
MATHCOUNTS®

2014
■ State Competition ■
Countdown Round
Problems 1–80

This booklet contains problems to be used
in the Countdown Round.

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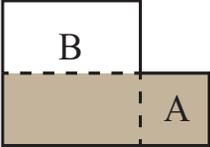
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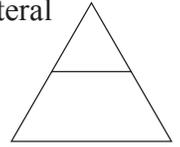
1. _____ What is the least possible denominator of a common fraction that lies strictly between $\frac{1}{3}$ and $\frac{1}{2}$?
2. _____ (even integers) In a list of 30 integers, 18 of the integers are multiples of 5. If 10 of the multiples of 5 are odd integers, what is the maximum number of even integers in the list?
3. _____ (units²) What is the largest possible area, in square units, of a triangle with perimeter 6 units? Express your answer in simplest radical form.
4. _____ (workers) A library system with 6 branches employs 60 workers. If no branch has fewer than 7 workers and no more than 11 workers, what is the minimum number of total workers in any two of the branches?
5. _____ (classes) At a school, grade point averages are calculated by making an A worth 4 points, a B worth 3, etc., and all courses are equally weighted. After 20 courses, Zach has a grade point average of exactly 3.9. If he can get As in all of his future classes, how many more classes must Zach take to raise his average to exactly 3.95?
6. _____ (feet) A rectangle of area 9 ft² is made by placing square A next to half of square B, as shown. In feet, what is the side length of square A? Express your answer in simplest radical form.


7. _____ If $b = \frac{3}{a}$, where $a \neq 0$ and $b \neq 1$, then $\frac{a-3}{b-1} = ka$. What is the value of k ?
8. _____ (integers) How many positive integers divide 6^{2014} but not 6^{2013} ?
9. _____ What is the sum of all integer values of x that satisfy $|x + 4| < 4$ and $-x < 3$?
10. _____ What integer is closest to $\sqrt{9800}$?
11. _____ The mean, median and mode of a collection of five positive integers are all 5. What is the largest possible number in the set?

12. _____ For what nonzero integer a is the point (a, a) on the graph of $f(x) = x^2 + 6x$?
13. _____ (units²) Centered at each vertex of a regular decagon of side length 2 units, a circle of radius 1 unit is drawn. In square units, what is the total area of the parts of the circles that are inside the decagon? Express your answer in terms of π .
14. _____ If $x + \frac{1}{x} = 6$, what is the value of $x^3 + \frac{1}{x^3}$?
15. _____ (values) A wallet contains exactly two each of \$5, \$10 and \$20 bills. Two bills are randomly selected from the wallet. How many different dollar values are possible?
16. _____ (units) Triangle ABC has B at the origin and \overline{AC} is parallel to the x -axis. $AC = 5$ units, $BC = 7$ units and the y -axis bisects \overline{AC} . What is the length of \overline{AB} , in units?
17. _____ (digits) How many digits are in the value of the product $8^5 \cdot 5^8$?
18. _____ (packages) Pencils can be purchased in packages of 12 or 25. If Masi purchased 8 packages to get 135 pencils, how many packages of 12 did he buy?
19. _____ (combinations) Caleb's Cupcakes sells red velvet, vanilla cream and chocolate chip cupcakes. How many different combinations are possible for a dozen cupcakes at Caleb's?
20. _____ For $y = kx^3$, $y = 36$ when $x = 2$. What is the value of y when $x = 4$?
21. _____ (units²) If ABCDEF is a regular hexagon of area 36 units², what is the area of $\triangle ACE$, in square units?
22. _____ (minutes) In a one-mile race, Blue Streak finished 30 seconds after The Flash. If Blue Streak had been given a $\frac{1}{8}$ -mile head start, Blue Streak would have finished 30 seconds before The Flash. How many minutes did it take for Blue Streak to run one mile?
23. _____ What is the absolute difference between the two real numbers x for which $(x + 1)(x - 1)(x - 2) = (x + 2)(x + 3)(x - 3)$? Express your answer in simplest radical form.

24. _____ (permutations) How many of the permutations that can be made using one or more letters in SOUTH CAROLINA also can be made using the letters in MAINE?

25. _____ (meters) An equilateral triangle is divided into a trapezoid and a smaller equilateral triangle as shown. The larger triangle has a side length of 24 m. If the area of the trapezoid is $\frac{3}{4}$ of the area of the larger triangle, in meters, how long is a side of the smaller triangle?



26. _____ (in²) Eight segments measuring 3, 5, 6, 9, 11, 13, 15 and 16 inches are used to construct a rectangle. In square inches, what is the largest possible area of the rectangle?

27. _____ The result of multiplying a number by m is the same as adding it to m . In terms of m , what is the number? Express your answer as a common fraction.

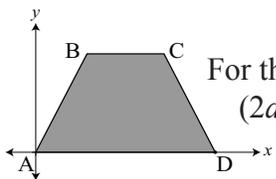
28. _____ What is the greatest positive 4-digit integer n for which the sum of its proper divisors is $n - 1$?

29. _____ If $3x + y + z = 15$ and $x - 2y - z = 30$, what is the value of $5x - 3y - z$?

30. _____ The fourth and fourteenth terms of an arithmetic progression are -14 and -4 , respectively. What is the 2014th term?

31. _____ When $(a + 2b + 3c)^6$ is expanded, what is the coefficient of a^3b^2c ?

32. _____ A number equals its double decreased by 1. What is the number?

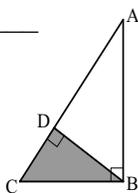
33. _____  For the isosceles trapezoid shown, three of the vertices are $(0, 0)$, $(2a, 2b)$ and $(2a + c, 2b)$. The coordinates of point D are $(ra + sb + tc, 0)$. What is the value of $r + s + t$?

34. _____ The quadratic equation $x^2 + ax + b = 0$ has roots -3 and -7 . What is the value of $a + b$?

35. _____ What is the value of $(11^2 - 10^2)^2$?
36. _____ The sum of the first 14 terms of an arithmetic sequence is 2014. If the sum of the first 28 terms of the same sequence is 2014, what is the sum of the first 42 terms of the sequence?
37. _____ (people) If two of the people in Demi's family are selected at random, there is a 50% chance that both people will have brown eyes. What is the minimum number of people in Demi's family?
38. _____ In a class of 99 students, 1 student represents 1% of the class (rounded to the nearest whole percent) and 99 students represent 100% of the class. What positive integer between 1 and 100 cannot be represented as a percentage of the class?
39. _____ Michael is two years older than three times Jennifer's age. If Jennifer is x years old, then, in years, Michael's age can be expressed as $mx + b$. What is the value of $m + b$?
40. _____ (ways) Alex has 3 books, and Eli has 4 books. If the 7 books are all different, in how many ways can they place their books on a shelf so that all of Alex's books are next to one another?
41. _____ The product $(x^{-2}y^3z^{-\frac{2}{3}})(xyz^{\frac{5}{3}})$ is equivalent to what common fraction that has only positive exponents?
42. _____ (dollars) A store sells fancy letters. There are no prices on the individual letters, but ADI buys the letters in her name for \$15, DIANE buys the letters in her name for \$21 and NADIA buys the letters in her name for \$23. In dollars, how much will ANNE pay for the letters in her name?

43. _____ What is the value of $9\left(\frac{1}{3} + 2 - \frac{2}{3}\right)$?

44. _____



Triangles ABC and BDC, shown here, are 30-60-90 right triangles, and $AC = 12$ cm. The area of $\triangle BDC$ can be written in simplest radical form as $\frac{a\sqrt{c}}{b}$ cm^2 , where a , b and c are positive integers. What is the value of $a + b + c$?

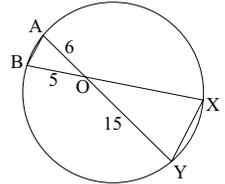
45. _____ (students) The mean score on a physics test for all students in the class was exactly 77. When Albert's score of 100 was removed, the mean decreased to exactly 76. How many students are in the class?
46. _____ What is the sum of all integers x such that $|2x + 5| \leq 4$?
47. _____ The two congruent sides of an isosceles triangle are each 3 inches longer than the base. The perimeter is 30 inches. What is the product of the numerical values of the lengths of the three sides of the triangle?
48. _____ Betty was counting the diagonals of a regular polygon with n sides. She stopped counting after 48 diagonals. What is the least possible value of n ?
49. _____ A weighted die has six sides, labeled 1 through 6, where the probability of rolling an n is directly proportional to n . What is the probability of rolling a 3? Express your answer as a common fraction.
50. _____ If x and y are nonzero real numbers such that $x^2 - 4xy + 4y^2 = 0$, what is the value of $\frac{y}{x}$? Express your answer as a common fraction.
51. _____ Jack and Jill both rolled two standard dice. What is the probability that Jill got the same two numbers as Jack, though not necessarily in the same order? Express your answer as a common fraction.
52. _____ (degrees) A regular pentagon has two interior diagonals originating at a vertex. What is the degree measure of the acute angle between the two diagonals?
53. _____ (ways) In how many ways can five integers from 1 to 7, inclusive, be placed in the boxes shown so that the row entries increase from left to right and the column entries increase from top to bottom?
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54. _____ Simeon calculated the sum of the first n positive integers, but he accidentally added one of those integers twice. If Simeon calculated the sum to be 100, what integer did he add twice?
55. _____ (reports) A 24-hour news station gives a weather report every 10 minutes. How many weather reports are given at the station each day?

56. _____ What is the value of $\sqrt{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3}$?

57. _____ (miles) A car travels 33 feet in $\frac{1}{4}$ second. How many miles will it travel in 10 minutes?

58. _____ If $f(x) = 3x - 10$ and $g(x) = 2x^2 - 10$, what is the value of $g(f(2))$?

59. _____ (units) In the figure, chords AY and BX intersect at O, AO = 6 units, BO = 5 units and YO = 15 units. What is XO, in units?



60. _____ (ways) One way to represent 60 as the sum of consecutive positive integers is $19 + 20 + 21$. Including this example, how many ways are there to represent 60 as the sum of two or more consecutive positive integers?

61. _____ (problems) Donald solved 5 more MATHCOUNTS problems than Janet. Janet solved 3 more than Ann. Ann solved twice as many as Randy. If Randy solved 7 problems, how many problems did Donald solve?

62. _____ The graphs of $y = -|x|$ and $y = |x| + a$ enclose an area of 72 units^2 , on a coordinate plane. What is the value of a ?

63. _____ A sack contains 7 red marbles and 3 black marbles. Two marbles are randomly selected, with replacement. What is the probability of choosing a red marble and then a black marble? Express your answer as a common fraction.

64. _____ If $2^x = 8,388,608$, what is the value of x ?

65. _____ (cents) Marika paid \$28.80 for a pound of gourmet coffee beans. One ounce of coffee beans makes 64 fluid ounces of brewed coffee. What is her cost, in cents, for 16 fluid ounces of brewed coffee?

66. _____ What is the units digit of the product $2^{2014} \cdot 3^{2014}$?

67. _____ When x is multiplied by 28, the result is a perfect square. If x is a positive integer, what is the smallest possible value of x ?

68. _____ (integers) How many positive integers are not less than the sum of their positive divisors?

69. _____ (ordered pairs) If x and y are integers and $y < 10$, for how many different ordered pairs (x, y) is $x^2 = y$?
70. _____ (percent) If the population of a city doubles in two years, what is its average yearly percentage growth? Express your answer to the nearest whole number.
71. _____ A line passes through the points $(3, 5)$ and $(-1, 3)$. What is the value of k such that the point $(k, 11)$ also is on the line?
72. _____ (miles) In miles, what is the perimeter of a $\frac{2}{3}$ -mile by $1\frac{3}{5}$ -mile rectangular piece of land? Express your answer as a mixed number.
73. _____ A set of consecutive even integers starts with -2 and ends with x . If the sum of the integers in the set is 108, what is the value of x ?
74. _____ (digits) A positive integer n is multiplied by 7. The resulting product contains just one digit repeated several times, and that digit is not 7. What is the least number of digits in n ?
75. _____ What is the sum of the prime factors of 102,102?
76. _____ (units) In isosceles trapezoid ABCD, with diagonals intersecting at E, $AB = 6$ units, $AE = BE = 4$ units and $CE = DE = 10$ units. What is CD, in units?
77. _____ (arrangements) Yasuko wants to arrange seven different family pictures in a row on her desk. If Yasuko wants the only picture of her parents to be in the middle, how many different arrangements of the pictures are possible?
78. _____ (ways) The squares in the figure are each to be painted either yellow or green. How many different ways can the five squares be painted?
79. _____ (votes) In the primary election, candidates Adams, Buchanan and Cleveland received votes in the ratio of 13:15:19, respectively. If there were 9447 votes cast, how many votes did Adams receive?
80. _____ A particular rectangle has length 7 units and perimeter 21 units. What is the ratio of its width to its length? Express your answer as a common fraction.

