1.4 Solve Problems Using Similar Triangles

One of the world's tallest totem poles was raised in Alert Bay, British Columbia in 1972. It would be very difficult to measure the height of this totem pole directly. One way to find its height is to use shadows. First, the length of the shadow of the totem pole is measured. At the same time, the shadow cast by a vertical object of known height is measured. Since the lengths of the two shadows and the length of the vertical object are known, similar triangles can be used to find the height of the totem pole. This totem pole stands approximately 173 feet tall!

Investigate

Find the Height of Your School's Flagpole

Tools

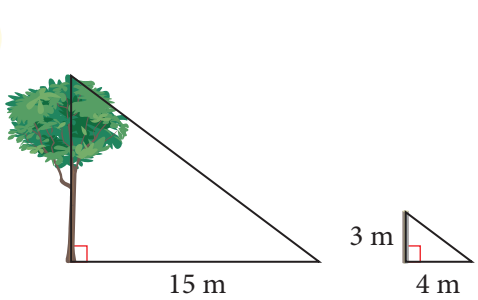
- long measuring tape
- metre or yard stick

Work in a group. Use a metre stick for the vertical object with known height.

1. Measure the length of the shadow of your school's flagpole.
2. Hold a metre stick at right angles to the ground. Have a member of the group measure the length of the shadow cast by the metre stick.
3. Sketch and label a diagram similar to the one shown. Include triangles showing the locations of the objects and their shadows.
4. Explain why the two triangles are similar.
5. Calculate the height of the flagpole. Show your work.

Example 1 Find the Height of a Tree

A pole 3 m tall casts a shadow 4 m long. A nearby tree casts a 15 m shadow. What is the height of the tree?



Solution

Use h to represent the height of the tree. The two triangles are similar, so corresponding sides are proportional.

$$\frac{h}{3} = \frac{15}{4}$$

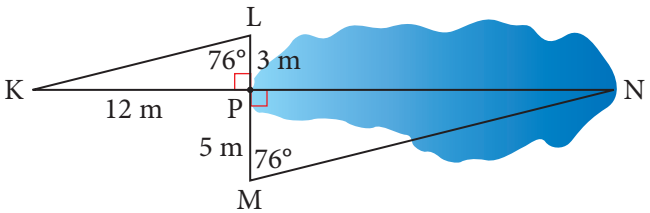
$$h = \frac{3 \times 15}{4}$$

$$h = 11.25$$

The tree is 11.25 m tall.

Example 2 Find the Length of a Pond

To find the length of a pond, a surveyor took some measurements. She recorded them on this diagram. What is the length of the pond?



Solution

The triangles are similar, so the corresponding sides are proportional.

$$\frac{NP}{12} = \frac{5}{3}$$ *Multiply both sides by 12.*

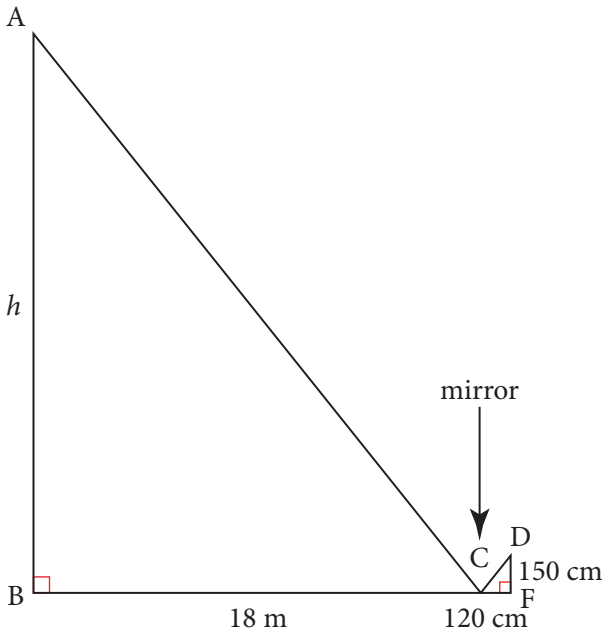
$$NP = \frac{12 \times 5}{3}$$

$$NP = 20$$

The pond is 20 m long.

Example 3 Use a Mirror to Find Height

Elizabeth's eyes are 150 cm from the floor. She places a mirror on the floor 18 m from the base of a climbing wall. She walks backward 120 cm, until she sees the top of the wall in the mirror. What is the height of the climbing wall?



Solution

Let h represent the height of the climbing wall. When a beam of light hits a mirror, the angle at which the light hits the mirror, $\angle ACB$, equals the angle at which the light reflects off the mirror, $\angle DCF$.

$\angle ABC = \angle DFC = 90°$

$\angle BCA = \angle FCD$

So, $\angle BAC = \angle FDC$

Since corresponding angles are equal, $\triangle ABC \sim \triangle DFC$.

So, $\frac{AB}{DF} = \frac{BC}{FC}$

$\frac{h}{150} = \frac{1800}{120}$

Make sure all measures are given in the same units. 18 m = 1800 cm.

$h = \frac{150 \times 1800}{120}$

$h = 2250$

The height of the climbing wall is 2250 cm, or 22.5 m.

Key Concepts

- Similar triangles can be used to find heights or distances that are difficult to measure.
- Similar triangles have many practical applications.

Discuss the Concepts

D1. On a sunny day, Sequoia and Banyan noticed that their shadows were different lengths. Banyan's shadow was 100 in. and Sequoia's shadow 92 in. long. Which student do you think is taller, Sequoia or Banyan? Why?

D2. Describe how you can find the height of a 200 year old tree.

Practise the Concepts A

1. List at least five objects whose measure could be found using similar triangles.

2. Choose one of the objects you listed in question 1. Explain how you would use similar triangles to measure this object.

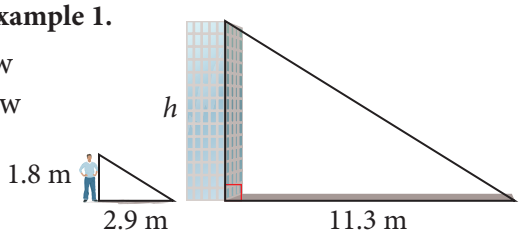
Literacy Connect

3. Sherlock Holmes uses similar triangles to determine the height of a tree in *The Adventure of the Musgrave Ritual*. Look up the story, and explain why it was necessary to use this method instead of measuring.

Apply the Concepts B

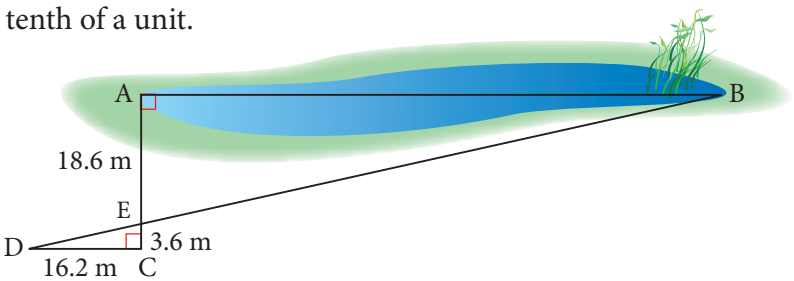
For help with question 4, refer to Example 1.

4. On a sunny day Josée's shadow is 2.9 m long, while the shadow of a tower is 11.3 m long. If Josée is 1.8 m tall, calculate the height of the tower.



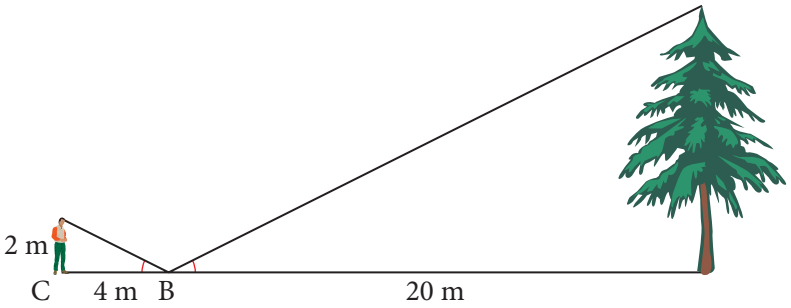
For help with question 5, refer to Example 2.

5. To calculate the length of a marsh, a surveyor produced the following diagram. Find the length of the marsh to the nearest tenth of a unit.

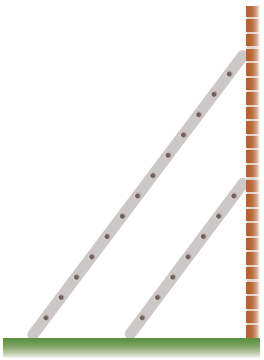


For help with question 6, refer to Example 3.

6. A hiker, whose eye level is 2 m above the ground, wants to find the height of a tree. He places a mirror horizontally on the ground 20 m from the base of the tree, and finds that if he stands at a point C, which is 4 m from the mirror B, he can see the reflection of the top of the tree. How tall is the tree?



7. Two ladders are leaned against a wall so that they make the same angle with the ground. The 10' ladder reaches 8' up the wall. How much further up the wall does the 18' ladder reach?

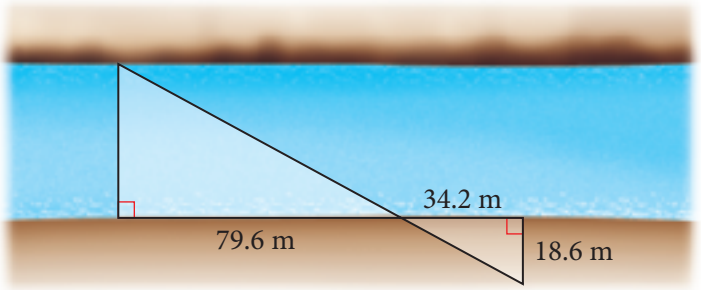


8. At a certain time of the day, the shadow of your friend who is 5 ft tall measures 8 ft. At the same time, the shadow of a tree measures 28 ft. Draw a diagram to represent the situation. How tall is the tree?

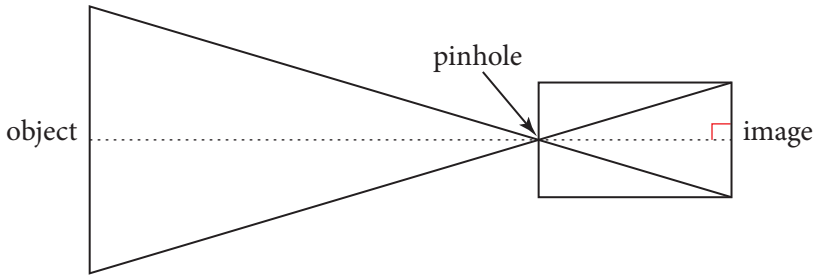
Chapter Problem

9. To find the height of a tree, Darren measures the shadow of a metre stick to be 90 cm and the shadow of the tree to be 3.2 m. Draw a diagram to represent the situation. How tall is the tree?

10. To find the width of a river, Jordan surveys the area and finds the following measures. Find the width of the river.



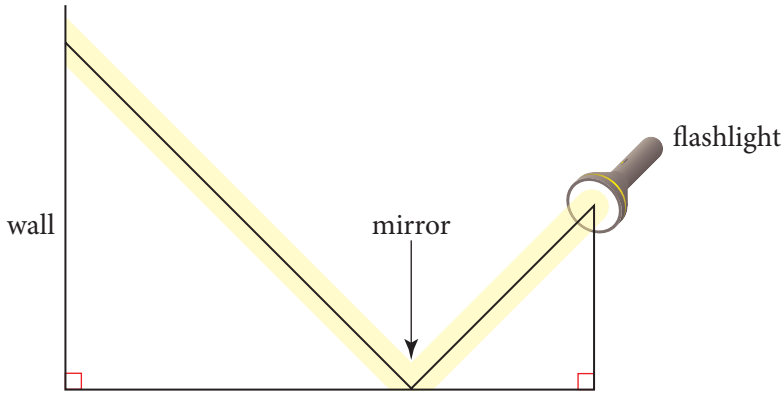
11. Light travels in a straight line. The pinhole camera, or camera obscura, makes use of this fact. When rays of light reflect off an object, and pass through the pinhole in a camera, they cross and form an upside-down image.



An object is 3.6 m from the pinhole. Its image is 4.2 cm from the opposite side of the pinhole. The height of the image is 0.8 cm. What is the height of the object?

For help with question 12, refer to Example 3.

12. Logan places a mirror on the floor 220 cm from the base of a wall. He holds a flashlight 130 cm above the ground, and shines the beam onto the mirror. How far must Logan stand back from the mirror so that the height at which the light shines on the wall is 100 cm greater than the height at which Logan holds the flashlight?



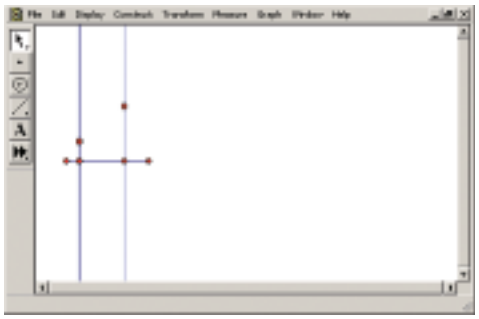
13. Use *The Geometer's Sketchpad*® to simulate finding the height of a tree.

a) Draw a horizontal line to represent the ground.

b) Place two points on the line; one will represent your feet, the other will represent the base of the tree trunk.

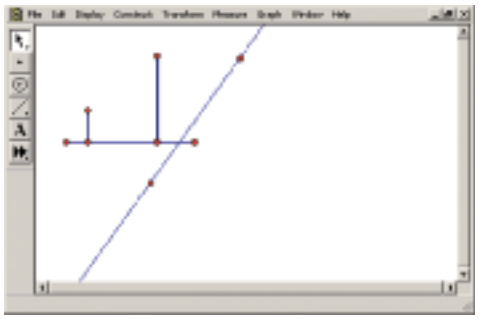
c) Construct perpendicular lines through these points.

d) Construct a point on each line, one to represent the top of your head, the other to represent the top of the tree.



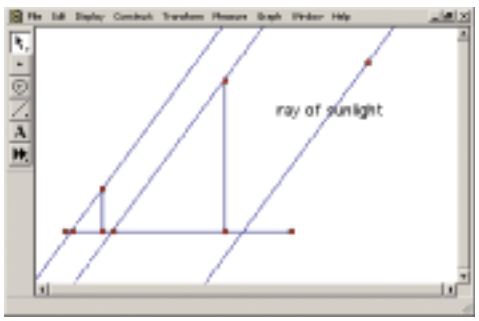
e) Construct segments joining the two points to the ground. Hide the original perpendicular lines.

f) Construct a line from the sky above the tree to the ground. This will represent a ray of sunlight.



g) Construct two lines parallel to the ray of sunlight, one through the point representing the top of the tree, the other through the point representing the top of your head.

h) Construct the points on the ground where the rays of sunlight intersect the ground.



i) Construct segments representing the shadows.

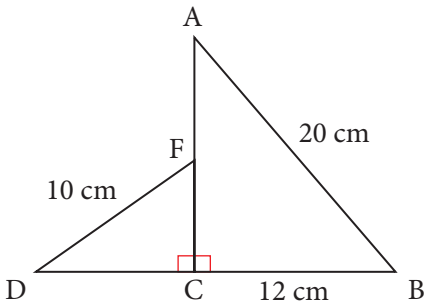
j) Use the **Measure** menu to measure the lengths of the line segments representing the heights of the person and of the tree, and the lengths of their shadows. Then, find the ratios comparing corresponding sides of the triangles.

k) Click and drag the line that represents the ray of sunlight to change the angle of elevation of the sun. What happens to the ratios from part j)? Explain.

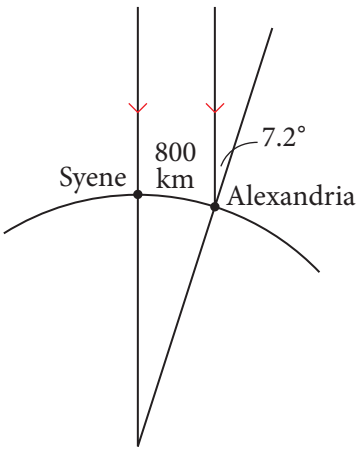
Extend the Concepts C

14. A ski tow rises 39.5 m over a horizontal distance of 118.8 m. What vertical distance have you risen if you have travelled 750.2 m horizontally?

15. In the diagram, $\angle D = \angle A$, AB = 20 cm, CB = 12 cm, and DF = 10 cm. What is the measure of AF?



16. Eratosthenes was a mathematician who lived around 230 B.C.E. While living in Egypt, he learned that at noon on the first day of summer (approximately June 21 on the modern calendar), the sun shone directly down into a deep well in the city of Syene. This meant the sun was directly overhead. At the same time, in Alexandria, approximately 800 km almost due north of Syene, the sun's rays hit the ground at an angle of 7.2° from the vertical.



Eratosthenes used this information to estimate the circumference of Earth.

He set up the proportion $\frac{7.2°}{360°} = \frac{800}{\text{circumference of Earth}}$

a) Solve the proportion to estimate the circumference of Earth.

b) The actual circumference of Earth is approximately 40 000 km. How close was Eratosthenes' estimate?

c) Use your answer from part a) to estimate the diameter of Earth.