# 2007-2008 MATHCOUNTS School Handbook: Volume II 

## Contains 200 creative math problems that meet NCTM standards for grades 6-8.

For questions about your local MATHCOUNTS program, please contact your local (chapter) coordinator. Coordinator contact information is available in the "Competition Information" section of www.mathcounts.org.

The printing of this handbook, accompanying registration materials and their distribution was made possible by

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1420 King Street, Alexandria, VA 22314
703-299-9006 info@mathcounts.org
www.mathcounts.org
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## Acknowledgments

The 2006-2007 MATHCOUNTS Question Writing Committee developed the questions for the 2007-2008 MATHCOUNTS School Handbook and competitions:

- Chair: Connie Laughlin, Mequon-Thiensville Schools, Muskego, Wis.
- Sam Baethge, San Antonio, Texas
- Chengde Feng, Oklahoma School of Science and Mathematics, Oklahoma City, Okl.
- Greg Murray, Dixie High School, St. George, Utah
- Sandra Powers, College of Charleston, Charleston, S.C.
- Joshua Zucker, Castilleja School, Palo Alto, Calif.
- Trevor Brown, Ontario, Canada (partial-term)

National Judges review competition materials, develop Masters Round questions and serve as arbiters at the National Competition:

- Richard Case, Computer Consultant, Greenwich, Conn.
- Flavia Colonna, George Mason University, Fairfax, Va.
- Peter Kohn, James Madison University, Harrisonburg, Va.
- Carter Lyons, James Madison University, Harrisonburg, Va.
- Monica Neagoy, Mathematics Consultant, Washington, D.C.
- Dave Sundin (STE 84), Statistical and Logistical Consultant, San Mateo, Calif.

National reviewers proofread and edit the MATHCOUNTS School Handbook and/or competition materials:

William Aldridge, Springfield, Va.
Mady Bauer, Bethel Park, Pa.
Susanna Brosonski, Orlando, Fla.
Lars Christensen (STE 89), Minneapolis, Minn.
Dan Cory (NAT 84, 85), Seattle, Wash.
Craig Countryman, San Diego, Calif.
Roslyn Denny, Valencia, Calif.
Edward Early (STE 92), Austin, Texas
Nancy English, Glendale, Mo.
Barry Friedman (NAT 86), Scotch Plains, N.J.
Joan M. Gell, Redondo Beach, Calif.
Dennis Hass, Westford, Mass.
Bonnie Hayman, St. Louis, Mo.
Helga Huntley (STE 91), Seattle, Wash.

Doug Keegan (STE 91, NAT 92), Austin, Texas David Kung (STE 85, NAT 86), St. Mary's City, Md.<br>Jane Lataille, Los Alamos, N.M.<br>Stanley Levinson, P.E., Lynchburg, Va.<br>Artie McDonald, P.E. (STE 88), Melbourne, Fla. Paul McNally, Haddon Heights, N.J. Randy Rogers, Cedar Rapids, Iowa Frank Salinas, Houston, Texas Laura Taalman (STE 87), Harrisonburg, Va. Craig Volden (NAT 84), Columbus, Ohio<br>Chaohua Wang, Bloomington, Ill.<br>Deborah Wells, Rockville, Md.<br>Judy White, Littleton, Mass.<br>Yiming Yao (STE 96), Vancouver, British Columbia

The Solutions were written by Kent Findell, Diamond Middle School, Lexington, Mass.

Editor and Contributing Author: Kristen L. Chandler, Deputy Director \& Program Director MATHCOUNTS Foundation

Introduction and Building a MATHCOUNTS Program: Joseph A. Bremner, Director of Marketing MATHCOUNTS Foundation

Executive Director: Louis DiGioia MATHCOUNTS Foundation

MathType software for handbook development contributed by Design Science Inc., www.dessci.com, Long Beach, Calif.

## Count Me In!

A contribution to the MATHCOUNTS Foundation will help us continue to make this worthwhile program available to middle school students nationwide

The MATHCOUNTS Foundation will use your contribution for programwide support to give thousands of students the opportunity to participate.

With your help, MATHCOUNTS will continue to:

Excite students about math by providing a fun and challenging experience that rewards their effort and achievement.

Teach young adults to be problem solvers and develop their competitive spirit.

Demonstrate how math is important to everyday life.

Build essential teamwork skills.

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To become a partner in
MATHCOUNTS, send
your contribution to:
MATHCOUNTS Foundation
P.O. Box }133
Merrifield, VA 22116-9706
Or give online at:
www.mathcounts.org
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Other ways to give:

- Ask your employer about matching gifts. Your donation could double.
- Remember MATHCOUNTS in your United Way and Combined Federal Campaign at work.
- Leave a legacy. Include MATHCOUNTS in your will.

For more information regarding contributions, call the director of development at 571-382-8896 or e-mail info@mathcounts.org.

The MATHCOUNTS Foundation is a 501(c)3 organization. Your gift is fully tax deductible.

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The National Association of Secondary School Principals has placed this program on the NASSP Advisory List of National Contests and Activities for 2007-2008.


The American Society of Association Executives has recognized MATHCOUNTS with a 2001 Award of Excellence for its innovative, society enriching activities

## CRITICAL 2007-2008 DATES

ㅁ Immediately

I SeptemberDec. 7

## Dec. 7

- Mid-January
$\square$ Feb. 1-24
ㅁ March 1-30
$\square$ March 14
- March 28
- May 8-11

For easy reference, write your local coordinator's address and phone number here. Contact information for coordinators is available in the "Competition Information" section of www.mathcounts.org or from the national office.

Send in your school's Request/Registration Form to receive Volume II of the handbook, the Club in a Box resource kit and/or your copy of the 2007 School Competition.
Items will ship shortly after receipt of your form, with mailing of the School Competition kit following this schedule:

Registration forms postmarked by Oct. 1: Kits mailed early November. Kits continue mailing every two weeks.
Registration forms postmarked by Dec. 7 deadline: Kits mailed early-January.
Mail or fax the MATHCOUNTS Request/Registration Form (with payment if participating in the competition) to:

MATHCOUNTS Registration, P.O. Box 441, Annapolis Junction, MD 20701
Fax: 301-206-9789 (Please fax or mail, but do not do both.)
Questions? Call 301-498-6141 or confirm your registration via the Registered
Schools database and/or MATHCOUNTS Club Schools list at www.mathcounts.org.

## Competition Registration Deadline

In some circumstances, late registrations may be accepted at the discretion of MATHCOUNTS and the local coordinator. Register on time to ensure participation by your students.

If you have not been contacted with details about your upcoming competition, call your local or state coordinator!

If you have not received your School Competition Kit by the end of January, contact MATHCOUNTS at 703-299-9006.

## Chapter Competitions

State Competitions
Deadline for Math Clubs to reach MATHCOUNTS Silver Level \& entry into drawing
Deadline for Math Clubs to reach MATHCOUNTS Gold Level \& entry into drawing
Lockheed Martin MATHCOUNTS National Competition - 2008 in Denver

## Interested in more coaching materials or MATHCOUNTS items?

Additional FREE resources are available at www.mathcounts.org.
Purchase items from the MATHCOUNTS store at www.mathcounts.org or contact Sports Awards at 800-621-5803.
Select items are also available at www.artofproblemsolving.com.

## INTRODUCTION

The mission of MATHCOUNTS is to increase enthusiasm for and enhance achievement in middle school mathematics throughout the United States. Currently celebrating our $25^{\text {th }}$ anniversary, MATHCOUNTS has helped more than 7 million students develop their mathematical abilities by tackling MATHCOUNTS problems.

The MATHCOUNTS Foundation administers a nationwide math enrichment, coaching and competition program. Each year, the MATHCOUNTS School Handbook is created and distributed free of charge to middle schools across the country. Consisting of 300 creative math problems meeting National Council of Teachers of Mathematics (NCTM) standards for grades 6-8, this handbook (Volume I and II) provides the basis for teachers and volunteers to coach student Mathletes ${ }^{\circledR}$ on problem-solving and mathematical skills. Teachers are encouraged to make maximum use of MATHCOUNTS materials by incorporating them into their classrooms or by using them with extracurricular math clubs. Teachers also are encouraged to share this material with other teachers at their schools as well as with parents.

The coaching season begins at the start of the school year. The competition season starts in the winter when participating schools administer their school competitions and select up to eight students (i.e., one team of four and up to four additional individuals) to compete at local competitions in February. Winners progress to state competitions in March. The top four competitors and top coach for each state earn the privilege to represent their state at the Lockheed Martin MATHCOUNTS National Competition in May 2008.

The new MATHCOUNTS Club Program runs from the start of the school year through March. However, clubs are encouraged to continue meeting until the end of the school year.

## Recent Changes

The 2007-2008 MATHCOUNTS School Handbook is being produced in two volumes. Volume I contains $\mathbf{1 0 0}$ math problems and Volume II contains $\mathbf{2 0 0}$ math problems. As in the past, these 300 FREE challenging and creative problems are designed to meet NCTM standards for grades 6-8.

Volume I is being sent directly to every U.S. school with 7th- and/or 8th-grade students and anyone else who registered for the MATHCOUNTS competition last year. This volume is also available for schools with 6th-grade students. Volume II of the handbook will also be provided to schools free of charge. However, Volume II will be sent only to those who request it, sign up a Math Club or register for the MATHCOUNTS competition.

Please use the Request/Registration Form in the back of this handbook to request Volume II of the handbook, sign up for the MATHCOUNTS Club Program and/or register for the MATHCOUNTS competition. You may also download this form or complete it online at www.mathcounts.org.

## MATHCOUNTS Launches Club Program

MATHCOUNTS is pleased to launch the MATHCOUNTS Club Program to coincide with its $25^{\text {th }}$ anniversary. This new program may be used by schools as a stand-alone program or incorporated into the student preparation for the MATHCOUNTS competition.

The MATHCOUNTS Club Program provides schools with the structure and activities to hold regular meetings of a math club. Depending on the level of student and teacher involvement, a school may receive a recognition plaque or banner and be entered into a drawing for prizes.

The Grand Prize, in the drawing for those schools that reach the highest level of this program, is a $\$ 500$ gift card for the teacher to use for student recognition (awards/party) and an all-expenses paid trip for four students and the teacher to witness the Lockheed Martin MATHCOUNTS National Competition - 2008 in Denver (May 8-11).

Further details on this exciting new program and the FREE resources for those who participate is available on page 17.

## Competition Highlights

Eight Competitors per School ( $6^{\text {th }}$-, $7^{\text {th }}$ - and $8^{\text {th }}$-grade students are eligible to compete)

- Each school is limited to one team of up to four students.
- Up to four students are eligible to compete as individuals, in addition to or in lieu of a school team. Full details regarding participation appear in the "Eligible Participants" section on pages 9-11.


## Fee Structure

- The cost to register a school team is $\$ 80$, and the cost to register an individual competitor is $\$ 20$. Reduced fees of $\$ 40$ per team and $\$ 10$ per individual are available to schools entitled to Title I funds.
Details appear in the "Registration" section on page 9.


## Competition Structure

- Sprint Round: 30 problems (Calculators are not permitted.)
- Target Round: 8 multi-step problems (Calculators are permitted.)
- Team Round: 10 problems (Calculators are permitted, and team members work together.)
- Countdown Round: One-on-one oral competition for the top-scoring students. (Calculators are not permitted.) Optional at the local and state levels.
- Masters Round: Top few students spend 15 minutes presenting and defending their solution to a topic to a group of judges. Conducted at National Competition and optional at state level.


## MATHCOUNTS Curriculum

MATHCOUNTS questions are written with the curricula for grades 6-8 in mind. In addition, many problems are designed to challenge and accelerate student learning, and questions become progressively more difficult at each level of competition. Possible topics include:

- Algebra
- Charts, Graphs \& Tables
- Computation
- Consumer Math
- Equations \& Inequalities
- Equivalent Expressions
- Estimation \& Approximation
- Geometry
- Logic
- Measurement
- Number Theory
- Probability
- Statistics


## Where to Find More Information

Problem-Solving Strategies are explained on page 39. Examples of the strategies being applied to previously published MATHCOUNTS problems are available on pages 29-39 in Volume I of the handbook. Answers to all problems in this handbook include one-letter codes indicating possible, appropriate problem-solving strategies.

Vocabulary and Formulas are listed on pages 41-42 of Volume I of the handbook.
Problem Index: To assist you in incorporating the MATHCOUNTS School Handbook problems into your curriculum, a problem index is included on page 62.

MATHCOUNTS Web Site: A variety of additional information and resources are available on www.mathcounts.org, including problems and answers from the prior year's Chapter and State Competitions, the MATHCOUNTS Coaching Kit, Club Program resources, forums and links to state programs.

MATHCOUNTS Registration Database: To confirm your school's registration, check the registration database at www.mathcounts.org. Other questions about the status of your registration should be directed to: MATHCOUNTS Registration, P.O. Box 441, Annapolis Junction, MD 20701. Telephone: 301-498-6141.

MATHCOUNTS Coordinators: Questions specific to a local or state program should be addressed to the coordinator in your area. Local and state coordinator information is available at www.mathcounts.org.

## BUILDING A MATHCOUNTS PROGRAM

## Recruiting Mathletes ${ }^{\circledR}$

Ideally, the materials in this handbook will be incorporated into the regular classroom curriculum so that all students learn problem-solving techniques and develop critical thinking skills. When a school MATHCOUNTS program is limited to extracurricular sessions, all interested students should be invited to participate regardless of their academic standing. Because the greatest benefits of the MATHCOUNTS program are realized at the school level, the more Mathletes involved, the better. Students should view their experience with MATHCOUNTS as fun, as well as challenging, so let them know from the very first meeting that the goal is to have a good time while learning.

Some suggestions from successful coaches on how to stimulate interest at the beginning of the school year:

- Build a display case using MATHCOUNTS shirts and posters. Include trophies and photos from previous years' coaching sessions or competitions.
- Post intriguing math questions (involving specific school activities and situations) in hallways, the library and the cafeteria, and refer students to the first meeting for answers.
- Make a presentation at the first pep rally or student assembly.
- Approach students through other extracurricular clubs (e.g., science club, computer club, chess club).
- Inform parents of the benefits of MATHCOUNTS participation via the school newsletter or parent-teacher organization.
- Create a MATHCOUNTS display for "Back to School Night."
- Have former Mathletes speak to students about the rewards of the program.
- Incorporate the "Problem of the Week" from the MATHCOUNTS Web site (www.mathcounts.org) into the weekly class schedule.
- Organize a MATHCOUNTS Math Club.


## Coaching Students

For students to reap the full benefits of MATHCOUNTS (and be prepared to compete at the local competition in February), it is important to begin coaching early in the school year. The Warm-Ups, Workouts and Stretches in this handbook should carry a coaching program from October through January. To encourage participation by the greatest number of students, postpone selection of your school's competitors until just before the local competition.

On average, MATHCOUNTS coaches meet with Mathletes for an hour one or two times a week at the beginning of the year and with increasing frequency as the competitions approach. Sessions may be held before school, during lunch, after school or on weekends-whatever works best with your school's schedule and limits scheduling conflicts with other activities.

Some suggestions for getting the most out of the Warm-Ups and Workouts at coaching sessions:

- Encourage discussion of the problems so that students learn from one another.
- Encourage a variety of methods for solving problems.
- Have students write problems for each other.
- Use the MATHCOUNTS "Problem of the Week." Based on current events, this problem is posted every Monday on the MATHCOUNTS Web site at www.mathcounts.org.
- Practice working in groups to develop teamwork (and to prepare for the Team Round).
- Practice oral presentations to reinforce understanding (and to prepare for the Masters Round).
- Take advantage of additional MATHCOUNTS coaching materials, such as previous years’ competitions, to provide an extra challenge or to prepare for competition. (See the "Additional Coaching Materials" section on the next page for information on what materials are available and how to order.)
- Provide refreshments and vary the location of your meetings to create a relaxing, fun atmosphere.
- Invite the school principal to a session to offer words of support.


## Maintaining a Strong Program

Keep the school program strong by soliciting local support and focusing attention on the rewards of MATHCOUNTS. Publicize success stories. Let the rest of the student body see how much fun Mathletes have. Remember, the more this year's students get from the experience, the easier recruiting will be next year. Here are some suggestions:

- Publicize MATHCOUNTS events in the school newspaper and local media. Let individual Mathletes tell their success stories.
- Inform parents of events through the PTA, open houses and the school newsletter.
- Schedule a special pep rally for the Mathletes.
- Recognize the achievements of Mathletes at a school awards program.
- Have a students versus teachers Countdown Round and invite the student body to watch.
- Solicit donations from local businesses to be used as prizes in practice competitions.
- Plan retreats or field trips for the Mathletes to area college campuses or hold an annual reunion.
- Take photos at coaching sessions and competitions and keep a scrapbook.
- Distribute MATHCOUNTS shirts to participating students.
- Start a MATHCOUNTS summer school program.
- Encourage teachers of lower grades to participate in mathematics enrichment programs.
- Organize a MATHCOUNTS Math Club and hold regular meetings throughout the school year.


## Calling on Volunteers

Volunteer assistance can be used to enrich the program and expand it to more students. Fellow teachers can serve as assistant coaches. Individuals such as MATHCOUNTS alumni and high school students, parents, community professionals and retirees also can help.

MATHCOUNTS has partnered with VolunteerMatch to recruit volunteers to work with individual schools and/or help at the local or state competitions. We expect this will result in additional volunteer support of MATHCOUNTS and assistance for those who are working directly with students to increase enthusiasm for and enhance achievement in middle school mathematics.

MATHCOUNTS coordinators will be able to post volunteer opportunities at http://mathcounts. volunteermatch.org and serve as a conduit to link interested volunteers with opportunities at schools and/or competitions, among other things.

## Additional Coaching Materials

MATHCOUNTS maintains a variety of resources on its Web site at www.mathcounts.org, including:

- A current events-based "Problem of the Week," posted every Monday morning;
- The "Go Figure! Math Challenge," where students can work problems from previous handbooks and competitions at their own pace;
- Discussion forums for students and coaches;
- Various sections of this handbook, MATHCOUNTS News and school registration information, as well as other program details.

In addition to this handbook, MATHCOUNTS offers a variety of coaching products to stimulate interest in the program and to enhance the educational experience. Materials include the Club in a Box resource kit, current and past MATHCOUNTS School Handbooks, Warm-Ups and Workouts and previous years' competitions. A wide selection of MATHCOUNTS items (T-shirts, hats, calculators, etc.) is also available.


Coaching materials and novelty items may be ordered through Sports Awards. An order form, with information on the full range of products, is available in the store area of www.mathcounts.org or by calling Sports Awards toll-free at 800-621-5803. Interested in placing an online order? A limited selection of MATHCOUNTS materials is also available at www.artofproblemsolving.com.

## MATHCOUNTS COMPETITIONS

A grassroots network of more than 17,000 volunteers organizes MATHCOUNTS competitions nationwide. Each year 500-plus local competitions and 57 "state" competitions are conducted, primarily by chapter and state societies of the National Society of Professional Engineers. All 50 states, the District of Columbia, Puerto Rico, Guam, Virgin Islands, Northern Mariana Islands, and U.S. Department of Defense and U.S. State Department schools worldwide participate in MATHCOUNTS.

The following procedures and rules govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these procedures and rules at any time. Coaches are responsible for being familiar with the rules and procedures outlined in this handbook. Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

## Registration

To participate in MATHCOUNTS competitions, a school representative is required to complete and return the Request/Registration Form (available at the back of this handbook and on the Web at www.mathcounts.org) along with a check, money order, purchase order or credit card authorization to be postmarked no later than Dec. 7, 2007, to: MATHCOUNTS Registration, P.O. Box 441, Annapolis Junction, MD 20701. The team registration fee is $\$ 80$. The individual registration fee is $\$ 20$ per student. Reduced fees of $\$ 40$ per team and $\$ 10$ per individual are available to schools entitled to receive Title I funds. Registration fees are nonrefundable.

By completing the registration form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

Academic centers or enrichment programs that do not function as students' official school of record are not eligible to register.

Each registered school receives a School Competition Kit (with instructions, School Competition and Answer Key, recognition ribbons and student participation certificates), a catalog of additional coaching materials, MATHCOUNTS News and the opportunity to send students to the local competition.

Registration materials must be postmarked by Dec. 7, 2007. In some circumstances, late registrations may be accepted at the discretion of MATHCOUNTS and the local coordinator. The sooner you register, the sooner you will receive your school competition materials and can start preparing your team. The first mailing of School Competition Kits will be sent in early November, and additional mailings will occur on a rolling basis.

Once processed, confirmation of your registration will be available through the registration database on the MATHCOUNTS Web site (www.mathcounts.org). Your state or local coordinator will be notified of your registration, and you then will be informed of the date and location of your local competition. If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org.

## Eligible Participants

Students enrolled in the $6^{\text {th }}, 7^{\text {th }}$ or $8^{\text {th }}$ grade are eligible to participate in MATHCOUNTS competitions. Students taking middle school mathematics classes who are not full-time $6^{\text {th }}, 7^{\text {th }}$ or $8^{\text {th }}$ graders are not eligible. Participation in MATHCOUNTS competitions is limited to three years for each student though there is no limit to the number of years a student may participate in the school-based coaching phase.

SCHOOL REGISTRATION: A school may register one team of four and up to four individuals for a total of eight participants. You must designate team members versus individuals prior to the start of the local (chapter) competition (i.e., a student registered as an "individual" may not help his/her school team advance to the next level of competition).

## Team Registration: Only one team (of up to four students) per school is eligible to compete.

Members of a school team will participate in the Sprint, Target and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see "Scoring" on page 15 for details). Consequently, teams of fewer than four students will be at a disadvantage.

Individual Registration: Up to four students may be registered in addition to or in lieu of a school team. Students registered as individuals will participate in the Sprint and Target Rounds but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels.

SCHOOL DEFINITIONS: Academic centers or enrichment programs that do not function as students' official school of record are not eligible to register. If it is unclear whether an educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.

School Enrollment Status: A student may compete only for his/her official school of record. A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his/her math course. If there is any doubt about a student's school of record, the local or state coordinator must be contacted for a decision before registering.

Small Schools: Schools with eight or fewer students in each of the $6^{\text {th }}, 7^{\text {th }}$ and $8^{\text {th }}$ grades are permitted to combine to form a MATHCOUNTS team. Only schools from the same or adjacent chapters within a state may combine to form a team. The combined team will compete in the chapter where the coach's school is located.

Homeschools: Homeschools in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete an affidavit verifying that students from the homeschool are in the $6^{\text {th }}, 7^{\text {th }}$ or $8^{\text {th }}$ grade and that the homeschool complies with applicable state laws. Completed affidavits must be submitted to the local coordinator prior to competition.

Virtual Schools: Any virtual school interested in registering students must contact the MATHCOUNTS national office at 703-299-9006 before Dec. 7, 2007, for registration details.

Substitutions by Coaches: Coaches may not substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional restrictions on substitutions (such as requiring parental release or requiring the substitution request to be submitted in writing) are at the discretion of the state coordinator. Coaches may not make substitutions for students progressing to the state competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed. The student being added to the team need not be a student who was registered for the Chapter Competition as an individual.

Religious Observances: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that school must take the exam at the same time. Advance testing will be done at the discretion of the local and state coordinators and under proctored conditions. If the student who is unable to attend the competition due to a religious observance is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other teams. The coordinator must be made aware of the team's decision before the advance testing takes place. Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

Special Needs: Reasonable accommodations may be made to allow students with special needs to participate. Requests for accommodation of special needs must be directed to local or state coordinators in writing at least three weeks in advance of the local or state competition. This written request should thoroughly explain the student's special need as well as what the desired accommodation would entail. Many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to, granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

## Levels of Competition

MATHCOUNTS competitions are organized at four levels: school, chapter (local), state and national. Competitions are written for the $6^{\text {th }}$ - through $8^{\text {th }}$-grade audience. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

The real success of MATHCOUNTS is influenced by the coaching sessions at the school level. This component of the program involves the most students (more than 500,000 annually), comprises the longest period of time and demands the greatest involvement.

SCHOOL COMPETITION: In January, after several months of coaching, schools registered for the competition phase of the program should administer the School Competition to all interested students. The School Competition is intended to be an aid to the coach in determining competitors for the chapter (local) competition. Selection of team and individual competitors is entirely at the discretion of coaches and need not be based solely on School Competition scores. The School Competition is sent to the coach of a school, and may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For additional announcements or edits, please check the Coaches Forum on the MATHCOUNTS Web site before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the local competitions.

CHAPTER COMPETITIONS: Held between Feb. 1 and Feb. 24, 2008, the Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The chapter and state coordinators determine the date and administration of the local competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highestranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or Mathletes also may progress at the discretion of the state coordinator. The policy for progression must be consistent for all chapters within a state.

STATE COMPETITIONS: Held between March 1 and March 30, 2008, the State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) and the optional Masters Round may or may not be included. The state coordinator determines the date and administration of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

LOCKHEED MARTIN MATHCOUNTS NATIONAL COMPETITION: Held Friday, May 9, 2008, in Denver, the National Competition consists of the Sprint, Target, Team, Countdown and Masters Rounds. Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches.All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

## Competition Components

MATHCOUNTS competitions are designed to be completed in approximately three hours:
The SPRINT ROUND (40 minutes) consists of 30 problems. This round tests accuracy, with time being such that only the most capable students will complete all of the problems. Calculators are not permitted.

The TARGET ROUND (approximately 30 minutes) consists of eight problems presented to competitors in four pairs ( 6 minutes per pair). This round features multi-step problems that engage Mathletes in mathematical reasoning and problem-solving processes. Problems assume the use of calculators.

The TEAM ROUND ( 20 minutes) consists of 10 problems that team members work together to solve. Team member interaction is permitted and encouraged. Problems assume the use of calculators. Note: Coordinators may opt to allow those competing as "individuals" to create a "squad" of four to take the Team Round for the experience, but the round should not be scored and is not considered official.

The COUNTDOWN ROUND is a fast-paced, oral competition for top-scoring individuals (based on scores in the Sprint and Target Rounds). In this round, pairs of Mathletes compete against each other and the clock to solve problems. Calculators are not permitted.

At Chapter and State competitions, a Countdown Round may be conducted officially, unofficially (for fun) or omitted. However, the use of an official Countdown Round will be consistent for all chapters within a state. In other words, all chapters within a state must use the round officially in order for any chapter within a state to use it officially. All students, whether registered as part of a school team or as an individual competitor, are eligible to qualify for the Countdown Round.

An official Countdown Round is defined as one that determines an individual's final overall rank in the competition. If the Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed.

If a Countdown Round is conducted unofficially, the official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners on the sole basis of students' scores in the Sprint and Target Rounds of the competition.

In an official Countdown Round, the top $25 \%$ of students, up to a maximum of 10, are selected to compete. These students are chosen based on their individual scores. The two lowest-ranked students are paired, a question is projected and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if $\mathrm{s} /$ he answers correctly, a point is scored; if a student answers incorrectly, the other student has the remainder of the 45 seconds to answer. Three questions are read to each pair of students, one question at a time, and the student who scores the most points (not necessarily 2 out of 3 ) captures the place, progresses to the next round and challenges the next highest-ranked student. (If students are tied after three questions [at $1-1$ or $0-0$ ], questions continue to be read until one is successfully answered.) This procedure continues until the fourth-ranked Mathlete and her/his opponent compete. For the final four rounds, the first student to correctly answer three questions advances. The Countdown Round proceeds until a first-place individual is identified. (More detailed rules regarding the Countdown Round procedure are identified in the "Instructions" section of the School Competition booklet.) Note: Rules for the Countdown Round change for the National Competition.

The Masters Round is a special round for top individual scorers at the state and national levels. In this round, top individual scorers prepare an oral presentation on a specific topic to be presented to a panel of judges. The Masters Round is optional at the state level; if held, the state coordinator determines the number of Mathletes that participate. At the national level, four Mathletes participate. (Participation in the Masters Round is optional. A student declining to compete will not be penalized.)

Each student is given 30 minutes to prepare his/her presentation. Calculators may be used. The presentation will be 15 minutes - up to 11 minutes may be used for the student's oral response to the problem, and the remaining time may be used for questions by the judges. This competition values creativity and oral expression as well as mathematical accuracy. Judging of presentations is based on knowledge, presentation and the responses to judges' questions.

## Additional Rules

All answers must be legible.
Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper.

Use of notes or other reference materials (including dictionaries) is not permitted.
Specific instructions stated in a given problem take precedence over any general rule or procedure.
Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint or Countdown Rounds, but they are permitted in the Target, Team and Masters Rounds. Where calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (i.e., typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Personal digital assistants (e.g., Palm Pilots ${ }^{\circledR}$ ) are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring that their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators, AC outlets or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak/dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator's malfunctioning.

Pagers, cell phones, radios and MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

## Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm \frac{a}{b}$, where $a$ and $b$ are natural numbers and $\operatorname{GCF}(a, b)=1$. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where $A$ and $B$ are algebraic expressions and $A$ and $B$ do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N \frac{a}{b}$, where $N, a$ and $b$ are natural numbers, $a<b$ and $\operatorname{GCF}(a, b)=1$. Examples:
Problem: Express 8 divided by 12 as a common fraction. Answer: $\frac{2}{3}$ Unacceptable: $\frac{4}{6}$ Problem: Express 12 divided by 8 as a common fraction. Answer: $\frac{3}{2} \quad$ Unacceptable: $\frac{12}{8}, 1 \frac{1}{2}$
Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of $\pi$.
Problem: Express 20 divided by 12 as a mixed number. Answer: $1 \frac{2}{3} \quad$ Unacceptable: $1 \frac{8}{12}, \frac{5}{3}$
Ratios should be expressed as simplified common fractions unless otherwise specified. Examples: Simplified, Acceptable Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6} \quad$ Unacceptable: $3 \frac{1}{2}, \frac{1}{4}, 3.5,2: 1$
Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are not in radical form. Examples: Problem: Evaluate $\sqrt{15} \times \sqrt{5}$. Answer: $5 \sqrt{3}$ Unacceptable: $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) $a . b c$, where $a$ is an integer and $b$ and $c$ are digits. The only exceptions to this rule are when $a$ is zero, in which case it may be omitted, or when $b$ and $c$ are both zero, in which case they may both be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, $5 \quad$ Unacceptable: 4.9, 8.0
Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $\$ 0.25$ will not be accepted.
Do not make approximations for numbers (e.g., $\pi, \frac{2}{3}, 5 \sqrt{3}$ ) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^{n}$ where $a$ is a decimal, $1 \leq|a|<10$, and $n$ is an integer. Examples:
Problem: Write 6895 in scientific notation. Answer: $6.895 \times 10^{3}$
Problem: Write 40,000 in scientific notation. Answer: $4 \times 10^{4}$ or $4.0 \times 10^{4}$
An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25 nor vice versa.
The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

## Scoring

Scores on the competition do not conform to traditional grading scales. Coaches and students should view an individual written competition score of 23 (out of a possible 46) as highly commendable.

The individual score is the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and 8 questions in the Target Round, so the maximum possible individual score is $30+2(8)=46$.

The team score is calculated by dividing the sum of the team members' individual scores by 4 (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible individual score is 46 . Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible team score is 66 since $((46+46+46+46) \div 4)+2(10)=66$.

If used officially, the Countdown Round yields final individual standings. The Masters Round is a competition for the top-scoring individuals that yields a separate winner and has no impact on progression to the National Competition.

Ties will be broken as necessary to determine team and individual prizes and to determine which individuals qualify for the Countdown Round. For ties among individuals, the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Sprint and Target Rounds are compared. For ties among teams, the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared. Note: These are very general guidelines. Please refer to the "General Instructions" accompanying each competition set for detailed procedures should a tie occur.

In general, questions in the Sprint, Target and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. The comparison of questions to break ties generally occurs such that those who correctly answer the more difficult questions receive the higher rank.

Protests concerning the correctness of an answer on the written portion of the competition must be registered with the room supervisor in writing by a coach within 30 minutes of the end of each round. Rulings on protests are final and may not be appealed. Protests will not be accepted during the Countdown or Masters Rounds.

## Results Distribution

Coaches should expect to receive the scores of their students, anonymous rankings of all scores and a list of the top $25 \%$ of students and top $40 \%$ of teams from their coordinator. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. Coordinators must wait for verification from the national office that all such competitions have been completed before distributing blank competition materials and answer keys. Both the problems and answers from Chapter and State competitions will be posted on the MATHCOUNTS Web site following the completion of all competitions at that level nationwide (Chapter - early March; State - early April). The previous year's problems and answers will be taken off the Web site at that time.

Student competition papers and answers will not be viewed by nor distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of the MATHCOUNTS Foundation.

## TEACHER'S SYLLABUS

The 300 original problems found in Volumes I and II of the MATHCOUNTS School Handbook are divided into three sections: Warm-Ups, Workouts and Stretches. Each Warm-Up and Workout contains problems that generally survey the grades 6-8 mathematics curricula. Workouts assume the use of a calculator; Warm-Ups do not. The Stretches are collections of problems centered around a specific topic.

The problems are designed to provide Mathletes with a large variety of challenges and prepare them for the MATHCOUNTS competition. These materials may be used as the basis for an exciting extracurricular mathematics club or may simply supplement the normal middle school mathematics curriculum.

Answers to all problems include one-letter codes indicating possible, appropriate problem-solving strategies. These strategies are explained on page 39. Examples of the strategies being applied to previously published MATHCOUNTS problems are available on pages 29-39 in Volume I of the handbook.

## WARM-UPS AND WORKOUTS

The Warm-Ups and Workouts are on pages 19-36 and are designed to increase in difficulty as students go through the handbook.
For use in the classroom, the problems in the Warm-Ups and Workouts serve as excellent additional practice for the mathematics that students are already learning. In preparation for competition, the Warm-Ups can be used to prepare students for problems they will encounter in the Sprint Round. It is assumed students will not be using calculators for Warm-Up problems. The Workouts can be used to prepare students for the Target and Team Rounds of competition. It is assumed students will be using calculators for Workout problems. Along with discussion and review of the solutions, it is recommended that Mathletes be provided with opportunities to present solutions to problems as preparation for the Masters Round.

All of the problems provide students with practice in a variety of problem-solving situations and may be used to diagnose skill levels, to practice and apply skills, or to evaluate growth in skills.

## STRETCHES

Page 37-38 contain the Number Theory and Patterns Stretches. The included problems cover a variety of difficulty levels. These exercises may be incorporated at any time.

## ANSWERS

Answers to all problems can be found on pages 41-46.

## SOLUTIONS

Complete solutions for the problems start on page 47 . These are only possible solutions. It is very possible you and/or your students will come up with more elegant solutions.

## SCHEDULE

The Stretch can be incorporated at any time. The following chart is the recommended schedule for using the Warm-Ups and Workouts (Volumes I and II of the handbook are required to complete this schedule.):

| October | Warm-Ups 1-4 | Workouts 1-2 |
| ---: | :--- | :--- |
| November | Warm-Ups 5-8 | Workouts 3-4 |
| December | Warm-Ups 9-12 | Workouts 5-6 |
| January | Warm-Ups 13-16 | Workouts 7-8 |
|  | School Level MATHCOUNTS Competition |  |
|  | Warm-Ups 17-18 |  |$\quad$ Workout 9

# MATHCOUNTS CLUB PROGRAM (NEW) 

MATHCOUNTS is pleased to launch the MATHCOUNTS Club Program to coincide with its $25^{\text {th }}$ anniversary. This new program may be used by schools as a stand alone program or incorporated into the student preparation for the MATHCOUNTS competition.

## What is it?

The MATHCOUNTS Club Program provides schools with the structure and activities to hold regular meetings of a math club. Depending on the level of student and teacher involvement, a school may receive a recognition plaque or banner and be entered into a drawing for prizes.

The 2007-2008 school year marks the debut of the Club Program.

## What are the different levels of the program?

| Level | Requirement | School Receives |
| :--- | :--- | :--- |
| Bronze | Sign up a math club with the <br> Request/Reg. Form (page 63) | - Club in a Box resource kit <br> - Volume II of the MATHCOUNTS School Handbook (200 problems) |
| Silver | 12 members of the math club <br> must take 5 of 6 monthly math <br> challenges (Sept. - Feb.) | - Plaque identifying school as a Silver Level MATHCOUNTS school <br> - Entry into drawing for one of ten \$250 gift cards for student <br> recognition (awards/party) |
| Gold | Silver Level requirements and <br> 12 members of the math club <br> must score an 80\% or better on <br> the Ultimate Math Challenge <br> (available in Feb.) | - Banner identifying school as a Gold Level MATHCOUNTS school <br> - Entry into drawing for: <br> 1) One of five \$500 gift cards for student recognition (awards/party) <br> 2) Grand Prize: \$500 gift card for student recognition (awards/party) <br> and trip for four students and teacher to witness the Lockheed <br> Martin MATHCOUNTS National Competition in Denver (May <br> $8-11)$ |
|  |  | 8- |

## How do schools get involved?

Simply sign up your math club with MATHCOUNTS. Shortly afterwards, we will send the Club in a Box resource kit and Volume II of the MATHCOUNTS School Handbook to the school. The Request/ Registration Form is available in the back of this handbook and at www.mathcounts.org.

## What resources do participating schools receive?

Schools participating in the Club Program will receive the Club in a Box resource kit. Among other items, this includes further details on the Club Program, the Club Resource Guide which outlines structured club activities, the first monthly math challenge, a poster and hands-on activity, 12 MATHCOUNTS pencils and a MATHCOUNTS lapel pin for the teacher. Additionally, five other monthly math challenges and an Ultimate Math Challenge will be made available online for use by math club students.

Depending on the level of student and teacher involvement, a school may receive a recognition plaque or banner and be entered into a drawing for prizes.

## Who is eligible to participate?

Anyone eligible for the MATHCOUNTS competition is eligible to participate in the Club Program. (The Club Program is open to all U.S. schools with 6th-, 7th- and/or 8th-grade students. Schools with 12 or fewer students in each of the 6th, 7th and 8th grades are permitted to combine for the purpose of reaching the Silver or Gold levels. Similarly, homeschools may combine for the purpose of reaching the Silver or Gold levels. See page 9 for details on eligibility for the MATHCOUNTS competition.)

## How many students can participate?

There is no limit to the number of students who may participate in the Club Program. Encourage every interested 6th-, 7th- and/or 8th-grade student to get involved.

## What does it cost to participate?

NOTHING. There is no fee to participate in the Club Program. Similar to the MATHCOUNTS School Handbook, the Club in a Box and other resources are free for all eligible schools that request them.

## Can a school participate in the Club Program and the MATHCOUNTS competition?

YES. A school may choose to participate in the Club Program, the competition or both programs. Since these programs can complement each other, any school that registers for the MATHCOUNTS competition will automatically be signed up for the Club Program and sent the Club in a Box resource kit.

## How is the Club Program different from the MATHCOUNTS competition?

The Club Program does not include a school versus school competition with the opportunity for top performers to advance. There are no fees to participate in the Club Program, and recognition is focused entirely on the school and math club.

## Why did MATHCOUNTS create the Club Program?

For one reason or another, some schools do not want students to participate in a math competition with other schools. Similarly, some students don't enjoy matching their mathematical abilities against students from other schools. The Club Program is designed specifically for these students and schools. Even so, many schools that participate in the MATHCOUNTS competition may find the Club Program a wonderful additional resource.

The Club Program encourages group activities and collaborative learning to develop students’ mathematical abilities. It also encourages those schools that might have overlooked MATHCOUNTS because of the competition program to use the free math problems in the annual MATHCOUNTS School Handbook.

## Warm-Up 7

1. minutes
2. $\qquad$ points

Rob's three assignment scores are 79,80 and 84 points. What is the arithmetic mean of these three scores?
3. sq units

What is the area of regular hexagon $A B C D E F$ if $A B=2$ units? Express your answer in simplest radical form.
4. $\qquad$ Anderson will roll two standard six-sided dice once. What is the probability that the two numbers rolled will be the same? Express your answer as a common fraction.
5. $\qquad$ Jerry worked for one day on a project that he could have completed alone in nine days. Bill joined Jerry the next day, and they worked together for exactly three days to complete the project. How much of the job did Bill do in those three days? Express your answer as a common fraction.
6. $\qquad$ What is the ones digit when $9^{55}$ is expressed as an integer?
7. $\qquad$ In the sequence of equilateral figures shown, the middle third of each segment is replaced with two segments that are each the same length as the replaced piece. Each side of the first figure (the triangle) is 27 units.
 What is the perimeter of the third figure in the sequence?
8. pennies

Sammy felt very generous. He gave Becky half of his pennies. Then an hour later, Sammy gave Janie one-fourth of his remaining pennies. Shortly after, Mary Beth borrowed half the pennies that Sammy had left. Sammy then had 12 pennies. How many pennies did Sammy give to Becky?

9. grams

A certain material's density is $2.5 \mathrm{~g} / \mathrm{cm}^{3}$. Given that density $=$ mass $\div$ volume, what is the mass, in grams, of $300 \mathrm{~cm}^{3}$ of this material?
10. $\$$

Alex parked his car at the Leggett Airport on Monday at 7 am. He removed his car from the parking lot on Friday of the same week at 10 am. The rates at Leggett are shown on this sign. How much did Alex pay for his parking during this time period?

## Parking

$\$ 1$ per hour
$\$ 7$ maximum per 24-hour period

## Warm-Up 8

1. $\qquad$ The faces of a cubical die are each labeled with a different prime number, and each of the six smallest prime numbers ( $2,3,5,7,11,13$ ) is on exactly one face of the die. The die will be rolled twice. What is the probability that the product of the two numbers rolled will be even? Express your answer as a common fraction.
2. sq units

What is the area of the triangle with vertices at $(0,2),(3,2)$ and $(3,0)$ on the coordinate plane?
3. $\qquad$ Gary can select any positive two-digit integer between 23 and 98 and write it as " $A B$ " with tens digit $A$ and ones digit $B$. When he subtracts the sum $A+B$ from his integer, the difference will be a new two-digit integer, "JK." What is the value of $\mathrm{J}+\mathrm{K}$ ?
4. runners

During the Great Falls Marathon, $50 \%$ of the runners dropped out before reaching the first water station. By the second water station, $\frac{1}{3}$ of the remaining runners dropped out. At the third water station, $25 \%$ of the remaining runners dropped out. By the fourth water station, $90 \%$ of the remaining runners dropped out. If the remaining six runners finished the marathon, how many runners started the race?

5. $\qquad$ If $\sqrt[n]{96}=2 \sqrt[n]{3}$, what is the value of $n$ ?
6. $\qquad$ Jo's circular wheel needs to complete eight full rotations, or $8 \times 360=2880^{\circ}$, to roll 12 meters. How many degrees must the wheel rotate to roll 0.5 meters?
7. sq inches

The area of this sign in the shape of a regular hexagon is $96 \sqrt{3}$ square inches. What is the total area of the two shaded regions? Express your answer in simplest radical form.

8. $\qquad$ Two sides of a particular isosceles triangle are 6 and 13 units. What is the area of this triangle? Express your answer in simplest radical form.
9. feet

A rectangular garden is 10 feet by 4 feet. A gravel border with uniform width along the sides and $90^{\circ}$ corners surrounds the garden, as shown. The area of the gravel border is six times the area of the garden. What is the perimeter of the outside of the gravel border?

10. degrees

In the figure shown, segments $A B$ and $C D$ are parallel. What is the sum of the measures of angles BAE, AEC and ECD?


## Workout 4

$\qquad$ A car that is traveling 90 feet per second is traveling how many miles per hour? There are 5280 feet in 1 mile. Express your answer to the nearest whole number.
2. $\qquad$ The product $(66)(9)(22)(39)$ has a prime factorization of the form $\left(2^{a}\right)\left(3^{b}\right)\left(11^{c}\right)\left(13^{d}\right)$. What is the value of $a c-b d$ ?
3. inches

Tony the Tortoise walked 10 inches during the first hour of his journey. He walked one and one-half times that distance during the second hour, and in general during the $n^{\text {th }}$ hour he walked $\left(1+\frac{1}{n}\right)$ times the distance walked during the $n-1^{\text {st }}$ hour. How many inches did Tony walk during the first three hours of his journey?
4. \$

Big Town Auto totaled \$437,184 in revenue in August. August's revenue was $8 \%$ greater than July's revenue. July's revenue was $12 \%$ below the revenue for June. What was the revenue for June?
5. $\qquad$ Let $B(n)$ denote the sum of the digits of the binary (base 2) representation of $n$. Let $T(n)$ denote the sum of the digits of the ternary (base 3) representation of n. For example, $B(9)=B\left(1001_{2}\right)=2$ and $T(9)=T\left(100_{3}\right)=1$. What is the smallest positive integer $n$ greater than 1 such that $\mathrm{B}(n)=\mathrm{T}(n)$ ?
6. items

Jordan buys pens for $\$ 3$ each and books for $\$ 5$ each, totaling exactly $\$ 2008$. What is the largest number of items (pens plus books) he could have bought?
7. blades

Terrell wondered how many blades of grass were in his 60-foot by 90-foot rectangular backyard. He picked a square region three inches on a side, which contained 520 blades of grass. If the grass was uniformly distributed throughout the backyard, how many blades were in the entire backyard?
8. inches


Each circular pulley shown has a radius of 12 inches. The shortest distance between the pulleys is 20 inches. What is the length of the continuous belt that loops around both pulleys? Express your answer to the nearest whole number.
9. meters

Two ladders, both 6 meters in length, are leaned up against opposite vertical walls in a 3-meter-wide corridor, as shown. How far above the ground do the two ladders cross? Express your answer as a decimal to the nearest tenth.

10. questions

To pass a 30-question test, Johnny needs to answer at least $60 \%$ of the questions correctly. When Johnny received his graded test back, he saw that he needed to have answered exactly two more questions correctly to have passed the test. How many questions did he answer correctly?

## Warm-Up 9

1. 

combos A jar contains 100 red marbles, 100 blue marbles and 100 white marbles. All 300 marbles are the same size. How many distinct color combinations are possible when three marbles are selected from the jar? The order in which the marbles are selected does not matter.
2. degrees

How many degrees are there in the smallest angle between the two hands of a clock at 2:30?

3. $\qquad$ A positive 16-digit integer is such that any two consecutive digits form a multiple of either 19 or 31. If the digit 2 appears only once, what is the sum of the 16 digits?
4. $\qquad$


Two non-congruent circles are externally tangent to each other. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 9 units, what is the area of the trapezoid?
5. $\qquad$ To create a unique house paint color, Melton mixes together a sample that is 12 gallons of red, 2.5 gallons of yellow and 0.5 gallons of blue paint. He then mixes a main batch of paint using 30 gallons of yellow paint and enough red and blue paint so as to maintain the original ratio. How many total gallons of paint did he use when making the sample and the main batch?

6. $\qquad$ In a math class, each student's final grade is the average of the scores on $n$ tests. If Alfred makes a 97 on the last test, his grade will be exactly 90 . If he makes a 73 on the last test, his grade will be exactly 87 . What is the value of $n$ ?
7. units The dimensions of two boxes are $a$ by $b$ by $c$ and $d$ by $e$ by $f$, respectively. If $a<c, d<b, e<a, b<e$ and $a<f$, what is the diameter of the largest ball that can fit into both boxes? Express your answer in terms of $a, b, c, d, e$ and/or $f$.
8. $\qquad$ The original price of an item is reduced by $20 \%$. This reduced price is then lowered by $10 \%$, and finally this newest price is reduced by $50 \%$ to get a final selling price. What fraction of the original price is the final selling price? Express your answer as a common fraction.
9. $\qquad$ In an arithmetic sequence, the sum of the $8^{\text {th }}$ and $9^{\text {th }}$ terms is 40 , and the sum of the $9^{\text {th }}$ and $10^{\text {th }}$ terms is 48 . What is the positive difference between the $8^{\text {th }}$ and $9^{\text {th }}$ terms in this sequence?
10. $\qquad$ ) The graphs of $y=2 x^{3}$ and $y=3 x^{2}$ intersect at $(0,0)$ and at what other point? Express your answer as an ordered pair of common-fraction coordinates.

## Warm-Up 10

1. $\qquad$ What is the value of $k$ in the equation $\left(21 \times 2^{4}\right) \times 5!=k!?$
2. $\qquad$ A rectangular block has faces with areas of $48 \mathrm{in}^{2}, 72 \mathrm{in}^{2}$ and $96 \mathrm{in}^{2}$. What is the volume of the block?
3. $\qquad$ An $8.5^{\prime \prime}$ by $11^{\prime \prime}$ piece of paper is cut entirely into $0.5^{\prime \prime}$ by $11^{\prime \prime}$ strips. What is the ratio of the total perimeter of all the strips of paper to the perimeter of the original paper? Express your answer as a common fraction.
4. \$
5. pounds

A math teacher explains that he weighs 100 pounds more than half his weight. How much does he weigh?
The cost at Modern Attire for a shirt and a tie is $\$ 60$. The cost for a tie and a pair of pants is $\$ 66$. The cost for a pair of pants and a sweatshirt is $\$ 72$. The cost for a sweatshirt and a jacket is $\$ 100$. The cost for a shirt and a jacket is $\$ 82$. What is the cost of one sweatshirt?
6. $\qquad$ For years Carl has kept a record of the number of deer he has spotted in his backyard. If the percent decrease is the same from 1998 to 2008 as it has been for the previous two 10 -year spans, as shown, how many deer can he anticipate seeing in 2008?

| Year | \# of Deer |
| :---: | :---: |
| 1978 | 1000 |
| 1988 | 900 |
| 1998 | 810 |
| 2008 | $?$ |

7. sq units


What is the area enclosed on the coordinate plane by the graph of the equation $|x|+|4 y|=20$ ?
8. $\qquad$ What is the $15^{\text {th }}$ term in the arithmetic sequence $7,10,13, \ldots$ ?
9. $\qquad$ Use each of the digits $3,4,6,8$ and 9 exactly once to create the greatest possible multiple of 6 . What is that multiple of 6 ?
10. $\qquad$ A tournament begins with 61 people. When two people play a game, the loser is eliminated from the tournament. Eventually, only one person remains as the tournament winner. How many games have been played when the tournament winner is determined?


## Workout 5

1. gallons

A tank contains 10,000 gallons of water at the beginning of the day on June 1. Each day, $1 \%$ of the water in the tank at the beginning of the day is lost to evaporation. How much water is left at the end of the last day of June? Express your answer to the nearest whole number.
2. sq units

An acute angle of a right triangle is $30^{\circ}$, and the hypotenuse is 40 units. What is the area of the triangle? Express your answer as a decimal to the nearest tenth.
3. $\qquad$ \% Larry bought two coats at a thrift store and then sold the coats a week later for $\$ 135$ each. He made a profit of $25 \%$ on the first coat but lost $25 \%$ on the second coat. What total percent did he lose on the sale of the two coats? Express your answer to the nearest hundredth.

4. feet

A Super-Duper bouncy ball is dropped straight down from a height of 80 feet. Each time the ball hits the ground it bounces straight back up $\frac{3}{4}$ of the height from which it just fell. How many total feet had the ball traveled when it hit the ground the third time?
5. ${ }^{\circ} \mathrm{F}$

A method for estimating the conversion between ${ }^{\circ} \mathrm{Celsius}$ and ${ }^{\circ}$ Fahrenheit is to double the Celsius temperature and then add $30^{\circ}$ to get the Fahrenheit temperature. The exact formula for the conversion is $F=\frac{9}{5} C+32$. If the Celsius temperature is $10^{\circ}$, what is the difference between the estimated Fahrenheit temperature and the actual Fahrenheit temperature?
6.

An integer is pseudoperfect if it is the sum of two or more of its positive divisors. (A divisor may be used only once in the sum.) For instance, 20 is pseudoperfect because its divisors $1,4,5$ and 10 have a sum of 20 . What is the sum of all the pseudoperfect integers between 50 and 60?
7. $\qquad$ The numbers 1,2,3 and 4 are placed in any order about a circle. At each turn of a game, a new circle is formed with four new entries, each of which is the square of the difference of each pair of adjacent numbers. Each new entry is placed
 between the two numbers from which it was calculated, and the old numbers are erased. What is the largest possible number ever to appear when playing the game if any initial ordering of the numbers $1,2,3$ and 4 may be used?
8. $\quad$ sq ft A 20-foot-high rectangular room has a floor that measures $18^{\prime}$ by $15^{\prime}$. Its doorway measures $3^{\prime}$ by $12^{\prime}$, and its only window measures $7^{\prime}$ by $10^{\prime}$. How many square feet of wall space does the room have?
9. hours

A boat has a speed of 6 mph in still water. The boat can travel 30 miles with the current in the same time in which it can travel 18 miles against the current. How many hours are necessary for the boat to travel 36 miles against the current?
10. $\qquad$ A bird collection has exactly four types of birds (eagles, doves, crows and sparrows). The eagles and doves make up $60 \%$ of the collection, and the doves and crows make up $20 \%$ of the collection. If the 18 crows in the collection represent $5 \%$ of the total number of birds, how many of the birds are sparrows?

## Warm-Up 11

1. $\qquad$ sq in

A square and a circle overlap such that a vertex of the square is at the center of the circle. The 4-inch radius of the circle is one-half the length of a side of the square. What is the area of the portion of the square region that is outside the circular region? Express your answer in terms of $\pi$.
2. $\qquad$ What is the remainder when $2008^{2007}$ is divided by 5 ?
3. triangles

How many non-congruent triangles, each with a perimeter of 15 units, can be constructed with all integral side lengths?
4. $\qquad$ This container is made from a right circular cylinder and a hemisphere of the same radius on top. The volume of the entire container is $108 \pi$ cubic meters. The base of the container has a radius of 3 meters. What is the height of the container from the bottom base to the top of the hemisphere?


Triangle $A B C$ has its vertices located on square $A X Y Z$, as shown. If $X B=7, B Y=3$ and $Y C=2$, what is the area of triangle $A B C$ ?
6. $\qquad$ Gary will roll two standard six-sided dice once. What is the probability that the difference between the two numbers rolled will be a multiple of 2? Express your answer as a common fraction.
$\qquad$ The point $(8,10)$ is the same distance from the point $(0, y)$ as it is from the $x$-axis. What is the greatest possible value of $y$ ?
$\qquad$ Twenty people each competed in a scavenger hunt. Twenty percent of the people each found $80 \%$ of the items, and $80 \%$ of the people each found $20 \%$ of the items. Those who found the most items found a dozen items each. How many items were on the list?
9. $\qquad$ A set of nine positive integers has a median of 5 , a mean of 6 and a unique mode of 8. What is the largest integer that can be a member of the set?
10. $\qquad$ My piggy bank has only nickels, dimes and dollar bills. The ratio of nickels to dimes is $2: 3$, and the ratio of dimes to dollar bills is 10:1. What is the ratio of coins to dollar bills? Express your answer in the form $a: b$, where $a$ and $b$ are positive integers with no common factors greater than 1.


## Warm-Up 12

1. $\qquad$ \% A study of 100 boys and 100 girls found that $60 \%$ of girls and $20 \%$ of boys enjoy the game Quirk. What percent of the children in the study who enjoy Quirk are girls?
2. degrees The measures of the four interior angles of a convex quadrilateral are $4 x, 3 x+20$, $2 x+40$ and $x+80$ degrees. What is the measure of the smallest interior angle of the quadrilateral?
3. sq units In the figure, quadrilateral $A B C D$ is a rectangle with integer side lengths. The areas of three smaller rectangles are given, in square units. What is the area of rectangle $A B C D$ ?

4. $\quad$ sq units

A rhombus of side length 5 units has a short diagonal of length 6 units. What is the area of the rhombus?
5. $\qquad$


A rectangular piece of paper is rolled, with no overlap, into the curved surface of a cylinder. The cylinder's volume is $1872 \pi$ cubic inches. The paper is then rolled tighter so that half the circumference of the new cylinder is overlapped paper, and the cylinder has the same height. What is the volume of the new cylinder? Express your answer in terms of $\pi$.
6. $\qquad$ If $x+\frac{1}{x}=3$, what is the value of $x^{4}+\frac{1}{x^{4}}$ ?
7. $\qquad$ The function $f(x)=3 x^{2}-6 x-11$ is graphed on a coordinate plane. What is the smallest $y$-coordinate of any points of the function?
8. $\qquad$ The mean of three numbers is 6 more than the least of the numbers, and it is 7 less than the greatest number. The median of the three numbers is 8 . What is the sum of the three numbers?
9. $\qquad$ Two liters of an alcohol/water mixture is $25 \%$ alcohol. How many liters of pure water should be added to make a $10 \%$ alcohol solution?
10. $\qquad$ In Figure 1, $A B C D$ is a rectangular piece of paper. Point $B$ is folded over onto its new location on side AD (see Figure 2), and the paper is creased from $Q$ to $C$. In Figure 2, $A B=8$ units, and the area of triangle QBA is 24 square units. What is the perimeter of rectangle $A B C D$ in Figure 1?

Figure 1



## Workout 6

1. $\qquad$ The sides of a quadrilateral have lengths $x, x+1, x+2$ and $x+3$ units. The value of the perimeter is less than 100 units and is a perfect cube. The mean of the side lengths is twice the square root of the perimeter. What is the value of $x$ ? Express your answer as a decimal to the nearest tenth.
2. 

The sum of $a$ and 4, the difference of $b$ and 4, the product of $c$ and 4 and the quotient of $d$ and 4 are all equal to the same integer value. If the sum of $a, b, c$ and $d$ is 100 , what is the ratio of $a$ to $d$ ? Express your answer as a common fraction.
3. times

Kelly's heart beats 60 times per minute when she is sleeping. If she sleeps 7 hours per day for all of 2009 and 2010, how many times will her heart beat while sleeping for those two years? Express your answer in scientific notation with five significant digits.
4. feet

A farmer's field is in the shape of regular hexagon $A B C D E F$. The distance from point $A$ to point $B$ is 420 feet. A fence post is placed at each vertex of the hexagon, and each side has 16 evenly spaced fence posts (counting the posts at the vertices). What is the distance from the center of one fence post to the center of an adjacent fence post on a side of the hexagonal field?
5. $\quad \mathrm{sq} \mathrm{cm}$

A circle with a radius of 2.5 cm is inscribed in a square. What is the area within the square region but outside the circular region? Express your answer as a decimal to the nearest tenth.
6. $\qquad$ According to this data, what is the positive difference between the mean number of yards for the top five rushers and the mean number of yards for the top five passers? Express your answer as a decimal to the nearest tenth.
$\qquad$ \% A square is inscribed in a circle that forms a dartboard. If a dart randomly hits the dartboard, what is the probability it will hit the region outside the square but inside the circle? Express your answer as a percent to the nearest tenth.
8. \$ $\qquad$ The 460 students and 20 staff at Bosney School are planning a bus trip. The bus company has large buses that can hold up to 64 passengers and small buses that can hold up to 40 passengers. The large buses cost $\$ 960$ each and the small buses cost $\$ 680$ each. What is the cost per passenger for the cheapest combination of buses?

cm
What is the total perimeter of a sector of a circle with a radius of 3 cm and a central angle measuring $90^{\circ}$ ? Express your answer as a decimal to the nearest tenth.

10. $\qquad$ If $\sqrt{5+\sqrt{1+x}}=\sqrt{2}+\sqrt{3}$, what is the value of $x$ ?

## Warm-Up 13

1. students

A teacher usually divides his class into six groups of $n$ students each. However, on Monday, three of the students were absent, so the teacher divided the remaining students into seven groups of $m$ students each. On Tuesday, four students were absent, so he went back to $n$ students per group, but there was one fewer group than he usually has. How many students are in the class?
2. sq units

There are two squares placed on sides of a right triangle, as shown to the right. The area of square $B$ is 100 square units, and the area of square $C$ is 64 square units. What is the area of right triangle $A$ ?

3. sq units


In rectangle $A B C D$, point $E$ lies on segment $B C$ such that $B E=\frac{1}{3} B C$. If $A B=8$ units and $D A=12$ units, what is the area of triangle $E C D$ ?
4. $\qquad$ What is the area of the region enclosed by the graphs of the lines $y=-2 x-3$, $y=2 x-3$ and the $x$-axis? Express your answer as a decimal to the nearest tenth.
5. $\qquad$ A math field day's budget consists of $\$ 432$ to pay problem-writers and $\$ \times$ per Mathlete for food, drinks, copying costs and trophies. A math field day for 100 Mathletes costs exactly half what a math field day for 248 Mathletes costs. What is the value of $x$ ?
6. $\qquad$ The radius of a particular circle inscribed in an equilateral triangle is 2 units. What is the perimeter of the triangle? Express your answer in simplest radical form.
7. units Triangle $A B C$ has a perimeter of 2007 units, and the sides have lengths that are all integers with $A B \leq B C \leq A C$. What is the positive difference between the largest possible length of segment $A B$ and the smallest possible length of segment $A B$ ?
8. $\qquad$ What is the probability that in a group of three friends no two of them were born on the same day of the week? Express your answer as a common fraction.
9. students

Ten percent of the male students at James HS are over six feet tall. The number of female students who are taller than six feet is equal to $10 \%$ of the number of male students over six feet tall. At James HS there are 250 female students and $20 \%$ more male students than female students. How many female students are taller than six feet?

10. $\quad$ values

A sequence of positive integers is formed by first selecting any positive two-digit integer as the first term of the sequence. Each term after the first term is the sum of twice the tens digit and twice the ones digit of the previous term. If the second term of the sequence is 16 and the third term is 14 , how many values are possible for the first term?

## Warm-Up 14

1. $\qquad$ The symbol * represents a sequence of mathematical operations. If 12 * $6=35$, $4 * 2=3,11 * 14=8,5 * 7=3$ and $10 * 7=8$, what is the value of 8 * 4 ?
2. $\qquad$ What is the value of $x$ in the equation $3^{12}+3^{12}+3^{12}=3^{x}$ ?
3. $\qquad$ If the graph of $f(x)=2 x^{2}+b x-3$ is symmetric about the line $x=3$, what is the value of $b$ ?
$\qquad$ Brenda gave Gail as many pieces of candy as Gail already had. Then Gail gave Brenda as many pieces of candy as Brenda needed to double her amount of candy. Now Brenda has three times as many pieces of candy as Gail. If Gail has at least one piece of candy, what is the minimum total number of pieces of candy needed for this to have occurred?
$\qquad$ Point $A$ is located at $(5,5)$ on the Cartesian plane. Point $B$ is also in the plane and has integer coordinates. If $0<A B \leq 4$, at how many points could $B$ be located?
4. sq units

A diagonal is drawn in rectangle ACEG, as shown. Segments $B F$ and $H D$ are drawn parallel to sides $C E$ and $G E$, respectively, with segments $A E, B F$ and $H D$ intersecting at point I. The area of rectangle BCDI is 36 square units. What is the area of rectangle FGHI?

7. sq units What is the area of a triangle with vertices at $(-5,-1),(3,5)$ and $(1,9)$ on the coordinate plane?
8. degrees

In the figure, the circles with centers $C$ and $H$ are tangent to line JK at points $J$ and $K$, respectively, and are externally tangent at point B. Points $\mathrm{T}, \mathrm{C}, \mathrm{B}$ and H are collinear. The measure of angle KHB is $110^{\circ}$. What is the measure of angle CJT?

9. $\qquad$ Two positive integers are relatively prime if 1 is their only common factor. How many sets of two relatively prime integers are there for which both integers in the set are greater than or equal to 2 and less than or equal to 9 ?
10. $\qquad$ In a study of the effects of time pressure on Mathletes, one-third of the Mathletes were given buzzers and the rest were not. All of the Mathletes answered a particular question. One-third of those with buzzers answered the question correctly, while one-fourth of those answering the question correctly had buzzers. What fraction of the students without buzzers answered the question correctly? Express your answer as a common fraction.

## Workout 7

1. $\qquad$ What is the value of $(1+2+3)+(2+3+4)+(3+4+5)+\ldots+(38+39+40)+$ $(39+40+41)$ ?
2. $\qquad$ If the sum of the interior angles of a particular convex polygon is $9720^{\circ}$, how many sides does the polygon have?
3. \$ $\qquad$ The 442 students and 40 staff of Beckwith School are planning a trip. Each person can go by bus or train. Using the information below, what is the least possible average cost per person for them to make this trip?

| Trailmore Bus Lines | Arca Trains |
| :--- | :--- |
| Large Bus: $\$ 1025$; holds up to 64 passengers | $\$ 20$ per person for first 100 passengers |
| Medium Bus: $\$ 792$; holds up to 36 passengers | $\$ 18$ per person for next 300 passengers |
|  | $\$ 16$ per person for all passengers over 400 |

4. $\qquad$ When a survey of 800 people was done, $55 \%$ of those surveyed said they ate candy, $65 \%$ said they ate chips and $90 \%$ said they ate hotdogs. What is the smallest possible number of people who said they ate candy, chips and hotdogs?
5. $\qquad$ A right circular cone has a base circumference of $24 \pi \mathrm{~cm}$ and a volume of $1512 \pi \mathrm{~cm}^{3}$. The cone is cut parallel to its base, and the newly formed shorter cone has a volume of $56 \pi \mathrm{~cm}^{3}$. What is the height of the shorter cone? Express your answer as a decimal to the nearest tenth.
6. sq feet

The front view of a metal sculpture is the region $A B C$ shown, with $A B=B C=22$ feet and segment $A B$ perpendicular to segment $B C$. Points $A$ and $C$ are the endpoints of a quarter-circle. What is the area of region $A B C$ ? Express your answer to the nearest whole number.

7. $\qquad$ Leonard wishes to purchase an air purifier for a small building. He needs to determine the volume of air inside the building in order to decide which model to purchase. The building is a triangularbased right prism on top of a right rectangular prism, with measurements as shown. How many cubic feet of air are in the entire building?

8. $\qquad$ What is the sum of the 13 smallest positive palindromes that have a tens digit of 3 and a ones digit of 7 ?
9. $\qquad$ Six pipes each having a radius of 0.5 feet are stacked in a triangular pile with three pipes on the ground tangent to each other, two in the next row and then one on top. What is the height of the pile? Express your answer in simplest radical form.

10. $\qquad$ The side lengths of a right triangle are each an integral number of units. If one of the legs is 13 units, what is the perimeter of the triangle?

## Warm-Up 15

1. $\quad \mathrm{cm}$ A ball is dropped from a height of 120 cm and always bounces upwards $\frac{2}{3}$ of the height from which it falls. How high does the ball go between the third and fourth bounces? Express your answer to the nearest whole number.
$\qquad$ Tom's graduating class has 288 students. At the graduation ceremony, the students will sit in rows with the same number of students in each row. If there must be at least 10 rows and at least 15 students in each row, then there can be $x$ students in each row. What is the sum of all possible values of $x$ ?
2. $\qquad$ Suppose a regular hexagon has a perimeter equal to the circumference of a circle. What is the ratio of a side of the hexagon to the radius of the circle? Express your answer as a common fraction in terms of $\pi$.
3. $\qquad$ There is a method traditionally used in some Russian villages to see which of the young women in the village are to be married the next year. Three blades of grass are folded in half and held in such a way that the six ends of the blades are visible but the rest of the blades are hidden. A young woman ties the ends together in pairs at random such that there are three knots and each end is tied to exactly one other end. If, on release, a three-blade loop is formed, the woman will be married the next year. What is the probability of getting a three-blade loop? Express your answer as a
 common fraction.


The point $A\left(\frac{5}{2}, 0\right)$ is reflected over the line $y=\frac{1}{2} x$ to the point $A^{\prime}$. What are the coordinates of $A^{\prime}$ ? Express any non-integer coordinate as a common fraction.
6. triangles

How many different right triangles with integer side lengths have one leg 15 units long?
7. $\qquad$ The sum of the reciprocals of three prime numbers is $\frac{167}{285}$. What is the sum of the three prime numbers?
8. $\qquad$ Points $M$ and $N$ are the midpoints of sides $A B$ and $B C$, respectively, of rectangle $A B C D$. If segments $A N$ and $C M$ intersect at point $X$, what fraction of the area of rectangle $A B C D$ lies in the quadrilateral BNXM? Express your answer as a common fraction.

9. arr

Diane has one stamp of each positive integer value 1 cent through 9 cents, inclusive. She wants to put 10 cents worth of postage in a row across the top of an envelope. If arrangements of the same stamps in a different order are
 considered different, how many arrangements are possible?
10. integers

How many positive three-digit integers have the property that the tens digit is greater than or equal to the hundreds digit and the ones digit is greater than or equal to the tens digit?

## Warm-Up 16

1. $\qquad$ Four circles are stacked vertically, as shown. Each of the top three circles has a diameter that is half the diameter of the circle just below it. The total area of the four circles is $765 \pi$ square inches. What is the height of the stack?

2. $\qquad$ The fourth Farey sequence, $F_{4}=\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$, is the list, written in increasing order, of all the common fractions with distinct values from 0 through 1, inclusive, that use only the integers $0,1,2,3$ or 4 as numerators and denominators. In the fifth Farey sequence, what is the product of the third and tenth terms? Express your answer as a common fraction.
3. $\qquad$ There are 64 identical-looking coins, one of which is slightly heavier than the others. A balance scale can be used to show which one of two groups of coins is heavier or that the two groups weigh the same. What is the minimum number of uses of the balance scale that is guaranteed to determine which of the coins is the heavier one?

4. $\qquad$ A square is inscribed in a right triangle with legs of 8 units and 15 units. If two of the vertices of the square lie on the hypotenuse and the other two vertices of the square lie on the legs of the triangle, what is the length of a side of the square? Express your answer as a common fraction.
5. meters

An apothem of a regular polygon is a line segment joining the center of the polygon to the midpoint of any side. What is the length of an apothem of a regular hexagon of side length 3 meters? Express your answer as a common fraction in simplest radical form.
6. $\qquad$ Suppose an equilateral triangle and a square have perimeters of equal length. What is the ratio of the area of the triangle to the area of the square? Express your answer as a common fraction in simplest radical form.
7. meters

Betty and Don are standing on point $A$ of rectangle $A B C D$. They walk away from each other on different sides of the rectangle. Betty walks in a straight line and stops 12 meters beyond point B. Don walks in a straight line through point $D$ and continues until point $C$ is directly between himself and Betty. The area of rectangle $A B C D$ is 348 square meters. How far beyond point $D$
 did Don walk?
8. $\qquad$ Suppose we replace each $x$ in the expression $\frac{x+1}{x-1}$ with the expression $\frac{x+1}{x-1}$. What is the value of the resulting expression when $x=\frac{4}{5}$ ? Express your answer as a common fraction.
9. sq units

What is the area of the region in the plane bounded by the lines with the equations $y=0, x=0, x=3$ and $y=2 x+1$ ?
10. degrees

In the figure, if $a+b+c=180$ degrees, what is the value of $u+v+w$ ?


## Workout 8

1. $\qquad$ \% The gravitational force on an asteroid varies inversely with the square of its distance from the sun. By what percent must the distance decrease in order that the gravitational force be multiplied by 3? Express your answer to the nearest tenth.
2. sq units Four unit squares contain some non-shaded circles, as shown. What is the total area of the shaded regions inside the squares? Express
 your answer in terms of $\pi$.
3. $\qquad$ If $a$ represents $\pi a^{4}$, then the volume of a sphere of radius 3 units could be represented by $x$ cu units for some positive value of $x$. What is the value of $x$ ? Express your answer in simplest radical form.
$\qquad$ What is the value of $x$ that satisfies the equation $2^{2!0!0!\cdot 8!}=2^{7} \cdot\left(2^{7}\right)^{x}$ ?
4. $\qquad$ Consider all of the positive five-digit integers that can be formed using each of the digits $3,4,5,6$ and 7 exactly once. What is the sum of these integers?
5. $\qquad$ Let $T$ be a positive integer whose only digits are 0 s and 1 s . If $X=T \div 12$, and $X$ is an integer, what is the smallest possible value of $X$ ?
$\qquad$ What is the maximum number of acute interior angles a convex pentagon can have?
6. $\qquad$ A "deletable prime" is a positive integer that (1) is prime and (2) is either a onedigit integer or, after removing one digit, results in another deletable prime. For example, 439 is deletable because 439 is prime and deleting the 9 results in another deletable prime, 43 , which is deletable because removing the 4 results in the prime 3. What is the smallest deletable prime larger than 443 ?
$\qquad$ What is the least positive common fraction that is an integral multiple of $\frac{9}{28}, \frac{12}{35}$ and $\frac{15}{56}$ ?
7. $\qquad$ \% A large tank contains a 400-kg mixture of water and alcohol. The mixture is $64 \%$ alcohol by weight. At each step, 100 kg of the mixture will be drained from the tank, replaced with 100 kg of water, and then stirred. After three steps, what percent of the final solution will be alcohol?

## Warm-Up 17

1. $\qquad$ Five girls (Alexandra, Betsy, Catherine, Deyola and Emily) travel with one boy (Frank) to a math contest. They have four hotel rooms, numbered 1 through 4. Each room can hold up to two people, and the boy has to have a room to
 himself. How many different ways are there to assign the students to the rooms, including the way shown here?
2. $\qquad$ Two different circles are each tangent to both the $x$-axis and the $y$-axis, as shown in the figure. One of the points of intersection of the circles is $(1,5)$. What is the product of the lengths of their radii?
3. (, ) What point on the line $y=2 x$ is closest to the point $(0,5)$ ?

4. $\qquad$ What is the area of regular octagon $A B C D E F G H$ divided by the area of quadrilateral ACEG? Express your answer in simplest radical form.
5. $\qquad$ A fair coin is tossed four times, and at least one of the tosses results in heads. What is the probability that exactly two tosses result in heads? Express your answer as a common fraction.
6. $\qquad$ Many numbers can be made by adding two or more consecutive terms of the arithmetic sequence $2,5,8,11, \ldots$. Two such examples are $7=2+5$ and $24=5+8+11$. What is the smallest number that can be made in at least two different ways by adding consecutive terms of this sequence?
7. $\quad$ sq units

Each of the six smaller squares in the sequence shown is formed by joining the midpoints of the sides of the next larger square. The center square has an area of 1 square unit. What is the total area of the shaded regions? Express your answer as a mixed number.

8. $\qquad$ The cities of Smallville and Largeville are 300 miles apart. Jim left from Smallville to go to Largeville at 10 a.m. Mickey left Largeville to go to Smallville at 10:30 a.m. on the same day. Jim traveled at a constant speed that was twice Mickey's constant speed, and they both arrived at a point 90 miles from Largeville at the same time. What was Mickey's constant speed, in miles per hour?
9. grids On a 5 by 5 grid of unit squares, one unit square is colored blue, one unit square is colored red, and the rest of the unit squares are white. Grids are considered different if no rotation could turn one into the other. How many different grids are there?
10. $\qquad$ Hypotenuse $B C$ in isosceles right triangle $A B C$ is $x$ units, where $x$ is a value that is a perfect square and a perfect cube greater than 1. If $A B$ can be expressed as $\frac{32 k \sqrt{5}}{5}$ units, what is the least possible value of $k$ ? Express your answer in simplest radical form.


## Warm-Up 18

1. $\qquad$ Rectangle $A B C D$ has points $E$ and $F$ on sides $A B$ and $C D$, respectively. If $A E=\frac{1}{3} A B$ and $C F=\frac{1}{4} C D$ and segments $D E$ and $B F$ intersect diagonal $A C$ at $G$ and $H$, respectively, what is the ratio $A G: G H: H C$ ? Express your answer in the form $a: b: c$, where $a, b$ and $c$ are relatively prime positive integers.
2. $\qquad$ A circle with radius 1 unit lies in the first quadrant and is tangent to both the $x$-and $y$-axes. A second larger circle lies in the first quadrant, is tangent to both axes and is externally tangent to the first circle. What is the radius of the second circle? Express your answer in simplest radical form.
3. terms

How many terms are in the expansion of the expression $\left[(3 x+2 y)^{2}(3 x-2 y)^{2}\right]^{3}$ after it is simplified to lowest terms?
4. $\qquad$ When 97,151 and 241 are each divided by a positive integer $K$, the remainder is the same. What is the largest possible value of $K$ ?
5. $\qquad$ Equilateral triangle $A B C$ has a side length of 6 units. Point $D$ lies on segment $B C$ such that $D C=2(B D)$. What is the length of the altitude of triangle $A D C$ from point C? Express your answer as a common fraction in simplest radical form.
6. $\qquad$


Billy can row upstream from point $A$ to point $B$ in three hours. Rowing at the same rate Billy needs only one hour to row from $B$ to $A$. What is the ratio of the rate of the current to the rate of Billy's rowing? Express your answer as a common fraction.
7. $\qquad$ A sequence begins $1,4,8,9,16, \ldots$ and consists of all the squares and cubes of positive integers written in ascending order. Numbers that are both squares and cubes, such as 1 , are written only once. What is the $50^{\text {th }}$ number in this sequence?
8. $\qquad$ Starting with the number 20, a list of increasing integers - not necessarily consecutive integers - has a product that is a perfect square. What is the least possible value for the last integer in the list?
9. $\qquad$ Kendra starts counting with $a$ and counts by $d$, where $a$ and $d$ are both positive integers. For example, for $a=5$ and $d=3$ the sequence would be $5,8,11$,.... The sum of two terms in Kendra's sequence is 10. How many different possible pairs ( $a, d$ ) are there?
10. paths

On an 8 by 8 grid of unit squares, a red marker starts in the unit square called $(3,1)$, which is the unit square in the third row and first column. Each separate move of the red marker is to a neighboring unit square horizontally, vertically or diagonally. How many paths of exactly four moves are there from $(3,1)$ to $(4,5)$ ?


## Workout 9

1. $\qquad$ In trapezoid $A B C D$, bases $A B$ and $C D$ are 13 and 39 units, respectively. Legs $B C$ and $D A$ are 24 and 10 units, respectively, and sides $B C$ and $D A$ lie on lines that are perpendicular to each other. What is the area of $A B C D$ ?
2. $\qquad$ An airplane started 8 miles south and 4 miles west of a radar station. The airplane travels due northeast at a speed of 3 miles per minute. In how many minutes is it again the same distance from the radar station as it started? Express your answer as a decimal to the nearest hundredth.
3. $\qquad$ On a 100-point test only three students had scores of 90 or above, and those scores were 97, 94 and 91. Exactly one student had a score of 59 and that was the lowest score. No score occurred more than two times. At least how many students must have had scores of 70 or above if the total of all the scores was 2431?
4. $\qquad$ What is the value of the sum $(1 \cdot 1+2 \cdot 3+3 \cdot 5+\ldots+100 \cdot 199)+(1 \cdot 3+2 \cdot 5+3 \cdot 7+\ldots+$ 100•201)? The first sum is of products of the form $(n)(2 n-1)$, and the second sum is of products of the form $(n)(2 n+1)$, with $n$ in each case going from 1 to 100 .
5. $\qquad$ What is the largest possible difference between the square of a three-digit integer and the square of the three-digit integer formed by reversing the digits of the original integer?
6. cylinders

Nicole needs to pour a cubic meter of water into cylinders that are each 1 meter tall and $\frac{1}{3}$ meter in diameter. How many cylinders does she need?
7. $\qquad$ Convex quadrilateral $A B C D$ has perpendicular diagonals. If $A B=25, B C=39$ and $C D=60$ units, what is the length of segment $D A$ ?
8. $\qquad$ What is the area of the circle that passes through $A(-8,0), B(0,8)$ and $C(12,0)$ ? Express your answer in terms of $\pi$.
9. $\qquad$ A set of dominoes consists of exactly one domino with every possible pair of integers $(x, y)$ with $0 \leq x \leq y \leq 6$. What is the sum of all the integers in the set of dominoes?

10. integers How many positive integers less than or equal to 50 are multiples of 3 or 4 , but not 5?

## Number Theory Stretch

1. $\qquad$ Using exponents when prime factors are used more than once, what is the prime factorization of 504?
2. factors How many positive factors does 504 have?
3. $\qquad$ What is the sum of the positive factors of 504?
4. $\qquad$ The base- 8 number $1342_{8}$ is equivalent to what base- 10 integer?
5. $\qquad$ What is the smallest positive integer that has a remainder of 7 when divided by 8 , a remainder of 8 when divided by 9 and a remainder of 11 when divided by 12?
6. integers

How many positive integers less than 101 are multiples of 3,4 or 7 ?
7. $\qquad$ What is the smallest positive integer value of $n$ such that $2^{n}+5^{n}+7^{n}$ is a multiple of 17 ?


What are the coordinates of the first-quadrant point that lies on the graph of $7 x+13 y=99$ and has coordinates that are both integers? Express your answer as an ordered pair $(x, y)$.
9. (, , )

Bill buys wigits for $\$ 3$, gigits for $\$ 8$ and pigits for $\$ 11$. If Bill bought exactly 100 aliens for $\$ 400$, and he purchased as few gigits as possible, what is the ordered triple (wigits, gigits, pigits) that represents his purchase?

wigit

gigit

pigit
10. $\qquad$ When the sum $2007^{2008}+2008^{2007}$ is simplified to an integer, what is the ones digit?

## Patterns Stretch

The following problems are from previous School Handbooks and competitions. Enjoy!

1. _ One digit of the decimal representation of $\frac{5}{7}$ will be chosen at random. What is the probability that the digit will be a 4? Express your answer as a common fraction. (1998 Chapter Countdown)
2. $\qquad$ A pentagon train is made by attaching regular pentagons with 1" sides so that each pentagon, except the two on the ends, is attached to exactly two other pentagons along sides, as shown. How many inches are in the perimeter of a pentagon train made from 85 pentagons? (1996 Chapter Target)
 ...

3. $\qquad$ What is the value of $x+y$ if the sequence $2,6,10, \ldots, x, y, 26$ is an arithmetic sequence? ('04-05 School Handbook)
4. $\qquad$ For what value of $x$ is the equation $x+2 x+3 x+\ldots+99 x+100 x=100$ true?
Express your answer as a common fraction. ('03-04 School Handbook)
5. $\qquad$ The first three towers in a sequence are shown. The $n^{\text {th }}$ tower is formed by stacking $n$ blocks on top of an $n$-by- $n$ square of blocks. How many blocks are in the 99 th tower?
(1997 State Sprint)

6. diagonals A diagonal of a polygon is a line containing two non-consecutive vertices. How many diagonals does a regular decagon have? ('02-03 School Handbook)
7. $\qquad$ What is the value of the expression $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) \ldots\left(1-\frac{1}{n+1}\right)$ when $n=12$ ? Express your answer as a common fraction. ('01-02 School Handbook)
8. $\qquad$ The $25^{\text {th }}$ day of the year 2003 fell on a Saturday. What day of the week did the $284^{\text {th }}$ day of the year 2003 fall? (1995 State Countdown)
9. $\qquad$ If the pattern shown is continued, what is the sum of the terms in Row 12? ('00-01 School Handbook)

Row 1 ... 2
Row $2 \ldots 2+4$
Row $3 \ldots 2+4+6$
Row $4 \ldots 2+4+6+8$
Row $5 \ldots 2+4+6+8+10$
10.

| $x$ | $y$ |
| :---: | :---: |
| -4 | 23 |
| 1 | 20 |
| 6 | 17 | According to the linear function represented in this table, what is the value of $x$ when $y=8$ ? ('04-'05 School Handbook)

20
17

## PROBLEM-SOLVING STRATEGIES

NCTM's Principles and Standards for School Mathematics recommends that the mathematics curriculum "include numerous and varied experiences with problem solving as a method of inquiry and application." There are many problems within the MATHCOUNTS program that may be considered difficult if attacked by setting up a series of equations, but quite simple when attacked with problem-solving strategies such as looking for a pattern, drawing a diagram, making an organized list and so on.

The problem-solving method that will be used in the following discussion consists of four basic steps:
FIND OUT Look at the problem.
Have you seen a similar problem before?
If so, how is this problem similar? How is it different?
What facts do you have?
What do you know that is not stated in the problem?
CHOOSE A STRATEGY How have you solved similar problems in the past?
What strategies do you know?
Try a strategy that seems as if it will work.
If it doesn't, it may lead you to one that will.
SOLVE IT
LOOK BACK
Use the strategy you selected and work the problem.
Reread the question.
Did you answer the question asked?
Is your answer in the correct units?
Does your answer seem reasonable?
Specific strategies may vary in name. Most, however, fall into these basic categories:

- Compute or Simplify (C)
- Use a Formula (F)
- Make a Model or Diagram (M)
- Make a Table, Chart or List (T)
- Guess, Check \& Revise (G)
- Consider a Simpler Case (S)
- Eliminate (E)
- Look for Patterns (P)

To assist in using these problem-solving strategies, the answers to the Warm-Ups and Workouts have been coded to indicate possible strategies. The single-letter codes above for each strategy appear in parentheses after each answer.

NOTE: Examples of these strategies being applied to previously published MATHCOUNTS problems are available on pages 29-39 in Volume I of the 2007-2008 MATHCOUNTS School Handbook.

## Warm-Up 7

## Answers

1. 360
(C)
2. $\frac{5}{9}$
(C, F, G)
3. 32
$(C, F, T)$
4. 81
(C, F)
5. $6 \sqrt{3}$
(C, F, M)
6. $\frac{1}{6}$
( $C, S, T$ )
7. 9
(C, P, S, T)
8. 750
(C)
9. 144
(C, P, S, T)
10. 31 or 31.00
(C)

## Warm-Up 8

## Answers

| 1. $\frac{11}{36}$ | ( $C, P, S, T)$ | 5. 5 | (C, E) | 8. $12 \sqrt{10}$ | ( $C, F, M)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2. 3 | $(C, F)$ | 6. 120 | (C, F, M) | 9. 68 | ( $C, F, M$ ) |
| 3. 9 | ( $C, G, P, S, T)$ | 7. $32 \sqrt{3}$ | $(C, F, M)$ | 10. 360 | ( $F, M, S$ ) |
| 4. 240 | $(C, M, T)$ |  |  |  |  |

## Workout 4

## Answers



## Warm-Up 9

## Answers

| 1. 10 | $(F, M, P, T)$ | 5.195 | $(C, M, T)$ | 8. | $\frac{9}{25}$ | $(C, S)$ |
| :--- | ---: | :--- | :--- | ---: | :--- | ---: | ---: |
| 2. 105 | $(C, F, M, S)$ | 6.8 | $(C, F, G, T)$ | 9. | 4 | $(C, F, G, P, S, T)$ |
| 3. | 72 | $(C, G, P, T)$ | $7 . d$ | $(C, E, M, P, S, T)$ | $10 .\left(\frac{3}{2}, \frac{27}{4}\right)$ | $(C, F, G, T)$ |
| 4. 81 | $(C, F, M)$ |  |  |  |  |  |

## Warm-Up 10

## Answers

| 1. 8 | $(C, F, G, P, S, T)$ | 5.200 | $(C, F, G)$ | 8.49 | $(C, F, P)$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2. 576 | $(C, E, F, G, M, T)$ | 6.229 | $(C, F, P)$ | $9.98,634$ | $(S)$ |
| 3. $\frac{391}{39}$ | $(C, F, M)$ | 7.200 | $(C, F, G, M, T)$ | 10.60 | $(C, P, S, T)$ |
| 4. 42 or 42.00 | $(C, F, T)$ |  |  |  |  |

## Workout 5

## Answers

$\left.\begin{array}{lr|ll|llr}\text { 1. } & 7397 & (C, F, T) & 5 . & 0 & (C) & 8 . \\ \text { 2. } & 346.4 & (C, F, M) & \text { 6. } 110 & (C, E, P, T) & 9 . & 8 \\ \text { 3. } & 6.25 & (C) & 7 . & 16,777,216 & (C, M, P, T) & 10.126\end{array}\right)(C, F, G, M)$

## Warm-Up 11

## Answers

| 1. $64-4 \pi$ | $(C, F, M)$ | 5.22 | $(C, F, M, S)$ | 8.15 | $(C, F, M, S)$ |  |
| :--- | :--- | :--- | :--- | ---: | :--- | ---: |
| 2. 2 | $(P, S, T)$ | 6. | $\frac{1}{2}$ | $(C, S, T)$ | 9.19 | $(E, G, M, T)$ |
| 3. 7 | $(E, P, T)$ | 7. 16 | $(C, F, M)$ | $10.50: 3$ | $(C, M, S)$ |  |
| 4. 13 | $(C, F, M)$ |  |  |  |  |  |

## Warm-Up 12

## Answers

1. 75

| $(C, F)$ | 5. | $832 \pi$ |
| ---: | :--- | ---: |
| $(C, F, M)$ | 6. | 47 |
| $(C, F, M)$ |  |  |
| $(C, F, F, M, T)$ | 7. | -14 |
| $(C, F, F, S)$ |  |  |

8. 27
$(C, G, M, T)$
9. 3
(C, M, T)
10. 128
(C, F, M)
11. 24
(C, F, M)

## Workout 6

## Answers

1. $14.5(C, E, F, G, M, T)$
2. $\frac{3}{16}$
$(C, E, G, M, T)$
3. $1.8396 \times 10^{7}$
(C)
4. 28
$(C, F, M)$
5. 5.4
(C, F, M)
6. 15.42
(C, G, M, T)
7. 402.2
( $C, F$ )
8. 10.7
(C, F, M)
9. 36.3
$(C, F, M)$
10. 23
$(C, G)$

## Warm-Up 13

## Answers

1. 24
$(C, G)$
2. 24
( $C, F, M$ )
3. 32
( $M, S$ )
4. 4.5
$(C, F, M)$
5. 9 or 9.00
6. $12 \sqrt{3}$
(C)
7. $\frac{30}{49}$
( $F, M$ )
8. 3
(C, F)
9. 668
( $C, E, G, M, S, T)$
10. 8
(C)

## Warm-Up 14

## Answers

| 1. | $15^{*}$ | $(G, P)$ | 5.48 | $(G, M, T)$ | 8.35 | $(C, M)$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 2. 13 | $(C, S)$ | 6.36 | $(C, F, M, P, S)$ | 9.19 | $(T)$ |  |
| 3. | -12 | $(C, F, G)$ | 7.22 | $(C, F, M)$ | $10 . \frac{1}{2}$ | $(C, G, T)$ |
| 4. 16 | $(C, E, T)$ |  |  |  |  |  |

*Because the rule was not defined, it may be possible to find another rule that works for all five examples given.

## Workout 7

## Answers

1. 2457
(C, F, P, S)
2. 10.5
(C, F, M)
3. $1,472,981$
( $C, M, P, T$ )
4. 56
(F)
5. 104
(C, F, M)
6. 4590
(C, F)
7. $1+\sqrt{3}$
(C, F, M)
8. 16.30
( $E, G, P$ )
9. 80
(C, F, M, P, T)
[^0]
## Warm-Up 15

## Answers

1. 36
$(C, M, T)$
2. $\frac{8}{15}$
$(C, F, M, P, T)$
3. 27
$(C, G, P, S)$
4. 58
$(C, G)$
5. $\frac{\pi}{3}$
$(C, F, M)$
6. $\left(\frac{3}{2}, 2\right)$
$(C, F, M)$
7. 4
$(C, E, F, G, P, T)$
8. $\frac{1}{6}$
9. 56
10. 165
(F, M, P, S)
$(C, E, S, T)$
$(C, P, S, T)$

## Warm-Up 16

## Answers

| 1. 90 | $(C, F, M)$ | $5 . \frac{3 \sqrt{3}}{2}$ | $(F, M)$ | $8 . \frac{4}{5}$ | $(C, F)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | ---: |
| 2. $\frac{1}{5}$ | $(C, G, P, T)$ | $6 . \frac{4 \sqrt{3}}{9}$ | $(C, F, M)$ | 9. 12 | $(C, F, M, P)$ |
| 3. 4 | $(G, P, S, T)$ | 7.29 | $(C, F, M)$ | 10.201 | $(C, F, M, P)$ |
| 4. $\frac{2040}{409}$ | $(C, F, M)$ |  |  |  |  |

## Workout 8

## Answers

| 1. | 42.3 | $(C, F)$ | $5.6,666,600$ | $(C, P, S, T)$ | 8.457 | $(C, E, G, T)$ |
| :--- | ---: | :--- | :--- | :--- | :--- | ---: |
| 2. $4-\pi$ | $(C, F, M, P)$ | 6.925 | $(C, G, P, S)$ | $9 . \frac{180}{7}$ | $(C, T)$ |  |
| 3. | $\sqrt{6}$ | $(C, F)$ | 7.3 | $(F, G, M)$ | 10.27 | $(C, F, T)$ |
| 4. | 11,519 | $(C, F, S)$ |  |  |  |  |

## Warm-Up 17

## Answers

| 1. 360 | $(C, F, M, S, T)$ | 5. | $\frac{2}{5}$ | $(F, M, T)$ | 8. | 30 | $(C, G, M)$ |
| :--- | ---: | :--- | :--- | ---: | :--- | ---: | ---: |
| 2. 26 | $(C, F, M)$ | 6. 55 | $(E, G, M, P, T)$ | 9. 150 | $(C, E, P, S, T)$ |  |  |
| 3. $(2,4)$ | $(C, F, M)$ | 7. $16 \frac{3}{4}$ | $(C, F, M, P, T)$ | $10 . \sqrt{10}$ | $(C, F, M)$ |  |  |
| 4. $\sqrt{2}$ | $(F, M, S)$ |  |  |  |  |  |  |

## Warm-Up 18

## Answers

1. $5: 11: 4$
(C, F, M, P, S)
2. $3+2 \sqrt{2}$
$(F, M)$

$$
\text { or } 2 \sqrt{2}+3
$$

3. 7
(C, F, S)
4. 18
( $C, E, F, G, T$ )
5. $\frac{6 \sqrt{21}}{7}$
$(C, F, M)$
6. 30
( $E, G, S, T$ )
7. $\frac{1}{2}$
8. 1728
$(C, G, M, T)$
9. 13
( $M, T$ )
( $M, T$ )

## Workout 9

## Answers

| 1. 240 | $(F, M)$ | $5.942,480$ | $(C, G)$ | $8.104 \pi$ | $(C, F, G, M)$ |
| :--- | ---: | ---: | ---: | :--- | ---: |
| 2. 5.66 | $(F, M)$ | 6.12 | $(C, F, M)$ | 9.168 | $(C, M, T)$ |
| 3. 13 | $(E, G, T)$ | 7.52 | $(C, F, M)$ | 10.19 | $(C, M, T)$ |
| 4. $1,353,400$ | $(C, F, P)$ |  |  |  |  |

## Number Theory Stretch

## Answers

1. $2^{3} \times 3^{2} \times 7$ or $(C, M)$
$2^{3} \times 3^{2} \times 7^{1}$
2. 24
( $C, F, P, T)$
3. 1560
(C, T)
(C, F)
4. 71
( $E, G, P, S, T)$
5. $(3,6)$
( $G, M, T$ )
6. 58
$(C, G, M, P, S, T)$
$(E, G, T)$
7. 3
8. $(86,4,10)$
( $C, E, G, P$ )
9. 3
( $\mathrm{P}, \mathrm{S}$ )

## Patterns Stretch

## Answers

1. $\frac{1}{6}$
(P)
2. 257
( $M, P$ )
3. 40
( $E, G, P$ )
4. $\frac{2}{101}$
( $C, F, P$ )

| 5. | 9900 | $(C, F, M, P, S)$ | 8. Saturday | $(C, G, P)$ |
| :---: | :---: | ---: | :--- | :--- |
| 6. | 35 | $(C, F, M, P)$ | 9.156 | $(C, P, S)$ |
| 7. | $\frac{1}{13}$ | $(C, P)$ | 10.21 | $(C, G, P)$ |

## PROBLEM INDEX

It is difficult to categorize many of the problems in the MATHCOUNTS School Handbook. It is very common for a MATHCOUNTS problem to straddle multiple categories and hit on multiple concepts. This list is intended to be a helpful resource, but it is in no way complete. Each problem has been placed in exactly one category.

| Algebraic | Measurement | Plane | Logic | Pattern |
| :---: | :---: | :---: | :---: | :---: |
| Expressions | WU 7-1 | Geometry | WU 9-7 | Recognition |
| \& Equations | WU 8-7 | WU 7-3 | WU 10-10 | WU 7-7 |
| WU 8-5 | WO 4-1 | WU 8-8 | WU 13-1 | WO 7-1 |
| WU 8-9 | WO 4-8 | WU 8-10 | WU 16-3 | WO 8-5 |
| WO 4-3 | WU 9-2 | WO 4-9 | WO 8-7 | WO 9-4 |
| WU 9-10 | WU 10-3 | WU 9-4 | WU 17-9 | *Stretch |
| WU 10-1 | WO 5-8 | WU 10-7 | WU 18-10 |  |
| WU 10-4 | WU 12-2 | WO 5-2 | WO 9-3 | Proportional |
| WO 5-1 | WU 12-3 | WU 11-1 | WO 9-9 | Reasoning |
| WO 5-5 | WO 6-4 | WU 11-5 |  |  |
| WO 5-9 | WU 13-7 | WU 12-4 | Probability, | WU 7-5 |
| WU 11-7 | WO 7-2 | WU 12-5 | Counting \& | WU 7-9 |
| WU 12-6 |  | WU 12-10 | Combinatorics | WU 8-6 WO 4-7 |
| WU 12-7 | Number | WO 6-5 | WU 7-4 | WU 9-5 |
| WO 6-1 | Theory | WO 6-9 | WU 8-1 | WU 11-10 |
| WO 6-2 |  | WU 13-2 | WU 9-1 | WU 11-10 |
| WO 6-10 | WU 7-6 | WU 13-3 | WU 9-1 |  |
| WU 13-5 | WU 8-3 | WU 13-6 | WU 11-3 | Problem |
| WU 14-2 | WO 4-2 | WU 14-6 | WU 11-6 | Solving |
| WU 14-3 | WO 4-5 | WU 14-8 | WO 6-7 | (Misc.) |
| WO 7-10 | WU 9-3 | WO 7-6 | WU 13-8 |  |
| WU 15-3 | WU 10-9 | WU 15-8 | WU 13-10 | WU 7-8 |
| WU 15-5 | WU 11-2 | WU 16-1 | WU 14-9 | WO 4-6 |
| WU 15-6 | WU 14-1 | WU 16-4 | WU 15-4 | WU 10-5 |
| WU 16-6 | WO 7-8 | WU 16-5 | WU 15-9 | WO 5-7 |
| WU 16-8 | WU 15-2 | WU 16-7 | WU 15-10 | WO 6-8 |
| WO 8-1 | WU 16-2 | WU 16-10 | WU 17-1 | WU 14-4 |
| WO 8-3 | WO 8-6 | WO 8-2 | WU 17-5 | WU 14-10 |
| WO 8-4 | WO 8-8 | WU 17-2 |  |  |
| WU 17-3 | WO 8-9 | WU 17-4 | Percents/ | General |
| WU 17-8 | WU 17-6 | WU 17-7 | Fraction | Math |
| WU 17-10 | WU 18-4 | WU 18-1 | WU 8-4 | WU 7-10 |
| WU 18-3 | WU 18-7 | WU 18-2 | WO 4-4 | WO 5-6 |
| WU 18-6 | WO 9-5 | WU 18-5 | WO 4-10 | WO 6-3 |
| WO 9-2 | WO 9-10 | WO 9-1 | WU 9-8 | WO 7-3 |
|  | *Stretch | WO 9-7 | WU 10-6 | WU 15-7 |
| Sequences |  |  | WO 5-3 |  |
| \& Series |  | Coordinate | WO 5-4 | Statistics |
| WU 9-9 | Solid | Geometry | WO 5-10 | WU 7-2 |
| WU 10-8 | Geometry | WU 8-2 | WU 11-8 | WU 9-6 |
| WU 18-9 | WU 10-2 | WU 13-4 | WU 12-1 | WU 11-9 |
|  | WU 11-4 | WU 14-5 | WU 12-9 | WU 12-8 |
|  | WO 7-5 | WU 14-7 | WU 13-9 |  |
|  | WO 7-7 | WU 16-9 | WO 7-4 | +0-6 |
|  | WO 7-9 | WO 9-8 | WU 15-1 |  |
|  | WO 9-6 |  | WO 8-10 |  |

[^1]
# MATHCOUNTS 2007-2008 

## Material Request \& Competition Registration Form

## Why do you need this form?

- Request Volume II of the MATHCOUNTS School Handbook, containing 200 math problems [FREE]
- Sign Up a Math Club and receive the Club in a Box resource kit [FREE]
- Register Your School to Participate in the MATHCOUNTS Competition [Competition Registration Deadline is Dec. 7, 2007]


## What is MATHCOUNTS?

The mission of MATHCOUNTS is to increase enthusiasm for and enhance achievement in middle school mathematics throughout the United States. Currently celebrating our $25^{\text {th }}$ anniversary, MATHCOUNTS has helped more than 7 million students develop their mathematical abilities by tackling MATHCOUNTS problems.

The MATHCOUNTS Foundation administers a nationwide math enrichment, coaching and competition program.

## Recent Program Changes

The 2007-2008 MATHCOUNTS School Handbook is being produced in two volumes. Volume I contains 100 math problems, and Volume II contains 200 math problems. As in the past, these 300 FREE challenging and creative problems are designed to meet NCTM standards for grades 6-8.

MATHCOUNTS is pleased to launch the new MATHCOUNTS Club Program, which may be used by schools as a stand-alone program or incorporated into the student preparation for the MATHCOUNTS competition. The MATHCOUNTS Club Program provides schools with the structure and activities to hold regular meetings of a math club. Depending on the level of student and teacher involvement, a school may receive a recognition plaque or banner and be entered into a drawing for prizes. Open to schools with 6th-, 7th- and 8th-grade students, the Club Program is free to all participants.

## MATHCOUNTS REQUEST/REGISTRATION FORM: 2007-2008 School Year

Mail or fax this completed form (with payment if choosing Option 3) to: Competition Registration (Option 3) must be postmarked by Dec. 7, 2007

## MATHCOUNTS Registration

P.O. Box 441, Annapolis Junction, MD 20701

Fax: 301-206-9789

| Teacher/Coach's Name |  |  | Principal's Name | $\square$ Previous MATHCOUNTS school |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| School Name |  |  |  |  |  |
| School Mailing Address |  |  |  |  |  |
| City, State ZIP |  |  |  | County |  |
| School Phone (__ ) |  |  | School Fax \# ( |  |  |
| Teacher/Coach Phone ( |  |  | Chapter (if known) |  |  |
| Teacher/Coach's E-mail |  |  |  |  |  |
| What type of school is this? | $\square$ Public | $\square$ Charter | $\square$ Religious | $\square$ Private | $\square$ Homeschool |

Send me Volume II of the 2007-2008 MATHCOUNTS School Handbook, which contains 200 math problems.
Why are you requesting this? $\square$ More Problems for Class $\quad \square$ More Problems for Club $\quad \square$ Preparing for Competition $\square$ It's free. Why not?
$\square$ Sign Up my Math Club and send me the Club in a Box resource kit and Volume II of the 2007-2008
MATHCOUNTS School Handbook, which contains 200 math problems. (There is NO COST for the Club Program.) (NOTE: You must complete the survey below.)

Please see page 17 in the handbook or visit www.mathcounts.org/club for details on this new program.


REGISTER MY SCHOOL for the MATHCOUNTS Competition series and send me the Club in a Box resource kit and Volume II of the 2007-2008 MATHCOUNTS School Handbook. (NOTE: You must complete the survey above.)

Competition Registration Fees:
Team Registration (up to four students):

$$
1 @ \$ 80=\$
$$

$\qquad$
$\square$ Individual Registration(s):
\# of students $\qquad$ (max. of 4)
@ $\$ 20$ each = \$ $\qquad$
By completing this registration form, you attest to the school administration's permission to register students for MATHCOUNTS under this school's name.

Total Due $=\$$

Title I Rate *
1 @ $\$ 40=\$$ $\qquad$ (a) $\$ 10$ each $=\$$

Total Due $=\$$

* Principal Signature is required to verify school qualifies for Title I fees:
Payment: $\square$ Check $\square$ Money order $\square$ Purchase order \# (p.o. must be included) $\square$ Credit card

Name on card: $\qquad$ Purchase order \# $\qquad$ (p.o. must be included) $\square$ Credit card Signature: $\qquad$ Card \#: $\qquad$ Exp:

[^2]
[^0]:    *The form $1+\sqrt{3}$ is preferred so that there is no question as to whether the " +1 " is under the radical.

[^1]:    *The Number Theory Stretch and Patterns Stretch have 20 more problems that also include geometry, algebra and measurement concepts.

[^2]:    Make checks payable to the MATHCOUNTS Foundation. Payment must accompany this registration form. All registrations will be confirmed with an invoice indicating payment received or payment due. Invoices will be sent to the school address provided. If a purchase order is used, the invoice will be sent to the address on the purchase order. Payment for purchase orders must include a copy of the invoice. Registration questions should be directed to the MATHCOUNTS Registration Office at 301-498-6141. Registration confirmation may be obtained at www.mathcounts.org.

