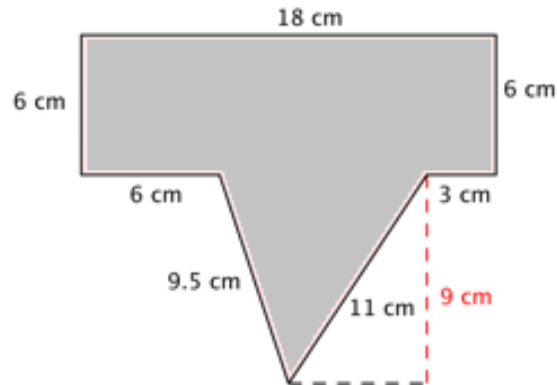


Problem 21

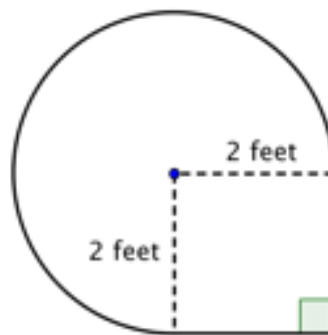
MEDIA EXAMPLE – Finding the Area of Non Standard Shapes

There are no formulas for finding the area of more complicated shapes, however we can use the strategies that were introduced in the beginning of this lesson to help us find areas.

- a) Find the area. Break up the areas into shapes that we recognize and add the area values together.



- b) Find the area of the given shape. Compute using 3.14 for π and round to the nearest hundredth.

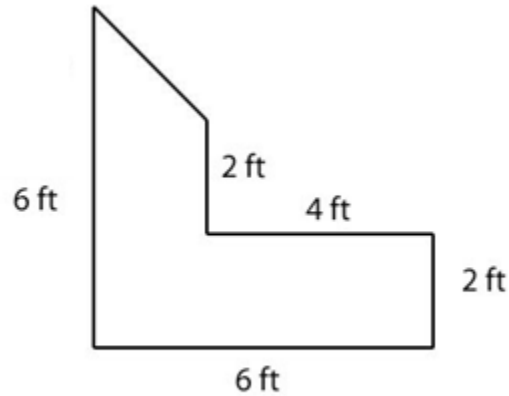




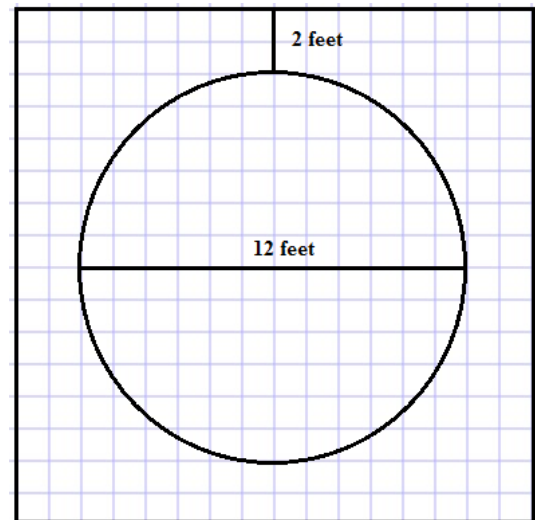
Problem 22

YOU TRY – Finding the Area of Non Standard Shapes

- a) Find the area. Break up the areas into shapes that we recognize and add the area values together.



- b) Jackson is putting an above ground swimming pool in his yard. The pool is circular, with a diameter of 12 ft. He wants to put a square deck around the pool that is at least two feet wider than the pool on each edge.
- i. How much space will the pool and deck take up in his yard?



- ii. What is the area of the surface of the pool?
- iii. What is the area of the deck that he is designing?

Summary of Formulas

Perimeter: The *perimeter* of a two dimensional figure is the one dimensional total distance around the edge of the figure.

To find the perimeter of any polygon (sides are lines)

1. Determine all of the side lengths
2. If one or more side lengths aren't given, use the other side lengths to determine the missing side lengths.
3. Find the sum of all the sides.

Circumference: The *circumference* is distance around the boundary of a circle. The circumference is equivalent to the perimeter of a polygon, but for circles. Since circles do not have lines as sides, we cannot add up the sides, and we need a special formula.

To find the circumference of a circle

1. Determine either the radius or the diameter. Make sure you know which one you are using.
2. If you know the radius, r ,
 $C = 2\pi r$ (exact) or $C = 2(3.14) \cdot r$ (approximation for π)
3. If you know the diameter, d ,
 $C = \pi d$ (exact) or $C = (3.14) \cdot d$ (approximation for π)

Area: The *area* is the number of square units that fills the inside of a figure. There are different formulas depending on the shape of the figure.

Area of a Square = $side \cdot side$	Area of a Trapezoid = $\frac{\text{sum of parallel bases}}{2} \cdot \text{height}$
Area of a Rectangle = $length \cdot width$	Area of a Triangle = $\frac{1}{2} \cdot \text{base} \cdot \text{height}$
Area of a Parallelogram = $base \cdot height$	Area of a Circle $A = \pi \cdot r^2$ (exact) or $A = 3.14 \cdot r^2$ (approximate)

Area of Composite Figures: If you need the area of an uncommon shape, you need to cut it into pieces so that you can find the area of the separate pieces with known formulas. Then you can use addition or subtraction to find the area. The principles below describe these methods.

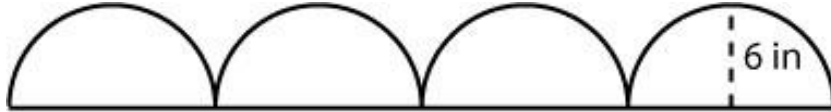
The moving principle – you can move a shape and its area doesn't change

The additivity principle – if you combine shapes without stretching or overlapping them, the area of the new shape is the sum of the area of the smaller shapes.

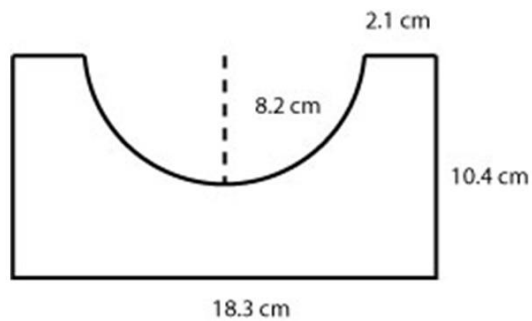
Unit 10 – Practice Problems

2. Find the circumference or perimeter given each described situation. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

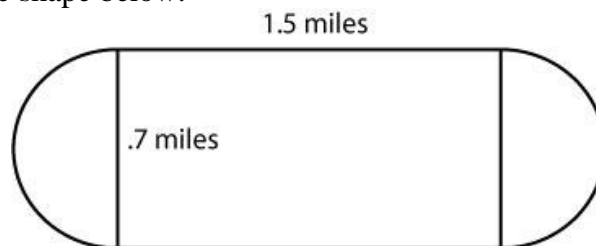
a) If the radius of each half circle is 6 inches, find the perimeter of the object.



b) Find the perimeter of the shape below.

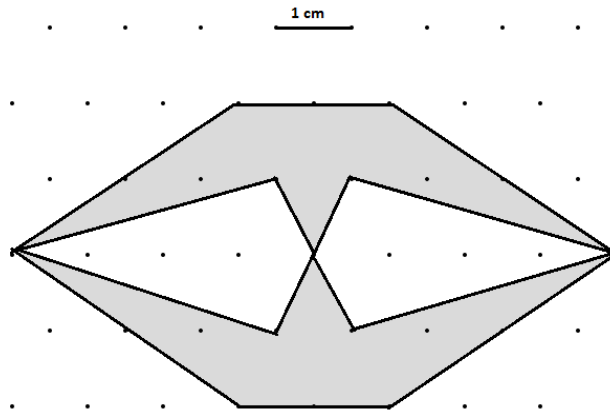


c) Find the perimeter of the shape below.



Unit 10 – Practice Problems

4. Use the moving and additivity principles to find the shaded area.

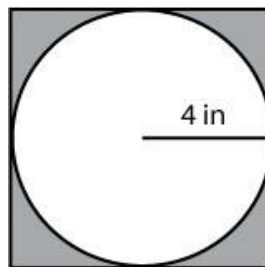


5. Find the area as requested below. Show all work. As in the examples, if units are included then units should be present in your final result. Use 3.14 for pi and round answers to tenths as needed.

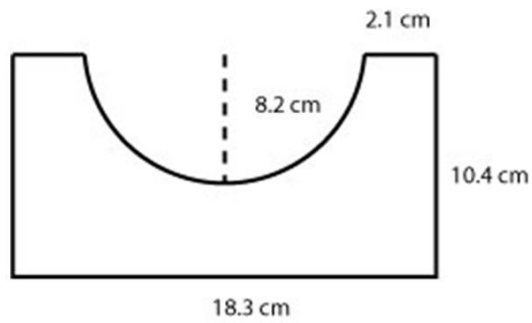
a) If the radius of each half circle is 6 inches, find the area of the object.



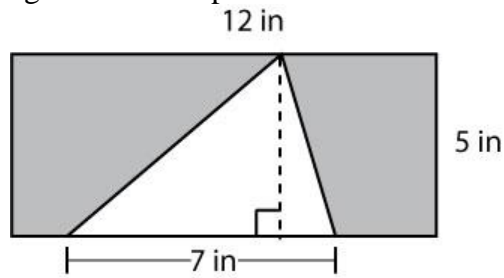
b) Find the area of the shaded region in the shape below.



c) Find the area of the shape below.

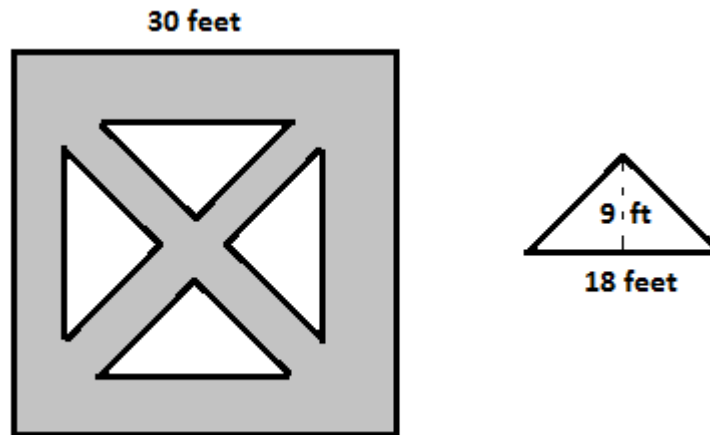


d) Find the area of the shaded region in the shape below.

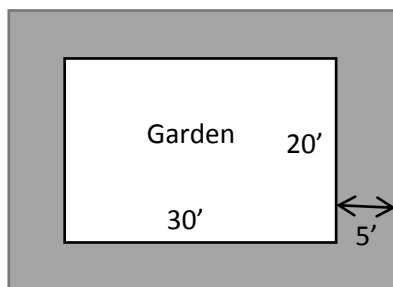


6. Draw 4 rectangles each that have area 24 square feet but different perimeters. Try to draw your rectangles with some relative accuracy to each other and include units.

11. The picture shows the design for an herb garden, with approximate dimensions shown. Four identical plots of land in the shape of right triangles are surrounded by paths. Use the moving and additivity principles to determine the area of the paths.

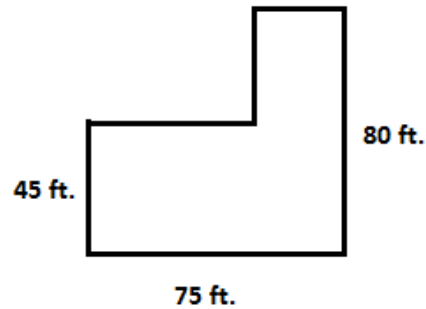


12. Wally wants to build a 5 ft. walkway around his garden that is 20 ft. wide and 30 ft. long. What will the area of the walkway (the shaded area in the drawing) be?



UNIT 10 – END OF UNIT ASSESSMENT

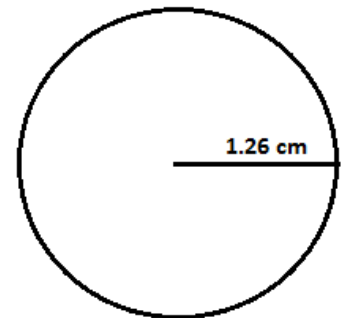
1. A shopping center has the shape and dimensions below. Find the perimeter of the shopping center. Include units in your answer.



2. Scott bought a 16 – inch pizza. The server said the 16 inch measurement referred to the diameter of the pizza. What is the circumference of the pizza? Include units in your answer.

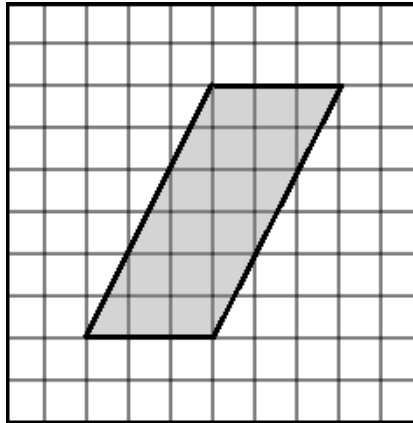
3. Use the diagram of the circle to answer the questions.

- i. Are you given the radius or diameter of the circle? How do you know?
- ii. Find the circumference of the circle in exact and rounded form.

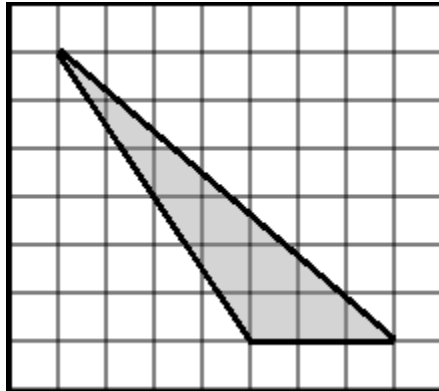


Unit 10 – Assessment

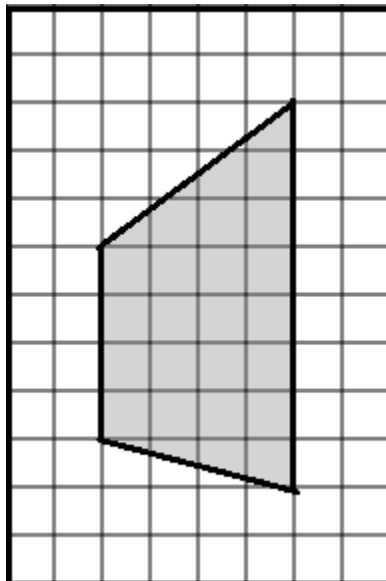
4. Determine the area of the parallelogram. Indicate any length measure units you use in your computation. Write your final answer in square units.



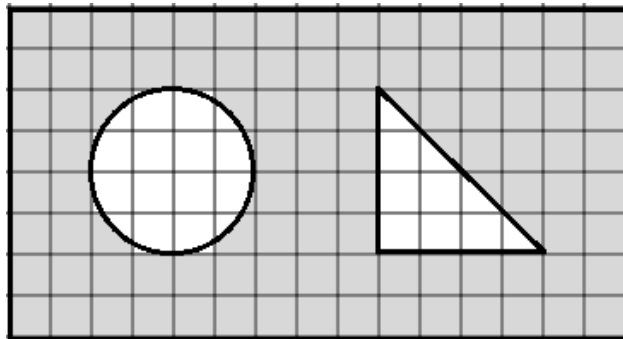
5. Determine the area of the triangle. Indicate any length measure units you use in your computation. Write your final answer in square units.



6. Determine the area of the trapezoid. Indicate any length measure units you use in your computation. Write your final answer in square units.



7. A circular pool has a diameter of 12 feet. What is the area of the bottom of the pool? Use 3.14 for π and round your answer to two decimal places.
8. An amusement park has a rectangular shape with a circular merry go round and a triangular concession stand. The shaded area represents the concrete sidewalks around the venue. Assuming the length of each square in the gridded diagram is 1 meter, what is the total area of the sidewalks?



UNIT 11 – VOLUME AND THE PYTHAGOREAN THEOREM

INTRODUCTION

In this Unit, we will use the idea of measuring volume that we studied to find the volume of various 3 dimensional figures. We will also learn about the Pythagorean Theorem, one of the most famous theorems in mathematics. We will use this theorem to find missing lengths of right triangles and solve problems.

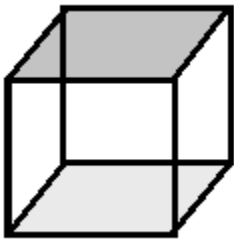
The table below shows the learning objectives that are the achievement goal for this unit. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

Learning Objective	Media Examples	You Try
Use the concept of stacking cubes to find the volume of a prism	1	2
Use the concept of stacking cubes to find the volume of a cylinder	3	4
Use formulas to find the volumes of spheres, cones, and pyramids	5	6
Use the additivity and moving principles to develop the concept behind Pythagorean Theorem	7	
Use grids and squares to find square roots and determine if a whole number is a perfect square	8	9
Apply the Pythagorean Theorem to find a missing side of a right triangle or solve an application problem	10	11

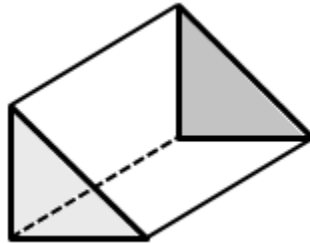
UNIT 11 –MEDIA LESSON

SECTION 11.1: VOLUME OF PRISMS

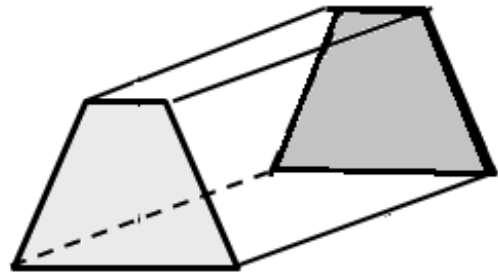
In this section, we will learn how to find the volume of a prism. Recall that when we measure the attribute of *volume*, we are finding the 3 dimensional space that a 3 dimensional object takes up or fills. A *prism* is a 3 dimensional object where two of its opposite sides are parallel and identical (called the bases), and the sides connecting them are squares, rectangles, or parallelograms. Here are some examples of prisms where the bases are shaded.



Square Prism (cube)



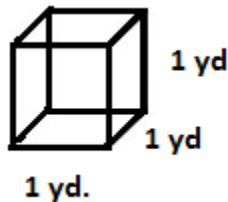
Triangular Prism



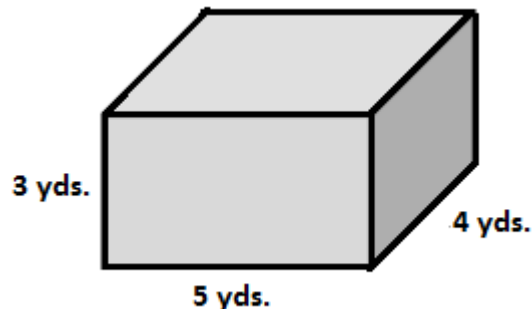
Trapezoidal Prism

Let's look at an example of finding the volume of a rectangular prism.

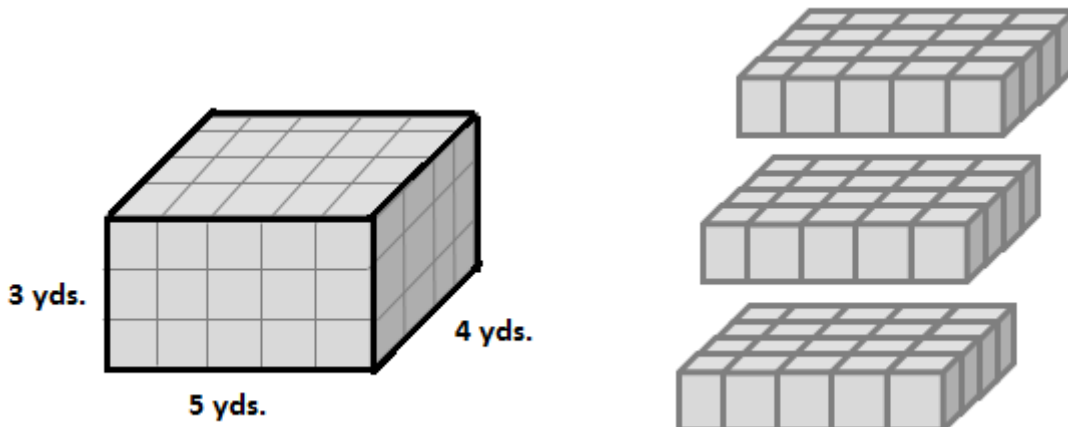
Example: Suppose you want to build a concrete patio, you will need to order the concrete in units of cubic yards. In unit 12, we learned that a cubic yard looks like a cube with a length of 1 yd, a width of 1 yd, and a depth of 1 yd. So 1 cubic yard of concrete is the amount of concrete that would fit in the box below.



When we calculate volume, we are finding how many unit cubes will fill up the space that we are calculating the volume of. If the concrete patio has the shape and dimensions below, we want to know how many cubic yard units will fill up the space.




When we find the volume of this solid, we are imagining filling the box with cubic yards, or cubes with length of 1 yard, width of 1 yard, and depth of 1 yard. It is a little easier to determine the number of cubic yards in the box if we think of the height representing the number of layers of cubes in our box. Now we might say that there are 3 layers of 4 by 5 arrays of cubes. So the total number of cubes must be $3 \cdot 4 \cdot 5$ cubes, or 60 cubic yards.



This strategy will always work when you are finding the volume of a prism. If you know how many cubes are in the bottom layer, then you can multiply that by the number of layers in the solid to find the volume.

Formally, we say that the volume of a prism is equal to the Area of the base times the height of the prism where the height is the distance between the two bases.

	Problem 1	MEDIA EXAMPLE – Volume of a Prism
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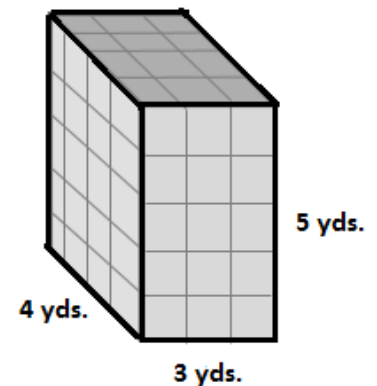
Write all the indicated measurements and attributes of the given prisms. Then find the volume of the solids. Include units in your answers.

- The figure to the right is the same shape as the previous example from the text, but rotated a quarter of a turn. Find its volume by using the top of the figure as the base.

Area of the Base:

Height of Prism (distance between two bases):

Volume of the Prism:



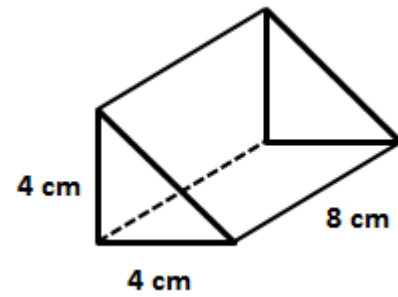
How does the volume of this figure compare to the volume of the previous example? Why do you think this relationship holds?

2. Shape of the Base:

Area of the Base:

Height of Prism (distance between two bases):

Volume of the Prism:

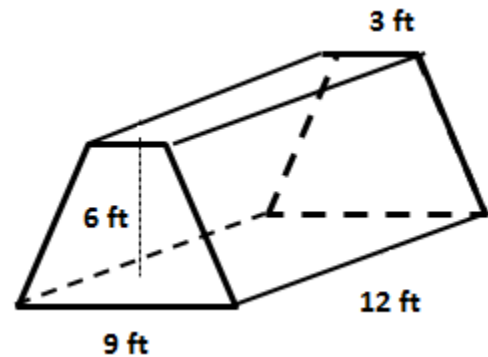


3. Shape of the Base:

Area of the Base:

Height of Prism (distance between two bases):

Volume of the Prism:



	Problem 2	YOU TRY – Volume of a Prism
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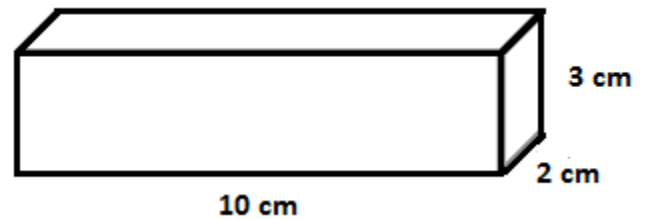
Answer the following questions. Include units in all of your answers when appropriate.

- a) Shade one of the sides that you are using as one of the bases of your prism (more than one correct answer).

Area of the Base:

Height of Prism (distance between two bases):

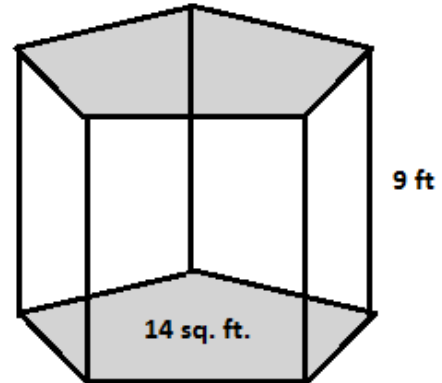
Volume of the Prism:



b) Area of the Base:

Height of Prism (distance between two bases):

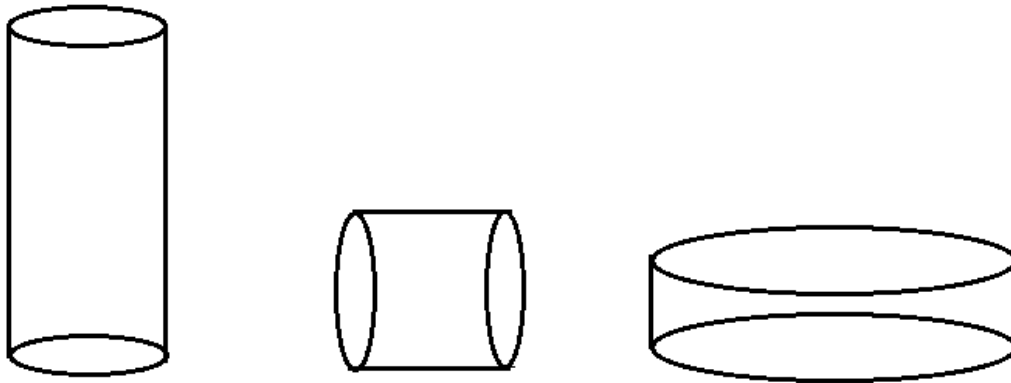
Volume of the Prism:



c) Gloria is making coffee themed gift baskets for her friends. She found some small boxes that she will fill with sugar cubes as one of the items in the basket. The boxes are 5 cm wide, 8 cm long, and 3 cm high. She measures the sugar cubes and finds that they are perfect centimeter cubes! How many sugar cubes will she need to fill each box? What is the volume of the box measured in cm^3 ?

SECTION 11.2: VOLUME OF A CYLINDER

A cylinder is similar to a prism in that they both have two parallel, identical bases. However, a cylinder's base is a circle, and the sides are not parallelograms, but are smooth like a circle. Some cylinders you may have seen in everyday life are soda cans, a tennis ball container, a paint can, or a candle. Here are some images of cylinders.

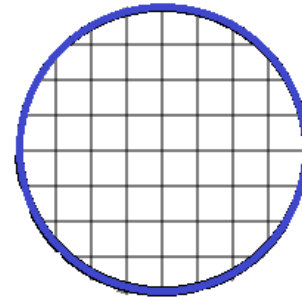


Unit 11 – Media Lesson

We can use the same reasoning that we used when we found the volume of a prism to find the volume of a cylinder. The image below is of the base of a cylinder. The interior, or area, is on grid paper so we can imagine stacking cubes on the base to find a volume.

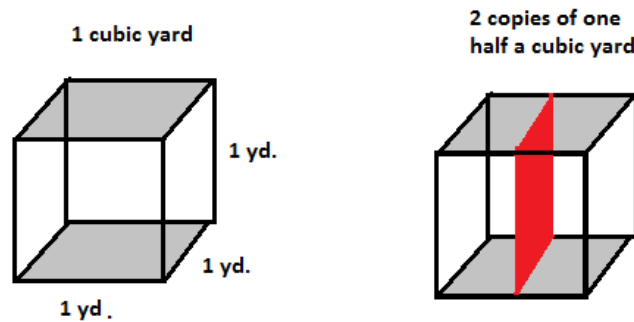
Radius of Base of Cylinder: 4 units

Area of base of cylinder: $\pi r^2 = \pi \cdot 16 = 16\pi \approx 50.24 \text{ units}^2$



Base of Cylinder

Since the base of the cylinder is a circle, some of the squares in the base are partial squares. However, we can still imagine stacking partial cubes with a base of the size of each of the partial squares and one unit high. For example, if we took 1 cubic yard, and split the bases in half, we would have 2 copies of $\frac{1}{2} \text{ yd}^3$ as shown in the image below.



So for any partial square in the base of a cylinder, we can stack a partial cube of height 1 with the base of the square and the result is the area of the square times 1 cubic units. This means that even for partial squares in the base, we can stack cubes with a height of 1 unit and attain a measure of volume.

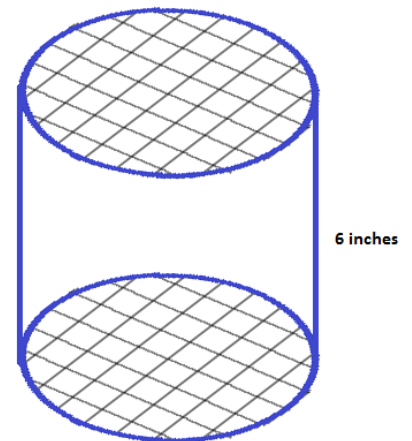
The image to the right is of a cylinder using the base given above and with a height of 6 inches. We'll now use a radius of 4 inches (as opposed to generic units). Notice how the squares and partial squares line up between the top and bottom bases. Now imagine stacking the cubic inches and partial cubic inches from bottom to top. The total number of these cubes will equal the volume of the cylinder.

Radius of Base of Cylinder: 4 inches

Area of base of cylinder: $\pi r^2 = \pi \cdot 16 = 16\pi \approx 50.24 \text{ in}^2$

Volume of cylinder: $6 \cdot \pi r^2 = 6 \cdot \pi \cdot 16 = 96\pi \approx 301.44 \text{ in}^3$

In general, like a prism, the volume of a cylinder is the area of its base times its height.





Problem 3

MEDIA EXAMPLE – Volume of a Cylinder

Write all the indicated measurements and attributes of the given cylinders. Then find the volume of the solids. Include units in your answers. Give your answer in exact form (using π) and approximate form using $\pi \approx 3.14$.

- a) Find the following measures for the figure to the right. The squares in the bases are square feet.

Area of the Base

Exact Form:

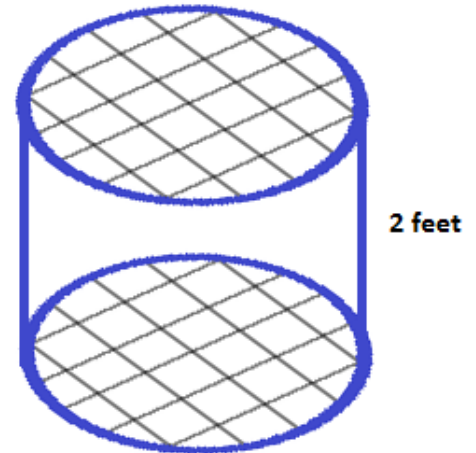
Approximate Form:

Height of Cylinder (distance between two bases):

Volume of the Cylinder

Exact Form:

Approximate Form:



- b) Find the following measures for the cylinder to the right.

Area of the Base

Exact Form:

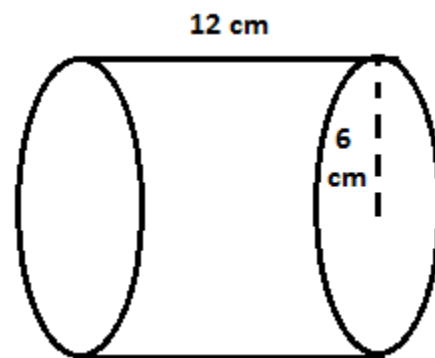
Approximate Form:

Height of Cylinder (distance between two bases):

Volume of the Cylinder

Exact Form:

Approximate Form:

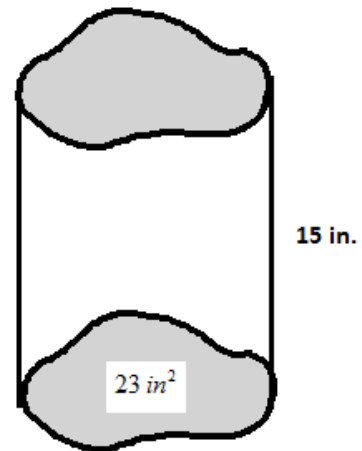


- c) The figure to the right is not a prism or a cylinder, but it has two identical parallel bases. Use the given information and the reasoning from this section to find the following.

Area of the Base:

Height of the Figure:

Volume of the Figure:



Problem 4

YOU TRY – Volume of a Cylinder

- a) Find the following measures for the cylinder to the right.

Area of the Base

Exact Form:

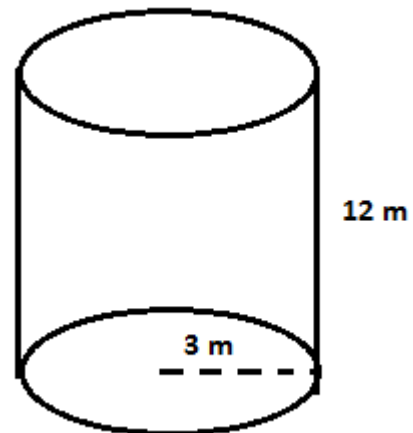
Approximate Form:

Height of Cylinder (distance between two bases):

Volume of the Cylinder

Exact Form:

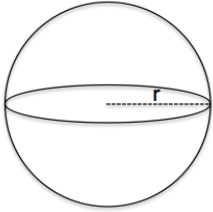
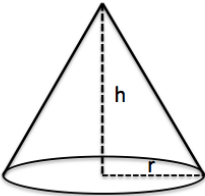
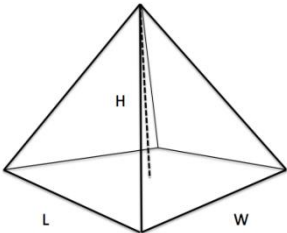
Approximate Form:



- b) Donna is making a cylindrical candle. She wants it to fit exactly in her candle holder which has a radius of 5.5 cm. She is going to make the candle 14 cm tall. How many cubic centimeters of wax will Donna need to make the candle? (Use 3.14 for π)

SECTION 11.3: VOLUMES OF OTHER SHAPES

It is helpful to know the formula for calculating the volume of some additional shapes. The mathematics for developing these formulas is beyond the scope of this class, but the formulas are easy to use. The chart below shows the formulas to find the volumes of some other basic geometric shapes.

Shape	Volume
Sphere with radius r 	$V = \frac{4}{3} \pi r^3$
Cone with height h and base radius r 	$V = \frac{1}{3} \pi r^2 h$
Pyramid 	$V = \frac{1}{3} l \cdot w \cdot h$

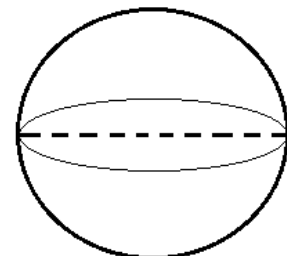


Problem 5

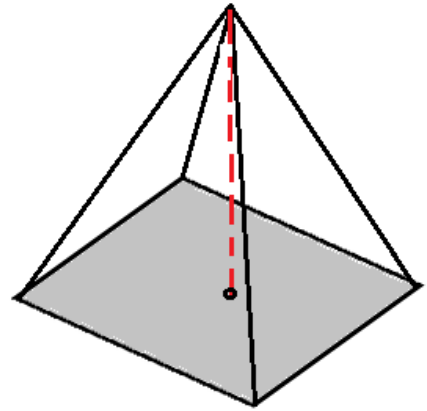
MEDIA EXAMPLE – Volumes of Other Shapes

Determine the volume of each of the following solids. Label any given information in the figure. Include units in your final result and round your answers to two decimal places.

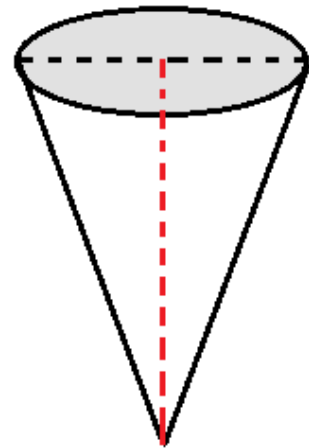
- a) A basketball has a diameter of approximately 9.55 inches. Find the volume of the basketball.




- b) The Great Pyramid of Giza in Egypt has a square base with side lengths of approximately 755.9 feet and a height of approximately 480.6 feet. Find the volume of the pyramid.



- c) An ice cream cone has a diameter of 8 cm and a height of 13 cm. What is the volume of the ice cream cone?



	Problem 6	YOU TRY – Volumes of Other Shapes
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The planetary object Pluto is approximately spherical. Its diameter is approximately 3300 miles. Find the volume of Pluto. Include units in your final result and round your answers to two decimal places.

SECTION 11.4: INTRODUCTION TO THE PYTHAGOREAN THEOREM

We discussed in Unit 13 that the perimeter of a shape is equal to the distance around the shape. We can only find the perimeter if we know the length of *all of the sides*. Sometimes we can use properties of the shape to find unknown side lengths. For example, if we know that the length of one side of a square is 5 inches, then we know that the other three lengths are 5 inches because a square has 4 equal side lengths. The Pythagorean Theorem is a useful formula that relates the side lengths of right triangles. In our first example, we will derive a result of the Pythagorean Theorem with special numbers and then use the information to determine the theorem in general.

	Problem 7	MEDIA EXAMPLE – Introduction to the Pythagorean Theorem
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Find the indicated areas requested below.

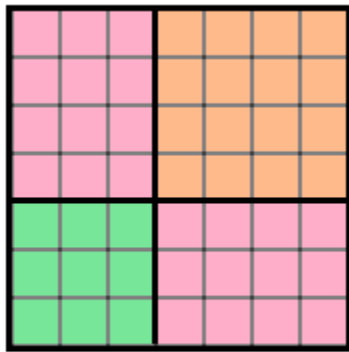


Figure A

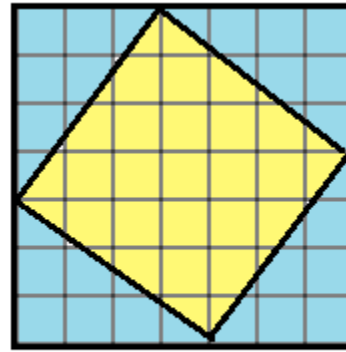


Figure B

- a) Find the total Area of Figure A.
- b) Find the total Area of Figure B.
- c) Find the corresponding Areas in Figures A and B and fill in the table below.

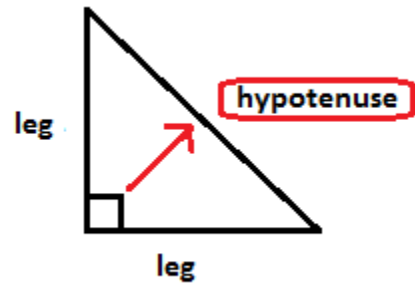
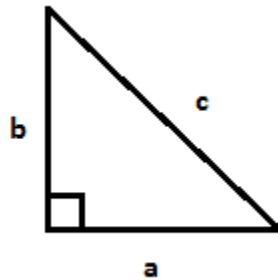
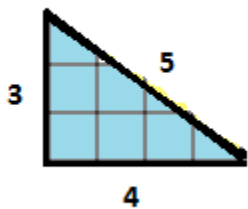
Figure	Computation	Simplified Result
Figure A: Pink rectangle		
Figure A: Green rectangle		
Figure A: Orange rectangle		
Figure B: Blue Triangle		

- d) Use the information in the table to find the area of the yellow shape in Figure B.
- e) The yellow shape is a square. How can you tell this from Figure B?
- f) Find the side length of the yellow square in Figure B.

RESULTS: PYTHAGOREAN THEOREM

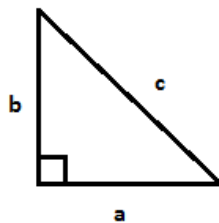
In the last example, we found that the area of the yellow square was the sum of the squares of the two known sides of the blue triangle. We also found that since the yellow shape was square, we could find the missing side length of the triangle by finding the number that when multiplied by itself gave us the area of the yellow square, namely, the missing side length was 5 since $5 \times 5 = 25$.

We can extend this idea to any right triangle and the result will always hold. The diagram below shows corresponding labels we use for right triangles in general when we discuss the Pythagorean Theorem. Notice that two of the sides of a right triangle are called *legs* and we label them with the letters *a* and *b*. It actually doesn't matter which we call *a* and which we call *b* as long as we are consistent in our computations. However, the third side has a special name called the *hypotenuse*. It is the side opposite the right angle in the rightmost diagram. When we use Pythagorean Theorem formulas, make sure you only use the hypotenuse for the letter *c*.

**The Pythagorean Theorem:**

The mathematician Pythagoras proved the Pythagorean Theorem. The theorem states that given any right triangle with sides *a*, *b*, and *c* as below, the following relationship is always true:

$$a^2 + b^2 = c^2$$



Notes about the Pythagorean Theorem:

- The triangle must be a RIGHT triangle (contains an angle that measures 90°).
- The side *c* is called the *Hypotenuse* and ALWAYS sits opposite from the right angle.
- The lengths *a* and *b* are interchangeable in the theorem but *c* cannot be interchanged with *a* or *b*. In other words, the location of *c* is very important and cannot be changed.

In the next section, we will learn about square roots and then write the Pythagorean Theorem in alternate formats to make our computations easier.

SECTION 11.5: SQUARE ROOTS

The **square root** of a number is that number which, when multiplied times itself, gives the original number. For example,

$$4 \cdot 4 = 4^2 = 16$$

So we say “the square root of 16 equals 4”. We denote square roots with the following notation.

$$\sqrt{16} = 4$$

A **perfect square** is a number whose square root is a whole number. The list below shows the first eight perfect squares.

$$1^2 = 1 \quad 2^2 = 4 \quad 3^2 = 9 \quad 4^2 = 16 \quad 5^2 = 25 \quad 6^2 = 36 \quad 7^2 = 49 \quad 8^2 = 64$$

We write the corresponding square root statements as shown below.

$$\sqrt{1} = 1 \quad \sqrt{4} = 2 \quad \sqrt{9} = 3 \quad \sqrt{16} = 4 \quad \sqrt{25} = 5 \quad \sqrt{36} = 6 \quad \sqrt{49} = 7 \quad \sqrt{64} = 8$$

The square root of a non-perfect square is a decimal value. For example, 19 is NOT a perfect square because $\sqrt{19} \approx 4.36$ is not a whole number.

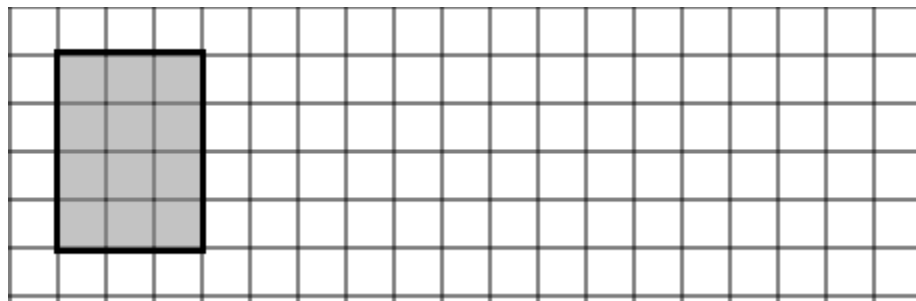


Problem 8

MEDIA EXAMPLE – Square Roots

Determine whether the given figures can be rearranged into squares with whole number side lengths. If so, determine the square root of the number. If not, determine what two perfect squares the number lies between.

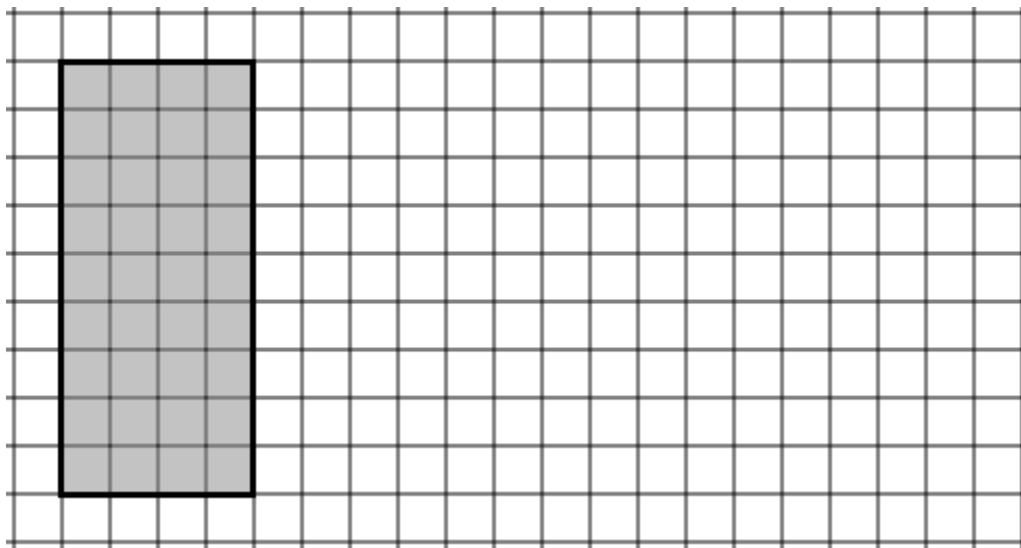
- a) 12 square units



- i. Is 12 a perfect square?
- ii. If 12 is a perfect square, what does $\sqrt{12}$ equal?
- iii. If 12 is not a perfect square, what two whole numbers does $\sqrt{12}$ lie between?

Unit 11 – Media Lesson

b) 36 square units



- i. Is 36 a perfect square?
 - ii. If 36 is a perfect square, what does $\sqrt{36}$ equal?
 - iii. If you added 1 more square unit, you would have 37 square units. Is 37 a perfect square? How do you know?
 - iv. What two whole numbers does the $\sqrt{37}$ lie between?
- c) Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

i. $\sqrt{81}$

ii. $\sqrt{20}$

iii. $\sqrt{9}$

iv. $\sqrt{60}$



Problem 9

YOU TRY – Square Roots

Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

a) $\sqrt{49}$

b) $\sqrt{17}$

c) $\sqrt{80}$

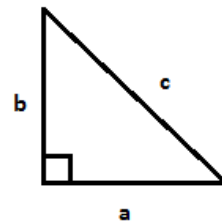
SECTION 11.6: APPLYING THE PYTHAGOREAN THEOREM

Now that we have learned about square roots, we are going to write the Pythagorean Theorem in some different forms that involve square roots so we can use the Pythagorean Theorem without algebra. The following are alternative forms of the Pythagorean Theorem and you will use them.

Pythagorean Theorem solved for a leg (*a or b*)

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$



Use either of these formulas when you are given either leg and the hypotenuse and need to find a missing leg. Again, the labeling of *a or b* is arbitrary (as long as they are both legs), but once you label your diagram with a specific letter, make sure you use it consistently.

Pythagorean Theorem solved for the hypotenuse (*c*)

$$c = \sqrt{a^2 + b^2}$$

Use this formula when you are given both values for the legs and need to find the hypotenuse.

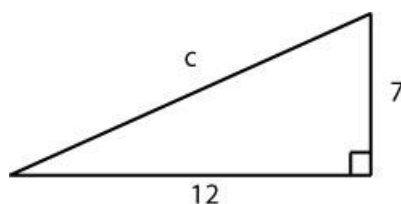


Problem 10

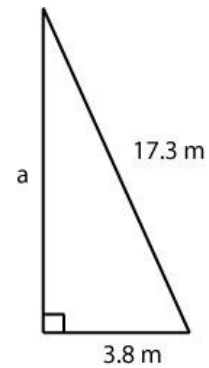
MEDIA EXAMPLE – Applying the Pythagorean Theorem

Use the Pythagorean Theorem to find the missing length of the given triangles. Round your answer to the tenth's place when needed.

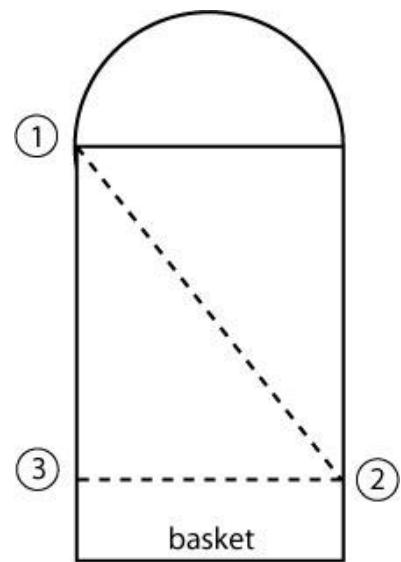
- a) Find the unknown side of the triangle.



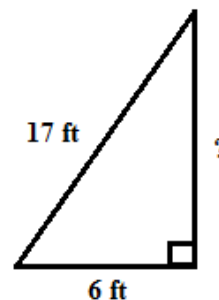
- b) Find the unknown side of the triangle.



- c) In NBA Basketball, the width of the free-throw line is 12 feet. A player stands at one exact corner of the free throw line (Player 1) and wants to throw a pass to his open teammate across the lane and close to the basket (Player 2). If his other teammate (Player 3 – heavily guarded) is directly down the lane from him 16 feet, how far is his pass to the open teammate? Fill in the diagram below and use it to help you solve the problem. (Source: <http://www.sportsknowhow.com>).



- d) Sara is flying her kite and it gets stuck in a tree. She knows the string on her kite is 17 feet long and she is 6 feet from the tree. How long of a ladder (in feet) will she need to get her kite out of the tree? Round your answer to the nearest hundredth as needed.



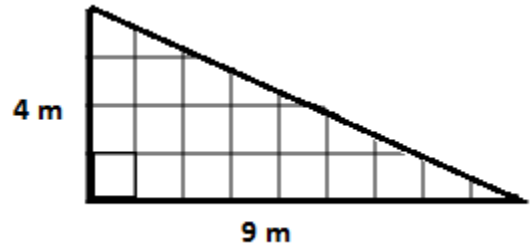


Problem 11

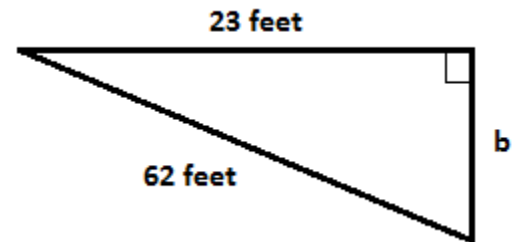
YOU TRY – Applying the Pythagorean Theorem

Use the Pythagorean Theorem to find the missing length of the given triangles. Round your answer to the tenth's place when needed.

- a) Find the unknown side of the triangle.



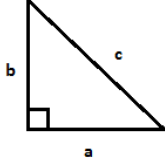
- b) Find the unknown side of the triangle.



- c) Given a rectangular field 105 feet by 44 feet, how far is it to walk from one corner of the field to the opposite corner? Draw a picture to represent this situation. Round your answer to the nearest tenth as needed.

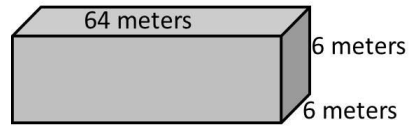
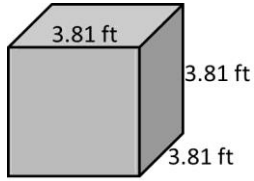
Summary of Formulas:

Volume: The *volume* of a three dimensional figure is the three dimensional space that fills the figure measured in cubic units.

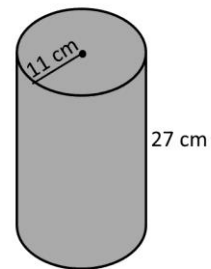
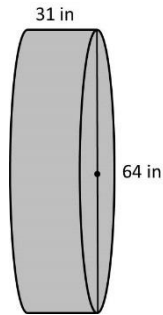
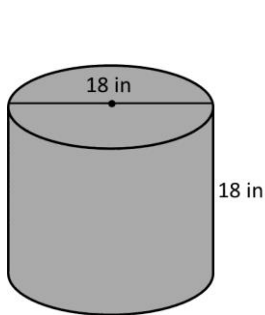
Volume of Prisms		
<p style="text-align: center;">Volume of a Cube (base is a square)</p> <p>Area of a Square = $side \cdot side$</p> <p>Volume of a Cube = Area of square base \cdot height of cube</p> <p>$V = side \cdot side \cdot side$</p>	<p style="text-align: center;">Volume of a Trapezoidal Prism (base is a trapezoid)</p> <p>Area of a Trapezoid = $\frac{sum\ of\ parallel\ bases}{2} \cdot height$</p> <p>Volume of a Trapezoidal Prism = Area of Trapezoid base \cdot height of prism</p> <p>$V = \left[\frac{sum\ of\ parallel\ bases}{2} \cdot height \right] \cdot (height\ of\ prism)$</p>	
<p style="text-align: center;">Volume of a Rectangular Prism (base is a rectangle)</p> <p>Area of a Rectangle = $length \cdot width$</p> <p>Volume of a Rectangular Prism = Area of Rectangle base \cdot height of prism</p> <p>$V = length \cdot width \cdot (height\ of\ prism)$</p>	<p style="text-align: center;">Volume of a Triangular Prism (base is a triangle)</p> <p>Area of a Triangle = $\frac{1}{2} \cdot base \cdot height$</p> <p>Volume of a Triangular Prism = Area of Triangle base \cdot height of prism</p> <p>$V = \frac{1}{2} \cdot base \cdot height \cdot (height\ of\ prism)$</p>	
<p style="text-align: center;">Volume of a Parallelogram Prism (base is a parallelogram)</p> <p>Area of a Parallelogram = $base \cdot height$</p> <p>Volume of a Parallelogram Prism = Area of Parallelogram base \cdot height of prism</p> <p>$V = base \cdot height \cdot (height\ of\ prism)$</p>	<p style="text-align: center;">Volume of a Cylinder (base is a circle)</p> <p>Area of a Circle $A = \pi \cdot r^2$ (exact) or $A = 3.14 \cdot r^2$ (approximate)</p> <p>Volume of a Cylinder = Area of Circle base \cdot height of Cylinder</p> <p>$V = \pi \cdot r^2 \cdot (height\ of\ cylinder)$</p>	
Special Volumes:		
<p>Pyramid: (base is a polygon)</p> <p>$\frac{1}{3} \cdot Area\ of\ the\ base \cdot height$</p>	<p>Cone: (base is a circle)</p> <p>$\frac{1}{3} \cdot \pi \cdot r^2 \cdot height$</p>	<p>Sphere: (ball)</p> <p>$\frac{4}{3} \cdot \pi \cdot r^3$</p>
Pythagorean Theorem:		
<p>Find a leg: $a = \sqrt{c^2 - b^2}$ or $b = \sqrt{c^2 - a^2}$</p> <p>Find the hypotenuse: $c = \sqrt{a^2 + b^2}$</p>		

UNIT 11 –PRACTICE PROBLEMS

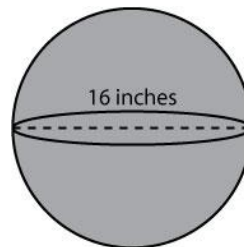
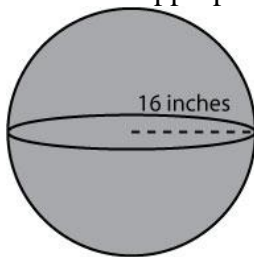
1. Determine the volume of each of the figures shown below. Round your answers to the nearest integer and include appropriate units of measure.



2. Determine the volume of each of the figures shown below. Use 3.14 for π . Round your answers to the nearest hundredth and include appropriate units of measure.



3. Determine the volume of the spheres shown below. Use 3.14 for π . Round your answers to the nearest hundredth and include appropriate units of measure.



Unit 11 – Practice Problems

4. Find the volume of a pyramid with a height of 27 cm and a rectangular base with dimensions of 3 cm and 7 cm. Round your answer to the nearest hundredth as needed

5. Sketch a cone with radius 5 feet and height 7 feet, then find the volume.

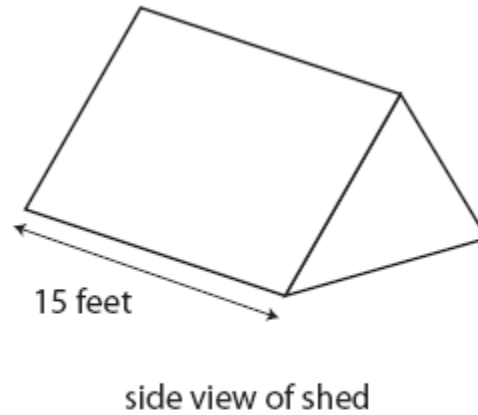
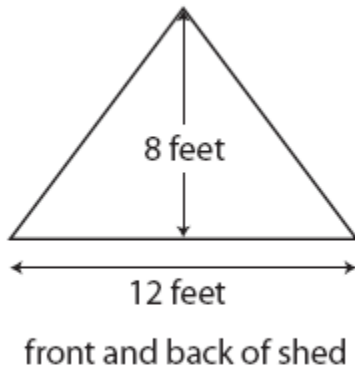
6. A box has length 4 feet, width 8 feet, and height 5 inches. Find the volume of the box in cubic feet *and* in cubic inches.

7. A marble has a radius of 12 cm. Find the volume of the marble.

8. A sports ball has a diameter of 11 cm. Find the volume of the ball.

9. A cone-shaped pile of sawdust has a base diameter of 20 feet, and is 6 feet tall. Find the volume of the pile.

10. The front and back of a storage shed are shaped like isosceles triangles with the dimensions shown. The storage shed is 15 feet long. What is the volume of the shed?



11. Renee is interested in buying a hot tub for her backyard and is looking at two models from the same company. Model B is roughly in the shape of a box with dimensions 3 ft x 10 ft x 4 ft. Model A is roughly in the shape of a cylinder with radius 3 ft and height 4 ft. Which one holds a greater volume of water and by how much?

12. A gumball has a radius that is 18 mm. The radius of the gumball's spherical hollow core is 5 mm. What is the volume of the gumball if you do not include its hollow core?

Unit 11 – Practice Problems

13. Mercury is the smallest planet with a radius of only 2,440 km at its equator. Jupiter is the largest of all the planets. It has a radius of 71,492 kilometers at the equator. Maureen makes models of these planets where 1000 km = 1 cm. Find the volume of the models of these planets. Round to the nearest tenth.

Source: <http://www.universetoday.com/37120/radius-of-the-planets/#ixzz2EirvutkL>

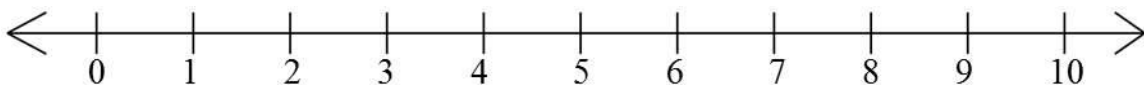
14. Square roots.

- a. Perfect Squares: Without using your calculator, fill in the blanks below.

$$\begin{array}{lll} \sqrt{1} = _ & \sqrt{_} = 5 & \sqrt{_} = 9 \\ \sqrt{4} = _ & \sqrt{_} = 6 & \sqrt{100} = _ \\ \sqrt{9} = _ & \sqrt{_} = 7 & \sqrt{_} = 11 \\ \sqrt{16} = _ & \sqrt{_} = 8 & \sqrt{144} = _ \end{array}$$

- b. Without using your calculator, place each of the following on the number line below.

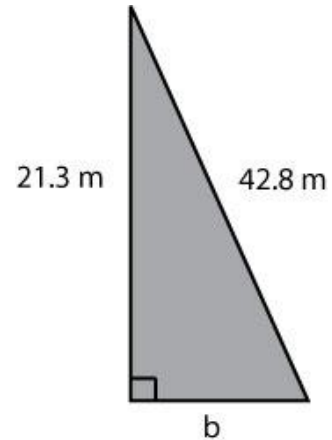
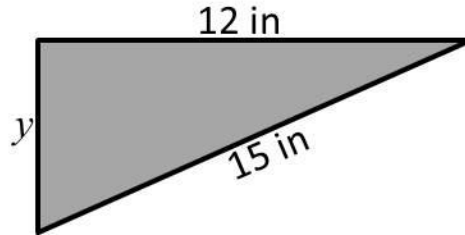
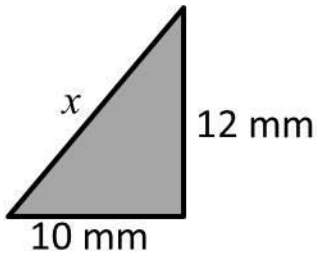
$$\sqrt{2} \quad \sqrt{11} \quad \sqrt{40} \quad \sqrt{60} \quad \sqrt{99}$$



- c. Use your calculator to evaluate each of the following. Round your answers to the nearest hundredth.

$$\sqrt{2} = ____ \quad \sqrt{11} = ____ \quad \sqrt{40} = ____ \quad \sqrt{60} = ____ \quad \sqrt{99} = ____$$

15. Use the Pythagorean Theorem to find the lengths of the missing sides of the triangles shown below. Round your answers to the nearest tenth and include appropriate units of measure.



16. Two trains left a station at exactly the same time. One train traveled south and one train traveled west. When the southbound train had gone 75 miles, the westbound train had gone 125 miles. How far apart were the trains at this time?

17. TV screens are measured on the diagonal. If we have a TV-cabinet that is 40-inches long and 34 –inches high, how large a TV could we put in the space (leave 2-inches on all sides for the edging of the TV).

18. Emma’s new rectangular smartphone is 12.5 cm in length and 6.5 cm in width. How long is its diagonal? Round to the nearest tenth.

UNIT 11 –END OF UNIT ASSESSMENT

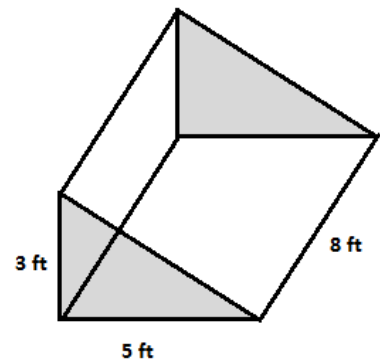
For 1 – 3: Write all the indicated measurements and attributes of the given prisms. Then find the volume of the solids. Include units in your answers. Use π for exact form and 3.14 for approximate form when needed.

1. Shape of the Base:

Area of the Base:

Height of Prism (distance between two bases):

Volume of the Prism:

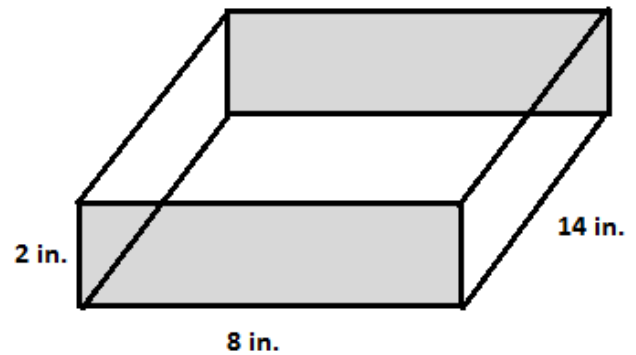


2. Shape of the Base:

Area of the Base:

Height of Prism (distance between two bases):

Volume of the Prism:



3. **Area of the Base**

Exact Form:

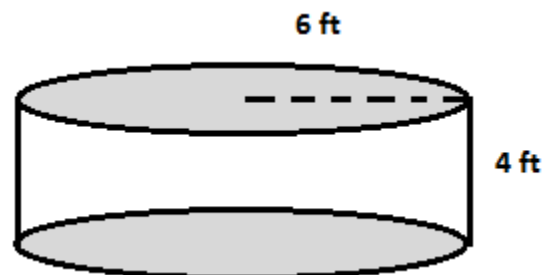
Approximate Form:

Height of Cylinder (distance between two bases):

Volume of the Cylinder

Exact Form:

Approximate Form:



Unit 11 – Assessment

4. A golf ball has a diameter of approximately 4.3 cm. Find the volume of a golf ball.
5. [Guinness World Records](#) reports that in 2015, a Norwegian ice cream company made the world's tallest ice cream cone. The cone was 3.8 meter high. If the cone's radius was 1.5 meters, what is the volume of the cone?
6. Find the square root of each of the following. Round to two decimal places if needed. Indicate those that are perfect squares and explain why.

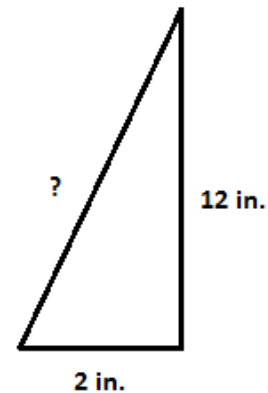
a) $\sqrt{100}$

b) $\sqrt{30}$

c) $\sqrt{1}$

d) $\sqrt{17}$

7. Use the Pythagorean Theorem to find the missing length of the given triangle. Round your answer to the tenth's place if needed.



8. Use the Pythagorean Theorem to find the missing length of the given triangle. Round your answer to the tenth's place if needed.

