1 Laboratory 7: Fourier Optics

1.1 Theory:

References: *Introduction to Optics*, Pedrottis, Chapters 11 and 21, *Optics*, E. Hecht, Chapters 10 and 11

The Fourier transform is an equivalent representation of a function or image in terms of the "amount" of each sinusoidal frequency that exists in the original function. The form of the transform is:

$$\begin{aligned}\mathcal{F}\{f[x,y]\} &\equiv F[\xi,\eta] = \iint_{-\infty}^{+\infty} f[x,y]\left(\exp\left[+2\pi i\left(\xi x+\eta y\right)\right]\right)^* \, dx\, dy \\ &= \iint_{-\infty}^{+\infty} f[x,y]\left(\exp\left[-2\pi i\left(\xi x+\eta y\right)\right]\right) \, dx\, dy\end{aligned}$$

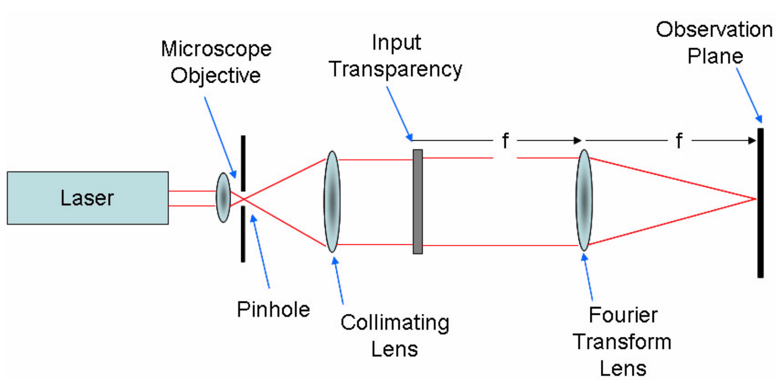
which evaluates the set of "projections" of $f[x,y]$ onto each of the complex-valued sinusoids $\exp[+2\pi i(\xi x+\eta y)]$. The transform is a 2-D array of "data" where the constant part of $f[x,y]$ maps to $F[0,0]$ and the oscillating parts of $f[x,y]$ to the other frequencies. The complex-valued "spectrum" $F[\xi,\eta]$ may be represented as its real and imaginary parts or as its magnitude and phase:

$$\begin{aligned}F[\xi,\eta] &= \mathrm{Re}\{F[\xi,\eta]\} + i\ \mathrm{Im}\{F[\xi,\eta]\} \\ &= |F[\xi,\eta]|\ \exp\left[+i\Phi\{F[\xi,\eta]\}\right]\end{aligned}$$

In the last lab exercise, we saw that diffraction of light in the Fraunhofer diffraction region is proportional to the Fourier transform of the 2-D input distribution.$f[x,y]$:

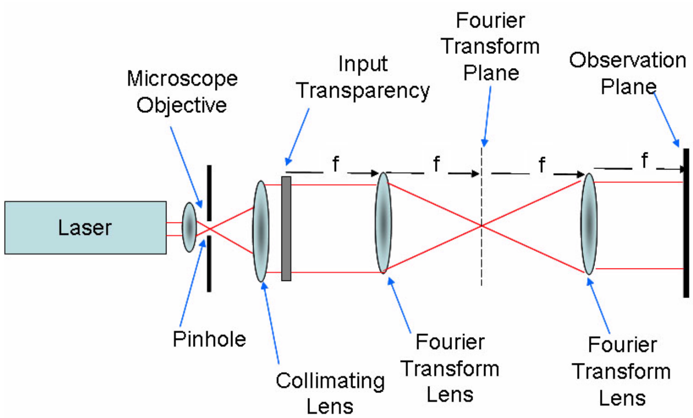
$$\begin{aligned}g[x,y] &\propto \mathcal{F}_2\{f[x,y]\}|_{\lambda_0 z_1\xi\to x,\lambda_0 z_1\eta\to y} \\ &\equiv \iint_{-\infty}^{+\infty} f[\alpha,\beta]\exp\left[-2\pi i\left(\alpha\frac{x}{\lambda_0 z_1}+\beta\frac{y}{\lambda_0 z_1}\right)\right] d\alpha\, d\beta \\ &= F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]\end{aligned}$$

where the propagation distance z_1 satisfies the condition $z_1 >> \frac{x^2+y^2}{\lambda_0}$. In other words, the "brightness" of the Fraunhofer diffraction pattern at each point $[x,y]$ is proportional to $\left|F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]\right|^2$. Since the propagation distance z_1 must be large, this is not a very practical means for evaluating the Fourier transform of the input function $f[x,y]$. However, you also saw how this large distance may be brought "close" by adding a lens after the input function to produce a practical system:



Apparatus for viewing Fraunhofer diffraction patterns.

In this lab, you will add a second lens to compute the "Fourier transform of the Fourier transform." The most obvious way to do this is shown:

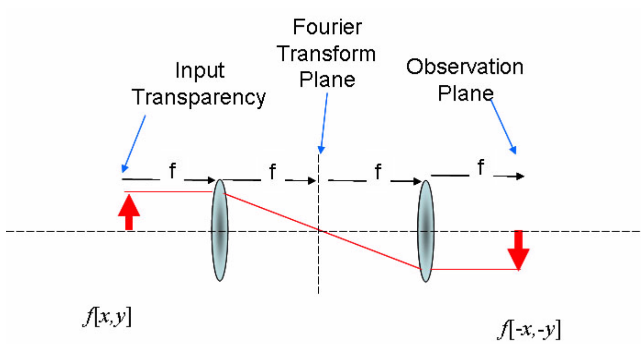


Apparatus for filtering Fraunhofer diffraction patterns to "modify" the object $f[x,y]$ as $g[x,y]$.

In words, the second lens is located one focal length away from the Fourier transform plane and the output is observed one focal length away from the lens. For an obvious reason, this is called a "4f" imaging system.

It is easy to trace a ray from an "arrow" located at the object plane parallel to the axis. The "image" of this ray will be inverted ("upside down"), which indicates that the "Fourier transform of the Fourier transform" is a reversed replica of the function:

$$\mathcal{F}_2\left\{\mathcal{F}_2\left\{f[x,y]\right\}\right\} = \mathcal{F}_2\left\{F\left[\frac{x}{\lambda_0\mathbf{f}},\frac{y}{\lambda_0\mathbf{f}}\right]\right\} \propto f[-x,-y]$$



Demonstration that the output of the $4\mathbf{f}$*-system is a reversed ("upside-down") replica $f[-x,-y]$ of the input function $f[x,y]$.*

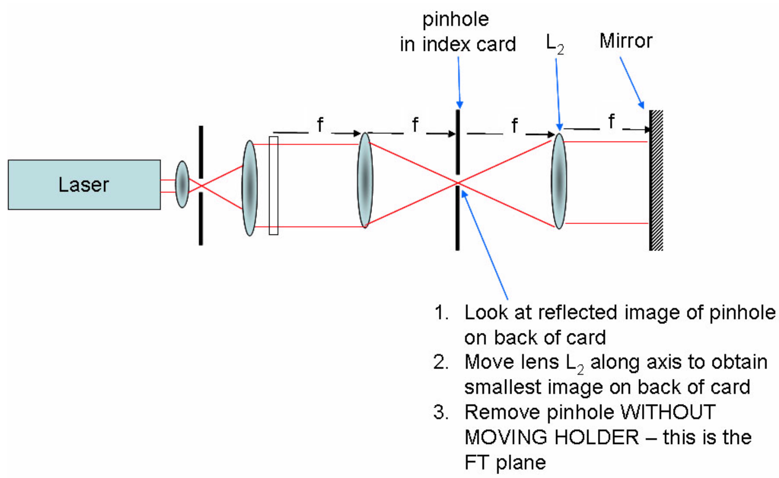
This imaging system makes the Fourier transform $F[\xi,\eta]$ of the input function $f[x,y]$ "accessible" where it can be modified ("filtered") before recomputing the Fourier transform to "reconstruct" the filtered image. A common filter placed in the Fourier transform plane includes a hole that passes the light in $F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]$ that "close to" the optical axis and blocks the light in $F\left[\frac{x}{\lambda_0 z_1},\frac{y}{\lambda_0 z_1}\right]$ that is "far" from the axis; the former light carries the information about the "slowly varying" sinusoids, so this is a *lowpass filter* that removes the information about rapidly oscillating sinusoids. Conversely, a transparency with a black "dot" can be placed to block the light from the constant part of $f[x,y]$ and from the low-frequency sinusoidal components, while passing light from high-frequency sinusoids; this will pass the information about the "edges" of the image and is a *highpass* filter.

1.2 Equipment:

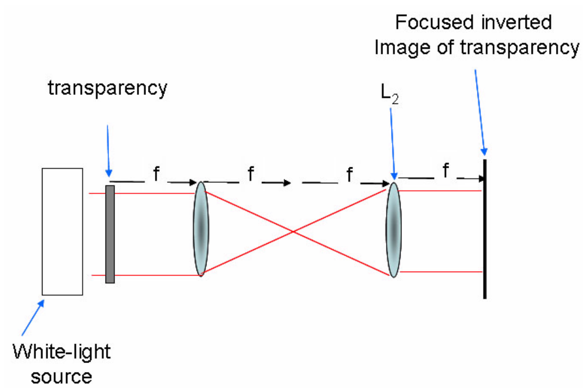
1. He:Ne laser
2. microscope objective to expand the beam; larger power (e.g., 60×) gives larger beam in shorter distance;
3. pinhole aperture to "clean up" the beam (if available)
4. positive lens with diameter $d \cong 50\,\mathrm{mm}$ and focal length $f \lessapprox 500\,\mathrm{mm}$, to collimate the beam;
5. positive lens with diameter $d \cong 50\,\mathrm{mm}$ and focal length $f \cong 200\,\mathrm{mm}$, to compute the Fourier transform;
6. aluminum foil, needles, and razor blades to make your own objects for diffraction;
7. mirror;
8. variable-diameter iris diaphragm;
9. set of Metrologic transparencies;
10. digital camera to record diffraction patterns and reconstructed images.

1.3 Procedure

1. Set up the experimental bench to see Fraunhofer diffraction with a lens, so that the output is the Fourier transform of the input.
2. Experiment with some of the prepared objects (Metrologic transparencies) and your own objects to confirm the Fourier transform relationship.
3. We now want to alter the Fourier transform of the object pattern at the Fourier plane. In other words, we can attenuate or remove some of the frequency components at that location. This process is called *filtering*; if the sinusoidal components with large spatial frequencies (sinusoidal components that oscillate "rapidly") are removed, the process is *lowpass filtering*; if the components that oscillate slowly are removed, we have *highpass filtering.* This process has been used since the invention of the laser to perform filtering "at the speed of light." To do so, we must add a second lens L_2 to compute the Fourier transform of the first Fourier transform, which would produce an "upside-down" replica of the object. There are several ways to place this lens, one uses a pinhole aperture with a needle and a white index card. This pinhole will be used to position lens L_2. Place the pinhole at the Fourier transform plane made by the first lens.
4. Place lens L_2 one focal length $\mathbf{f}_2$ from the Fourier transform plane; note that $\mathbf{f}_2$ need not be equal to $\mathbf{f}_1$. Place a mirror at a distance $\mathbf{f}_2$ beyond the lens. Look at the image of the pinhole on the rear side of the the index card while moving lens L_2 along the optical axis. The correct location of lens L_2 occurs where the image of the pinhole is smallest. Remove the index card with the pinhole without moving its holder; this is the location of the Fourier transform plane.



5. Replace the mirror by a viewing screen and insert a white light source as shown:



The image of the transparency should be in focus.

6. Replace the white-light source with the laser system. Observe the images of the following Metrologic slides: #10 (medium grid), #13 (concentric circles with variable widths; this is a *Fresnel zone plate)*; #19 (fan pattern).

7. Put Metrologic slide #10 (medium grid) at the input plane. The slides #3 (circular aperture), #15 (narrow slit), and a square aperture from #16 will be used as filters placed at the Fourier transform plane. You also may want to use a small pinhole as a filter; pierce a piece of aluminum foil with a needle and place at the Fourier transform plane.

   (a) With no filter, observe and/or photograph the output.
   (b) For the medium grid, allow only the central "dot" to pass; observe and/or photograph the output.
   (c) Allow the other dots to pass, one at a time; observe and/or photograph the output.
   (d) Allow the central 3×3 set of nine dots to pass; observe and/or photograph the output.
   (e) Allow the central vertical row of dots to pass; observe and/or photograph the output.
   (f) Allow the central horizontal row of dots to pass; observe and/or photograph the output.
   (g) Allow an off-center horizontal row of dots to pass; observe and/or photograph the output.
   (h) Allow a diagonal row of dots to pass; observe and/or photograph the output.

8. Use the same setup, but replace the input with Metrologic slide #7 (concentric wide circles)

    (a) With no filter, observe and/or photograph the output.

    (b) Use the horizontal slit (slide #15) to allow part of the diffracted light to pass; observe and/or photograph the output.

9. Use slide #22 (simulation of cloud-chamber photograph) as the object and slide #26 (transparent bar with obstruction at center) as the filter. Position the filter so that the transparent bar is perpendicular to the lines in the input.

10. Use slide #25 as the input and #26 as the filter.

    (a) Orient the filter bar in the vertical direction; observe the output.

    (b) Orient the filter bar in the horizontal direction; observe the output

11. Use slide #26 (halftone image of Albert Einstein) as the input and a variable-diameter iris or slides #17 and #18 (circular apertures) as the filter. Experiment with the filter.