MATHCOUNTS®

2016
■ Chapter Competition ■
Sprint Round
Problems 1−30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____   Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon

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1. _____ Sheryl is counting backwards by 4. If the first number she says is 155, and the second number she says is 151, what is the 9th number Sheryl says?
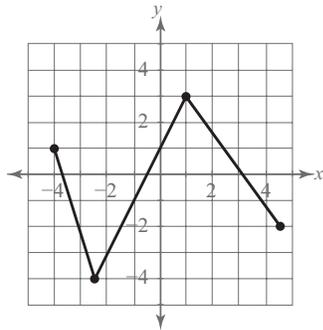
2. _____ What is the difference between the maximum and minimum values of this function graphed on the coordinate plane?

3. _____ If $\dfrac{1}{b} = \dfrac{b}{a}$ and $b = -1$, what is the value of a?

4. _____ What is the sum of all two-digit multiples of 3 that have units digit 1?

5. _____ What is the sum of the first eight terms in the sequence beginning -4, 5, …, where each subsequent term is the sum of the previous two terms?

6. $ _____

SHIPPING RATES
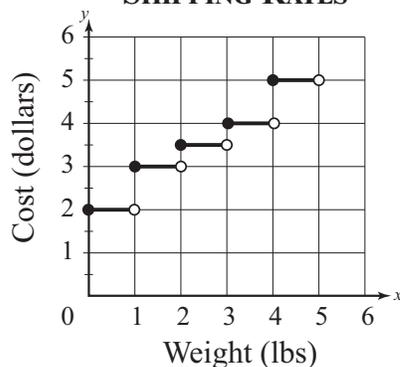
According to the graph, what is the total amount Blythe will be charged to ship three packages weighing 1.8 pounds, 2 pounds and 4.4 pounds?

7. _____ degrees   The graph shows the cookie dough ingredients for Mel's secret cookie recipe. What is the degree measure of the portion of the graph that indicates the percent of the cookie dough that is flour?

MEL'S COOKIE DOUGH

Flour 40%
Eggs 4%
Butter 16%
Sugar 22%
Chocolate Chips 18%
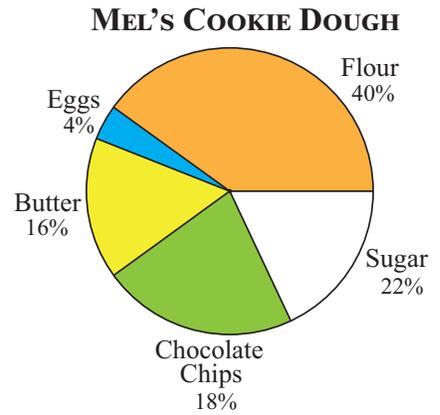
8. \$ _____   If 3 zoguts and 4 gimuns together cost $18, and 2 zoguts and 3 gimuns together cost $13, what is the combined cost of 1 zogut and 1 gimun?

9. _____ inches

40
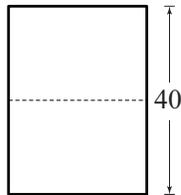
A rectangular piece of paper measures 40 inches on its long side. It is folded in half along the dotted line. The ratio of the long side of the original sheet to the short side of the original sheet is the same as the ratio of the long side of the folded sheet to the short side of the folded sheet. What is the length of the short side of the original sheet? Express your answer in simplest radical form.

10. _____   What is the greatest multiple of 3 that can be formed using one or more of the digits 2, 4, 5 and 8, assuming no digit is used more than once?

11. _____   If $\dfrac{a^6 b}{a^3 b^5} = \dfrac{a^m}{b^n}$ for positive integers m and n, what is the value of $m + n$?

12. _____   For what positive integer x, does 8% of x% of 200 have a value of 4?

13. _____ The table shows the number of students enrolled for each grade at East-West High School. Every 9th- and 10th-grade student at East-West is automatically entered into a drawing to win a new tablet computer. One winning student will be chosen randomly. What is the probability that a 10th-grader at East-West will win the drawing? Express your answer as a common fraction.

Grade	Enrollment
9	192
10	136
11	129
12	93

14. _____ If $2^2 \cdot 4^4 = 2^k$, what is the value of k?

15. _____ units^2 In the xy-plane, a circle has center P(5, 10) and intersects the x-axis at a single point Q(5, 0). What is the area of circle P? Express your answer in terms of π.

16. _____ When 3 is subtracted from $2x$ and the difference is divided by 5, the result is 7. What is the value of x?

17. _____ percent In a basketball game, Melina's ratio of 2-point shot attempts to 3-point shot attempts was 4:1. What percent of Melina's attempted shots were 3-point shots?

18. _____ What is the sum of the distinct prime factors of 2016?

19. _____ The opposite faces of a standard six-sided die add up to seven. Two identical standard six-sided dice are placed side by side as shown. What is the sum of the numbers of dots on the two faces that touch each other?
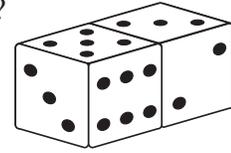
20. _____ sets How many sets of two or three distinct positive integers have a sum of 8?

21. _____ If a, b and c are integers such that $\dfrac{a+b}{2} = 3$, $\dfrac{b+c}{2} = 4$ and $\dfrac{a+c}{2} = 5$, what is the value of $a + b + c$?

22. _____ The mean of a set of five numbers is $3k$. When a sixth number is added to the set, the mean increases by k. What is the ratio of the sixth number to the sum of the first five numbers? Express your answer as a common fraction.
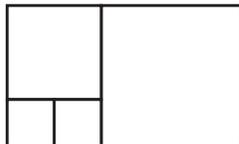
23. _____ inches The large rectangle shown here is composed of four squares. If the area of the large rectangle is 240 in², what is its perimeter?

24. _____ What is the least positive integer n such that $n!$ is divisible by 1000?

25. _____ degrees   In isosceles triangle ABC, shown here, AB = AC and $m\angle A = 32°$. Triangles ABC and PQR are congruent. Side AC intersects side PR at X so that $m\angle PXC = 114°$, and side PQ intersects side BC at Y as shown. What is the degree measure of $\angle PYC$?
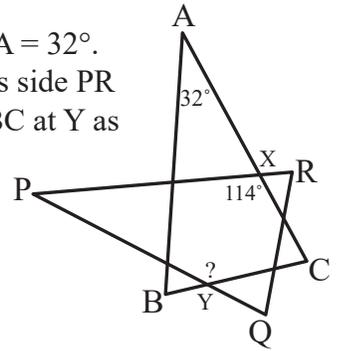
A	B	C	D	E
1	2	3		
		6	5	4
7	8	9		
		12	11	10
⋮	⋮	⋮	⋮	⋮

26. _____   The integers 1 through 66 are arranged in five columns A to E as shown, where column C contains the multiples of 3. What is the sum of the numbers in column D?

27. _____ minutes   On Monday, a worker began painting a fence at 8:00 a.m. At 10:00 a.m., two more workers showed up and helped the first worker paint. The three workers finished the job at 11:30 a.m. On Tuesday, the first worker began painting an identical fence at 8:00 a.m. The other two workers showed up earlier this time, and the job was finished at 10:54 a.m. All three workers paint at the same constant rate. How many minutes did the first worker paint alone on Tuesday?
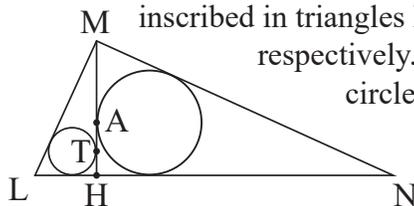
28. _____   Triangle LMN, shown here, has altitude MH drawn to side LN. Circles are inscribed in triangles MNH and MLH, tangent to altitude MH at A and T, respectively. If MA:AT:TH = 4:2:1, what is the ratio of the smaller circle's area to the larger circle's area? Express your answer as a common fraction.

29. _____ mi/h   A bug crawls a distance of n miles at a speed of $n + 1$ miles per hour one day and then crawls $2n + 1$ miles at a speed of $n^2 + n$ miles per hour the next day. If the total time for the trip was 6 hours, what was the bug's average speed? Express your answer as a common fraction.

30. _____   What is the greatest 5-digit palindrome n such that $7n$ is a 6-digit palindrome?

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction?   *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction?   *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit

expressed as a common fraction in terms of π?   *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number?   *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is the value of $\sqrt{15}\times\sqrt{5}$?   *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* both are zero, in which case they both may be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5   *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation?   *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation?   *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.