# 2.1 Rational Numbers

# Essential Question How can you use a number line to order

rational numbers?

### The Meaning of a Word 🕒 Rational

The word **rational** comes from the word *ratio*. Recall that you can write a ratio using fraction notation.

If you sleep for 8 hours in a day, then the ratio of your sleeping time to the total hours in a day can be written as



A **rational number** is a number that can be written as the ratio of two integers.

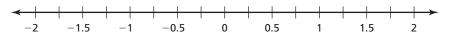
8 h

 $2 = \frac{2}{1} \qquad -3 = \frac{-3}{1} \qquad -\frac{1}{2} = \frac{-1}{2} \qquad 0.25 = \frac{1}{4}$ 

#### ACTIVITY: Ordering Rational Numbers

#### Work in groups of five. Order the numbers from least to greatest.

• Use masking tape and a marker to make a number line on the floor similar to the one shown.



- Write the numbers on pieces of paper. Then each person should choose one.
- Stand on the location of your number on the number line.
- Use your positions to order the numbers from least to greatest.
- **a.** -0.5, 1.25,  $-\frac{1}{3}$ , 0.5,  $-\frac{5}{3}$  **b.**  $-\frac{7}{4}$ , 1.1,  $\frac{1}{2}$ ,  $-\frac{1}{10}$ , -1.3 **c.** -1.4,  $-\frac{3}{5}$ ,  $\frac{9}{2}$ ,  $\frac{1}{4}$ , 0.9 **d.**  $\frac{5}{4}$ , 0.75,  $-\frac{5}{4}$ , -0.8, -1.1

#### Rational Numbers

In this lesson, you will

- understand that a rational number is an integer divided by an integer.
- convert rational numbers to decimals.

# Laurie's Notes



# Introduction

### **Standards for Mathematical Practice**

• MP1 Make Sense of Problems and Persevere in Solving Them: In this lesson, students will write fractions as decimals and vice versa. Students should always check the reasonableness of their answers. For instance,  $\frac{7}{11}$  is greater than  $\frac{1}{2}$ . So, when you write  $\frac{7}{11}$  as a decimal, the result should be greater than 0.5.

### Motivate

· A key skill for both activities today will be the ability to compare fractions and decimals. Try a warm up where students need to fill in the following table.

Fraction	$\frac{1}{2}$		$\frac{3}{5}$		$\frac{3}{4}$	
Decimal		0.1		0.8		1.4

- Check for understanding of the process of converting between these two forms of numbers.
- Students have studied operations with whole numbers, fractions, decimals, and integers.
- $\ref{eq: 1}$  "Do you think there is a number halfway between -3 and -4? What is that number?"
- Explain that in this chapter, they will perform operations on numbers such as -3.5. Define rational numbers.

# Activity Notes

### **Activity 1**

- MP4 Model with Mathematics: In preparing for this activity, be sure to leave sufficient space between the number line marks so that students are able to stand at their locations comfortably. If there is enough space in the classroom, make multiple number lines on the floor. Consider different orientations so that students may see some as vertical number lines and some as horizontal number lines. If space is limited, pairs of students could do the same problem on the board or on a piece of paper.
- MP3a Construct Viable Arguments: When the first set of numbers has been located, spend time having students give their reasoning as to why they located the numbers as they did. For instance, how did they know
  - that  $-\frac{5}{3}$  was to the left of -0.5 versus to the right of it?
- So that all students have an opportunity to use the number line on the floor, rotate groups at the end of each set of numbers.
- Extension: If time permits, ask students to name a decimal between two fractions  $\left(-\frac{1}{2} \text{ and } -\frac{3}{4}\right)$  and to name a fraction between two decimals (-0.6 and -0.7).

#### **Common Core State Standards**

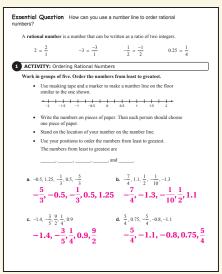
7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts. 7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

#### **Previous Learning**

Students should know how to convert between common fractions (halves. fourths, fifths, and tenths) and decimals. They should also be able to graph common fractions and decimals on a number line.



#### 2.1 Record and Practice Journal

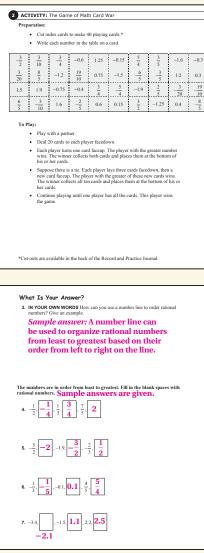


#### **English Language Learners**

#### Vocabulary

Let English language learners know that the pronunciation of rational numbers that end in –th such as fourths and fifths is sometimes difficult for native English speakers.

#### 2.1 Record and Practice Journal



# Laurie's Notes

### Activity 2

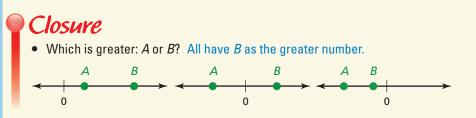
- You may want to make the game cards ahead of time or have students create the game cards.
- **Management Tip:** To preserve cards for multiple uses, make cards on colored cardstock and store individual sets in sealable plastic bags.
- The card game *War* is familiar to many students. The question asked each play is, "Which number is greater?" The player with the greater value collects both cards. If the cards have an equivalent value, there is a tie. As stated in the text, each player lays 3 cards face down and then 1 card face up. The player with the card of greater value collects all of the cards.
- **Comparing Cards:** The key component of this activity is when students actually compare the two rational numbers. Discuss with students how they will compare the numbers. When both numbers are positive or the signs are different, students will have less difficulty. If both numbers are negative, students need to remember that the farther the number is to the right on the number line, the greater its value.

For example,  $-\frac{3}{5} > -0.75$  because  $-\frac{3}{5}$  is to the right of -0.75 on a number line.

- To start play, give students the opportunity to preview the cards. Explain the rules and let students begin. If one group finishes early, have them shuffle the cards and play again.
- Extension: The cards can also be used to play the game *Memory*. Put the fraction cards in one group and the decimal cards in another group. Place all cards face down in two grids. Students select one card from each group. If the cards match, (meaning they are equivalent), then the student keeps the cards. If they do not match, the cards are put back face down. A deck of 40 cards is too many! Reduce the deck to 24 (12 in each group). Make sure the equivalent decimals and fractions are in each deck.

#### What Is Your Answer?

- Listen for the big idea, namely that the farther to the right the number is on the number line, the greater the value of that number.
- MP3 Construct Viable Arguments and Critique the Reasoning of Others: For Questions 4–7, students should work with partners. Have students share their results and their reasoning. Answers will vary, so the explanation is important to hear.



### 2 ACTIVITY: The Game of Math Card War

#### Preparation:

- Cut index cards to make 40 playing cards.
- Write each number in the table on a card.

#### **To Play:**

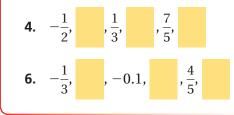
- Play with a partner.
- Deal 20 cards to each player facedown.
- Each player turns one card faceup. The player with the greater number wins. The winner collects both cards and places them at the bottom of his or her cards.
- Suppose there is a tie. Each player lays three cards facedown, then a new card faceup. The player with the greater of these new cards wins. The winner collects all ten cards and places them at the bottom of his or her cards.
- Continue playing until one player has all the cards. This player wins the game.

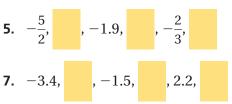
$-\frac{3}{2}$	$\frac{3}{10}$	$-\frac{3}{4}$	-0.6	1.25	-0.15	$\frac{5}{4}$	$\frac{3}{5}$	-1.6	-0.3
$\frac{3}{20}$	$\frac{8}{5}$	-1.2	$\frac{19}{10}$	0.75	-1.5	$-\frac{6}{5}$	$-\frac{3}{5}$	1.2	0.3
1.5	1.9	-0.75	-0.4	$\frac{3}{4}$	$-\frac{5}{4}$	-1.9	$\frac{2}{5}$	$-\frac{3}{20}$	$-\frac{19}{10}$
$\frac{6}{5}$	$-\frac{3}{10}$	1.6	$-\frac{2}{5}$	0.6	0.15	$\frac{3}{2}$	-1.25	0.4	$-\frac{8}{5}$

# -What Is Your Answer?

**3. IN YOUR OWN WORDS** How can you use a number line to order rational numbers? Give an example.

The numbers are in order from least to greatest. Fill in the blank spaces with rational numbers.







Use what you learned about ordering rational numbers to complete Exercises 28–30 on page 48.



Consider Similar Problems What are some ways to determine which number is greater?

# 2.1 Lesson



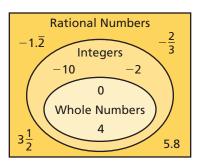
Key Vocabulary

p. 46 terminating decimal, p. 46 repeating decimal, p. 46



#### **Rational Numbers**

A **rational number** is a number that can be written as  $\frac{a}{b}$  where *a* and *b* are integers and  $b \neq 0$ .



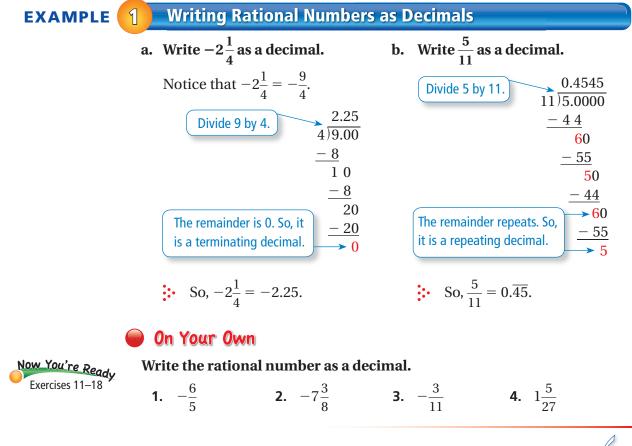
Because you can divide any integer by any nonzero integer, you can use long division to write fractions and mixed numbers as decimals. These decimals are also rational numbers and will either *terminate* or *repeat*.

A **terminating decimal** is a decimal that ends.

1.5, -0.25, 10.625

A **repeating decimal** is a decimal that has a pattern that repeats.

 $-1.333... = -1.\overline{3}$  $0.151515... = 0.\overline{15}$ Use bar notation to show which of the digits repeat.



Multi-Language Glossary at BigIdeasMath com

# Laurie's Notes

### Introduction

#### Connect

- Yesterday: Students ordered rational numbers. (MP1, MP3, MP4)
- Today: Students will extend this knowledge to include repeating decimals.

#### Motivate

- Ask students to form a "name fraction," where the numerator is the number of letters in their first name and the denominator is the number of letters in their last name.
- Before class, go through your class roster and select two students whose name fractions are nearly equivalent, but one is a terminating decimal and the other is a repeating decimal. Discuss writing the fractions as decimals.
- When you look at the repeating decimal, share that today's lesson is about writing rational numbers, which may be repeating decimals.

## Lesson Notes

#### Key Idea

• **Discuss:** Students have worked with fractions and decimals before. Explain that when negative fractions and decimals are included, we refer to these numbers as rational numbers. Also point out that the definition includes the word *can*, meaning that the rational numbers do not have to

be written in the form  $\frac{a}{b}$ , but they *can* be.

- Define terminating and repeating decimals. Give examples of each.
- **Common Error:** Some students will write  $\frac{1}{3}$  as 0.333 and think that is

sufficient. They do not realize what the repeat bar represents.

• **Big Idea:** In this lesson students should gain an understanding that every quotient of integers (with a non-zero divisor) is a rational number.

### **Example 1**

- "How do you write a fraction as a decimal?" Listen for 3 methods:
   1) benchmark fractions you know, 2) write the fraction as an equivalent fraction with a denominator as a power of 10 and use the place value, or
   3) divide the numerator by the denominator.
- MP1a Make Sense of Problems: Mathematically proficient students are able to plan a solution. Choosing between methods may help students be more efficient and accurate when writing fractions as decimals.
- Complete part (a) as a class. The first step is to write the mixed number as the equivalent improper fraction. Then divide the numerator by the denominator. Point out that the negative sign is simply placed in the answer after the calculations are complete.
- Complete part (b) as a class. Remind students that you always divide the numerator by the denominator, regardless of the size of the numbers!

#### On Your Own

• Neighbor Check: Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

**Goal** Today's lesson is writing fractions as decimals, including **repeating decimals**, and writing decimals as fractions in simplest form.

Technology for the Feacher Dynamic Classroo

Lesson Tutorials Lesson Plans Answer Presentation Tool

#### **Extra Example 1**

Write the rational number as a decimal.

**a.** 
$$4\frac{3}{16}$$
 4.1875  
**b.**  $-3\frac{4}{9}$  -3. $\overline{4}$ 

# Laurie's Notes

#### Extra Example 2

Write -2.625 as a mixed number in simplest form.  $-2\frac{5}{9}$ 

# On Your Own **5.** $-\frac{7}{10}$ **6.** $\frac{1}{8}$ **7.** $-3\frac{1}{10}$ **8.** $-10\frac{1}{4}$

#### **Extra Example 3**

Order the rational numbers  $-\frac{5}{9}$ ,  $-1\frac{3}{4}$ ,  $-\frac{13}{8}$ , and -0.6 from least to greatest.  $-1\frac{3}{4}$ ,  $-\frac{13}{8}$ , -0.6,  $-\frac{5}{9}$ 

#### On Your Own

**9.** All of the sea creatures (anglerfish, squid, shark, and whale) are deeper than the dolphin.

#### **Differentiated Instruction**

#### Auditory

Writing terminating decimals as rational numbers is easier if the students read the decimal using place value as opposed to reading the digits.

Terminating decimal: -0.26

Read as place value: negative twenty-six hundredths

Read as digits: negative zero point two six

### Example 2

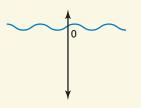
- "How do you write a decimal as a fraction?" Look at the place value of the last digit in the decimal and that will be the denominator.
- Work through Example 2.
- "How was the fraction simplified?" Both the numerator and the denominator were divided by a common factor of 2.
- Be sure to discuss the Study Tip.
- Extension: Write -0.026 and -2.6 as fractions. This helps students focus on the importance of place value and where the last digit is located.

#### On Your Own

- Neighbor Check: Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- In Questions 7 and 8, the whole number portion of the decimal can be a problem.

#### Example 3

- Discuss the unit of measure, kilometers.
- Work through the problem. When doing this problem in class, draw the number line vertically and identify sea level.

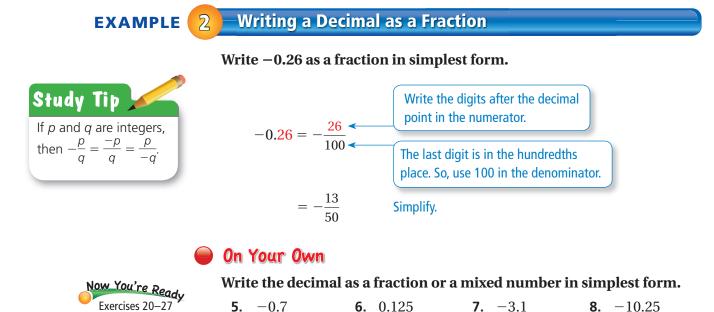


#### On Your Own

- Neighbor Check: Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.
- Extension: If calculators are available to students, explore the repeating patterns for certain sets of fractions (thirds, ninths, elevenths, etc.).

### Closure

- Exit ticket:
- Write  $-\frac{5}{6}$  as a decimal.  $-0.8\overline{3}$  Write -0.56 as a fraction.  $-\frac{56}{100} = -\frac{14}{25}$



#### **EXAMPLE 3** Ordering Rational Numbers

Creature	Elevation (kilometers)
Anglerfish	$-\frac{13}{10}$
Squid	$-2\frac{1}{5}$
Shark	$-\frac{2}{11}$
Whale	-0.8

The table shows the elevations of four sea creatures relative to sea level. Which of the sea creatures are deeper than the whale? Explain.

Write each rational number as a decimal.

$$-\frac{13}{10} = -1.3$$
$$-2\frac{1}{5} = -2.2$$
$$-\frac{2}{11} = -0.\overline{18}$$

Then graph each decimal on a number line.



■ Both −2.2 and −1.3 are less than −0.8. So, the squid and the anglerfish are deeper than the whale.

#### On Your Own



**9.** WHAT IF? The elevation of a dolphin is  $-\frac{1}{10}$  kilometer. Which of the sea creatures in Example 3 are deeper than the dolphin? Explain.

# 2.1 Exercises





# Vocabulary and Concept Check

- 1. VOCABULARY Is the quotient of two integers always a rational number? Explain.
- 2. WRITING Are all terminating and repeating decimals rational numbers? Explain.

Tell whether the number belongs to each of the following number sets: *rational numbers, integers, whole numbers.* 

<b>3.</b> -5	<b>4.</b> $-2.1\overline{6}$	<b>5.</b> 12	<b>6.</b> 0
Tell whether the de	ecimal is <i>terminating</i> (	or repeating.	
<b>7.</b> -0.4848	<b>8.</b> -0.151	<b>9.</b> 72.72	<b>10.</b> -5.236

# Practice and Problem Solving

Write the rational number as a decimal.

<b>11.</b> $\frac{7}{8}$	<b>12.</b> $\frac{1}{11}$	<b>13.</b> $-\frac{7}{9}$	<b>14.</b> $-\frac{17}{40}$
<b>15.</b> $1\frac{5}{6}$	<b>16.</b> $-2\frac{17}{18}$	<b>17.</b> $-5\frac{7}{12}$	<b>18.</b> $8\frac{15}{22}$

**19. ERROR ANALYSIS** Describe and correct the error in writing the rational number as a decimal.

#### Write the decimal as a fraction or a mixed number in simplest form.

<b>2 20.</b> −0.9	<b>21.</b> 0.45	<b>22.</b> -0.258	<b>23.</b> -0.312
<b>24.</b> -2.32	<b>25.</b> -1.64	<b>26.</b> 6.012	<b>27.</b> -12.405

Order the numbers from least to greatest.

- **31.** 2.1,  $-\frac{6}{10}$ ,  $-\frac{9}{4}$ , -0.75,  $\frac{5}{3}$  **29.**  $\frac{9}{5}$ , -2.5, -1.1,  $-\frac{4}{5}$ , 0.8 **30.** -1.4,  $-\frac{8}{5}$ , 0.6, -0.9,  $\frac{1}{4}$  **31.** 2.1,  $-\frac{6}{10}$ ,  $-\frac{9}{4}$ , -0.75,  $\frac{5}{3}$  **32.**  $-\frac{7}{2}$ , -2.8,  $-\frac{5}{4}$ ,  $\frac{4}{3}$ , 1.3 **33.**  $-\frac{11}{5}$ , -2.4, 1.6,  $\frac{15}{10}$ , -2.25
  - **34. COINS** You lose one quarter, two dimes, and two nickels.
    - **a.** Write the amount as a decimal.
    - **b.** Write the amount as a fraction in simplest form.

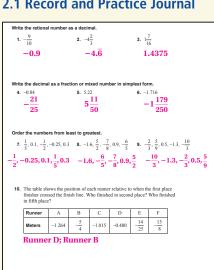
# **35. HIBERNATION** A box turtle hibernates in sand at $-1\frac{5}{8}$ feet. A spotted turtle hibernates at $-1\frac{16}{25}$ feet. Which turtle is deeper?

### **Assignment Guide and Homework Check**

Level	Day 1 Activity Assignment	Day 2 Lesson Assignment	Homework Check
Basic	28–30, 48–52	1–10, 15–23 odd, 31–35 odd	15, 21, 31, 35
Average	28–30, 48–52	1–10, 15–23 odd, 31–35 odd, 43–45	15, 21, 31, 44
Advanced	28–30, 48–52	1–10, 19, 32–42 even, 45–47	28, 36, 42, 46
Accelerated	1–10, 19, 28–4	28, 36, 42, 46	

### **Common Errors**

- Exercises 11–18 Students may forget to carry the negative sign through the division operation. Tell them to create a space for the final answer and to write the sign of the number in the space at the beginning.
- Exercises 20–27 Students may try to put the decimal number over the denominator. Remind them to remove the decimal point before they write it as a fraction. They can also write the whole number in front of the fraction while they are reducing it.
- Exercises 28-33 Students may just order the fractions or decimals without the negative signs. Remind them that some numbers are negative and will be less than the positive numbers.



#### 2.1 Record and Practice Journal

# Vocabulary and Concept Check

- **1.** no; The denominator cannot be 0.
- 2. yes; These decimals can be written as  $\frac{a}{b}$  where a and *b* are integers and  $b \neq 0$ .
- 3. rational numbers, integers
- **4.** rational numbers
- **5.** rational numbers, integers, whole numbers
- **6.** rational numbers, integers, whole numbers
- 7. repeating
- 8. terminating
- **9.** terminating
- 10. repeating

Practice and Problem Solving					
11. (	).875	12.	0.09		
13	$-0.\overline{7}$	14.	-0.425		
<b>15.</b> 1	.83	16.	$-2.9\overline{4}$		

- **17.** -5.583 **18.** 8.681
- **19.** The bar should be over both digits to the right of the decimal point.

$$-\frac{7}{11} = -0.\overline{63}$$
20.  $-\frac{9}{10}$ 
21.  $\frac{9}{20}$ 
22.  $-\frac{129}{500}$ 
23.  $-\frac{39}{125}$ 
24.  $-2\frac{8}{25}$ 
25.  $-1\frac{16}{25}$ 
26.  $6\frac{3}{250}$ 
27.  $-12\frac{81}{200}$ 
28.  $-\frac{7}{3}, -\frac{3}{4}, 0.5, \frac{2}{3}, 1.2$ 

2

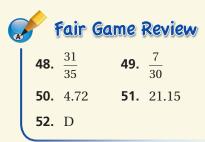


**29.** 
$$-2.5$$
,  $-1.1$ ,  $-\frac{4}{5}$ ,  $0.8$ ,  $\frac{9}{5}$   
**30.**  $-\frac{8}{5}$ ,  $-1.4$ ,  $-0.9$ ,  $\frac{1}{4}$ ,  $0.6$   
**31.**  $-\frac{9}{4}$ ,  $-0.75$ ,  $-\frac{6}{10}$ ,  $\frac{5}{3}$ ,  $2.1$   
**32.**  $-\frac{7}{2}$ ,  $-2.8$ ,  $-\frac{5}{4}$ ,  $1.3$ ,  $\frac{4}{3}$   
**33.**  $-2.4$ ,  $-2.25$ ,  $-\frac{11}{5}$ ,  $\frac{15}{10}$ ,  $1.6$   
**34. a.**  $-0.55$ 

**b.** 
$$-\frac{11}{20}$$

**38.** 
$$\frac{15}{8} = 1\frac{7}{8}$$
 **39.**  $-4\frac{6}{10} > -4.65$ 

- 40-45. See Additional Answers.
- 46. See Taking Math Deeper.
- 47. See Additional Answers.



### Mini-Assessment

Write the rational number as a decimal.

**1.**  $\frac{8}{9}$  0.8 **2.**  $-\frac{11}{10}$  -1.1 **3.**  $\frac{4}{125}$  0.032 **4.**  $-\frac{13}{15}$  -0.8 $\overline{6}$ 

5. When your cousin was born, she was  $21\frac{4}{5}$  inches long. When your friend was born, he was  $21\frac{5}{6}$  inches long. Who was longer at birth? your friend

# Taking Math Deeper

### **Exercise 46**

Students have already learned that it is easier to order numbers in decimal form than in fraction form. This problem gives students practice with this skill using negative numbers. The challenge in this problem is that students need to decide what place values to use for all four decimals.

1	Write	each	number	as a	decimal.
---	-------	------	--------	------	----------

Week	1	2	3	4
Change (inches)	$-\frac{7}{5}$	$-1\frac{5}{11}$	-1.45	$-1\frac{91}{200}$
Decimal	-1.4000	-1.4545	-1.4500	-1.4550





This problem is more difficult than it appears.



2

Write the numbers in order from least to greatest.

 $-1\frac{91}{200}$   $-1\frac{5}{11}$  -1.45  $-\frac{7}{5}$ 



The U.S. Geological Survey (USGS) records the water levels at various locations in the United States. You can track these measurements by going to www.usgs.org.

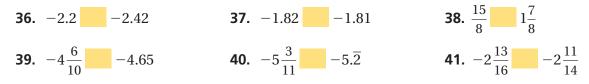
## Project

Create a chart showing the water levels at various locations in the Great Lakes during the same week. What is the range in water levels? Why do you think the levels vary?

### **Reteaching and Enrichment Strategies**

If students need help	If students got it
Resources by Chapter • Practice A and Practice B • Puzzle Time Record and Practice Journal Practice Differentiating the Lesson Lesson Tutorials Skills Review Handbook	Resources by Chapter • Enrichment and Extension • Technology Connection Start the next section

Copy and complete the statement using <, >, or =.



**42. OPEN-ENDED** Find one terminating decimal and one repeating decimal between  $-\frac{1}{2}$  and  $-\frac{1}{3}$ .

Player	Hits	At Bats	
Eva	42	90	
Michelle	38	80	

**43. SOFTBALL** In softball, a batting average is the number of hits divided by the number of times at bat. Does Eva or Michelle have the higher batting average?

- **44. PROBLEM SOLVING** You miss 3 out of 10 questions on a science quiz and 4 out of 15 questions on a math quiz. Which quiz has a higher percent of correct answers?
- **45. SKATING** Is the half pipe deeper than the skating pool? Explain.

