3 Parallel and Perpendicular Lines

3.1 Pairs of Lines and Angles
3.2 Parallel Lines and Transversals
3.3 Proofs with Parallel Lines
3.4 Proofs with Perpendicular Lines
3.5 Equations of Parallel and Perpendicular Lines

Bike Path *(p. 161)*

Crosswalk *(p. 154)*

Kiteboarding *(p. 143)*

Gymnastics *(p. 130)*

Tree House *(p. 130)*

Maintaining Mathematical Proficiency

Finding the Slope of a Line

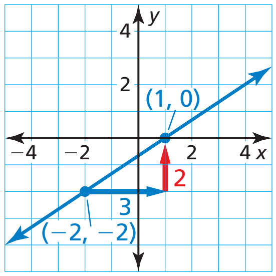
Example 1 Find the slope of the line shown.

Let $(x_1, y_1) = (-2, -2)$ and $(x_2, y_2) = (1, 0)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Write formula for slope.}$$

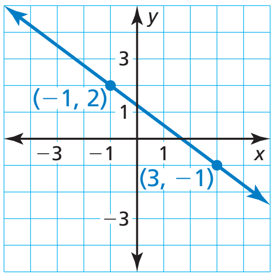
$$= \frac{0 - (-2)}{1 - (-2)} \qquad \text{Substitute.}$$

$$= \frac{2}{3} \qquad \text{Simplify.}$$

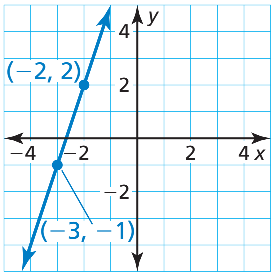


Find the slope of the line.

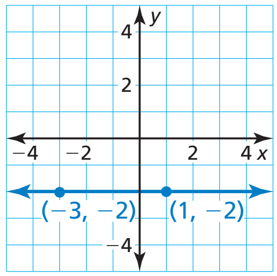
1.



2.



3.



Writing Equations of Lines

Example 2 Write an equation of the line that passes through the point $(-4, 5)$ and has a slope of $\frac{3}{4}$.

$y = mx + b$ — Write the slope-intercept form.

$5 = \frac{3}{4}(-4) + b$ — Substitute $\frac{3}{4}$ for m, -4 for x, and 5 for y.

$5 = -3 + b$ — Simplify.

$8 = b$ — Solve for b.

▶ So, an equation is $y = \frac{3}{4}x + 8$.

Write an equation of the line that passes through the given point and has the given slope.

4. $(6, 1)$; $m = -3$
5. $(-3, 8)$; $m = -2$
6. $(-1, 5)$; $m = 4$
7. $(2, -4)$; $m = \frac{1}{2}$
8. $(-8, -5)$; $m = -\frac{1}{4}$
9. $(0, 9)$; $m = \frac{2}{3}$
10. **ABSTRACT REASONING** Why does a horizontal line have a slope of 0, but a vertical line has an undefined slope?

Mathematical Practices

Mathematically proficient students use technological tools to explore concepts.

Characteristics of Lines in a Coordinate Plane

Core Concept

Lines in a Coordinate Plane

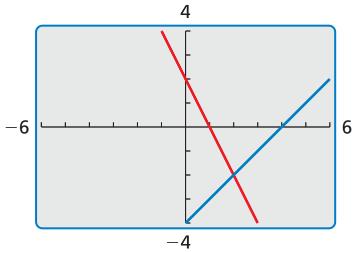
1. In a coordinate plane, two lines are *parallel* if and only if they are both vertical lines or they both have the same slope.
2. In a coordinate plane, two lines are *perpendicular* if and only if one is vertical and the other is horizontal or the slopes of the lines are negative reciprocals of each other.
3. In a coordinate plane, two lines are *coincident* if and only if their equations are equivalent.

EXAMPLE 1 Classifying Pairs of Lines

Here are some examples of pairs of lines in a coordinate plane.

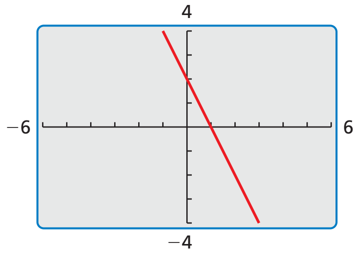
a. $2x + y = 2$
$x - y = 4$

These lines are not parallel or perpendicular. They intersect at $(2, -2)$.



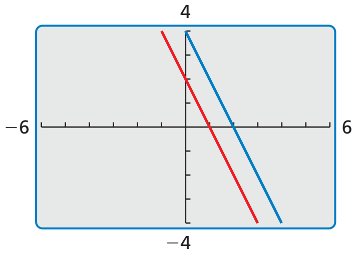
b. $2x + y = 2$
$4x + 2y = 4$

These lines are coincident because their equations are equivalent.



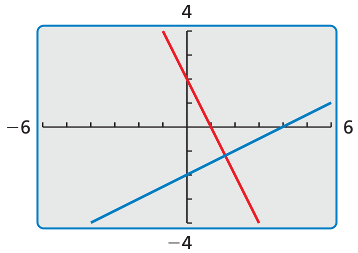
c. $2x + y = 2$
$2x + y = 4$

These lines are parallel. Each line has a slope of $m = -2$.



d. $2x + y = 2$
$x - 2y = 4$

These lines are perpendicular. They have slopes of $m_1 = -2$ and $m_2 = \frac{1}{2}$.



Monitoring Progress

Use a graphing calculator to graph the pair of lines. Use a square viewing window. Classify the lines as parallel, perpendicular, coincident, or nonperpendicular intersecting lines. Justify your answer.

1. $x + 2y = 2$
$2x - y = 4$

2. $x + 2y = 2$
$2x + 4y = 4$

3. $x + 2y = 2$
$x + 2y = -2$

4. $x + 2y = 2$
$x - y = -4$

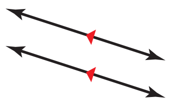
3.1 Pairs of Lines and Angles

Essential Question What does it mean when two lines are parallel, intersecting, coincident, or skew?

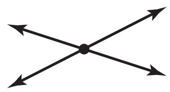
EXPLORATION 1 Points of Intersection

Work with a partner. Write the number of points of intersection of each pair of coplanar lines.

a. parallel lines



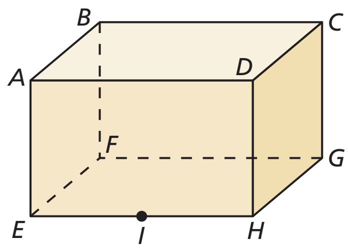
b. intersecting lines



c. coincident lines

EXPLORATION 2 Classifying Pairs of Lines

Work with a partner. The figure shows a *right rectangular prism*. All its angles are right angles. Classify each of the following pairs of lines as *parallel*, *intersecting*, *coincident*, or *skew*. Justify your answers. (Two lines are **skew lines** when they do not intersect and are not coplanar.)



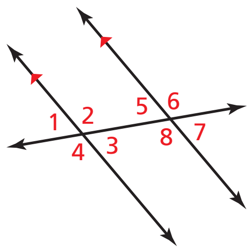
Pair of Lines	Classification	Reason
a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$		
b. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$		
c. $\overleftrightarrow{EI}$ and $\overleftrightarrow{IH}$		
d. $\overleftrightarrow{BF}$ and $\overleftrightarrow{EH}$		
e. $\overleftrightarrow{EF}$ and $\overleftrightarrow{CG}$		
f. $\overleftrightarrow{AB}$ and $\overleftrightarrow{GH}$		

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results.

EXPLORATION 3 Identifying Pairs of Angles

Work with a partner. In the figure, two parallel lines are intersected by a third line called a *transversal*.



a. Identify all the pairs of vertical angles. Explain your reasoning.

b. Identify all the linear pairs of angles. Explain your reasoning.

Communicate Your Answer

4. What does it mean when two lines are parallel, intersecting, coincident, or skew?

5. In Exploration 2, find three more pairs of lines that are different from those given. Classify the pairs of lines as *parallel*, *intersecting*, *coincident*, or *skew*. Justify your answers.

3.1 Lesson

Core Vocabulary

parallel lines, *p. 126*
skew lines, *p. 126*
parallel planes, *p. 126*
transversal, *p. 128*
corresponding angles, *p. 128*
alternate interior angles, *p. 128*
alternate exterior angles, *p. 128*
consecutive interior angles, *p. 128*

Previous
perpendicular lines

What You Will Learn

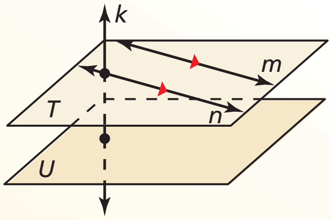
- Identify lines and planes.
- Identify parallel and perpendicular lines.
- Identify pairs of angles formed by transversals.

Identifying Lines and Planes

Core Concept

Parallel Lines, Skew Lines, and Parallel Planes

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** when they do not intersect and are coplanar. Two lines are **skew lines** when they do not intersect and are not coplanar. Also, two planes that do not intersect are **parallel planes**.



Lines m and n are parallel lines ($m \parallel n$).

Lines m and k are skew lines.

Planes T and U are parallel planes ($T \parallel U$).

Lines k and n are intersecting lines, and there is a plane (not shown) containing them.

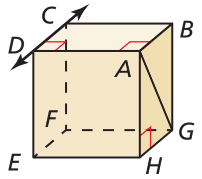
Small directed arrows, as shown in red on lines m and n above, are used to show that lines are parallel. The symbol $\parallel$ means "is parallel to," as in $m \parallel n$.

Segments and rays are parallel when they lie in parallel lines. A line is parallel to a plane when the line is in a plane parallel to the given plane. In the diagram above, line n is parallel to plane U.

EXAMPLE 1 Identifying Lines and Planes

REMEMBER
Recall that if two lines intersect to form a right angle, then they are perpendicular lines.

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?



a. line(s) parallel to $\overleftrightarrow{CD}$ and containing point A

b. line(s) skew to $\overleftrightarrow{CD}$ and containing point A

c. line(s) perpendicular to $\overleftrightarrow{CD}$ and containing point A

d. plane(s) parallel to plane EFG and containing point A

SOLUTION

a. $\overleftrightarrow{AB}$, $\overleftrightarrow{HG}$, and $\overleftrightarrow{EF}$ all appear parallel to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AB}$ contains point A.

b. Both $\overleftrightarrow{AG}$ and $\overleftrightarrow{AH}$ appear skew to $\overleftrightarrow{CD}$ and contain point A.

c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{FC}$ all appear perpendicular to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AD}$ contains point A.

d. Plane ABC appears parallel to plane EFG and contains point A.

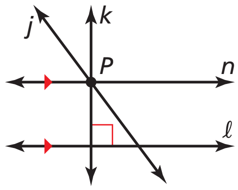
Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Look at the diagram in Example 1. Name the line(s) through point F that appear skew to $\overleftrightarrow{EH}$.

Identifying Parallel and Perpendicular Lines

Two distinct lines in the same plane either are parallel, like line ℓ and line n, or intersect in a point, like line j and line n.

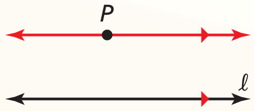


Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line. For example, line k is the line through point P perpendicular to line ℓ, and line n is the line through point P parallel to line ℓ.

Postulates

Postulate 3.1 Parallel Postulate

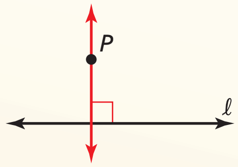
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

Postulate 3.2 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

EXAMPLE 2 Identifying Parallel and Perpendicular Lines

The given line markings show how the roads in a town are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.

SOLUTION

a. $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$

b. $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$

c. $\overleftrightarrow{FE}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{MD}$ is parallel to $\overleftrightarrow{FE}$, and by the Parallel Postulate, there is exactly one line parallel to $\overleftrightarrow{FE}$ through M.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

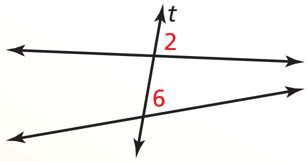
2. In Example 2, can you use the Perpendicular Postulate to show that $\overleftrightarrow{AC}$ is *not* perpendicular to $\overleftrightarrow{BF}$? Explain why or why not.

Identifying Pairs of Angles

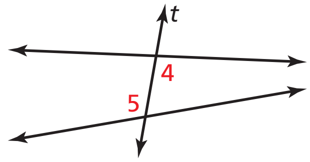
A **transversal** is a line that intersects two or more coplanar lines at different points.

Core Concept

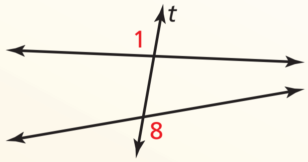
Angles Formed by Transversals



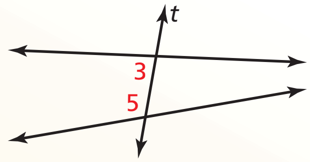
Two angles are **corresponding angles** when they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal *t*.



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal *t*.



Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal *t*.

EXAMPLE 3 Identifying Pairs of Angles

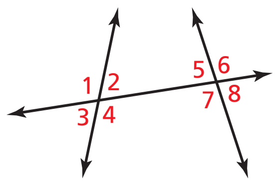
Identify all pairs of angles of the given type.

a. corresponding

b. alternate interior

c. alternate exterior

d. consecutive interior



SOLUTION

a. ∠1 and ∠5
∠2 and ∠6
∠3 and ∠7
∠4 and ∠8

b. ∠2 and ∠7
∠4 and ∠5

c. ∠1 and ∠8
∠3 and ∠6

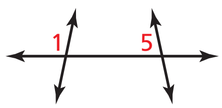
d. ∠2 and ∠5
∠4 and ∠7

Monitoring Progress

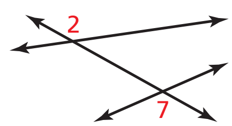
Help in English and Spanish at *BigIdeasMath.com*

Classify the pair of numbered angles.

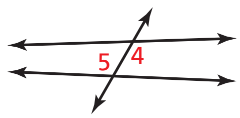
3.



4.



5.



3.1 Exercises

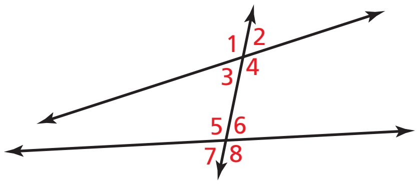
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Two lines that do not intersect and are also not parallel are ________ lines.

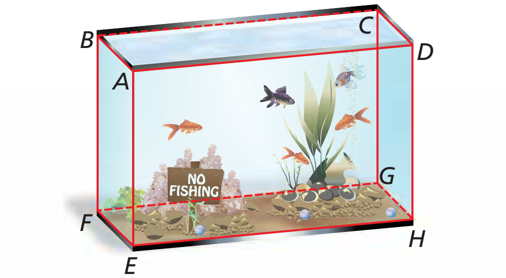
2. **WHICH ONE DOESN'T BELONG?** Which angle pair does *not* belong with the other three? Explain your reasoning.

$\angle 2$ and $\angle 3$	$\angle 4$ and $\angle 5$
$\angle 1$ and $\angle 8$	$\angle 2$ and $\angle 7$



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, think of each segment in the diagram as part of a line. All the angles are right angles. Which line(s) or plane(s) contain point *B* and appear to fit the description? *(See Example 1.)*



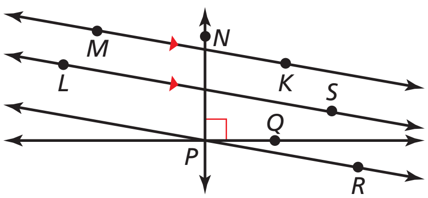
3. line(s) parallel to $\overleftrightarrow{CD}$

4. line(s) perpendicular to $\overleftrightarrow{CD}$

5. line(s) skew to $\overleftrightarrow{CD}$

6. plane(s) parallel to plane *CDH*

In Exercises 7–10, use the diagram. *(See Example 2.)*



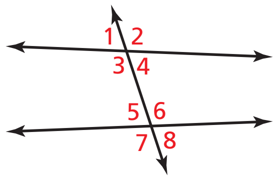
7. Name a pair of parallel lines.

8. Name a pair of perpendicular lines.

9. Is $\overleftrightarrow{PN} \parallel \overleftrightarrow{KM}$? Explain.

10. Is $\overleftrightarrow{PR} \perp \overleftrightarrow{NP}$? Explain.

In Exercises 11–14, identify all pairs of angles of the given type. *(See Example 3.)*



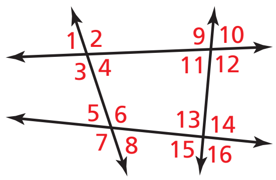
11. corresponding

12. alternate interior

13. alternate exterior

14. consecutive interior

USING STRUCTURE **In Exercises 15–18, classify the angle pair as *corresponding*, *alternate interior*, *alternate exterior*, or *consecutive interior* angles.**



15. $\angle 5$ and $\angle 1$

16. $\angle 11$ and $\angle 13$

17. $\angle 6$ and $\angle 13$

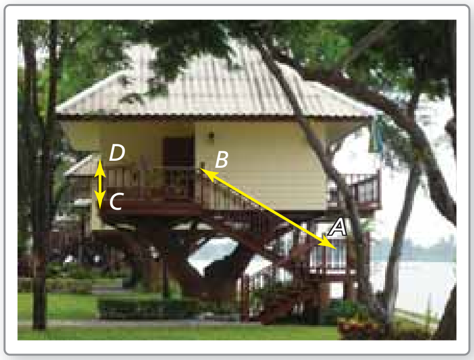
18. $\angle 2$ and $\angle 11$

ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in the conditional statement about lines.**

19. ✗ If two lines do not intersect, then they are parallel.

20. ✗ If there is a line and a point not on the line, then there is exactly one line through the point that intersects the given line.

21. **MODELING WITH MATHEMATICS** Use the photo to decide whether the statement is true or false. Explain your reasoning.



a. The plane containing the floor of the tree house is parallel to the ground.

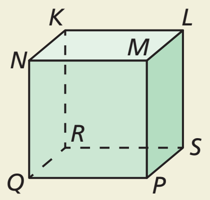
b. The lines containing the railings of the staircase, such as $\overleftrightarrow{AB}$, are skew to all lines in the plane containing the ground.

c. All the lines containing the balusters, such as $\overleftrightarrow{CD}$, are perpendicular to the plane containing the floor of the tree house.

22. **THOUGHT PROVOKING** If two lines are intersected by a third line, is the third line necessarily a transversal? Justify your answer with a diagram.

23. **MATHEMATICAL CONNECTIONS** Two lines are cut by a transversal. Is it possible for all eight angles formed to have the same measure? Explain your reasoning.

24. **HOW DO YOU SEE IT?** Think of each segment in the figure as part of a line.



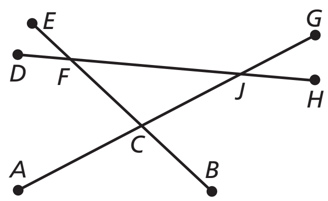
a. Which lines are parallel to $\overleftrightarrow{NQ}$?

b. Which lines intersect $\overleftrightarrow{NQ}$?

c. Which lines are skew to $\overleftrightarrow{NQ}$?

d. Should you have named all the lines on the cube in parts (a)–(c) except $\overleftrightarrow{NQ}$? Explain.

In Exercises 25–28, copy and complete the statement. List all possible correct answers.



25. $\angle BCG$ and ____ are corresponding angles.

26. $\angle BCG$ and ____ are consecutive interior angles.

27. $\angle FCJ$ and ____ are alternate interior angles.

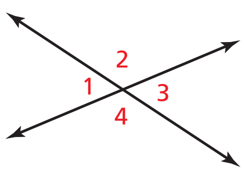
28. $\angle FCA$ and ____ are alternate exterior angles.

29. **MAKING AN ARGUMENT** Your friend claims the uneven parallel bars in gymnastics are not really parallel. She says one is higher than the other, so they cannot be in the same plane. Is she correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the diagram to find the measures of all the angles. *(Section 2.6)*



30. $m\angle 1 = 76°$

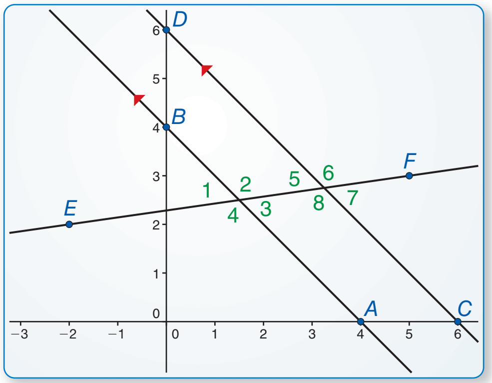
31. $m\angle 2 = 159°$

3.2 Parallel Lines and Transversals

Essential Question When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

EXPLORATION 1 Exploring Parallel Lines

Work with a partner. Use dynamic geometry software to draw two parallel lines. Draw a third line that intersects both parallel lines. Find the measures of the eight angles that are formed. What can you conclude?



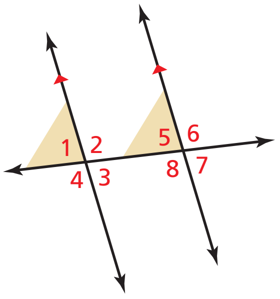
ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

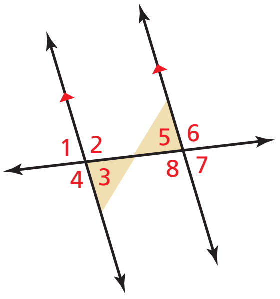
EXPLORATION 2 Writing Conjectures

Work with a partner. Use the results of Exploration 1 to write conjectures about the following pairs of angles formed by two parallel lines and a transversal.

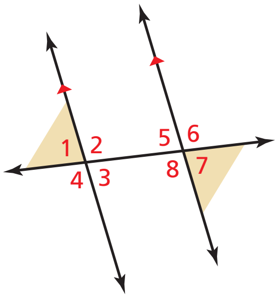
a. corresponding angles



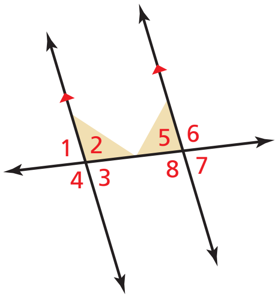
b. alternate interior angles



c. alternate exterior angles



d. consecutive interior angles



Communicate Your Answer

3. When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

4. In Exploration 2, $m\angle 1 = 80°$. Find the other angle measures.

3.2 Lesson

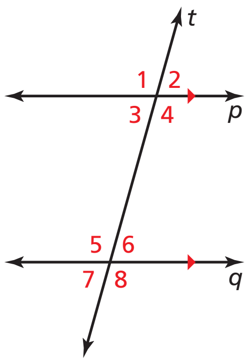
Core Vocabulary

Previous
corresponding angles
parallel lines
supplementary angles
vertical angles

What You Will Learn

- Use properties of parallel lines.
- Prove theorems about parallel lines.
- Solve real-life problems.

Using Properties of Parallel Lines



Theorems

Theorem 3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Proof Ex. 36, p. 180

Theorem 3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Proof Example 4, p. 134

Theorem 3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Ex. 15, p. 136

Theorem 3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

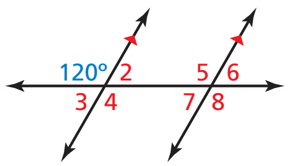
Proof Ex. 16, p. 136

ANOTHER WAY

There are many ways to solve Example 1. Another way is to use the Corresponding Angles Theorem to find $m\angle 5$ and then use the Vertical Angles Congruence Theorem (Theorem 2.6) to find $m\angle 4$ and $m\angle 8$.

EXAMPLE 1 Identifying Angles

The measures of three of the numbered angles are 120°. Identify the angles. Explain your reasoning.



SOLUTION

By the Alternate Exterior Angles Theorem, $m\angle 8 = 120°$.

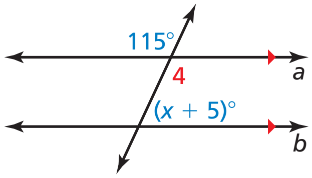
$\angle 5$ and $\angle 8$ are vertical angles. Using the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 5 = 120°$.

$\angle 5$ and $\angle 4$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 4 = 120°$.

- So, the three angles that each have a measure of 120° are $\angle 4$, $\angle 5$, and $\angle 8$.

EXAMPLE 2 Using Properties of Parallel Lines

Find the value of x.



SOLUTION

By the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

▶ So, the value of x is 60.

Check

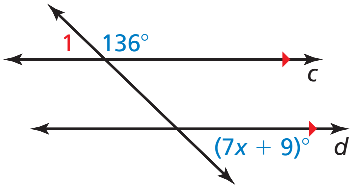
$115° + (x + 5)° = 180°$

$115 + (60 + 5) \stackrel{?}{=} 180$

$180 = 180$ ✓

EXAMPLE 3 Using Properties of Parallel Lines

Find the value of x.



SOLUTION

By the Linear Pair Postulate (Postulate 2.8), $m\angle 1 = 180° - 136° = 44°$. Lines c and d are parallel, so you can use the theorems about parallel lines.

$m\angle 1 = (7x + 9)°$	Alternate Exterior Angles Theorem
$44° = (7x + 9)°$	Substitute 44° for $m\angle 1$.
$35 = 7x$	Subtract 9 from each side.
$5 = x$	Divide each side by 7.

▶ So, the value of x is 5.

Check

$44° = (7x + 9)°$

$44 \stackrel{?}{=} 7(5) + 9$

$44 = 44$ ✓

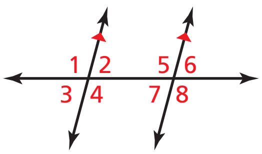
Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the diagram.

1. Given $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which theorem you use in each case.

2. Given $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.



Proving Theorems about Parallel Lines

EXAMPLE 4 Proving the Alternate Interior Angles Theorem

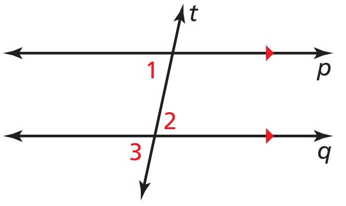
Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

STUDY TIP

Before you write a proof, identify the **Given** and **Prove** statements for the situation described or for any diagram you draw.

SOLUTION

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.



Given $p \parallel q$

Prove $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Theorem
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem (Theorem 2.6)
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence (Theorem 2.2)

Monitoring Progress

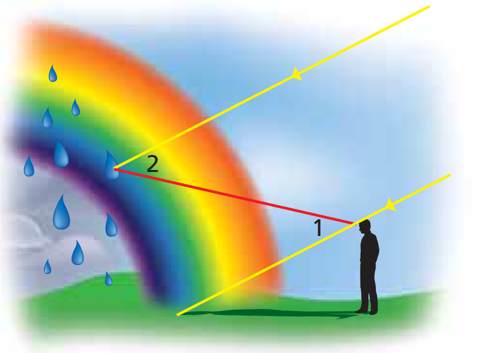
Help in English and Spanish at *BigIdeasMath.com*

3. In the proof in Example 4, if you use the third statement before the second statement, could you still prove the theorem? Explain.

Solving Real-Life Problems

EXAMPLE 5 Solving a Real-life Problem

When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



SOLUTION

Because the Sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$.

▶ So, by the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. **WHAT IF?** In Example 5, yellow light leaves a drop at an angle of $m\angle 2 = 41°$. What is $m\angle 1$? How do you know?

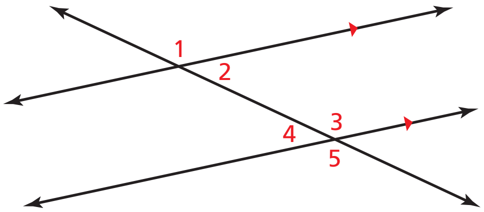
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are the Alternate Interior Angles Theorem (Theorem 3.2) and the Alternate Exterior Angles Theorem (Theorem 3.3) alike? How are they different?

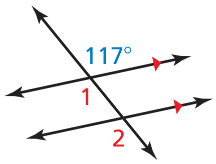
2. **WHICH ONE DOESN'T BELONG?** Which pair of angle measures does *not* belong with the other three? Explain.

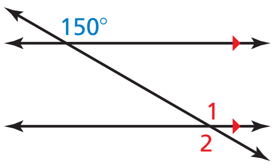
$m\angle 1$ and $m\angle 3$	$m\angle 2$ and $m\angle 4$
$m\angle 2$ and $m\angle 3$	$m\angle 1$ and $m\angle 5$

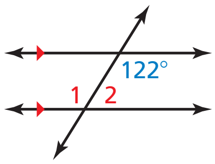


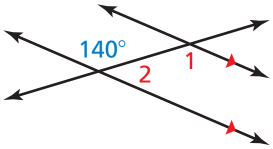
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. *(See Example 1.)*

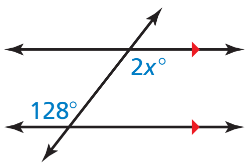
3. 

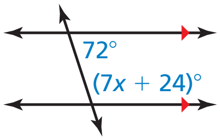
4. 

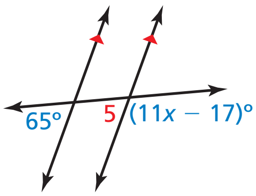
5. 

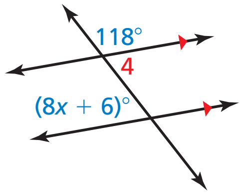
6. 

In Exercises 7–10, find the value of x. Show your steps. *(See Examples 2 and 3.)*

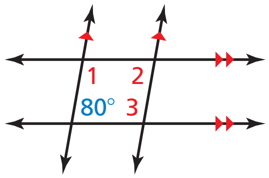
7. 

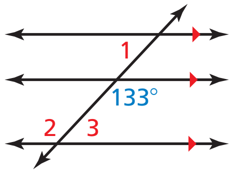
8. 

9. 

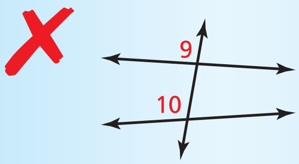
10. 

In Exercises 11 and 12, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

11. 

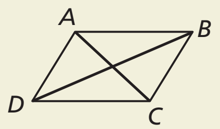
12. 

13. **ERROR ANALYSIS** Describe and correct the error in the student's reasoning.



$\angle 9 \cong \angle 10$ by the Corresponding Angles Theorem (Theorem 3.1).

14. HOW DO YOU SEE IT? Use the diagram.



a. Name two pairs of congruent angles when $\overline{AD}$ and $\overline{BC}$ are parallel. Explain your reasoning.

b. Name two pairs of supplementary angles when $\overline{AB}$ and $\overline{DC}$ are parallel. Explain your reasoning.

19. CRITICAL THINKING Is it possible for consecutive interior angles to be congruent? Explain.

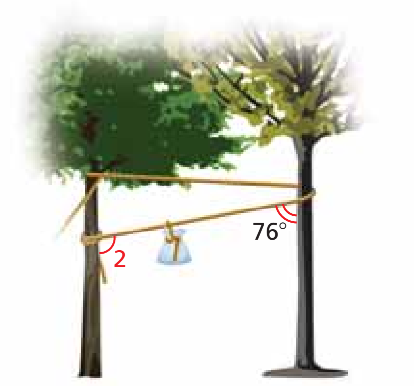
20. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible that a transversal intersects two parallel lines? Explain your reasoning.

PROVING A THEOREM In Exercises 15 and 16, prove the theorem. *(See Example 4.)*

15. Alternate Exterior Angles Theorem (Thm. 3.3)

16. Consecutive Interior Angles Theorem (Thm. 3.4)

17. PROBLEM SOLVING A group of campers tie up their food between two parallel trees, as shown. The rope is pulled taut, forming a straight line. Find $m\angle 2$. Explain your reasoning. *(See Example 5.)*



18. DRAWING CONCLUSIONS You are designing a box like the one shown.

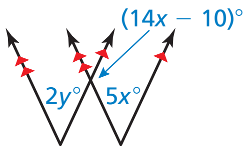
a. The measure of $\angle 1$ is 70°. Find $m\angle 2$ and $m\angle 3$.

b. Explain why $\angle ABC$ is a straight angle.

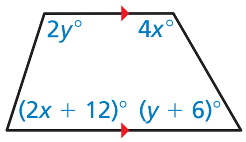
c. If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? Explain.

MATHEMATICAL CONNECTIONS In Exercises 21 and 22, write and solve a system of linear equations to find the values of x and y.

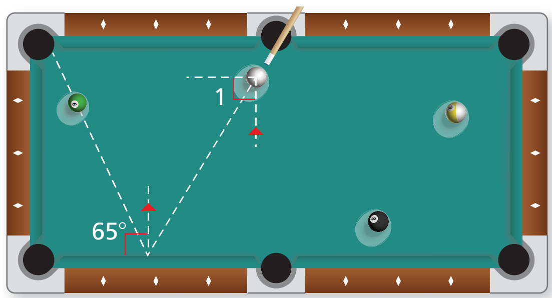
21.



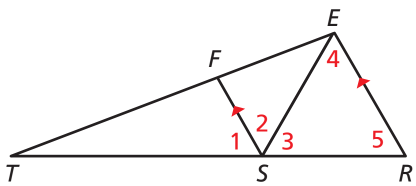
22.



23. MAKING AN ARGUMENT During a game of pool, your friend claims to be able to make the shot shown in the diagram by hitting the cue ball so that $m\angle 1 = 25°$. Is your friend correct? Explain your reasoning.



24. REASONING In the diagram, $\angle 4 \cong \angle 5$ and $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. Explain your reasoning.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the converse of the conditional statement. Decide whether it is true or false. *(Section 2.1)*

25. If two angles are vertical angles, then they are congruent.

26. If you go to the zoo, then you will see a tiger.

27. If two angles form a linear pair, then they are supplementary.

28. If it is warm outside, then we will go to the park.

3.3 Proofs with Parallel Lines

Essential Question For which of the theorems involving parallel lines and transversals is the converse true?

EXPLORATION 1 Exploring Converses

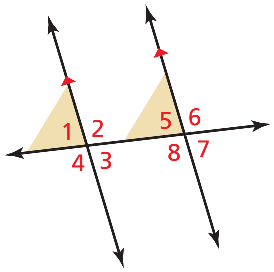
Work with a partner. Write the converse of each conditional statement. Draw a diagram to represent the converse. Determine whether the converse is true. Justify your conclusion.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

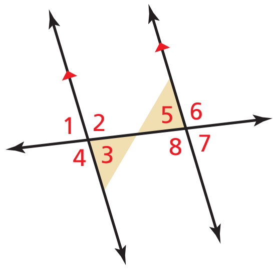
a. Corresponding Angles Theorem (Theorem 3.1)
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Converse



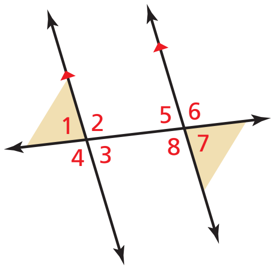
b. Alternate Interior Angles Theorem (Theorem 3.2)
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Converse



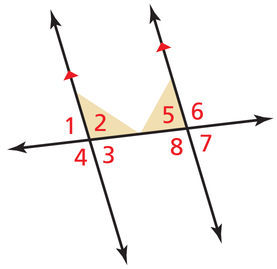
c. Alternate Exterior Angles Theorem (Theorem 3.3)
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Converse



d. Consecutive Interior Angles Theorem (Theorem 3.4)
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Converse



Communicate Your Answer

2. For which of the theorems involving parallel lines and transversals is the converse true?

3. In Exploration 1, explain how you would prove any of the theorems that you found to be true.

3.3 Lesson

Core Vocabulary

Previous
converse
parallel lines
transversal
corresponding angles
congruent
alternate interior angles
alternate exterior angles
consecutive interior angles

What You Will Learn

- Use the Corresponding Angles Converse.
- Construct parallel lines.
- Prove theorems about parallel lines.
- Use the Transitive Property of Parallel Lines.

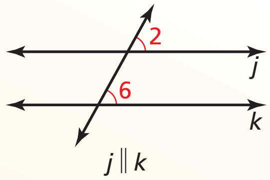
Using the Corresponding Angles Converse

Theorem 3.5 below is the converse of the Corresponding Angles Theorem (Theorem 3.1). Similarly, the other theorems about angles formed when parallel lines are cut by a transversal have true converses. Remember that the converse of a true conditional statement is not necessarily true, so you must prove each converse of a theorem.

Theorem

Theorem 3.5 Corresponding Angles Converse

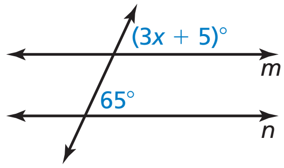
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



Proof Ex. 36, p. 180

EXAMPLE 1 **Using the Corresponding Angles Converse**

Find the value of x that makes $m \parallel n$.



SOLUTION

Lines m and n are parallel when the marked corresponding angles are congruent.

$(3x + 5)° = 65°$ Use the Corresponding Angles Converse to write an equation.

$3x = 60$ Subtract 5 from each side.

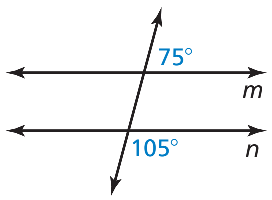
$x = 20$ Divide each side by 3.

▶ So, lines m and n are parallel when $x = 20$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

1. Is there enough information in the diagram to conclude that $m \parallel n$? Explain.



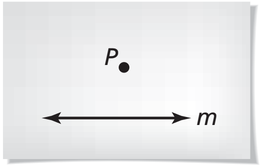
2. Explain why the Corresponding Angles Converse is the converse of the Corresponding Angles Theorem (Theorem 3.1).

Constructing Parallel Lines

The Corresponding Angles Converse justifies the construction of parallel lines, as shown below.

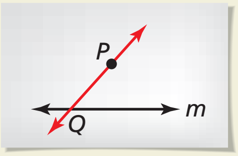
CONSTRUCTION **Constructing Parallel Lines**

Use a compass and straightedge to construct a line through point P that is parallel to line m.



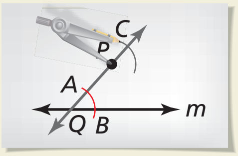
SOLUTION

Step 1



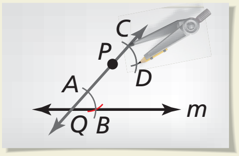
Draw a point and line Start by drawing point P and line m. Choose a point Q anywhere on line m and draw $\overleftrightarrow{QP}$.

Step 2



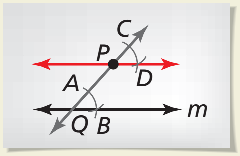
Draw arcs Draw an arc with center Q that crosses $\overleftrightarrow{QP}$ and line m. Label points A and B. Using the same compass setting, draw an arc with center P. Label point C.

Step 3



Copy angle Draw an arc with radius AB and center A. Using the same compass setting, draw an arc with center C. Label the intersection D.

Step 4

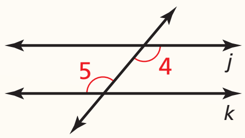


Draw parallel lines Draw $\overleftrightarrow{PD}$. This line is parallel to line m.

Theorems

Theorem 3.6 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

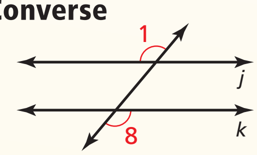


$j \parallel k$

Proof Example 2, p. 140

Theorem 3.7 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

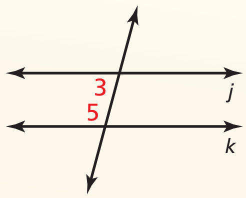


$j \parallel k$

Proof Ex. 11, p. 142

Theorem 3.8 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

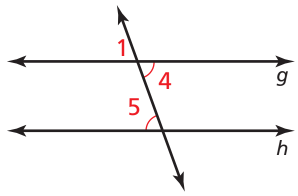
Proof Ex. 12, p. 142

Proving Theorems about Parallel Lines

EXAMPLE 2 Proving the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

SOLUTION

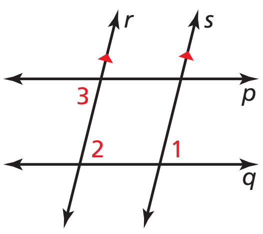


Given $\angle 4 \cong \angle 5$

Prove $g \parallel h$

STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem (Theorem 2.6)
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence (Theorem 2.2)
4. $g \parallel h$	**4.** Corresponding Angles Converse

EXAMPLE 3 Determining Whether Lines Are Parallel



In the diagram, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

SOLUTION

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

Plan for Proof **a.** Look at $\angle 1$ and $\angle 2$. $\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$. If $\angle 2 \cong \angle 3$, then $p \parallel q$.

Plan for Action **a.** It is given that $r \parallel s$, so by the Corresponding Angles Theorem (Theorem 3.1), $\angle 1 \cong \angle 2$.

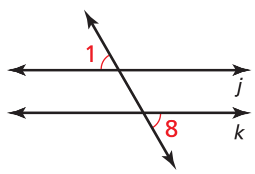
b. It is also given that $\angle 1 \cong \angle 3$. Then $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence (Theorem 2.2).

▶ So, by the Alternate Interior Angles Converse, $p \parallel q$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. If you use the diagram below to prove the Alternate Exterior Angles Converse, what **Given** and **Prove** statements would you use?



4. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 2.

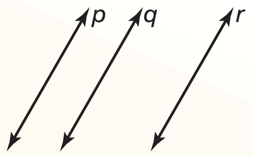
It is given that $\angle 4 \cong \angle 5$. By the _______, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence (Theorem 2.2), _______. So, by the _______, $g \parallel h$.

Using the Transitive Property of Parallel Lines

Theorem

Theorem 3.9 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

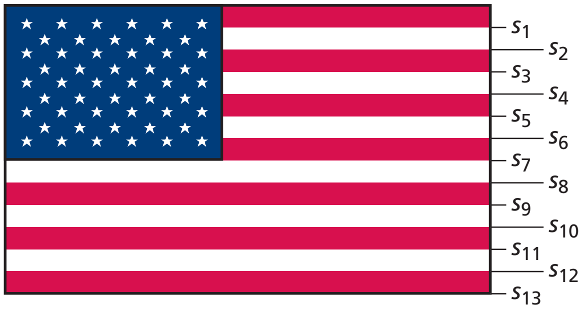


Proof Ex. 39, p. 144; Ex. 48, p. 162

If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

EXAMPLE 4 Using the Transitive Property of Parallel Lines

The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



SOLUTION

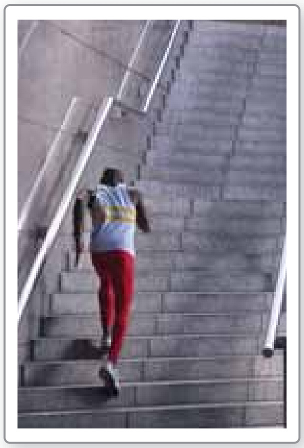
You can name the stripes from top to bottom as $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one immediately below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$.

▶ So, the top stripe is parallel to the bottom stripe.

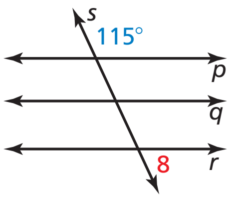
Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

5. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. Explain why the top step is parallel to the ground.



6. In the diagram below, $p \parallel q$ and $q \parallel r$. Find $m\angle 8$. Explain your reasoning.



3.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

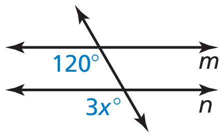
1. **VOCABULARY** Two lines are cut by a transversal. Which angle pairs must be congruent for the lines to be parallel?

2. **WRITING** Use the theorems from Section 3.2 and the converses of those theorems in this section to write three biconditional statements about parallel lines and transversals.

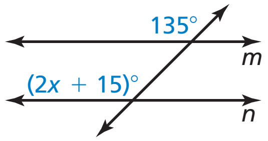
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the value of x that makes $m \parallel n$. Explain your reasoning. *(See Example 1.)*

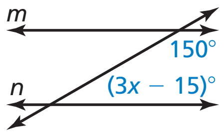
3.



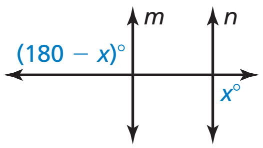
4.



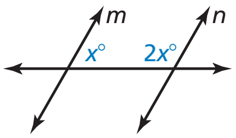
5.



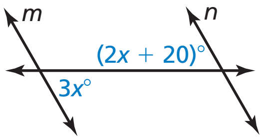
6.



7.

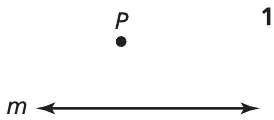


8.

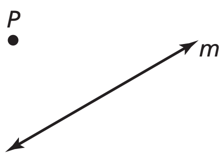


In Exercises 9 and 10, use a compass and straightedge to construct a line through point P that is parallel to line m.

9.



10.



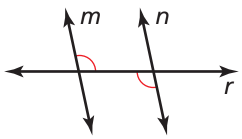
PROVING A THEOREM **In Exercises 11 and 12, prove the theorem.** *(See Example 2.)*

11. Alternate Exterior Angles Converse (Theorem 3.7)

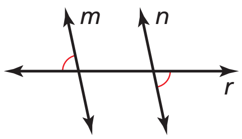
12. Consecutive Interior Angles Converse (Theorem 3.8)

In Exercises 13–18, decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use. *(See Example 3.)*

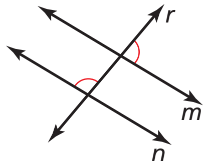
13.



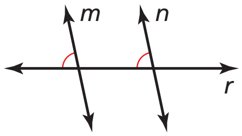
14.



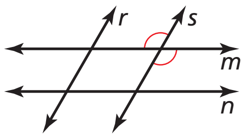
15.



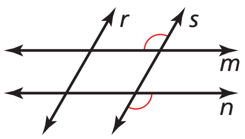
16.



17.

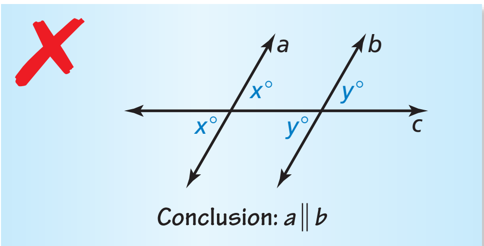


18.

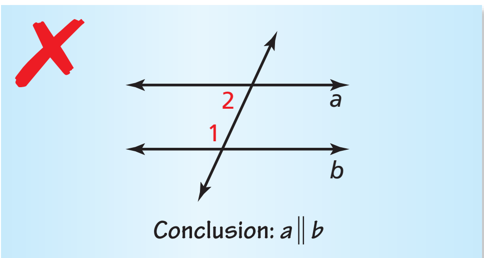


ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in the reasoning.**

19.

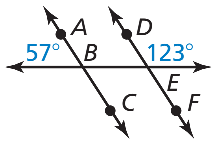


20.

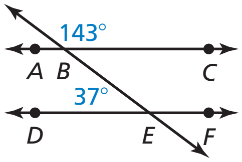


In Exercises 21–24, are $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ parallel? Explain your reasoning.

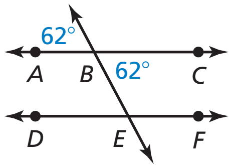
21.



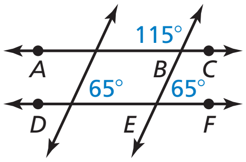
22.



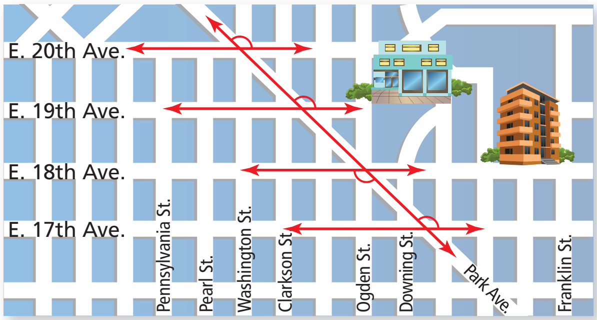
23.



24.

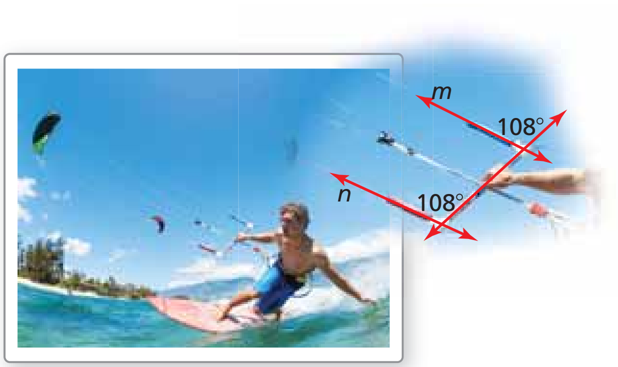


25. **ANALYZING RELATIONSHIPS** The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? Explain your reasoning. *(See Example 4.)*

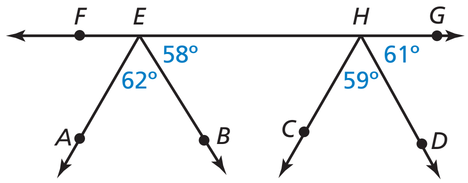


26. **ANALYZING RELATIONSHIPS** Each rung of the ladder is parallel to the rung directly above it. Explain why the top rung is parallel to the bottom rung.

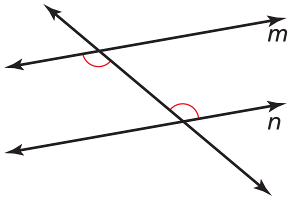
27. **MODELING WITH MATHEMATICS** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m?



28. **REASONING** Use the diagram. Which rays are parallel? Which rays are not parallel? Explain your reasoning.



29. **ATTENDING TO PRECISION** Use the diagram. Which theorems allow you to conclude that $m \parallel n$? Select all that apply. Explain your reasoning.



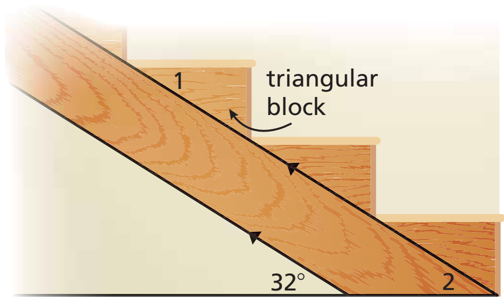
Ⓐ Corresponding Angles Converse (Thm. 3.5)

Ⓑ Alternate Interior Angles Converse (Thm. 3.6)

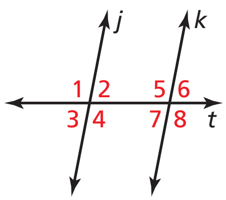
Ⓒ Alternate Exterior Angles Converse (Thm. 3.7)

Ⓓ Consecutive Interior Angles Converse (Thm. 3.8)

30. **MODELING WITH MATHEMATICS** One way to build stairs is to attach triangular blocks to an angled support, as shown. The sides of the angled support are parallel. If the support makes a 32° angle with the floor, what must $m\angle 1$ be so the top of the step will be parallel to the floor? Explain your reasoning.



31. **ABSTRACT REASONING** In the diagram, how many angles must be given to determine whether $j \parallel k$? Give four examples that would allow you to conclude that $j \parallel k$ using the theorems from this lesson.

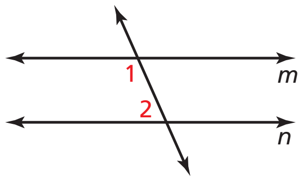


32. THOUGHT PROVOKING Draw a diagram of at least two lines cut by at least one transversal. Mark your diagram so that it cannot be proven that any lines are parallel. Then explain how your diagram would need to change in order to prove that lines are parallel.

PROOF **In Exercises 33–36, write a proof.**

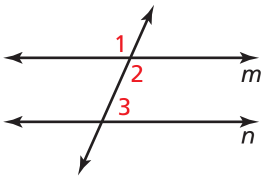
33. Given $m\angle 1 = 115^\circ$, $m\angle 2 = 65^\circ$

Prove $m \parallel n$



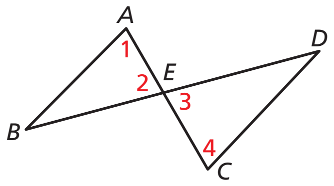
34. Given $\angle 1$ and $\angle 3$ are supplementary.

Prove $m \parallel n$



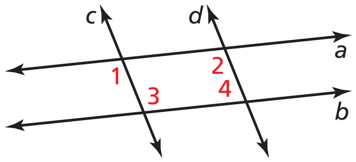
35. Given $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\overline{AB} \parallel \overline{CD}$

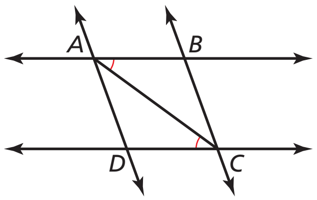


36. Given $a \parallel b$, $\angle 2 \cong \angle 3$

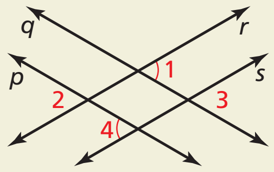
Prove $c \parallel d$



37. MAKING AN ARGUMENT Your classmate decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram. Is your classmate correct? Explain your reasoning.



38. HOW DO YOU SEE IT? Are the markings on the diagram enough to conclude that any lines are parallel? If so, which ones? If not, what other information is needed?



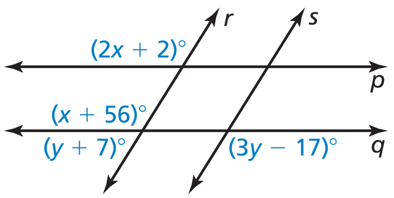
39. PROVING A THEOREM Use these steps to prove the Transitive Property of Parallel Lines Theorem (Theorem 3.9).

a. Copy the diagram with the Transitive Property of Parallel Lines Theorem on page 141.

b. Write the **Given** and **Prove** statements.

c. Use the properties of angles formed by parallel lines cut by a transversal to prove the theorem.

40. MATHEMATICAL CONNECTIONS Use the diagram.



a. Find the value of x that makes $p \parallel q$.

b. Find the value of y that makes $r \parallel s$.

c. Can r be parallel to s and can p be parallel to q at the same time? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the Distance Formula to find the distance between the two points. *(Section 1.3)*

41. $(1, 3)$ and $(-2, 9)$

42. $(-3, 7)$ and $(8, -6)$

43. $(5, -4)$ and $(0, 8)$

44. $(13, 1)$ and $(9, -4)$

3.1–3.3 What Did You Learn?

Core Vocabulary

parallel lines, *p. 126*
skew lines, *p. 126*
parallel planes, *p. 126*
transversal, *p. 128*
corresponding angles, *p. 128*
alternate interior angles, *p. 128*
alternate exterior angles, *p. 128*
consecutive interior angles, *p. 128*

Core Concepts

Section 3.1

Parallel Lines, Skew Lines, and Parallel Planes, *p. 126*
Postulate 3.1 Parallel Postulate, *p. 127*
Postulate 3.2 Perpendicular Postulate, *p. 127*
Angles Formed by Transversals, *p. 128*

Section 3.2

Theorem 3.1 Corresponding Angles Theorem, *p. 132*
Theorem 3.2 Alternate Interior Angles Theorem, *p. 132*
Theorem 3.3 Alternate Exterior Angles Theorem, *p. 132*
Theorem 3.4 Consecutive Interior Angles Theorem, *p. 132*

Section 3.3

Theorem 3.5 Corresponding Angles Converse, *p. 138*
Theorem 3.6 Alternate Interior Angles Converse, *p. 139*
Theorem 3.7 Alternate Exterior Angles Converse, *p. 139*
Theorem 3.8 Consecutive Interior Angles Converse, *p. 139*
Theorem 3.9 Transitive Property of Parallel Lines, *p. 141*

Mathematical Practices

1. Draw the portion of the diagram that you used to answer Exercise 26 on page 130.
2. In Exercise 40 on page 144, explain how you started solving the problem and why you started that way.

Study Skills

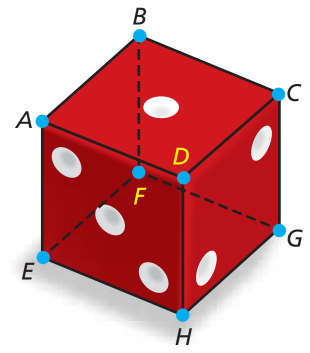
Analyzing Your Errors

Misreading Directions

- **What Happens:** You incorrectly read or do not understand directions.
- **How to Avoid This Error:** Read the instructions for exercises at least twice and make sure you understand what they mean. Make this a habit and use it when taking tests.

3.1–3.3 Quiz

Think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point G and appear to fit the description? *(Section 3.1)*



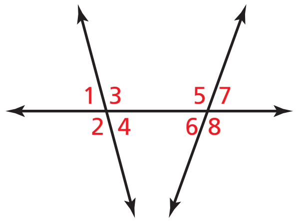
1. line(s) parallel to $\overleftrightarrow{EF}$

2. line(s) perpendicular to $\overleftrightarrow{EF}$

3. line(s) skew to $\overleftrightarrow{EF}$

4. plane(s) parallel to plane ADE

Identify all pairs of angles of the given type. *(Section 3.1)*



5. consecutive interior

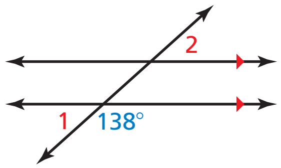
6. alternate interior

7. corresponding

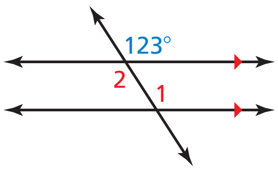
8. alternate exterior

Find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. *(Section 3.2)*

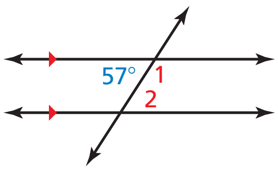
9.



10.

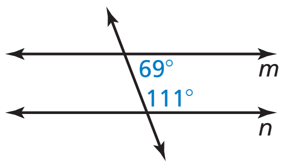


11.

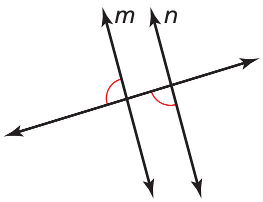


Decide whether there is enough information to prove that $m \parallel n$. If so, state the theorem you would use. *(Section 3.3)*

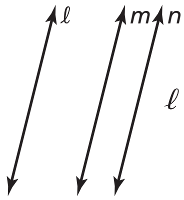
12.



13.



14.

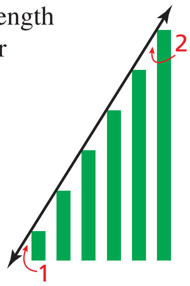


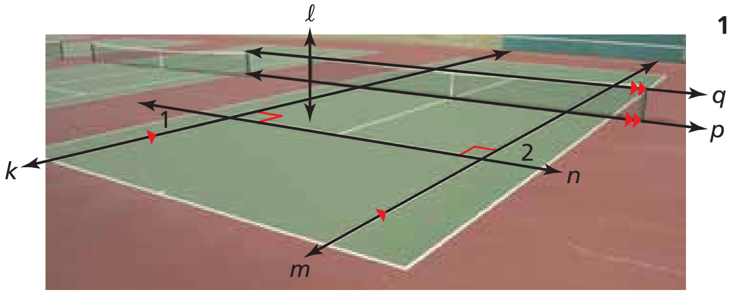
$\ell \parallel m$ and $\ell \parallel n$

15. Cellular phones use bars like the ones shown to indicate how much signal strength a phone receives from the nearest service tower. Each bar is parallel to the bar directly next to it. *(Section 3.3)*

a. Explain why the tallest bar is parallel to the shortest bar.

b. Imagine that the left side of each bar extends infinitely as a line. If $m\angle 1 = 58°$, then what is $m\angle 2$?





16. The diagram shows lines formed on a tennis court. *(Section 3.1 and Section 3.3)*

a. Identify two pairs of parallel lines so that each pair is in a different plane.

b. Identify two pairs of perpendicular lines.

c. Identify two pairs of skew lines.

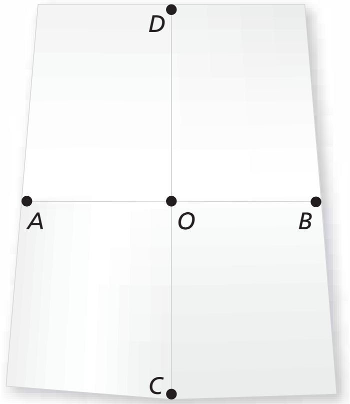
d. Prove that $\angle 1 \cong \angle 2$.

3.4 Proofs with Perpendicular Lines

Essential Question What conjectures can you make about perpendicular lines?

EXPLORATION 1 Writing Conjectures

Work with a partner. Fold a piece of paper in half twice. Label points on the two creases, as shown.

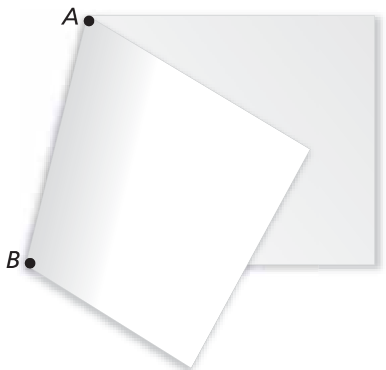


a. Write a conjecture about $\overline{AB}$ and $\overline{CD}$. Justify your conjecture.

b. Write a conjecture about $\overline{AO}$ and $\overline{OB}$. Justify your conjecture.

EXPLORATION 2 Exploring a Segment Bisector

Work with a partner. Fold and crease a piece of paper, as shown. Label the ends of the crease as A and B.



a. Fold the paper again so that point A coincides with point B. Crease the paper on that fold.

b. Unfold the paper and examine the four angles formed by the two creases. What can you conclude about the four angles?

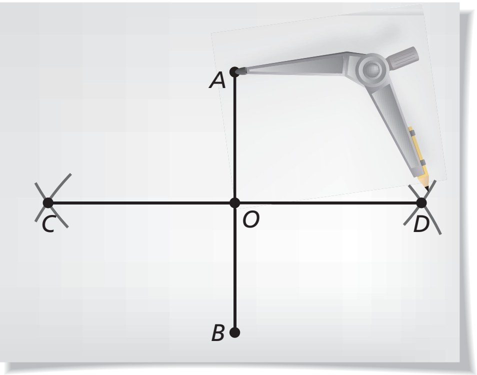
EXPLORATION 3 Writing a Conjecture

Work with a partner.

a. Draw $\overline{AB}$, as shown.

b. Draw an arc with center A on each side of $\overline{AB}$. Using the same compass setting, draw an arc with center B on each side of $\overline{AB}$. Label the intersections of the arcs C and D.

c. Draw $\overline{CD}$. Label its intersection with $\overline{AB}$ as O. Write a conjecture about the resulting diagram. Justify your conjecture.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

Communicate Your Answer

4. What conjectures can you make about perpendicular lines?

5. In Exploration 3, find AO and OB when $AB = 4$ units.

3.4 Lesson

Core Vocabulary

distance from a point to a line, *p. 148*
perpendicular bisector, *p. 149*

What You Will Learn

- Find the distance from a point to a line.
- Construct perpendicular lines.
- Prove theorems about perpendicular lines.
- Solve real-life problems involving perpendicular lines.

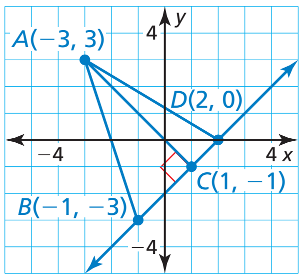
Finding the Distance from a Point to a Line

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB.

distance from a point to a line

EXAMPLE 1 **Finding the Distance from a Point to a Line**

Find the distance from point A to $\overleftrightarrow{BD}$.



REMEMBER

Recall that if $A(x_1, y_1)$ and $C(x_2, y_2)$ are points in a coordinate plane, then the distance between A and C is

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

SOLUTION

Because $\overline{AC} \perp \overleftrightarrow{BD}$, the distance from point A to $\overleftrightarrow{BD}$ is AC. Use the Distance Formula.

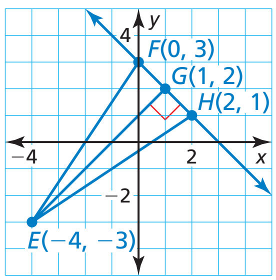
$$AC = \sqrt{(-3-1)^2 + [3-(-1)]^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{32} \approx 5.7$$

- So, the distance from point A to $\overleftrightarrow{BD}$ is about 5.7 units.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

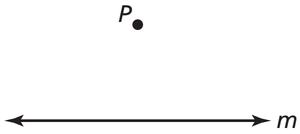
1. Find the distance from point E to $\overleftrightarrow{FH}$.



Constructing Perpendicular Lines

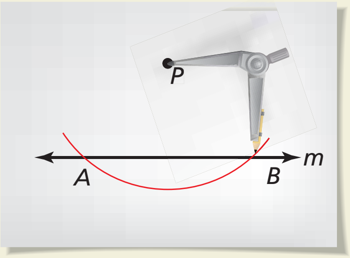
CONSTRUCTION **Constructing a Perpendicular Line**

Use a compass and straightedge to construct a line perpendicular to line m through point P, which is not on line m.



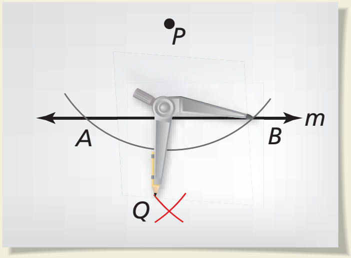
SOLUTION

Step 1



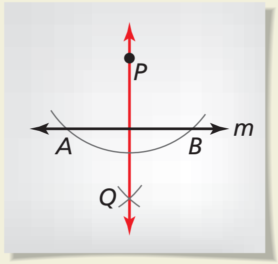
Draw arc with center *P* Place the compass at point P and draw an arc that intersects the line twice. Label the intersections A and B.

Step 2

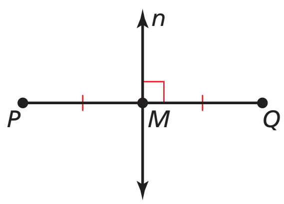


Draw intersecting arcs Draw an arc with center A. Using the same radius, draw an arc with center B. Label the intersection of the arcs Q.

Step 3



Draw perpendicular line Draw $\overleftrightarrow{PQ}$. This line is perpendicular to line m.



The **perpendicular bisector** of a line segment $\overline{PQ}$ is the line n with the following two properties.

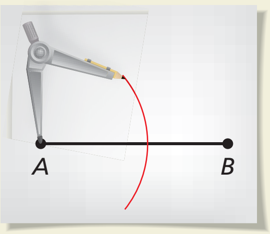
- $n \perp \overline{PQ}$
- n passes through the midpoint M of $\overline{PQ}$.

CONSTRUCTION **Constructing a Perpendicular Bisector**

Use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.

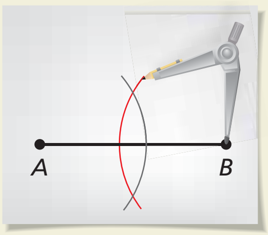
SOLUTION

Step 1



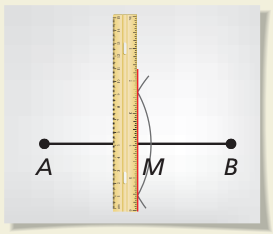
Draw an arc Place the compass at A. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

Step 2



Draw a second arc Keep the same compass setting. Place the compass at B. Draw an arc. It should intersect the other arc at two points.

Step 3



Bisect segment Draw a line through the two points of intersection. This line is the perpendicular bisector of $\overline{AB}$. It passes through M, the midpoint of $\overline{AB}$. So, $AM = MB$.

Proving Theorems about Perpendicular Lines

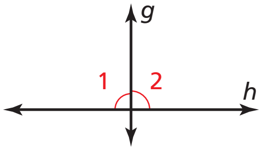
Theorems

Theorem 3.10 Linear Pair Perpendicular Theorem

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof Ex. 13, p. 153

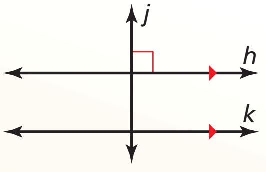


Theorem 3.11 Perpendicular Transversal Theorem

In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

Proof Example 2, p. 150; Question 2, p. 150

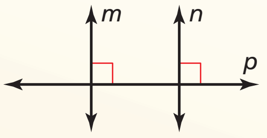


Theorem 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

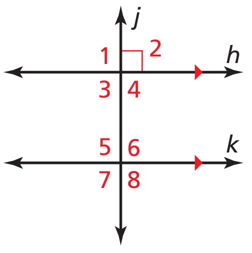
If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Proof Ex. 14, p. 153; Ex. 47, p. 162



EXAMPLE 2 Proving the Perpendicular Transversal Theorem

Use the diagram to prove the Perpendicular Transversal Theorem.



SOLUTION

Given $h \parallel k, j \perp h$

Prove $j \perp k$

STATEMENTS	REASONS
1. $h \parallel k, j \perp h$	**1.** Given
2. $m\angle 2 = 90°$	**2.** Definition of perpendicular lines
3. $\angle 2 \cong \angle 6$	**3.** Corresponding Angles Theorem (Theorem 3.1)
4. $m\angle 2 = m\angle 6$	**4.** Definition of congruent angles
5. $m\angle 6 = 90°$	**5.** Transitive Property of Equality
6. $j \perp k$	**6.** Definition of perpendicular lines

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

2. Prove the Perpendicular Transversal Theorem using the diagram in Example 2 and the Alternate Exterior Angles Theorem (Theorem 3.3).

Solving Real-Life Problems

EXAMPLE 3 **Proving Lines Are Parallel**

The photo shows the layout of a neighborhood. Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

SOLUTION

Lines p and q are both perpendicular to s, so by the Lines Perpendicular to a Transversal Theorem, $p \parallel q$. Also, lines s and t are both perpendicular to q, so by the Lines Perpendicular to a Transversal Theorem, $s \parallel t$.

▶ So, from the diagram you can conclude $p \parallel q$ and $s \parallel t$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Use the lines marked in the photo.

3. Is $b \parallel a$? Explain your reasoning.

4. Is $b \perp c$? Explain your reasoning.

3.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The perpendicular bisector of a segment is the line that passes through the ________ of the segment at a ________ angle.

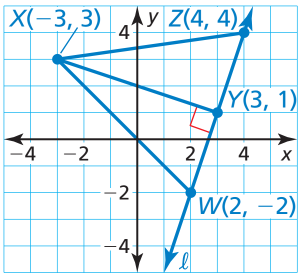
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the distance from point X to line $\overleftrightarrow{WZ}$.

Find XZ.

Find the length of $\overline{XY}$.

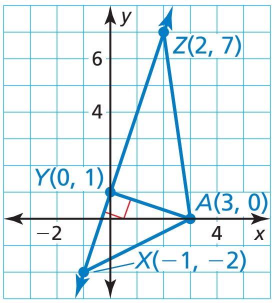
Find the distance from line ℓ to point X.



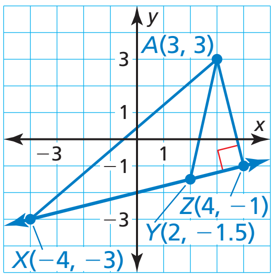
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the distance from point A to $\overleftrightarrow{XZ}$. *(See Example 1.)*

3.

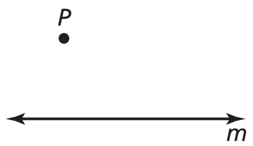


4.

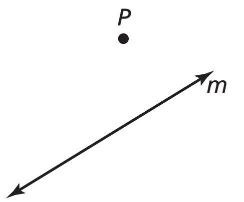


CONSTRUCTION **In Exercises 5–8, trace line m and point P. Then use a compass and straightedge to construct a line perpendicular to line m through point P.**

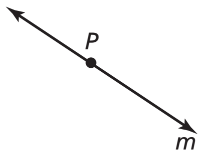
5.



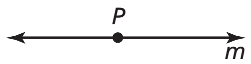
6.



7.

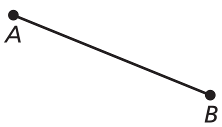


8.

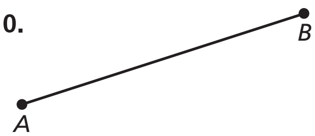


CONSTRUCTION **In Exercises 9 and 10, trace $\overline{AB}$. Then use a compass and straightedge to construct the perpendicular bisector of $\overline{AB}$.**

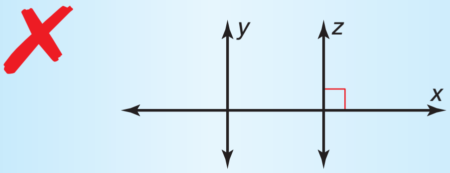
9.



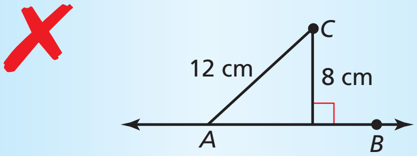
10.



ERROR ANALYSIS **In Exercises 11 and 12, describe and correct the error in the statement about the diagram.**

11. 

Lines *y* and *z* are parallel.

12. 

The distance from point *C* to $\overleftrightarrow{AB}$ is 12 centimeters.

PROVING A THEOREM **In Exercises 13 and 14, prove the theorem.** *(See Example 2.)*

13. Linear Pair Perpendicular Theorem (Thm. 3.10)

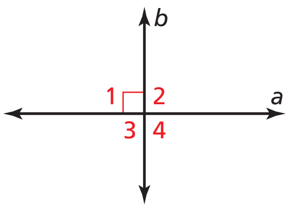
14. Lines Perpendicular to a Transversal Theorem (Thm. 3.12)

PROOF **In Exercises 15 and 16, use the diagram to write a proof of the statement.**

15. If two intersecting lines are perpendicular, then they intersect to form four right angles.

Given $a \perp b$

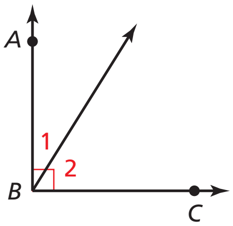
Prove $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.



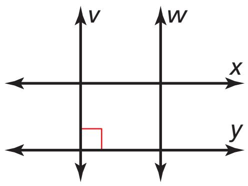
16. If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

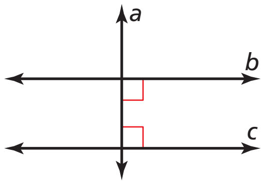
Given $\overrightarrow{BA} \perp \overrightarrow{BC}$

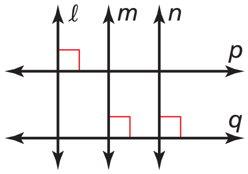
Prove $\angle 1$ and $\angle 2$ are complementary.

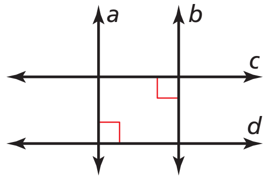


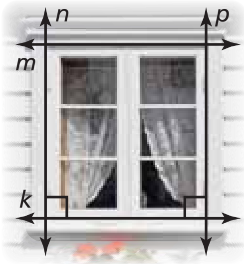
In Exercises 17–22, determine which lines, if any, must be parallel. Explain your reasoning. *(See Example 3.)*

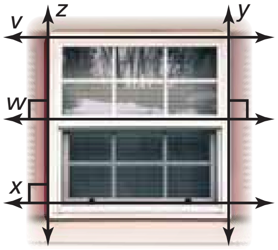
17. 

18. 

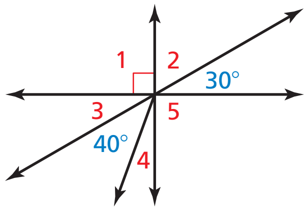
19. 

20. 

21. 

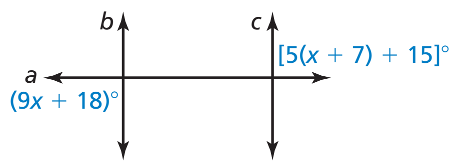
22. 

23. **USING STRUCTURE** Find all the unknown angle measures in the diagram. Justify your answer for each angle measure.

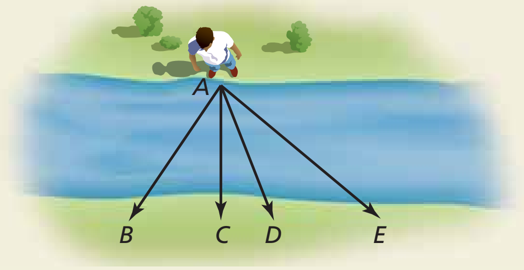


24. **MAKING AN ARGUMENT** Your friend claims that because you can find the distance from a point to a line, you should be able to find the distance between any two lines. Is your friend correct? Explain your reasoning.

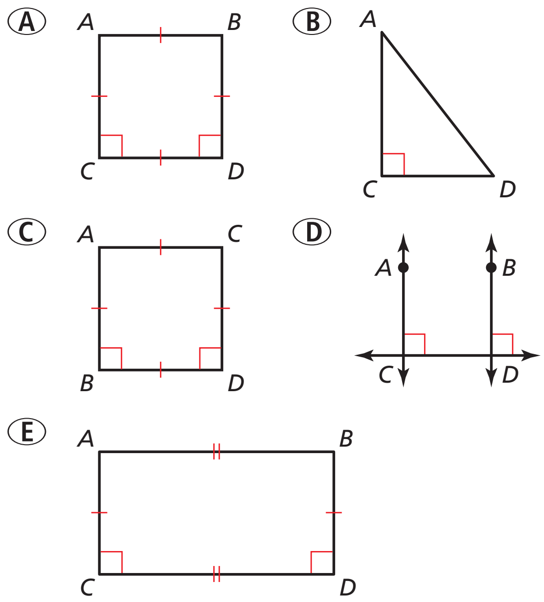
25. **MATHEMATICAL CONNECTIONS** Find the value of x when $a \perp b$ and $b \parallel c$.



26. HOW DO YOU SEE IT? You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? Explain your reasoning.



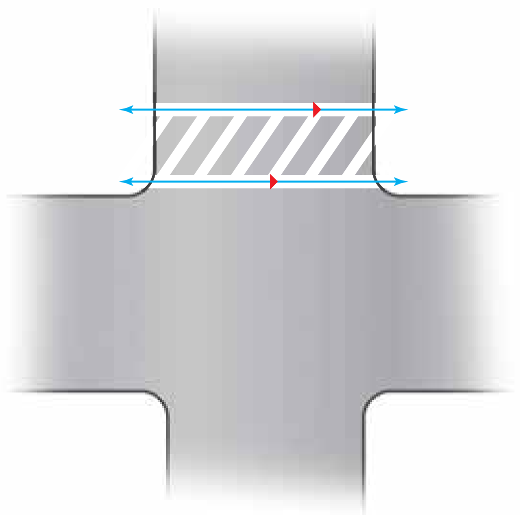
27. ATTENDING TO PRECISION In which of the following diagrams is $\overline{AC} \parallel \overline{BD}$ and $\overline{AC} \perp \overline{CD}$? Select all that apply.



28. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, how many right angles are formed by two perpendicular lines? Justify your answer.

29. CONSTRUCTION Construct a square of side length AB.

30. ANALYZING RELATIONSHIPS The painted line segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments in the photo would look like if they were perpendicular to the crosswalk. Which type of line segment requires less paint? Explain your reasoning.



31. ABSTRACT REASONING Two lines, a and b, are perpendicular to line c. Line d is parallel to line c. The distance between lines a and b is x meters. The distance between lines c and d is y meters. What shape is formed by the intersections of the four lines?

32. MATHEMATICAL CONNECTIONS Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$.

33. WRITING Describe how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the ratio. *(Skills Review Handbook)*

34. $\dfrac{6 - (-4)}{8 - 3}$ **35.** $\dfrac{3 - 5}{4 - 1}$ **36.** $\dfrac{8 - (-3)}{7 - (-2)}$ **37.** $\dfrac{13 - 4}{2 - (-1)}$

Identify the slope and the *y*-intercept of the line. *(Skills Review Handbook)*

38. $y = 3x + 9$ **39.** $y = -\frac{1}{2}x + 7$ **40.** $y = \frac{1}{6}x - 8$ **41.** $y = -8x - 6$

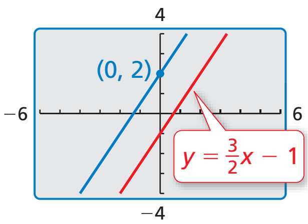
3.5 Equations of Parallel and Perpendicular Lines

Essential Question How can you write an equation of a line that is parallel or perpendicular to a given line and passes through a given point?

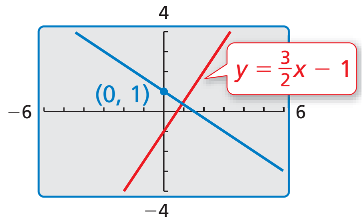
EXPLORATION 1 Writing Equations of Parallel and Perpendicular Lines

Work with a partner. Write an equation of the line that is parallel or perpendicular to the given line and passes through the given point. Use a graphing calculator to verify your answer. What is the relationship between the slopes?

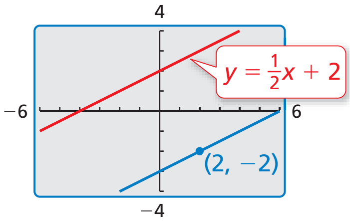
a.



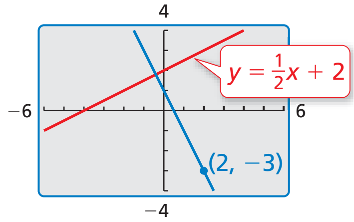
b.



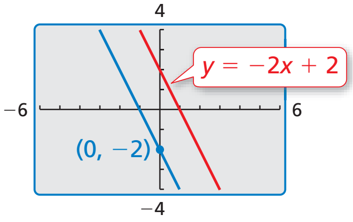
c.



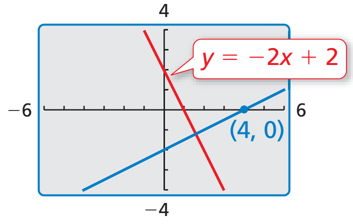
d.



e.



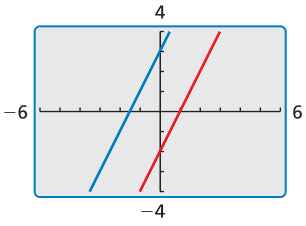
f.



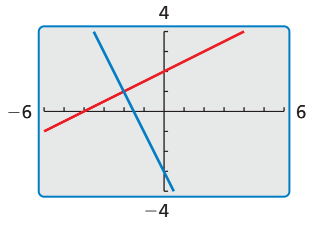
EXPLORATION 2 Writing Equations of Parallel and Perpendicular Lines

Work with a partner. Write the equations of the parallel or perpendicular lines. Use a graphing calculator to verify your answers.

a.



b.



MODELING WITH MATHEMATICS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

Communicate Your Answer

3. How can you write an equation of a line that is parallel or perpendicular to a given line and passes through a given point?

4. Write an equation of the line that is (a) parallel and (b) perpendicular to the line $y = 3x + 2$ and passes through the point $(1, -2)$.

3.5 Lesson

Core Vocabulary

directed line segment, *p. 156*

Previous
slope
slope-intercept form
y-intercept

What You Will Learn

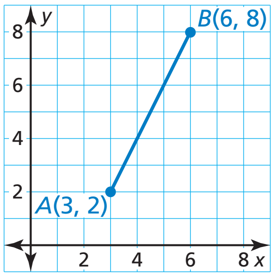
- Use slope to partition directed line segments.
- Identify parallel and perpendicular lines.
- Write equations of parallel and perpendicular lines.
- Use slope to find the distance from a point to a line.

Partitioning a Directed Line Segment

A **directed line segment** AB is a segment that represents moving from point A to point B. The following example shows how to use slope to find a point on a directed line segment that partitions the segment in a given ratio.

EXAMPLE 1 Partitioning a Directed Line Segment

Find the coordinates of point P along the directed line segment AB so that the ratio of AP to PB is 3 to 2.



REMEMBER

Recall that the slope of a line or line segment through two points, (x_1, y_1) and (x_2, y_2), is defined as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

You can choose either of the two points to be (x_1, y_1).

SOLUTION

In order to divide the segment in the ratio 3 to 2, think of dividing, or *partitioning*, the segment into 3 + 2, or 5 congruent pieces.

Point P is the point that is $\frac{3}{5}$ of the way from point A to point B.

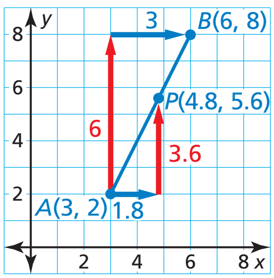
Find the rise and run from point A to point B. Leave the slope in terms of rise and run and do not simplify.

$$\text{slope of } \overline{AB}\text{:} \quad m = \frac{8-2}{6-3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point P, add $\frac{3}{5}$ of the run to the x-coordinate of A, and add $\frac{3}{5}$ of the rise to the y-coordinate of A.

$$\text{run: } \frac{3}{5} \text{ of } 3 = \frac{3}{5} \cdot 3 = 1.8$$

$$\text{rise: } \frac{3}{5} \text{ of } 6 = \frac{3}{5} \cdot 6 = 3.6$$



So, the coordinates of P are

$$(3 + 1.8, 2 + 3.6) = (4.8, 5.6).$$

The ratio of AP to PB is 3 to 2.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio.

1. $A(1, 3)$, $B(8, 4)$; 4 to 1

2. $A(-2, 1)$, $B(4, 5)$; 3 to 7

Identifying Parallel and Perpendicular Lines

In the coordinate plane, the x-axis and the y-axis are perpendicular. Horizontal lines are parallel to the x-axis, and vertical lines are parallel to the y-axis.

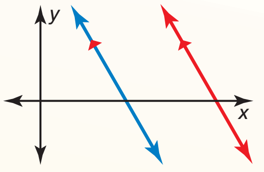
Theorems

Theorem 3.13 Slopes of Parallel Lines

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Proof p. 439; Ex. 41, p. 444



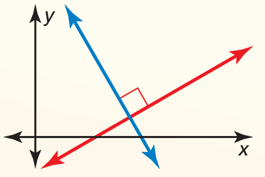
$m_1 = m_2$

Theorem 3.14 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Horizontal lines are perpendicular to vertical lines.

Proof p. 440; Ex. 42, p. 444



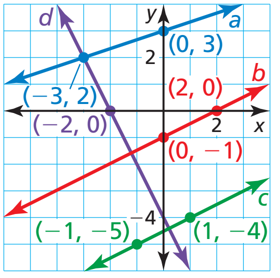
$m_1 \cdot m_2 = -1$

READING

If the product of two numbers is -1, then the numbers are called *negative reciprocals*.

EXAMPLE 2 Identifying Parallel and Perpendicular Lines

Determine which of the lines are parallel and which of the lines are perpendicular.



SOLUTION

Find the slope of each line.

$$\text{Line } a\text{: } m = \frac{3-2}{0-(-3)} = \frac{1}{3}$$

$$\text{Line } b\text{: } m = \frac{0-(-1)}{2-0} = \frac{1}{2}$$

$$\text{Line } c\text{: } m = \frac{-4-(-5)}{1-(-1)} = \frac{1}{2}$$

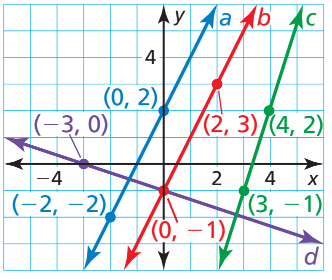
$$\text{Line } d\text{: } m = \frac{2-0}{-3-(-2)} = -2$$

Because lines b and c have the same slope, lines b and c are parallel. Because $\frac{1}{2}(-2) = -1$, lines b and d are perpendicular and lines c and d are perpendicular.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

3. Determine which of the lines are parallel and which of the lines are perpendicular.



Writing Equations of Parallel and Perpendicular Lines

You can apply the Slopes of Parallel Lines Theorem and the Slopes of Perpendicular Lines Theorem to write equations of parallel and perpendicular lines.

EXAMPLE 3 Writing an Equation of a Parallel Line

Write an equation of the line passing through the point $(-1, 1)$ that is parallel to the line $y = 2x - 3$.

REMEMBER

The linear equation $y = 2x - 3$ is written in slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept.

SOLUTION

Step 1 Find the slope m of the parallel line. The line $y = 2x - 3$ has a slope of 2. By the Slopes of Parallel Lines Theorem, a line parallel to this line also has a slope of 2. So, $m = 2$.

Step 2 Find the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

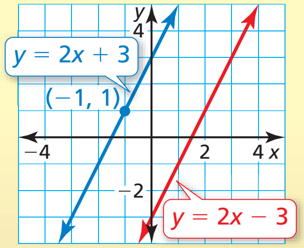
$y = mx + b$ Use slope-intercept form.

$1 = 2(-1) + b$ Substitute for m, x, and y.

$3 = b$ Solve for b.

▶ Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$. Use a graph to check that the line $y = 2x - 3$ is parallel to the line $y = 2x + 3$.

Check



EXAMPLE 4 Writing an Equation of a Perpendicular Line

Write an equation of the line passing through the point $(2, 3)$ that is perpendicular to the line $2x + y = 2$.

SOLUTION

Step 1 Find the slope m of the perpendicular line. The line $2x + y = 2$, or $y = -2x + 2$, has a slope of -2. Use the Slopes of Perpendicular Lines Theorem.

$-2 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = \frac{1}{2}$ Divide each side by -2.

Step 2 Find the y-intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

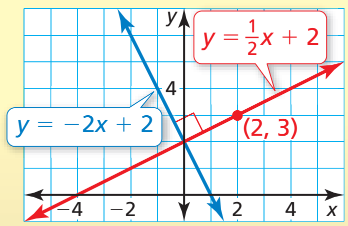
$y = mx + b$ Use slope-intercept form.

$3 = \frac{1}{2}(2) + b$ Substitute for m, x, and y.

$2 = b$ Solve for b.

▶ Because $m = \frac{1}{2}$ and $b = 2$, an equation of the line is $y = \frac{1}{2}x + 2$. Check that the lines are perpendicular by graphing their equations and using a protractor to measure one of the angles formed by their intersection.

Check



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

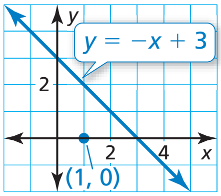
4. Write an equation of the line that passes through the point $(1, 5)$ and is (a) parallel to the line $y = 3x - 5$ and (b) perpendicular to the line $y = 3x - 5$.

5. How do you know that the lines $x = 4$ and $y = 2$ are perpendicular?

Finding the Distance from a Point to a Line

Recall that the distance from a point to a line is the length of the perpendicular segment from the point to the line.

EXAMPLE 5 Finding the Distance from a Point to a Line



Find the distance from the point $(1, 0)$ to the line $y = -x + 3$.

SOLUTION

Step 1 Find an equation of the line perpendicular to the line $y = -x + 3$ that passes through the point $(1, 0)$.

First, find the slope m of the perpendicular line. The line $y = -x + 3$ has a slope of -1. Use the Slopes of Perpendicular Lines Theorem.

$-1 \cdot m = -1$ — The product of the slopes of $\perp$ lines is -1.

$m = 1$ — Divide each side by -1.

Then find the y-intercept b by using $m = 1$ and $(x, y) = (1, 0)$.

$y = mx + b$ — Use slope-intercept form.

$0 = 1(1) + b$ — Substitute for x, y, and m.

$-1 = b$ — Solve for b.

Because $m = 1$ and $b = -1$, an equation of the line is $y = x - 1$.

REMEMBER

Recall that the solution of a system of two linear equations in two variables gives the coordinates of the point of intersection of the graphs of the equations.

There are two special cases when the lines have the same slope.

- When the system has no solution, the lines are parallel.
- When the system has infinitely many solutions, the lines coincide.

Step 2 Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

$y = -x + 3$ — Equation 1

$y = x - 1$ — Equation 2

Substitute $-x + 3$ for y in Equation 2.

$y = x - 1$ — Equation 2

$-x + 3 = x - 1$ — Substitute $-x + 3$ for y.

$x = 2$ — Solve for x.

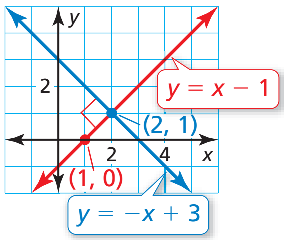
Substitute 2 for x in Equation 1 and solve for y.

$y = -x + 3$ — Equation 1

$y = -2 + 3$ — Substitute 2 for x.

$y = 1$ — Simplify.

So, the perpendicular lines intersect at $(2, 1)$.



Step 3 Use the Distance Formula to find the distance from $(1, 0)$ to $(2, 1)$.

$$\text{distance} = \sqrt{(1-2)^2 + (0-1)^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \approx 1.4$$

▶ So, the distance from the point $(1, 0)$ to the line $y = -x + 3$ is about 1.4 units.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

6. Find the distance from the point $(6, 4)$ to the line $y = x + 4$.
7. Find the distance from the point $(-1, 6)$ to the line $y = -2x$.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A ________ line segment *AB* is a segment that represents moving from point *A* to point *B*.

2. **WRITING** How are the slopes of perpendicular lines related?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the coordinates of point *P* along the directed line segment *AB* so that *AP* to *PB* is the given ratio. *(See Example 1.)*

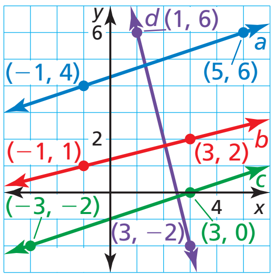
3. $A(8, 0)$, $B(3, -2)$; 1 to 4

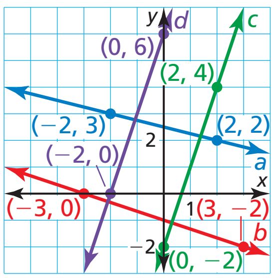
4. $A(-2, -4)$, $B(6, 1)$; 3 to 2

5. $A(1, 6)$, $B(-2, -3)$; 5 to 1

6. $A(-3, 2)$, $B(5, -4)$; 2 to 6

In Exercises 7 and 8, determine which of the lines are parallel and which of the lines are perpendicular. *(See Example 2.)*

7.


8.


In Exercises 9–12, tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. Justify your answer.

9. **Line 1:** $(1, 0)$, $(7, 4)$
Line 2: $(7, 0)$, $(3, 6)$

10. **Line 1:** $(-3, 1)$, $(-7, -2)$
Line 2: $(2, -1)$, $(8, 4)$

11. **Line 1:** $(-9, 3)$, $(-5, 7)$
Line 2: $(-11, 6)$, $(-7, 2)$

12. **Line 1:** $(10, 5)$, $(-8, 9)$
Line 2: $(2, -4)$, $(11, -6)$

In Exercises 13–16, write an equation of the line passing through point *P* that is parallel to the given line. Graph the equations of the lines to check that they are parallel. *(See Example 3.)*

13. $P(0, -1)$, $y = -2x + 3$

14. $P(3, 8)$, $y = \frac{1}{5}(x + 4)$

15. $P(-2, 6)$, $x = -5$

16. $P(4, 0)$, $-x + 2y = 12$

In Exercises 17–20, write an equation of the line passing through point *P* that is perpendicular to the given line. Graph the equations of the lines to check that they are perpendicular. *(See Example 4.)*

17. $P(0, 0)$, $y = -9x - 1$

18. $P(4, -6)$, $y = -3$

19. $P(2, 3)$, $y - 4 = -2(x + 3)$

20. $P(-8, 0)$, $3x - 5y = 6$

In Exercises 21–24, find the distance from point *A* to the given line. *(See Example 5.)*

21. $A(-1, 7)$, $y = 3x$

22. $A(-9, -3)$, $y = x - 6$

23. $A(15, -21)$, $5x + 2y = 4$

24. $A\left(-\frac{1}{4}, 5\right)$, $-x + 2y = 14$

25. ERROR ANALYSIS Describe and correct the error in determining whether the lines are parallel, perpendicular, or neither.

> ✗
> Line 1: (3, −5), (2, −1)
> Line 2: (0, 3), (1, 7)
>
> $m_1 = \dfrac{-1-(-5)}{2-3} = -4 \qquad m_2 = \dfrac{7-3}{1-0} = 4$
>
> Lines 1 and 2 are perpendicular.

26. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through the point (3, 4) and is parallel to the line $y = 2x + 1$.

> ✗
> $y = 2x + 1, (3, 4)$
>
> $4 = m(3) + 1$
>
> $1 = m$
>
> The line $y = x + 1$ is parallel to the line $y = 2x + 1$.

In Exercises 27–30, find the midpoint of $\overline{PQ}$. Then write an equation of the line that passes through the midpoint and is perpendicular to $\overline{PQ}$. This line is called the *perpendicular bisector*.

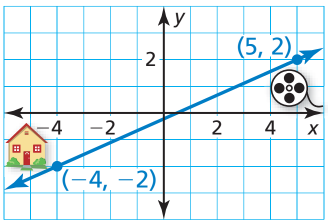
27. $P(-4, 3)$, $Q(4, -1)$

28. $P(-5, -5)$, $Q(3, 3)$

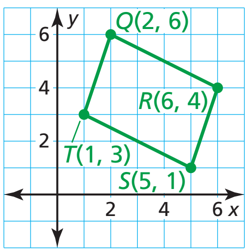
29. $P(0, 2)$, $Q(6, -2)$

30. $P(-7, 0)$, $Q(1, 8)$

31. MODELING WITH MATHEMATICS Your school lies directly between your house and the movie theater. The distance from your house to the school is one-fourth of the distance from the school to the movie theater. What point on the graph represents your school?

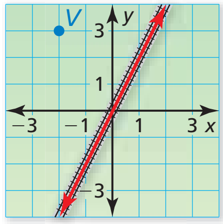


32. REASONING Is quadrilateral $QRST$ a parallelogram? Explain your reasoning.



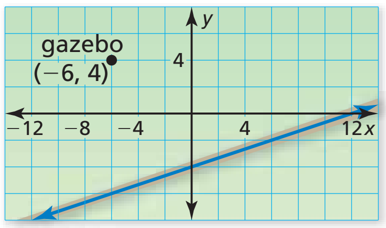
33. REASONING A triangle has vertices $L(0, 6)$, $M(5, 8)$, and $N(4, -1)$. Is the triangle a right triangle? Explain your reasoning.

34. MODELING WITH MATHEMATICS A new road is being constructed parallel to the train tracks through point V. An equation of the line representing the train tracks is $y = 2x$. Find an equation of the line representing the new road.



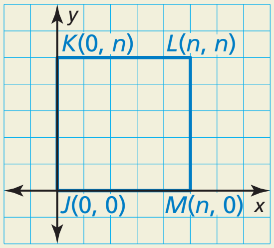
35. MODELING WITH MATHEMATICS A bike path is being constructed perpendicular to Washington Boulevard through point $P(2, 2)$. An equation of the line representing Washington Boulevard is $y = -\frac{2}{3}x$. Find an equation of the line representing the bike path.

36. PROBLEM SOLVING A gazebo is being built near a nature trail. An equation of the line representing the nature trail is $y = \frac{1}{3}x - 4$. Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the gazebo from the nature trail?



37. CRITICAL THINKING The slope of line ℓ is greater than 0 and less than 1. Write an inequality for the slope of a line perpendicular to ℓ. Explain your reasoning.

38. **HOW DO YOU SEE IT?** Determine whether quadrilateral $JKLM$ is a square. Explain your reasoning.



39. **CRITICAL THINKING** Suppose point P divides the directed line segment XY so that the ratio of XP to PY is 3 to 5. Describe the point that divides the directed line segment YX so that the ratio of YP to PX is 5 to 3.

40. **MAKING AN ARGUMENT** Your classmate claims that no two nonvertical parallel lines can have the same y-intercept. Is your classmate correct? Explain.

41. **MATHEMATICAL CONNECTIONS** Solve each system of equations algebraically. Make a conjecture about what the solution(s) can tell you about whether the lines intersect, are parallel, or are the same line.

a. $y = 4x + 9$
$4x - y = 1$

b. $3y + 4x = 16$
$2x - y = 18$

c. $y = -5x + 6$
$10x + 2y = 12$

42. **THOUGHT PROVOKING** Find a formula for the distance from the point (x_0, y_0) to the line $ax + by = 0$. Verify your formula using a point and a line.

MATHEMATICAL CONNECTIONS In Exercises 43 and 44, find a value for k based on the given description.

43. The line through $(-1, k)$ and $(-7, -2)$ is parallel to the line $y = x + 1$.

44. The line through $(k, 2)$ and $(7, 0)$ is perpendicular to the line $y = x - \frac{28}{5}$.

45. **ABSTRACT REASONING** Make a conjecture about how to find the coordinates of a point that lies beyond point B along $\overrightarrow{AB}$. Use an example to support your conjecture.

46. **PROBLEM SOLVING** What is the distance between the lines $y = 2x$ and $y = 2x + 5$? Verify your answer.

PROVING A THEOREM In Exercises 47 and 48, use the slopes of lines to write a paragraph proof of the theorem.

47. Lines Perpendicular to a Transversal Theorem (Theorem 3.12): In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

48. Transitive Property of Parallel Lines Theorem (Theorem 3.9): If two lines are parallel to the same line, then they are parallel to each other.

49. **PROOF** Prove the statement: If two lines are vertical, then they are parallel.

50. **PROOF** Prove the statement: If two lines are horizontal, then they are parallel.

51. **PROOF** Prove that horizontal lines are perpendicular to vertical lines.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the point in a coordinate plane. *(Skills Review Handbook)*

52. $A(3, 6)$

53. $B(0, -4)$

54. $C(5, 0)$

55. $D(-1, -2)$

Copy and complete the table. *(Skills Review Handbook)*

56.

x	-2	-1	0	1	2
$y = x + 9$					

57.

x	-2	-1	0	1	2
$y = x - \frac{3}{4}$					

3.4–3.5 What Did You Learn?

Core Vocabulary

distance from a point to a line, *p. 148*
perpendicular bisector, *p. 149*
directed line segment, *p. 156*

Core Concepts

Section 3.4

Finding the Distance from a Point to a Line, *p. 148*
Constructing Perpendicular Lines, *p. 149*
Theorem 3.10 Linear Pair Perpendicular Theorem, *p. 150*
Theorem 3.11 Perpendicular Transversal Theorem, *p. 150*
Theorem 3.12 Lines Perpendicular to a Transversal Theorem, *p. 150*

Section 3.5

Partitioning a Directed Line Segment, *p. 156*
Theorem 3.13 Slopes of Parallel Lines, *p. 157*
Theorem 3.14 Slopes of Perpendicular Lines, *p. 157*
Writing Equations of Parallel and Perpendicular Lines, *p. 158*
Finding the Distance from a Point to a Line, *p. 159*

Mathematical Practices

1. Compare the effectiveness of the argument in Exercise 24 on page 153 with the argument "You can find the distance between any two parallel lines." What flaw(s) exist in the argument(s)? Does either argument use correct reasoning? Explain.

2. Look back at your construction of a square in Exercise 29 on page 154. How would your construction change if you were to construct a rectangle?

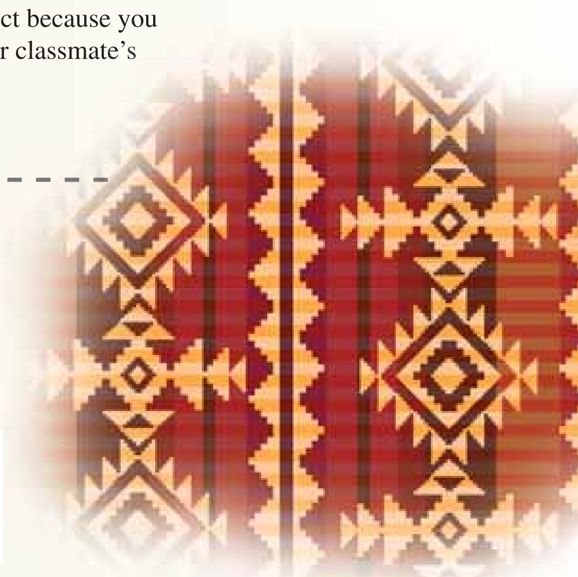
3. In Exercise 31 on page 161, a classmate tells you that your answer is incorrect because you should have divided the segment into four congruent pieces. Respond to your classmate's argument by justifying your original answer.

Performance Task

Navajo Rugs

Navajo rugs use mathematical properties to enhance their beauty. How can you describe these creative works of art with geometry? What properties of lines can you see and use to describe the patterns?

To explore the answers to this question and more, go to ***BigIdeasMath.com***.

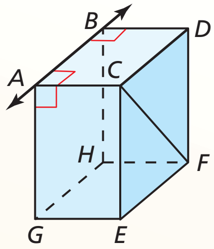


3 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

3.1 Pairs of Lines and Angles *(pp. 125–130)*

Think of each segment in the figure as part of a line.



a. Which line(s) appear perpendicular to $\overleftrightarrow{AB}$?

▶ $\overleftrightarrow{BD}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BH}$, and $\overleftrightarrow{AG}$ appear perpendicular to $\overleftrightarrow{AB}$.

b. Which line(s) appear parallel to $\overleftrightarrow{AB}$?

▶ $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$, and $\overleftrightarrow{EF}$ appear parallel to $\overleftrightarrow{AB}$.

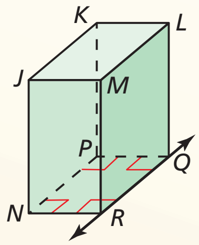
c. Which line(s) appear skew to $\overleftrightarrow{AB}$?

▶ $\overleftrightarrow{CF}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{DF}$, $\overleftrightarrow{FH}$, and $\overleftrightarrow{EG}$ appear skew to $\overleftrightarrow{AB}$.

d. Which plane(s) appear parallel to plane *ABC*?

▶ Plane *EFG* appears parallel to plane *ABC*.

Think of each segment in the figure as part of a line. Which line(s) or plane(s) appear to fit the description?

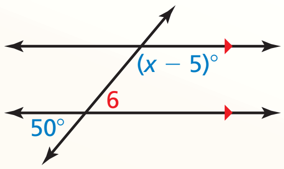


1. line(s) perpendicular to $\overleftrightarrow{QR}$
2. line(s) parallel to $\overleftrightarrow{QR}$
3. line(s) skew to $\overleftrightarrow{QR}$
4. plane(s) parallel to plane *LMQ*

3.2 Parallel Lines and Transversals *(pp. 131–136)*

Find the value of *x*.

By the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 6 = 50°$.

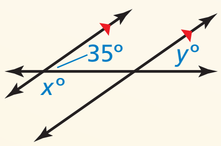


$(x - 5)° + m\angle 6 = 180°$	Consecutive Interior Angles Theorem (Thm. 3.4)
$(x - 5)° + 50° = 180°$	Substitute 50° for $m\angle 6$.
$x + 45 = 180$	Combine like terms.
$x = 135$	Subtract 45 from each side.

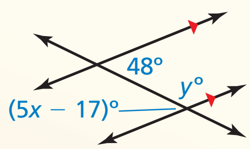
▶ So, the value of *x* is 135.

Find the values of *x* and *y*.

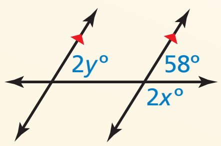
5.



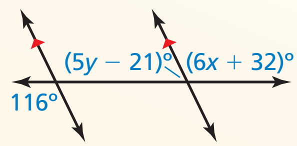
6.



7.

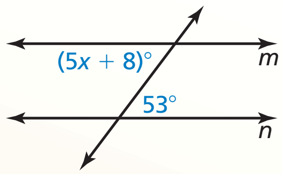


8.



3.3 Proofs with Parallel Lines *(pp. 137–144)*

Find the value of x that makes $m \parallel n$.



By the Alternate Interior Angles Converse (Theorem 3.6), $m \parallel n$ when the marked angles are congruent.

$$(5x + 8)° = 53°$$

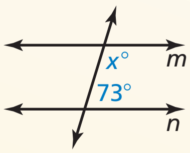
$$5x = 45$$

$$x = 9$$

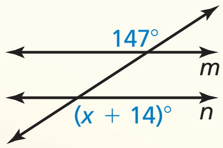
▶ The lines m and n are parallel when $x = 9$.

Find the value of x that makes $m \parallel n$.

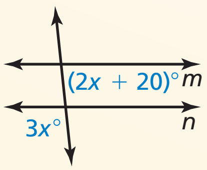
9.



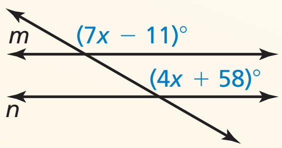
10.



11.

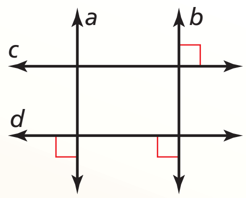


12.



3.4 Proofs with Perpendicular Lines *(pp. 147–154)*

Determine which lines, if any, must be parallel. Explain your reasoning.

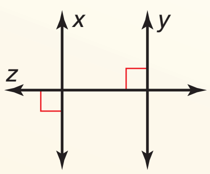


Lines a and b are both perpendicular to d, so by the Lines Perpendicular to a Transversal Theorem (Theorem 3.12), $a \parallel b$.

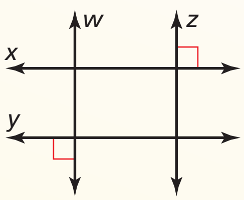
Also, lines c and d are both perpendicular to b, so by the Lines Perpendicular to a Transversal Theorem (Theorem 3.12), $c \parallel d$.

Determine which lines, if any, must be parallel. Explain your reasoning.

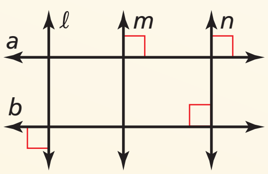
13.



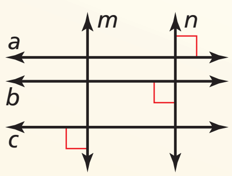
14.



15.



16.



3.5 Equations of Parallel and Perpendicular Lines (pp. 155–162)

a. Write an equation of the line passing through the point $(-2, 4)$ that is parallel to the line $y = 5x - 7$.

Step 1 Find the slope m of the parallel line. The line $y = 5x - 7$ has a slope of 5. By the Slopes of Parallel Lines Theorem (Theorem 3.13), a line parallel to this line also has a slope of 5. So, $m = 5$.

Step 2 Find the y-intercept b by using $m = 5$ and $(x, y) = (-2, 4)$.

$y = mx + b$ Use slope-intercept form.

$4 = 5(-2) + b$ Substitute for m, x, and y.

$14 = b$ Solve for b.

Because $m = 5$ and $b = 14$, an equation of the line is $y = 5x + 14$.

b. Write an equation of the line passing through the point $(6, 1)$ that is perpendicular to the line $3x + y = 9$.

Step 1 Find the slope m of the perpendicular line. The line $3x + y = 9$, or $y = -3x + 9$, has a slope of -3. Use the Slopes of Perpendicular Lines Theorem (Theorem 3.14).

$-3 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = \frac{1}{3}$ Divide each side by -3.

Step 2 Find the y-intercept b by using $m = \frac{1}{3}$ and $(x, y) = (6, 1)$.

$y = mx + b$ Use slope-intercept form.

$1 = \frac{1}{3}(6) + b$ Substitute for m, x, and y.

$-1 = b$ Solve for b.

Because $m = \frac{1}{3}$ and $b = -1$, an equation of the line is $y = \frac{1}{3}x - 1$.

Write an equation of the line passing through the given point that is parallel to the given line.

17. $A(3, -4)$, $y = -x + 8$

18. $A(-6, 5)$, $y = \frac{1}{2}x - 7$

19. $A(2, 0)$, $y = 3x - 5$

20. $A(3, -1)$, $y = \frac{1}{3}x + 10$

Write an equation of the line passing through the given point that is perpendicular to the given line.

21. $A(6, -1)$, $y = -2x + 8$

22. $A(0, 3)$, $y = -\frac{1}{2}x - 6$

23. $A(8, 2)$, $y = 4x - 7$

24. $A(-1, 5)$, $y = \frac{1}{7}x + 4$

Find the distance from point A to the given line.

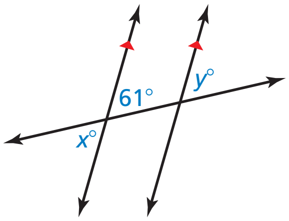
25. $A(2, -1)$, $y = -x + 4$

26. $A(-2, 3)$, $y = \frac{1}{2}x + 1$

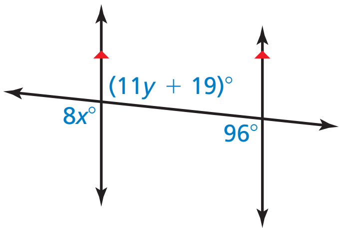
3 Chapter Test

Find the values of x and y. State which theorem(s) you used.

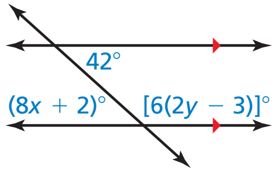
1.



2.



3.



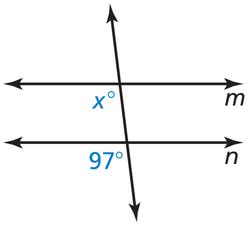
Find the distance from point A to the given line.

4. $A(3, 4)$, $y = -x$

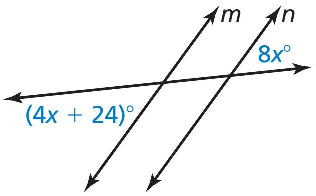
5. $A(-3, 7)$, $y = \frac{1}{3}x - 2$

Find the value of x that makes $m \parallel n$.

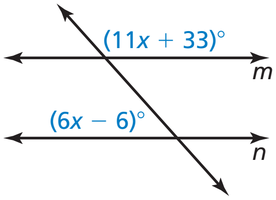
6.



7.



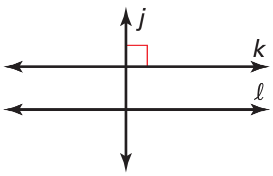
8.



Write an equation of the line that passes through the given point and is (a) parallel to and (b) perpendicular to the given line.

9. $(-5, 2)$, $y = 2x - 3$

10. $(-1, -9)$, $y = -\frac{1}{3}x + 4$



11. A student says, "Because $j \perp k$, $j \perp \ell$." What missing information is the student assuming from the diagram? Which theorem is the student trying to use?

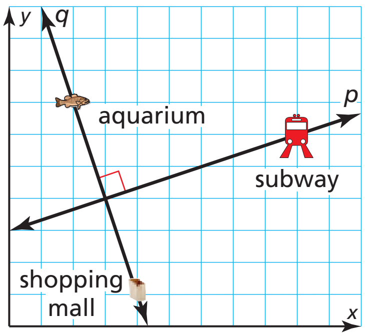
12. You and your family are visiting some attractions while on vacation. You and your mom visit the shopping mall while your dad and your sister visit the aquarium. You decide to meet at the intersection of lines q and p. Each unit in the coordinate plane corresponds to 50 yards.

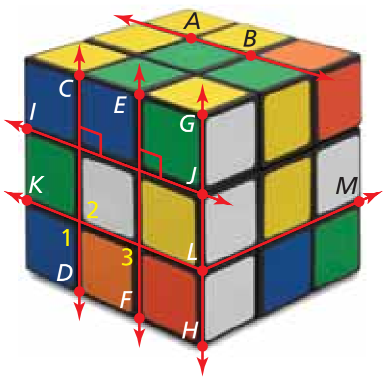
a. Find an equation of line q.

b. Find an equation of line p.

c. What are the coordinates of the meeting point?

d. What is the distance from the meeting point to the subway?





13. Identify an example on the puzzle cube of each description. Explain your reasoning.

a. a pair of skew lines

b. a pair of perpendicular lines

c. a pair of parallel lines

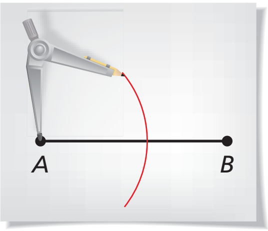
d. a pair of congruent corresponding angles

e. a pair of congruent alternate interior angles

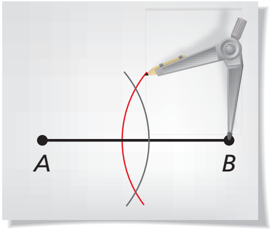
3 Cumulative Assessment

1. Use the steps in the construction to explain how you know that $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.

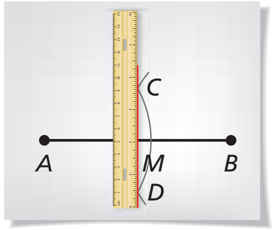
Step 1



Step 2



Step 3



2. The equation of a line is $x + 2y = 10$.

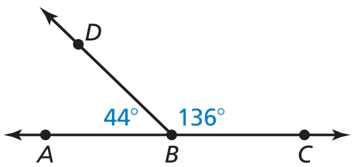
a. Use the numbers and symbols to create the equation of a line in slope-intercept form that passes through the point $(4, -5)$ and is parallel to the given line.

b. Use the numbers and symbols to create the equation of a line in slope-intercept form that passes through the point $(2, -1)$ and is perpendicular to the given line.

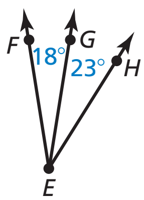
y	x	$=$	$+$	$-$	-9	-2	-1
$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	3	4	5

3. Classify each pair of angles whose measurements are given.

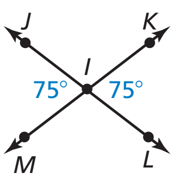
a.



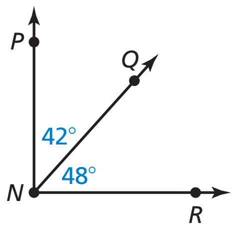
b.



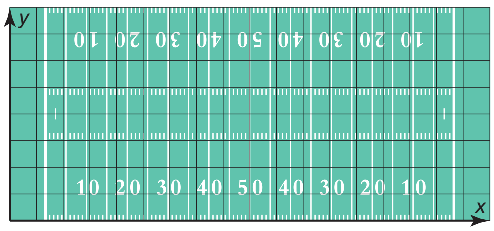
c.



d.



4. Your school is installing new turf on the football field. A coordinate plane has been superimposed on a diagram of the football field where 1 unit = 20 feet.

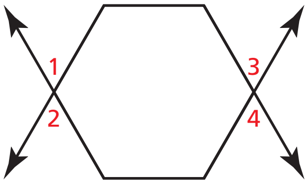


a. What is the length of the field?

b. What is the perimeter of the field?

c. Turf costs \$2.69 per square foot. Your school has a \$150,000 budget. Does the school have enough money to purchase new turf for the entire field?

5. Enter a statement or reason in each blank to complete the two-column proof.

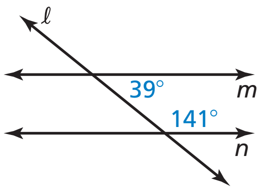


Given $\angle 1 \cong \angle 3$

Prove $\angle 2 \cong \angle 4$

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 3$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** ______________________
3. $\angle 2 \cong \angle 3$	**3.** ______________________
4. ____________	**4.** Vertical Angles Congruence Theorem (Thm. 2.6)
5. $\angle 2 \cong \angle 4$	**5.** ______________________

6. Your friend claims that lines m and n are parallel. Do you support your friend's claim? Explain your reasoning.



7. Which of the following is true when $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are skew?

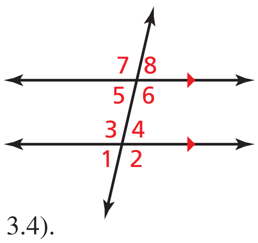
Ⓐ $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel.

Ⓑ $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect.

Ⓒ $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular.

Ⓓ A, B, and C are noncollinear.

8. Select the angle that makes the statement true.



a. $\angle 4 \cong$ ___ by the Alternate Interior Angles Theorem (Thm. 3.2).

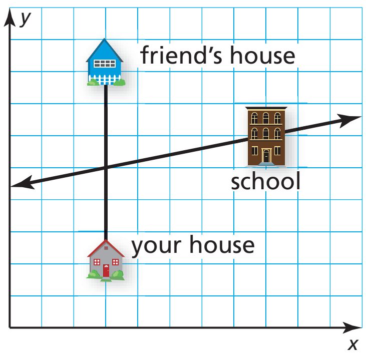
b. $\angle 2 \cong$ ___ by the Corresponding Angles Theorem (Thm. 3.1).

c. $\angle 1 \cong$ ___ by the Alternate Exterior Angles Theorem (Thm. 3.3).

d. $m\angle 6 + m$ _____ $= 180°$ by the Consecutive Interior Angles Theorem (Thm. 3.4).

$\angle 1$ $\angle 2$ $\angle 3$ $\angle 4$ $\angle 5$ $\angle 6$ $\angle 7$ $\angle 8$

9. You and your friend walk to school together every day. You meet at the halfway point between your houses first and then walk to school. Each unit in the coordinate plane corresponds to 50 yards.



a. What are the coordinates of the midpoint of the line segment joining the two houses?

b. What is the distance that the two of you walk together?