Aircraft Altimetry and its Application to Glaciology
by Anthony Arendt for the UAF Summer School in Glaciology, June 2010

1 Overview

Aircraft laser altimetry is a relatively new tool in glaciology that allows us to monitor the evolution of the surface elevation of glaciers and ice sheets. Altimetry flights usually follow a profile along the centerline of a glacier in an effort to characterize the elevation change of an entire basin. Extrapolation of elevation changes to broader regions allows for an assessment of glacier contribution to rising sea level.

Some of the earliest assessments of the mass balance of the Greenland ice sheet was acquired using the NASA Airborne Topographic Mapping (ATM) laser altimeter (Krabill et al., 1995, 1999, 2000, 2002). The ATM system has also been used to monitor glaciers in Iceland (Garvin and R.S. Williams, 1993), Arctic Canada (Abdalati et al., 2004), Alaska (Arendt et al., 2008) and other regions. Aircraft laser altimetry using small aircraft has also been applied extensively to mountain glacier systems in the Swiss Alps (Favey et al., 1999), the Canadian Rocky Mountains (Hopkinson and Demuth, 2006), Svalbard (Arnold et al., 2006) and Alaska/northwestern Canada (Echelmeyer et al., 1996; Aðalgeirsdóttir et al., 1998; Sapiano et al., 1998; Rabus and Echelmeyer, 1998; Arendt et al., 2002, 2006).

This goal of this document is to explain the mass balance equations relevant to elevation change measurements, followed by a description of altimetry technology. It also describes methods for calculating elevation, volume and mass changes from altimetric measurements.

2 Mass Balance Equations

The continuity equation illustrates the way in which altimetry measurements of elevation change relate to the surface mass balance

$$\dot{h} = b - \nabla \cdot q \tag{1}$$

where $\dot{h}$ is the rate of change of glacier surface elevation, b is the mass balance, and q is the ice flux per unit cross section of the glacier. This equation shows that changes in glacier surface elevation can result not only from accumulation and ablation, but also from the flow of ice as it redistributes mass throughout the glacier system.

When direct mass balance measurements (b) are available, and assuming all variations occur with elevation alone, the glacier-wide mass balance B is

$$B = \int_{Z} b(z)a(z)\,dz \tag{2}$$

where Z is the elevation range of the glacier and a is the area distribution function.

Aircraft altimetry measures changes in surface elevation $\dot{h}$. In this case the glacier-wide balance is calculated as above, again assuming variations in z alone

$$B = \rho_i \int_Z \dot{h}(z)\, a(z)\, dz \tag{3}$$

This includes an additional term ρ_i, the mean density of the changing snow/ice volume, required to obtain a mass balance. Both $\dot{h}$ as well as b, when integrated over the entire glacier basin, yield the glacier-wide balance. This occurs because the flux divergence term $\nabla \cdot q$ (Equation 1) sums to zero over the entire glacier basin.

The most crucial thing to remember from these equations is that point measurements of elevation changes or mass balances both provide the glacier-wide balance when integrated over the entire glacier surface. However, at a point, the mass balance and elevation changes tell us very different things about what is happening at the glacier surface. A common mistake is to infer climatological information from the elevation change signal. For example, if a glacier is thinning in its accumulation zone, that could be due either to an increase in surface melting/decrease in accumulation (i.e. changes in b) or due to an increase in ice flux divergence from that location (i.e. more ice flow downstream than what is received upstream). It is entirely possible that $b = 0$ at that location and all changes are due to dynamic effects alone.

3 Aircraft Laser Altimetry

Laser altimetry is a remote sensing technique in which surface elevations are determined by recording the travel time of a pulse of electromagnetic radiation transmitted from and received by a precisely located altimeter. Altimeter trajectory is measured via kinematic processing of Global Positioning System (GPS) data, and altimeter attitude is measured using an onboard Inertial Navigation System (INS). GPS data are collected at one or several ground base stations concurrently with aircraft GPS data, and kinematic processing is used to obtain the aircraft position. GPS solution quality generally decreases as the distance between the base station and the aircraft (GPS baseline length) increases, and laser accuracy decreases as aircraft roll angles increase. Pitch and roll angles must also be minimized to reduce loss of GPS signal reception during flight.

Laser altimeters operate at 532 nm (green) or 1024 nm (near infrared) wavelengths. There are both profiling and scanning laser systems. A profiling system is nadir-pointing and only measures points directly below the sensor. A scanning system uses a rotating mirror, or a series of transmitter/sensors arrayed in a parallel (pushbroom) configuration, to obtain a swath of laser measurements and a more dense array of across-track elevations. Aircraft laser altimeter footprints generally vary from about 1-3 m, depending on the altitude of the aircraft. The width of the swath is a function of the aircraft altitude, while the density of measurements depends on the frequency of the laser pulse.

4 Calculating Elevation Changes

Altimetry measurements are compared with elevations obtained at identical points in space obtained at an earlier time, to calculate elevation changes $\dot{h}$. The earlier elevation measurement is commonly obtained from a previous altimetry pass, but can also be derived from other sources such as topographic maps. Repeat mapping of an earlier altimetry pass requires expert aviation to position the aircraft along an earlier trajectory. As in other geodetic methods, elevation changes

must be determined over an entire glacier basin in order to assess the glacier-wide balance. Due to this restriction, altimetry missions designed for glacier mass balance assessment should at a minimum include a trajectory along the main central flowline(s) of the glacier.
It is essential to ensure that the vertical and horizontal datum of each elevation dataset are the same before carrying out elevation change calculations. Altimetry measurements are calculated using GPS which provides a height above ellipsoid (HAE). The ellipsoid is a mathematical representation of the Earth's surface. Many other datasets, in particular topographic maps, are distributed as heights above mean sea level (MSL). The difference between MSL and HAE is easily determined using one of several models of Earth's geoid. This correction, illustrated in Figure 1, must be carried out before calculating elevation differences.

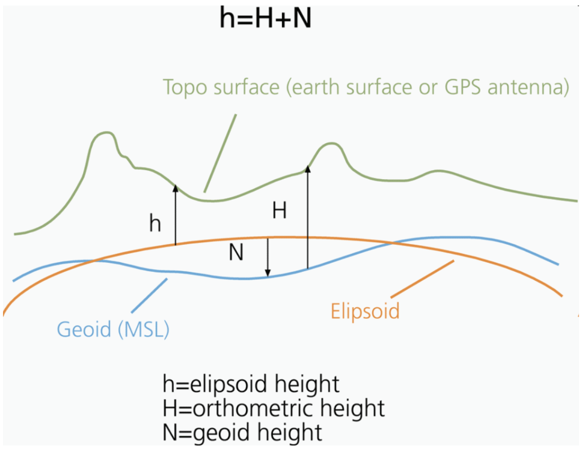

Figure 1: Difference between geoid and ellipsoid heights.

A second consideration is that the surface of the Earth is constantly changing in response to the loading history imposed on it from previous glaciations, and due to tectonic events. In some regions such as Alaska and Patagonia, the Earth is uplifting at rates up to 30 mm/yr in response to the rapid loss of ice after the Little Ice Age (e.g. Larsen et al. (2004)). This introduces a systematic error in elevation change calculations that are often ignored over short time periods, but that should be considered for multi-decadal elevation change observations.

5 Calculating Volume Changes

In the absence of a complete map of the entire glacier surface, glacier volume changes are determined by assuming the change in elevation along a centerline represents the entire width of a glacier at that elevation (Fig. 2). Usually the glacier area-elevation distribution (hypsometry) is reclassified into a series of equal-interval elevation "bins", so that the glacier surface is treated as a series of rectangular steps. Accurate determination of the glacier hypsometry at the time of the earlier measurement is required.
The glacier surface area at the earlier and later time are necessary for accurate volume change calculations. The extrapolation of centerline data should be done over the surface area of the glacier at the later time, however special consideration of terminus changes is required. If the glacier has

retreated then the elevation of the surface at the later time is the bedrock elevation. For tidewater glaciers, note that changes in the portion of ice that is displacing ocean water does not contribute to changes in sea level. In fact, if the tidewater glacier retreats, a portion of the formerly submerged terminus gets replaced with ocean water. Because ice is less dense than water, there is actually a slight drop in sea level due to that portion of the tidewater volume change. Such calculations require knowledge of the fjord bathymetry.

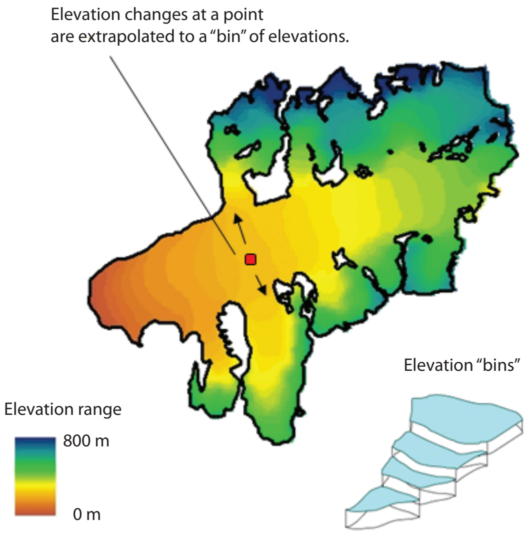

Figure 2: Elevation changes calculated at a point are commonly assumed to represent an entire bin of elevations.

6 Calculating Mass Changes

As shown in Equation 3, conversion of volume changes to mass changes requires knowledge of the density of the changing snow/ice volume. Because it is difficult to measure changes in the near-surface density of the glacier over time, nearly all studies assume some value for the density. It is most common to invoke what is termed "Sorge's Law", which states that a glacier with a time-invariable accumulation rate and no melting will have a time-invariable profile of density as a function of depth beneath the surface; by extension, the density profile also remains unchanged when the rates of melting near the surface and refreezing at depth are constant and equal (Bader, 1954) (Fig. 3). It follows from Sorge's Law that a change of glacier thickness can be converted to an equivalent change of mass by multiplying by the density of glacier ice.

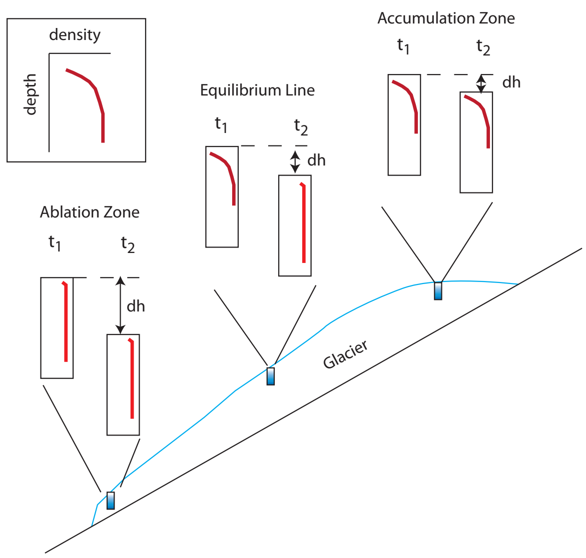

Figure 3: illustration of Sorge's Law of densification as it applies to the calculation of glacier volume changes. t_1 and t_2 describe two different altimetry measurement periods. If the near-surface density profile does not change during this time, then the total glacier volume change has a density equal to that of ice. In practice this rarely occurs, and the density profile changes in response to climate change. The effects of these changes on the mass balance calculation are largest near the equilibrium line, where there is the greatest potential for density variations.

7 Calculating Area-Averaged Elevation Changes

Following Finsterwalder (1954), the area-averaged elevation change is:

$$\bar{h} = \frac{\Delta V}{\frac{(A_2+A_1)}{2}} \tag{4}$$

8 Errors

Recent work has shown that some of the key assumptions used in centerline aircraft altimetry may need to be reassessed. Berthier et al. (2010) compared elevation changes determined from SPOT5, ASTER and USGS Digital Elevation Models (DEMs) with centerline altimetry estimate of glacier volume changes in Alaska. They show that, especially for large glacier systems, the assumption that a centerline profile represents all areas of a glacier at that elevation is not correct. A good example is the Columbia Glacier, a large and rapidly-retreating glacier in the Western Chugach Mountains of Alaska (Fig. 4). Aircraft measurements on this glacier followed a profile on the main glacier trunk. Berthier et al. (2010) show that the surrounding tributaries to this main trunk have thinning rates that do not match the large rates of thinning along the main trunk, resulting in an overestimation of glacier volume loss using centerline altimetry.
Several factors have been proposed to explain why centerline altimetry might not represent changes across the entire glacier. On many glaciers there is considerable debris along the margins that

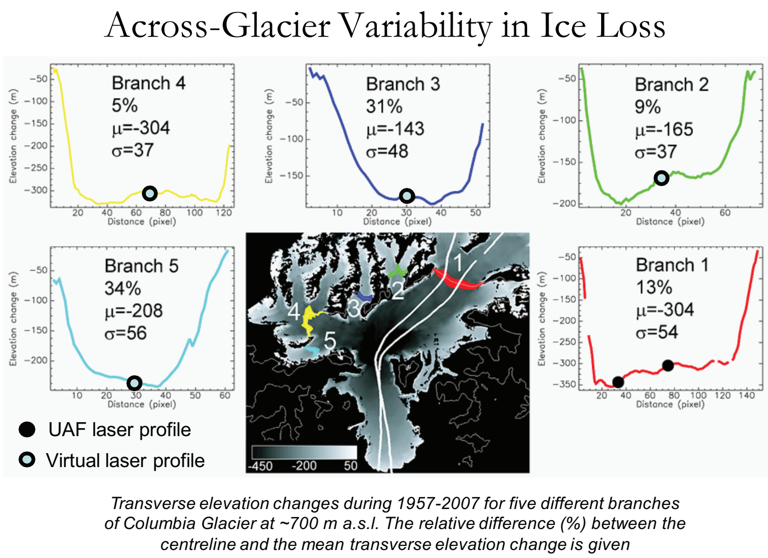

Figure 4: Illustration of potential errors when using centerline altimetry to represent all tributaries of a large and dynamic glacier (Columbia Glacier, Alaska). Reproduced from Berthier et al. (2010)

insulates the surface from melting, and this might reduce thinning rates there relative to the center of the glacier. Also, the assumption that elevation changes occur as a "step", as assumed when the hypsometry is represented in equal-elevation bins, is in general not correct because it does not account for the sloping bedrock at the glacier margins. However, the largest source of error in centerline altimetry is probably that illustrated in Fig. 4, where for large glaciers a single branch does not represent other tributaries to that system.

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