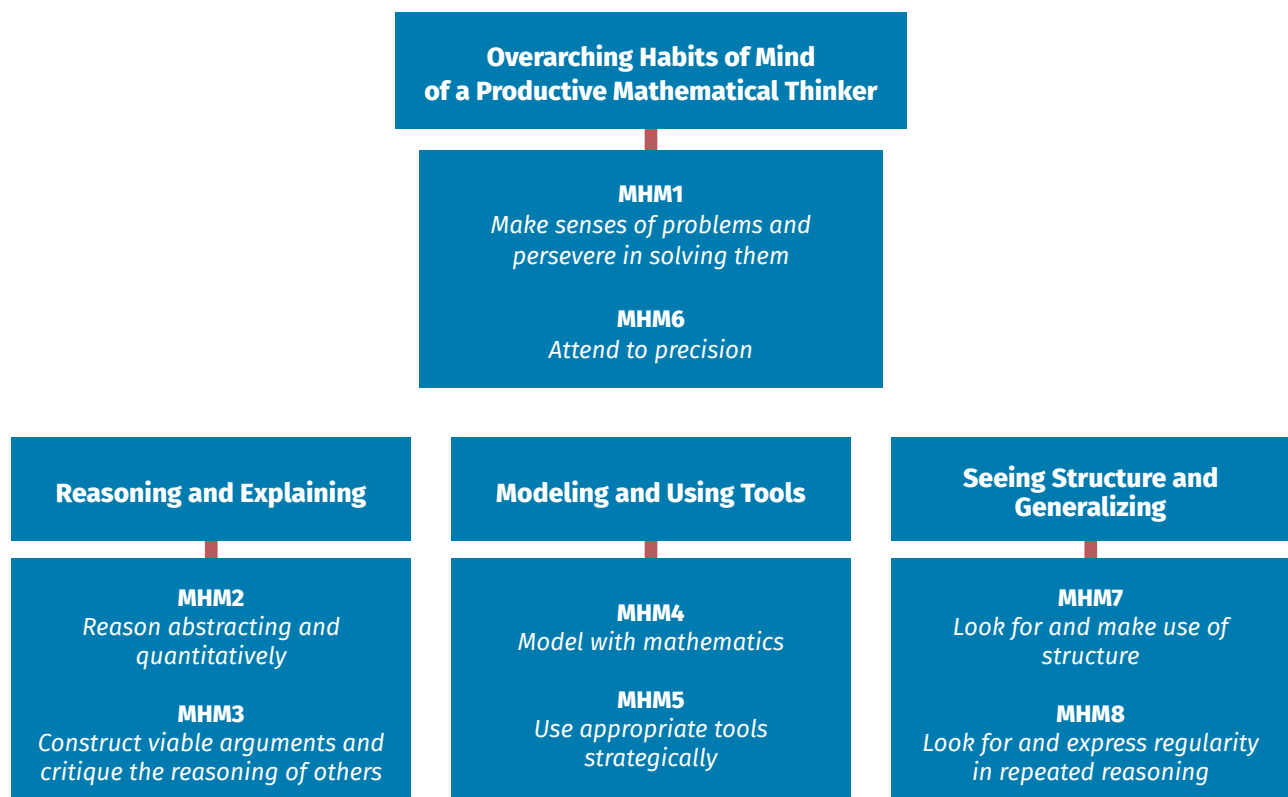West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstracting and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

West Virginia DEPARTMENT OF EDUCATION

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Advanced Mathematical Modeling

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students will learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As students solve problems in various applied situations, they will develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they will rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students will use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Developing College and Career Skills	**Finance**
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**
• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and consider the advantages/disadvantages of each method.	• Solve geometric problems involving inaccessible distances. • Use vectors to solve applied problems.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Developing College and Career Skills	
Math as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Finance	
Understanding financial models.	Standards 4-6
Personal use of finance.	Standards 7-8
Probability	
Analyzing information using probability and counting.	Standards 9-10
Managing uncertainty.	Standards 11-12
Statistics	
Critiquing statistics.	Standards 13-16
Conducting statistical analysis.	Standards 17-21
Communicating statistical information.	Standards 22-23

Modeling	
Managing numerical data.	Standards 24-25
Modeling data and change with functions.	Standards 26-30
Networks	
Networking for decision making.	Standards 31-32
Social Decision Making	
Making decisions using ranking and voting.	Standards 33-34
Geometry	
Concrete geometric representation (physical modeling).	Standards 35-36
Abstract geometric representation (matrix modeling).	Standards 37-38

Developing College and Career Skills

Cluster	**Math as a language**
M.AMM.1	Demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.AMM.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

Cluster	**Tools for problem solving**
M.AMM.3	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Finance

Cluster	**Understanding financial models**
M.AMM.4	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Auto, Mortgage, and/or Credit Card).
M.AMM.5	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit.)
M.AMM.6	Determine, represent, and analyze mathematical models for Inflation and the Consumer Price Index using concepts of rate of change and percentage growth.

Cluster	Personal use of finance
M.AMM.7	Research and analyze personal budgets based on given parameters (e.g., Fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission), career choice, geographic region, retirement and/or investment planning, etc.).
M.AMM.8	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.

Probability

Cluster	Analyzing information using probability and counting
M.AMM.9	Use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.AMM.10	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.

Cluster	Managing uncertainty
M.AMM.11	Use probabilities to make and justify decisions about risks in everyday life.
M.AMM.12	Calculate expected value to analyze mathematical fairness, payoff and risk.

Statistics

Cluster	Critiquing statistics
M.AMM.13	Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.
M.AMM.14	Interpret and compare the results of polls, given a margin of error.
M.AMM.15	Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.
M.AMM.16	Describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics and other results appearing in a study, including reports published in the media.

Cluster	Conducting statistical analysis
M.AMM.17	Identify the population of interest, select an appropriate sampling technique and collect data.
M.AMM.18	Identify the variables to be used in a study.

M.AMM.19	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.
M.AMM.20	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features.
M.AMM.21	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.

Cluster	Communicating statistical information
M.AMM.22	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.
M.AMM.23	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Modeling

Cluster	Managing numerical data
M.AMM.24	Solve problems involving large quantities that are not easily measured.
M.AMM.25	Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.

Cluster	Modeling data and change with functions
M.AMM.26	Determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale).
M.AMM.27	Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights, diurnal cycle, and/or music).
M.AMM.28	Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates).
M.AMM.29	Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay).
M.AMM.30	Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.

Networks

Cluster	Networking for decision making
M.AMM.31	Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, kolam drawings of India, traveling salesman problem, and/or map coloring).
M.AMM.32	Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures.

Social Decision Making

Cluster	Making decisions using ranking and voting
M.AMM.33	Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results).
M.AMM.34	Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting).

Geometry

Cluster	Concrete geometric representation (physical modeling)
M.AMM.35	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.AMM.36	Solve geometric problems involving inaccessible distances.
Cluster	**Abstract geometric representation (matrix modeling)**
M.AMM.37	Use vectors to represent and solve applied problems.
M.AMM.38	Use matrices to represent geometric transformations and solve applied problems.