A Response to Intervention Solution

RAMP-UP MATHEMATICS
RESEARCH BASIS

ALWAYS LEARNING
PEARSON

The United States had a larger percentage of students score at the lowest levels on the Programme for International Student Assessment (PISA) than the overall average for countries belonging to the Organization for Economic Co-operation and Development [OECD].

—National Center for Education Statistics (2004)

RAMP-UP MATHEMATICS

RESEARCH BASIS

Background

In 2003 the Programme for International Student Assessment (PISA) focused on students' abilities to use their skills and mathematical understanding in a wide range of contexts to answer questions and solve problems—the type of problems encountered in everyday life. On average, U.S. 15-year-olds performed worse than their international counterparts, tying with Latvia to share the dubious distinction of placing 28th out of 39 countries in the category of Mathematics Literacy (Lemke et al., 2004). In addition, the United States had a larger percentage of students score at the lowest levels than the overall average for countries belonging to the Organization for Economic Co-operation and Development [OECD] (NCES, 2004a).

The United States has a high school graduation rate of 73%, while most industrialized countries achieve 80% (NCES, 2006a). Poor, black, and Hispanic students, as well as English language learners in the U.S. face even worse prospects. For the twelve months ending in October 2001, students in the lowest 20% of income-earning families were six times more likely to drop out of high school than students from the top 20% of income-earning families (NCES, 2004b). Researchers following a group of 8th-grade students to graduation in 1994 found that 64% of low socio-economic status (SES) students earned a diploma in 2000, while 83% of mid-SES students and 97% of high-SES students did the same (NCES, 1996). English language learners face an even greater challenge. The number of 5- to 17-year-old students who speak a second language at home rose from 9% to 19% in the last twenty years (NCES, 2006b). For these students, the ramp up is even steeper, because they must overcome a language deficit while simultaneously struggling to comprehend important mathematical concepts.

When it comes to college performance, the picture is not much better. Sixty-one percent of students who first enrolled in two-year public colleges and 25% who first enrolled in four-year institutions took at least one remedial course (NCES, 2004a). A number of studies have shown a direct relationship between socio-economic status and attainment of advanced degrees. The higher their SES, the more likely students are to complete college. However, if determined, low-SES students were able to continue with advanced mathematics in high school, they would be ten times more likely than other low-SES students to earn a college degree by 2000 (NCES 2004b).

Robert Moses and Charles Cobb (2002) insist on a new "civil right" for all students—the right to be adequately prepared for advanced mathematics like algebra and geometry. Algebra has long been considered the "gatekeeper course"; without it, students may have trouble earning a high school diploma, entering a two-year college, or completing a technical program. And without a more sustained program including Geometry, Algebra 2, and Trigonometry, students may not be prepared for college level work or apply to more competitive universities. Recent research states that "…in spite of work experience, it is evident that education is the largest factor influencing average earnings. On average, persons possessing a bachelor's degree would earn…nearly twice as much as workers who have only a high school diploma." (EducationAtlas.com, 2006). The future of U.S. students depends on their getting the opportunity to study advanced mathematics and prepare for college, and that opportunity is not being provided. Gaston Caperton (2004), president of the College Board, remarked in a press release dated 2004, "We need to make rigorous academics available to all students, not just a select group. Too often, students who aspire to participate in college-level courses in high school, such as AP, are limited by earlier curriculum. In some cases, it is too late to catch up."

Introduction to Ramp-Up Mathematics

For over a decade America's Choice has been working to improve mathematics performance and broaden the college opportunity for all students. America's Choice has conducted a systematic study of international and domestic testing data; an exhaustive comparison of U.S. curriculum, texts, classrooms, and teaching techniques with those of leading nations such as Singapore and Japan; and numerous observations and interviews with hundreds of classroom teachers. As a result of this effort, America's Choice has defined the parameters for a powerful and comprehensive program in mathematics recovery. Ramp-Up Mathematics programs drop a ladder down from algebra to students who are years behind in mathematics so they can climb up to grade level and finally see over the horizon.

Ramp-Up to Pre-Algebra was created specifically to accelerate the learning of students entering middle school one to two years behind their peers, and to prepare them to complete Algebra I by the end of 8th or 9th grade.

Ramp-Up to Algebra was created specifically to accelerate the learning of students entering high school two to three years behind in mathematics. Students who successfully complete Ramp-Up to Algebra in 9th grade could complete Algebra II by the end of 12th grade.

Ramp-Up to Pre-Algebra and Ramp-Up to Algebra are double-period, comprehensive mathematics programs. Each year-long course is designed to accelerate learning, fill in gaps in understanding, and correct lingering misconceptions. Too many remedial programs try to build up from the lowest levels by reteaching, but students need more than to repeat earlier mathematics instruction. Students need to study critical mathematical concepts from a more useful perspective—the perspective of algebra. The Ramp-Up Mathematics programs were designed by working from algebra down, to create a clear and efficient path from arithmetic to algebra, which also builds a foundation in geometry.

Ramp-Up Mathematics focuses on a carefully selected set of concepts that strengthen students' conceptual foundation while preparing them for work with algebraic equations the following year. Ramp-Up Mathematics is composed of a logical sequence of progressive units centered on the concepts. Lessons within each unit are scaffolded deliberately to facilitate deeper comprehension and transferability of the concepts. Each unit in Ramp-Up Mathematics offers a suite of expertly-crafted problems and tasks that require students to push their reasoning and challenge their own assumptions while interacting deeply with the mathematics. While the teacher's role is still a guiding factor, the students' activities focus primarily on student-math interactions rather than the traditional student-teacher interactions.

Ramp-Up Mathematics emphasizes a good balance of skills, concepts, and applications. Lessons include regular opportunities not only to practice computation skills, but also to discuss abstract mathematical theories, master sophisticated algebraic concepts, and solve real-world problems.

Learning is active in the Ramp-Up Mathematics classroom; students are engaged in developing and articulating their mathematical reasoning as they work through problems. The focus of the work is on mathematical strategies and the reasoning behind those strategies. Answers are important because they validate successful methods, but student strategies—wrong or right—are what hold the potential for real learning.

Ramp-Up Mathematics is specifically designed to help the lowest-performing students make rapid progress in arithmetic and algebra and to ensure that schools meet their targets for Adequate Yearly Progress.

Introduction to the Six Main Premises

1. A Clear Path to Algebra

Algebra is said to be the "language of math," and yet that is never made explicit in American textbooks. In the most successful countries, emphasis is placed on the algebraic structure of arithmetic. The connection between algebra and the rest of mathematics should and could be made quite easily, because the underlying structure of arithmetic is the very same structure that governs algebra. Researcher Liping Ma (1999) says that "…even concepts in early arithmetic, such as addition and division, can be presented and developed in ways that lead to easier transitions to algebra."

2. Explicit Concepts Balanced by Problem Solving and Skills

Many traditional mathematics programs tend to focus on one area to the detriment of another. Research demonstrates the importance of teaching skills, problem solving, and concepts for students' future success in algebra: "Just as algebra must be more than a disparate course in the curriculum, prealgebra must not be a single entity but rather a collection of knowledge, skills, and dispositions prerequisite for understanding algebraic concepts" (Lodholz,1990).

3. Communicate Mathematical Thinking

In *Classroom Discussions: Using Math Talk*, Suzanne Chapin et al. (2003) discuss the importance of sharing one's thinking process. "As students talk through their reasoning... they learn the importance of communicating clearly and being understood by others, and begin to use language in precise and explicit ways. Often students will recognize flaws in their reasoning or computational procedures, as they make their solution methods public." Lily Wong Fillmore and Catherine Snow (2002) suggest that "For the most part, academic English is learned over the course of schooling through frequent engagement in classroom talk, reading textbooks, and writing."

4. Deeper, Richer Interactions with the Mathematics

Trends in International Math and Science Study (TIMSS) comparisons found that the leading countries use curricula that are more focused. These curricula concentrate on fewer topics in greater depth, an approach advocated by Zal Usiskin (1980), who states that "The curriculum [United States] is quite overcrowded, and there is no alternative but to take things out." This approach to content is supported by a recent study comparing Singapore, the top scoring nation in TIMSS, to the United States: "Textbook publishers should consider reducing the number of topics and lessons in their textbooks to give more textbook space for developing a rich mathematical treatment of each topic. Publishers could also improve their textbooks by cutting down on the number of illustrations and text largely unrelated to helping students learn mathematics. In this manner, U.S. textbooks would more closely model the logical topic organization, rich problem-based approach, and varied pictorial representations of mathematics concepts found in Singaporean textbooks" (Ginsburg et al., 2005).

5. Better Learners of Mathematics

Lev Vygotsky's work shows that learning is directly influenced by the social and cultural contexts in which it takes place. He illustrates the fact that knowledge gains that normally cannot be achieved alone become possible when learning takes place alongside a more capable peer or adult (Vygotsky, 1962, 1978). This view of learning is similar in some respects to the centuries-old practice of apprenticeship that, according to Lauren Resnick, allows learners "to acquire the complex interdisciplinary knowledge, practical abilities, and appropriate forms of social behavior that went with high levels of skilled performance" (Resnick & Nelson-Le Gall, 1997).

6. Professional Development

In a 2005 issue of *Essential Information for Education Policy,* the American Educational Research Association stated that "teacher professional development can improve student achievement when it focuses on teachers' knowledge of the subject matter and how students understand and learn it" (AERA, 2005).

It is not uncommon for instructors to teach English language learners or special needs students without the benefit of a special credential or advanced knowledge of the students' primary languages. Meanwhile, "the number of children ages 5 to 17 who spoke a language other than English at home more than doubled between 1979 and 2004" (NCES, 2006b). One thing is clear: instructors need professional development and instructional support in teaching English language learners and special needs students.

Research Details for the Six Main Premises

1. A Clear Path to Algebra

What the Research Says

In order to progress to more advanced mathematics, students must establish a deep and unambiguous hold on a coherent set of concepts—concepts that lead directly to the fundamentals of algebra. In the most successful countries, mathematics instruction emphasizes the algebraic structure of arithmetic, even (and especially) in early elementary instruction. Researchers at the National Center for Improving Student Learning in Mathematics [NCISLA] state that in the U.S., concepts in arithmetic are not usually developed in ways that transfer readily to algebra—but there is no reason they should not be. Algebra and arithmetic share the same fundamental properties (Early Algebra Project, 2006).

Lesley Booth's research underscores the benefit of connecting arithmetic to algebra and enlisting active engagement of students' reasoning abilities. "In arithmetic, the focus of activity is the finding of particular numerical answers. In algebra, however, this is not so. In algebra, the focus is on the derivation of procedures and relationships and the expression of these in general, simplified form. Many students do not realize this; they assume that what is required is a numerical answer" (Booth, 1989).

A simple yet profound example of the disconnected nature of arithmetic and algebra is found in the research of Carpenter, et al., wherein it is revealed that many U.S. students develop misconceptions about the "equals" (=) sign. It is probably unfair to expect otherwise, since it is nearly impossible for students to possess complete understanding of a mathematical symbol whose use in early arithmetic offers no connection to its true meaning or to its later use in algebra. American students become convinced that the = sign is a signal to "do something" or "to compute," something like punching the button on a simple adding machine. They miss altogether that = is a mathematical symbol denoting the concept of equivalency (Carpenter et al., 2003).

Researchers have found that young students have implicit knowledge of properties in mathematics. Many students recognize that numbers in a simple addition problem are interchangeable—e.g., 5 + 2 is equivalent to 2 + 5. Yet if this simple number property has not been taught explicity, many students do not make the connection to other contexts. For instance, they may not understand that the commutative property of numbers applies to more "complicated math" like larger numbers or fractions. Researchers are finding that "students can learn to think about arithmetic in ways that both enhance their early learning of arithmetic and provide a foundation for learning algebra" (Wisconsin Center for Education Research, 2000).

What Ramp-Up Mathematics Does

In the Ramp-Up Mathematics classroom, students learn mathematics in a way that familiarizes them with the algebraic structure of mathematics. Ramp-Up Mathematics prepares students for more advanced mathematics by introducing the *foundations of algebra* in the first module and subsequently teaching the arithmetic *using elementary algebraic structures.*

Foundations of Algebra

Both Ramp-Up Mathematics courses start with a unit on the foundations of algebra. The overall purpose of this first unit is to introduce students to the foundations of mathematical reasoning—including justifying statements and answering "say why" questions—with mathematical expressions using letters and numbers. During this unit, students learn the language of mathematics, which is algebra. Students study the number properties such as the commutative property and the distributive property, the conventions for using letters, and the concept of equivalence. Students learn to use diagrams, tables, formulas, and graphs to represent mathematical situations. This introduction to the fundamentals of algebra helps students move from working with numeric values to using variables, which allows them to more easily generalize and connect the mathematical concepts as they work through the curriculum.

Using Elementary Algebraic Structures

Ramp-Up Mathematics focuses on the most fundamental aspects of algebra throughout all of its modules, including using horizontal notation, the number properties, variables, and graphical representations.

Students are introduced to letters (variables) in the first lesson and then, in subsequent lessons, students learn to generalize mathematical concepts and statements. For instance, students are first introduced to the distributive property with numeric examples and then they work with the general form: $a(b + c) = ab + ac$. When working with fractions, students first work on numeric problems like $\frac{1}{2} \bullet \frac{3}{2} = \frac{3}{4}$, and then generalize to $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$. Generalizing to algebraic representations or solving problems with variables is an integral part of every unit in the Ramp-Up Mathematics courses.

The ultimate goal of Ramp-Up Mathematics is to get students back on track and progressing toward algebra.

2. Explicit Concepts Balanced by Problem Solving and Skills

What the Research Says

The alarming results of TIMSS in 2003 and earlier in 1999 caused school districts, state boards of education, and textbook publishers to engage in an unproductive argument dubbed the "math wars." In an effort to improve mathematics instruction and learning, these entities took turns advocating two alternate approaches, neither to great success. The first was to "get back to basics," an idea that revolved around the importance of skills and the repeated practice of those skills. The skills-based curricula resulted in a rather shallow computational expertise that required little in the way of real thinking and reasoning. As a result, students didn't see much of a connection between the mathematics and their everyday lives. The second approach to the mathematics was all about problem solving. Math in context, real-world math, and authentic math were all names for this type of curriculum. This idea fared no better; students did not automatize math skills that need to be rote (Kelly, 2004).

There is a large body of literature demonstrating the limits of learning math through facts and procedures. At the same time, a number of studies have demonstrated that conceptual understanding can be produced through a variety of pedagogical techniques, including carefully sequenced direct instruction of the underlying principles, practice with a wide variety of problem types, and exposure to worked examples (Whitehurst, 2003).

What Ramp-Up Mathematics Does

America's Choice has created a coherent and balanced program. Students work with a discrete set of concepts that have been chosen to build a deep and logical foundation in mathematics while clearing a path for future work in algebra. *Concepts, problem solving, and skills* are recognized as interdependent. To reinforce the emphasis on concepts and help students interact with the mathematics, each student receives a *Concept Book,* which explains the mathematical concepts addressed in the course.

Concepts, Problem Solving, and Skills

America's Choice believes that it is all three of these elements—concepts, problem solving, and skills—that make students successful math learners. Skills are required to implement procedures accurately and automatically. Problem solving is critical to developing mathematical reasoning. And conceptual knowledge is required to see how the structure of mathematics is built on a web of interconnected ideas.

Ramp-Up Mathematics provides conceptual development supported by rich opportunities for problem solving and regular practice in computation to build reliable skills. Skills practice takes place 5 to 10 minutes daily. Skills problems are constructed to reveal patterns that emphasize the underlying mathematical concepts while building procedural automaticity. Rich tasks prompt students to move away from simple solutions and intuitive methods to deeper thinking from which they can generalize to more complex problems.

Concept Book

In Ramp-Up Mathematics, all students receive a *Concept Book*. This reference book is modeled after the Japanese texts and provides all the mathematics covered in Ramp-Up to Pre-Algebra and Ramp-Up to Algebra. It is organized by concept, not lesson, and presents the concepts with explanations, examples, and diagrams. The *Concept Book* helps students and teachers build a broader, more general sense of the concepts. Students will learn to rely on it in every workshop session, as well as when they are doing homework outside the classroom.

3. Communicate Mathematical Thinking

What the Research Says

Students create knowledge by being active in their own learning and comprehension. Malcom Swan surveyed 750 mathematics learners, asking them to describe the kinds of things they do. Their answers included: I listen while the teacher explains; I copy down the method from the board or textbook; I only do questions I am told to do; I work on my own; I practice the same method repeatedly" (Swan, 2005). Of course, none of these answers describe the type of mental activity one hopes to see.

Students create knowledge by communicating their learning in the language of math. Researchers seem to agree that acquiring the new language of mathematics is not easy. Even highly-skilled native English speakers have trouble adopting and routinely using some mathematical terms. According to Hilary Shuard and Andrew Rothery (1984), research shows that "many Mathematical English words which teachers and authors and examiners expect to be understood seem to be absent from the vocabulary of a large proportion of pupils, or at best, they have only a vague or confused idea of what the words mean." Lily Wong Fillmore and Catherine Snow (2002) state that "Teachers need to recognize that all students need support to acquire the structures and vocabulary associated with academic English, and they need to know how to provide it."

Building technical vocabulary is important; students should be expected to discuss mathematics using the academic language of the discipline and talk that is accountable. "Accountable talk...puts forth and demands knowledge that is accurate and relevant to the issue under discussion...uses evidence appropriate to the discipline... and follows established norms of good reasoning" (Resnick, 1999).

Suzanne Chapin argues that talking and interacting with other students around the mathematics is an important part of the learning process. "Talk can be used to assist students in organizing what they already know into larger and more powerful conceptual structures—the big ideas of mathematics…. Discussing the concepts on which procedures are based as well as the reasons behind the steps in any procedure can serve to strengthen students' understanding of both" (Chapin et al., 2003).

What Ramp-Up Mathematics Does

The Ramp-Up Mathematics courses use an *extended workshop model* that is focused on student communication of mathematical ideas. In Ramp-Up Mathematics, students improve their ability to communicate about the mathematics by *learning and using mathematical vocabulary*. During the workshop, students expose and revise their mathematical thinking through *sharing and discussing thinking and strategies*.

Extended Workshop Model

Ramp-Up Mathematics uses a highly verbal and collaborative workshop structure to create an environment in which the goal is to learn and use the academic language of math as soon as possible. The Extended Workshop model consists of an Opening, Work Time, Closing, Skills, and Review and Consolidation. The routines of the Ramp-Up Mathematics workshop enable students to move from task to task independently, and allow the teacher to focus more on the math and less on classroom management.

During the Work Time, students first think through and attempt the math on their own, then share strategies and ideas with partners. Students are encouraged to use accountable talk while exchanging explanations, evidence, and justifications. Students learn how to interact with their classmates and the instructor around the mathematics, and how to conduct themselves during partner, small-group, and whole-group discussions.

During the Closing, students present and discuss strategies and solutions as a whole group. The teacher selects students to present the work that will be most useful for group discussion. The teacher's role is to guide the discussion and engage students in a process of uncovering their misconceptions and actively resolving them. During this time, the teacher should be modeling the thoughts, actions, and work style of real mathematicians.

The Ramp-Up Mathematics Extended Workshop is about talking, reading, writing, and working with mathematics. Students need to really grapple with problems by sharing solutions and comparing schemes. The goal is for students to focus on the concepts and on the exploratory process of mathematics.

Learning and Using Mathematical Vocabulary

Ramp-Up Mathematics includes explicit instruction for vocabulary, introduction and repeated emphasis of new terms, and many opportunities for students to use the language aloud, write the words, label them, cluster them, etc. The Teacher Edition includes extensive support for teaching and reinforcing the use of academic language.

Sharing and Discussing Thinking and Strategies

Discussing the mathematics is an important part of Ramp-Up Mathematics. Partner work engages students in high-level discussions of their mathematical reasoning that include comparing strategies and explaining or justifying their solution steps. Students are active participants in revising the cognitive structure of their mathematics framework. In order to uncover and address misconceptions, students must make their thinking visible. During partner and whole-group work, students are in a safe group setting in which they can share their thinking and strategies about the mathematics. Students benefit by gaining insight into their own thinking, as well as gaining confidence about the advantages of sharing their thoughts with their peers and the instructor.

4. Deeper, Richer Interactions with the Mathematics

What the Research Says

America's Choice researchers were struck by the math textbooks they found in Japan, Singapore, Finland, and other successful countries they studied. The Japanese volumes were especially remarkable for their size. An American middle school math book is typically 600 pages; the Japanese books are about 150 pages, yet the amount of real mathematics presented in the Japanese texts is greater. Japanese students find concepts in a concise format with explicit definitions and clear examples. The mathematics is straightforward and unhampered by superfluous information or procedures. American textbooks are notorious for their length; they are often filled with a plethora of topics, thousands of exercises, and over 340 days of instruction. The net result is that important and closely-related concepts become separated and lost among pages and pages of inconsequential activities (Lewis, 2000).

Zal Usiskin (1980) describes this problem. "The curriculum is quite overcrowded, and there is no alternative but to take things out."

International comparisons from the TIMSS studies (Schmidt et al., 2002; Stigler, et al., 1999, Mullis et al., 2000, 2004) and the SIMSS studies a few years earlier (McKnight et al., 1987) criticize the U.S. curriculum for being a mile wide and an inch deep. Alan Ginsburg and others agree: "Textbook publishers should consider reducing the number of topics and lessons in their textbooks to give more textbook space for developing a rich mathematical treatment of each topic" (Ginsburg et al., 2005). Building a foundation of mathematical knowledge must be done carefully, and in America it is not. The curriculum is poorly prepared, numerous topics are scattered haphazardly, and concepts are covered thinly. What students need, and what students from Singapore, Japan, and other Asian countries have proven is sufficient to succeed, is a study of a discrete set of concepts that are studied more deeply than in the United States
(Ma, 1999; Usiskin, 1980).

Working intensively with fewer but richer problem situations enables students to manipulate and probe the mathematics from every angle. Rich collaborative tasks prompt students to move away from simple solutions and intuitive methods to deeper thinking in which they "generalize to more complex problems" (Swan, 2004).

What Ramp-Up Mathematics Does

Ramp-Up Mathematics tasks focus on a *deeper study of fewer concepts* and *making connections between concepts*. *Using multiple representations* allows students to see the mathematics from different perspectives.

Deeper Study of Fewer Concepts

In Ramp-Up Mathematics, thinking and reasoning are valued over the number of problems completed or the number of correct answers. Students push their thinking beyond the rote activity of solving for answers. The answers, in and of themselves, are not important; the product is not a list of problems checked off or a neat page of computation. The product is deepened mathematical knowledge. Logical and methodical reasoning such as categorizing methods, structuring a solution, anticipating problems, estimating answers, considering alternatives, and weighing options are all tasks in which students participate.

Making Connections Between Concepts

Students who are successful in math are able to transfer their mathematical knowledge from one context to another. In Ramp-Up Mathematics, students are continually being challenged to apply familiar concepts to new situations. Whenever possible, Ramp-Up Mathematics teachers will ask students to think about the mathematics and generalize. Questions like "Does this always work?", "Why or why not?", "Is this always true, sometimes true, or never true?", or "What is similar and different about those two problems? (or those two solutions)" prompt students to think more deeply about the concepts and see how the concepts are connected.

The Preparing for the Closing section of each lesson helps students make those connections by asking questions that generalize beyond the Work Time problems to deeper concepts. The Preparing for the Closing questions help students put a frame around the concepts they have been learning. Since these problems are designed to help students consolidate their learning, students are encouraged to justify their solutions to their peers. The process of connecting concepts involves collaborating with peers to summarize, analyze, problem solve, explain, and apply new knowledge.

The Closing is also an important time for teachers and students to tie together the concepts of the lesson and connect those concepts to previous learning. Teachers are given suggested questions for the Closing to help focus the discussion and connect the concepts.

Using Multiple Representations

Ramp-Up Mathematics tasks are designed to approach each concept from many different angles, offering students a variety of ways to perceive and comprehend the mathematics. Students routinely use number lines, area diagrams, tables, graphs, fractions, decimals, and words to represent the mathematics. Using these different representations of the same concepts helps students see the connections and transferability of mathematical concepts.

5. Better Learners of Mathematics

What the Research Says

Intelligence is perceived by many as a static condition or capacity that an individual has to acquire, retain, and apply knowledge. But researchers such as Lev Vygotsky and Lauren Resnick argue that there are social factors that affect intelligence, and suggest that some aspects of intelligence are in fact learned. Such ideas have far-reaching implications for the design of educational programs aimed at raising the overall cognitive competence and academic achievement of educationally disadvantaged students.

Lev Vygotsky emphasizes the role of culture in "…mediating learning—that is, in providing the tools (words, conventions, symbols, signs, etc.) through which knowledge is communicated. Knowledge is not something that individuals gain from the outside; rather it is something that they gain through their own active experiences. This means that learning and knowledge are to a large extent culturally and socially influenced" (Vygotsky, 1978).

Lauren Resnick and Sharon Nelson-Le Gall state that "interpreting intelligence as a social practice requires a critical expansion of the definition of the construct to include not just the cognitive skills and forms of knowledge that have classically been considered the essence of intelligence, but also a cluster of social performances, such as asking questions, striving to master new problems, and seeking help in problem solving…" (Resnick & Nelson-LeGall, 1997). These researchers put forth one definition of intelligence: "the ability to learn well." This definition implies that intelligence is not just knowledge or cognitive ability, but rather the process by which one acquires knowledge and cognitive ability.

Similarly, researchers from Project Zero state that "Even though in U.S. culture we tend to separate the how from the what of learning, we think the two are integrally linked. We suggest that the focus...should be making learning visible...both the acts and products of learning..." (Project Zero, 2006).

Finally, research has shown that social collaboration promotes deep conceptual insights and shifts in perspective that lead to increases in student understanding and retention of concepts (Damon & Phelps, 1989; Slavin, 1996; Webb, 1989).

What Ramp-Up Mathematics Does

America's Choice believes that intelligence can be learned. As a result, Ramp-Up Mathematics is structured to facilitate and encourage the social as well as the intellectual aspects of learning. By creating a *predictable and safe learning environment*, students can become better learners of mathematics. Within this safe environment, students participate in *peer and group discussions* and are encouraged to *view mistakes as learning opportunities*.

Predictable and Safe Learning Environment

It is critical that students feel safe enough in the Ramp-Up Mathematics workshop to share their ideas, strategies, and, most importantly, mistakes. Most students are not used to talking about mathematics, yet in higher-level courses, student success might depend on participation in study groups and the students' ability to share ideas. The structure of Ramp-Up Mathematics frees students to interact more meaningfully with the mathematics and their fellow students, which leads to better learning.

Peer and Group Discussions

An integral part of the learning process is exchanging ideas. In Ramp-Up Mathematics, students experience rich opportunities for collaboration. During partner and small-group work, teachers help students work together by observing groups, interjecting questions or leading comments, and encouraging students to use accountable talk. During whole-group discussions, groups of students present their work and the teacher facilitates a whole-class discussion of strategies and ideas about the mathematics. This type of collaboration provides immediate feedback and motivates the group to act as a community of learners.

Collaboration and shared goals motivate students to delve deeper into the mathematics. Intensive partner work is especially key to Ramp-Up Mathematics, because students push one another for stronger evidence and justifications. Being able to communicate one's thoughts and ideas is important in any discipline, and students must learn how to do this in a positive and meaningful way. Being a good learner involves asking questions that will bring the appropriate feedback, as well as being able to effectively evaluate and provide feedback on the thinking of others. Students become more productive learners who share the attitudes, habits of mind, and learning techniques of effective critical thinkers.

The rituals and routines, instructor guidance, and the nature of the tasks in the sessions all support the development of these collaborative learning skills.

View Mistakes as Learning Opportunities

The Ramp-Up Mathematics philosophy is that failed solution routes should be followed as closely as successful ones, because within them lie mistaken or incomplete ideas that must be addressed in order for students to move forward. These incorrect or failed strategies are often the key to breakthroughs in understanding.

Ramp-Up Mathematics presents tasks that are deliberately designed to elicit mistakes that are indicative of deeper underlying misconceptions. And the structure and timing of the tasks provide students with opportunities to discover their flawed thinking and correct it. The Ramp-Up Mathematics workshop takes place in a 90-minute double period, which allows for intensive study of the concepts. Students can take their discussions further and stay with a conceptual idea longer without interruption. The double period gives students time to work backward from a mistake to root out flawed thinking.

6. Professional Development

What the Research Says

A 1996 National Commission on Teaching and America's Future [NCTAF] report stated that, annually, over 50,000 untrained people enter teaching on either emergency or substandard licenses. Almost one third (over 30%) of all secondary mathematics teachers do not have even a college minor in math. And the proportion of teachers who are inadequately prepared is even higher in high-poverty schools and in lower-track classes (NCTAF, 1996).

Recent research has also demonstrated that what teachers "know" has substantial influence on what students learn (Darling-Hammond & McLaughlin, 1999). Thus, it follows that effective professional development increases the chances that what students learn is meaningful and relevant (Hawley & Valli, 1999). A professional development program should not just briefly cover material, but provide potential teachers with an understanding of core concepts (Loucks-Horsley, Stiles, & Hewson, 1996).

David Kennedy found that schools in which teachers had opportunities to learn mathematics the way students learned it had significant increases in learning. Successful programs should not address "generic" learning, but instead focus on the learning of particular mathematical ideas (Kennedy et al., 1999). Teachers must become mathematics learners and be challenged at their own level of competence (Loucks-Horsley, Hewson, Love, & Stiles, 1998).

The No Child Left Behind Act of 2001 guarantees ELL students an equal opportunity to achieve their learning goals. Researchers have elicited a number of important concepts in working with ELL students. Christine Denmark and Adele Neuberg point out that instructors must "continuously check for understanding by their students," and keep language "adjusted according to child's needs" (Denmark & Neuberg, 2005). Research also indicates that "cooperative learning provides the conditions for nonnative English speakers who have different degrees of bilingualism to work in both languages" (Lee & Jung, 2004). And, according to Wendy Schwartz, "Asking students to devise math problems from their own experiences increases their interest, concretizes the subject, and demonstrates math's usefulness. It also promotes multiculturalism" (Schwartz, 1991).

Finally, providing meaningful assessment is a concern in any learning program. "Studies have...documented that the practice of frequent, informal monitoring (formative assessment) can produce significant learning gains, especially with low achievers" (Black & William, 1998; Black et al., 2004). Paul Black also suggests that "Assessment for learning is any assessment for which the first priority in its design and practice is to serve the purpose of promoting the pupil's learning" (Black et al., 2004). Formative assessment, in contrast to some standardized tests or district-wide exams, is meant to measure the status of a particular group of students against another, to measure a specific outcome, or to test competency, such as technical licensing exams.

What Ramp-Up Mathematics Does

Ramp-Up Mathematics is designed to provide teachers with comprehensive support by offering *professional development*, *ELL support*, and a variety of both formal and informal *assessments*.

Professional Development

Ramp-Up Mathematics offers eight days of professional development for teachers as well as a one-day orientation for administrators and support personnel. The professional development institutes are delivered throughout the year by certified mathematics trainers, typically with a three-day institute prior to the start of the course and two additional sessions scheduled early in the fall and after the first semester is completed. The focus on professional development is three fold:

- A focus on effective teaching strategies that are grounded in research. Lessons are modeled for and by teachers using the appropriate strategies to engage all learners. The support for ELL students is made explicit as are the structures of the classroom that allow students to think deeply about the mathematics.

- A focus on building deeper mathematics understanding for teachers. Even teachers who are very confident in teaching their specific course content may be lacking the deep understanding of topics that allows them to connect the prior knowledge of the students to the new mathematics they will be learning. Teachers explore the concepts in the student materials and *Concept Book* to better understand how to build on student strengths and address misconceptions.

- A focus on building a community of learners, both with students and teachers. By the time students enter a Ramp-Up Mathematics classroom they have had years of nonsuccess and frustration in learning mathematics. These students do not come with the disposition and habits of successful mathematics students and must learn how to read, write, and talk the language of math. Teachers are given guidance in creating an environment that supports questioning and facilitates skills that allow students to take responsibility for their learning. Teachers, too, need support in moving from comfortable, yet ineffective, classroom practices to ones that challenge, support and promote their students in learning mathematics. The institutes work on building that network of support for these teachers.

ELL Support

The Teacher Edition is embedded with suggestions about how to support ELL math students. The Teacher Edition offers strategies to track English language learners' engagement in the workshop, encourage their participation, and check for comprehension.

The Ramp-Up Mathematics tasks are designed around research that has proven effective with ELL students. Students are often asked to modify problems or create their own problems in new contexts to share with partners. Ramp-Up Mathematics tasks use contexts and math situations that are meaningful and understandable to English language learners. Students explore concepts by comparing work and exchanging ideas. Collaboration occurs with partners, in small groups, and in whole-class discussions. Ramp-Up Mathematics workshops are language-rich rather than language-free; math discussions occur daily and are ongoing. The Ramp-Up Mathematics Teacher Edition offers effective strategies to encourage ELL students to share their knowledge.

Assessments

Multiple choice pre-tests, post-tests, and four cumulative assessments have been developed in conjunction with the Australian Council on Education Research (ACER). ACER is an international leader in the field of assessment and has been involved in both the Trends in Math and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). The Ramp-Up Mathematics courses provide periodic measures focused on course content including end-of-unit assessment, quizzes, observational assessment through class profiles, projects with rubrics, and self-assessment in the "Assessing Your Work" questions for students.

The purpose of assessment in Ramp-Up Mathematics is to facilitate the planning and delivery of differentiated teaching. Investing time in the thoughtful use of Ramp-Up Mathematics assessments will help a teacher figure out the next constructive instructional step, and will provide an opening for conversation, affirming the fact that students need information in order to have a responsible role in their own learning.

References

A

Abedi, J., Courtney, M., & Leon, S. (2003). *Research-Supported Accommodation for English Language Learners in NAEP* (CSE Technical Report 586). Los Angeles: Center for the Study of Evaluation; National Center for Research on Evaluation, Standards, and Student Testing; Graduate School of Education and Information Studies; University of California, Los Angeles.

Aleksandrov, A. D., Kolmogorov, A. N., & Lavrent'ev, M. A. (1956). *Mathematics: Its Content, Methods, and Meaning.* Cambridge, MA: The MIT Press.

America's Choice. (2006). *Mathematics Navigator, an Intervention to Repair Gaps and Misconceptions.* Mathematics Navigator: Elementary School / Middle School / High School. Washington, DC. Downloadable PDF brochure at web site: http://www.ncee.org/acsd/math/220.jsp

American Association for the Advancement of Science. (2000). *Middle Grades Mathematics Textbooks: A Benchmarks-Based Evaluation* (A study included in *Project 2061*). Washington, DC.

The American Diploma Project. (2004). *Ready or Not: Creating a High School Diploma That Counts.* Web site: http://www.achieve.org/files/ADPreport_7.pdf

American Educational Research Association. (2004). English Language Learners: Boosting Academic Achievement. L. B. Resnick (Ed.), C. E. Snow, Issue Researcher. *AERA Research Points: Essential Information for Educational Policy*, *2*(1) (Winter 2004).

American Educational Research Association. (2005). Teaching Teachers: Professional Development to Improve Student Achievement. L. B. Resnick (Ed.), J. Hill & D. Cohen, Issue Researchers. *AERA Research Points: Essential Information for Educational Policy*, *3*(1) (Summer 2005).

Askew, M. (2002). The changing primary mathematics classroom—the challenge of the National Numeracy Strategy. In L. Haggerty (Ed.), *Aspects of Teaching Secondary Mathematics: Perspectives on Practice.* London: Routledge Falmer.

B

Ball, D. L. (2002). Knowing mathematics for teaching: Relations between research and practice. *Mathematics and Education Reform Newsletter*, *14*(3) 1–5.

Ball, D. L., & Cohen, D. K. (1999). Developing Practice, Developing Practitioners: Toward a Theory of Professional Education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the Learning Profession: Handbook of Policy and Practice* (pp. 3–32). San Francisco: Jossey-Bass.

Bell, A. (1993). Principles in the Design of Teaching. *Education Studies in Mathematics*, *24*, 5–34.

Bell, A. (2005). Introduce Diagnostic Teaching. Alan Bell and the Toolkit Team. *A Strategy in the Toolkit for Change Agents.* MARS, Michigan State University.

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of Operation in Verbal Arithmetic Problems: The Effects of Number Size, Problem Structure, and Context. *Educational Studies in Mathematics*, *15*, 129–147.

Bell, A., & Swan, M. (2006). *Mathematics Assessment Resource Service* (MARS Project). Collaborative project with Michigan State University, funded by the National Science Foundation. Washington, DC.

Bell, A., Swan, M., & Taylor, G. (1981). Choice of Operation in Verbal Problems with Decimal Numbers. *Educational Studies in Mathematics*, *12*, 399–420.

Biancarosa, G., & Snow, C. E. (2004). *Reading Next: A Vision for Action and Research in Middle and High School Literacy.* Washington, DC: Alliance for Excellent Education.

Black, P., Harrison, C., Lee, C., Marshall, C., & William, D. (2002). *Working Inside the Black Box: Assessment for Learning in the Classroom.* London: King's College London, School of Education.

Black, P., Harrison, C., Lee, C., Marshall, B., & William, D. (2004). *The Nature and Value of Formative Assessment for Learning.* London: King's College London, Assessment for Learning Group. Web site: http://www.kcl.ac.uk/content/1/c4/73/57/formative.pdf

Black, P., & William, D. (1998). Inside the Black Box: Raising Standards through Classroom Assessment. *Phi Delta Kappan*, *80*(2), 139–148.

Booth, L. (1989). A Question of Structure. In S. Wagner & C. Kieran (Eds.), *Research Issues in the Learning and Teaching of Algebra.* Reston, VA: National Council of Teachers of Mathematics & Lawrence Erlbaum.

Booth, L. (1999). Children's difficulties in beginning algebra. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (pp. 299–307). Reston, VA: National Council of Teachers of Mathematics.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How People Learn: Brain, Mind, Experience, and School.* Committee on Developments in the Science of Learning Research and Educational Practice, Commission on Behavioral and Social Sciences and Education. Washington, DC: National Research Council. Web site: http://www.nap.edu

C

California State Department of Education. (1999). *Mathematics Content Standards for California Public Schools Kindergarten Through Grade Twelve.* Sacramento, CA.

Caperton, G. (2004). *The College Board Launches Integrated Learning Program for Grades 6–12, SpringBoard™, and Unveils College Board Standards for College Success™* (Press Release: Nov. 6, 2004). New York: The College Board. Web site: http://www.collegeboard.com/press/releases/40652.html

Carpenter, T. P., Fennema, E., & Romberg, T. A. (Eds.). (1993). *Rational Numbers: An Integrated Research.* Hillsdale, NJ: Erlsbaum.

Carpenter, T. P., Loef Frank, M., & Levi, L. et al. (2003). *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School.* Portsmouth, NH: Heinemann.

Champigny, H. (2003). *North Middlesex Regional School District Singapore Mathematics Program: A District-scale Study of Program Effectiveness.* Presentation before January 2003 "An Invitation to a Singapore Mathematics Information Session" held by Massachusetts Department of Education. Worcester, MA.

Chapin, S., O'Connor, C., & Canavan Anderson, N. (2003). *Classroom Discussions: Using Math Talk to Help Students Learn. Grades K–6.* Sausalito, CA: Math Solutions Press.

Chazan, D. (2000). *Beyond Formulas in Mathematics and Teaching: Dynamics of the High School Algebra Classroom.* New York: Teachers College Press.

Colvin, R. L. (1997, February 23). Why tiny Singapore is at top of the class. *Los Angeles Times.*

Council of Chief State School Officers. (1995). *State Curriculum Frameworks in Mathematics and Science: How Are They Changing Across the States?* Washington, DC

Council of Chief State School Officers. (2004). *The SCASS Exchange.* June 2004. Web site: http://www.ccsso.org/content/PDFs/EXJune04.pdf

Cummins, J. (1984). *Bilingualism and special education: Issues in assessment and pedagogy.* Clevedon, England: Multilingual Matters.

D

Damon, W., & Phelps, E. (1989). Strategic Uses of Peer Learning in Children's Education. In T. J. Berndt & G. W. Ladd (Eds.), *Peer Relationships in Child Development.* New York: John Wiley and Sons.

Darling-Hammond, L., & McLaughlin, M. W. (1999). Investing in teaching as a learning profession: Policy problems and prospects. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 376–411). San Francisco: Jossey-Bass.

Daro, P., & Resnick, L. (2006). *Accountable Talk: Teaching Students to be Mathematically Smart. Adapted from Phil Daro and Lauren Resnick.* University of Texas, El Paso, Mathematical Sciences. Web site: http://www.math.utep.edu/Faculty/duval/class/random/mathaccttalk.html

Denmark, C., & Neuberg, A. (2005). *Using Instructional Strategies to Make Mathematics Accessible for English Language Learners* (an America's Choice professional development workshop). Washington, DC: National Center on Education and the Economy.

Devlin, K. (2000). Finding Your Inner Mathematician. *The Chronicle of Higher Education*, *46*, B5.

Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How Students Learn: History, Mathematics, and Science in the Classroom.* Board on Behavioral, Cognitive, and Sensory Sciences and Education. Washington, DC: National Academy Press.

Dossey, J. A. (1997). *Essential Skills in Mathematics: A Comparative Analysis of American and Japanese Assessments of Eighth-graders.* (NCES97-885). L. Peak & D. Nelson, project officers. Washington, DC: National Center for Education Statistics.

E

Early Algebra Project. (2006). National Center for Improving Student Learning and Achievement in Mathematics and Science (NCISLA).

EducationAtlas.com. (2006). *Value of Higher Education.* Web site: http://www.educationatlas.com/value-of-higher-education.html

F

Ferrini-Mundy, J., & Schram, T. (Eds.). (1997). The Recognizing and Recording Reform in Mathematics Education Project: Insights, Issues, and Implications. *Journal for Research in Mathematics Education.* Monograph No. 8. Reston, VA: National Council of Teachers of Mathematics.

Fillmore, L. W., & Snow, C. (2002). What Teachers Need to Know about Language. C. Temple Adger, C. E. Snow, D. Christian (Eds.), *Language in Education*, *97*. Delta Systems Co., Inc.

Foxman, D., et al. (1980). *Great Britain Assessment of Performance Unit.* Mathematical development: secondary survey report. London, HMSO.

Freeman, R. (2006). *Higher Education for a Highly Competitive World.* Prepared for the United States Department of Education. Retrieved September 2006. Last modified June 16, 2006. Web site: http://www.ed.gov/print/teachers/how/prep/higher/higher-ed.html

G

Gelfand, I. M., & Shen, A. (1993). *Algebra*. New York: Birkhauser.

Ginsburg, A., Leinwand, S., Anstrom, T., Pollock, E., & Witt, E. (Eds.). (2005). *What the United States Can Learn From Singapore's World-Class Mathematics System (and What Singapore Can Learn from the United States): An Exploratory Study.* Prepared for the United States Department of Education by American Institutes for Research. Washington, DC: U.S. Dept. of Education. Web site: http://www.air.org/news/documents/singapore%20edweek.htm

Gonzales, P., Guzman J. C., Partelow, L. et al. (2004). *Highlights from the Trends in International Mathematics and Science Study (TIMSS) 2003.* U.S. Department of Education, National Center for Education Statistics. Washington, DC: U.S. Government Printing Office.

Grissmer, D. W., Flanagan, A., Kawata, J., & Williamson, S. (2000). *Improving Student Achievement: What State NAEP Test Scores Tell Us.* Santa Monica: Rand.

H

Hart, K. (1980). *Secondary School Children's Understanding of Mathematics.* A report of the Mathematics Component of the CSMS Programme. Chelsea College, University of London.

Hart, K. (1981). *Children's Understanding of Mathematics 11–16.* London, England: John Murray.

Hawley, W. D., & Valli, L. (1999). The essentials of professional development. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the Learning Profession: Handbook of Policy and Practice* (pp. 127–50). San Francisco: Jossey-Bass.

Hiebert, J., Carpenter, T. P., Fennema, E., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making Sense: Teaching and Learning Mathematics With Understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., Gallimore, R., Garnier, H., Bogard Givvin, K., Hollingsworth, H., Jacobs, J., Chui, A., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P., & Stigler, J. (2003). *Highlights from the TIMSS 1999 Video Study of Eighth-Grade Mathematics Teaching.* Washington, DC: NCES.#2003011.

Hill, H., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*(5), 330–351.

Hill, H., Rowan, B., & Ball, D. (in press). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal.*

I

The Ingenuity Project. (2003). Web site: http://www.ingenuityproject.org

J

Johnson, M. (1976). *How to Solve Word Problems in Algebra: A Solved Problem Approach.* New York: McGraw-Hill.

K

Karplus, R., Pulos, S., & Stage, E. K. (1983a). Early Adolescents' Proportional Reasoning on 'Rate' Problems. *Educational Studies in Mathematics*, *14*, 219–233.

Karplus, R., Pulos, S., & Stage, E. K. (1983b). Proportional Reasoning of Early Adolescents. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 45–90). New York: Academic Press.

Kelly, K. (2004). The "Math Wars" A Short Analysis of the Controversy. Web site: http://www.newfoundations.com/ConAnalysis/KellyMathWars.html

Kennedy, D. M., Ip, A., Adams, C., & Eizenberg, N. (1999). *Developing Generic Interactive Learning Tools to Engage Students: The Text Analysis Object for Web and CD-ROM.* Paper presented at the EdMedia99.

Kennedy, L. M., & Tipps, S. (1998). *Guiding Children's Learning of Mathematics* (8th ed.). Belmont, CA: Wadsworth.

Kieran, C., and Chalouh, L. (1993). Prealgebra: The Transition from Arithmetic to Algebra. In B. Moses (Ed.), *Algebraic Thinking, grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 59–70). Reston, VA: National Council of Teachers of Mathematics.

Kodaira, K. (Ed.). (1992a). *Japanese Grade 7 Mathematics*, UCSMP Textbook Translations. Chicago: The University of Chicago School Mathematics Project.

Kodaira, K. (Ed.). (1992b). *Japanese Grade 8 Mathematics*, UCSMP Textbook Translations. Chicago: The University of Chicago School Mathematics Project.

Kodaira, K. (Ed.). (1992c). *Japanese Grade 9 Mathematics*, UCSMP Textbook Translations. Chicago: The University of Chicago School Mathematics Project.

L

Lamon, S. (1999). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Mahwah, NJ: Erlbaum.

Lee, H. J., & Jung, W. S. (2004). Limited English Proficient (LEP) Students: Mathematical Understanding. *Mathematics Teaching in the Middle School*, *9*(5). Reston, VA: National Council of Teachers of Mathematics.

Lemke, M., Sen, A., Pahlke, E., Partelow, L., Miller, D., Williams, T., Kastberg, D., Jocelyn, L. (2004). *International Outcomes of Learning in Mathematics Literacy and Problem Solving: PISA 2003 Results from the U.S. Perspective.* (NCES 2005–003). Washington, DC: U.S. Department of Education, National Center for Education Statistics. Web site: http://nces.ed.gov/pubs2005/2005003.pdf

Lesh, R., Post, T., & Behr, M. (1988). Proportional Reasoning. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades.* (pp. 93–118). Reston, VA: National Council of Teachers of Mathematics.

Lewis, C. (2000). *Lesson Study: The Core of Japanese Professional Development.* Invited Address to the Special Interest Group on Research in Mathematics Education American Educational Research Association Meetings, New Orleans, April 28, 2000.

Lochhead, J., & Mestre, J. P. (1988). From Words to Algebra: Mending Misconceptions. In *The Ideas of Algebra, K–12, The 1988 Yearbook of the National Council of Teachers of Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Lodholz, R. D. (1990). The Transition from Arithmetic to Algebra. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999). (pp. 52–58). Reston, VA: National Council of Teachers of Mathematics.

Loucks-Horsley, S., Hewson, P. W., Love, N., & Stiles, K. E. (1998). *Designing Professional Development for Teachers of Science and Mathematics.* Thousand Oaks, CA: Corwin Press for the National Institute for Science Education.

Loucks-Horsley, S., Stiles, K., & Hewson, P. (1996). Principles of Effective Professional Development for Mathematics and Science Education: A Synthesis of Standards. NISE Brief, University of Wisconsin-Madison.

Loveless, T. (2003). *Do students have too much homework?* (Brown Center Report on American Education). Washington, DC: Brookings Institution.

M

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics.* Mahwah, NJ: Erlbaum.

Maccini, P., & Gagnon, J. C. (2000). Best Practices for Teaching Mathematics to Secondary Students with Special Needs. *Focus on Exceptional Children*, *32*, 1–21.

Marshall, H. H. (2001). Cultural Influences on the Development of Self Concept: Updating Our Thinking. *Young Children* (November 2001).

Maryland State Department of Education. (2000). *Maryland State Content Standards for Math.* Web site: http://mdk12.org/mspp/standards/math/index.html

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. T., & Cooney, T. J. (1987). *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective.* Champaign, IL: Stripes Publishing.

Ministry of Education (MOE), Singapore. (1997). *Primary mathematics.* Singapore: Curriculum Development Institute of Singapore.

Ministry of Education (MOE), Singapore. (2001). *Primary mathematics syllabus.* Curriculum Planning and Development Division. Web site: http://www.moe.gov.sg/cpdd/pri_maths_links.htm

Ministry of Education (MOE), Singapore. (2002). *Primary mathematics* (3rd ed.). Singapore: Federal Publications.

Ministry of Education (MOE), Singapore. (2003a). Mathematics Department. Web site: http://www.moe.edu.sg/schools/rulang/org_mathematics.htm#LSP

Ministry of Education (MOE), Singapore. (2003b). *Primary School Leaving Exam.* Web site: http://www.seab.gov.sg/SEAB/psle/generalInfo.html

Ministry of Education, Singapore. (2004). *Singapore Tops the Trends in International Mathematics and Science Study* (Press Release # 1214). December 2004. Web site: http://www.moe.gov.sg/press/2004/pr20041214.htm

Moses, R. P., & Cobb, C. E. (2002). *Radical Equations: Civil Rights from Mississippi to the Algebra Project*. Boston: Beacon Press.

Mullis, I., Martin, M., Gonzalez, E., Gregory, K., Garden, R., O'Connor, K., Chrostowski, S., & Smith, T. (2000). *TIMSS 1999 International Mathematics Report: Findings from IEA's Repeat of the Third International Mathematics and Science Study at the Eighth Grade.* Boston: TIMSS and PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I., Martin, M., Gonzalez, E., & Chrostowski, S. (2004). *TIMSS 2003 International Mathematics Report.* Boston: TIMSS and PIRLS International Study Center, Lynch School of Education, Boston College.

N

National Assessment of Educational Progress. (2004a). *The State Assessment.* Web site: http://nces.ed.gov/nationsreportcard/about/state.asp

National Assessment of Educational Progress. (2004b) *Released Questions Data Tool.* Web site: http://nces.ed.gov/nationsreportcard/itmrls/

National Center for Education Statistics, United States Department of Education. (1996). *High School Completion and Life Experiences, Dropout Rates in the United States: 1994*. Web site: http://nces.ed.gov/pubs96/r94/r9412a.asp

National Center for Education Statistics, United States Department of Education. (2000). *Pursuing Excellence: Comparisons of International Eighth-grade Mathematics and Science Achievement from a U.S. Perspective, 1995 and 1999.* Office of Educational Research and Improvement. U.S. Department of Education: NCES 2001–028.

National Center for Education Statistics, United States Department of Education. (2003). Postsecondary attainment of 1988 8th-graders. In *The Condition of Education, Student Effort and Educational Progress.* Indicator 22 (2003). Web site: http://nces.ed.gov/programs/coe/2003/section3/indicator22.asp

National Center for Education Statistics, United States Department of Education. (2004a). *U.S. Performance in Mathematics Literacy and Problem Solving 2004*. Web site: http://nces.ed.gov/surveys/pisa/pisa2003highlights_2.asp

National Center for Education Statistics, United States Department of Education. (2004b). Event Dropout Rates by Family Income, 1972–2001. In *The Condition of Education, Student Effort and Educational Progress*. Web site: http://nces.ed.gov/programs/coe/2004/section3/indicator16.asp

National Center for Education Statistics, United States Department of Education. (2004c). Learning Opportunities: Remedial Coursetaking. In *The Condition of Education, Contexts of Post-secondary Education.* Indicator 31. Retrieved September, 2006. Web site: http://nces.ed.gov/programs/coe/2004/section5/indicator31.asp

National Center for Education Statistics, United States Department of Education. (2006a). U.S. Student and Adult Performance on International Assessment of Educational Achievement. *The Condition of Education, Special Analysis 2006.* Web site: http://nces.ed.gov/programs/coe/2006/analysis/sa02.asp

National Center for Education Statistics, United States Department of Education. (2006b). Language Minority School-Age Children. In *The Condition of Education, Participation in Education.* Indicator 7(2006). Web site: http://nces.ed.gov/programs/coe/2006/section1/indicator07.asp

National Center for Education Statistics, United States Department of Education. (2006c). Learning Opportunities: Remedial Coursetaking. In *The Condition of Education, Contexts of Post-secondary Education.* Indicator 31. Retrieved September, 2006. Web site: http://nces.ed.gov/programs/coe/2004/section5/indicator31.asp

National Commission on Teaching and America's Future. (1996). *What Matters Most: Teaching for America's Future.* New York.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA.

National Council of Teachers of Mathematics. (1994). *Addenda Series Grades 5–8: Understanding Rational Numbers and Proportion.* (pp. 61–75). Reston, VA.

National Council of Teachers of Mathematics. (1999). *Algebraic Thinking, grades K–12: Readings from NCTM's School-based Readings and Other Publications.* B. Moses (Ed.). Reston, VA.

National Council of Teachers of Mathematics. (2000). *Principals and Standards for School Mathematics.* Reston, VA. Web site: http://www.nctm.org/standards

National Mathematics Advisory Panel. (2006). *Strengthening Math Education through Research.* Washington, DC: United States Department of Education. Web site: http://www.ed.gov/about/bdscomm/list/mathpanel/factsheet.html

National Research Council. (2001a). *Adding It Up: Helping Children Learn Mathematics.* Kilpatrick, J. & Swafford, B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2001b). *Knowing What Students Know: The Science and Design of Educational Assessment.* J. Pellegrino, N. Chudowsky, & R. Glaser (Eds.). Committee on the Foundations of Assessment, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

New Jersey Grade 8 Middle School Statewide Assessment. Web site: http://www.state.nj.us/njded/assessment/ms/sample/test_book_math.pdf

Newmann, F. M., Smith, B., Allensworth, E., & Bryk, A. (2001). Instructional program coherence: What it is and why it should guide school improvement policy. *Educational Evaluation and Policy Analysis*, *23*(4) (Winter 2001), 297–321.

Ng, M. M., Guthrie, J. T., Van Meter, P., McCann, A., & Alao, S. (1998). How Do Classroom Characteristics Influence Intrinsic Motivation for Literacy? *Reading Psychology*, *19*(4), 319–398.

Nisbett, R. (2003). *The Geography of Thought: How Asians and Westerners Think Differently…and Why.* New York: The Free Press.

No Child Left Behind Act. 2001. [Public Law 107-110, Sec. 1111(b)(3)(A)]North Carolina End-of-Grade Test, Sample Items for Grades 6 and 8. Web site: http://www.ncpublicschools.org/accountability/testing/eog/

O

Ohio Sixth Grade Proficiency Test, Sample Items. Web site: http://www.ode.state.oh.us/proficiency/samples/sixth_previous.asp

P

Palincsar, A. S., & Herrenkohl, L. R. (2002.) Designing Collaborative Learning Contexts. *Theory into Practice*, *41*(1), 26–32.

Partnership for 21st Century Skills. (2003). *Learning for the 21st century.* Washington, DC. Web site: http://www.21stcenturyskills.org

Post, T., Harel, G., Behr, M., & Lesh, R. (1991). Intermediate teachers' knowledge of rational number concepts. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating Research on Teaching and Learning Mathematics* (pp. 177–198). Madison, WI: Center for Educational Research.

Programme for International Student Assessment. (2003). *Problem Solving for Tomorrow's World—First Measure of Cross-Curricular Competencies.* Organization for Economic Co-operation and Development. Web site: http://www.pisa.oecd.org/dataoecd/25/12/34009000.pdf

Project Zero. N. Goodman, founder. (2006). *Making Learning Visible: Children as Individual and Group Learners* (Project Zero research project). Cambridge, MA: Harvard Graduate School of Education. Web site: http://pzweb.harvard.edu/

R

Raimi, R. A., & Braden, L.S. (1998). *State Mathematics Standards: An Appraisal of Math Standards in Forty-six States, the District of Columbia, and Japan.* Washington, DC: Thomas B. Fordham Foundation.

Resnick, L. (1999). Making America Smarter: A Century's Assumptions About Innate Ability Give Way to a Belief in the Power of Effort. *Education Week*. (June 16), 38–40.

Resnick, L., (Ed.). (2005). American Educational Research Association. Research Points: Essential Information for Education Policy. Teaching Teachers to Improve Student Achievement. *Professional Development*, *3*(1) (Summer 2005). Washington, DC.

Resnick, L., & Nelson-Le Gall, S. (1997). Socializing Intelligence. In L. Smith, J. Dockrell, & P. Tomlinson (Eds.), *Piaget, Vygotsky and Beyond.* London and New York: Routledge.

Rosenbaum Foundation. (2001). *Singapore Primary Mathematics Teacher's Guide.* Chicago.

Rosenstein, J. G., Caldwell, J. H., & Crown, W. D. (1996). *New Jersey mathematics curriculum framework.* New Jersey Mathematics Coalition and New Jersey Department of Education. Web site: http://www.state.nj.us/njded/frameworks/math/math.pdf

Rosnick, P. (1981). Some Misconceptions Concerning the Concept of Variable. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 313–315). Reston, VA: National Council of Teachers of Mathematics.

The Royal Society/Joint Mathematical Council of the United Kingdom. (1995). *Teaching and Learning Algebra pre-19.* London.

Ryan, J., & Williams, J. (2000). *Mathematical Discussions with Children*. Manchester, England: University of Manchester Press.

S

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum. *American Educator* (Summer 2002), 1–17.

Schoenfeld, A. H. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense-making in Mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: Macmillan.

Schoenfeld, A. H., & Arcavi, A. (1988). On the Meaning of Variable. *Mathematics Teacher* (September 1988). Reston, VA: National Council of Teachers of Mathematics.

Schwartz, W. (1991). *Teaching Limited English Proficient Students to Understand and Use Mathematics.* ERIC/CUE Digest No. 70. ERIC Clearinghouse on Urban Education. New York: NY. Eric Identifier: ED334310.

Shuard, H., & Rothery, A. (1984). *Children Reading Mathematics.* Language and Reading in Mathematics Group. London: J. Murray.

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher* (February 1986).

Silver, E. A., Smith, M. S., & Nelson, B. S. (1995). The QUASAR project: Equity concerns meet mathematics education reform in middle school. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New Directions in Equity in Mathematics Education* (pp. 9–56). New York: Cambridge University Press.

Slavin, R. L. (1996). Perception of self and attitude towards one's college class environment. *Perceptual and Motor Skills*, *83*, 211–217.

Smith, M. S., Silver, E. A., & Stein, M. K., with Boston, M., Henningsen, M. A., & Hillen, A. F. (2005). *Improving Instruction in Rational Numbers and Proportionality.* New York: Teachers College Press.

Snipes, J., Doolittle, F., & Herlihy, C. (2002). *Foundations for success: case studies of how urban school systems improve achievement.* MDRC and Great City Schools.

Solano-Flores, G., Lara, J., Sexton, U., & Navarrete, C. (2001). *Testing English Language Learners: A Sampler of Student Responses to Science and Mathematics Test Items.* Washington, DC: Council of Chief State School Officers.

Sowder, J., Armstrong, B., Lamon, S., Simon, M., Sowder, L., & Thompson, A. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education*, *1*, 127–155.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*, 455–488.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development.* New York: Teachers College Press.

Stigler, J. W., & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York: The Free Press.

Swan, M. (1985). *The Language of Functions and Graphs, An Examination Module for Secondary Schools.* Manchester, England: Joint Matriculation Board, Shell Centre for Mathematical Education.

Swan, M. (2004). *Learning Mathematics Through Discussion and Reflection* (First Draft) (Standards Unit). Department for Education and Skills Standards Unit. University of Nottingham.

Swan, M. (2005). *Improving Learning in Mathematics: Challenges and Strategies* (Standards Unit). Department for Education and Skills Standards Unit. University of Nottingham.

T

Texas Assessment of Knowledge and Skills (TAKS), Sample Items for 6th and 8th Grade. Web site: http://scotthochberg.com/files/taas/math6.pdf

Thorpe, J. A. (1989). Algebra: What Should We Teach and Why Should We Teach It. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 31–39). Reston, VA: National Council of Teachers of Mathematics.

Times Media Private Limited. (2003). *Primary Mathematics.* Content prepared by Curriculum, Planning and Publishing Division, Ministry of Education, Singapore.

Tomlinson, C. A. (2001). *Differentiation of Instruction in the Elementary Grades.* ERICDigests.org. Web site: http://www.ericdigests.org/2001-2/elementary.html

Tournaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics*, *16*, 181–204.

Treilibs, V., Burkhardt, H., & Low, B. (1980). *Formulation Processes in Mathematical Modeling.* Nottingham, England: Shell Centre for Mathematical Education.

Tucker, A., Fey, J., Schifter, D., & Sowder, J. (2001). *The Mathematical Education of Teachers.* Volume 11 in the Conference Board of the Mathematical Sciences series. Web site: http://www.cbmsweb.org/MET_Document/

Tyson, H., & Woodward, A. (1989). Why students aren't learning very much from textbooks. *Educational Leadership*, *47*, 14–17.

Tyson-Bernstein, H. (1988). *A Conspiracy of Good Intentions: America's Textbook Fiasco.* Washington, DC: Council for Basic Education.

U

Usiskin, Z. (1980). What Should Not Be in the Algebra Curriculum of Average College-Bound Students. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 76–81). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1987). Why Elementary Algebra Can, Should, and Must Be an Eighth-Grade Course for Average Students. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 40–48). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1988). Conceptions of School Algebra and Uses of Variables. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 7–13). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1995). Why is Algebra Important to Learn? In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 22–29). Reston, VA: National Council of Teachers of Mathematics.

V

Vail, K. (2004). Remaking High School. *American School Board Journal*, *191*(11). Retrieved July, 2006. Web site: http://asbj.com/2004/11/1104coverstory.html

Valverde, G., & Schmidt, W. (1997). Refocusing U.S. math and science education. *Issues in Science and Technology* (Winter, 1997–98).

Van de Walle, J. A. (2001). *Elementary and Middle School Mathematics: Teaching Developmentally* (4th ed.). New York: Addison Wesley Longman, Inc.

Van Dyke, F., & Craine, T. V. (1997). Equivalent Representations in the Learning of Algebra. In B. Moses (Ed.), *Algebraic Thinking, Grades K–12: Readings from NCTM's School-based Readings and Other Publications* (1999) (pp. 215–219). Reston, VA: National Council of Teachers of Mathematics.

Viadero, D. (2000). U.S. schools importing Singaporean texts. *Education Week* (September 27, 2000).

Virginia Department of Education, Division of Instruction. (2004). *Mathematics: Strategies for Teaching Limited English Proficient (LEP) Students.* A Supplemental Resource to the K–12 Mathematics Standards of Learning. April 2004. Web site: http://www.pen.k12.va.us/VDOE/Instruction/ESL/LEPmathResource.pdf

Vygotsky, L. S. (1962). *Thought and Language.* Cambridge, MA: MIT Press.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes.* M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.). Cambridge, MA: Harvard University Press.

W

Webb, N.M. (Ed.). (1989). Peer interaction, problem-solving, and cognition: Multidisciplinary perspectives [special issue]. *International Journal of Education Research*, *13*(1).

Whimbey, A., & Lochhead, J. (1986). *Problem Solving and Comprehension* (4th ed.). Hillsdale, NJ: Erlsbaum.

Whitehurst, G. J. (2003). *Research on Mathematics Education.* Paper presented at Secretary's Summit on Mathematics, U.S. Department of Education.

Wisconsin Center for Education Research. (2000). Building a Foundation for Learning Algebra in the Elementary Grades. *In Brief*, *1*(2) (Fall 2000). Madison, WI: University of Wisconsin Madison.

Wu, H. (1999). Basic skills versus conceptual understanding: A bogus dichotomy. *Mathematics Education*, *20*(4) (Fall 1999), 1–7.

Wu, H. (2002). Chapter 2: *Fractions* (Draft) (June 20, 2001; revised September 3, 2002). Web site: http://math.Berkeley.edu/~wu/

Contact us to learn more about Ramp-Up Mathematics:

800.848.9500 | pearsonschoolimprovement.com

Pearson School Achievement Services

1919 M Street, NW, Suite 310 | Washington, DC 20036

ISBN: 978-9-90800-723-6