

Constructing Parallel and Perpendicular Lines

1. Plan

What You'll Learn

- To construct parallel lines
- To construct perpendicular lines

... And Why

To construct the shortest segment from a point to a line, as in Example 4

Check Skills You'll Need

Use a straightedge to draw each figure. Then use a straightedge and compass to construct a figure congruent to it. **1-3. See back of book.**

- a segment
- an obtuse angle
- an acute angle

Use a straightedge to draw each figure. Then use a straightedge and compass to bisect it. **4-6. See back of book.**

- a segment
- an acute angle
- an obtuse angle

GO for Help Lesson 1-7

Objectives

- To construct parallel lines
- To construct perpendicular lines

Examples

- Constructing $\ell \parallel m$
- Constructing a Special Quadrilateral
- Perpendicular at a Point on a Line
- Perpendicular From a Point to a Line



Math Background

The method in this lesson for constructing parallel lines is based on the Converse of the Corresponding Angles Postulate in Lesson 3-2. An alternative method might base a construction on the Converse of the Alternate Interior Angles Theorem or on the theorem *In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.* The method for constructing perpendicular lines is based on the method for constructing the perpendicular bisector of a segment in Lesson 1-7.

More Math Background: p. 124D

Lesson Planning and Resources

See p. 124E for a list of the resources that support this lesson.



Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Constructing Segments and Angles

Lesson 1-7: Examples 1 and 2
Extra Skills, Word Problems, Proof Practice, Ch. 1

Constructing Bisectors

Lesson 1-7: Examples 3 and 5
Extra Skills, Word Problems, Proof Practice, Ch. 1

1

Constructing Parallel Lines

You can use what you know about parallel lines, transversals, and corresponding angles to construct parallel lines.

1 EXAMPLE Constructing $\ell \parallel m$

Construct the line parallel to a given line and through a given point that is not on the line.

Given: line ℓ and point N not on ℓ

Construct: line m through N with $m \parallel \ell$



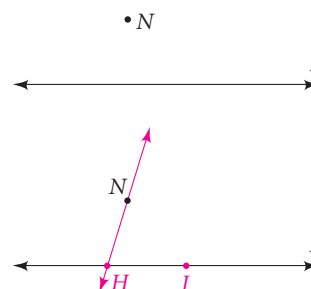
Real-World Connection

Careers Architects construct parallel and perpendicular lines when they build models of the buildings they design.

Step 1

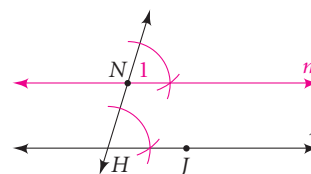
Label two points H and J on ℓ .

Draw \overrightarrow{HN} .



Step 2

Construct $\angle 1$ with vertex at N so that $\angle 1 \cong \angle NHJ$ and the two angles are corresponding angles. Label the line you just constructed m .



$m \parallel \ell$

Quick Check

1 Critical Thinking Explain why lines ℓ and m must be parallel.

If corr. \triangle are \cong , the lines are \parallel by the Converse of Corr. \triangle Postulate.

For many constructions, you will find it helpful to first visualize or sketch what the final figure should look like. This will often suggest the construction steps. In Example 2, a sketch is shown at the left of the example.

Differentiated Instruction Solutions for All Learners

Special Needs L1

For students who lack manual dexterity, seek out and use alternative compasses.

learning style: tactile

Below Level L2

Ask volunteers to demonstrate and explain how to copy an angle and a segment and how to construct the perpendicular bisector of a line segment.

learning style: visual

2. Teach

Guided Instruction

Visual Learners

Do the constructions in this lesson on the board.

1 EXAMPLE Error Prevention

In Step 2, make sure that students realize that the angle constructed at vertex N must be a corresponding angle. If the congruent angle were constructed on the opposite side of \overleftrightarrow{HN} , the lines would not be parallel.

2 EXAMPLE Math Tip

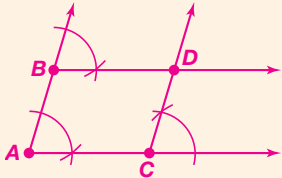
The constructed quadrilateral is a trapezoid.

PowerPoint

Additional Examples

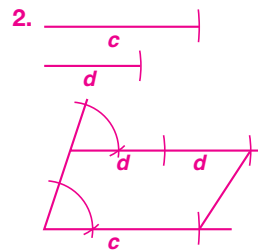
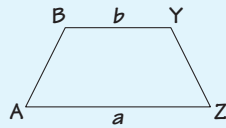
1 Draw a vertical line and a point not on the line. Demonstrate the construction of Example 1. **Check students' constructions.**

2 Construct a quadrilateral with both pairs of sides parallel.



Problem Solving Hint

First, draw a sketch of the figure.

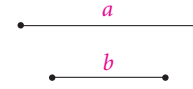


Quick Check

2 EXAMPLE Constructing a Special Quadrilateral

Construct a quadrilateral with one pair of parallel sides of lengths a and b .

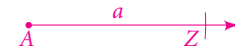
Given: segments of lengths a and b



Construct: quadrilateral $ABYZ$ with $AZ = a$, $BY = b$, and $\overleftrightarrow{AZ} \parallel \overleftrightarrow{BY}$

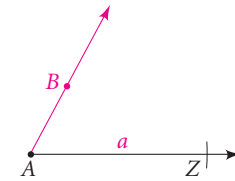
Step 1

Construct \overleftrightarrow{AZ} with length a .



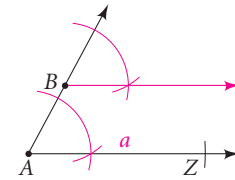
Step 2

Draw a point B not on \overleftrightarrow{AZ} . Then draw \overleftrightarrow{AB} .



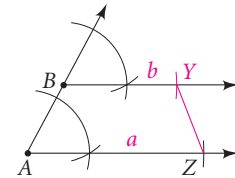
Step 3

Construct a ray parallel to \overleftrightarrow{AZ} through B .



Step 4

Construct Y so that $BY = b$. Then draw \overleftrightarrow{YZ} .



Quadrilateral $ABYZ$ has $AZ = a$, $BY = b$, and $\overleftrightarrow{AZ} \parallel \overleftrightarrow{BY}$.

2 Draw two segments. Label their lengths c and d . Construct a quadrilateral with one pair of parallel sides of lengths c and $2d$. **See above left.**

2

Constructing Perpendicular Lines

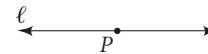
You can construct perpendicular lines using a compass and a straightedge.

3 EXAMPLE Perpendicular at a Point on a Line

Construct the perpendicular to a given line at a given point on the line.

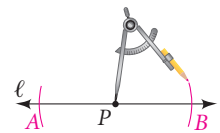
Given: point P on line ℓ

Construct: \overleftrightarrow{CP} with $\overleftrightarrow{CP} \perp \ell$



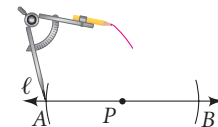
Step 1

Put the compass point on point P . Draw arcs intersecting ℓ in two points. Label the points A and B .



Step 2

Open the compass wider. With the compass tip on A , draw an arc above point P .



For: Construction Activity
Use: Interactive Textbook, 3-8

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Differentiated Instruction Solutions for All Learners

Advanced Learners L4

Have students use the methods in Example 3 to construct a square as simply as possible.

English Language Learners ELL

In Example 3, the word *perpendicular* is a noun. Point out that *perpendicular* may mean a perpendicular line or segment. Similarly, *parallel* may mean a parallel line or segment.

Auditory Learners

Have students work with partners to do the constructions in Examples 3 and 4, taking turns explaining the steps in each construction.

4 EXAMPLE Teaching Tip

The compass setting in step 2 does not have to be the same as that in step 1. However, the compass settings must be the same in steps 2 and 3 and must be large enough that the arcs constructed in these two steps intersect.

PowerPoint Additional Examples

3 Why does step 2 instruct you to open the compass wider? **With the compass tip on A and then on B, the same compass setting would make arcs that intersect at point P on line ℓ .**

4 Examine the construction. At what special point does \overrightarrow{RG} meet line ℓ ? **the midpoint of EF**

Resources

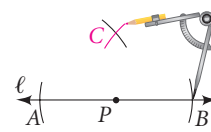
- Daily Notetaking Guide 3-8 **L3**
- Daily Notetaking Guide 3-8—Adapted Instruction **L1**

Closure

Explain how to construct a line parallel to a given line. Tell which theorem or postulate you use. **Construct congruent corresponding angles; the Converse of the Corresponding Angles Post.**

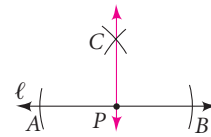
Step 3

Without changing the compass setting, place the compass point on point B. Draw an arc that intersects the arc from Step 2. Label the point of intersection C.



Step 4

Draw \overrightarrow{CP} .



• $\overrightarrow{CP} \perp \ell$



3 Use a straightedge to draw \overrightarrow{EF} . Construct \overrightarrow{FG} so that $\overrightarrow{FG} \perp \overrightarrow{EF}$ at point F. **See margin.**

You will prove in Chapter 5 that the perpendicular segment is the shortest segment from a point to a line. Here is its construction.

4 EXAMPLE Perpendicular From a Point to a Line

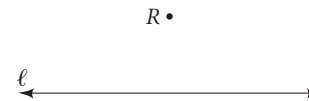
Construct the perpendicular to a given line through a given point not on the line.

Given: line ℓ and point R not on ℓ

Construct: \overrightarrow{RG} with $\overrightarrow{RG} \perp \ell$

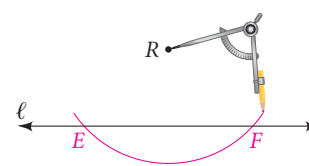
Step 1

Open your compass to a size greater than the distance from R to ℓ . With the compass point on point R, draw an arc that intersects ℓ at two points. Label the points E and F.



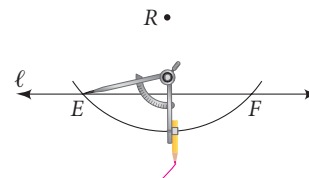
Step 2

Place the compass point on E and make an arc.



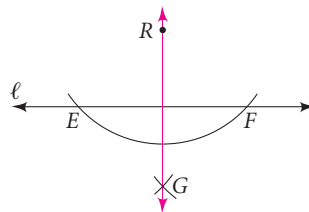
Step 3

Keep the same compass setting. With the compass tip on F, draw an arc that intersects the arc from Step 2. Label the point of intersection G.



Step 4

Draw \overrightarrow{RG} .



• $\overrightarrow{RG} \perp \ell$



Real-World Connection

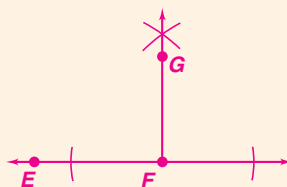
You can draw large circles using a simple, large compass.



4 Draw a line \overrightarrow{CX} and a point Z not on \overrightarrow{CX} . Construct \overrightarrow{ZB} so that $\overrightarrow{ZB} \perp \overrightarrow{CX}$. **See back of book.**

Quick Check

3.



3. Practice

Assignment Guide

- 1** A B 1-7, 14-16
2 A B 8-13, 17-26
C Challenge 27-36
 Test Prep 37-40
 Mixed Review 41-47

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 4, 10, 15, 21, 24.

Alternative Method

Exercise 14 This exercise presents another way to construct a line parallel to a given line.

Exercise 25 Have students discuss the significance of the phrase *must be true*.

Differentiated Instruction Resources

GPS Guided Problem Solving L3

Enrichment L4

Reteaching L2

Adapted Practice L1

Practice L3

Practice 3-7 Constructing Parallel and Perpendicular Lines

Construct a line perpendicular to line l through point Q .

1. $\bullet Q$ 2. $\bullet Q$ 3. $\bullet Q$

Construct a line perpendicular to line l at point F .

4. l 5. l 6. l

Construct a line parallel to line l and through point K .

7. $\bullet K$ 8. $\bullet K$ 9. $\bullet K$

For Exercises 10–15, use the segments at the right.

10. Construct a quadrilateral with one pair of parallel sides of lengths a and b .

11. Construct a quadrilateral with one pair of parallel sides of lengths b and c .

12. Construct a square with side lengths of b .

13. Construct a right triangle with leg lengths of a and c .

14. Construct a right triangle with leg lengths of b and c .

15. Construct an isosceles right triangle with leg lengths of a .

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

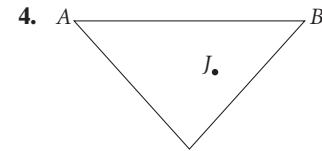
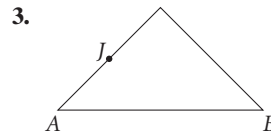
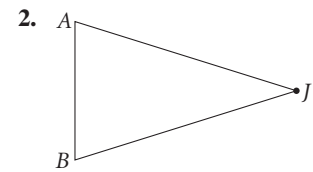
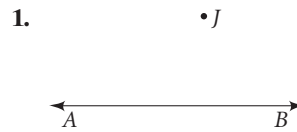
Practice and Problem Solving

A Practice by Example

Example 1
(page 181)



In Exercises 1–4, draw a figure like the given one. Then construct the line through point J and parallel to \overleftrightarrow{AB} . 1–4. See margin pp. 184–185.



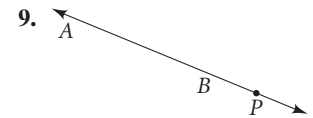
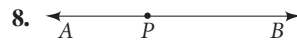
Example 2
(page 182)

For Exercises 5–7, draw two segments. Label their lengths a and b . Construct a quadrilateral with one pair of parallel sides as described. 5–7. See back of book.

5. The sides have lengths a and b .
 6. The sides have lengths $2a$ and b .
 7. The sides have lengths a and $\frac{1}{2}b$.

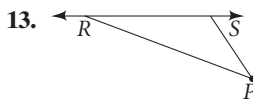
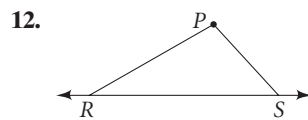
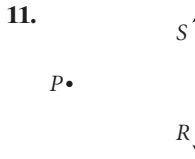
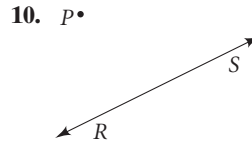
Example 3
(pages 182, 183)

In Exercises 8–9, draw a figure like the given one. Then construct the line perpendicular to \overleftrightarrow{AB} at point P . 8–9. See back of book.



Example 4
(page 183)

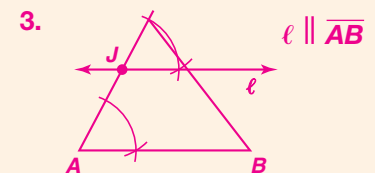
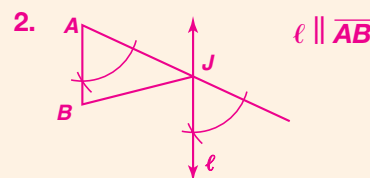
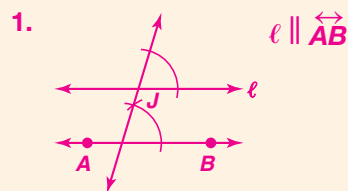
In Exercises 10–13, draw a figure like the given one. Then construct the line through point P and perpendicular to \overleftrightarrow{RS} . 10–13. See back of book.



B Apply Your Skills



14. Draw an acute angle. Construct an angle congruent to your angle so that the two angles are alternate interior angles. (*Hint*: Think of the letter Z.) See margin, p. 185.
15. **Writing** Explain how to use the Converse of the Alternate Interior Angles Theorem to construct a line parallel to a given line through a point not on the line. (*Hint*: See Exercise 14.) Construct a \cong alt. int. \angle ; then draw the \parallel line.

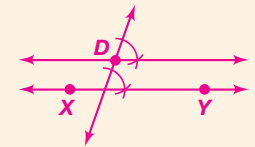
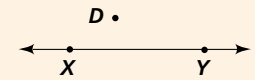




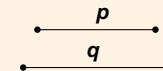
Lesson Quiz

Draw a figure similar to the one given. Then complete the construction. **Check students' constructions.**

1. Construct a line through D that is parallel to \overleftrightarrow{XY} .

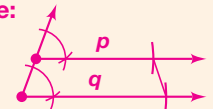


2. Construct a quadrilateral with one pair of parallel sides of lengths p and q .

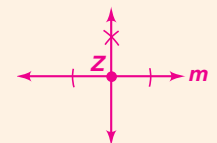
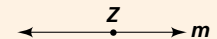


Quadrilaterals may vary.

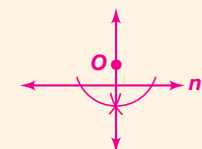
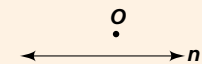
Sample:



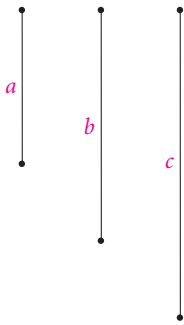
3. Construct the line perpendicular to line m at point Z .



4. Construct the perpendicular to line n through point O .



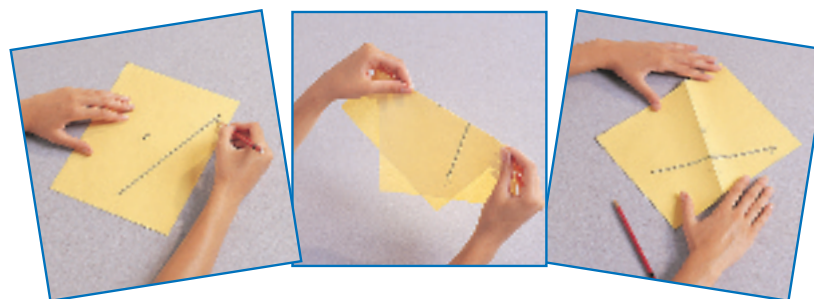
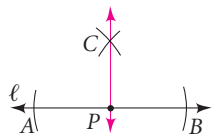
16. Draw obtuse $\triangle ABC$ with obtuse $\angle B$.
 a. Construct line ℓ through point A so that $\ell \parallel \overline{BC}$. **a-b. See back**
 b. Construct line m through point C so that $m \parallel \overline{AB}$. **of book.**



Exercises 17–24

For Exercises 17–24, use the segments at the left. **17–24. See back of book.**

17. Draw a line m . Construct a segment of length b that is perpendicular to line m .
 18. Construct a rectangle with base b and height c .
 19. Construct a square with sides of length a .
 20. Construct a rectangle with one side length a and a diagonal length b .
 21. a. Construct a quadrilateral with a pair of parallel sides of length c .
 b. **Make a Conjecture** What appears to be true about the other pair of sides in the quadrilateral you constructed?
 c. Use a protractor, a ruler, or both to check the conjecture you made in part (b).
 22. Construct a right triangle with legs of lengths a and b .
 23. Construct a right triangle with legs of lengths b and $\frac{1}{2}b$.
 24. a. Construct a triangle with sides of lengths a , b , and c .
 b. Construct the midpoint of each side of the triangle.
 c. Form a new triangle by connecting the midpoints.
 d. **Make a Conjecture** How do the sides of the smaller triangle and the sides of the larger triangle appear to be related?
 e. Use a protractor, a ruler, or both to check the conjecture you made in part (d).
 25. **Multiple Choice** The diagram at the left shows the construction of line \overleftrightarrow{CP} perpendicular to line ℓ through point P . Which of the following *must* be true? **D**
 (A) $\overleftrightarrow{CB} \parallel \overleftrightarrow{AB}$ (B) $\overleftrightarrow{AC} \perp \overleftrightarrow{CB}$
 (C) $CP = \frac{1}{2}AB$ (D) $\overline{AC} \cong \overline{BC}$
 26. **Paper Folding** You can use paper folding to create a perpendicular to a given line through a given point (Activity Lab, page 102). Fold the paper so that the line folds onto itself and the fold line contains the given point.



- a. Draw a line m and a point W not on the line. Use paper folding to create the perpendicular to m through W . Label this fold line k . **a-b. Check students'**
 b. Next, fold the line perpendicular to k through W . Label this fold line p . **work.**
 c. What is true of p and m ? Justify your answer. **$p \parallel m$; in a plane, two lines \perp to a third are \parallel .**

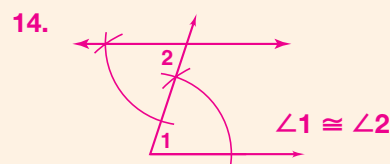
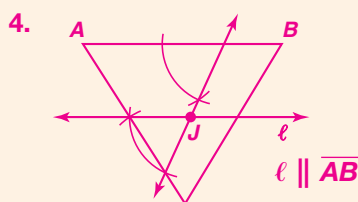
Challenge

Draw a segment, \overline{DG} . Construct a quadrilateral whose diagonals are both congruent to \overline{DG} , bisect each other, and meet the additional condition given below. Describe the quadrilateral that you get. **27–28. See back of book.**

27. The diagonals are not perpendicular. 28. The diagonals are perpendicular.

GO Online Homework Help

Visit: PHSchool.com
 Web Code: aue-0308



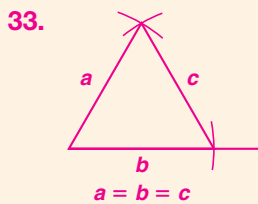
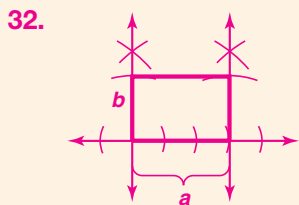
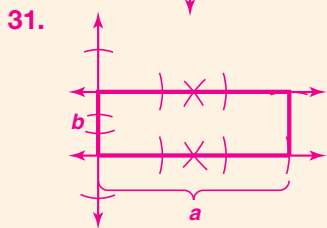
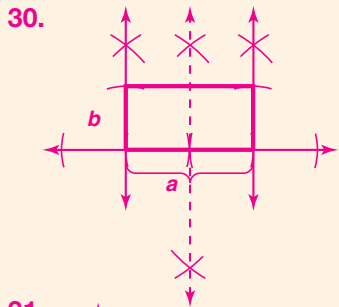
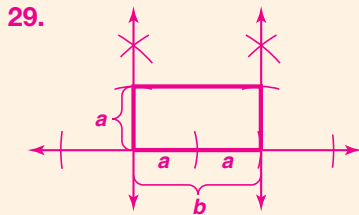
Alternative Assessment

Have students work in pairs to construct a rectangle. Their work should include a construction diagram and a written explanation of the steps they used.

Test Prep

Resources

- For additional practice with a variety of test item formats:
- Standardized Test Prep, p. 193
 - Test-Taking Strategies, p. 188
 - Test-Taking Strategies with Transparencies



29–32. See margin.

Construct a rectangle whose side lengths a and b meet the given condition.

29. $b = 2a$ 30. $b = \frac{1}{2}a$ 31. $b = \frac{1}{3}a$ 32. $b = \frac{2}{3}a$

Construct a triangle whose side lengths a , b , and c meet the given conditions. If such a triangle is not possible, explain. 33, 35. See margin. 34. See back of book.

33. $a = b = c$ 34. $a = b = 2c$ 35. $a = 2b = 2c$ 36. $a = b + c$

Not possible; The shorter sides would meet at a point on the longer side, forming a segment.

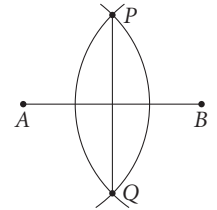


Test Prep

Multiple Choice

37. In the construction shown at the right, the two arcs with centers A and B have the same radius. What must be true of \overline{PQ} ? **A**

- A. \overline{PQ} bisects \overline{AB} . B. $\overline{PQ} \parallel \overline{AB}$
C. $\overline{PQ} \cong \overline{AB}$ D. $\overline{PQ} \cong \overline{AQ}$



38. Suppose you construct lines ℓ , m , and n so that $\ell \perp m$ and $\ell \parallel n$. Which of the following is true? **J**

- F. $m \parallel n$ G. $m \parallel \ell$ H. $n \perp \ell$ J. $n \perp m$

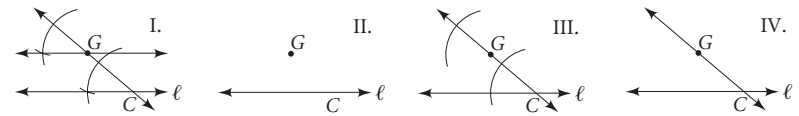
Short Response

39. Use a compass and straightedge to construct the following figure.

a. Draw a line ℓ and a point G not on ℓ . Construct an arc centered at point G to intersect ℓ in two points. Label the points R and T . Draw \overline{GR} and \overline{GT} .

b. Classify $\triangle RGT$. Justify your response. **a–b. See back of book.**

40. These pictures show steps for constructing a line parallel to a given line, but they are not necessarily in order.



- a. List the construction steps in the correct order. **a–b. See margin.**
b. For any step that uses a compass, describe the location(s) of the compass point.

Mixed Review



Lesson 3-7

Are the lines parallel? Explain.
No; the slopes are different.

41. $y = -4x - 3$
 $y = 4x + 3$

42. $y = \frac{1}{2}x + 1$
 $y = -2x - 1$

Yes; the slopes are both $-\frac{1}{3}$, and the y-intercepts are different.

43. $x + 3y = -6$
 $4x + 12y = -6$

No; the slopes are different.

Lesson 1-8

Find the distance between the points to the nearest tenth.

44. $W(8, -2)$ and $Z(2, 6)$ **10**

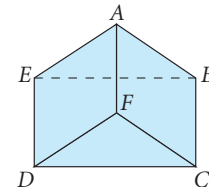
45. $W(-4.5, 1.2)$ and $Z(3.5, -2.8)$ **8.9**

Lesson 1-3

Name the intersection of the planes.

46. plane ABE and plane $EBCD$ **\overleftrightarrow{EB}**

47. plane $AFDE$ and plane FCD **\overleftrightarrow{DF}**



35. Not possible; if $a = 2b = 2c$, then $2a = 2b + 2c$ or $a = b + c$. The shorter sides would meet at the midpoint of the longer side, forming a segment.

40. [2] a. II, IV, III, I

b. (III): location of compass at points C and G ; (I): same as III and the intersection points

of \overleftrightarrow{CG} with arcs drawn in (III)

[1] incorrect sequence OR incorrect location of compass point