Revisiting Mathematics manipulative materials

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revisit the use of manipulatives. They look at the different types and the ways in which they are used by teachers.

Introduction

It is over 12 years since APMC published Bob Perry and Peter Howard's research on the use of mathematics manipulative materials in primary mathematics classrooms (Perry & Howard, 1997). Since then the availability of virtual manipulatives and associated access to computers and interactive whiteboards have caused educators to rethink the use of mathematics manipulative materials. In addition, the introduction of national testing (NAPLAN) in 2008, in which pictures of mathematics manipulative materials are included, but no access to them is given, is likely to impact on how mathematics manipulatives are used. It seems timely then to revisit the use of mathematics manipulative materials in primary and, in Western Australia, designated middle schools.

What is a manipulative?

The issue of defining what is meant by the term "manipulative" continues to be problematic. Perry and Howard used Hynes' definition of manipulatives as "concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p. 11). The current authors were concerned that the use of mathematics

manipulative materials is often justified on the basis that the students are involved in "hands on" learning. This justification is simply not enough, and so it was decided to create a new definition of a mathematics manipulative material that encompassed the idea that students need to engage with the manipulative and that thinking should be stimulated. Our definition is:

A mathematics manipulative material is an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered.

Consequently, a mathematics manipulative object has the potential to lead to an awareness and development of concepts and ideas linked with mathematics and they would most likely be purpose designed. We do not consider the above definition all-embracing. After careful consideration, we believe that there are also tools (such as calculators), teaching tools (demonstration models) and teaching aids (e.g., fraction charts); but these are somewhat different from mathematics manipulative materials. Within our definition, structured and unstructured mathematics manipulative materials are recognised. Under both Hynes' and our definitions, virtual manipulatives are not included. We believe it best to delay the use of virtual manipulatives until students have had experience of the "real thing". We have noted in observations of classes where physical and virtual manipulatives are used, younger children experience difficulty understanding two-dimensional representations of three-dimensional objects. It was also noted that when virtual manipulatives are used on an interactive whiteboard, student access is somewhat limited. In many cases the interactive whiteboard is used mainly for demonstration (Mildenhall, Swan, Northcote & Marshall, 2008). This may become less of an issue as hand-held technologies such as the i-pad become more available. Initially this is more likely to be the case in secondary and upper primary classes.

This study

Perry and Howard (1997) based their findings on responses from 249 primary teachers in New South Wales. To mirror their investigation, a four-page survey was sent to all primary and designated middle schools in Western Australia. Responses were received from over 820 teachers across 250 schools. That is at least one teacher in each of approximately one-third of all Western Australian schools, responded to the survey. In some schools many more teachers, up to a maximum of 15, responded. The responses were from teachers in large metropolitan primary schools, district high schools (Years K–10), and remote Aboriginal community schools. They encompassed many religious and educational philosophies, from Catholic, Anglican, Lutheran and Islamic colleges to Montessori and alternative schools.

Space was provided on the survey for teachers to write extended comments. To probe further, semi-structured interviews were conducted with a sample of teachers who volunteered via the survey.

What manipulatives are used?

Teachers were asked to identify the manipulatives they used in their mathematics classes. This involved ticking boxes with information about eight different mathematics manipulatives that we felt were likely to be used in most schools. They were given space to describe how they used the particular manipulatives. The manipulatives were listed alphabetically: Attribute Blocks, Base Ten blocks (MAB), Cuisenaire rods, Multilink cubes, Pattern Blocks, Polydrons/Geoshapes, square tiles and Unifix cubes. Drawings were provided of each manipulative to avoid

confusion. A brief summary of the data is shown in Table 1 below.

Table 1. Percentage use of particular manipulatives across all Year levels.

Manipulative	Yes	No
Pattern Blocks	84.3	15.7
Base Ten blocks	81.9	18.1
Attribute Blocks	77.5	22.5
Polydrons/Geoshapes	71.2	28.8
Unifix cubes	66.5	33.5
Multilink cubes	43.3	56.7
Square tiles	35.3	64.7
Cuisenaire rods	35.1	64.9

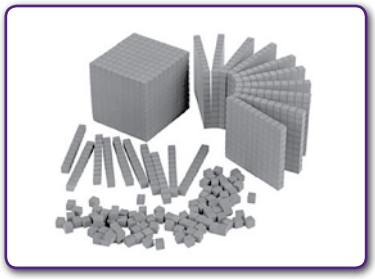
Teachers were asked to list other mathematics manipulatives they used regularly. Among those nominated were: popsticks, Klicko, Lego, dice, Mobilo, "found" materials, trundle wheels, fraction cakes, tangrams, counters, Miras, dominoes, straws, and clocks. It could be argued that, according to our definitions, some of these materials are teaching tools rather than mathematics manipulatives, but as we deliberately chose not to provide a definition of the term "manipulative" on the survey, teachers felt free to include all equipment they used in their mathematics lessons.

The most used manipulatives

Teachers were also asked to nominate the three manipulatives they used most. The most commonly named manipulatives were, in order: Base Ten blocks (MAB), counters, and Unifix cubes. It is interesting that in the previous question, Pattern Blocks were the most nominated the manipulatives in our list, yet when asked to identify the manipulatives that they used most, and not provided with a list, the results were quite different. The calculation to determine the most used manipulatives was done by giving a value of 3 to the first nominated, 2 to the second, and 1 to the third. This weighting and the fact that counters were not included on the original list may account for the difference in results.

Pattern Blocks

Unifix Cubes



Base Ten Blocks

Counters

Why manipulatives were used

"Two overwhelming reasons for the use of manipulatives are teachers believe that the materials benefit children's mathematics learning and that children enjoy using them" (Perry & Howard, 1997, p. 27). In our survey, most participants described more than one advantage of using manipulatives. Responses were grouped into the categories as shown in Table 2 below.

Table 2. Advantages of using manipulatives.

Comments	Number
Heighten interest; helped engage students; enjoyment; 'fun'; provide motivation	191
They are a visual aid; assist in concrete visualisation	188
Provide hands-on learning	135
Build a better understanding	126
Help children grasp concepts; or reinforce them	61
Applied to all styles of learning	48
Can be used to introduce concepts	36
Teacher can more easily note what the child is 'thinking'	27
Encouraged oral language	23

Space was provided on the surveys for teachers to write extended answers. It was encouraging to find that some teachers had given a great deal of thought to the effectiveness of manipulative materials. This was evidenced by comments such as:

- "The students sometimes misunderstand the point of the lesson if it is always explained using the same manipulatives."
- "Sometimes kids will pick up a "wrong" concept from a manipulative so their use needs guidance and supervision and follow-up, then builds great understanding and concepts."
- "All children need to learn mathematics with manipulatives. A lot of children need concrete materials to aid in all maths activities for some time. As they become more adept in mental strategies they are able to dispose of the concrete materials. They are essential to all mathematical learning."

Even more succinct was the comment, "Ask Piaget"!

Very little reference was made to learning theory. Vague clichés such as "hands on learning" are not sufficient to justify the use of manipulatives. The authors believe that unless teachers have a clear understanding of how manipulatives assist children learn they are likely to make only token use of them, which may be detrimental to learning.

Implications

The survey generated a wealth of data and so only selected findings that relate to the work of Perry and Howard are reported here.

Perry and Howard (1997) listed eight implications for teaching and learning resulting from their work. All but two of these implications will be addressed. Those not addressed concerned ESL teachers. Some suggestions are included for improving the way mathematics manipulative materials are used.

The use of manipulatives is supported by almost all primary teachers across all years and for all areas of mathematics.

Perry and Howard (1997) commented that teachers felt they should be utilising manipulatives in their mathematics lessons. Organisation of mathematics manipulative materials and ready access to them was an issue that was raised in the 1997 study. The current study showed that professional development that deepens teachers' knowledge of the materials and their uses needs to be undertaken.

We suggest that schools organise materials in three ways:

- classroom kits containing essential materials that remain in every classroom;
- rotated or shared materials that remain in a classroom for about three weeks, and then get moved to another classroom, to be returned to the classroom in the next rotation; and
- less frequently-used or bulky materials to be kept in the storeroom.

For further details on this form of manipulatives use and storage, see White, Swan and Marshall (2009).

Teachers use manipulatives because they believe that they benefit children's mathematics learning and children enjoy using them.

Older students can be involved in rotating materials around the school.

Perry and Howard (1997) made the point that the use of manipulatives must be encouraged by teachers, parents, children and others involved in the learning process. The data from the current survey suggest that the majority of teachers believe that mathematics manipulative materials assist learning, however written comments and subsequent interviews revealed that teachers could not identify exactly what it is about manipulatives that assists in the learning of mathematics. This is of concern, because without a clear understanding of how or even if mathematics manipulatives enhance learning of mathematics, teachers may either lack conviction when using them or abandon them at the first sign of any problems associated with using them.

Perry and Howard (1997) found that teachers reported a decrease in the use of mathematics manipulative materials from Kindergarten to Year 6. This was also the case in this current survey. Table 3 below highlights the percentage of teachers who used manipulatives with various frequencies at different year levels.

This finding matches earlier research by Gilbert and Bush (1988). Based on written comments made by the teachers on the survey and comments made during follow up interviews, it appears that teachers associate the use of mathematics manipulatives with concept formation and and hence

Table 3. Percentage teachers using manipulatives at different year levels.

Year Level	Daily	Several times a week	Once a week	Every couple of weeks	About once a month	Never	Other
PP	82.6	17.4					
Year 1	47.8	47.8	3.3	1.1			
Year 2	29.9	58.4	6.5	5.2			
Year 3	19.6	63	4.3	6.5	2.2	2.2	2.2
Year 4	19.5	29.3	31.7	9.8	2.4		7.3
Year 5	20.9	32.6	25.6	7	7		7
Year 6	9.1	38.6	15.9	25	9.1		2.3
Year 7	4.5	45.5	4.5	13.6	31.8		
Year 8		20	10	50	10		10
Year 9					40		60

to be abandoned when the mathematics becomes more complex. While this may seem reasonable, consider how children are exposed to some of the more difficult concepts in mathematics such as fractions later in their schooling and how various manipulatives can support the development of fraction concepts. Unfortunately, the fact that mathematics manipulative materials cannot be used in NAPLAN testing, which begins at Year 3, reinforces the belief that mathematics manipulative materials are the domain of the early years.

Perry and Howard (1997) found that teachers used manipulatives in different ways at different year levels. Similarly teachers in the current survey indicated a mix of ways in which manipulative materials were used ranging from self-discovery to teacher directed. Many teachers gave comments similar to this one: "Initially it may be teacher-directed, but then we move on to self-discovery".

Perry and Howard (1997) described a perception among older children that it may be "babyish" to use manipulatives, and many teachers made similar comments in the current survey and interviews. Perry and Howard's comment that, "Years 5 and 6 are preparatory years for secondary school and that there is little use of manipulatives in secondary school mathematics" (p. 29) also came through in this study.

A significant number of teachers indicated that they would like more training in the use of manipulatives. This is in spite of feeling confident about the use of the materials available to them.

Perry and Howard (1997) found that over 40% of teachers of all year groups indicated that they would like some professional development on the use of mathematics manipulatives, with the number rising to 63% of the "Other" category (Other referred to teachers not involved in teaching a particular year level), of whom the majority were ESL teachers. In the current survey fewer than 10% of respondents indicated that they had undertaken any professional development on the use of manipulatives. Despite much training being offered by departmental officers, it was surprising that many recalled having professional training offered by a private mathematics materials supplier.

Of interest in the current study was that only 19% of respondents said that they would like further help with manipulatives, and although they were given the opportunity to nominate a particular manipulative they would like to work with, most gave no details of what they wanted. Several suggested that they desired to be trained in "what's new", but overall very little in terms of specific needs was indicated.

Stein and Bovalino (2001) found that good maths lessons do not just happen by themselves. Much of the groundwork for good teaching has taken place years before when teachers may have received professional development on the incorporation of manipulatives into their teaching. When discussing professional learning, Stein and Bovalino did not mean the "easy-fix" series of activities for teachers to take away and use the next day. Of far greater importance, was professional learning that gave insights into the way that manipulatives can assist with children's learning.

Conclusion

The conclusions made in the study by Perry and Howard (1997) are still relevant.

> Manipulatives benefit the learning and teaching of mathematics. Teacher use of manipulatives needs to be strengthened through appropriate professional development within the overall context of the student's learning of mathematics. There is strong teacher support for manipulative use in the earlier grades of primary school. However, all children need access to and availability of a wide range of manipulatives as they meet new mathematical concepts and continue to construct

> mathematical meanings. … Schools and education systems need to recognise that the aspirations of their teachers to benefit children's learning to as great an extent as possible mean that manipulatives need to be available to all teachers and all children as they need them. … This will have budgetary, organisational and professional development implications for the schools and systems (pp. 29–30).

Given the considerable amount of money spent on purchasing mathematics manipulative materials; the time spent sorting, organising and storing maths materials; and all the effort put into managing lessons involving manipulatives, it is surprising that teachers have not been more inclined to question how and if mathematics manipulative materials actually help children learn mathematics. Equally surprising is the fact that only 19% of teachers (compared with 40% and 63% in the two categories of the Perry and Howard study) would like further professional development in the use of manipulatives, but were vague as to what they required.

The authors believe that there are potential gains to be made by using mathematics manipulative materials where appropriate and in a systematic manner. To be effective, however, simply placing one's hands on the manipulative materials will not magically impart mathematical understanding. Without the appropriate discussion and teaching to make the links to the mathematics explicit, the very opposite may be true; children may end up with mathematical misconceptions.

Postscript

During the writing of this paper, our colleague Geoff White, who was an original member of the research group, passed away. At the time we were completing a booklet to help teachers make more effective use of mathematics manipulatives materials. Any teachers wishing to obtain a copy of this free publication are welcome to contact this article's authors.

References

Gilbert, R., & Bush, W., (1988). Familiarity, availability, and use of manipulative devices in mathematics at the primary level. *School Science and Mathematics, 88*(6), 459–469.

Hynes, M., (1986). Selection criteria. *Arithmetic Teacher, 33*(6), 11–13.

Mildenhall, P., Swan, P., Northcote, M., & Marshall, L., (2008). Virtual manipulatives on the interactive whiteboard: A preliminary investigation. *Australian Primary Mathematics Classroom, 13*(1), 9–14.

Perry, B., & Howard, P., (1997). Manipulatives in primary mathematics: Implications for teaching and learning. *Australian Primary Mathematics Classroom, 2*(2), 25–30.

Stein, M. N., & Bovalino, J. W., (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 359.

White, G., Swan, P., & Marshall, L., (2009). *Hands on Heads on: The effective use of mathematics manipulative materials: A mathematics manipulatives continuum.* Perth: R.I.C. Publications.

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