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# Algebra 2 - Semester 2 - Final Exam Review 

Your final exam will be 60 multiple choice questions coving the following content. This review is intended to show examples of problems you may see on the final. This packet is your last assignment of the year and will be due the day of the exam. Completing this ENTIRE packet will earn your 5\% extra credit on the exam.

## Topics:

- Simplifying Rational Expressions
- Rational Equations
- Inverse Functions
- Exponential Functions
- Logarithms
- Sequences and Series


## Simplifying Rational Expressions

Simplify each of the following expressions. Circle your answer.
1.) $\frac{12 x(x-4)}{6 x}$
2.) $\frac{2 x^{2}+3 x-5}{x^{2}-8 x-9}$
3.) $\frac{6\left(x^{2}-6\right)(x+1)}{(x+2)\left(x^{2}-6\right)}$

Simplify the expressions by multiplying. Simplify all answers as best you can.
4.) $\frac{(x+9)(5 x-2)}{2(3 x-1)} \cdot \frac{(3 x-1)}{(x+9)(2 x-3)}$
5.) $\frac{x^{2}-4}{x+2} \cdot \frac{x^{2}+6 x+8}{x-2}$
$\qquad$
6.) $\frac{5 x}{21 x+21} \cdot \frac{21 x+21}{20}$

Simplify the expressions by dividing.
7.) $\frac{6 b}{8(3 b+5)} \div \frac{2}{8(3 b+5)}$
8.) $\frac{4 r^{2}}{12 r^{2}} \div \frac{1}{r-2}$
9.) $\frac{\frac{1}{4}}{\frac{2}{8}}$

Simplify by adding/subtracting the rational expressions:
10.) $\frac{2 x+y}{6 y^{2}}+\frac{5 x+6 y}{6 y^{2}}$
11.) $\frac{4}{2 y}+\frac{6 x}{2 x y}$
12.) $\frac{3}{x^{2}+6 x+5}+\frac{6 x}{x^{2}+7 x+10}$

## Rational Equations

13.) What is a rational equation?
14.) What is an extraneous solution?
15.) What is an identity?
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16.) Solve the rational equation using cross multiplication. Be sure to check for extraneous solutions. Show your work.
a.) $\frac{y-1}{y-3}=\frac{2}{y-3}$
b.) $\frac{-6}{b-3}=\frac{5}{b+8}$

For numbers 17-18, determine the LCM. Show all your work - no bucket necessary.
17.) $\frac{1}{5 n-1}+\frac{3 n+3}{5 n^{2}-31 n+6}=\frac{3 n-9}{5 n^{2}-31 n+6}$
18.) $\frac{3}{n^{2}+5 n}=\frac{1}{n+5}-\frac{1}{n^{2}+5 n}$

Solve the following rational equations using the LCM. Be sure to check for extraneous solutions.
19.) $\frac{1}{y^{2}+5 y+6}+\frac{1}{y+2}=\frac{6}{y^{2}+5 y+6}$

LCM: $\qquad$
Solution: $\mathrm{x}=$ $\qquad$
20.) $\frac{3}{p-6}=1-\frac{1}{p-6}$

LCM:
Solution: $\mathrm{x}=$ $\qquad$
$\qquad$
21.) Graph the following rational equation. Be sure to also fill out the table, show points on the graph, and plot your asymptote.
$y=\frac{1}{x-2}$


| $X$ | $Y$ |
| :--- | :---: |
| -5 |  |
| -4 |  |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

## Inverse Functions

22.) Determine the inverse of each of the following equations.
a.) $y=\frac{3 x-6}{9}$
b.) $y=\sqrt{x}+11$
c.) $y=x^{2}-8$
d.) $y=\frac{1}{4} x-2$
23.) Determine the inverse of the equation.

Then graph the function and its inverse. Circle your answer.
$y=\frac{1}{2} x-2$

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24.) Evaluate the following functions using: $\quad h(x)=5 x-2$ and $g(x)=3 x+1$
a.) $h(g(1))$
b.) $g(h(1))$
25.) Determine the composition function $f[g(x)]$ using the following:

$$
f(x)=4 x-6 \quad g(x)=3 x-2
$$

26.) Show that the following are inverses using the inverse composition property.
**show $f(g(x))=x=g(f(x))^{* *}$
$f(x)=\frac{x-5}{3}$
$g(x)=3 x+5$
27.) Graph the function given by $y=(1.5)^{x}$

| $X$ | $Y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |


$\qquad$

## Exponential Functions

28.) Do the following show exponential growth or decay?
a.) $y=0.005(9)^{x}$
b.) $y=2(.205)^{x}$
c.) $y=7\left(\frac{7}{3}\right)^{x}$

For numbers 29-30, use the exponential growth/decay model to determine the answers to the following:

$$
y=A(1+r)^{t}
$$

29.) Suppose that in 2020 there will be 3 zombies. If the number of zombies increases by $98 \%$ each year. About how many zombies will there be in 2035?
30.) A new jet ski costs $\$ 2400$. The value decreases by $10 \%$ each year. How much is it worth after 2 years?

For numbers 31-32, use compound interest to determine the answer to the following:

$$
A=P\left(1+\frac{r}{n}\right)^{n t}
$$

31.) Suppose you put $\$ 17,000$ into an account that pays $2.95 \%$ annual interest. Find the balance after 2 years if the interest is compounded monthly.
32.) Suppose you owe $\$ 16,280$ for student loans. Your bank charges you $24.5 \%$ interest. Determine how much you owe if you don't pay the bill for 2 years and the interest is compounded weekly ( 52 weeks=1 year).
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## Logarithms

Rewrite each equation in exponential form. $\log _{b} y=x \leftrightarrow b^{x}=y$
34.) $\log _{23} 529=2$
35.) $\log _{81} 9=\frac{1}{2}$

Rewrite each equation in logarithmic form. $\log _{b} y=x \leftrightarrow b^{x}=y$
36.) $7^{-3}=\frac{1}{343}$
37.) $9^{x}=188$
38.) Evaluate the expression. Round to the nearest hundredth. $\log _{7}(30.5)$

- Product Property: $\log _{b} m n=\log _{b} m+\log _{b} n$
- Quotient Property: $\log _{b} \frac{m}{n}=\log _{b} m-\log _{b} n$
- Power Property: $\log _{b} m^{n}=\boldsymbol{n l o g} g_{b} m$

Expand each of the following logarithms using the properties of logs.
39.) $\log _{2} 9 a b$
40.) $\log _{3} \frac{x^{5}}{y}$

Condense each expression to a single logarithm using the properties of logarithms.
41.) $\log _{8} 1-5 \log _{8} x$
42.) $\frac{1}{2} \ln 6+\frac{1}{2} \ln x$
$\qquad$

Solve each equation using inverse operations. Round your answer to the nearest thousandth.
43.) $\log x=1$
44.) $5^{x}=261$

Solve each equation using inverse operations. Round your answer to the nearest thousandth.
45.) $\log _{14} x=3$
46.) $2.3^{x+1}=25$

Solve the following equations.
47.) $e^{4 x}=e^{64}$
48.) $2^{x+1}=4^{x}$

Solve the following equations.
49.) $\log _{3}(4 x+7)=\log _{3}(2 x-9)$
50.) $\ln \left(x^{2}-13\right)=\ln \left(x^{2}+x\right)$

Solve the following equations.
51.) $\ln 6+\ln 5=\ln 10 \mathrm{x}$
52.) $\ln x-\ln 2=5$
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## Sequences and Series

53.) Given the following sequence, find $a_{3}$, then determine $a_{7} .1,6,11,16,21,26, \ldots$
54) Write the first six terms for the sequence $a_{n}=n+4$.
55) Write a rule for the nth term of the sequence. $1,8,27,64, \ldots$
56) $13,16,19,22,25,28,31,34, \ldots$ Write a rule that can be used to find $\mathrm{a}_{100}$.
57) Find $\mathrm{S}_{5}$. $3,5,7,9,11,13, \ldots$
58.) Write the following series using sigma notation
a.) $2+4+6+8$
b.) $2+4+6+8+\ldots$
c.) $25+50+75+\ldots+250$
59.) Determine the value of the series given by $\sum_{i=1}^{5}\left(3+n^{2}\right)$
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## Special series:

$$
\sum_{i=1}^{n} 1=\mathrm{n} \quad \sum_{i=1}^{n} i=\frac{n(n+1)}{2} \quad \sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2 n+1)}{6}
$$

Compute the value of the series using the formulas for special series.
60.) $\quad \sum_{i=1}^{34} 1$
61.) $\sum_{i=1}^{20} i$
62.) $\sum_{i=1}^{15} i^{2}$
63.) Determine if the following sequences are arithmetic, geometric, or neither.
a.) $1,4,7,10, \ldots$
b.) $27,9,3,1, \ldots$
64.) Write the rule for the following arithmetic sequence: $60,52,44,36, \ldots$
65.) What is the sum of the arithmetic series $\sum_{i=1}^{20}(3+5 i)$ ?
66.) Write a rule for the nth term of the sequence, then find $a_{7}$.
a.) $4,20,100,500, \ldots$
b.) $152,-76,38,-19, \ldots$
67.) Find the sum of the geometric series $\sum_{i=1}^{16} 4(3)^{i-1}$
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The Sum of an Infinite Geometric Series:

$$
S=\frac{a_{1}}{1-r} \text { provided that }|r|<1 \text {, if }|r|>1 \text {, then the limit doesn't exist. }
$$

68.) Find the sum of the infinite geometric series.
a.) $\sum_{i=1}^{\infty} 2(0.9)^{i-1}$
b.) $1+.5+.25+.125+\ldots$
69.) Rewrite the following repeating decimals as a fraction in lowest terms.
a.) $0.512512512 . .$.
b.) $0.99999999999 \ldots$
70.) Write the first 6 terms of the sequence with the following rules:
a.) $a_{0}=1, \quad a_{n}=a_{n-1}+9$
b.) $a_{1}=8, \quad a_{n}=2 a_{n-1}$

Recursive formulas for arithmetic and geometric sequences:
Arithmetic Sequences: $a_{n}=a_{n-1}+d$, where $d$ is the common difference
Geometric Sequences: $a_{n}=r \cdot a_{n-1}$, where $r$ is the common ratio
71.) Write a recursive rule for the following sequences:
a.) $-32,0,32,64, \ldots$
b.) $8,12,18,27$, ...
72.) Find the first three iterates $x_{1}, x_{2}$, and $x_{3}$ of the function $f(x)=7 x-5$ for an initial value of $x_{0}=1$

